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4-1b) $T(n) = T(7n/10) + n$

$\therefore a = 1,\ b = 10/7,\ f(n) = n$

$\Rightarrow n^{\log_b a} = n^{\log_{10/7} 1} = n^0 = 1$, hence 3rd case, $(n^{\log_b a} = 1) < (f(n) = n)$

$af(n/b) = 7n/10 \leq (7/10)n = cf(n)$, for $c = 7/10$.

Consequently, by case 3, the solution of the recurrence is $T(n) = \Theta(n)$

4-1c) $T(n) = 16T(n/4) + n^2$

$a = 16,\ b = 4,\ f(n) = n^2$

$\Rightarrow n^{\log_b a} = n^{\log_4 16} = n^2$, hence 2nd case, $(n^{\log_b a} = n^2) == (f(n) = n^2)$

Hence, the tight asymptotic bound include $T(n) = \Theta(n^2 \cdot \lg n)$

4-1d) $T(n) = 7T(n/3) + n^2$

$a = 7,\ b = 3,\ f(n) = n^2$

$\Rightarrow n^{\log_b a} = n^{\log_3 7} = n^{1.77\ldots}$, where $1.77 < \log_3 7 < 1.78$.

Hence, $f(n) = \Omega(n^{\log_3 7 + \varepsilon})$, where $\varepsilon = 2 - \log_3 7$, $\therefore T(n) = \Theta(n^2)$

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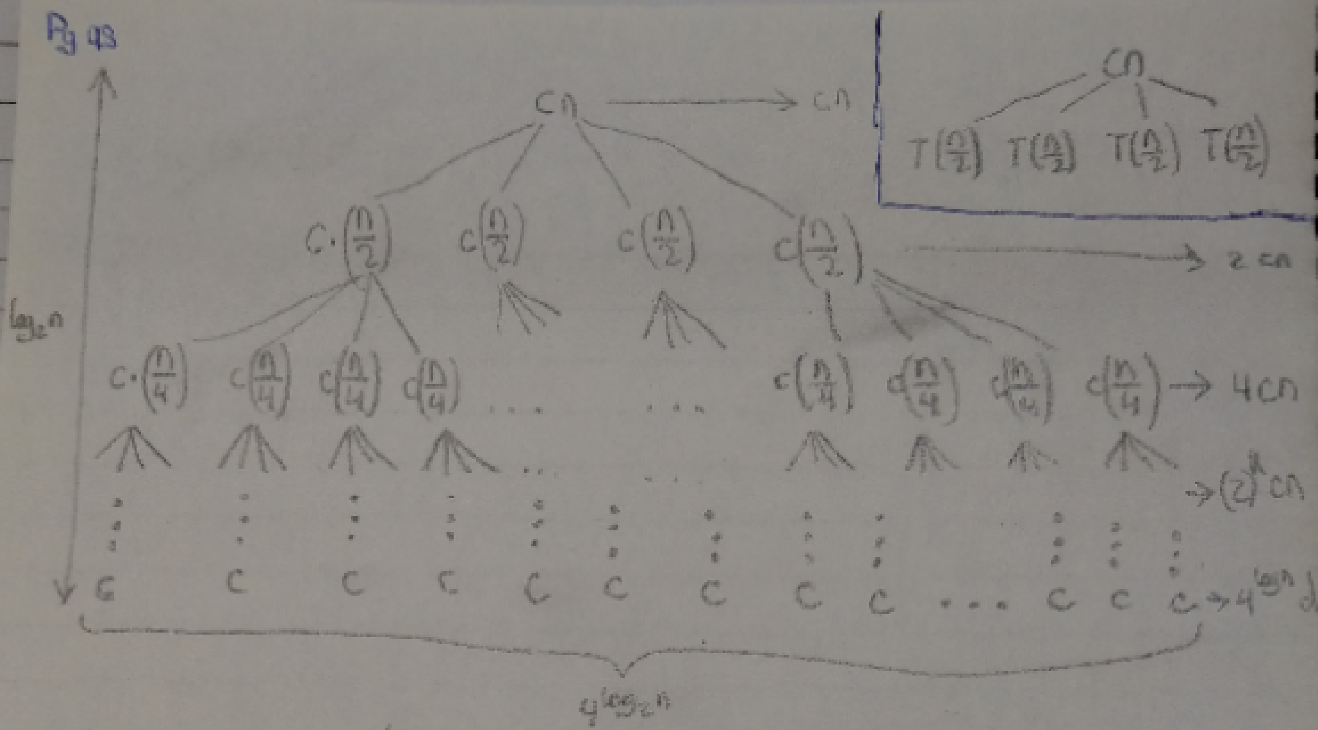

$$\therefore \text{Complexity} = cn \cdot \sum_{k=0}^{\log n} (2)^k$$

$$2^{\log(n)} = n^{\log(2)} = n$$

$$= cn \cdot \left[\frac{1 - 2^{\log n + 1}}{-1}\right]$$

$$= cn \cdot \left[2 \cdot 2^{\log n} - 1\right]$$

$$= 2 \cdot cn^2 - cn$$

$$\cong \Theta(n^2)$$

4.-2a) Binary Search: array passed by pointer

$T(n) = T(n/2) + \Theta(1)$

$a = 1,\ b = 2,\ f(n) = 1$

$\Rightarrow n^{\log_b a} = n^{\log_2 1} = n^0 = 1$, hence 2nd case, $(n^{\log_b a} = 1) == (f(n) = 1)$

Consequently, by case 2, the solution for worst case is $T(N) = \Theta(n^{\log_b a} \cdot \log N)$
$= \Theta(1 \cdot \log N) = \Theta(\log N)$

Binary Search: array passed by copying entire array (N)

By a factor of N?

$T(n) = T(n/2) + \Theta(N)$

$a = 1,\ b = 2,\ f(n) = N$

Tree Method

$$\log_2 N \left\{ \begin{array}{c} N \\ \downarrow \\ N \\ \downarrow \\ \vdots \\ N \end{array} \right.$$

Hence, solving the recurrence relation we get that
$T(N) = \Theta(N \cdot \log N)$

Binary Search: copying subrange from array that might be accessed

$T(n) = T(n/2) + \Theta(n/2)$

$$\left(\frac{1}{2}\right)^{\log_2 N} = N^{\log\left(\frac{1}{2}\right)} = N^{(\log 1 - \log_2 2)}$$
$$= N^{-1}$$

$$\log_2 N \left\{ \begin{array}{c} n/2 \\ \downarrow \\ n/4 \\ \downarrow \\ \vdots \\ n/2^k \end{array} \right.$$

$$n \cdot \sum_{k=0}^{\log_2 N} \left(\frac{1}{2}\right)^k$$
$$= \left(\frac{1 - \left(\frac{1}{2}\right)^{\log_2 N + 1}}{1 - \frac{1}{2}}\right) \cdot N$$
$$= \left(\frac{1 - N^{-1}}{1/2}\right) \cdot N$$
$$= \left(2 - \frac{2}{N}\right) \cdot N$$
$$= 2N - 2$$
$$\therefore T(N) = \Theta(N)$$