

~~1) $G(x) = \left(\frac{x}{1!} + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots \right) \left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots \right) \left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \right)^3$~~

~~1) $G(x) = \left(\frac{x}{1!} + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots \right) \left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots \right) \left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \right)^3$~~

Since $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$, $e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots$

$$\therefore \left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots \right) = \frac{1}{2} (e^x + e^{-x})$$

$$\therefore \left(x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots \right) = \frac{1}{2} (e^x - e^{-x})$$

$$G(x) = \frac{1}{2} (e^x - e^{-x}) \cdot \frac{1}{2} (e^x + e^{-x}) e^{3x}$$

$$= \frac{1}{4} (e^{5x} - e^{-x})$$

$$a_n = \frac{1}{4} (5^n - 1)$$

$$G(x) = \frac{1}{4} \left(\sum_{n=0}^{\infty} (5^n - 1) \frac{x^n}{n!} \right)$$

$$a_6 = \frac{1}{4} (5^6 - 1) = 3096$$

$$2) n=3$$

or

2) Problem involving catalan numbers. One person must return the book before another one borrows it. Therefore

$$C_3 = C(6,3) - C(6,2) = 5$$