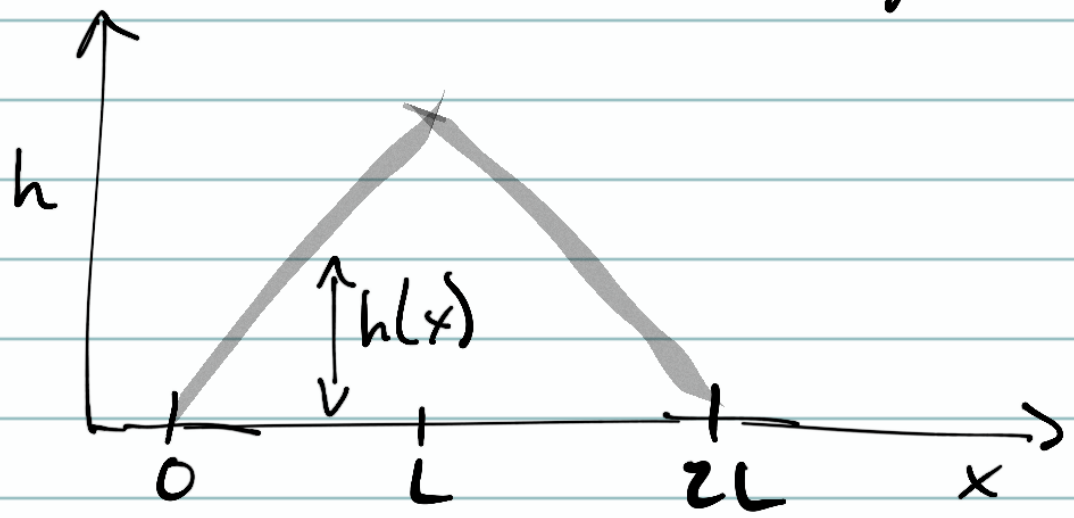
Advection-diffusion examples

Geomorphology - evolution of landscapes over time

Consider a hill of height, h



→ Small-scale processes like erosion and sedimentation smooth out bumpiness in topography by moving material down (in the direction of gravity)
→ Can be modeled as diffusive process

If we presume mass flux, q, is proportional to local slope:

$$q = -\rho D \frac{\partial h}{\partial x}$$

ρ: density of sediments

D: diffusivity of erosion processes

$$\frac{\partial h}{\partial t} = -\frac{1}{\rho}\frac{\partial q}{\partial x} = D\frac{\partial^2 h}{\partial x^2}$$

→ Diffusion equation

If there is a river/water body maintaing a set height at bottom of hill, this could serve as boundary condition:

$$h(x=0) = h(x=2L) = 0$$

Perhaps we might consider a river that also transports mass through advection → add uh_x term

Another sort of example

Advection-diffusion of a tracer in the ocean (salt?)