Coarse-graining black holes out of equilibrium with boundary observables on time slice

Daichi Takeda (Kyoto U)
Based on arXiv:2403.07275 (JHEP 2024, 319, (2024))

May. 31, 2024
seminar @ YITP

Thermodynamics constrains gravity?

Origin of spacetime is unknown

BH is thermodynamic = macroscopic

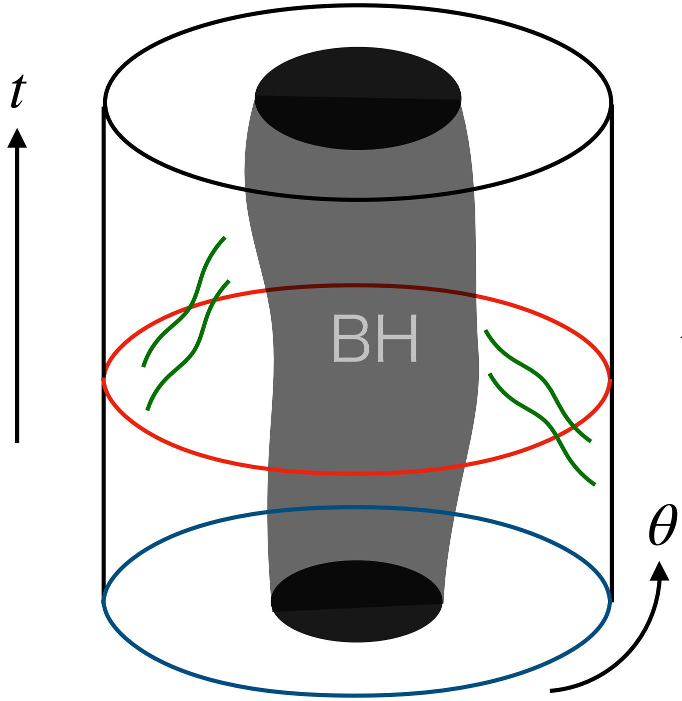
BH is statistical mechanics of QM??

Strominger-Vafa (1996)

BH thermodynamics will give macroscopic clues to QG!

A definition of entropy obeying 1st and 2nd law

$(d+1)$ dim dynamical BH



Stationary at $t=0$

mass M_t

(angular) momentum P_t

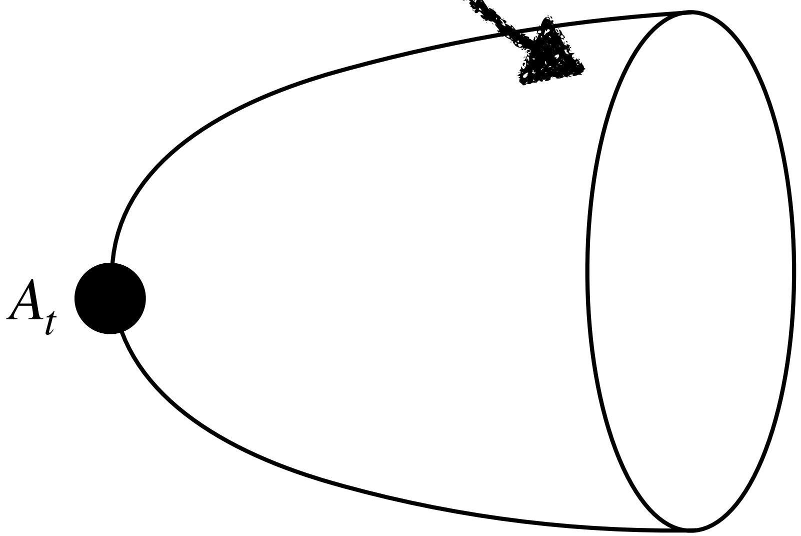
Normalizable modes $\pi_{I,t}(\theta)$

Stationary Euclid BH with same values

Coarse-grained entropy: $S_t := \dfrac{A_t}{4G}$

1st law within GR: $\dot{S}_t = \beta_t \dot{M}_t + \cdots$

2nd law by AdS/CFT: $S_t \geq S_0$



A definition of entropy obeying 1st and 2nd law

1. BH thermodynamics and problems
2. Coarse-grained entropy and 2nd law in CFT
3. Rewrite in gravity by AdS/CFT
4. 2nd law implies null energy condition
5. (Generalized) 1st law in GR

A definition of entropy obeying 1st and 2nd law

1. BH thermodynamics and problems
2. Coarse-grained entropy and 2nd law in CFT
3. Rewrite in gravity by AdS/CFT
4. 2nd law implies null energy condition
5. (Generalized) 1st law in GR

4 laws in thermodynamics

0th law: Existence of intensive variables

Temperature T, chemical potential μ, …

1st law: thermodynamic relation

$$\mathrm{d}E = T\mathrm{d}S + \mu\mathrm{d}N + \cdots$$

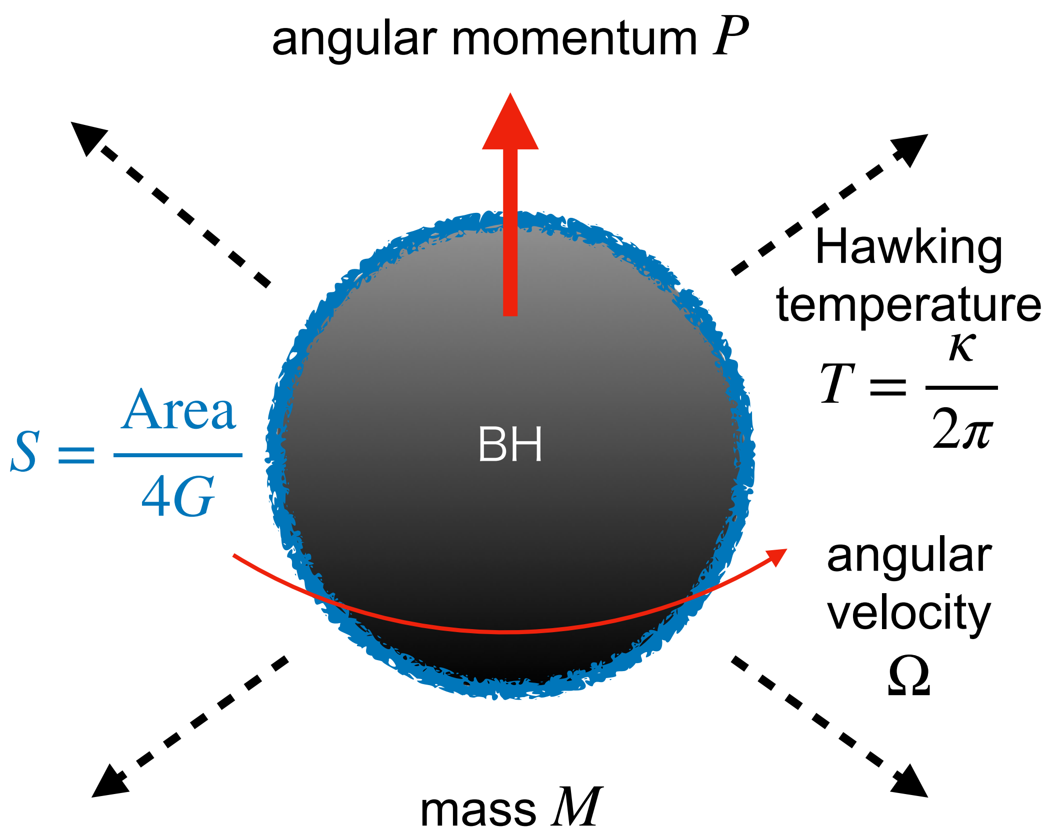
2nd law: Iff adiabatic process $X \to Y$ is possible,

$$S_X \leq S_Y$$

3rd law: Entropy vanishes at $T = 0$

Unnecessary to construct thermodynamics

BHT is almost parallel



0th law: intensive variables

$$T, \Omega, \ldots$$

1st law: thermodynamic relation

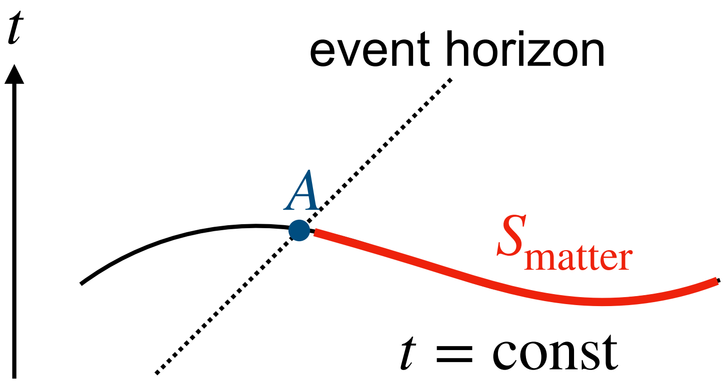
$$\mathrm{d}M = T\mathrm{d}S + \Omega \mathrm{d}P + \cdots$$

2nd law: still under debate

Hawking area law?
Generalized second law?
Other candidates?

2nd law is under debate

Generalized entropy?



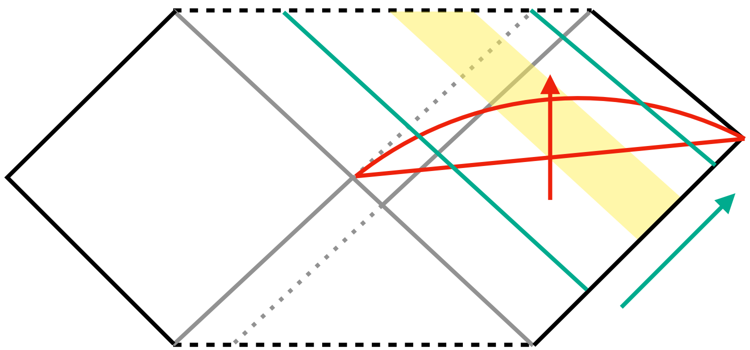
$$S_{\mathrm{gen}} := \frac{A}{4G} + S_{\mathrm{matter}}$$

Various attempts to show 2nd law:
S_{gen} increases monotonically

I think, this claim is different from thermodynamic 2nd law…

2nd law: Iff $X \to Y$ is possible, $S_X \leq S_Y$

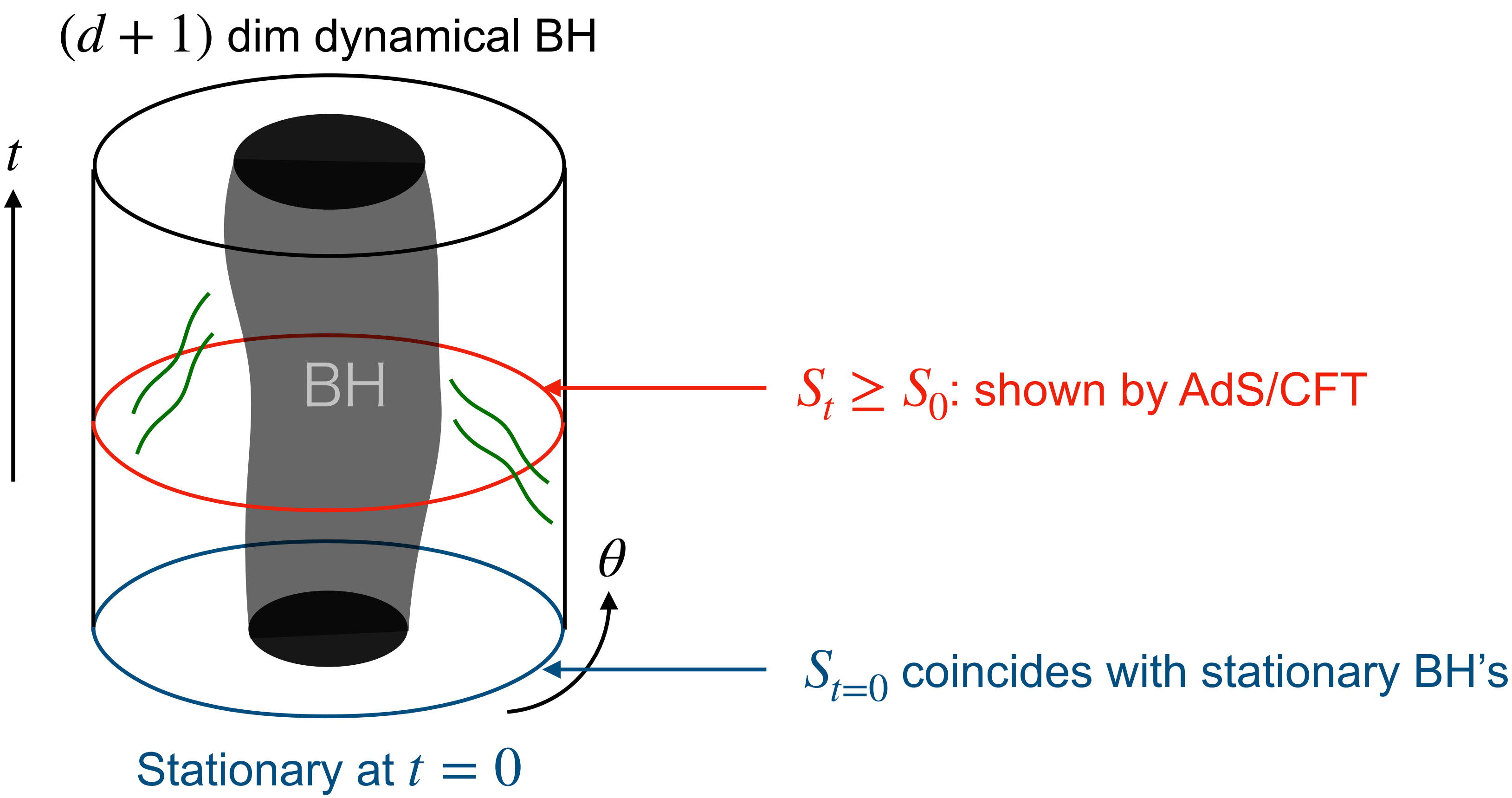
ex) Vaidya



always out of equilibrium

$S_{\mathrm{gen}} \neq S_{\mathrm{Sch}}$ for initial state

To do: Find entropy obeying 1st and 2nd law



Generalized 1st law $\dot{S}_t = \beta_t(\dot{M}_t - \Omega_t \dot{P}_t) - \int \mathrm{d}^{d-1}\theta \cdots$

local contributions
from matter fields

A definition of entropy obeying 1st and 2nd law

Coarse-graining is respecting some aspects

Canonical ensemble

Maximize $S(\rho) = -\operatorname{Tr}\rho\ln\rho$
under $\operatorname{Tr}(\rho H) = E$ and $\operatorname{Tr}\rho = 1$

$\rightarrow$ $\rho_{\text{can}} \propto e^{-\beta H}$ ($\beta = \beta(E)$: Lagrange multiplier)

$$\boxed{S_{\text{can}} = -\operatorname{Tr}\rho_{\text{can}}\ln\rho_{\text{can}}}$$

Coarse-graining is respecting some aspects

Coarse-grained state ρ_{cg}

$\{H, P_A, O_I(\theta)\}$: operator set to be respected

Maximize $S(\rho) = -\,\mathrm{Tr}\rho \ln \rho$
under $\mathrm{Tr}(\rho H) = h,\ \mathrm{Tr}(\rho P_A) = p_A,\ \mathrm{Tr}(\rho O_I(\theta)) = o_I(\theta),\ \mathrm{Tr}\rho = 1$

$$\boxed{\rho_{\text{cg}} = \frac{1}{Z} \exp\left[-\beta\left(H - \omega^A P_A - \int \mathrm{d}^{d-1}\theta\, \lambda^I(\theta) O_I(\theta)\right)\right]}$$

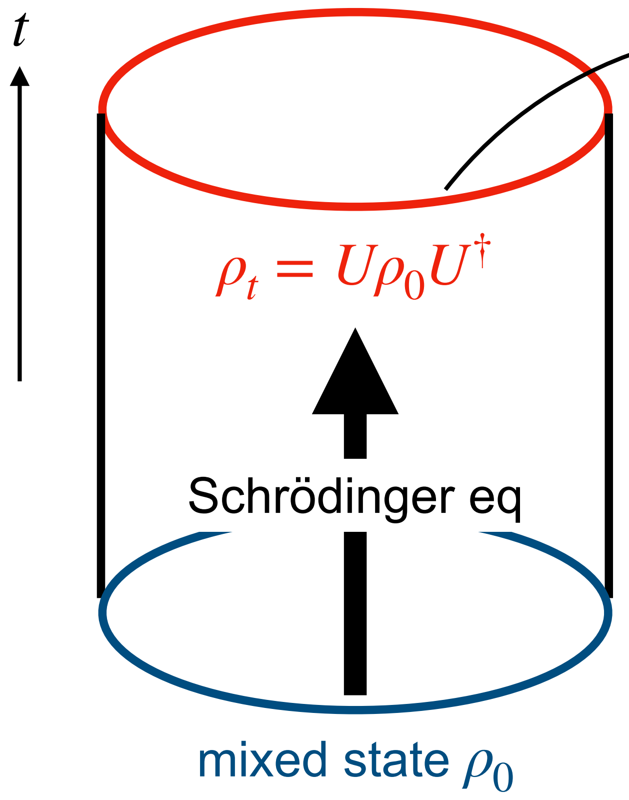
Coarse-grained entropy

$$\boxed{S := -\,\mathrm{Tr}\rho_{\text{cg}} \ln \rho_{\text{cg}}}$$

Coarse-grained entropy of time t

Coarse-graining conditions:

$$\mathrm{Tr}(\rho H) = h,\ \mathrm{Tr}(\rho P_A) = p_A,\ \mathrm{Tr}(\rho O_I(\theta)) = o_I(\theta)$$



Coarse-grained state $\rho_{\mathrm{cg},t}$ defined by

$$\boxed{h = \mathrm{Tr}(\rho_t H),\ p_A = \mathrm{Tr}(\rho_t P_A),\ o_I = \mathrm{Tr}(\rho_t O_I(\theta))}$$

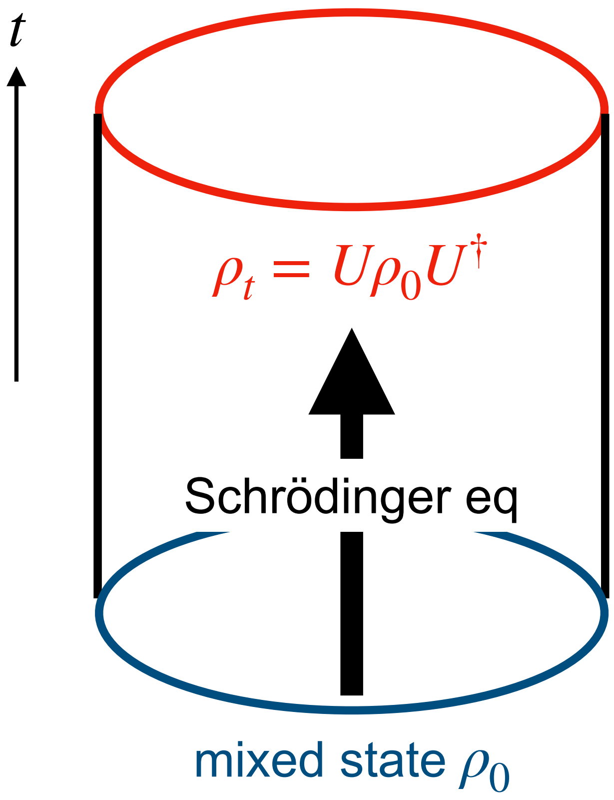
Coarse-grained entropy S_t

$$\boxed{S_t := -\mathrm{Tr}\rho_{\mathrm{cg},t} \ln \rho_{\mathrm{cg},t}}$$

2nd law from relative entropy

Positivity of relative entropy

$$\mathrm{Tr}\,\rho_t \left(\ln\rho_t - \ln\rho_{\mathrm{cg},t}\right) \geq 0 \quad \Longleftrightarrow \quad \text{2nd law} \quad S_t \geq S_0$$



$$\rho_t$$

$$H(t) = H - \int \mathrm{d}^{d-1}\theta\, j^I(t,\theta) O_I(\theta)$$

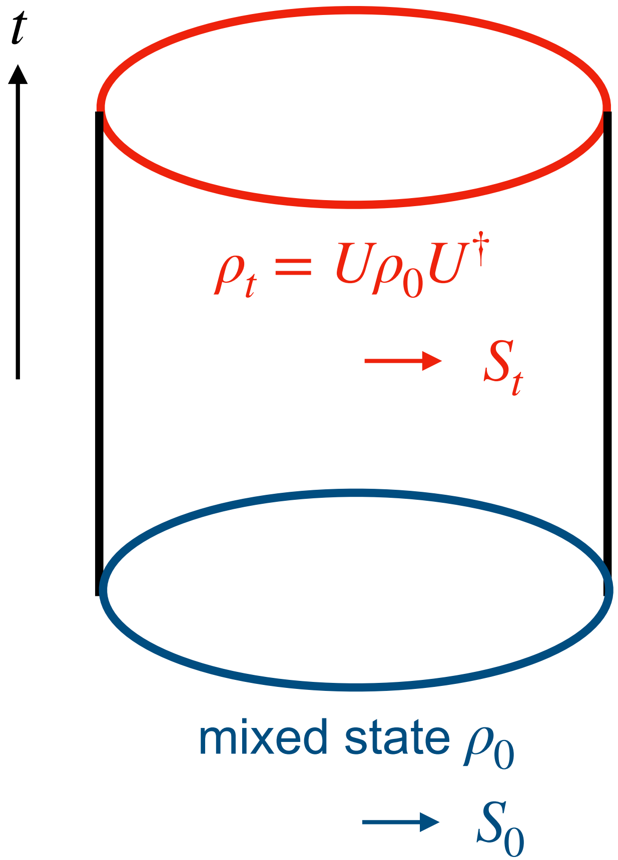
$$\rho_0 \propto \exp\left[-\beta_0\left(H - \omega_0^A P_A - \int \mathrm{d}^{d-1}\theta\, \lambda_0^I(\theta) O_I(\theta)\right)\right]$$

$$\rho_{\mathrm{cg},0} = \rho_0$$

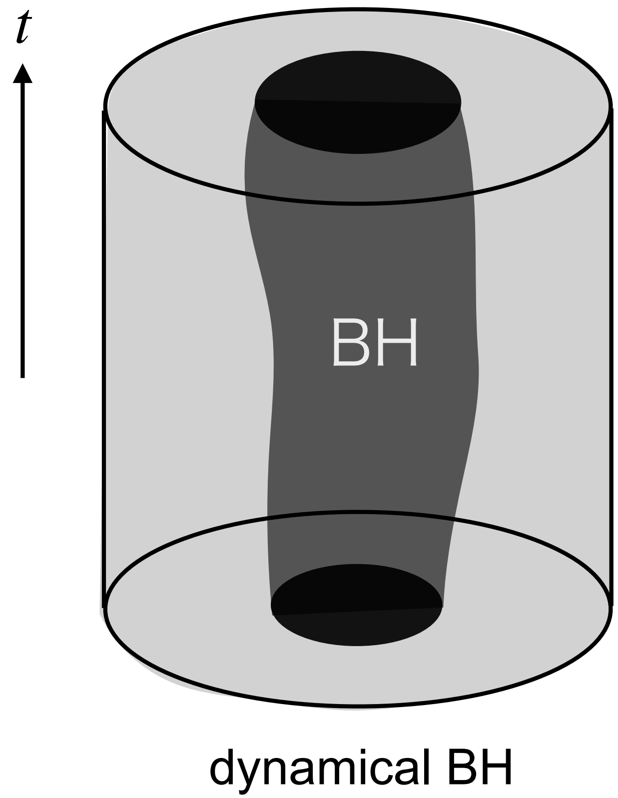
A definition of entropy obeying 1st and 2nd law

1. BH thermodynamics and problems
2. Coarse-grained entropy and 2nd law in CFT
3. Rewrite in gravity by AdS/CFT
4. 2nd law implies null energy condition
5. (Generalized) 1st law in GR

AdS/CFT constrains BH dynamics



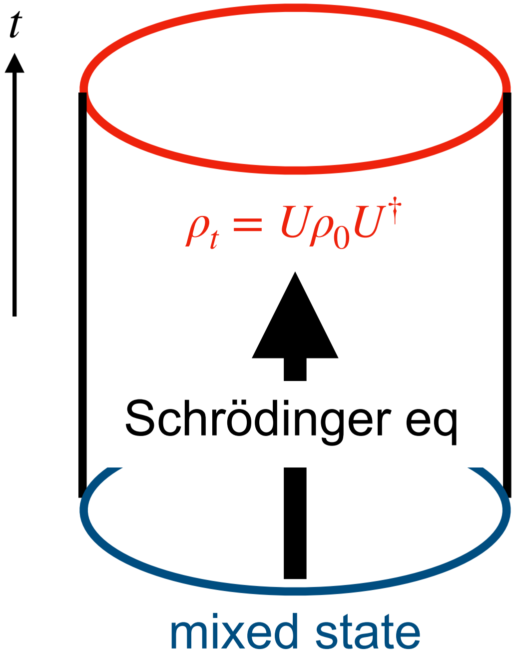
AdS/CFT
=



AdS/CFT

$S_0 \leq S_t$ $\Longrightarrow$ Constraint on BH spacetime

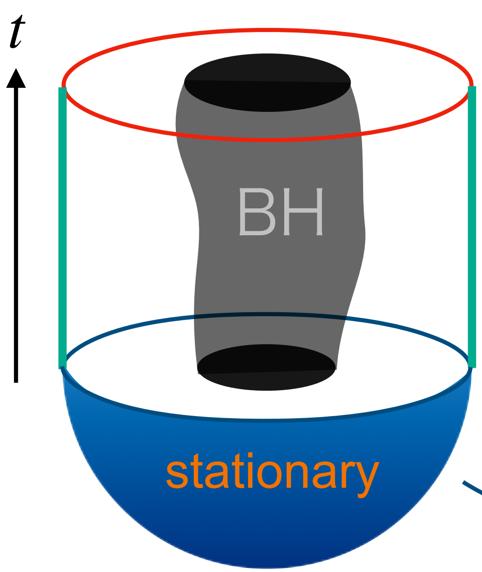
Setup: equilibrium to non-equilibrium



ρ_t

$$H(t) = H - \int \mathrm{d}^{d-1}\theta\, j^I(t,\theta) O_I(\theta)$$

$$\rho_0 \propto \exp\left[-\beta_0 \left(H - \omega_0^A P_A - \int \mathrm{d}^{d-1}\theta\, \lambda_0^I(\theta) O_I(\theta)\right)\right]$$

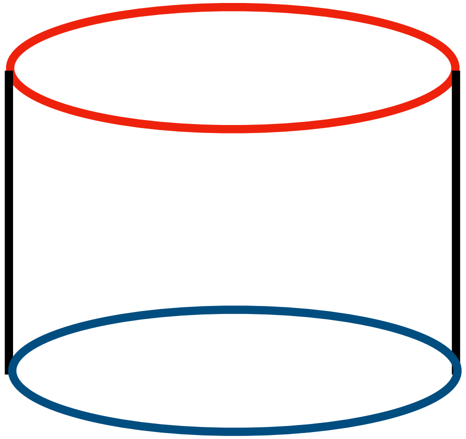


EOM

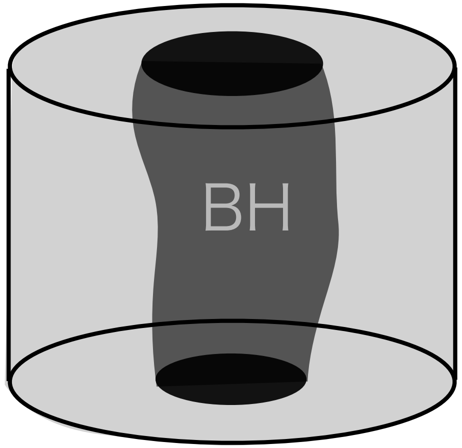
B.C. $\Phi(x) \sim r^{\Delta_I - d} j^I(t,\theta)$

Prepare initial config.
by continuing Euclid BH

GKPW formula and 1pt functions



$=$



$$\left\langle e^{i\int \mathrm{d}^{d-1}\theta\, j^I(t,\theta) O_I(\theta)} \right\rangle = e^{iI_{\rm grav}[\Phi]}$$ with $\Phi(x) \sim r^{\Delta_I - d} j^I(t,\theta)$

$$\mathrm{Tr}(\rho_t O_I(\theta)) = \frac{\delta}{\delta j^I(t,\theta)} I_{\rm grav}[\Phi] =: \pi_{I,t}(\theta)$$

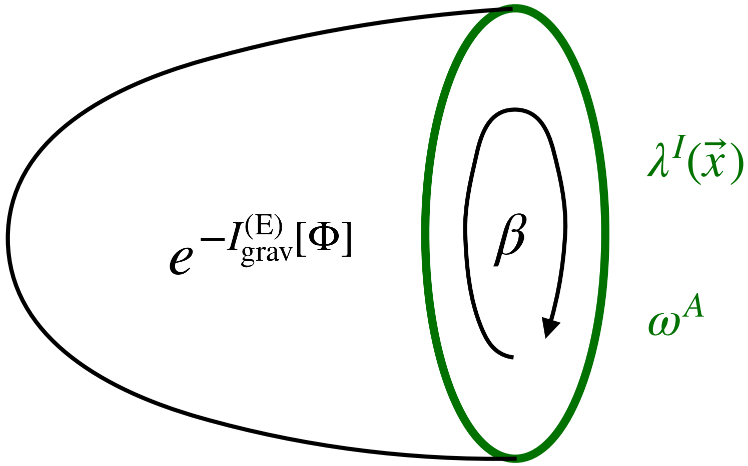
$\mathrm{Tr}(\rho_t H)$, $\mathrm{Tr}(\rho_t P_A)$ $=$ ADM mass, momenta

Computed from Brown-York tensor

Coarse-grained state = Euclid BH

$Z[\beta, \Omega, \lambda] =$

$$\mathrm{Tr}\,\exp\left[-\beta\left(H - \omega^A P_A - \int \mathrm{d}^{d-1}\theta\,\lambda^I(\theta) O_I(\theta)\right)\right] \overset{\text{AdS/CFT}}{=}$$



$$\mathrm{Tr}(\rho_{\mathrm{cg}} O_I(\theta)) = -\beta^{-1}\frac{\delta}{\delta\lambda^I(\theta)} I^{(\mathrm{E})}_{\mathrm{grav}}[\Phi] =: \pi^{(\mathrm{E})}_I(\theta)$$

$\mathrm{Tr}(\rho_{\mathrm{cg}} H),\ \mathrm{Tr}(\rho_{\mathrm{cg}} P_A)$ = ADM mass, momenta

At each time t, coarse-graining conditions become

$\boxed{\pi_{I,t}(\theta) = \pi^{(\mathrm{E})}_I(\theta)\text{, and matching of mass and momenta}}$

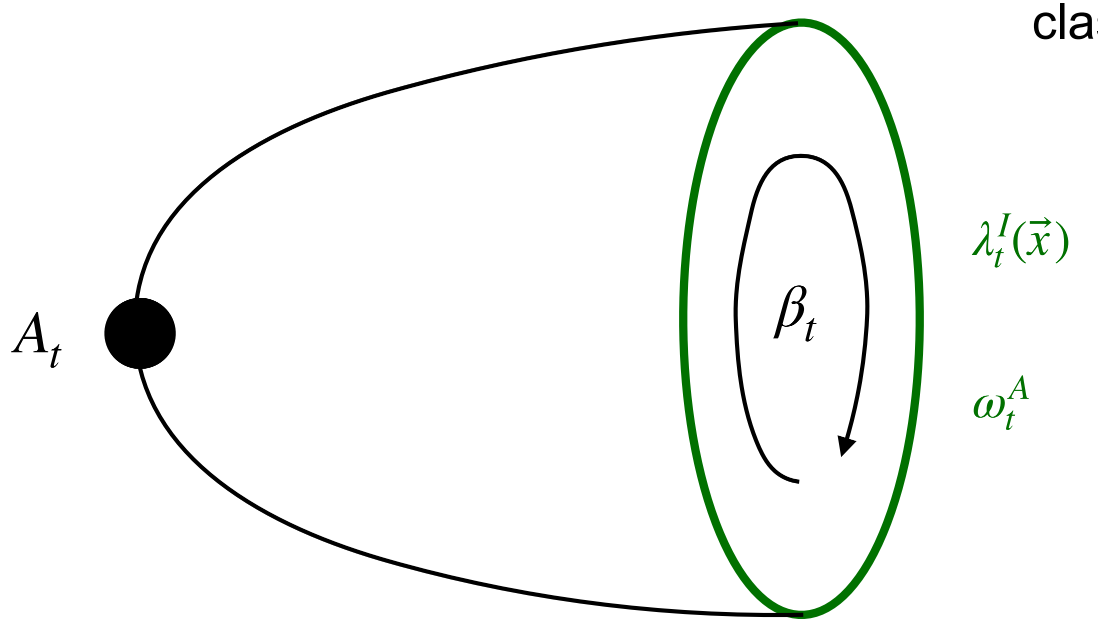
Solution: $(\beta, \omega, \lambda) = (\beta_t, \omega_t, \lambda_t)$

Entropy is the cigar tip area

$$S_t = -\mathrm{Tr}\rho_{\mathrm{cg},t} \ln \rho_{\mathrm{cg},t}$$

$$= -\beta^2 \frac{\partial}{\partial \beta}(\beta^{-1} \ln Z[\beta, \omega, \lambda])\Big|_{(\beta,\omega,\lambda)\to(\beta_t,\omega_t,\lambda_t)} = \frac{A_t}{4G}$$

classical Einstein



$A_t \geq A_0$ via AdS/CFT

Positivity of relative entropy

$$\mathrm{Tr}\,\rho_t\left(\ln\rho_t - \ln\rho_{\mathrm{cg},t}\right) \geq 0 \quad \Longleftrightarrow \quad \text{2nd law} \quad S_t \geq S_0$$

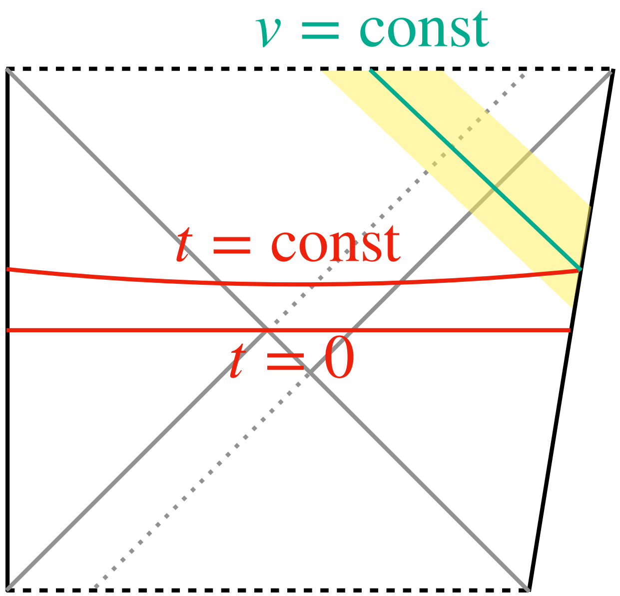
$$\Downarrow$$

$$A_t \geq A_0$$

A definition of entropy obeying 1st and 2nd law

1. BH thermodynamics and problems
2. Coarse-grained entropy and 2nd law in CFT
3. Rewrite in gravity by AdS/CFT
4. 2nd law implies null energy condition
5. (Generalized) 1st law in GR

Sch-AdS



$$ds^2 = -f(v,r)dv^2 + \frac{dr^2}{f(v,r)} + r^2 d\Omega^2,$$

$$f(v,r) = 1 + \frac{r^2}{L^2} - \frac{2\mu(v)}{r^{d-2}}$$

on boundary $v = t$

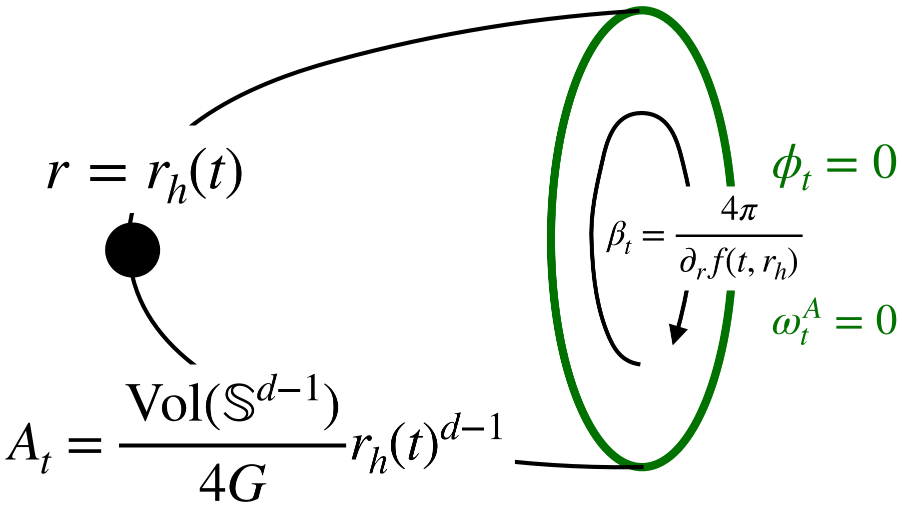
The values to be respected: mass M_t, angular momenta $P_{A,t}$, charge Q_t

$$M_t = \frac{d-1}{8\pi G}\mathrm{Vol}(\mathbb{S}^{d-1}) \times \mu(t) + (\mu - \text{indep.})$$

$$P_{A,t} = 0$$

$$Q_t = 0$$

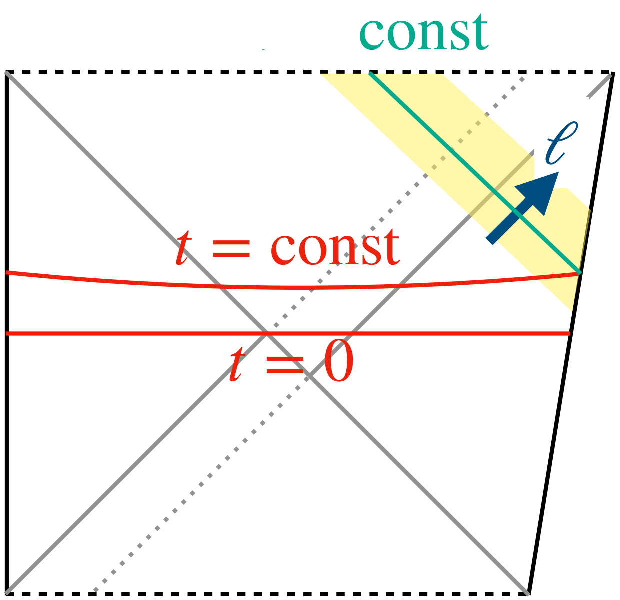
Sch-AdS



Euclid BH having same $M_t, P_{A,t}, Q_t$

$$ds^2 = f(v, r)d\tau^2 + \frac{dr^2}{f(v, r)} + r^2 d\Omega^2$$

$$f(v, r) = 1 + \frac{r^2}{L^2} - \frac{2\mu(v)}{r^{d-2}}$$



AdS/CFT says

$A_t \geq A_0$

This does not hold for all $\mu(v)$

$\boxed{\text{But it holds if } T_{\ell\ell} \geq 0}$

The same thing holds for other cases

AdS/CFT says

$A_t \geq A_0$

This does not hold always

$\boxed{\text{But it holds if } T_{\ell\ell} \geq 0}$

I also confirmed in $P_\phi \neq 0$ case and $\underline{Q \neq 0 \text{ case}}$

asymptotically flat

A definition of entropy obeying 1st and 2nd law

1. BH thermodynamics and problems
2. Coarse-grained entropy and 2nd law in CFT
3. Rewrite in gravity by AdS/CFT
4. 2nd law implies null energy condition
5. (Generalized) 1st law in GR

Entropy vs Euclid action: Legendre tr

Entropy S_t and free energy $F_t := \beta_t^{-1} I^{(\mathrm{E})}_{\mathrm{grav}}[\beta_t, \omega_t, \lambda_t]$

$$S_t = -I^{(\mathrm{E})}_{\mathrm{grav}}[\beta_t, \omega_t, \lambda_t] + \beta_t \left(M_t - \omega_t^A P_{A,t} - \int \mathrm{d}^{d-1}\vec{x}\, \lambda_t^I(\theta) \pi_{I,t}(\theta) \right)$$

Values to be respected

$(\beta_t, \omega_t, \lambda_t)$ are viewed as functions of (M_t, P_t, π_t)

c.f.) CFT description

$$\begin{aligned} S_t &= -\mathrm{Tr}\rho_{\mathrm{cg},t} \ln \rho_{\mathrm{cg},t} \\ &= \ln Z[\beta_t, \omega_t, \lambda_t] + \beta_t \left(\langle H \rangle_t - \omega^A \langle P_A \rangle_t - \int \mathrm{d}^{d-1}\theta\, \lambda^I(\theta) \langle O_I(\theta) \rangle_t \right) \end{aligned}$$

First law is generalized

$$S_t = -I^{(\mathrm{E})}_{\mathrm{grav}}[\beta_t, \omega_t, \lambda_t] + \beta_t \left(M_t - \omega_t^A P_{A,t} - \int \mathrm{d}^{d-1}\theta\, \lambda_t^I(\theta)\pi_{I,t}(\theta) \right)$$

$(\beta_t, \omega_t, \lambda_t)$ are viewed as functions of (M_t, P_t, π_t)

The time dependence of S_t is through $(M_t, P_{A,t}, \pi_{I,t})$

The variation of $I^{(\mathrm{E})}_{\mathrm{grav}}[\beta, \omega, \lambda]$

$$\delta I^{(\mathrm{E})}_{\mathrm{grav}}[\beta, \omega, \lambda] = M\delta\beta - P_A \delta(\beta\omega) - \beta \int \mathrm{d}^{d-1}\theta\, \delta\lambda^I(\theta)\pi_I(\theta) + (\cancel{\mathrm{EOM}})$$

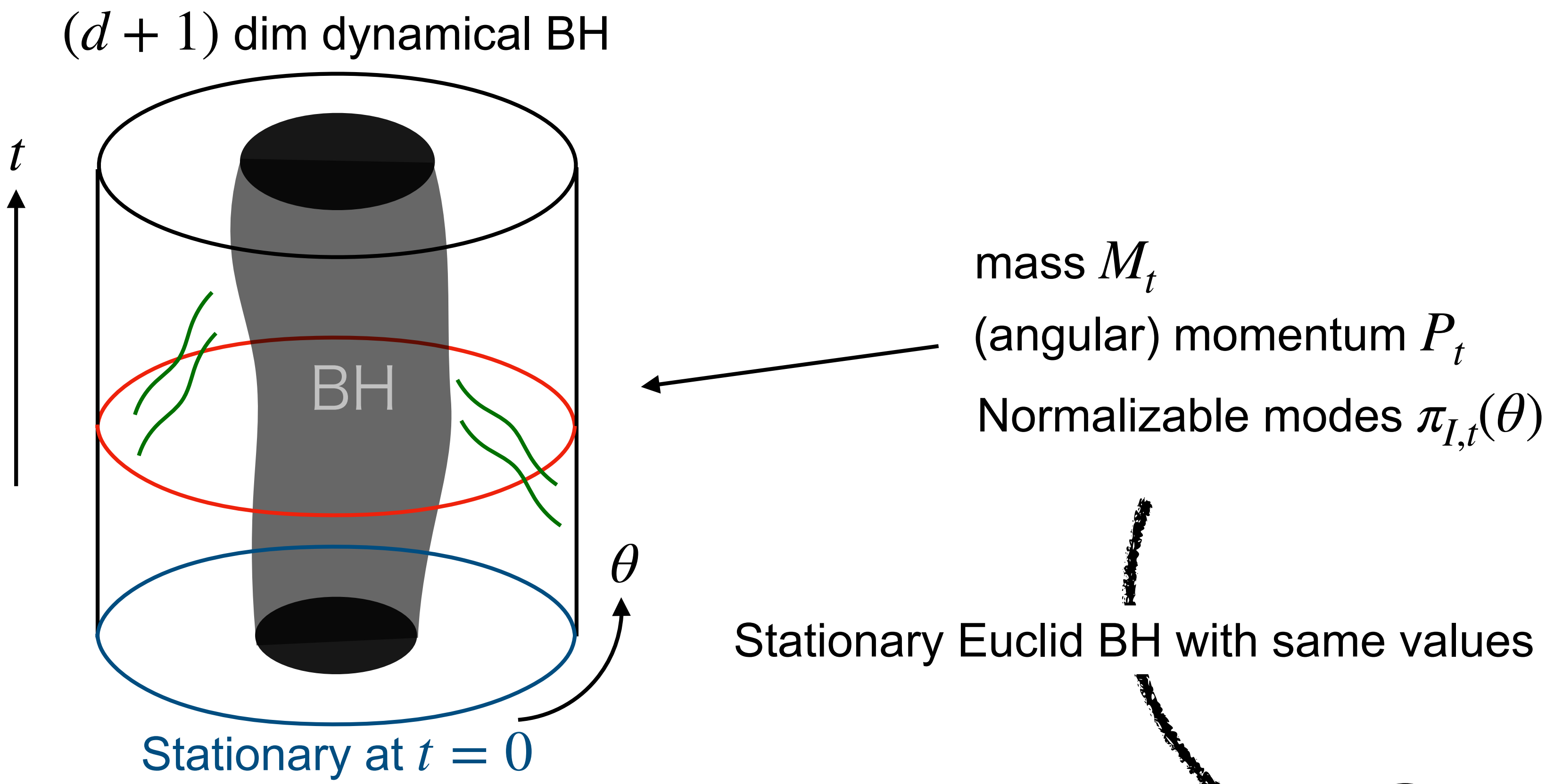
Setting $\delta = \dfrac{\mathrm{d}}{\mathrm{d}t}$, we obtain

$$\boxed{\dot{S}_t = \beta_t(\dot{M}_t - \omega_t^A \dot{P}_{A,t}) - \int \mathrm{d}^{d-1}\theta\, \lambda_t^I(\theta)\, \dot{\tilde{\pi}}_{I,t}(\theta), \quad \tilde{\pi}_{I,t} = \beta_t \pi_{T,t}}$$

A definition of entropy obeying 1st and 2nd law

1. BH thermodynamics and problems
2. Coarse-grained entropy and 2nd law in CFT
3. Rewrite in gravity by AdS/CFT
4. 2nd law implies null energy condition
5. (Generalized) 1st law in GR

A definition of entropy obeying 1st and 2nd law



Coarse-grained entropy: $S_t := \dfrac{A_t}{4G}$

1st law within GR: $\dot{S}_t = \beta_t \dot{M}_t + \cdots$

2nd law by AdS/CFT: $S_t \geq S_0$