Tsinghua University C&A Final Exam –Fall 2010 part I: Combinatorics

Answer as many problems as you can. Show your work. An answer with no explanation will receive no credit. Write your name on the top right corner of each page.
[Total time: 1.5 hours]

Name:

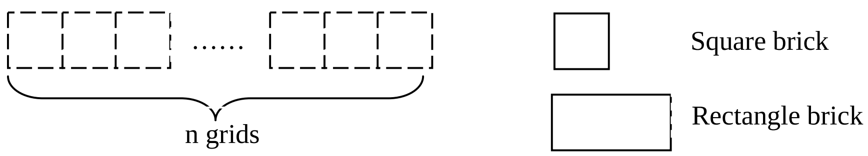
Student ID:

1. Count the number of permutations of ***eight*** letters: “A,B,C,D,E,F,G and H” that A, C, E and G are not in its natural positions. （4 points）

2. How many inequivalent ways are there to color the ***edges*** of a cube with the 3 different colors? （6 points）

3. Randomly pick n+1 numbers from integers between 1 to 2n, please prove that there are at least two integers such that one of them is divisible by the other. (5 points)

4. A worker is tiling a road with n square grids, there are two different kinds of bricks available, one is the square brick and the other is the rectangle brick which covers 2 grids. Please count the total different ways the road can be tiled. (7 points)



5. How many ways to put 8 ***identical*** balls into 4 ***different*** boxes that no empty box is allowed. (3 points)

Tsinghua University C&A Final Exam –Fall 2011 part I: Combinatorics

Answer as many problems as you can. Show your work. An answer with no explanation will receive no credit. Write your name on the top right corner of each page.
[Total time: 2 hours]

Name:

Student ID:

1. Count the number of distinct permutations $i_1 i_2 i_3 i_4 i_5$ of {1, 2, 3, 4, 5}, where $i_1 \neq 1$; $i_2 \neq 2, 3$; and $i_3 \neq 4, 5$ (4 points)

2. How many in-equivalent ways to paint the faces on a **tetrahedron** in 3 different colors? (5 points)

3. A bag contains 10 apples, 12 bananas, 14 oranges, and 16 pears. If a boy picks one piece of fruit each time, how many picks are needed to make sure at least a dozen pieces of fruit of the same kind? (3 points)

4. Solve the following recurrence relations. (6 points)

$$f(n+1) = 1 + \sum_{i=0}^{n-1} f(i),\ f(0) = 1.$$

5. How many integral solutions of $x_1 + x_2 + x_3 + x_4 = 14$, satisfying $x_1 \geqslant -2$, $5 \geqslant x_2 \geqslant 0$, $x_3 \geqslant -10$, $x_4 \geqslant 8$. (4 points)

6. Transform the following problems into augmented form.

$$\begin{aligned} &\min\ z = 3x_1 + 6x_2 + 2x_3 \\ &s.t.\ \ 3x_1 + 4x_2 - x_3 \geq -2 \\ &\qquad x_1 - 3x_2 + 2x_3 \leq 4 \\ &\qquad x_1 \leq 0, x_2 \geq 1. \end{aligned}$$

(3 points)

Tsinghua University C&A Final Exam –Fall 2011 part I: Combinatorics

Answer as many problems as you can. Show your work. An answer with no explanation will receive no credit. Write your name on the top right corner of each page.
[Total time: 2 hours]

Name:

Student ID:

1. Count the number of permutations $i_1i_2i_3i_4i_5$ of {1, 2, 3, 4, 5}, where $i_1 \neq 1$; $i_2 \neq 2, 3$; and $i_3 \neq 4, 5$（4 points）

r1=5, r2=3*2+1+0=7,r3=1*2=2

5!-5*4!+7*3!-2*2!=38

2. We have regular triangle stickers in three colors (red, blue and green), how many in-equivalent ways to paste the stickers on a **tetrahedron**?（5 points）

   $(1)^4$ 1, (1)(3) 8, $(2)^2$ 3.

   $(8*3^2+3*3^2+3^4)/12=15$

3. A bag contains 10 apples, 12 bananas, 14 oranges, and 16 pears. If a boy picks one piece of fruit each time, how many picks are needed to make sure at least a dozen pieces of fruit of the same kind? (3 points)

   10+11+11+11+1=44

4. Solve the following recurrence relations. (6 points)

$$f(n+1)=1+\sum_{i=0}^{n-1} f(i),\ f(0)=1.$$

f(n+2)-f(n+1)-f(n)=0 f(0)=1 f(1)=2

$$a_n = F_{n+2} = \frac{1}{\sqrt{5}}(\alpha^{n+2}-\beta^{n+2})$$

$$A=\frac{3\sqrt{5}+5}{10} \quad B=\frac{5-3\sqrt{5}}{10}$$

5. How many integral solutions of $x_1+x_2+x_3+x_4=14$, satisfying $x_1 \geqslant -2$, $5 \geqslant x_2 \geqslant 0$, $x_3 \geqslant -10$, $x_4 \geqslant 8$. (4 points)

y1+x2+y3+y4=10

C(10+4-1,10)=C(13,10) = C(13,3)

x2⩾6
y1+y2+y3+y4=4
C(4+4-1,4)=C(7,4)
C(13,3)-C(7,4)

6. Transform the following problems into augmented form.

$$\min\ z = 3x_1 + 6x_2 + 2x_3$$
$$s.t.\ \ 3x_1 + 4x_2 - x_3 \geq -2$$
$$x_1 - 3x_2 + 2x_3 \leq 4$$
$$x_1 \leq 0, x_2 \geq 1.$$

(3 points)

$$\max\ z = -3x_1 - 6x_2 - 2x_4 + 2x_5$$
$$s.t.\ \ 3x_1 - 4x_2 - x_4 + x_5 + x_6 = 6$$
$$-x_1 - 3x_2 + 2x_4 - 2x_5 + x_7 = 4$$
$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0,.$$

Tsinghua University C&A Final Exam –Fall 2010 part I: Combinatorics

Answer as many problems as you can. Show your work. An answer with no explanation will receive no credit. Write your name on the top right corner of each page.
[Total time: 1.5 hours]

Name:

Student ID:

1. Count the number of permutations of eight letters: “A, B, C, D, E, F, G and H” that A, C, E and G are not in its natural positions. （4 points）

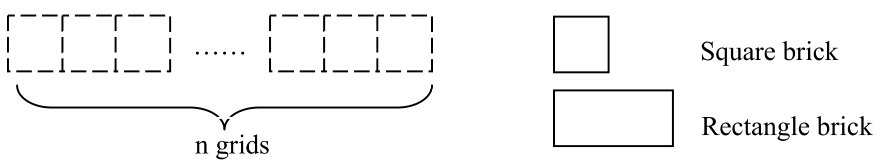
$$\begin{aligned}\left|\overline{A_1}\cap\overline{A_2}\cap\overline{A_3}\cap\overline{A_4}\right| &= 8! - C(4,1)7! + C(4,2)6! \\ &\quad - C(4,3)5! + C(4,4)4! \\ &= 24024\end{aligned}$$

2. How many inequivalent ways are there to color the edges of a cube with the 3 different colors? （6 points）

$$((3)^{12}+6(3)^{3}+3(3)^{6}+6(3)^{7}+8*3^{4})/24 = 22815$$

3. Randomly pick n+1 numbers from integers between 1 to 2n, please prove that there are at least two integers such that one of them is divisible by the other. (4 points)

4. A worker is tiling a road with n square grids, there are two different kinds of bricks available, one is the square brick and the other is the rectangle brick which covers 2 grids. Please count the total different ways the road can be tiled. (7 points)



$a_n = a_{n-1} + a_{n-2}$ $\quad a_0 = 1,\ a_1 = 1,\ a_2 = 2,\ a_3 = 3,\ a_4 = 5,\ a_5 = 8,$

$$a_n = F_{n+1} = \frac{1}{\sqrt{5}}(\alpha^{n+1} - \beta^{n+1})$$

5. How many ways to put 8 ***identical*** balls into 4 ***different*** boxes that no empty box is allowed. (4 points)

   C(8-1,4-1) = C(7,3)=35