Guard in Art gallery.

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Contents

Introduction

- The "Art galley theorem" is referred to, was originated from real world situation. There are G amount of guards guarding the galley, and the goal is the compute the minimal amount of guards needed to guard the entire art gallery.

- A "polygon" is generally defined as an oredered sequence of at least three points $v_1, v_2, v_3, \cdots v_n$ in the plane, called vertices.

Problem statement

- Given a polygon art gallery P, how many possible minimum guards will be needed? and where to place them , so we can be sure it will be guided. (in our case, a 24-sided polygon that we have).

Methodology

The tools that must be used in order to solve the problem includes:

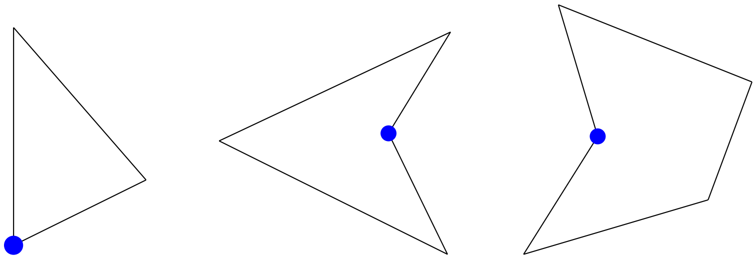
- Define the Parameters
- Use Geometry Software
- Draw the Polygon
- Customize the Shape
- Visualize and Analyze
- Documentation

Why An art Gallery problem?

- This problem is important in various fields, including security, surveillance, and robotics, where efficient guarding or monitoring of an area is crucial.

Discussion

Taking blue bullets as guards, n as number of sides and G as their numbers

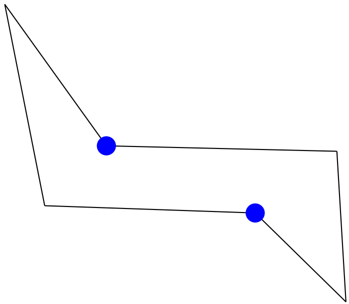


(a) $n = 3$ (b) $n = 4$ (c) $n = 5$

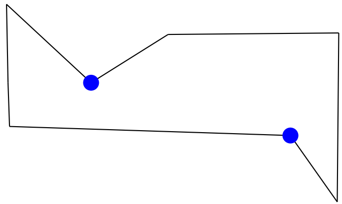
Figure: Three figures from $3 \leq n \leq 5$

$$G = \left\lfloor \frac{3}{3} \right\rfloor = \left\lfloor \frac{4}{3} \right\rfloor = \left\lfloor \frac{5}{3} \right\rfloor = 1$$

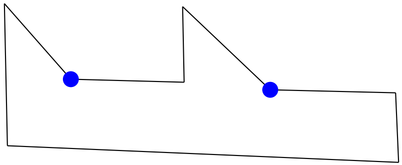
Discussion



(a) $n = 6$



(b) $n = 7$

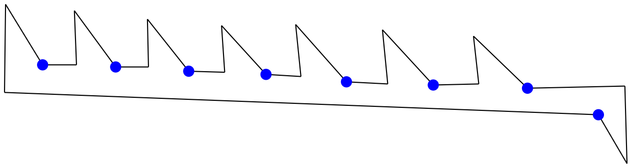


(c) $n = 8$

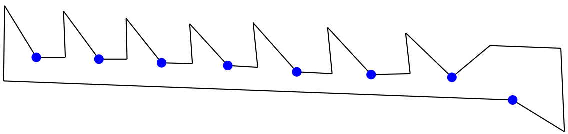
Figure: Three figures from $6 \leq n \leq 8$

$$G = \left\lfloor \frac{6}{3} \right\rfloor = \left\lfloor \frac{7}{3} \right\rfloor = \left\lfloor \frac{8}{3} \right\rfloor = 2$$

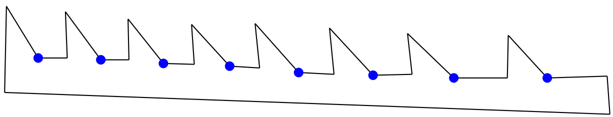
Discussion



(a) $n = 24$



(b) $n = 25$



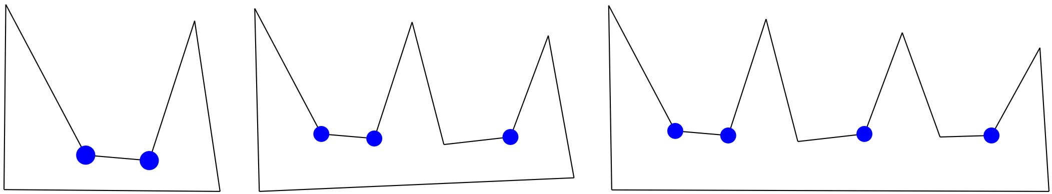
(c) $n = 26$

Figure: Three figures from $24 \leq n \leq 26$

$$G = \left\lfloor \frac{24}{3} \right\rfloor = \left\lfloor \frac{24}{3} \right\rfloor = \left\lfloor \frac{26}{3} \right\rfloor = 8$$

Discussion

From the above figures we can generalise that $G(n) = \left\lfloor \frac{n}{3} \right\rfloor$



(a) $G(6) = 3$ (b) $G(9) = 3$ (c) $G(12) = 4$

In each instance of the crown gallery, every additional crown requires a guard, so it would appear that for any instance of $G(n)$ crown gallery where $n = 3k$, we would require $\frac{n}{3}$ guards.

Proof

Proof.

Thus we have $G(3K) = G(3K+1) = G(3K+2):$ True

$$\text{we have that } G(n) \geq \left\lfloor \frac{n}{3} \right\rfloor \text{ For maximum guards}$$
$$G(n) \leq \left\lfloor \frac{n}{3} \right\rfloor \text{ For minimum guards}$$

$$\text{Therefore } G(n) = \left\lfloor \frac{n}{3} \right\rfloor$$

by the property that if $a \geq b$ and $b \leq a$

then $a = b$

$\square$

Conclusion

- In conclusion, it is always possible to any polygon with n sides, and therefore to keep polygonal art gallery with $\left\lfloor \frac{n}{3} \right\rfloor$ guards.

Bibliography

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End

Thank You