

# Tree

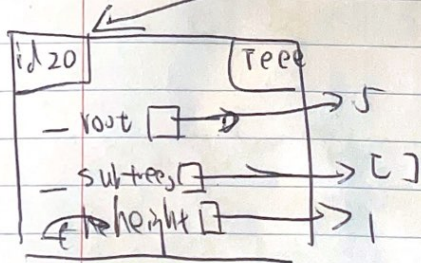
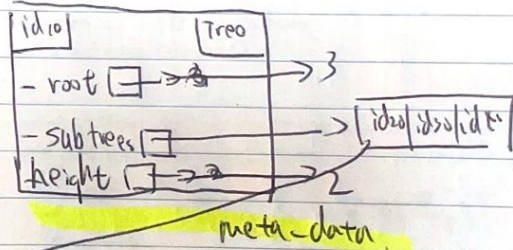
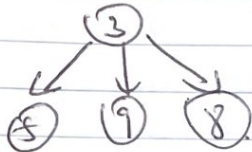
## 课后作业

both private

- root : ~~int~~ Any | None.
- subtrees : ~~list~~ list[Tree]

RI prohibits.

Empty node & Not null subtree



## CSC148 - Tree method practice

Here is an abbreviated version of the Tree class we're studying this week.

```
class Tree:
    • _root: Any | None
    • _subtrees: list[Tree]

    • def __init__(self, root: Any | None, subtrees: list[Tree]) -> None:
    • def is_empty(self) -> bool:
```

Your goal for this worksheet is to implement the following new Tree method, using the recursive Tree code template.

```
class Tree:
    def leaves(self) -> list:
        """Return a list of all of the leaf values in this tree.
        """
        if self.is_empty():
            ...
        elif self._subtrees == []: # self is a leaf
            ...
        else:
            ...
            for subtree in self._subtrees:
                ... subtree.leaves() ...
            ...
```

Handwritten notes for the recursive case:

```
e-t
>>> empty tree = Tree(None, [])
>>> e-t.leaves()
None []
>>> t = Tree(1, [t1, t2])
t1 = Tree(1, [])
t2 = Tree(2, [])
```

1. (base cases, examples) First, let's consider some base cases for this function. In the space below, write two doctest examples, one that calls this method on an empty tree, and one that calls this method on a leaf.

Handwritten doctest examples:

```
>>> t = Tree(None, []) # empty tree
>>> t.leaves()
[]
>>> t = Tree(148, []) # one-node tree
>>> t.leaves()
[148]
```

2. (base cases, implementation) Implement the base cases of the Tree.leaves method below.

```
class Tree:
    def leaves(self) -> list:
        """Return a list of all of the leaf values in this tree.

        The leaf values are returned in left-to-right order.
        """
        if self.is_empty():
```

Handwritten note for the empty tree case:

```
>>> Tree(None, []).leaves()
[]
```

Handwritten note for the empty tree case:

```
return []
```

Handwritten note for the empty tree case:

```
return []
```

Handwritten note for the leaf case:

```
return [self._root]
```

```
elif self._subtrees == []: # self is a leaf
```

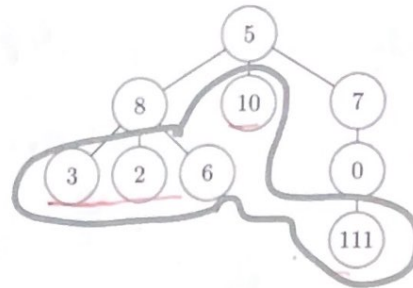
Handwritten note for the leaf case:

```
# know: self._root is not None. ∴ a one-node tree
return [self._root]
```

```
else:
    ... # Will do this later
```



3. (recursive step, example) Now suppose we have a variable `tree` that refers to the tree on the right.



- (a) Complete the doctest example below.

```
>>> tree.leaves()
```

```
[3, 2, 6, 10, 111]
```

leaf is the bottom

对 Node 的 all subtrees, subtree.leaves() 加起来

- (b) Complete the following recursion table to show each of the subtrees of this tree, as well as what the recursive call to `leaves` will return for that subtree, assuming the recursive call is correct. We have started the table for you.

subtree	subtree.leaves()
	<p>[3, 2, 6]</p> <p>[3, 4]</p>
	<p>[10]</p> <p>[0]</p>
	<p>[111]</p> <p>[11]</p>

My job? (besides making the recursive calls)

build my answer by "adding in" each of these.  
Use extend.

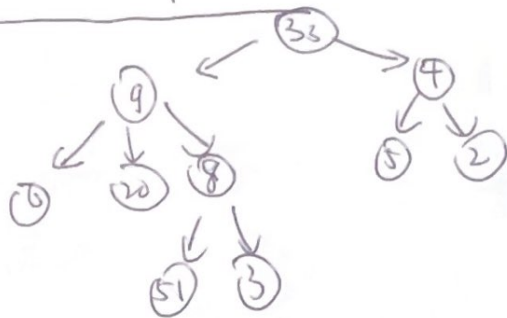
4. (recursive step, implementation) Implement the recursive step for the `Tree.leaves` method.

```
class Tree:
    def leaves(self) -> list:
        ...
        else: # Recursive step!
            res = []
            for subtree in self._subtrees:
                res = []
                res.append(subtree.leaves())
            return res
```

5. Recall that in many `Tree` methods, the "leaf" case from our recursive code template is redundant and can be removed. Is this the case here? Why or why not?

# Traversal

pre-order, post-order traversal  
 visit root before children → root after children



pre-order

33 9 6 20 8 51 3 4 5 2

child1 child2

post-order

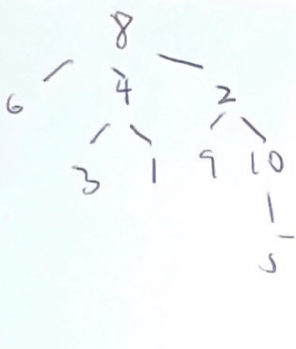
6 20 51 3 8 9 5 2 4 33

def average(self) → float:

Base case:  
 empty tree  
 one node

int, int ← # of nodes  
 sum  
 return 0.  
 return 19.0.

Recursive case:

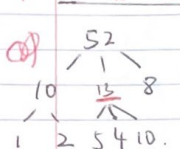


recursive calls	return
6	6, 1
4	8, 3
2	26, 4

my job?

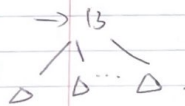
→ ~~19.0~~  
 6 + 8 + 26 + 8.  
 1 + 3 + 4 + 1

- delete\_item from tree
- delete\_item(self, item: Any) → bool

Scenario	Recursive calls	return	Our job
a) Empty tree	/	/	return False
b) One-node tree + item = 13 (13) handle with e?	/	/	return True
c) (31) handle with d?	/	/	return False
d) 	<div>10 False</div> <div>13 True Remove 13</div> <div>8 False</div>	<div>→</div> <div>→</div> <div>→</div>	<div>return if any is 13</div> <div>stop if we have True,</div> <div>return True</div> <div>otherwise, return False</div> <div>if tree has 13, marks this logic works</div>

General recursive structure

e) root is 13.



递归处理

if self.empty():

a) → return False

elif self.root == item:

e) and maybe b) one node tree with item at root

item at root

self.\_delete\_root() ← helper

else: # d) big tree where root is not item

# c) one node tree, root != item

for subtree in self.\_subtrees:

if subtree.delete\_item(item):

return False return True

bb?

+26 +8.

+4 +1



2/13/11 Changmin

## CSC148 - Tree Deletion Algorithms

We've seen that when deleting an item from a tree, the bulk of the work comes when you've already found the item, that is, you are "at" a subtree where the item is the root value, and you need to delete it. Our goal is to complete our code from lecture by implementing the helper `Tree.delete_root`.

```
class Tree:
    def delete_root(self) -> None:
        """Remove the root of this tree. Precondition: not self.is_empty()"""
```

1. We can't just set the `self.root` attribute to `None`. Why not?

**No, violates an I.I.**

Instead, we will give `_root` a new value from somewhere else in the tree. Let's look at two different ways we can do this.

2. Approach 1: "Promote" a subtree

**Idea:** to delete the root, take the rightmost subtree  $t_1$ , and make the root of  $t_1$  the new root of the full tree, and make the subtrees of  $t_1$  become subtrees of the full tree. (Note: we could have promoted the leftmost subtree, or any other subtree.)

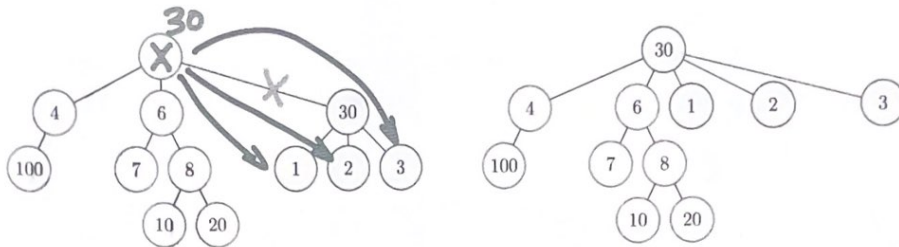


Figure 1: Before and after images of `Tree.delete_root` using Approach 1.

Implement `Tree.delete_root` using this approach.

```
class Tree:
    def delete_root(self) -> None:
        """Remove the root of this tree. Precondition: not self.is_empty()"""
```

# one node tree  
if `self._subtree == []`:  
    `self._root = None`

else:

# Find right most subtree  
    `right_tree = self._subtree[-1].pop()`  
    `self._root.val = right_tree._root.val`  
    for `sub_tree` in `right_tree._subtree`:  
        `self._subtree.append(sub_tree)`

### 3. Approach 2: Replace the root with a leaf

Idea: to delete the root, find the leftmost leaf of the tree, and make the leaf value the new root value. No other values in the tree should move. (Note: we could have replaced the root with the value in rightmost leaf, or any other leaf.)

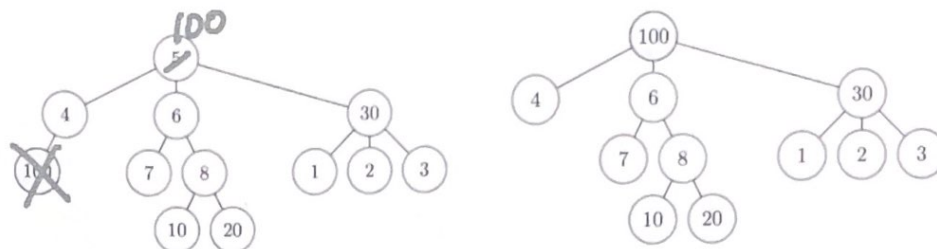


Figure 2: Before and after images of `Tree.delete_root` using Approach 2.

Implement `Tree.delete_root` using this approach. We recommend defining an additional helper method to recursively remove and return the leftmost leaf in a tree.

class Tree:

def delete\_root(self) -> None:

"""Remove the root of this tree. Precondition: not self.is\_empty()"""

self.\_find\_leftmost\_leaf()

val = subtree.\_subtree.pop(0)

self.\_root = val

① find leftmost val

② its val

③ replace leftmost leaf

④ assign val to root

! keep track of parent

if not self.\_subtree:  
self.\_root = None

Re:

self.\_root =  
self.\_find(self)

def \_find\_leftmost\_leaf(self) -> int: (self, tree)

# one more / base case

if self.\_subtree == []:  
return self.\_root

else:  
leftmost\_child = self.\_subtree.pop(0)

if leftmost\_child.\_subtree == []:

tree, val = leftmost\_child.\_find\_leftmost(self)

return leftmost\_child, val

if not tree.\_subtree:  
val = self.\_root  
self.\_root = None  
return val

is\_empty():  
if tree.\_subtree[0].is\_empty():  
tree.\_subtree.pop(0) (remove leaf)

return self.\_find(self, tree.\_subtree[0])