Advection-Diffusion Equation

→ Combines two processes:

$$\frac{\partial \bar{J}}{\partial t} + u\frac{\partial \bar{J}}{\partial x} = D\frac{\partial^2 \bar{J}}{\partial x^2}$$

Often used to capture things carried in a turbulent geophysical flow (e.g. ocean, atmosphere) because small-scale turbulence is hard to model, but has the effect of mixing things up on small scales and transporting them on large scales

Consider two discretizations:

FE in time, CFE in advection and diffussion

$$\frac{\bar{J}_i^{k+1} - \bar{J}_i^k}{\Delta t} + \frac{u}{2\Delta x}\left(\bar{J}_{i+1}^k - \bar{J}_{i-1}^k\right) = \frac{D}{\Delta x^2}\left(\bar{J}_{i+1}^k - 2\bar{J}_i^k + \bar{J}_{i-1}^k\right)$$

$$\bar{J}_i^{k+1} = \bar{J}_i^k - \frac{C}{2}\left(\bar{J}_{i+1}^k - \bar{J}_{i-1}^k\right) + C_D\left(\bar{J}_{i+1}^k - 2\bar{J}_i^k + \bar{J}_{i-1}^k\right)$$

Could also consider upwind in advection term

$$- C\left(\bar{J}_i^k - \bar{J}_{i-1}^k\right)$$

May be less diffusive w.r.t advection

Peclet number of method $Pe = \frac{C}{C_D} = \frac{u\Delta x}{D}$