Line-Following Differential Drive Robot with PID Controller
(with Simulation using ROS 2 and Gazebo)

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1. Background

In my year 3 undergraduate studies, I have taken a course called control theory. This course really ignites my passion to control system and I realized that our daily lives are full of control systems. Since my interest is in robotics. Hence, I have decided to enhance my previous robotics project by utilizing the knowledge that I have learned from control theory class.

I have developed a line-following robot car for my course project during my year 2 undergraduate studies, but it was not stable then. Now, I am going to add on a PID controller for this robot, which can reduce the error during runtime to stabilize the robot so that it can run in a more stable condition with higher speed.

At this stage, I haven't implemented my upgraded robot to the hardware. However, thank to the wonderful robotics tools called ROS 2 and Gazebo as they allow robotics software engineers to simulate our robots in a virtual world that has accounted the Physics of the real world including the inertia and friction.

Please refer to the next section for the comparison of the before and after.

2. Project Demonstration

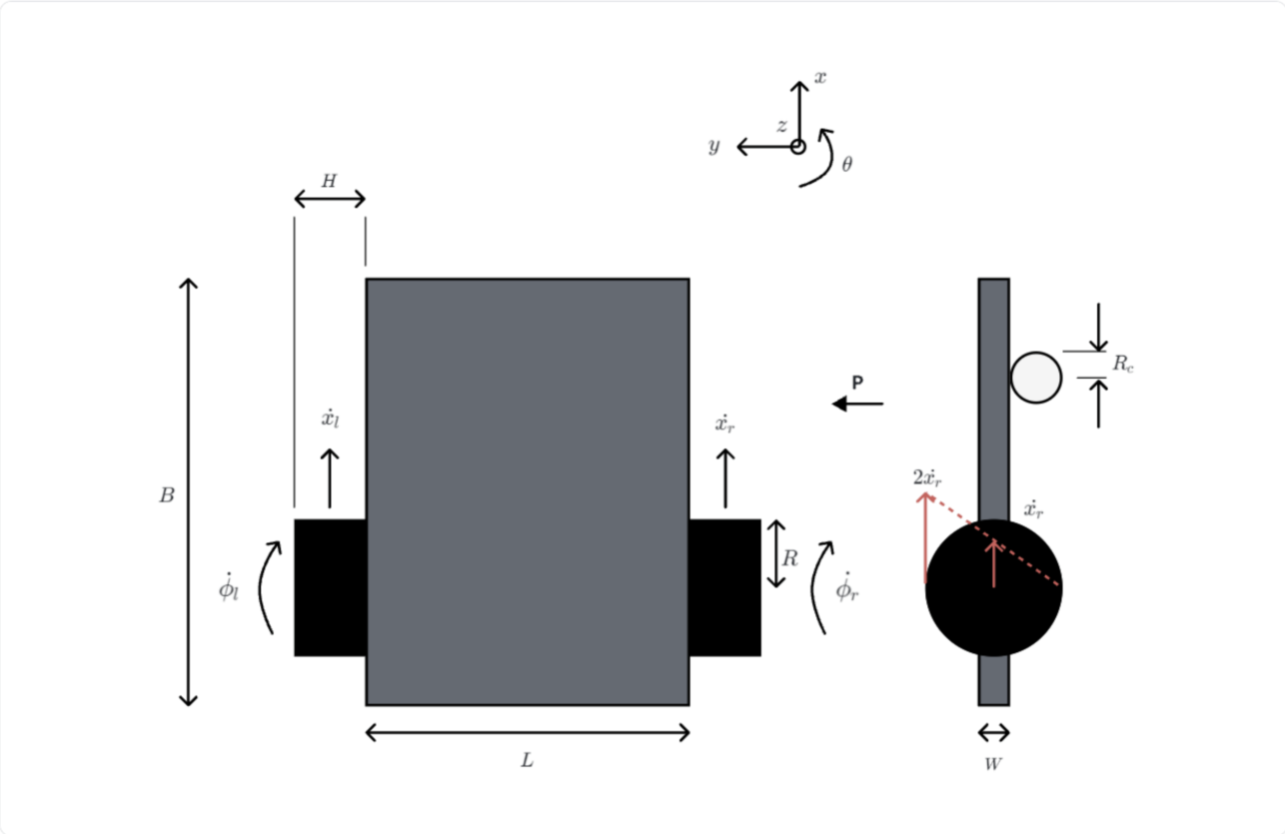
Without PID controller:

https://youtu.be/v8UCGJjRuuA?si=I2rcmbpK7iDtBp1M

With PID controller (Gazebo Simulation):

https://youtu.be/cY7xIWWwvhk

3. Kinematics Analysis



Consider right wheel as a system,

$$\begin{aligned} v &= r\dot{\phi} \\ \dot{x}_r &= R\dot{\phi}_r \\ \dot{\phi}_r &= \frac{\dot{x}_r}{R} \end{aligned} \tag{1}$$

Similarly, for left wheel,

$$\dot{\phi}_l = \frac{\dot{x}_l}{R} \tag{2}$$

Consider the entire robot as a system,

$$\begin{aligned} v &= r\dot{\theta} \\ \dot{x}_r - \dot{x}_l &= \frac{L}{2}\dot{\theta} \\ \dot{\theta} &= \frac{2}{L}(\dot{x}_r - \dot{x}_l) \end{aligned} \tag{3}$$

$$\dot{x} = \frac{\dot{x}_r + \dot{x}_r}{2} \tag{4}$$

Solve (1), (3), (4),

From (4):

$$\dot{x_r} + \dot{x}_l = 2\dot{x} \tag{4'}$$

(3) + (4'):

$$2\dot{x}_r = 2\dot{x} + \frac{L}{2}\dot{\theta} \tag{5}$$

Sub (5) into (1),

$$\dot{\phi}_r = \frac{2\dot{x} + \frac{L}{2}\dot{\theta}}{2R}$$

$$\therefore \dot{\phi}_r = \frac{1}{R}\dot{x} + \frac{L}{4R}\dot{\theta} \tag{6}$$

$$\therefore \ddot{\phi}_r = \frac{1}{R}\ddot{x} + \frac{L}{4R}\ddot{\theta} \tag{7}$$

Solve (2), (3), (4),

From (4):

$$\dot{x_r} + \dot{x}_l = 2\dot{x} \tag{4''}$$

(3) + (4″):

$$2\dot{x}_l = 2\dot{x} + \frac{L}{2}\dot{\theta} \tag{8}$$

Sub (7) into (2),

$$\dot{\phi}_l = \frac{2\dot{x} + \frac{L}{2}\dot{\theta}}{2R}$$

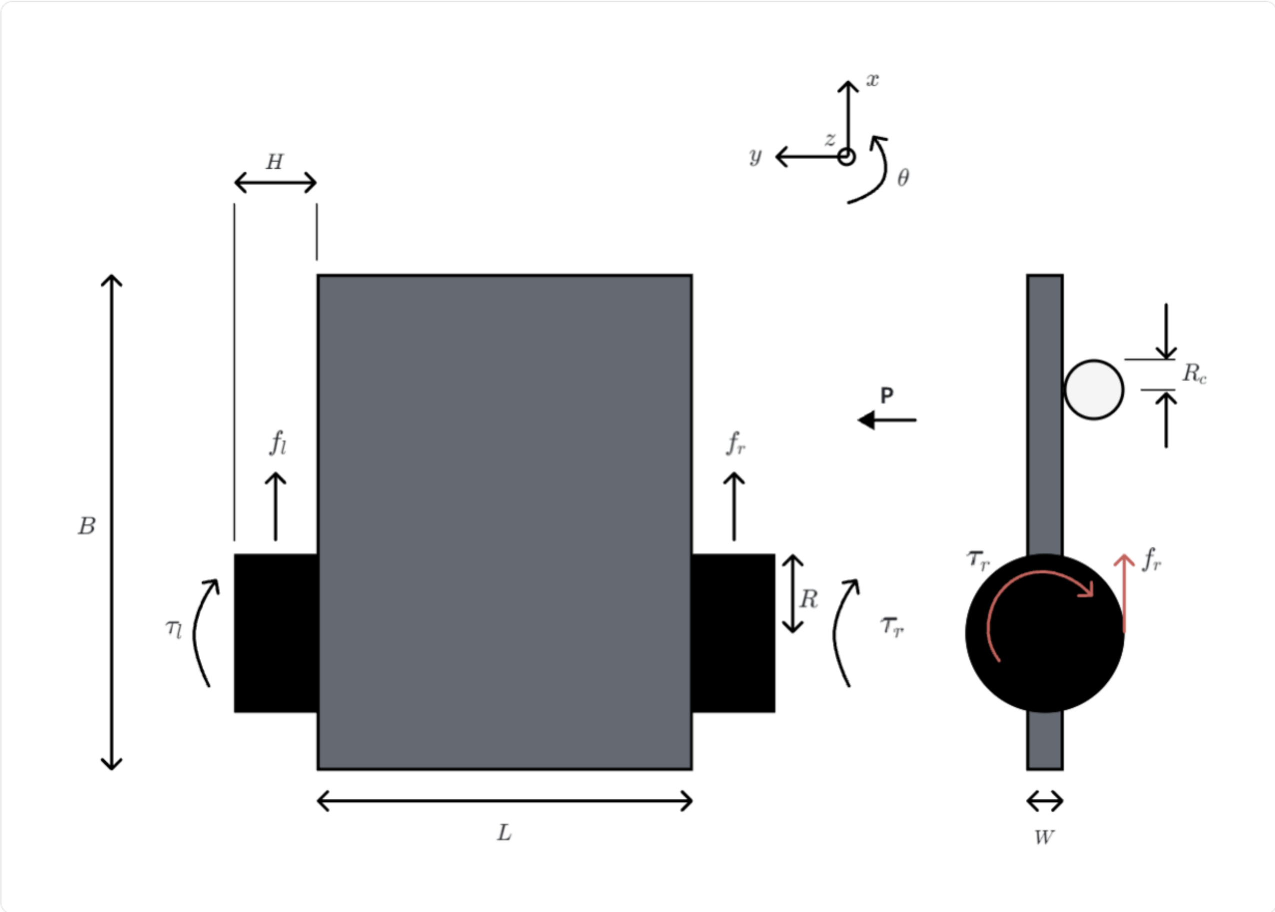
$$\therefore \dot{\phi}_l = \frac{1}{R}\dot{x} + \frac{L}{4R}\dot{\theta} \tag{9}$$

$$\therefore \ddot{\phi}_l = \frac{1}{R}\ddot{x} + \frac{L}{4R}\ddot{\theta} \tag{10}$$

$$\therefore \begin{Bmatrix} \dot{\phi}_r \\ \dot{\phi}_l \end{Bmatrix} = \begin{bmatrix} \frac{1}{R} & \frac{1}{R} \\ \frac{L}{4R} & -\frac{L}{4R} \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{\theta} \end{Bmatrix} \tag{11}$$

$$\therefore \begin{Bmatrix} \ddot{\phi}_r \\ \ddot{\phi}_l \end{Bmatrix} = \begin{bmatrix} \frac{1}{R} & \frac{1}{R} \\ \frac{L}{4R} & -\frac{L}{4R} \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} \tag{12}$$

4. Dynamics Analysis



Consider right wheel as a system,

$$\begin{aligned} \Sigma\vec{\tau} &= J\vec{\alpha} \\ \tau_r - f_r R &= J\ddot{\phi}_r \\ \frac{\tau_r}{R} - f_r &= \frac{J_w}{R}\ddot{\phi}_r \end{aligned} \tag{13}$$

Similarly, for left wheel,

$$\begin{aligned} \tau_l - f_l R &= J\ddot{\phi}_l \\ \frac{\tau_l}{R} - f_l &= \frac{J_w}{R}\ddot{\phi}_l \end{aligned} \tag{14}$$

Consider the entire robot as a system,

$$\begin{aligned} \Sigma\vec{\tau} &= J\vec{\alpha} \\ (f_r - f_r)\frac{L}{2} &= J\ddot{\theta} \\ f_r - f_r &= \frac{2J}{L}\ddot{\theta} \end{aligned} \tag{15}$$

$$\begin{aligned} \Sigma\vec{F} &= m\vec{a} \\ f_r + f_r &= m\ddot{x} \end{aligned} \tag{16}$$

Solve (13), (14), (16) with (7), (10)

$$\frac{\tau_r}{R}+\frac{\tau_l}{R}=\frac{J_w}{R}\left(\ddot{\phi}_r+\ddot{\phi}_l\right)+m\ddot{x}$$

$$\frac{1}{R}(\tau_r+\tau_l)=\frac{J_w}{R}\left(\frac{2}{R}\ddot{x}\right)+m\ddot{x}$$

$$\frac{1}{R}(\tau_r+\tau_l)=\left(\frac{2J_w}{R^2}+m\right)\ddot{x} \tag{17}$$

Solve (13), (14), (15) with (7), (10)

$$\frac{\tau_r}{R}-\frac{\tau_l}{R}=\frac{J_w}{R}\left(\ddot{\phi}_r-\ddot{\phi}_l\right)+\frac{2J}{L}\ddot{\theta}$$

$$\frac{1}{R}(\tau_r-\tau_l)=\frac{J_w}{R}\left(\frac{L}{2R}\ddot{\theta}\right)+\frac{2J}{L}\ddot{\theta}$$

$$\frac{1}{R}(\tau_r-\tau_l)=\left(\frac{J_wL}{2R^2}+\frac{2J}{L}\right)\ddot{\theta}$$

$$\tau_r-\tau_l=\left(\frac{J_wL}{2R}+\frac{2JR}{L}\right)\ddot{\theta} \tag{18}$$

$$\therefore \begin{bmatrix}\frac{1}{R} & \frac{1}{R}\\ 1 & -1\end{bmatrix}\begin{Bmatrix}\tau_r\\ \tau_l\end{Bmatrix}=\begin{bmatrix}\frac{2J_w}{R^2}+m & 0\\ 0 & \frac{J_wL}{2R}+\frac{2JR}{L}\end{bmatrix}\begin{Bmatrix}\ddot{x}\\ \ddot{\theta}\end{Bmatrix} \tag{19}$$

$$\therefore \begin{Bmatrix}f_{eq}\\ \tau_{eq}\end{Bmatrix}=\begin{bmatrix}\frac{2J_w}{R^2}+m & 0\\ 0 & \frac{J_wL}{2R}+\frac{2JR}{L}\end{bmatrix}\begin{Bmatrix}\ddot{x}\\ \ddot{\theta}\end{Bmatrix} \tag{20}$$

From (19), (20):

$$f_{eq}=\left(\frac{2J_w}{R^2}+m\right)\ddot{x}$$

By Laplace Transform:

$$F_{eq}=\left(\frac{2J_w}{R^2}+m\right)s^2X$$

$$X=\frac{1}{\left(\frac{2J_w}{R^2}+m\right)s^2}F_{eq} \tag{21}$$

$$\tau_{eq}=\left(\frac{J_wL}{2R}+\frac{2JR}{L}\right)\ddot{\theta}$$

By Laplace Transform:

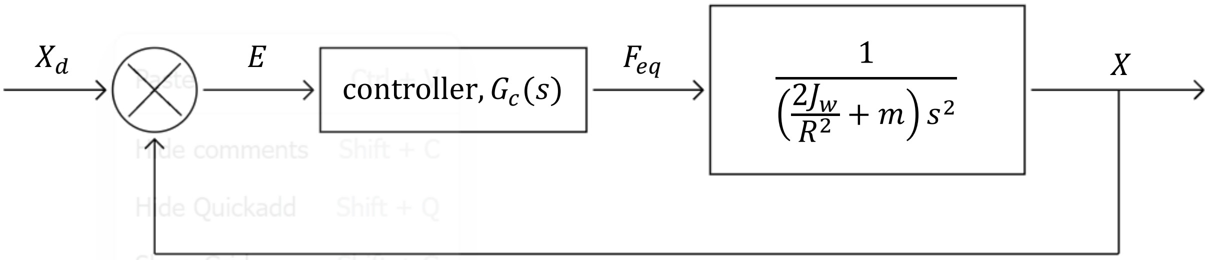
$$T_{eq}=\left(\frac{J_wL}{2R}+\frac{2JR}{L}\right)s^2\Theta$$

$$\Theta=\frac{1}{\left(\frac{J_wL}{2R}+\frac{2JR}{L}\right)s^2}T_{eq} \tag{22}$$

5. Block Diagram

From (21):

Subsystem 1:



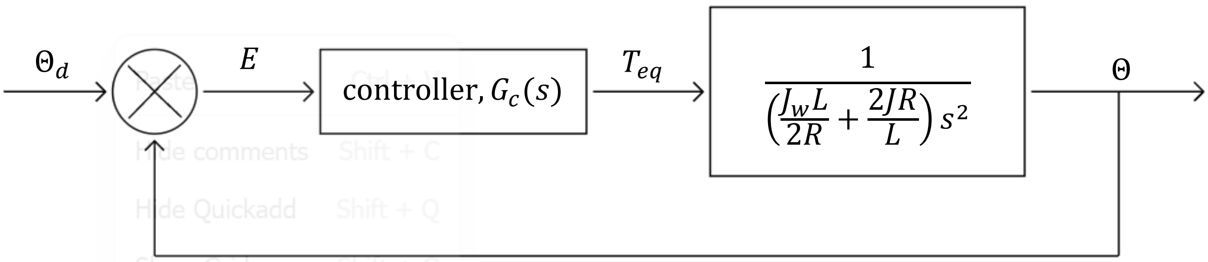
$$f_{eq} = K_P e + K_D \frac{de}{dt}$$

By Laplace Transform,

$$F_{eq} = K_P E + K_D sE$$
$$= E(K_P + K_D s)$$
$$\frac{F_{eq}}{E} = K_P + K_D s$$
$$\therefore G_c(s) = K_P + K_D s \tag{23}$$

From (22):

Subsystem 2:



$$\tau_{eq} = K_P e + K_D \frac{de}{dt}$$

By Laplace Transform,

$$T_{eq} = K_P E + K_D sE$$
$$= E(K_P + K_D s)$$
$$\frac{T_{eq}}{E} = K_P + K_D s$$
$$\therefore G_c(s) = K_P + K_D s \tag{23'}$$

6. Design of PID Controller

Assumptions:

2% settling time, $t_s = 0.5s$

maximum overshoot, $m = 10\%$

The differential drive robot can be treated as a second order system, let the transfer function of this system be $G(s)$.

$$\therefore G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2} \tag{24}$$

ζ and ω_n in equation (24) are damping ratio and natural frequency of the differential drive robot, respectively. They can be found using equations (25) and (26) below.

$$m = e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} = 0.1 \tag{25}$$

$$t_s = \frac{4}{\zeta\omega_n} = 0.5 \tag{26}$$

Having this information, we can move on to locate the desired poles for this system using the characteristics equation.

$$s^2 + 2\zeta\omega_n + \omega_n^2 = 0 \tag{27}$$

Solve equation (27), and the roots will be the desired poles for this system.

$$s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} \tag{28}$$

Construct the closed-loop transfer function of this system from the block diagrams and substitute the desired poles (i.e., equation (28)) into equations (29) and (30).

For subsystem 1:

$$G_{CL} = \frac{\dfrac{K_P + K_D s}{\left(\frac{2J_w}{R^2} + m\right)s^2}}{1 + \dfrac{K_P + K_D s}{\left(\frac{2J_w}{R^2} + m\right)s^2}} \tag{29}$$

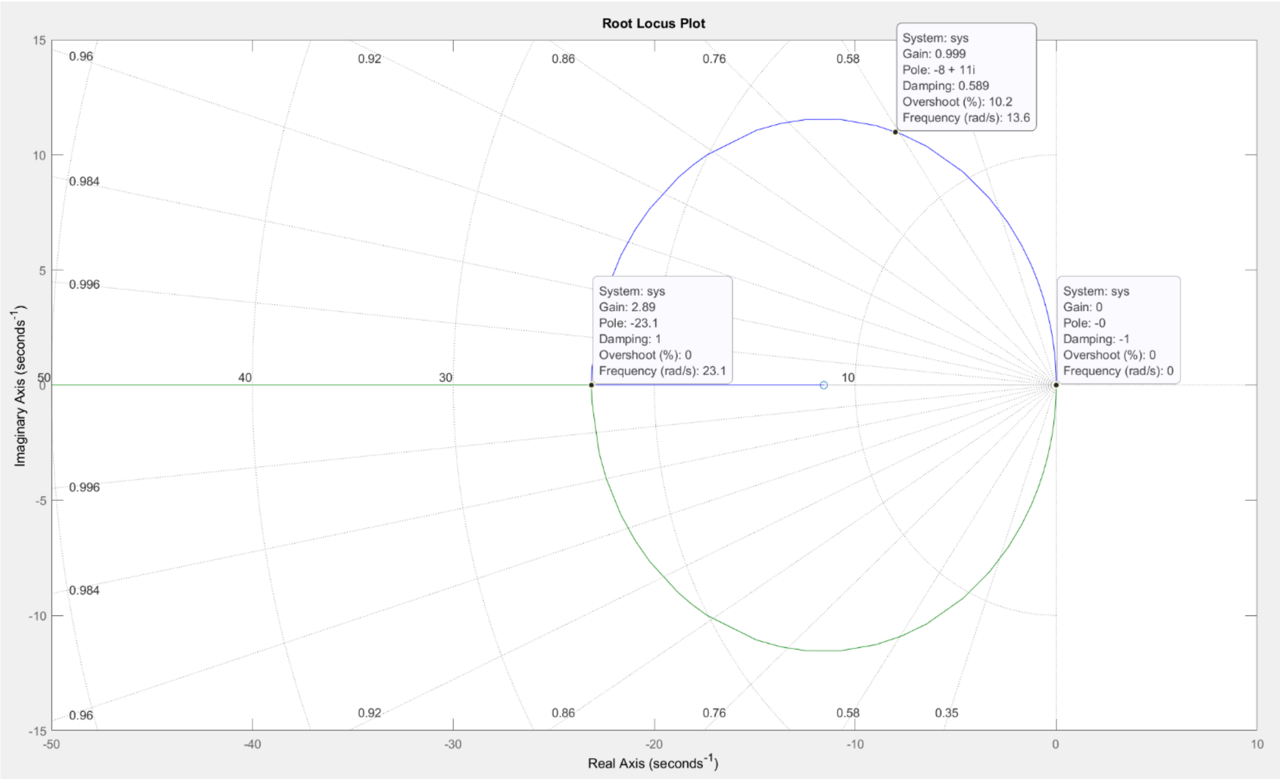
For subsystem 2:

$$G_{CL} = \frac{\dfrac{K_P + K_D s}{\left(\frac{J_w L}{2R} + \frac{2JR}{L}\right)s^2}}{1 + \dfrac{K_P + K_D s}{\left(\frac{J_w L}{2R} + \frac{2JR}{L}\right)s^2}} \tag{30}$$

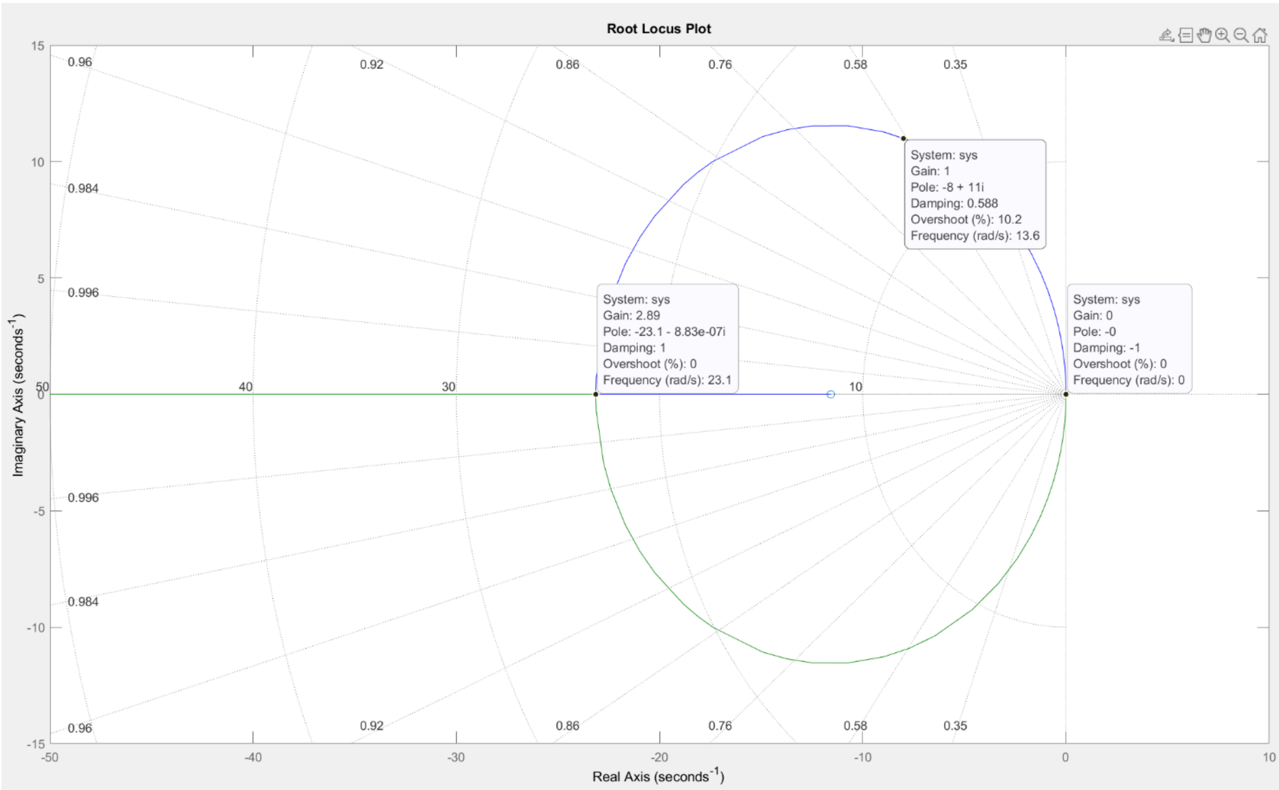
Now, all the constants in equations (29) and (30) should have been found. Solve the characteristics equations of equations (29) and (30). We can visualize the root locus of the system using MATLAB.

7. MATLAB Visualization for Root Locus

Subsystem 1:



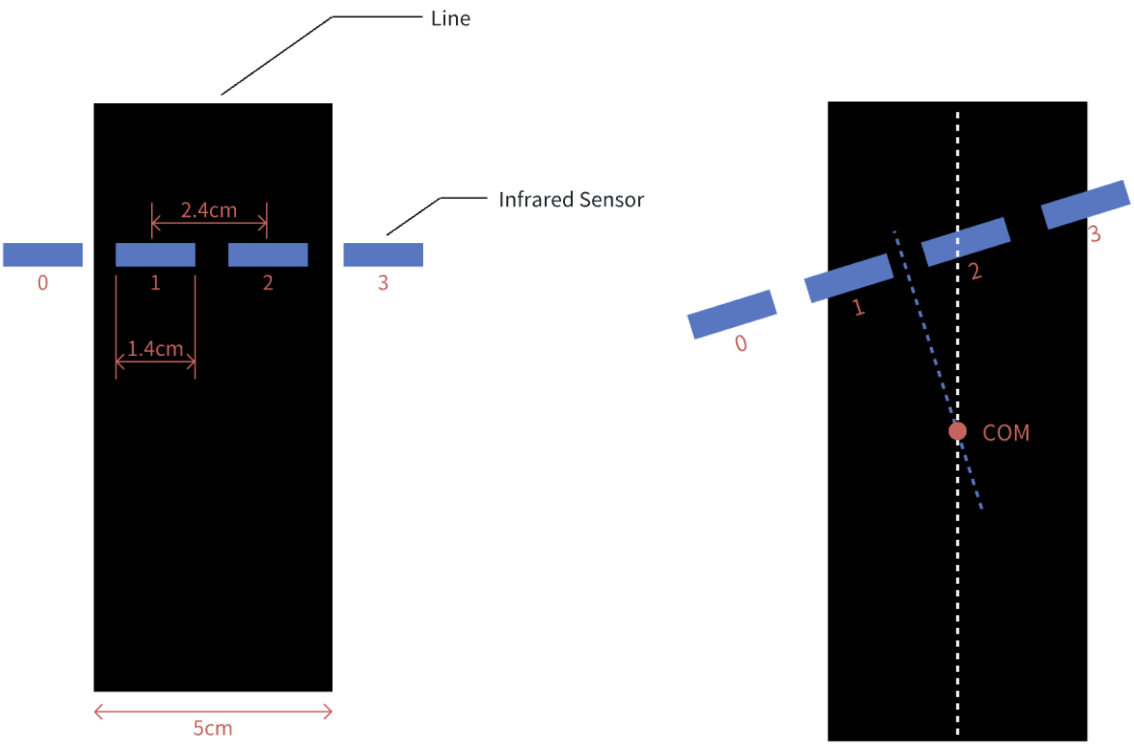
Subsystem 2:



The trend of the root locus moving to the left side of the S plane proves that the system is stable.

8. Implementation

The robot has four infrared sensors to interact with the surrounding in order to know how much it is deviating from the line.



Robot centered in the middle of the line:

$$\begin{aligned} sensor\ position &= 3 - \frac{0 \times 1023 + 1 \times 0 + 2 \times 0 + 3 \times 1023}{1023 + 0 + 0 + 1023} \\ &= 1.5 \ \ \text{(sensor 1.5 is above the middle line)} \end{aligned}$$

Robot deviates to the left:

$$\begin{aligned} sensor\ position &= 3 - \frac{0 \times 1023 + 1 \times 100 + 2 \times 0 + 3 \times 500}{1023 + 100 + 0 + 500} \\ &= 2 \ \ \text{(sensor 2 is above the middle line)} \end{aligned}$$

Since we have decoupled the system into two independent single-input-single-output subsystems, therefore we can view the rotation independently from the translation. Hence, the center of mass (COM) of the robot can be viewed as being fixed on the middle line.

$$\begin{aligned} \text{error chord length, y} &= (1.5 - 2) \times \frac{2.4}{2} \\ &= -0.6cm \end{aligned}$$

$$\text{angle of deviation, } \theta_d - \theta = \tan^{-1}\frac{y}{0.5L}$$