

Name:

Student ID:

1. Find the number of **derangements** of $\{1,2,3,4\}$ and $i_1 \neq 2; i_3 \neq 4$.
2. Here we have 3 red beads and 3 blue beads. We could choose 4 beads to embed them at the corners of a **tetrahedron**. How many in-equivalent ways to embed the beads?
3. List $\{a_1, a_2, \dots, a_n\}$ is an ordered permutation of $1, 2, \dots, n$. Please prove that if n is odd, then $(a_1-1)(a_2-2)\dots(a_n-n)$ should be even?
If the multiplication result was odd then it means that half of them are even and half of them are odd. But there are total odd number of numbers. Therefore the multiplication should be even.
4. How many different ways to color n grids in a line with red, white or blue colors but no two adjacent grids are colored with red?
5. Transform the following problems into augmented form and solve it by simplex method.
$$\max z = 2x_1 + 2x_2 + 2x_3$$
$$s.t. \quad -2x_1 + x_2 - x_3 \geq -3$$
$$2x_1 - x_2 + 2x_3 \leq 4$$
$$2x_1 - 2x_3 \leq 6$$
$$x_1 \geq 1, x_2 \leq 2.$$

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2. Find the number of **derangements** of $\{1,2,3,4\}$ and $i_1 \neq 2; i_3 \neq 4$. (4 points)

$$r_1=6, r_2=3*3+1+1=11, r_3=2*1+1*2, r_4=1$$
$$4!-6*3!+11*2!-4*1=6$$

6. Here we have 3 red beads and 3 blue beads. We could choose 4 beads to embed them at the corners of a **tetrahedron**. How many in-equivalent ways to embed the beads? (5 points)

Corner to the opposite side :

a) $(1)^1(3)^1$ rotate ± 120 degree, 4 corners, totally 8;

Middle of edge- Middle of edges :

b) $(2)^2$ 3 pairs;

No move:

c) $(1)^4$ 1;

Total 12 permutations

$$l=1/12[8*2^2+3*2^2+2^4]=5$$

$5-2=3$ except two cases that all four beads are in the same color.

7. List $\{a_1, a_2, \dots, a_n\}$ is an ordered permutation of $1, 2, \dots, n$. Please prove that if n is odd, then $(a_1-1)(a_2-2)\dots(a_n-n)$ should be even? (3 points)

If the multiplication result was odd then it means that half of them are even and half of them are odd. But there are total odd number of numbers. Therefore the multiplication should be even.

8. How many different ways to color n grids in a line with red, white or blue colors but no two adjacent grids are colored with red? (5 points)

$$A_n = 2A_{n-1} + A_{n-2}$$

$$A_1=3, A_2=8, A_3=22$$

$$A_0=(8-6)/2=1$$

$$A_n = p*(1+\sqrt{3})^n + q*(1-\sqrt{3})^n$$

$$A_0 = p+q=1$$

$$A_1 = p(1+\sqrt{3}) + q(1-\sqrt{3})=3$$

$$q = (-2*\sqrt{3} + 3) / 6$$

$$p = (2*\sqrt{3} + 3) / 6$$

6. Transform the following problems into augmented form and solve it by simplex method.

$$\max z = 2x_1 + 2x_2 + 2x_3$$

$$s.t. -2x_1 + x_2 - x_3 \geq -3$$

$$2x_1 - x_2 + 2x_3 \leq 4 \quad (4 \text{ points})$$

$$2x_1 - 2x_3 \leq 6$$

$$x_1 \geq 1, x_2 \leq 2.$$

$$Z=10, X_1=2, x_2=2, z=1$$