$$A + B = 1$$

$$A\left(\frac{1-\sqrt{5}}{2}\right) + B\left(\frac{1+\sqrt{5}}{2}\right) = 1$$

$$A = 1 - B$$

$$(1-B)\left(\frac{1-\sqrt{5}}{2}\right) + B\left(\frac{1+\sqrt{5}}{2}\right) = 1$$

$$\frac{1 - B - \sqrt{5} + \sqrt{5}B}{2} + B\left(\frac{1+\sqrt{5}}{2}\right) = 1$$

$$\frac{1 - \cancel{B} - \sqrt{5} + \sqrt{5}\cdot B + \cancel{B} + \sqrt{5}\cdot B}{2} = 1$$

$$\frac{1 - \sqrt{5} + 2B\sqrt{5}}{2} = 1$$

$$B\sqrt{5} + \frac{1}{2} - \frac{\sqrt{5}}{2} = 1$$

$$B\sqrt{5} = 1 - \frac{1}{2} + \frac{\sqrt{5}}{2}$$

$$B\sqrt{5} = 1 - \frac{1-\sqrt{5}}{2}$$

$$B = \frac{\sqrt{5}+5}{10}$$

$$\Rightarrow A = 1 - \frac{\sqrt{5}+5}{10}$$

$$= 1 - \frac{1+\sqrt{5}}{2\sqrt{5}}$$

$$\therefore a_n = \left(1 - \frac{1+\sqrt{5}}{2\sqrt{5}}\right)\cdot\left(\frac{1-\sqrt{5}}{2}\right)^n + \left(\frac{\sqrt{5}+5}{10}\right)\left(\frac{1+\sqrt{5}}{2}\right)^n$$

2) Two cases: ~~either~~

a) If n^{th} term is blue or white, the rest can be done in a_{n-1} ways $\Rightarrow 2a_{n-1}$

b) If n^{th} term is red, then $(n-1)^{th}$ is blue or white (two ways). The rest can be done in a_{n-2} ways $\Rightarrow 2a_{n-2}$

$$a_n = 2a_{n-1} + 2a_{n-2}, \quad n \geq 2, \quad a_0 = 1, \quad a_1 = 3$$

$$x^2 - 2x - 2 = 0 \qquad x = \frac{2 \pm \sqrt{4+8}}{2} = \frac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}$$

$$a_n = A(1+\sqrt{3})^n + B(1-\sqrt{3})^n$$

$$a_0 = A + B = 1$$

$$a_1 = A(1+\sqrt{3}) + B(1-\sqrt{3}) = 3$$

It is known

(n=2) $a_2 = 3 \cdot 3 - 1 = 8$

(n=3) $a_3 = 3 \cdot 3 \cdot 3 - 5 = 22$

Compute a_1

$$a_3 = 2a_2 + 2a_1$$

$$22 = 16 + 2a_1 \Rightarrow a_1 = 3$$

Compute a_0

$$a_2 = 2a_1 + 2a_0$$

$$8 = 6 + 2a_0 \Rightarrow a_0 = 1$$

$$\boxed{a_n = \left(1 + \frac{2\sqrt{3}-3}{6}\right)(1+\sqrt{3})^n - \left(\frac{2\sqrt{3}-3}{6}\right)(1-\sqrt{3})^n}$$

$$\left.\begin{array}{l} A + B = 1 \\ A(1+\sqrt{3}) + B(1-\sqrt{3}) = 3 \end{array}\right] A = 1 - B$$

$$(1-B)(1+\sqrt{3}) + B(1-\sqrt{3}) = 3$$

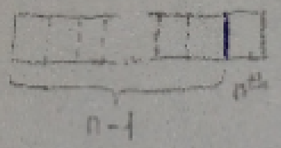
$$1 + \sqrt{3} - B - B\sqrt{3} + B - B\sqrt{3} = 3$$

$$-2B\sqrt{3} = 2 - \sqrt{3}$$

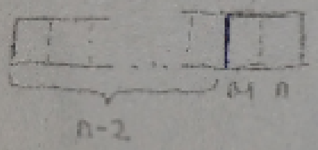
$$A = 1 + \frac{2\sqrt{3}-3}{6} \qquad B = -\frac{2-\sqrt{3}}{2\sqrt{3}} = -\frac{2\sqrt{3}-3}{6}$$

Combinatorics HW6-2 2019280366 Albert Sebastia Millan

1) There are two cases in a sequence of n length of square blocks: (1) either the last space is occupied by a square brick or (2) the last 2 spaces are occupied by a rectangle brick:



Square brick



Rectangle brick

Thus, the total number of arrangements to pave a road of length n (a_n) would be the sum of the number of arrangements for a road of length n-1 and one single square brick (a_{n-1}) plus the number of arrangements for a road of length n-2 plus a rectangle brick (a_{n-2}).

$\therefore a_n = a_{n-1} + a_{n-2}$,

where $a_0 = 1$, $a_1 = 1$, $n \geq 2$

$x^2 - x - 1 = 0$ $\qquad r = \frac{1 \pm \sqrt{5}}{2}$

$$a_n = A\left(\frac{1-\sqrt{5}}{2}\right)^n + B\left(\frac{1+\sqrt{5}}{2}\right)^n$$

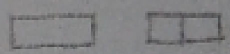
$$a_0 = A + B = 1$$
$$a_1 = +A\left(\frac{1-\sqrt{5}}{2}\right) + B\left(\frac{1+\sqrt{5}}{2}\right) = 1$$

See next page for step-by-step computation

$$\boxed{\therefore a_n = \left(1 - \frac{1+\sqrt{5}}{2\sqrt{5}}\right)\left(\frac{1-\sqrt{5}}{2}\right)^n + \left(\frac{\sqrt{5}+5}{10}\right)\left(\frac{1+\sqrt{5}}{2}\right)^n}$$

It is known

$a_2 = 2$



$a_3 = 3$

Compute a_1

$a_3 = a_2 + a_1$

$3 = 2 + a_1 \Rightarrow a_1 = 1$

Compute a_0

$a_2 = a_1 + a_0$

$2 = 1 + a_0 \Rightarrow a_0 = 1$