Q4) Having equation $x_1 + x_2 + x_3 = 30$, where $x_1 \geq 5$, $x_2 \geq -8$, $x_3 \geq 5$, we can perform a chan
of variable such that $y_1 = x_1 - 5$, $y_2 = x_2 + 8$, $y_3 = x_3 - 5$

$\therefore$ $y_1 + y_2 + y_3 = 30 - 5 + 8 - 5 = 28$

Hence, the # of integral solutions should be

$$\binom{28+3-1}{28} = \binom{30}{28} = 435$$

Combinatorics HW 2.1 2019280366 Albert Sebastià Millan Trujols

Q1) Combinatorics

The word 'Combinatorics' has 13 characters, such that $S = \{c, 2\cdot o, m, b, 2\cdot i, n, a, t, r, c, s\}$
Characters 'o' and 'i' are repeated twice, so we need to remove those identical permutations where these characters' positions are exchanged by those of the same type, using the division principle. Order does matter, hence permutations are required.

$$\therefore \frac{13!}{2!2!} = 1556755200$$

Q2) The binomial coefficient can be derived using Pascal's triangle for $(x+z)^n$. Since there are 3 terms in $(2a+b+c)^6$, we say that $z = b+c$, $\therefore (2a+z)^6$

$$(2a+z)^6 = \binom{6}{0}(2a)^6 + \binom{6}{1}(2a)^5 \cdot z + \binom{6}{2}(2a)^4 z^2 + \binom{6}{3}(2a)^3 z^3 + \binom{6}{4}(2a)^2 z^4 + \binom{6}{5}(2a)\cdot z^5 + \binom{6}{6} z^6$$

In the above equation, the term of interest is $\binom{6}{4}(2a)^2 z^4$ as the power of 'a' is 2. Using Pascal formula we substitute back: $z^4 = (b+c)^4$

$$\therefore (b+c)^4 = \binom{4}{0}b^4 + \binom{4}{1}b^3c^1 + \binom{4}{2}b^2c^2 + \binom{4}{3}b^1c^3 + \binom{4}{4}c^4$$

Again, the term of interest is $\binom{4}{2}b^2c^2$ as both the powers of 'b' and 'c' is 2. Putting all together, we have:

$$\binom{6}{4}\cdot(2a)^2\cdot\binom{4}{2}b^2c^2 = 15\cdot 4\cdot 6\cdot a^2b^2c^2 = \boxed{360}\cdot a^2b^2c^2$$

There are 12 identical apples that must be split amongst 3 kids where each at least receives one apple. This can be represented as:

$$x_1 + x_2 + x_3 = 12, \text{ where } x_1, x_2, x_3 \geq 1$$

We then perform a change of variable, such that $y_1 = x_1 - 1$, $y_2 = x_2 - 1$, $y_3 = x_3 - 1$, in order to account for the case where each child receives an apple:

$$y_1 + y_2 + y_3 = 12 - 1 - 1 - 1 = 9$$

Hence, the solution is then the number of nonnegative integral solutions of the equation

$$\binom{9+3-1}{9} = \binom{11}{9} = \frac{11!}{9!2!} = \boxed{55}$$