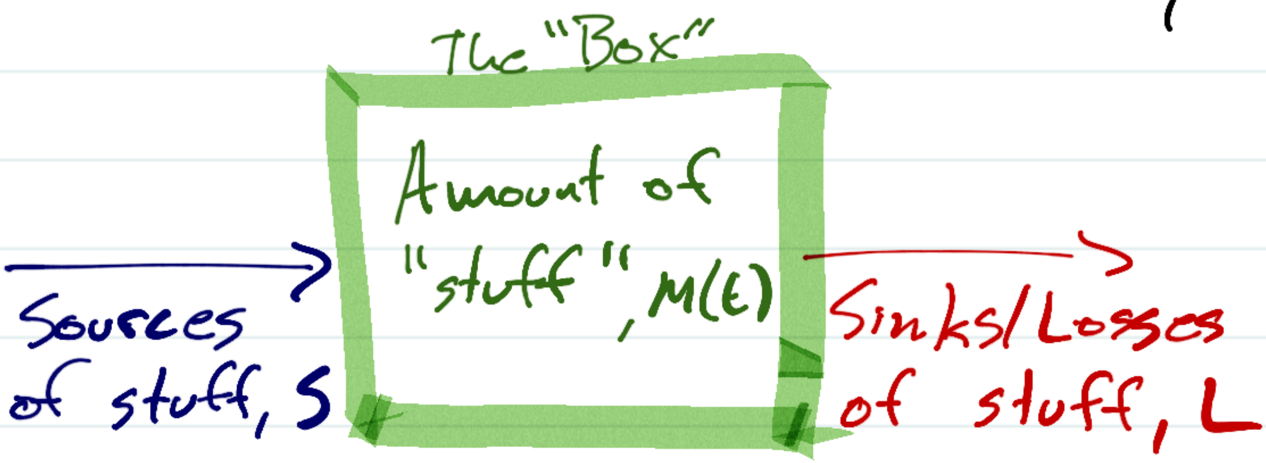
Box Models

A method for building simple models - very common in Earth sciences

- → Typically formulated as a coupled set of ODEs - use one of our ODE solving methods

Idea tracking how some quantity moves between reservoirs within a system



For this model $\boxed{\frac{dM}{dt} = S - L}$

→ If S, L do not depend on time, then for there to be a steady-state, $S = L$ $\left(\frac{dM}{dt} = 0\right)$

-> A more common assumption is that

$L = kM$ -> loss rate of stuff is proportional to amount of stuff
(e.g. rate of flow out of a spigot in a water container depends on amount of water)

$$\frac{dM}{dt} = S - kM$$

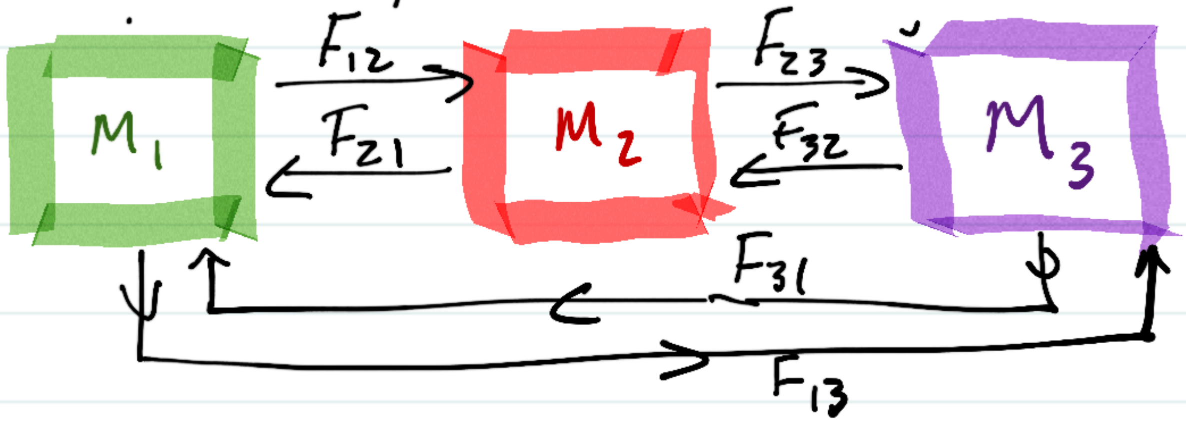
Steady-state

$$\frac{dM}{dt} = 0 = S - kM$$

$$\boxed{M = \frac{S}{k}}$$

There is always a s-s if $S \geq 0, k \geq 0$

More complex systems with many boxes



System of ODEs describing this system

$$\frac{dM_1}{dt} = F_{21} + F_{31} - F_{12} - F_{13}$$

$$\frac{dM_2}{dt} = F_{12} + F_{32} - F_{21} - F_{23}$$

$$\frac{dM_3}{dt} = F_{23} + F_{13} - F_{32} - F_{31}$$

This system is thought to be closed if all sources and sinks are within the system (i.e. $\sum F_{ij} = 0$)

↳ Closed systems must conserve M → so $\frac{dM_j}{dt} = 0$ eventually

① Definition ①: Closed systems and conservation

② The residence time of stuff in a box is the average time that stuff spends in a box:

$$t_j = M_j / \sum_j F_{ji} = \frac{\text{amount}}{\text{loss rate}}$$

Come back to matrix formulation of this problem during lin. alg weeks

If F_{ij} depend linearly on M_i, we can write this system as a matrix equation:

$$\frac{d}{dt}\begin{bmatrix} M_i \\ \vdots \end{bmatrix} = \begin{bmatrix} & A & \end{bmatrix}\begin{bmatrix} M_i \\ \vdots \end{bmatrix}$$

Which, with the FE method is re-written

$$\begin{bmatrix} M_i(t+1) \\ \vdots \end{bmatrix} = \begin{bmatrix} & A' & \end{bmatrix}\begin{bmatrix} M_i(t) \\ \vdots \end{bmatrix}$$