Ordinary differential equations: review

What is an ODE?

Remember:

$$\frac{dx}{dt} = \lim_{t_0,\ \Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

Draw graph rep

In essence:

rate of change = process + process + process

ODEs are a central part of modeling in the Earth sciences, because change is everywhere in the Earth system and we are often interested in determining why something changed and predicting how it will change in the future.

In terms of equations: $\frac{dx}{dt} = f(x,t)$

(quant of interest; time; process causing change)

describes a generic ODE

→ ODEs on their own usually have an infinite number of solutions (similar to how an indefinite integral has a generic solution often w/ integration constants)

→ Solving ODEs for particular solutions (typical in GeoSci) involves turning the ODE into either:

(a) Initial value problem (IVP)
- Set one or more condition at a single value of the independent variable: e.g., $y(x=a) = y_0$, $\left.\frac{dy}{dx}\right|_{x=a} = d_0$

(b) Boundary value problem (BVP)
→ Set conditions at more than a single value of the ind. var
e.g., $y(x=a) = y_1$ $\quad y(x=b) = y_2$