Review of PDEs

What makes ODEs ordinary is that they only involve derivatives in one variable!

$$\frac{dx}{dt} = f(x,t)$$

derivative in t

Partial differential equations (PDEs) involve derivatives in more than one variable:

Example: For a variable $T = T(x,t)$

$$\frac{\partial T}{\partial t} = D \frac{\partial^2 T}{\partial x^2}$$

The diffusion equation (will return to this)

time derivative (rate of change of T in time)

constant

spatial derivative (i.e. slope of T in space)

Remember: when you are considering derivates in multiple variables, use partial deriv: ∂

"curly"

→ We might consider derivatives in more than one variable because change can be caused by:

→ Time-dependent processes at one location ($\frac{\partial T}{\partial t}$)

→ Spatially-dependent processes which involve change over space ($\frac{\partial^2 T}{\partial x^2}$)

→ We want to understand and model the relationship between these derivates, typically in time and space, and we can do so by combining what we learned about solving coupled systems of ODEs with some new tricks

Classification of PDEs

→ Most PDEs of interest in Earth Science involve 2nd order derivatives.

→ They can generally be classified into three categories, with diff num methods to solve each type

For a variable $u = u(x,t)$

$u_x = \partial u / \partial x \quad u_{xx} = \frac{\partial^2 u}{\partial x^2} \quad u_t = \frac{\partial u}{\partial t}$

Consider the most general PDE:

$$\boxed{G = Au_{xx} + Bu_{xt} + Cu_{tt} + Du_x + Eu_t + Fu}$$

If $G=0$, PDE is homogenous

Types of PDEs

① Parabolic : $B^2 - 4AC = 0$

Example $Au_{xx} = u_t$ → Diffusion Equation

$B = C = 0 \rightarrow B^2 - 4AC = 0$

② Hyperbolic: $B^2 - 4AC > 0$

Example $u_{xx} - Cu_{tt} = 0$ → Wave Equation

$A = 1 \quad B = 0 \rightarrow B^2 - 4AC > 0$

③ Elliptic Equations: $B^2-4AC<0$

Example Poisson's/Laplace's Equation

$$u_{xx}+u_{yy}=0 \qquad A=C=1$$
$$B=0$$
$$B^2-4AC<0$$

→ Some combination of these PDEs is used in some way or another to describe many processes in Earth science.
↳ space and time-dependent

→ We will extend what we have learned about ODE numerical methods to solve PDEs using numerical methods.