Advection-diffusion equation

$$\frac{\partial J}{\partial t} + u\frac{\partial J}{\partial x} = D\frac{\partial^2 J}{\partial x^2}$$

advection term

diffusion term

FE disc

$$\frac{J_i^{k+1} - J_i^k}{\Delta t}$$

FE

$$, \quad \frac{J_{i+1}^k - 2J_i^k + J_{i-1}^k}{(\Delta x)^2}$$

① CFE

$$u\,\frac{J_{i+1}^k - J_{i-1}^k}{\Delta x}$$

Artificial diffusion

② Upwind

$$u\,\frac{J_i^k - J_{i-1}^k}{\Delta x}$$

If we pick $C=1$ then no artificial diff

$$C = \frac{u\Delta t}{\Delta x} \qquad C_D = \frac{D\Delta t}{(\Delta x)^2}$$

Peclet number of the method

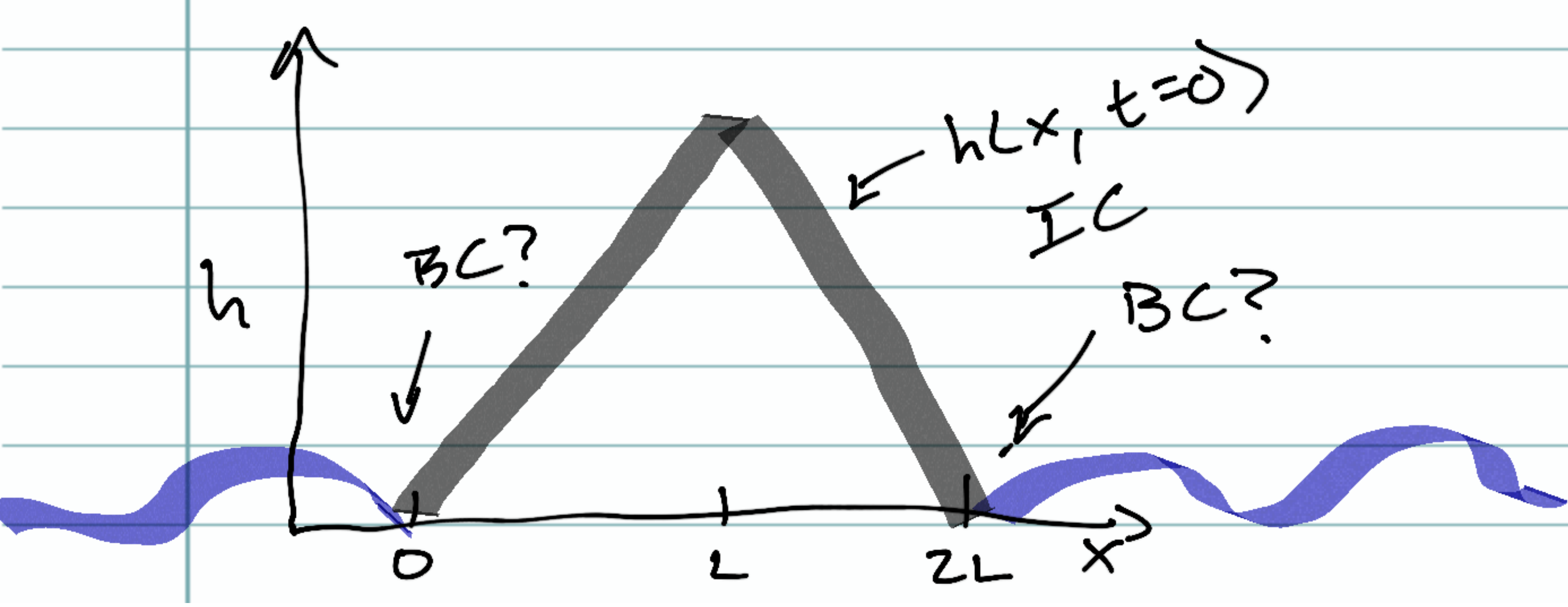
$$Pe = \frac{c}{c_D} = \frac{u\,\Delta x}{D}$$

Pe large → dominated by advection
Pe small → dominated by diffusion

Demo - Erosion of a hill slope

Diffusion equation can represent any small scale process that moves "stuff" down gradient

Geomorphology - evolution of landscapes over time



→ Small-scale processes like erosion and sedimentation smooth out bumpiness in topography by moving downslope in the direction of gravity

mass flux

Assumption #1: $q = -\rho D \frac{\partial h}{\partial x}$

density of sediments

erosion processes

thickness of sediment

Advection equation $\frac{\partial h}{\partial t} = -\frac{1}{\rho}\frac{\partial}{\partial x}(\underbrace{uh}_{q})$

q → mass flux

Diffusion equation for thickness of sediments

$$\boxed{\frac{\partial h}{\partial t} = D\frac{\partial^2 h}{\partial x^2}}$$

BCs?

$h(x=0) = 0$

$h(x=2L) = 0$

River?

Basement rock?

→ Parameters

→ Set Δx, Δt

→ Pre-allocate solution vector, matrix

→ Define the matrix

→ for loop for time stepping

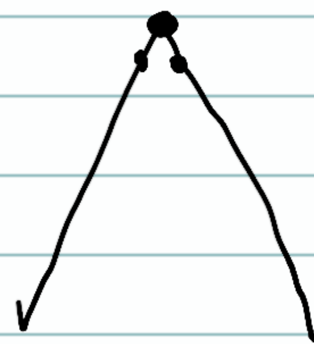
$\boxed{\text{FE method for solving diffusion equation}}$

$D = 1 \frac{m^2}{s}$ $L = 10\ m$

$H(x=L) = 20\ m$

$\Delta x = \frac{2L}{201}$

$C_D = \frac{D \Delta t}{(\Delta x)^2} \sim 1$



$t_f = 100\ s$