Coupled ODEs

→ So far, we have only considered single variables with a single ODE to describe their evolution.

→ However, in modeling processes in Earth sciences, we will often want to understand many interacting variables.

→ The good news → nothing really changes!
  → We can use the same methods for solving ODEs that we've already talked about, we just need to be careful about how we write code to solve them.

Example: $\frac{dx}{dt} = f(x, y, t)$ $\frac{dy}{dt} = g(x, y, t)$

$x(t=0) = x_0$ $y(t=0) = y_0$

Pseudocode for FE solution:

$x(1) = x_0$ $\quad \Delta t = t_f / n$

$y(1) = y_0$

for $i = 1, \ldots, n$

$$x(i+1) = x(i) + f(x(i), y(i), t(i))\,\Delta t$$
$$y(i+1) = y(i) + g(x(i), y(i), t(i))\,\Delta t$$
$$t(i+1) = t(i) + \Delta t$$

end

This solution also makes it possible to solve some higher-order ODEs:

Example: $\frac{d^2x}{dt^2} = f(x,t) \rightarrow \frac{dy}{dt} = f(x,t)$

↳ Introduce $\frac{dx}{dt} = y$