Combinatorics HW 1.1 2019280366 Albert Sebastia Nullan Trujols

Q1) $\boxed{\text{A: 568 matches}}$

There is a one-to-one correspondence between the number of matches and number of players eliminated. That is, for every match, there is one player eliminated. Hence, a total of 568 matches is required to have a single winner.

Q2) The following proof by disjunction aims to show that it is not possible to create a magic square such that

$$A = \begin{bmatrix} 2 & 3 & a & b \\ 4 & c & d & e \\ f & g & h & i \\ j & k & l & m \end{bmatrix},$$

where the variables $a, b, c, d, e, f, g, h, i, j, k, l$ and m correspond to a single, non-repeating value in the set $S = \{1, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$.

$$\text{Magic Constant } M = \frac{n \cdot (n^2+1)}{2} = \frac{4 \cdot (16+1)}{2} = 34$$

$$* \; n \cdot M = \text{Sum}(A) \therefore M = \frac{\text{Sum}(A)}{n}$$

I start by assuming that there is a matrix such that of A that is a magic square. In such case, rows, columns and diagonals should each add up to the magic constant M. Also, the variables listed above can only take a single, non-repeating value from the set S. From the first column and row respectively, we have:

1st column: $2 + 4 + f + j = 34$
$f + j = 28$

1st row: $2 + 3 + a + b = 34$
$a + b = 29$

Therefore, there are only two subsets of values from set S that the variables 'a' and 'b' can take, including $S_r = [\{13, 16\}, \{14, 15\}]$. Similarly, variables 'f' and 'j' can only take the values of two subsets from set S, including $S_c = [\{13, 15\}, \{12, 16\}]$. However, if $a = 13$ & $b = 16$ or vice-versa, there is no subset S_c that does not contain the values 13 and 16, involving repetition, thus, resulting in invalid values for 'a' and 'b'. For the case where $a = 14$ and $b = 15$ or vice-versa, the only subset of values that complies with the rules of repetition and magic constant are $\{12, 16\}$. Hence, by disjunction, the only posible values for 'a' and 'b' are those obtained from the permutation of $\{14, 15\}$. Similarly the only possible values for 'f' and 'j' are those obtained from the permutation $\{12, 16\}$. This generates the following 4 cases:

Case B:

$$\begin{bmatrix} 2 & 3 & 14 & 15 \\ 4 & c & d & e \\ 12 & g & h & i \\ 16 & k & l & m \end{bmatrix}$$

In the following cases, the set of single, non-repeating values is consequently reduced to $S = \{1, 5, 6, 7, 8, 9, 10, 11, 13\}$. In case B, the anti-diagonal must add to the magic constant M. That is:

$$16 + 15 + g + d = 34$$
$$g + d = 3$$

There is no pair of values from set S that adds up to 3, hence this case is discarded.

Case C:

$$\begin{bmatrix} 2 & 3 & 15 & 14 \\ 4 & c & d & e \\ 12 & g & h & i \\ 16 & k & l & m \end{bmatrix}$$

The ~~diagonal~~ anti-diagonal must add up to equal M. That is:

$$16 + 14 + g + d = 34$$
$$g + d = 4$$

There is no pair of values from set S that add up to 4, hence this case is discarded

Case D:

$$\begin{bmatrix} 2 & 3 & 14 & 15 \\ 4 & c & d & e \\ 16 & g & h & i \\ 12 & k & l & m \end{bmatrix}$$

Anti-diagonal must add-up to M. That is:

$$15 + 12 + g + d = 34$$
$$g + d = 7$$

There is a single pair of values (1, 6) from set S that adds up to 7. We replace 'g' and 'd' by these values, generating two more cases:

D_1
$$\begin{bmatrix} 2 & 3 & 14 & 15 \\ 4 & c & 6 & e \\ 16 & 1 & h & i \\ 12 & k & l & m \end{bmatrix}$$

Set S reduced to:
$S = \{5, 7, 8, 9, 10, 11, 13\}$
Adding up the values in the 2nd column, there is no pair of values from set S for 'c' and 'k' that adds up to 34. Case D_1 discarded

$$3 + c + 1 + k = 34$$
$$c + k = 30$$

D_2
$$\begin{bmatrix} 2 & 3 & 14 & 15 \\ 4 & c & 1 & e \\ 16 & 6 & h & i \\ 12 & k & l & m \end{bmatrix}$$

Set S reduced to:
$S = \{5, 7, 8, 9, 10, 11, 13\}$
Adding up the values in the 2nd ro[w] $(4 + c + 1 + e)$ there is no pair of value[s] in S for 'c' and 'e' that adds u[p] to 34. Case D_2 is therefore discarded.

$$4 + 1 + c + e = 34$$
$$c + e = 29$$

Case E:

$$\begin{bmatrix} 2 & 3 & 15 & 14 \\ 4 & c & d & e \\ 16 & g & h & i \\ 12 & k & l & m \end{bmatrix}$$

Ant-diagonal must add to M:

$$12 + g + d + 14 = 34$$
$$g + d = 8$$

There is a single pair of values (1, 7) from se[t] S that adds up to 8. We replace 'g' & 'd' by these values, generating two more cases:

E_1
$$\begin{bmatrix} 2 & 3 & 15 & 14 \\ 4 & c & 7 & e \\ 16 & 1 & h & i \\ 12 & k & l & m \end{bmatrix}$$

The 2nd column is identical to that in case D_1. We already showed it not possible to add-up to M from the set of remaining numbers ∴ discarded.

$$\begin{bmatrix} 2 & 3 & 15 & 14 \\ 4 & c & 1 & e \\ 16 & 7 & h & i \\ 12 & k & l & m \end{bmatrix}$$

The 2nd row is identical to that [in] case D_2. We have showed it is not possible to add-up to M from [the] remaining set of numbers. Hence, c[ase] E_2 is discarded.

There are no more cases left. By disjunction, I have therefore shown it is not possible to genera[te] a magic square fro[m] matrix A.