[Brief Review*]

# Towards a Reconstruction of General Bulk Metrics[1]

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A new approach for the bulk reconstruction, the method using the light-cone cuts, is introduced. The past (future) light-cone cut of the bulk point $p$ is defined as

$$C^-(p) = \partial J^-(p) \cap \partial M \ (C^+(p) = \partial J^+(p) \cap \partial M), \tag{1}$$

where $M$ is the spacetime assumed to be asymptotically $\mathrm{AdS}_n$ with some natural conditions for mathematical reasons. From now on, we focus on past cuts, but the same discussion can be applied to future cuts.

First, using the "bulk-point singularity" [2], we can get the past cuts of the points in $J^+(\partial M) \cap J^-(\partial M)$. The divergence of a boundary to boundary correlator, whose fields are massless, is caused by that the interaction vertex is a bulk point which is null-related to all points in the correlator, under certain conditions. The divergence of this class is called bulk-point singularity. Let us consider $n$ boundary points $x_1, \cdots, x_n$ which are spacelike separated each other, and two boundary points $z_1$ and $z_2$ in the past of $x_i$. Then moving $z_1$ and $z_2$ while keeping

$$\langle \mathcal{O}(z_1)\mathcal{O}(z_2)\mathcal{O}(x_1)\cdots\mathcal{O}(x_n)\rangle \to \infty \tag{2}$$

for fixed $x_i$ traces out the past cut of the unique bulk point which is null-separated from $x_i$. Since we do not know about the bulk geometry, we cannot know whose past cut it is. In principle, however, the past cut can be labeled by some $n$ parameters related to $x_i$. Let us set the parameters $\lambda = (\lambda^1, \cdots, \lambda^n)$ and denote the cut by $C^-_\lambda$. By repeating this step, all past cuts related one-to-one to points in $J^+(\partial M) \cap J^-(\partial M)$ can be collected, then we define the vector space of $\lambda$'s, $\mathcal{M}^- := \{\lambda\}$.

Let $\lambda_1, \lambda_2, \cdots$ be vectors on $\mathcal{M}^-$, meaning that $C^-_{\lambda_1}, C^-_{\lambda_2} \cdots$ are the past cuts. In the appendix, it is shown that, if $C^-_{\lambda_1} = C^-(p)$ and $C^-_{\lambda_2} = C^-(q)$ are tangent each other precisely at one point, then $p$ and $q$ are null related. Therefore, taking a boundary point $x \in C^-_{\lambda_1}$ and collecting the past cuts tangent to $C^-_{\lambda_1}$ at $x$, the unique null geodesic from $p$ to $x$ via $q$ is obtained. Then we can identify this null geodesic to the one connecting $\lambda_1$ and other $\lambda$'s corresponding to other cuts tangent to $C^-_{\lambda_1}$ at $x$, because of the existence of the one-to-one map from $J^+(\partial M) \cap J^-(\partial M)$ to $\mathcal{M}^-$. Repeating this process for points on $C^-_{\lambda_1}$, we get null geodesics through $\lambda_1$, and they form light-cone of $\lambda_1$ in $\mathcal{M}^-$.

Once the causal structure is introduced in $\mathcal{M}^-$, the conformal metric (the metric up to conformal factors) can be determined. The null generators at $\lambda \in \mathcal{M}^-$ are obtained by looking for other past cuts tangent to $C^-_\lambda$. Let $\sigma = (\sigma^1, \cdots, \sigma^{n-2})$ be the parameter describing the points on $C^-_\lambda$ as $C^-_\lambda(\sigma) \in \partial M$. The condition which $C^-_{\lambda_2}$ is tangent to $C^-_{\lambda_1}$ is as follows:

$$\exists\sigma_1,\ \exists\sigma_2, \quad C^-_{\lambda_1}(\sigma_1) = C^-_{\lambda_2}(\sigma_2) \quad \text{and} \quad \nabla_\sigma C^-_{\lambda_1}(\sigma)\big|_{\sigma=\sigma_1} = \nabla_\sigma C^-_{\lambda_2}(\sigma)\big|_{\sigma=\sigma_2}. \tag{3}$$

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A null generator $N$ at $\lambda$ satisfies the above condition for $\lambda_1 = \lambda$ and $\lambda_2 = \lambda + \varepsilon N$, where $\varepsilon$ is any infinitesimal real variable. By solving the conditions, we can find out $n(n+1)/2$ generators at $\lambda$. Imposing the condition that the $n(n+1)/2$ null generators are null at $\lambda$ determines the metric at $\lambda$ up to conformal factors. Since $\mathcal{M}^-$ can be regarded as the copy of $J^+(\partial M) \cap J^-(\partial M)$, the reconstruction of the bulk metric has been completed now.

# References


[1] N. Engelhardt and G.T. Horowitz, *Towards a Reconstruction of General Bulk Metrics*, *Class. Quant. Grav.* **34** (2017) 015004 [1605.01070].

[2] J. Maldacena, D. Simmons-Duffin and A. Zhiboedov, *Looking for a bulk point*, *JHEP* **01** (2017) 013 [1509.03612].