Combinatorics HW6.1 Albert Millan Tragadu 2019280366

1) We know from a Fibonacci series that $F_k = F_{k-1} + F_{k-2}$. This rule is paramount to proof it by induction.

$$1 = 1$$

For $n=1$, $F_{2(1)-1} = F_{2(1)} \Rightarrow F_1 = F_2 \therefore$ True

For $n=2$, $F_{2(2)-1} + F_1 = F_{2(2)} \Rightarrow F_1 + F_3 = F_4 \Rightarrow F_2 + F_3 = F_4$

For $n=3$, $F_1 + F_3 + F_{2(3)-1} = F_{2(3)} \Rightarrow F_1 + F_3 + F_5 = F_6 \Rightarrow F_4 + F_5 = F_6$

Assume true for n

$$F_1 + F_3 + \dots + F_{2n-1} = F_{2n}$$

Show true for $n+1$

$$F_1 + F_3 + \dots + F_{2n-1} + F_{2n+1} = F_{2n+2}$$

$$F_{2n} + F_{2n+1} = F_{2n+2}$$

$\therefore$ We obtain the property of the Fibonacci series where the n^{th} term is the sum of the previous two terms. Hence by induction, we can state true for all n.

2) $a_n = 2a_{n-1} + 4a_{n-2} - 5a_{n-3}$

$$a_n - 2a_{n-1} - 4a_{n-2} + 5a_{n-3} = 0$$

$$x^3 - 2x^2 - 4x + 5 = 0$$

3.) Characteristic eq: $x^2 - 2x - 8 = 0$

$$x = \frac{2 \pm \sqrt{4+32}}{2} = \frac{2 \pm 6}{2} = \begin{cases} 4 \\ -2 \end{cases}$$

$$h_n = A(4)^n + B(-2)^n$$

$$\left.\begin{aligned} h_0 &= A(4)^0 + B(-2)^0 = A + B = 1 \\ h_1 &= A(4)^1 + B(-2)^1 = 4A - 2B = 10 \end{aligned}\right\} \begin{aligned} A &= 2 \\ B &= -1 \end{aligned}$$

$$\boxed{\therefore h_n = 2(4)^n - (-2)^n}$$