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Combinatorics HW5 Albert Millan Trujols

1) The result can be obtained by adding-up the orderly partitions resulting from r_i-splittings where $r = 2, 3, \ldots, n-1$. Each splitting r_i can be computed such that $C(n-1, r_i-1)$, where i is the i^{th}-splitting.

$$\therefore C(5,5) + C(4,3) + C(4,2) + C(4,1) = 1+4+6+4 = \boxed{15}$$

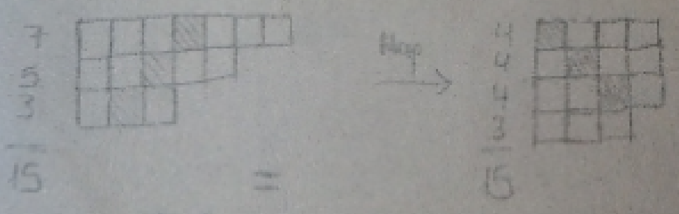
2) $G(x) = (1+x^1+x^2+x^3+\ldots)(1+x^2+x^4+\ldots)(1+x^3+x^6+\ldots)\ldots$

$$= \frac{1}{1-x} \cdot \frac{1}{1-x^2} \cdot \frac{1}{1-x^3} \cdots$$

$$= \prod_{k=1} \left(\frac{1}{1-x^k}\right)$$

The solution is the coefficient of x^n term upon expanding the generating function.

3) The proof is trivial based on the fact that there is a 1-to-1 correspondence between any given set of different odd numbers that adds up to 'n' and a self-conjugated Ferrer's diagram. Suppose $n=15$ and are given 7, 5, and 3. Considering they are odd numbers, they can be split into two identical parts by taking the middle square. Using this middle square of each part as the diagonal, ~~we would obtain~~ of the Ferrer's diagram, we obtain a self-conjugated Ferrer diagram.



By folding through the term in the middle, we obtain two parts of equal length as long as the original part is an odd number. Hence, for every partition of 'n' with distinct odd numbers, there is a self-conjugated Ferrer diagram. This implies a one-to-one correspondence therefore the partition number for integer 'n' using distinct odd numbers should always equal to the partition number of 'n' being partitioned into the self-conjugated Ferrer's diagram.