Seismic Tomography : solving inverse problems to estimate model parameter(s)

Ashim Rijal

Department of Earth and Environment Sciences, LMU Munich

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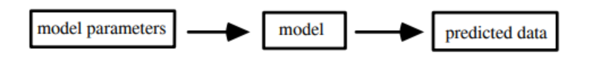
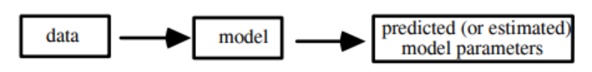
Forward and Inverse problem



The process of predicting data based on some mathematical (or physical) model with a given set of model parameters. eg: travel time based on reference velocity structure.



The process of predicting (or estimating) the numerical values of a set of model parameters of an assumed model based on observation. eg: invert travel time to determine the velocity structure.

Inverse problem

- We want to find a velocity structure of the Earth from observed seismic data. Given the observed arrival times, find a model that fits them.

$$\mathbf{d} = A\mathbf{m}$$

d_i is a data vector (arrival times) and m_j is assumed model vector (eg. slowness or 1/velocity)

- Start with the initial model $\mathbf{m}^0$ (a guess, eg. 1D Earth velocity model).
- Predictions from starting model

$$d_i^0 = A(m_j^0)$$

Inverse problem cont...

$$d_i^0 = A(m_j^0)$$

- This doesn't predict the actual observation. Hence, we seek to change the initial model by $\Delta\mathbf{m}$ amount.

$$m_j^1 = m_j^0 + \Delta m_j^0$$

- Further, our data do not depend linearly on the model parameters.
- Linearize the problem by expanding the data in a Taylor series about the starting model

$$d_i \approx d_i^0 + \sum_j \frac{\partial d}{\partial m}|_{m^0}.\Delta m_j^0$$

Inverse problem cont...

$$d_i \approx d_i^0 + \sum_j \frac{\partial d}{\partial m}|_{m^0}.\Delta m_j^0$$

- Let's write this in terms of the difference between the observed data and those predicted,

$$\Delta d_i^0 \equiv d_i - d_i^0 \approx \sum_j \frac{\partial d_i}{\partial m_j}|_{m^0}.\Delta m_j^0$$

$$\mathbf{\Delta d} = \mathbf{G\Delta m}$$

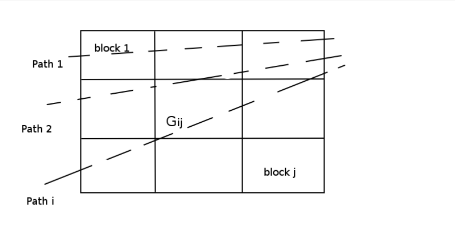
- Where, $\mathbf{G} = G_{ij} = \frac{\partial d_i}{\partial m_j}$ is the partial derivative matrix. This is a linear algebra problem.

Inverse problem cont...

- In our seismic tomography example, this matrix would be

$$G_{ij} = \frac{\partial T_i}{\partial s_j}$$

T_i being the travel-time perturbation- i^{th} ray and u_j is the slowness perturbation- j^{th} box



Source: Stein ands Wysession

Inverse problem cont...

$$\Delta \mathbf{d} = \mathbf{G} \Delta \mathbf{m}$$

- Because there are many more equations (ray paths) than unknowns (model parameters), the system of equations is overdetermined.

- Also inconsistent because our data contain noise (instrument, measurement related, or unknown velocity structures in earthquake location problem)

- The least-squares problem

$$arg_{min} ||\Delta \mathbf{d} - \mathbf{G} \Delta \mathbf{m}||_2$$

has a unique minimum $||\mathbf{r}||_2$ and a unique residual $\mathbf{r} = \Delta \mathbf{d} - \mathbf{G} \Delta \mathbf{m}^*$ iff $\Delta \mathbf{m}^*$ solves the normal equation

$$\mathbf{G}^{\mathbf{T}} \mathbf{G} \Delta \mathbf{m} = \mathbf{G}^{\mathbf{T}} \Delta \mathbf{d}$$

Inverse problem cont...

-Then in the second iteration, we use

$$m_j^1 = m_j^0 + \Delta m_j^0$$
$$......$$
$$m_j^n = m_j^{n-1} + \Delta m_j^{n-1}$$

- Define a tolerance level and stop iterating.

- In finite precision arithmetics $\mathbf{G^T G}$ may become singular.

- If $\mathbf{G}$ is dense - LAPACK routines (eg. DGELS)

- If $\mathbf{G}$ is sparse - iterative LSQR algorithm

Jupyter notebook