Approach B:

Suppose $|S| = a_1, a_2, a_3, \ldots a_{52}$, where $a_i$ is a unique integer number.

If numbers in $S$ are divided by 100, they will have a remainder such that

$$r_i = a_i \bmod 100$$

If two numbers $a_i$ and $a_k$ exist such that $r_i == r_k$, then $a_i - a_k$ is divisible by 100.

Lets consider the case where $|R| = r_1, r_2, r_3, \ldots, r_{52}$ are also unique. Thus ranging from 0-99.

It follows that there cannot be a remainder pair in $|R|$ such that $r_i = (100 - r_k) \bmod 100$. These two elements are complimentary, adding up to 100 ($r_i + r_k = 100$), Hence being divisible by 100.

However, there are 49 combinations of pairs that add-up to 100, such that:

$\{1, 99\}, \{2, 98\}, \{3, 97\}, \ldots, \{49, 51\}$

There are two more that are self-complimentary: 0 & 50

$$(100 - 0) \bmod 100 = 0$$
$$(100 - 50) \bmod 100 = 50$$

In total there are thus 51 possible distinct complimentary remainder pairs (pigeonholes) and we have 52 remainders (pigeons). By the pigeon principle, it is not possible to fit $r_{52}$ without putting it with its complimentary remainder pair ($100 - r_{52}$) as there are more unique remainders in $|R|$ than those that form a complimentary pair (51). In turn, there is at least 1 complimentary pair and therefore, two integers whose values' sum or difference is divisible by 100.

Combinatorics HW 3.2 Albert Millan Truyols 2019280306

1) Considering unlimited supply of oranges, apples and bananas, the worst case scenario occurs when we have 7 apples, 5 bananas or 8 oranges. Only in this case will drawing on a new fruit guarantee that the statement proposed in the question is true. Hence,

$$7+5+8+1 = \boxed{21 \text{ fruits}}$$

2) We have 52 integers and have to find 2 of these whose sum or difference is divisible by 100. We know the following statements are true:

- A number is divisible by 100 if the remainder equals zero.
- Sum of two numbers ~~is~~ divisible by 100 if the sum of the remainders equals 100, such as:

$$a \bmod 100 + b \bmod 100 = 100$$

- Difference of two numbers is divisible by 100 if the difference of the remainders equals zero, such that:

$$a \bmod 100 - b \bmod 100 = 0$$

Approach A

There are 49 different combinations of pairs that add-up to 100 without repeated values

$$\{1, 99\}, \{2, 98\}, \{3, 97\}, \ldots, \{49, 51\}$$

There are 2 more that are self-complimentary: 0 and 50

$$(100-0) \bmod 100 = 0$$

$$(100-50) \bmod 100 = 50$$

In total there are thus 51 cases (pigeonholes). Since there are 51+1 pigeons, even in the worst case scenario (first 51 integers having different remainders), the 52nd integer remainder would either have its complementary in one of the boxes, suming 100, or otherwise already exist in one of the boxes, thus equaling zero when substracting them.