[Brief Review*]

Interior Product, Lie derivative and Wilson Line in the *KBc* Subsector of Open String Field Theory

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KBc sector is a subspace of bosonic open string field theory (SFT) expanded by

$$K = \int_{-i\infty}^{i\infty} \frac{\mathrm{d}z}{2\pi i} T(z), \quad B = \int_{-i\infty}^{i\infty} \frac{\mathrm{d}z}{2\pi i} b(z), \quad c = c(0), \tag{1}$$

where T is the energy-momentum tensor, b is the anti-ghost field, and c is the ghost field. Here operators are defined in sliver frame z, which is defined through the map $z = (2/\pi)\arctan w$ with the coordinate w used in radial quantization. The following relations called KBc algebra are satisfied by K, B, c and BRST operator Q_{B}:

$$[K,\, B] = 0, \quad \{B, c\} = 1, \quad B^2 = 0, \quad c^2 = 0, \tag{2}$$

$$Q_{\mathrm{B}}K = 0, \quad Q_{\mathrm{B}}B = K, \quad Q_{\mathrm{B}}c = cKc. \tag{3}$$

Since any CFT has KBc sector, any solution in KBc sector is universal. Analytic solutions are studied by combining K, B, c and some matter sectors as well. In this case, the solutions are not universal in general. The review paper [1] is useful to learn KBc sector and universal solutions including tachyon vacuum.

We established the notion of the manifold in KBc sector in the paper. A top-down approach is taken in this review, but a heuristic approach in the paper. Let $\xi = (\xi^1, \xi^2)$ a two-component function, $\xi : \mathbb{R}_{\geq 0} \to \mathbb{C}^2$.[1)] Then it is easily confirmed that the new triad $(K(\xi), B(\xi), c(\xi))$ defined as follows also satisfies KBc algebra:

$$K(\xi) := e^{\xi^1(K)}K, \quad B(\xi) := e^{\xi^1(K)}B, \quad c(\xi) = e^{-i\xi^2(K)}ce^{-\xi^1(K)}Bce^{i\xi^2(K)}. \tag{4}$$

It is known that the eigen value of K runs over non-negative real number. We define KBc manifold $\mathcal{K}$ as follows:

- The triad $(K(\xi), B(\xi), c(\xi))$ is a point of $\mathcal{K}$.
- The coordinate is the function ξ.

Next, we define the interior product $\mathcal{I}_X$, where $X = (X^1, X^2)$ is two-component function as the coordinate ξ is. It is known that the action of bosonic open SFT (called Witten's action) has some similarities to the Chern-Simons (CS) action. Especially, the ghost number in bosonic open SFT corresponds to the form number in CS theory. Therefore, we assume that $\mathcal{I}_X$ lower the ghost number by one. In addition, KBc algebra should be broken by the

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1) $\mathbb{R}_{\geq 0} = \{x \in \mathbb{R} | x \geqq 0\}$

operation of $\mathcal{I}_X$ at each point of $\mathcal{K}$. Under the two conditions,[2] the general form of the operation of $\mathcal{I}_X$ at ξ is given by[3]

$$\mathcal{I}_X K(\xi) = iB(\xi)X^1(K), \quad \mathcal{I}_X B(\xi) = 0, \quad \mathcal{I}_X c(\xi) = \left\{ \frac{X^2(K)}{K(\xi)} B(\xi), c(\xi) \right\}. \tag{5}$$

We further impose on the interior product Leibniz-rule

$$\mathcal{I}_X(PQ) = (I_X P)Q + (-1)^{|P|} P(\mathcal{I}_X Q), \tag{6}$$

for the operation of $\mathcal{I}_X$ to be defined for any quantities in KBc sector. Here $|P|$ is 0 (1) when P is Grassmann-even (odd).

By analogy with the ordinary manifold ($L_V = \{\mathrm{d}, i_X\}$), we define the Lie derivative as

$$\pounds_X = -i\{Q_{\mathrm{B}}, \mathcal{I}_X\}. \tag{7}$$

We have used a known fact that BRST operator corresponds to the exterior derivative in the similarities described above. The concrete forms of the operation of $\pounds_X$ at ξ are given by

$$\begin{aligned} &\pounds_X K(\xi) = K(\xi)X^1(K), \quad \pounds_X B(\xi) = B(\xi)X^1(K), \\ &\pounds_X c(\xi) = -c(\xi)X^1(K)B(\xi)c(\xi) - i[X^2(K), c(\xi)]. \end{aligned} \tag{8}$$

From the definition and (6), $\pounds_X$ follows Liebniz-rule

$$\pounds_X(PQ) = (\pounds_X P)Q + P(\pounds_X Q). \tag{9}$$

The ordinary properties and formulas satisfied by the ordinary interior product, exterior product and Lie derivative still hold here, by replacing the exterior derivative with BRST operator. For example, $\{I_X, I_Y\} = 0$, $[Q_B, \pounds_X] = 0$ and $[\pounds_X, \pounds_Y] = \pounds_{[X,Y]}$, where $[X, Y]$ is a Lie bracket properly defined in the oroginal paper. As other benefits, the followings are introduced in the paper.

- The KBc triad at $\xi + \delta\xi$ can be obtained by acting $\pounds_{\delta\xi}$ on the triad at ξ for any infinitesimal $\delta\xi$. Then any two points (in other words, triads) on a continuous curve on $\mathcal{K}$ are related by the continuous operation of Lie derivative.
- If Ψ is a solution described only by KBc sector, then $\Psi(\xi)$, which is obtained by the replacement of (K, B, c) with $(K(\xi), B(\xi), c(\xi))$, is also a solution.
- Wilson line is also defined by analogy with CS theory. Although some similar properties to the ordinary one still hold, the construction appears to be incomplete.

[2] We imposed more conditions in the original paper, for example nilpotency, but now it was turned out for these to be sufficient.

[3] Here $X^{1,2}$ can depend on the original K not through $K(\xi)$, and the factor $1/K(\xi)$ accompanying X^2 is just for convenience to simplify the expression of the Lie derivative introduced in the next paragraph.

Possible future works are as follows.

- The physical interpretation of KBc manifold.
- Improving the Wilson line.
- The extension to the remaining sector of bosonic open SFT.
- The case of super SFT.

References

[1] Y. Okawa, *Analytic methods in open string field theory*, *Prog. Theor. Phys.* **128** (2012) 1001.