ESCOM Escuela Superior de Cómputo

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Algorithm Competition Template Library
new ESCOM version 2022-12-02
"LATIN AMERICA REGIONAL FINALS" EDITION

Contents

Contest (1)

1.1 template.cpp

19 lines

```
#include <bits/stdc++.h>
using namespace std;
#define int long long
#define endl '\n'
#define __ ios_base::sync_with_stdio(false),cin.tie(NULL);
#define rep(i, a, b) for(int i = a; i < (b); ++i)
#define all(x) begin(x), end(x)
#define sz(x) (int)(x).size()
typedef long long ll;
typedef pair<int, int> pii;
typedef vector<int> vi;
signed main() { __
  int n;
  cin>>n;
  vector<int> nums(n);
  for(auto &c:nums)cin>>c;
  return 0;
}
```

1.2 .bashrc

3 lines

```
alias c='g++ -Wall -Wconversion -Wfatal-errors -g -std=c++17 \
  -fsanitize=undefined,address'
```

1.3 .vimrc

7 lines

```
set cin aw ai is ts=4 sw=4 tm=50 nu noeb bg=dark ru cul
sy on |  im jk <esc> | im kj <esc> | no ; :
" Select region and then type :Hash to hash your selection.
" Useful for verifying that there aren't mistypes.
ca Hash w !cpp -dD -P -fpreprocessed \| tr -d '[:space:]' \
 \| md5sum \| cut -c-6
```

1.4 hash.sh

3 lines

```
# Hashes a file, ignoring all whitespace and comments. Use for
# verifying that code was correctly typed.
cpp $1 -dD -P -fpreprocessed | tr -d '[:space:]'| md5sum |cut -c-6
```

1.5 troubleshoot.txt

49 lines

```
Pre-submit:
Write a few simple test cases if sample is not enough.
Are time limits close? If so, generate max cases.
Is the memory usage fine?
Could anything overflow?
Make sure to submit the right file.
Wrong answer:
Print your solution! Print debug output, as well.
Are you clearing all data structures between test cases?
Can your algorithm handle the whole range of input?
Read the full problem statement again.
Do you handle all corner cases correctly?
Have you understood the problem correctly?
Any uninitialized variables?
Any overflows?
Confusing N and M, i and j, etc.?
Are you sure your algorithm works?
What special cases have you not thought of?
Are you sure the STL functions you use work as you think?
Add some assertions, maybe resubmit.
Create some testcases to run your algorithm on.
Go through the algorithm for a simple case.
Go through this list again.
Explain your algorithm to a teammate.
Ask the teammate to look at your code.
Go for a small walk, e.g. to the toilet.
Is your output format correct? (including whitespace)
Rewrite your solution from the start or let a teammate do it.
Runtime error:
Have you tested all corner cases locally?
Any uninitialized variables?
Are you reading or writing outside the range of any vector?
Any assertions that might fail?
Any possible division by 0? (mod 0 for example)
Any possible infinite recursion?
Invalidated pointers or iterators?
Are you using too much memory?
Debug with resubmits (e.g. remapped signals, see Various).
Time limit exceeded:
Do you have any possible infinite loops?
What is the complexity of your algorithm?
Are you copying a lot of unnecessary data? (References)
```

How big is the input and output? (consider scanf)

Avoid vector, map. (use arrays/unordered_map)

What do your teammates think about your algorithm?

Memory limit exceeded:

What is the max amount of memory your algorithm should need?

Are you clearing all data structures between test cases?

Mathematics (2)

2.1 Equations and series

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\begin{matrix} ax + by = e \\ cx + dy = f \end{matrix} \Rightarrow \begin{matrix} x = \dfrac{ed - bf}{ad - bc} \\ y = \dfrac{af - ec}{ad - bc} \end{matrix}$$

In general, given an equation $Ax = b$, the solution to a variable x_i is given by

$$x_i = \frac{\det A_i'}{\det A}$$

where A_i' is A with the i'th column replaced by b.

2.2 Recurrences

If $a_n = c_1 a_{n-1} + \cdots + c_k a_{n-k}$, and $r_1, \ldots, r_k$ are distinct roots of $x^k + c_1 x^{k-1} + \cdots + c_k$, there are $d_1, \ldots, d_k$ s.t.

$$a_n = d_1 r_1^n + \cdots + d_k r_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n = (d_1 n + d_2) r^n$.

2.3 Series

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \ \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \ \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

In general:

$$\sum_{i=1}^{n} i^m = \frac{1}{m+1}\left[(n+1)^{m+1} - 1 - \sum_{i=1}^{n}((i+1)^{m+1} - i^{m+1} - (m+1)i^m)\right]$$

$$\sum_{i=1}^{n-1} i^m = \frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k} B_k n^{m+1-k}$$

Geometric series:

$$\sum_{i=0}^{n} c^i = \frac{c^{n+1}-1}{c-1}, \ c \neq 1, \ \sum_{i=0}^{\infty} c^i = \frac{1}{1-c}, \ \sum_{i=1}^{\infty} c^i = \frac{c}{1-c}, |c| < 1$$

$$\sum_{i=0}^{n} ic^i = \frac{nc^{n+2} - (n+1)c^{n+1} + c}{(c-1)^2}, \ c \neq 1, \ \sum_{i=0}^{\infty} ic^i = \frac{c}{(1-c)^2}, |c| < 1$$

Harmonic series:

$$H_n = \sum_{i=1}^{n} \frac{1}{i}, \ \sum_{i=1}^{n} iH_i = \frac{n(n+1)}{2} H_n - \frac{n(n-1)}{4}$$

$$\sum_{i=1}^{n} H_i = (n+1)H_n - n, \ \sum_{i=1}^{n}\binom{i}{m} H_i = \binom{n+1}{m+1}(H_{n+1} - \frac{1}{m+1})$$

$$H_n = \sum_{k=1}^{n} \frac{1}{k} \approx \ln(n) + \gamma + \frac{1}{2n} - \frac{1}{12n^2}$$

$$\gamma \approx 0.577215664901532860606512$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots, \ (-\infty < x < \infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots, \ (-1 < x \leq 1)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \ldots, \ (-1 \leq x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots, \ (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots, \ (-\infty < x < \infty)$$

---

1. $\binom{n}{k} = \frac{n!}{(n-k)!k!}$ **2.** $\sum_{k=0}^{n}\binom{n}{k} = 2^n$ **3.** $\binom{n}{k} = \binom{n}{n-k}$ **4.** $\binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1}$

5. $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$ **6.** $\binom{n}{m}\binom{m}{k} = \binom{n}{k}\binom{n-k}{m-k}$ **7.** $\binom{r+k}{k} = \binom{r+n+1}{n}$

i	2^i	p_i
1	2	2
2	4	3
3	8	5
4	16	7
5	32	11
6	64	13
7	128	17
8	256	19
9	512	23
10	1,024	29
11	2,048	31
12	4,096	37
13	8,192	41
14	16,384	43
15	32,768	47
16	65,536	53
17	131,072	59
18	262,144	61
19	524,288	67
20	1,048,576	71
21	2,097,152	73
22	4,194,304	79
23	8,388,608	83
24	16,777,216	89
25	33,554,432	97
26	67,108,864	101
27	134,217,728	103

$n=0$	1
$n=1$	1 1
$n=2$	1 2 1
$n=3$	1 3 3 1
$n=4$	1 4 6 4 1
$n=5$	1 5 10 10 5 1
$n=6$	1 6 15 20 15 6 1
	0 1 2 3 4 5 6

2.1 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x. It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x x p_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent X and Y,

$$V(aX + bY) = a^2V(X) + b^2V(Y).$$

2.1.1 Discrete distributions

Binomial distribution

The number of successes in n independent yes/no experiments, each which yields success with probability p is $\mathrm{Bin}(n,p)$, $n = 1, 2, \ldots$, $0 \le p \le 1$.

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu = np,\ \sigma^2 = np(1-p)$$

$\mathrm{Bin}(n,p)$ is approximately $\mathrm{Po}(np)$ for small p.

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each wich yields success with probability p is $\mathrm{Fs}(p)$, $0 \le p \le 1$.

$$p(k) = p(1-p)^{k-1},\ k = 1, 2, \ldots$$

$$\mu = \frac{1}{p},\ \sigma^2 = \frac{1-p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\mathrm{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda}\frac{\lambda^k}{k!}, k = 0, 1, 2, \ldots$$

$$\mu = \lambda,\ \sigma^2 = \lambda$$

2.0.1 Continuous distributions

Uniform distribution

If the probability density function is constant between a and b and 0 elsewhere it is $\mathrm{U}(a,b)$, $a < b$.

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a+b}{2},\ \sigma^2 = \frac{(b-a)^2}{12}$$

Exponential distribution

The time between events in a Poisson process is $\mathrm{Exp}(\lambda)$, $\lambda > 0$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \ge 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda},\ \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution

Most real random values with mean μ and variance σ^2 are well described by $\mathcal{N}(\mu, \sigma^2)$, $\sigma > 0$.

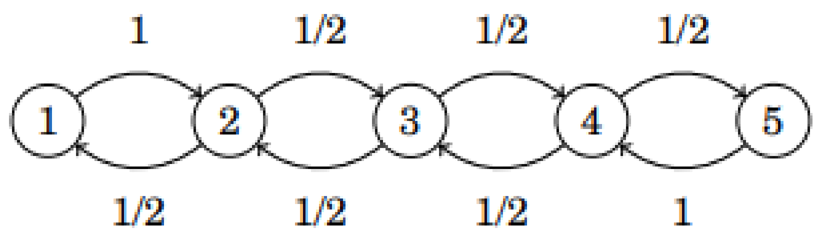
$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

2.1 Markov chains

A Markov chain is a random process that consists of states and transitions between them. For each state, we know the probabilities for moving to other states. A Markov chain can be represented as a graph whose nodes are states and edges are transitions. As an example, consider a problem where we are in floor 1 in an n floor building. At each step, we randomly walk either one floor up or one floor down, except that we always walk one floor up from floor 1 and one floor down from floor n. What is the probability of being in floor m after k steps? In this problem, each floor of the building corresponds to a state in a Markov chain. For example, if $n = 5$, the graph is as follows:



2.4 Number theory

2.4.1 Bézout's identity

For $a \neq$, $b \neq 0$, then $d = gcd(a,b)$ is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

If (x, y) is one solution, then all solutions are given by

$$\left(x + \frac{kb}{\gcd(a,b)}, y - \frac{ka}{\gcd(a,b)}\right), \quad k \in \mathbb{Z}$$

$$(f * e)(n) = f(n)$$
$$(\varphi * \mathbf{1})(n) = n$$
$$(\mu * \mathbf{1})(n) = e(n)$$
$$\varphi(n^k) = n^{k-1}\varphi(n)$$
$$\sum_{\substack{k=1\\ \gcd(k,n)=1}}^{n} k = \frac{n\varphi(n)}{2} \quad , \quad n \geq 2$$

$$\sum_{k=1}^{n} \operatorname{lcm}(k,n) = \frac{n}{2} + \frac{n}{2}\sum_{d|n} d\varphi(d) = \frac{n}{2} + \frac{n}{2}\prod_{p^a|n} \frac{p^{2a+1}+1}{p+1}$$

$$\sum_{k=1}^{n} \gcd(k,n) = \sum_{d|n} d\varphi\left(\frac{n}{d}\right) = \prod_{p^a|n} p^{a-1}(1 + (a+1)(p-1))$$

Lifting the exponent: sea p un primo, x, y enteros y n un entero positivo tal que $p \mid x - y$ pero $p \nmid x$ ni $p \nmid y$. Entonces:

- Si p es impar: $v_p(x^n - y^n) = v_p(x-y) + v_p(n)$
- Si $p = 2$ y n es par: $v_p(x^n - y^n) = v_p(x-y) + v_p(n) + v_p(x+y) - 1$

donde $v_p(n)$ es el exponente de p en la factorización en primos de n. Suma de dos cuadrados: sea $\chi_4(n)$ una función multiplicativa igual a 1 si $n \equiv 1 \mod 4$, -1 si $n \equiv 3 \mod 4$ y cero en otro caso. Entonces, el número de soluciones enteras (a, b) de la ecuación $a^2 + b^2 = n$ es $4(\chi_4 * 1)(n) = 4\sum_{d|n} \chi_4(d)$.

Sean $a, b, c \in \mathbb{Z}$ con $a \neq 0$ y $b \neq 0$. La ecuación $ax + by = c$ tiene como soluciones:

$$x = \frac{x_0 c - bk}{d}$$
$$y = \frac{y_0 c + ak}{d}$$

para toda $k \in \mathbb{Z}$ si y solo si $d|c$, donde $ax_0 + by_0 = \gcd(a,b) = d$. Si a y b tienen el mismo signo, hay exactamente $\max\left(\left\lfloor \frac{x_0 c}{|b|} \right\rfloor + \left\lfloor \frac{y_0 c}{|a|} \right\rfloor + 1, 0\right)$ soluciones no negativas. Si tienen el signo distinto, hay infinitas soluciones no negativas.

Dada una función aritmética f con $f(1) \neq 0$, existe otra función aritmética g tal que $(f * g)(n) = e(n)$, dada por:

$$g(1) = \frac{1}{f(1)}$$
$$g(n) = -\frac{1}{f(1)} \sum_{d|n, d<n} f\left(\frac{n}{d}\right) g(d) \quad , \quad n > 1$$

Sean $h(n) = \sum_{k=1}^{n} f\left(\left\lfloor \frac{n}{k} \right\rfloor\right) g(k)$, $G(n) = \sum_{k=1}^{n} g(k)$ y $m = \lfloor \sqrt{n} \rfloor$, entonces:

$$h(n) = \sum_{k=1}^{\lfloor n/m \rfloor} f\left(\left\lfloor \frac{n}{k} \right\rfloor\right) g(k) + \sum_{k=1}^{m-1} \left(G\left(\left\lfloor \frac{n}{k} \right\rfloor\right) - G\left(\left\lfloor \frac{n}{k+1} \right\rfloor\right)\right) f(k)$$

Sean $F(n) = \sum_{k=1}^{n} f(k)$, $G(n) = \sum_{k=1}^{n} g(k)$, $h(n) = (f * g)(n) = \sum_{d|n} f(d) g\left(\frac{n}{d}\right)$ y $H(n) = \sum_{k=1}^{n} h(k)$, entonces:

$$H(n) = \sum_{k=1}^{n} f(k) G\left(\left\lfloor \frac{n}{k} \right\rfloor\right)$$

Sean $\Phi_p(n) = \sum_{k=1}^{n} k^p \varphi(k)$ y $M_p(n) = \sum_{k=1}^{n} k^p \mu(k)$. Aplicando lo anterior, podemos calcular $\Phi_p(n)$ y $M_p(n)$ con complejidad $O(n^{2/3})$ si precalculamos con fuerza bruta los primeros $\lfloor n^{2/3} \rfloor$ valores, y para los demás, usamos las siguientes recurrencias (DP con `map`):

$$\Phi_p(n) = S_{p+1}(n) - \sum_{k=2}^{\lfloor n/m \rfloor} k^p \Phi_p\left(\left\lfloor \frac{n}{k} \right\rfloor\right) - \sum_{k=1}^{m-1} \left(S_p\left(\left\lfloor \frac{n}{k} \right\rfloor\right) - S_p\left(\left\lfloor \frac{n}{k+1} \right\rfloor\right)\right) \Phi_p(k)$$

$$M_p(n) = 1 - \sum_{k=2}^{\lfloor n/m \rfloor} k^p M_p\left(\left\lfloor \frac{n}{k} \right\rfloor\right) - \sum_{k=1}^{m-1} \left(S_p\left(\left\lfloor \frac{n}{k} \right\rfloor\right) - S_p\left(\left\lfloor \frac{n}{k+1} \right\rfloor\right)\right) M_p(k)$$

En general, si queremos hallar $F(n)$ y existe una función mágica $g(n)$ tal que $G(n)$ y $H(n)$ se puedan calcular en $O(1)$, entonces:

$$F(n) = \frac{1}{g(1)} \left[H(n) - \sum_{k=2}^{\lfloor n/m \rfloor} g(k) F\left(\left\lfloor \frac{n}{k} \right\rfloor\right) - \sum_{k=1}^{m-1} \left(G\left(\left\lfloor \frac{n}{k} \right\rfloor\right) - G\left(\left\lfloor \frac{n}{k+1} \right\rfloor\right)\right) F(k)\right]$$

2.1.1 Mobius Function

$$\mu(n) = \begin{cases} 0 & n \text{ is not square free} \\ 1 & n \text{ has even number of prime factors} \\ -1 & n \text{ has odd number of prime factors} \end{cases}$$

Mobius Inversion:

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d) g(n/d)$$

Other useful formulas/forms:

$\sum_{d|n} \mu(d) = [n = 1]$ (very useful)

$g(n) = \sum_{n|d} f(d) \Leftrightarrow f(n) = \sum_{n|d} \mu(d/n) g(d)$

$g(n) = \sum_{1 \leq m \leq n} f(\lfloor \frac{n}{m} \rfloor) \Leftrightarrow f(n) = \sum_{1 \leq m \leq n} \mu(m) g(\lfloor \frac{n}{m} \rfloor)$

Example of first form:

$$f(n) = \sum_{i=1}^{n} \sum_{j=1}^{n} [\gcd(i,j) == 1]$$

Apply the Mobius inversion to $[\gcd(i,j) == 1]$

$$f(n) = \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{d|gcd(i,j)} \mu(d) \tag{2.1}$$
$$= \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{d=1,n} [d|gcd(i,j)] \mu(d) \tag{2.2}$$
$$= \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{d=1,n} [d|i][d|j] \mu(d) \tag{2.3}$$
$$= \sum_{d=1}^{n} \mu(d) \sum_{i=1}^{n} [d|i] \sum_{j=1}^{n} [d|j] \tag{2.4}$$
$$= \sum_{d=1}^{n} \mu(d) (\frac{n}{d})^2 \tag{2.5}$$

2.5 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \;\; b = k \cdot (2mn), \;\; c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

2.6 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^{\times}$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

2.6.1 Some primes

$10^2 + 1$, $10^3 + 9$, $10^4 + 7$, $10^5 + 3$, $10^6 + 3$, $10^7 + 19$, $10^8 + 7$, $10^9 + 7$, $10^{10} + 19$, $10^{11} + 3$, $10^{12} + 39$, $10^{13} + 37$, $10^{14} + 31$, $10^{15} + 37$, $10^{16} + 61$, $10^{17} + 3$, $10^{18} + 3$.

2.6.2 Números primos de Mersenne

Números primos de la forma $M_p = 2^p - 1$ con p primo. Todos los números perfectos pares son de la forma $2^{p-1} M_p$ y viceversa.

Los primeros 47 valores de p son: 2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107, 127, 521, 607, 1279, 2203, 2281, 3217, 4253, 4423, 9689, 9941, 11213, 19937, 21701, 23209, 44497, 86243, 110503, 132049, 216091, 756839, 859433, 1257787, 1398269, 2976221, 3021377, 6972593, 13466917, 20996011, 24036583, 25964951, 30402457, 32582657, 37156667, 42643801, 43112609.

$\varphi(ip) \;=\; p\varphi(i)$
$\sum_{d|n} \phi(d) = n$
A number in base 10 has non-repeating decimals if its reduced fraction s/t has a denominator in the form t = $2^v \cdot 5^w$

Prime factors of $n!$

if p is prime the highest power p^k of p that divides $n!$ is given by

$$k = \left\lfloor \frac{n}{p} \right\rfloor + \left\lfloor \frac{n}{p^2} \right\rfloor + \left\lfloor \frac{n}{p^3} \right\rfloor + \cdots$$

Función φ de Euler

The most famous and important property of Euler's totient function is expressed in **Euler's theorem:**

$$\alpha^{\phi(m)} \equiv 1 (mod \quad m) \tag{2.1}$$

if α and $\boldsymbol{m}$ are relative prime.
In the particular case when m is prime, Euler's theorem turns into:

Fermat's little theorem

$$\alpha^{m-1} \equiv 1 (mod \quad m) \tag{2.2}$$

$$\alpha^n \equiv \alpha^{n \;\; mod \;\; \phi(m)} \quad (mod \quad m) \tag{2.3}$$

This allows computing $x^n mod \quad m$ for very big n, especially if n is the result of another computation, as it allows to compute n under a modulo.

2.1.2 Estimates

$\sum_{d|n} d = O(n \log \log n)$.
The number of divisors of some number is not to big
Here are the maximum number of divisors of any number of n digits

4	6	8	10	12	14	16	18	
64	240	768	2304	6720	17280	41472	103680	

Usefull to get all divisors of some big number like 10^{18} if you know 2 more small factors like $a \leq 10^9 b \leq 10^9$

2.2 Functions

2.2.1 Fórmula de Faulhaber

$$S_p(n) = \sum_{k=1}^{n} k^p = \frac{1}{p+1} \sum_{k=0}^{p} \binom{p+1}{k} B_k{}^+ n^{p+1-k}$$

2.2.2 Función Beta

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)} = 2\int_0^{\pi/2} \sin^{2x-1}(\theta) \cos^{2x-1}(\theta) d\theta$$
$$= \int_0^1 t^{x-1}(1-t)^{y-1} dt = \int_0^{\infty} \frac{t^{x-1}}{(1+t)^{x+y}} dt$$

2.2.3 Función zeta de Riemann

La siguiente fórmula converge rápido para valores pequeños de n ($n \approx 20$):

$$\zeta(s) \approx \frac{1}{d_0(1-2^{1-s})} \sum_{k=1}^{n} \frac{(-1)^{k-1} d_k}{k^s}$$
$$d_k = \sum_{j=k}^{n} \frac{4^j}{n+j} \binom{n+j}{2j}$$

2.2.4 Funciones generadoras

$$\sum_{n=0}^{\infty} \left(\sum_{k=0}^{n} a_k \right) x^n = \frac{1}{1-x} \sum_{n=0}^{\infty} a_n x^n$$

$$\sum_{n=0}^{\infty} \binom{n+k-1}{k-1} x^n = \frac{1}{(1-x)^k}$$

$$\sum_{n=0}^{\infty} p_n x^n = \frac{1}{\prod_{k=1}^{\infty} (1-x^k)} = \frac{1}{\sum_{n=-\infty}^{\infty} (-1)^n x^{\frac{1}{2}n(3n+1)}}$$

$$\sum_{p=0}^{\infty} \frac{S_p(n)}{p!} x^p = \frac{e^{x(n+1)} - e^x}{e^x - 1}$$

$$\sum_{n=0}^{\infty} n^k x^n = \frac{\sum_{i=0}^{k-1} \left\langle {k \atop i} \right\rangle x^{i+1}}{(1-x)^{k+1}} \quad , \quad k \geq 1$$

Sean $a_1, a_2, \ldots, a_n$ números complejos. Sean $p_m = \sum_{i=1}^{n} a_i^m$ y s_m el m-ésimo polinomio elemental simétrico de $a_1, a_2, \ldots, a_n$.
Entonces se cumple que $xS'(x) + P(x)S(x) = 0$, donde
$P(x) = \sum_{m=1}^{\infty} p_m x^m$ y
$S(x) = \prod_{i=1}^{n}(1 - a_i x) = \sum_{m=0}^{n} (-1)^m s_m x^m$.

2.6.3 Funciones generatrices

Una lista de funciones generatrices para secuencias útiles:

$(1, 1, 1, 1, 1, 1, \ldots)$	$\frac{1}{1-z}$
$(1, -1, 1, -1, 1, -1, \ldots)$	$\frac{1}{1+z}$
$(1, 0, 1, 0, 1, 0, \ldots)$	$\frac{1}{1-z^2}$
$(1, 0, \ldots, 0, 1, 0, 1, 0, \ldots, 0, 1, 0, \ldots)$	$\frac{1}{1-z^2}$
$(1, 2, 3, 4, 5, 6, \ldots)$	$\frac{1}{(1-z)^2}$
$(1, \binom{m+1}{m}, \binom{m+2}{m}, \binom{m+3}{m}, \ldots)$	$\frac{1}{(1-z)^{m+1}}$
$(1, c, \binom{c+1}{2}, \binom{c+2}{3}, \ldots)$	$\frac{1}{(1-z)^c}$
$(1, c, c^2, c^3, \ldots)$	$\frac{1}{1-cz}$
$(0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots)$	$\ln \frac{1}{1-z}$

Truco de manipulación:

$$\frac{1}{1-z} G(z) = \sum_n \sum_{k \leq n} g_k z^n$$

2.6.4 Aproximación de Stirling

$$\ln(n!) \approx n \ln(n) - n + \frac{1}{2} \ln(2\pi n)$$

$$\#\text{ de dígitos de } n! = 1 + \left\lfloor n \log\left(\frac{n}{e}\right) + \frac{1}{2}\log(2\pi n) \right\rfloor \quad (n \geq 30)$$

2.6.5 Árbol de Stern–Brocot

Todos los racionales positivos se pueden representar como un árbol binario de búsqueda completo infinito con raíz $\frac{1}{1}$.

- Dado un racional $q = [a_0; a_1, a_2, \ldots, a_k]$ donde $a_k \neq 1$, sus hijos serán $[a_0; a_1, a_2, \ldots, a_k + 1]$ y $[a_0; a_1, a_2, \ldots, a_k - 1, 2]$, y su padre será $[a_0; a_1, a_2, \ldots, a_k - 1]$.
- Para hallar el camino de la raíz $\frac{1}{1}$ a un racional q, se usa búsqueda binaria iniciando con $L = \frac{0}{1}$ y $R = \frac{1}{0}$. Para hallar M se supone que $L = \frac{a}{b}$ y $R = \frac{c}{d}$, entonces $M = \frac{a+c}{b+d}$.

2.2 Grafos

- Sea d_n el número de grafos con n vértices etiquetados: $d_n = 2^{\binom{n}{2}}$.
- Sea c_n el número de grafos conexos con n vértices etiquetados. Tenemos la recurrencia: $c_1 = 1$ y $d_n = \sum_{k=1}^{n} \binom{n-1}{k-1} c_k d_{n-k}$. También se cumple, usando funciones generadoras exponenciales, que $C(x) = 1 + \ln(D(x))$.
- Sea t_n el número de torneos fuertemente conexos en n nodos etiquetados. Tenemos la recurrencia $t_1 = 1$ y $d_n = \sum_{k=1}^{n} \binom{n}{k} t_k d_{n-k}$. Usando funciones generadoras exponenciales, tenemos que $T(x) = 1 - \frac{1}{D(x)}$.
- Número de spanning trees en un grafo completo con n vértices etiquetados: n^{n-2}.
- Número de bosques etiquetados con n vértices y k componentes conexas: kn^{n-k-1}.
- Para un grafo no dirigido simple G con n vértices etiquetados de 1 a n, sea $Q = D - A$, donde D es la matriz diagonal de los grados de cada nodo de G y A es la matriz de adyacencia de G. Entonces el número de spanning trees de G es igual a cualquier cofactor de Q.
- Sea G un grafo. Se define al polinomio $P_G(x)$ como el polinomio cromático de G, en donde $P_G(k)$ nos dice cuántas k-coloraciones de los vértices admite G. Ejemplos comunes:
  - Grafo completo de n nodos: $P(x) = x(x-1)(x-2)\ldots(x-(n-1))$
  - Grafo vacío de n nodos: $P(x) = x^n$
  - Árbol de n nodos: $P(x) = x(x-1)^{n-1}$
  - Ciclo de n nodos: $P(x) = (x-1)^n + (-1)^n(x-1)$

2.2.1 Teorema de Euler

si un grafo conexo, plano es dibujado sobre un plano sin intersección de aristas, y siendo v el número de vértices, e el de aristas y f la cantidad de caras (regiones conectadas por aristas, incluyendo la región externa e infinita), entonces

$$v - e + f = 2$$

El numero de maneras de conectar un grafo con k componentes conexas con el minimo número de aristas donde s_i es el tamaño del componente $(s_i \cdot s_2 \cdot \cdots \cdot s_k) \cdot n^{k-2}$ $\sum_{i=1}^{n/2} \binom{n}{i \cdot 2} = 2^(n-1)$

Kirchhoff Matrix Tree Theorem

Count the number of spanning trees in a graph, as the determinant of the Laplacian matrix of the graph.
Laplacian Matrix :
Given a simple graph G with n vertices, its Laplacian matrix $L_{n \times n}$ is defined as

$$L = D - A$$

The elements of L are given by

$$L_{i,j} = \begin{cases} deg(v_i) & \text{if } i == j \\ -1 & \text{if } i \neq j \text{and } v_i \text{ is adjacent to } v_j \\ 0 & \text{otherwise} \end{cases}$$

define $\tau(G)$ as number of spanning trees of a grap G

$$\tau(G) = \det L_{n-1 \times n-1}$$

Where $L_{n-1 \times n-1}$ is a laplacian matrix deleting any row and any column

$$\det \begin{pmatrix} deg(v_1) & L_{1,2} & \cdots & L_{1,n-1} \\ L_{2,1} & deg(v_2) & \cdots & L_{2,n-1} \\ \vdots & \vdots & \ddots & \vdots \\ L_{n-1,1} & L_{n-1,2} & \cdots & deg(v_{n-1}) \end{pmatrix}$$

Generalization for a multigraph $K_n^m \pm G$
define $\tau(K_n^m \pm G)$ as number of spanning trees of a grap $K_n^m \pm G$

$$\tau(K_n^m \pm G) = n * (nm)^{n-p-2} \det(B)$$

where $B = mnI_p + \alpha * L(G)$is a $p \times p$ matrix, $\alpha = \pm$ according $(K_n^m \pm G)$, and $L(G)$ is the Kirchhoff matrix of G

2.7 Fibonacci

$$f_0 = 0,\ f_1 = 1$$

$$f_n = \frac{1}{\sqrt{5}}(\frac{1+\sqrt{5}}{2})^n - \frac{1}{\sqrt{5}}(\frac{1-\sqrt{5}}{2})^n$$

$$f_n = \frac{\varphi - (1-\varphi)^n}{\sqrt{5}}$$

$$\varphi = \frac{1+\sqrt{5}}{2}$$

$$f_{n+1} = f_n * 2 - f_{n-2}$$

$$f_0 + f_1 + f_2 + \cdots + f_n = f_{n+2} - 1$$

$$f_0 - f_1 + f_2 - \cdots + (-1)^n f_n = (-1)^n f_{n-1} - 1$$

$$f_1 + f_3 + f_5 + \cdots + f_{2n-1} = f_{2n}$$

$$f_0 + f_2 + f_4 + \cdots + f_{2n} = f_{2n+1} - 1$$

$$f_0^2 + f_1^2 + f_2^2 + \cdots + f_n^2 = f_n f_{n+1}$$

$$f_1 f_2 + f_2 f_3 + f_3 f_4 + \cdots + f_{2n-1} f_n = f_{2n}^2$$

$$f_1 f_2 + f_2 f_3 + f_3 f_4 + \cdots + f_{2n} f_{2n+1} = f_{2n+1}^2 - 1$$

$$k \geq 1 \Rightarrow f_{n+k} = f_k f_{n+1} + f_{k-1} f_n \forall n \geq 0$$

Identidad de Cassini: $f_{n+1} fn - 1 - f_n^2 = (-1)^n$

$$f_{n+1}^2 + f_n^2 = f_{2n+1}$$

$$f_{n+2}^2 - f_n^2 = f_{2n+2}$$

$$f_{n+2}^2 - f_{n+1}^2 = f_n f_{n+3}$$

$$f_{n+2}^3 - f_{n+1}^3 - f_n^3 = f_{3n+3}$$

$$mcd(f_n, f_m) = f_{mcd(n,m)}$$

$$f_{n+1} = \sum_{j=0}^{\lfloor \frac{n}{2} \rfloor} \binom{n-j}{j}$$

$$f_{3n} = \sum_{j=0}^{n} \binom{n}{j} 2^j f_j$$

El último dígito de cada número se repite periódicamente cada 60 números. Los dos últimos, cada 300; a partir de ahí, se repiten cada $15 * 10^{n-1}$ números.

2.2.2 Propiedades de logaritmos

$$\log(a-b) = log(a) + log(1 - \frac{b}{a})$$

$$\log(ab) = log(a) + log(b)$$

$$\log(a^n) = n \times log(a)$$

$$\log_c(a) = \frac{\log(a)}{\log(c)}$$

$$\log(\sqrt[n]{a}) = \frac{1}{n}\log(a)$$

$$\log(\frac{a}{b}) = \log(a) - \log(b)$$

$$((x\%a)\%a * b) = ((x\%a * b)\%a) = x\%a$$

$$a + b = a^b + 2 \times a\&b$$

$$a + b = (a|b) + (a\&b)$$

$$a1 = (a01 + a12 - a02)/2$$

$$lcm(a, gcd(b,c)) = gcd(lcm(a,b), lcm(a,c)) - -(1)$$

$$gcd(a, lcm(b,c)) = lcm(gcd(a,b), gcd(a,c)) - -(2)$$

$$gcd(a,b) = gcd(a, b-a)$$

2.8 Trigonometry

$$\sin(v+w) = \sin v \cos w + \cos v \sin w$$
$$\cos(v+w) = \cos v \cos w - \sin v \sin w$$

$$\tan(v+w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}$$
$$\sin v + \sin w = 2 \sin \frac{v+w}{2} \cos \frac{v-w}{2}$$
$$\cos v + \cos w = 2 \cos \frac{v+w}{2} \cos \frac{v-w}{2}$$
$$(V+W)\tan(v-w)/2 = (V-W)\tan(v+w)/2$$

where V, W are lengths of sides opposite angles v, w.

$$a \cos x + b \sin x = r \cos(x - \phi)$$
$$a \sin x + b \cos x = r \sin(x + \phi)$$

where $r = \sqrt{a^2+b^2}, \phi = \operatorname{atan2}(b, a)$.

2.9 Geometry

2.9.1 Triangles

Side lengths: a, b, c

Semiperimeter: $p = \dfrac{a+b+c}{2}$

Area: $A = \sqrt{p(p-a)(p-b)(p-c)}$

Circumradius: $R = \dfrac{abc}{4A}$

Inradius: $r = \dfrac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two):

$$s_a = \sqrt{bc\left[1 - \left(\frac{a}{b+c}\right)^2\right]}$$

Law of sines: $\dfrac{\sin\alpha}{a} = \dfrac{\sin\beta}{b} = \dfrac{\sin\gamma}{c} = \dfrac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc\cos\alpha$

Law of tangents: $\dfrac{a+b}{a-b} = \dfrac{\tan\frac{\alpha+\beta}{2}}{\tan\frac{\alpha-\beta}{2}}$

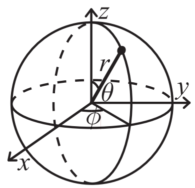
2.2.3 Quadrilaterals

With side lengths a, b, c, d, diagonals e, f, diagonals angle θ, area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin\theta = F \tan\theta = \sqrt{4e^2f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is 180°, $ef = ac + bd$, and $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$.

2.2.4 Spherical coordinates



$$\begin{aligned} x &= r\sin\theta\cos\phi & r &= \sqrt{x^2+y^2+z^2} \\ y &= r\sin\theta\sin\phi & \theta &= \operatorname{acos}(z/\sqrt{x^2+y^2+z^2}) \\ z &= r\cos\theta & \phi &= \operatorname{atan2}(y, x) \end{aligned}$$

2.2.5 Ángulo entre dos vectores

Sea α el ángulo entre $\vec{a}$ y $\vec{b}$:

$$\cos\alpha = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|\|\vec{b}\|}$$

2.2.6 Proyección de un vector

Proyección de $\vec{a}$ sobre $\vec{b}$:

$$\operatorname{proy}_{\vec{b}} \vec{a} = \left(\frac{\vec{a} \cdot \vec{b}}{\vec{b} \cdot \vec{b}}\right)\vec{b}$$

2.3 Derivatives/Integrals

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\arccos x = -\frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\tan x = 1 + \tan^2 x \qquad \frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$
$$\int \tan ax = -\frac{\ln|\cos ax|}{a} \qquad \int x\sin ax = \frac{\sin ax - ax\cos ax}{a^2}$$
$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2}\operatorname{erf}(x) \qquad \int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax-1)$$

Integration by parts:

$$\int_a^b f(x)g(x)dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x)dx$$

Combinatorial (3)

3.1 Permutations

3.1.1 Factorial

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800

n	11	12	13	14	15	16	17
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14

n	20	25	30	40	50	100	150	171
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX

3.1.2 factoradic

El sistema factorádico es un sistema numérico de raíz mixta basado en factoriales en el que el n-ésimo dígito, empezando desde la derecha, debe ser multiplicado por n!

Hay una relación natural entre los enteros 0, ..., n! − 1 (o de manera equivalente los números factorádicos con n elementos) en orden lexicográfico, cuando los enteros son expresados en forma factorádica. Esta relación ha sido llamada código Lehmer o código Lucas-Lehmer (tabla invertida). Por ejemplo, con n = 3, dicha relación es

Decimal	Factoradic	Permutation
0_{10}	$0_2 0_1 0_0$	(0,1,2)
1_{10}	$0_2 1_1 0_0$	(0,2,1)
2_{10}	$1_2 0_1 0_0$	(1,0,2)
3_{10}	$1_2 1_1 0_0$	(1,2,0)
4_{10}	$2_2 0_1 0_0$	(2,0,1)
5_{10}	$2_2 1_1 0_0$	(2,1,0)

3.2 IntPerm

Description: Permutation -> integer conversion. (Not order preserving.) Integer -> permutation can use a lookup table.
Time: $\mathcal{O}(n)$

6 lines

```
int permToInt(vi& v) {
  int use = 0, i = 0, r = 0;
  for(int x:v) r = r * ++i + __builtin_popcount(use & -(1<<x)),
    use |= 1 << x;              // (note: minus, not ~!)
  return r;
}
```

3.3 permutation.cpp

64 lines

```
#include <bits/stdc++.h>
using namespace std;
vector<int> bit;
int n;
int sum(int idx) {
  int ans = 0;
  for(++idx;idx>0 ;idx-= idx&-idx)ans+=bit[idx];
  return ans;
}
void add(int idx,int val) {
  for(++idx;idx<n;idx+= idx&-idx)bit[idx]+=val;
}
int bit_search(int s) {
  int sum = 0;
  int pos = 0;
  for(int i = ceil(log2(n));i>=0;i--) {
    if((pos+(1<<i))<n && (sum+bit[pos+(1<<i)])<s) {
      sum+=bit[pos+(1<<i)];
      pos+=(1<<i);
    }
  }
  return pos;
}
int main() {
  int x;
  cin>>n;
  vector<int> factoradicA(n);
  vector<int> factoradicB(n);
  bit.resize(n);
  for(int i = 0;i<n;i++)
    add(i,1);
  for(int i = 0;i<n;i++) {
    cin>>x;
    factoradicA[i] = sum(x-1);
    add(x,-1);
  }
  bit.assign(n,0);
  for(int i = 0;i<n;i++)
    add(i,1);
  for(int i = 0;i<n;i++) {
    cin>>x;
    factoradicB[i] = sum(x-1);
    add(x,-1);
  }
  vector<int> final(n);
  int carry= 0;
  for(int i = n-1;i>=0;i--) {
    int fact = (n-1)-i;
    final[i] = (factoradicA[i]+factoradicB[i])+carry;
    if(final[i]>=fact+1) {
      final[i]-=fact+1;
      carry = 1;
    }
    else carry = 0;
  }
  for(int i = 0;i<n;i++)add(i,1);
  for(int i = 0;i<n;i++) {
    x = bit_search(final[i]+1);
    cout<<x<<" ";
    add(x,-1);
  }
  cout<<endl;
  return 0;
}
```

3.3.1 General

- Principio de las casillas: al colocar n objetos en k lugares hay al menos $\lceil \frac{n}{k} \rceil$ objetos en un mismo lugar.
- Número de funciones: sean A y B conjuntos con $m = |A|$ y $n = |B|$. Sea $f : A \to B$:
  - Si $m \leq n$, entonces hay $m!\binom{n}{m}$ funciones inyectivas f.
  - Si $m = n$, entonces hay $n!$ funciones biyectivas f.
  - Si $m \geq n$, entonces hay $n!\left\{{m \atop n}\right\}$ funciones suprayectivas f.
- Barras y estrellas: ¿cuántas soluciones en los enteros no negativos tiene la ecuación $\sum_{i=1}^{k} x_i = n$? Tiene $\binom{n+k-1}{k-1}$ soluciones.
- ¿Cuántas soluciones en los enteros positivos tiene la ecuación $\sum_{i=1}^{k} x_i = n$? Tiene $\binom{n-1}{k-1}$ soluciones.
- Desordenamientos: $a_0 = 1$, $a_1 = 0$, $a_n = (n-1)(a_{n-1} + a_{n-2}) = na_{n-1} + (-1)^n$.
- Sea $f(x)$ una función. Sea $g_n(x) = xg'_{n-1}(x)$ con $g_0(x) = f(x)$. Entonces $g_n(x) = \sum_{k=0}^{n} \left\{{n \atop k}\right\} x^k f^{(k)}(x)$.
- Supongamos que tenemos $m+1$ puntos: $(0, y_0)$, $(1, y_1)$, $\ldots$, (m, y_m). Entonces el polinomio $P(x)$ de grado m que pasa por todos ellos es:

$$P(x) = \left[\prod_{i=0}^{m}(x-i)\right](-1)^m \sum_{i=0}^{m} \frac{y_i(-1)^i}{(x-i)i!(m-i)!}$$

- Sea $a_0, a_1, \ldots$ una recurrencia lineal homogénea de grado d dada por $a_n = \sum_{i=1}^{d} b_i a_{n-i}$ para $n \geq d$ con términos iniciales $a_0, a_1, \ldots, a_{d-1}$. Sean $A(x)$ y $B(x)$ las funciones generadoras de las sucesiones a_n y b_n respectivamente, entonces se cumple que $A(x) = \frac{A_0(x)}{1 - B(x)}$, donde $A_0(x) = \sum_{i=0}^{d-1} \left[a_i - \sum_{j=0}^{i-1} a_j b_{i-j} \right] x^i$.
- Si queremos obtener otra recurrencia c_n tal que $c_n = a_{kn}$, las raíces del polinomio característico de c_n se obtienen al elevar todas las raíces del polinomio característico de a_n a la k-ésima potencia; y sus términos iniciales serán $a_0, a_k, \ldots, a_{k(d-1)}$.

3.3.2 Cycles

Let $g_S(n)$ be the number of n-permutations whose cycle lengths all belong to the set S. Then

$$\sum_{n=0}^{\infty} g_S(n) \frac{x^n}{n!} = \exp\left(\sum_{n \in S} \frac{x^n}{n} \right)$$

3.3.3 The twelvefold way

¿Cuántas funciones $f\colon N \to X$ hay?

N	X	Any f	Injective	Surjective
dist.	dist.	x^n	$(x)_n$	$x!\left\{ {n \atop x} \right\}$
indist.	dist.	$\binom{x+n-1}{n}$	$\binom{x}{n}$	$\binom{n-1}{n-x}$
dist.	indist.	$\left\{ {n \atop 1} \right\} + \ldots + \left\{ {n \atop x} \right\}$	$[n \leq x]$	$\left\{ {n \atop k} \right\}$
indist.	indist.	$p_1(n) + \ldots p_x(n)$	$[n \leq x]$	$p_x(n)$

Where $\binom{a}{b} = \frac{1}{b!}(a)_b$ and $p_x(n)$ is the number of ways to partition the integer n using x summands.

3.3.4 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left\lfloor \frac{n!}{e} \right\rceil$$

$$!n = (n-1)(!(n-1)+!(n-2)); !1 = 0, !2 = 1$$

$$!n = n! \sum_{k=0}^{n} \frac{(-1)^k}{k!}$$

3.3.5 Burnside's lemma

Given a group G of symmetries and a set X, the number of elements of X *up to symmetry* equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts "configurations" (of some sort) of length n, we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n,k)) = \frac{1}{n} \sum_{k|n} f(k)\phi(n/k).$$

3.4 Partitions and subsets

3.4.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1,\ p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	∼2e5	∼2e8

3.4.2 Lucas' Theorem

Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + \ldots + n_1 p + n_0$ and $m = m_k p^k + \ldots + m_1 p + m_0$. Then

$$\binom{m}{n} \equiv \prod_{i=0}^{k} \binom{m_i}{k_i} \pmod{p}$$

$$m = \sum_{i=0}^{k} m_i p^i \quad , \quad n = \sum_{i=0}^{k} n_i p^i$$

$$0 \leq m_i, n_i < p$$

3.4.3 Binomials

3.5 multinomial.h

Description: Computes $\binom{k_1 + \cdots + k_n}{k_1, k_2, \ldots, k_n} = \frac{(\sum k_i)!}{k_1!k_2!...k_n!}$.

6 lines

```
ll multinomial(vi& v) {
  ll c = 1, m = v.empty() ? 1 : v[0];
  rep(i,1,sz(v)) rep(j,0,v[i])
    c = c * ++m / (j+1);
  return c;
}
```

3.6 BinomialCoeficients

Description: A few ways to calc a binomial coeficient with diferent complexityes
Time: Varios complexities

87 lines

```
/*********************************************************
   Binomial coeficients
 * Computes ${N}\choose{k}$
*********************************************************
/*+ If you can know that the result don't overflow and don't need MOD */
long binomial_Coeff_without_MOD(int n,int r){
   long ans = 1;
   for(int i = 1;i<=min(n-k,k);i++){
      ans = (ans* (n-(i-1)))/i;
    }
   return ans;
 }
/* O(n) solutions
   Based in the prof of C(n,k) = C(n-1,k-1) + C(n-1,k)
   Also calc all C(n,i) for 0<=i<=n
*/
long binomial_Coeff(int n,int m){
  int i,j;
  long bc[MAXN][MAXN];
  for (i=0; i<=n; i++) bc[i][0] = 1;
  for (j=0; j<=n; j++) bc[j][j] = 1;
  for (i=1; i<=n; i++)
     for (j=1; j<i; j++)
        bc[i][j] = bc[i-1][j-1] + bc[i-1][j];
  return bc[n][m];
 }
/*
   O(k) solution
   Only calc C(n,k)
*/
int binomial_Coeff_2(int n, int k)  {
   int res = 1;
   if ( k > n - k )
      k = n - k;
   for (int i = 0; i < k; ++i){
      res *= (n - i);
      res /= (i + 1);
```

```cpp
    }
    return res;
}
/*  Factorial modulo P */
int factmod(int n, int p) {
    int res = 1;
    while (n > 1) {
        res = (res * ((n/p) % 2 ? p-1 : 1)) % p;
        for (int i = 2; i <= n%p; ++i)
            res = (res * i) % p;
        n /= p;
    }
    return res % p;
}
/** O(1) binomial coeficient with precalc in O(n) */
const int M = 1e6;
const lli mod = 986444681;
vector<lli> fact(M+1, 1), inv(M+1, 1), invfact(M+1, 1);
lli ncr(lli n, lli r) {
  if(r < 0 || r > n) return 0;
  return fact[n] * invfact[r] % mod * invfact[n - r] % mod;
}
void calc() {
    for(int i = 2; i <= M; ++i) {
    fact[i] = (lli)fact[i-1] * i % mod;
    inv[i] = mod - (lli)inv[mod % i] * (mod / i) % mod;
    invfact[i] = (lli)invfact[i-1] * inv[i] % mod;
  }
}
/** Lucas Theorem: Computes C(N,R)%p in O(log(n)) if P is prime */
/** call calc() first */
lli Lucas(lli N,lli R) {
  if(R<0||R>N)
    return 0;
  if(R==0||R==N)
    return 1ll;
  if(N>=mod)
    return (1ll*Lucas(N/mod,R/mod)*Lucas(N%mod,R%mod))%mod;
  return fact[n] * invfact[r] % mod * invfact[n - r] % mod;
}
/* Using calc() we can also calculate P(n,k) (permutations) */
lli permutation(int n,int k) {
    return (1ll*fact[n]* invfact[n-k])%mod;
}
/** Cayley's formula: Computes all posibles trees whit n nodes */
lli cayley(int n ,int k) {
    if(n-k-1<0)
        return (1ll*k*modpow(n,mod-2))%mod;
    return (1ll*k*modpow(n,n-k-1))%mod;
}
```

3.7 General purpose numbers

3.7.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t-1}$ (FFT-able).
$B[0,\ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$

Sums of powers:

$$\sum_{i=1}^{n} n^m = \frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k}B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^{\infty} f(x)dx - \sum_{k=1}^{\infty}\frac{B_k}{k!}f^{(k-1)}(m)$$

$$\approx \int_m^{\infty} f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

3.7.2 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\ \ c(0,0) = 1$$
$$\textstyle\sum_{k=0}^{n} c(n,k)x^k = x(x+1)\ldots(x+n-1)$$

$c(8,k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \ldots$

3.7.3 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k}(-1)^j\binom{n+1}{j}(k+1-j)^n$$

3.7.4 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!}\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}j^n$$

3.7.5 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \ldots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

3.7.6 Labeled unrooted trees

\# on n vertices: n^{n-2}
\# on k existing trees of size n_i: $n_1 n_2 \cdots n_k n^{k-2}$
\# with degrees d_i: $(n-2)!/((d_1-1)!\cdots(d_n-1)!)$

3.7.7 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2}C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

3.7.8 Números de Catalán

están definidos por la recurrencia:

$$C_{n+1} = \sum_{i=0}^{n} C_i C_{n-i}$$

3.7.9 Números de Stirling del primer tipo

$\left[n \atop k \right]$ representa el número de permutaciones de n elementos en exactamente k ciclos disjuntos.

$$\left[0 \atop 0 \right] = 1$$

$$\left[0 \atop n \right] = \left[n \atop 0 \right] = 0 \qquad , \quad n > 0$$

$$\left[n \atop k \right] = (n-1)\left[n-1 \atop k \right] + \left[n-1 \atop k-1 \right] \qquad , \quad k > 0$$

$$\sum_{k=0}^{n} \left[n \atop k \right] = n!$$

$$\sum_{k=0}^{\infty} \left[n \atop k \right] x^k = \prod_{k=0}^{n-1} (x+k)$$

3.7.10 Números de Stirling del segundo tipo

$\left\{ n \atop k \right\}$ representa el número de formas de particionar un conjunto de n objetos distinguibles en k subconjuntos no vacíos.

$$\left\{ 0 \atop 0 \right\} = 1$$

$$\left\{ 0 \atop n \right\} = \left\{ n \atop 0 \right\} = 0 \qquad , \quad n > 0$$

$$\left\{ n \atop k \right\} = k\left\{ n-1 \atop k \right\} + \left\{ n-1 \atop k-1 \right\} \qquad , \quad k > 0$$

$$= \sum_{j=0}^{k} \frac{j^n}{j!} \cdot \frac{(-1)^{k-j}}{(k-j)!}$$

3.7.11 Números de Euler

$\left\langle n \atop k \right\rangle$ representa el número de permutaciones de 1 a n en donde exactamente k números son mayores que el número anterior, es decir, cuántas permutaciones tienen k "ascensos".

$$\left\langle 1 \atop 0 \right\rangle = 1$$

$$\left\langle n \atop k \right\rangle = (n-k)\left\langle n-1 \atop k-1 \right\rangle + (k+1)\left\langle n-1 \atop k \right\rangle \qquad , \quad n \geq 2$$

$$= \sum_{j=0}^{k} (-1)^j \binom{n+1}{j} (k+1-j)^n$$

$$\sum_{k=0}^{n-1} \left\langle n \atop k \right\rangle = n!$$

3.7.12 Números de Catalan

$$C_0 = 1$$

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \sum_{j=0}^{n-1} C_j C_{n-1-j}$$

$$\sum_{n=0}^{\infty} C_n x^n = \frac{1-\sqrt{1-4x}}{2x}$$

3.7.13 Números de Bell

B_n representa el número de formas de particionar un conjunto de n elementos.

$$B_n = \sum_{k=0}^{n} \left\{ n \atop k \right\} = \sum_{k=0}^{n-1} \binom{n-1}{k} B_k$$

$$\sum_{n=0}^{\infty} \frac{B_n}{n!} x^n = e^{e^x - 1}$$

3.7.14 Números de Bernoulli

$$B_0{}^{+} = 1$$

$$B_n{}^{+} = 1 - \sum_{k=0}^{n-1} \binom{n}{k} \frac{B_k{}^{+}}{n-k+1}$$

$$\sum_{n=0}^{\infty} \frac{B_n{}^{+} x^n}{n!} = \frac{x}{1-e^{-x}} = \frac{1}{\frac{1}{1!} - \frac{x}{2!} + \frac{x^2}{3!} - \frac{x^3}{4!} + \cdots}$$

3.8 BinomialCoeficients

Description: A few ways to calc a binomial coeficient with diferent complexityes
Time: Varios complexities

87 lines

```
/**********************************************************
   Binomial coeficients
 * Computes ${N}\choose{k}$
**********************************************************
/** If you can know that the result don't overflow and don't need MOD */
long binomial_Coeff_without_MOD(int n,int r) {
   long ans = 1;
   for(int i = 1;i<=min(n-k,k);i++) {
      ans = (ans* (n-(i-1)))/i;
   }
   return ans;
}
/* O(n) solutions
   Based in the prof of C(n,k) = C(n-1,k-1) + C(n-1,k)
   Also calc all C(n,i) for 0<=i<=n
*/
long binomial_Coeff(int n,int m) {
 int i,j;
 long bc[MAXN][MAXN];
 for (i=0; i<=n; i++) bc[i][0] = 1;
 for (j=0; j<=n; j++) bc[j][j] = 1;
 for (i=1; i<=n; i++)
    for (j=1; j<i; j++)
       bc[i][j] = bc[i-1][j-1] + bc[i-1][j];
 return bc[n][m];
}
/*
   O(k) solution
   Only calc C(n,k)
*/
int binomial_Coeff_2(int n, int k)  {
   int res = 1;
   if ( k > n - k )
      k = n - k;
   for (int i = 0; i < k; ++i) {
      res *= (n - i);
      res /= (i + 1);
    }
   return res;
}
/*  Factorial modulo P */
int factmod(int n, int p)  {
   int res = 1;
   while (n > 1)  {
      res = (res * ((n/p) % 2 ? p-1 : 1)) % p;
      for (int i = 2; i <= n%p; ++i)
         res = (res * i) % p;
      n /= p;
    }
   return res % p;
}
/** O(1) binomial coeficient with precalc in O(n) */
const int M = 1e6;
const lli mod = 986444681;
vector<lli> fact(M+1, 1), inv(M+1, 1), invfact(M+1, 1);
lli ncr(lli n, lli r) {
  if(r < 0 || r > n) return 0;
  return fact[n] * invfact[r] % mod * invfact[n - r] % mod;
}
void calc() {
   for(int i = 2; i <= M; ++i) {
   fact[i] = (lli)fact[i-1] * i % mod;
```

```
    inv[i] = mod - (lli)inv[mod % i] * (mod / i) % mod;
    invfact[i] = (lli)invfact[i-1] * inv[i] % mod;
  }
}
/** Lucas Theorem: Computes C(N,R)%p in O(log(n)) if P is prime */
/** call calc() first */
lli Lucas(lli N,lli R) {
  if(R<0||R>N)
    return 0;
  if(R==0||R==N)
    return 1ll;
  if(N>=mod)
    return (1ll*Lucas(N/mod,R/mod)*Lucas(N%mod,R%mod))%mod;
  return fact[n] * invfact[r] % mod * invfact[n - r] % mod;
}
/* Using calc() we can also calculate P(n,k) (permutations) */
lli permutation(int n,int k) {
    return (1ll*fact[n]* invfact[n-k])%mod;
}
/** Cayley's formula: Computes all posibles trees whit n nodes */
lli cayley(int n ,int k) {
    if(n-k-1<0)
        return (1ll*k*modpow(n,mod-2))%mod;
    return (1ll*k*modpow(n,n-k-1))%mod;
}
```

3.9 IntPerm

Description: Permutation -> integer conversion. (Not order preserving.) Integer -> permutation can use a lookup table.
Time: $\mathcal{O}(n)$

6 lines

```
int permToInt(vi& v) {
  int use = 0, i = 0, r = 0;
  for(int x:v) r = r * ++i + __builtin_popcount(use & -(1<<x)),
    use |= 1 << x;              // (note: minus, not ~!)
  return r;
}
```

3.10 StirlingFirst.cpp

108 lines

```
#include<bits/stdc++.h>
using namespace std;
const int N = 1 << 18;
const int mod = 998244353;
const int root = 3;
int lim, rev[N], w[N], wn[N], inv_lim;
void reduce(int &x) { x = (x + mod) % mod; }
int POW(int x, int y, int ans = 1) {
  for (; y; y >>= 1, x = (long long) x * x % mod) if (y & 1) ans = (long long) ans * x % mod;
  return ans;
}
void precompute(int len) {
  lim = wn[0] = 1; int s = -1;
  while (lim < len) lim <<= 1, ++s;
  for (int i = 0; i < lim; ++i) rev[i] = rev[i >> 1] >> 1 | (i & 1) << s;
  const int g = POW(root, (mod - 1) / lim);
  inv_lim = POW(lim, mod - 2);
  for (int i = 1; i < lim; ++i) wn[i] = (long long) wn[i - 1] * g % mod;
}
void ntt(vector<int> &a, int typ) {
  for (int i = 0; i < lim; ++i) if (i < rev[i]) swap(a[i], a[rev[i]]);
  for (int i = 1; i < lim; i <<= 1) {
    for (int j = 0, t = lim / i / 2; j < i; ++j) w[j] = wn[j * t];
    for (int j = 0; j < lim; j += i << 1)
      for (int k = 0; k < i; ++k) {
        const int x = a[k + j], y = (long long) a[k + j + i] * w[k] % mod;
        reduce(a[k + j] += y - mod), reduce(a[k + j + i] = x - y);
      }
  }
  if (!typ) {
    reverse(a.begin() + 1, a.begin() + lim);
    for (int i = 0; i < lim; ++i) a[i] = (long long) a[i] * inv_lim % mod ;
  }
}
vector<int> multiply(vector<int> &f, vector<int> &g) {
  int n=(int)f.size() + (int)g.size() - 1;
  precompute(n);
  vector<int> a = f, b = g;
  a.resize(lim); b.resize(lim);
  ntt(a, 1), ntt(b, 1);
  for (int i = 0; i < lim; ++i) a[i] = (long long) a[i] * b[i] % mod;
  ntt(a, 0);
  //while((int)a.size() && a.back() == 0) a.pop_back();
  return a;
}
int fact[N], ifact[N];
vector<int> shift(vector<int> &f, int c) { //f(x + c)
  int n=(int)f.size();
  precompute(n + n - 1);
  vector<int> a = f; a.resize(lim);
  for (int i = 0; i < n; ++i) a[i] = (long long) a[i] * fact[i] % mod;
  reverse(a.begin(), a.begin()+n);
  vector<int> b; b.resize(lim); b[0] = 1;
  for (int i = 1; i < n; ++i) b[i] = (long long) b[i - 1] * c % mod;
  for (int i = 0; i < n; ++i) b[i] = (long long) b[i] * ifact[i] % mod;
  ntt(a, 1), ntt(b, 1);
  for (int i = 0; i < lim; ++i) a[i] = (long long) a[i] * b[i] % mod;
  ntt(a, 0), reverse(a.begin(), a.begin() + n);
  vector<int> g; g.resize(n);
  for (int i = 0; i < n; ++i) g[i] = (long long) a[i] * ifact[i] % mod;
  return g;
}
// (x+1)*(x+2)*(x+3) ... (x+n)
// O(n log n) only for ntt friendly primes
// otherwise use divide and conquer in O(n log^2 n)
vector<int> range_mul(int n) {
  if (n == 0) return vector<int>( { 1 } );
  if (n & 1) {
    vector<int> f = range_mul(n - 1);
    f.push_back(0);
    for (int i = (int)f.size()-1; i; --i) f[i] = (f[i - 1] + (long long) n * f[i]) % mod;
    f[0] = (long long) f[0] * n % mod;
    return f;
  }
  else {
    int n_ = n >> 1;
    vector<int> f = range_mul(n_);
    vector<int> tmp = shift(f, n_);
    f.resize(n_ + 1);
    tmp.resize(n_ + 1);
    return multiply(f, tmp);
  }
}
// returns stirling1st(n, i) for 0 <= i <= n
vector<int> stirling(int n) {
  if (n == 0) return {1};
  vector<int> ans = range_mul(n - 1);
  ans.resize(n + 1);
  for (int i = n - 1; i >= 0; i--) {
    ans[i + 1] = ans[i];
  }
  ans[0] = 0;
  return ans;
}
int32_t main() {
  ios_base::sync_with_stdio(0);
  cin.tie(0);
  fact[0] = 1;
  for (int i = 1; i < N; ++i) fact[i] = (long long) fact[i - 1] * i % mod ;
  ifact[N - 1] = POW(fact[N - 1], mod - 2);
  for (int i = N - 1; i; --i) ifact[i - 1] = (long long) ifact[i] * i % mod;
  int n; cin >> n;
  auto ans = stirling(n);
  for (int i = 0; i <= n; i++) {
    cout << ans[i] << ' ';
  }
  return 0;
```

```
}
```

3.11 multinomial.h

Description: Computes $\binom{k_1+\cdots+k_n}{k_1,k_2,\ldots,k_n} = \frac{(\sum k_i)!}{k_1!k_2!...k_n!}$.

6 lines

```
ll multinomial(vi& v) {
 ll c = 1, m = v.empty() ? 1 : v[0];
 rep(i,1,sz(v)) rep(j,0,v[i])
  c = c * ++m / (j+1);
 return c;
}
```

number-theory (4)

4.1 CRT

Description: Chinese Remainder Theorem. crt(a, m, b, n) computes x such that $x \equiv a \pmod m$, $x \equiv b \pmod n$. If $|a| < m$ and $|b| < n$, x will obey $0 \le x < \mathrm{lcm}(m,n)$. Assumes $mn < 2^{62}$. given a set of congruence equations -> $a \equiv a_1(mod p_1)$ $a \equiv a_2(mod p_2)$ ... $a \equiv a_k(mod p_k)$ Return a if p_i are pairwise coprimes
Time: $\log(n)$

42 lines

```
// #include "ModInverse.h"
// #include "euclid.h"
int crt(int a, int m, int b, int n) {
 if (n > m) swap(a, b), swap(m, n);
 int x, y, g = euclid(m, n, x, y);
 assert((a - b) % g == 0); // else no solution
 x = (b - a) % n * x % n / g * m + a;
 return x < 0 ? x + m*n/g : x;
}
int lcm(int a,int b){
 return a*b/__gcd(a,b);
}
vector<int>nums;
vector<int>rem;
int CRT() {
  int prod = 1;
  for (int i = 0; i < nums.size(); i++)
    prod *= nums[i];
  int result = 0;
  for (int i = 0; i < nums.size(); i++) {
    int pp = prod / nums[i];
    result += rem[i] * inverse(pp, nums[i]) * pp;
  }
  return result % prod;
}
/** general CRT if pi,p2,p3 no coprimes, return 0 if no solution */
inline int normalize(int x, int mod) { x %= mod; if (x < 0) x += mod;
    ↪ return x; }
vector<int> a;
vector<int> p;
int LCM;
int CRT(int &ans) {
  int t =a.size();
  ans = a[0];
  LCM = p[0];
  for(int i = 1; i < t; i++) {
    int x1,d= gcd(LCM, p[i],x1,d);
    if((a[i] - ans) % d != 0) return 0;
    ans = normalize(ans + x1 * (a[i] - ans) / d % (p[i] / d) * LCM,
        ↪ LCM * p[i] / d);
    LCM = lcm(LCM, p[i]); // you can save time by replacing above LCM
        ↪ * n[i] /d by LCM = LCM * n[i] / d
  }
  return 1;
}
```

4.2 CarmichaelLambda.cpp

Description: lambda(n) is a smallest number that satisfies $a^l ambda(n) = 1(mod n)$ for all a coprime with n. This is also known as an universal totien tunction psi(n).

58 lines

```
typedef long long ll;
ll gcd(ll a, ll b)
{
 while (a) swap(a, b %= a);
 return b;
}
ll lcm(ll a, ll b)
{
 return a * (b / gcd(a, b));
}
ll carmichael_lambda(ll n)
{
 ll lambda = 1;
 if (n % 8 == 0)
  n /= 2;
 for (ll d = 2; d * d <= n; ++d)
  if (n % d == 0)
  {
   n /= d;
   ll y = d - 1;
   while (n % d == 0)
   {
    n /= d;
    y *= d;
   }
   lambda = lcm(lambda, y);
  }
 if (n > 1)
  lambda = lcm(lambda, n - 1);
 return lambda;
}
// lambda(n) for all n in [lo, hi)
vector<ll> carmichael_lambda(ll lo, ll hi)
{
 vector<ll> ps = primes(sqrt(hi) + 1);
 vector<ll> res(hi - lo), lambda(hi - lo, 1);
 iota(res.begin(), res.end(), lo);
 for (ll k = ((lo + 7) / 8) * 8; k < hi; k += 8)
  res[k - lo] /= 2;
 for (ll p : ps)
  for (ll k = ((lo + (p - 1)) / p) * p; k < hi; k += p)
  {
   if (res[k - lo] < p)
    continue;
   ll t = p - 1;
   res[k - lo] /= p;
   while (res[k - lo] > 1 && res[k - lo] % p == 0)
   {
    t *= p;
    res[k - lo] /= p;
   }
   lambda[k - lo] = lcm(lambda[k - lo], t);
  }
 for (ll k = lo; k < hi; ++k)
  if (res[k - lo] > 1)
   lambda[k - lo] = lcm(lambda[k - lo], res[k - lo] - 1);
 return lambda; // lambda[k-lo] = lambda(k)
}
```

4.3 ContinuedFractions

Description: Given N and a real number $x \ge 0$, finds the closest rational approximation p/q with $p, q \le N$. It will obey $|p/q - x| \le 1/qN$.
For consecutive convergents, $p_{k+1}q_k - q_{k+1}p_k = (-1)^k$. ($p_k/q_k$ alternates between $> x$ and $< x$.) If x is rational, y eventually becomes ∞; if x is the root of a degree 2 polynomial the a's eventually become cyclic.
Time: $\mathcal{O}(\log N)$

21 lines

```
typedef double d;
pair<ll, ll> approximate(d x, ll N) {
 ll LP = 0, LQ = 1, P = 1, Q = 0, inf = LLONG_MAX; d y = x;
 for (;;) {
  ll lim = min(P ? (N-LP) / P : inf, Q ? (N-LQ) / Q : inf),
     a = (ll)floor(y), b = min(a, lim),
     NP = b*P + LP, NQ = b*Q + LQ;
  if (a > b) {
   // If b > a/2, we have a semi-convergent that gives us a
   // better approximation; if b = a/2, we *may* have one.
   // Return {P, Q} here for a more canonical approximation.
```

```
    return (abs(x - (d)NP / (d)NQ) < abs(x - (d)P / (d)Q)) ?
      make_pair(NP, NQ) : make_pair(P, Q);
   }
  if (abs(y = 1/(y - (d)a)) > 3*N) {
    return {NP, NQ};
   }
  LP = P; P = NP;
  LQ = Q; Q = NQ;
 }
}
```

4.4 Eratosthenes

Description: Prime sieve for generating all primes up to a certain limit. isprime[i] is true iff i is a prime.
Time: lim=100'000'000 $\approx$ 0.8 s. Runs 30% faster if only odd indices are stored.

11 lines

```
const int MAX_PR = 5'000'000;
bitset<MAX_PR> isprime;
vi eratosthenesSieve(int lim) {
 isprime.set(); isprime[0] = isprime[1] = 0;
 for (int i = 4; i < lim; i += 2) isprime[i] = 0;
 for (int i = 3; i*i < lim; i += 2) if (isprime[i])
   for (int j = i*i; j < lim; j += i*2) isprime[j] = 0;
 vi pr;
 rep(i,2,lim) if (isprime[i]) pr.push_back(i);
 return pr;
}
```

4.5 Euclidjav.h

Description: Finds {x, y, d} s.t. ax + by = d = gcd(a, b).

11 lines

```
static BigInteger[] euclid(BigInteger a, BigInteger b) {
 BigInteger x = BigInteger.ONE, yy = x;
 BigInteger y = BigInteger.ZERO, xx = y;
 while (b.signum() != 0) {
   BigInteger q = a.divide(b), t = b;
   b = a.mod(b); a = t;
   t = xx; xx = x.subtract(q.multiply(xx)); x = t;
   t = yy; yy = y.subtract(q.multiply(yy)); y = t;
  }
 return new BigInteger[]{x, y, a};
}
```

4.6 Factor

Description: Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).
Time: $\mathcal{O}\left(n^{1/4}\right)$, less for numbers with small factors.

19 lines

```
// #include "MillerRabin.h"
ull pollard(ull n) {
 auto f = [n](ull x) { return modmul(x, x, n) + 1; };
 ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
 while (t++ % 40 || __gcd(prd, n) == 1) {
   if (x == y) x = ++i, y = f(x);
   if ((q = modmul(prd, max(x,y) - min(x,y), n))) prd = q;
   x = f(x), y = f(f(y));
  }
 return __gcd(prd, n);
}
vector<ull> factor(ull n) {
 if (n == 1) return {};
 if (isPrime(n)) return {n};
 ull x = pollard(n);
 auto l = factor(x), r = factor(n / x);
 l.insert(l.end(), all(r));
 return l;
}
```

4.7 FastCountDivisors

Description: Count the number of divisors of a large number
Time: $\mathcal{O}\left(n^{\frac{1}{3}}\right)$

95 lines

```
// #include "MillerRabin"
// #include "primes"
/**  Need primes[],lp[],N= $10^6 $ */
#define lli long long
bool isSquare(lli val){
 lli lo = 1, hi = val;
 while(lo <= hi){
   lli mid = lo + (hi - lo) / 2;
   lli tmp = (val / mid) / mid; // be careful with overflows!!
   if(tmp == 0)hi = mid - 1;
   else if(mid * mid == val)return true;
   else if(mid * mid < val)lo = mid + 1;
  }
 return false;
}
lli countDivisors(lli n) {
   lli ans = 1;
 for(int i = 0; i < primes.size(); i++){
   if(n == 1)break;
   int p = primes[i];
   if(n % p == 0){  // checks whether p is a divisor of n
     int num = 0;
     while(n % p == 0){
       n /= p;
       ++num;
     }
     // p^num divides initial n but $p^{num + 1}$ does not divide initial val
     // => p can be taken 0 to num times => num + 1 possibilities!!
     ans *= num + 1;
    }
  }
   if(n == 1)return ans; // first case
 else if(isPrime(n))return ans * 2; // second case
 else if(isSquare(n))return ans * 3; // third case but with p == q
 else return ans * 4; // third case with p != q
}
using uint32 = unsigned int;
using uint64 = unsigned long long;
using uint128 = __uint128_t;
// compute $\sum_{i=1}^{n} \sigma (i)$ in $~O(n^{1/3})$ time.
// it is also equal to \sum_{i=1}^{n} floor(n / i)
// takes ~100 ms for n = 1e18
uint128 sum_sigma0(uint64 n) {
 auto out = [n] (uint64 x, uint32 y) {
   return x * y > n;
  };
 auto cut = [n] (uint64 x, uint32 dx, uint32 dy) {
   return uint128(x) * x * dy >= uint128(n) * dx;
  };
 const uint64 sn = sqrtl(n);
 const uint64 cn = pow(n, 0.34); //cbrtl(n);
 uint64 x = n / sn;
 uint32 y = n / x + 1;
 uint128 ret = 0;
 stack<pair<uint32, uint32>> st;
 st.emplace(1, 0);
 st.emplace(1, 1);
 while (true) {
   uint32 lx, ly;
   tie(lx, ly) = st.top();
   st.pop();
   while (out(x + lx, y - ly)) {
   ret += x * ly + uint64(ly + 1) * (lx - 1) / 2;
   x += lx, y -= ly;
    }
   if (y <= cn) break;
   uint32 rx = lx, ry = ly;
   while (true) {
   tie(lx, ly) = st.top();
   if (out(x + lx, y - ly)) break;
   rx = lx, ry = ly;
   st.pop();
    }
   while (true) {
   uint32 mx = lx + rx, my = ly + ry;
```

```
    if (out(x + mx, y - my)) {
      st.emplace(lx = mx, ly = my);
    }
    else {
      if (cut(x + mx, lx, ly)) break;
      rx = mx, ry = my;
    }
    }
  }
  for (--y; y > 0; --y) ret += n / y;
  return ret * 2 - sn * sn;
}
auto ans = sum_sigma0(n);
string s = "";
while (ans > 0) {
  s += char('0' + ans % 10);
  ans /= 10;
}
reverse(s.begin(), s.end());
cout << s << '\n';
```

4.8 FastEratosthenes

Description: Prime sieve for generating all primes smaller than LIM.
Time: LIM=1e9 ≈ 1.5s also other fast sieves for diferent purposes

107 lines

```
const int LIM = 1e7;
bitset<LIM> isPrime;
vector<int> eratosthenes() {
  const int S = (int)round(sqrt(LIM)), R = LIM / 2;
  vector<int> pr = {2}, sieve(S+1); pr.reserve(int(LIM/log(LIM)*1.1));
  vector<pii> cp;
  for (int i = 3; i <= S; i += 2) if (!sieve[i]) {
    cp.push_back({i, i * i / 2});
    for (int j = i * i; j <= S; j += 2 * i) sieve[j] = 1;
  }
  for (int L = 1; L <= R; L += S) {
    array<bool, S> block{};
    for (auto &[p, idx] : cp)
      for (int i=idx; i < S+L; idx = (i+=p)) block[i-L] = 1;
    rep(i,0,min(S, R - L))
      if (!block[i]) pr.push_back((L + i) * 2 + 1);
  }
  for (int i : pr) isPrime[i] = 1;
  return pr;
}
// More easy linear sieve also gets sieve of function $\mu$
const int N = 1000007;
vector<int> m(N+1);
void criba() {
  vector<int> lp(N+1);
  vector<int> primes;
  m[1]= 1;
  for(int i = 2;i<=N;i++) {
    if(lp[i]== 0) {
      primes.push_back(i);
      lp[i]= i;
      m[i] = -1;
    }
    for(int j = 0;j<primes.size()&& primes[j]<=lp[i] && primes[j]*i<=N;j++) {
      lp[primes[j]*i] = primes[j];
      if(lp[i]==primes[j])m[primes[j]*i]= 0;
      else m[primes[j]*i] = m[primes[j]]*m[i];
    }
  }
}
// Greatest prime sieve
vector<int> gp;
void greatestPrimeSieve(int n) {
  gp.resize(n + 1, 1);
  gp[0] = gp[1] = 0;
  for(int i = 2; i <= n; ++i) gp[i] = i;
  for(int i = 2; i <= n; i++)
    if(gp[i] == i)
      for(int j = i; j <= n; j += i)
        gp[j] = i;
  }
// Segmented sieve , get primes in range[L,R] with complexity O(max(sqrt(R)log(sqrt(R)),R-L)) ??? ;
// Also is one of the fastest sieve get all primes in range [1-n] with n = $1^9$ in 8.62s and for
// n = $1^8$ in 0.76s
vector<int> PrimesInRange;
void calcPrimes(int l ,int r) {
  auto sum = l<=2?2:0;
  if(l<=2)PrimesInRange.push_back(2);
  int cnt = 1;
  const int S = round(sqrt(r));
  vector<char> sieve(S + 1, true);
  vector<array<int, 2>> cp;
  for (int i = 3; i <= S; i += 2) {
    if (!sieve[i])
      continue;
    cp.push_back({i, (i * i - 1) / 2});
    for (int j = i * i; j <= S; j += 2 * i){
      sieve[j] = false;
    }
  }
  vector<char> block(S);
  int high = (r - 1) / 2;
  int x = l/S;
  int L = (x/2)*S;
  for(auto &i:cp) {
    int p = i[0],idx = i[1];
    if(idx>L) {
      i[1]-=L;
    }
    else {
      int X = (L-idx)/p;
      if((L-idx)%p)X++;
      if(X>=1 && idx<=L)
        i[1] = (idx+(p*X))-L;
    }
  }
  for (int low =(x/2)*S; low <= high; low += S) {
    fill(block.begin(), block.end(), true);
    for (auto &i : cp) {
      int p = i[0], idx = i[1];
      for (; idx < S; idx += p){
        block[idx] = false;
      }
      i[1] = idx - S;
    }
    if (low == 0)
      block[0] = false;
    for (int i = 0; i < S && low + i <= high; i++){
      if (block[i] && (((low+i)*2)+1)>=l){
        // push the primes here if needed
        ++cnt, sum += (low + i) * 2 + 1;
      }
    }
  };
  // cout << "sum = " << sum << endl;
  // cout << "cnt = " << cnt << endl;
}
```

4.9 FracBinarySearch

Description: Given f and N, finds the smallest fraction $p/q \in [0,1]$ such that $f(p/q)$ is true, and $p,q \leq N$. You may want to throw an exception from f if it finds an exact solution, in which case N can be removed.
Usage: `fracBS([](Frac f) { return f.p>=3*f.q; }, 10); // {1,3}`
Time: $\mathcal{O}(\log(N))$

24 lines

```
struct Frac { ll p, q; };
template<class F>
Frac fracBS(F f, ll N) {
  bool dir = 1, A = 1, B = 1;
  Frac lo{0, 1}, hi{1, 1}; // Set hi to 1/0 to search (0, N]
  if (f(lo)) return lo;
  assert(f(hi));
  while (A || B) {
    ll adv = 0, step = 1; // move hi if dir, else lo
```

```
    for (int si = 0; step; (step *= 2) >>= si) {
      adv += step;
      Frac mid{lo.p * adv + hi.p, lo.q * adv + hi.q};
      if (abs(mid.p) > N || mid.q > N || dir == !f(mid)) {
        adv -= step; si = 2;
      }
    }
    hi.p += lo.p * adv;
    hi.q += lo.q * adv;
    dir = !dir;
    swap(lo, hi);
    A = B; B = !!adv;
  }
  return dir ? hi : lo;
}
```

4.10 LinealDiophantine.cpp

Description: A Linear Diophantine Equation (in two variables) is an equation of the general form: $ax + by = c$ where a,b,c are given integers, and , are unknown integers.

63 lines

```
int gcd(int a, int b, int& x, int& y) {
    if (b == 0) {
        x = 1;
        y = 0;
        return a;
    }
    int x1, y1;
    int d = gcd(b, a % b, x1, y1);
    x = y1;
    y = x1 - y1 * (a / b);
    return d;
}
bool find_any_solution(int a, int b, int c, int &x0, int &y0, int &g) {
    g = gcd(abs(a), abs(b), x0, y0);
    if (c % g) {
        return false;
    }
    x0 *= c / g;
    y0 *= c / g;
    if (a < 0) x0 = -x0;
    if (b < 0) y0 = -y0;
    return true;
}
void shift_solution(int & x, int & y, int a, int b, int cnt) {
    x += cnt * b;
    y -= cnt * a;
}
int find_all_solutions(int a, int b, int c, int minx, int maxx, int miny, int maxy) {
    int x, y, g;
    if (!find_any_solution(a, b, c, x, y, g))
        return -1;
    a /= g;
    b /= g;
    int sign_a = a > 0 ? +1 : -1;
    int sign_b = b > 0 ? +1 : -1;
    shift_solution(x, y, a, b, (minx - x) / b);
    if (x < minx)
        shift_solution(x, y, a, b, sign_b);
    if (x > maxx)
        return -1;
    int lx1 = x;
    shift_solution(x, y, a, b, (maxx - x) / b);
    if (x > maxx)
        shift_solution(x, y, a, b, -sign_b);
    int rx1 = x;
    shift_solution(x, y, a, b, -(miny - y) / a);
    if (y < miny)
        shift_solution(x, y, a, b, -sign_a);
    if (y > maxy)
        return -1;
    int lx2 = x;
    shift_solution(x, y, a, b, -(maxy - y) / a);
    if (y > maxy)
        shift_solution(x, y, a, b, sign_a);
    int rx2 = x;
    if (lx2 > rx2)
        swap(lx2, rx2);
    int lx = max(lx1, lx2);
    int rx = min(rx1, rx2);
    if (lx > rx)
        return -1;
    return lx;
}
```

4.11 MillerRabin

Description: Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to $7 \cdot 10^{18}$; for larger numbers, use Python and extend A randomly.
Time: 7 times the complexity of $a^b \mod c$.

23 lines

```
typedef unsigned long long ull;
ull modmul(ull a, ull b, ull M) {
  ll ret = a * b - M * ull(1.L / M * a * b);
  return ret + M * (ret < 0) - M * (ret >= (ll)M);
}
ull modpow(ull b, ull e, ull mod) {
  ull ans = 1;
  for (; e; b = modmul(b, b, mod), e /= 2)
    if (e & 1) ans = modmul(ans, b, mod);
  return ans;
}
bool isPrime(ull n) {
  if (n < 2 || n % 6 % 4 != 1) return (n | 1) == 3;
  ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022},
      s = __builtin_ctzll(n-1), d = n >> s;
  for (ull a : A) { // ^count trailing zeroes
    ull p = modpow(a%n, d, n), i = s;
    while (p != 1 && p != n - 1 && a % n && i--)
      p = modmul(p, p, n);
    if (p != n-1 && i != s) return 0;
  }
  return 1;
}
```

4.12 ModFloorDivision

Description: Sum of aritmetic floor division
`f(a, b, c, n)` $= \sum_{i=0}^{n} \lfloor \frac{(ai+b)}{c} \rfloor$.
Time: $\log(a)$,

16 lines

```
int f(int a, int b, int c, int n){
  int m = (a*n + b)/c;
  if(n==0 || m==0) return b/c;
  if(n==1) return b/c + (a+b)/c;
  if(a<c && b<c) return m*n - f(c, c-b-1, a, m-1);
  else return (a/c)*n*(n+1)/2 + (b/c)*(n+1) + f(a%c, b%c, c, n);
}
// $\sum_{k=1}^{n}{\lfloor \frac{n}{k} \rfloor)}$
int floor_sum(int n) {
  int sum = 0;
  for (int i = 1, last; i <= n; i = last + 1) {
    last = n / (n / i);
    sum += (n / i) * (last - i + 1);
  }
  return sum;
}
```

4.13 ModInverse.h

Description: Pre-computation of modular inverses. Assumes LIM $\leq$ mod and that mod is a prime.

15 lines

```
int inverse(int a, int m){
  int x, y ;
    if isPrime(m)return mod_pow(a,m-2,m);
  if(gcd( a, m, x, y ) != 1) return 0; // not all numbers has inverse modulo m
  return (x%m + m) % m;
}
/** All inverse (1 to p-1)%p p is prime*/
```

```
vector<int> allinverse(int p) {
   vector<int> ans(p);
   ans[1] = 1;
   for(int i = 2;i<p;i++) {
      ans[i] = p-(p/i)*ans[p%i]%p;
   }
   return ans;
}
```

4.14 ModLog

Description: Returns the smallest $x > 0$ s.t. $a^x = b \pmod{m}$, or -1 if no such x exists. modLog(a,1,m) can be used to calculate the order of a.
Time: $\mathcal{O}\left(\sqrt{m}\right)$

11 lines

```
ll modLog(ll a, ll b, ll m) {
  ll n = (ll) sqrt(m) + 1, e = 1, f = 1, j = 1;
  unordered_map<ll, ll> A;
  while (j <= n && (e = f = e * a % m) != b % m)
    A[e * b % m] = j++;
  if (e == b % m) return j;
  if (__gcd(m, e) == __gcd(m, b))
    rep(i,2,n+2) if (A.count(e = e * f % m))
      return n * i - A[e];
  return -1;
}
```

4.15 ModMulLL

Description: Calculate $a \cdot b \bmod c$ (or $a^b \bmod c$) for $0 \le a, b \le c \le 7.2 \cdot 10^{18}$.
Time: $\mathcal{O}(1)$ for `modmul`, $\mathcal{O}(\log b)$ for `modpow`

11 lines

```
typedef unsigned long long ull;
ull modmul(ull a, ull b, ull M) {
  ll ret = a * b - M * ull(1.L / M * a * b);
  return ret + M * (ret < 0) - M * (ret >= (ll)M);
}
ull modpow(ull b, ull e, ull mod) {
  ull ans = 1;
  for (; e; b = modmul(b, b, mod), e /= 2)
    if (e & 1) ans = modmul(ans, b, mod);
  return ans;
}
```

4.16 ModPow.h

10 lines

```
const int mod = 1e9+7;
int modpow(int a,int b) {
  int x = 1;
  while(b) {
    if(b&1) (x*=a)%=mod;
    (a*=a)%=mod;
    b>>=1;
  }
  return x;
}
```

4.17 ModSqrt

Description: Tonelli-Shanks algorithm for modular square roots. Finds x s.t. $x^2 = a \pmod{p}$ ($-x$ gives the other solution).
Time: $\mathcal{O}\left(\log^2 p\right)$ worst case, $\mathcal{O}(\log p)$ for most p

26 lines

```
// #include "ModPow.h"
ll sqrt(ll a, ll p) {
  a %= p; if (a < 0) a += p;
  if (a == 0) return 0;
  assert(modpow(a, (p-1)/2, p) == 1); // else no solution
  if (p % 4 == 3) return modpow(a, (p+1)/4, p);
  // $\frac{a^{n+3}}{8}$ or $\frac{2^{n+3}}{8} * \frac{2^{n-1}}{4}$ works if p % 8 == 5
  ll s = p - 1, n = 2;
  int r = 0, m;
  while (s % 2 == 0)
    ++r, s /= 2;
  /// find a non-square mod p
  while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
  ll x = modpow(a, (s + 1) / 2, p);
  ll b = modpow(a, s, p), g = modpow(n, s, p);
  for (;; r = m) {
    ll t = b;
    for (m = 0; m < r && t != 1; ++m)
      t = t * t % p;
    if (m == 0) return x;
    ll gs = modpow(g, 1LL << (r - m - 1), p);
    g = gs * gs % p;
    x = x * gs % p;
    b = b * g % p;
  }
}
```

4.18 ModSum

Description: Sums of mod'ed arithmetic progressions.
`f(a, b, c, n)` $= \sum_{i=0}^{\text{to}-1} (ki + c)\%m$.
Time: $\log(m)$, with a large constant.

15 lines

```
typedef unsigned long long ull;
ull sumsq(ull to) { return to / 2 * ((to-1) | 1); }
/// ^written in a weird way to deal with overflows correctly
ull divsum(ull to, ull c, ull k, ull m) {
  ull res = k / m * sumsq(to) + c / m * to;
  k %= m; c %= m;
  if (!k) return res;
  ull to2 = (to * k + c) / m;
  return res + (to - 1) * to2 - divsum(to2, m-1 - c, m, k);
}
ll modsum(ull to, ll c, ll k, ll m) {
  c = ((c % m) + m) % m;
  k = ((k % m) + m) % m;
  return to * c + k * sumsq(to) - m * divsum(to, c, k, m);
}
```

4.19 ModularArithmetic.h

Description: Operators for modular arithmetic. You need to set `mod` to some number first and then you can use the structure.

19 lines

```
#include "euclid.h"
const ll mod = 17; // change to something else
struct Mod {
  ll x;
  Mod(ll xx) : x(xx) {}
  Mod operator+(Mod b) { return Mod((x + b.x) % mod); }
  Mod operator-(Mod b) { return Mod((x - b.x + mod) % mod); }
  Mod operator*(Mod b) { return Mod((x * b.x) % mod); }
  Mod operator/(Mod b) { return *this * invert(b); }
  Mod invert(Mod a) {
    ll x, y, g = euclid(a.x, mod, x, y);
    assert(g == 1); return Mod((x + mod) % mod);
  }
  Mod operator^(ll e) {
    if (!e) return Mod(1);
    Mod r = *this ^(e / 2); r = r * r;
    return e&1 ? *this * r : r;
  }
};
```

4.20 NumberTheory.cpp

428 lines

```
 #include <bits/stdc++.h>
using namespace std;
using Integer128 = __int128;
typedef __int128 Intsote;
typedef long long int lli;
// ______________________________INTSOTE______________________________ //
/*
   Función para imprimir el __int128
*/
ostream& operator<<( ostream& dest, __int128_t value ) {
```

```
  ostream::sentry s( dest );
  if ( s ) {
    __uint128_t tmp = value < 0 ? -value : value;
    char buffer[ 128 ];
    char* d = std::end( buffer );
    do {
      -- d;
      *d = "0123456789"[ tmp % 10 ];
      tmp /= 10;
    } while ( tmp != 0 );
    if ( value < 0 ) {
      -- d;
      *d = '-';
    }
    int len = end( buffer ) - d;
    if ( dest.rdbuf()->sputn( d, len ) != len ) {
      dest.setstate( ios_base::badbit );
    }
  }
  return dest;
}
//__________Exponenciacion y multiplicacion modular_________//
/*
 [Tested Timus 1141,1204, uva 1230,374,11029]
*/
//Ed1.- Do Not use mod_mult if it gives TLE
//Ed2.- Usually in small base and exponent
//Ed3.- Al chile ya no lo uses
lli mod_mult(lli a, lli b, lli mod) {
 lli x = 0;
 while(b) {
  if(b & 1) x = (x + a) % mod;
  a = (a << 1) % mod;
  b >>= 1;
 }
 return x;
}
lli mod_pow(lli a, lli n, lli mod) {
 lli x = 1;
 while(n) {
  if(n & 1) x = mod_mult(x, a, mod);
  a = mod_mult(a, a, mod);
  n >>= 1;
 }
 return x;
}
/*
  C++ is stupid and pow(10,2) is 99
  better use mod_pow whit mod very large
*/
int trail(lli a,lli b,int n) {
  lli ntrail = mod_pow(10,n,10000000000);
  return mod_pow(a,b,ntrail);
}
int leading(lli a,lli b,lli n) {
  n--;
  lli nleading = mod_pow(10,n,10000000000);
  return (int)(pow(10, fmod(b*log10(a), 1))*nleading);
}
//_________Euclides extendido___________//
/*
  Solve ax+by = (a,b)
 // Algoritmo de Euclides extendido entre a y b. Ademas de devolver el
     ↪ gcd(a, b), resuelve la ecuación diofantica con el par (x, y).
 //Si el parametro mod no es especificado, se resuelve con aritmetica
     ↪ normal; si mod se especifica, la identidad se resuelve modulo
     ↪ mod.
 [Tested Timus 1141,1204]
*/
int gcd(int a,int b,int &x,int &y) {
 if(b==0) { x = 1; y = 0; return a; }
 int r = gcd(b, a%b, y, x);
 y -= a/b*x;
 return r;
}
//________Inverso modular_________//
int inverse(int a, int m) {
 int x, y ;
  if isPrime(m)return mod_pow(a,m-2,m);
 if(gcd( a, m, x, y ) != 1) return 0;
 return (x%m + m) % m;
}
/*
  All inverse (1 to p-1)%p
*/
vector<lli> allinverse(lli p) {
  vector<lli> ans(p);
  ans[1] = 1;
  for(lli i = 2;i<p;i++) {
    ans[i] = p-(p/i)*ans[p%i]%p;
  }
  return ans;
}
//__________Linear Diophantine Equation__________//
/*
  Use gcd -Extended euclides-
  Solve ax+by=c
  -Find any solution
  -Getting all solutions
  -Finding the number of solutions and the solutions in a given
      ↪ interval
  -Find the solution with minimum value of x+y
  [Tested Spoj - Crucial Equation,SGU 106,Codeforces - Ebony and Ivory,
      ↪ Codechef
  - Get AC in one go, LightOj - Solutions to an equation]
*/
bool find_any_solution(int a, int b, int c, int &x0, int &y0, int &g) {
  g = gcd(abs(a), abs(b), x0, y0);
  if (c % g)
    return false;
  x0 *= c / g;
  y0 *= c / g;
  if (a < 0) x0 = -x0;
  if (b < 0) y0 = -y0;
  return true;
}
//___________PHI de euler_____________//
int phi(int n) {
  int result = n;
  for (int i = 2; i * i <= n; i++) {
    if(n % i == 0) {
      while(n % i == 0)
        n /= i;
      result -= result / i;
    }
  }
  if(n > 1)
    result -= result / n;
  return result;
}
//_________GCD and LCM__________//
/*
  [Tested ??]
*/
lli gcd(vector<lli> &nums) {
  lli ans =0;
  for(lli &num:nums)ans =__gcd(ans,num);
  return ans;
}
lli lcm(lli a,lli b) {
  return b*(a/__gcd(a,b));
}
lli lcm(vector<lli> & nums) {
  lli ans = 1;
  for(lli & num : nums) ans = lcm(ans, num);
  return ans;
}
//_________Teorema chino del residuo____________//
/*
  [Tested ???]
  if p1,p2,p3 are coprime
*/
vector<int>nums;
```

```cpp
vector<int>rem;
int CRT() {
   int prod = 1;
   for (int i = 0; i < nums.size(); i++)
      prod *= nums[i];
   int result = 0;
   for (int i = 0; i < nums.size(); i++) {
      int pp = prod / nums[i];
      result += rem[i] * inverse(pp, nums[i]) * pp;
   }
   return result % prod;
}
/*
   general CRT if pi,p2,p3 no coprimes
   return 0 if no solution
*/
inline lli normalize(lli x, lli mod) { x %= mod; if (x < 0) x += mod; return x; }
vector<int> a;
vector<int> n;
lli LCM;
lli CRT(lli &ans) {
   int t =a.size();
   ans = a[0];
   LCM = n[0];
   for(int i = 1; i < t; i++) {
      int x1,d= gcd(LCM, n[i],x1,d);
      if((a[i] - ans) % d != 0) return 0;
      ans = normalize(ans + x1 * (a[i] - ans) / d % (n[i] / d) * LCM, LCM * n[i] / d);
      LCM = lcm(LCM, n[i]); // you can save time by replacing above LCM * n[i] /d by LCM = LCM * n[i] / d
   }
   return 1;
}
//__________Factorial modulo p___________//
/*
   O(P logp n)
*/
int factmod(int n, int p) {
   int res = 1;
   while (n > 1) {
      res = (res * ((n/p) % 2 ? p-1 : 1)) % p;
      for (int i = 2; i <= n%p; ++i)
         res = (res * i) % p;
      n /= p;
   }
   return res % p;
}
//________Discrete Log________//
int dlog(int a, int b, int m) {
   int n = (int) sqrt (m + .0) + 1;
   int an = 1;
   for (int i = 0; i < n; ++i)
      an = (an * a) % m;
   map<int, int> vals;
   for (int p = 1, cur = an; p <= n; ++p) {
      if (!vals.count(cur))
         vals[cur] = p;
      cur = (cur * an) % m;
   }
   for (int q = 0, cur = b; q <= n; ++q) {
      if (vals.count(cur)) {
         int ans = vals[cur] * n - q;
         return ans;
      }
      cur = (cur * a) % m;
   }
   return -1;
}
int powmod(int a, int b, int m) {
   int res = 1;
   while (b > 0) {
      if (b & 1) {
         res = (res * 1ll * a) % m;
      }
      a = (a * 1ll * a) % m;
      b >>= 1;
   }
   return res;
}
int dislog(int a, int b, int m) {
   int n = (int) sqrt (m + .0) + 1;
   map<int, int> vals;
   for (int p = n; p >= 1; --p)
      vals[powmod(a, p * n, m)] = p;
   for (int q = 0; q <= n; ++q) {
      int cur = (powmod(a, q, m) * 1ll * b) % m;
      if (vals.count(cur)) {
         int ans = vals[cur] * n - q;
         return ans;
      }
   }
   return -1;
}
//_________Discrete root__________//
int generator(int p) {
   vector<int> fact;
   int phi = p-1, n = phi;
   for (int i = 2; i * i <= n; ++i) {
      if (n % i == 0) {
         fact.push_back(i);
         while (n % i == 0)
            n /= i;
      }
   }
   if (n > 1)
      fact.push_back(n);
   for (int res = 2; res <= p; ++res) {
      bool ok = true;
      for (int factor : fact) {
         if (mod_pow(res, phi / factor, p) == 1) {
            ok = false;
            break;
         }
      }
      if (ok) return res;
   }
   return -1;
}
void discrete_root(int n, int k ,int a) {
   int g = generator(n);
   // Baby-step giant-step discrete logarithm algorithm
   int sq = (int) sqrt (n + .0) + 1;
   vector<pair<int, int>> dec(sq);
   for (int i = 1; i <= sq; ++i)
      dec[i-1] = {mod_pow(g, i * sq * k % (n - 1), n), i};
   sort(dec.begin(), dec.end());
   int any_ans = -1;
   for (int i = 0; i < sq; ++i) {
      int my = mod_pow(g, i * k % (n - 1), n) * a % n;
      auto it = lower_bound(dec.begin(), dec.end(), make_pair(my, 0));
      if (it != dec.end() && it->first == my) {
         any_ans = it->second * sq - i;
         break;
      }
   }
   if (any_ans == -1) {
      puts("0");
      return 0;
   }
   // Print all possible answers
   int delta = (n-1) / gcd(k, n-1);
   vector<int> ans;
   for (int cur = any_ans % delta; cur < n-1; cur += delta)
      ans.push_back(mod_pow(g, cur, n));
   sort(ans.begin(), ans.end());
   printf("%d\n", ans.size());
   for (int answer : ans)
      printf("%d ", answer);
}
//__________________________Oerador moudulo_________________________//
//(A + B) mod C = (A mod C + B mod C) mod C
```

```cpp
//(A * B) mod C = (A mod C * B mod C) mod C
// xy (mod a) ≡((x (mod a) * y) (mod a))
// A^B mod M = ( A^(M-1) * A^(M-1) *.......* A^(M-1) * A^(x) ) mod M
// a^(p-1) mod p = 1, When p is prime.
//Mod for negatives,also work in positives ,(a%mod+mod)%mod;
//________________Trailing zeors in factorial in any base______________//
/*
  [Tested Codeforces round 538-C]
*/
lli trail_fact(lli n,lli b) {
  lli ans = 1000000000000000000LL;
 for (lli i=2; i<=b; i++)  {
  if (1LL * i * i > b) i = b;
  int cnt = 0;
  while (b % i == 0)  { b /= i; cnt++; }
  if (cnt == 0) continue;
  lli tmp = 0, mul = 1;
  while (mul <= n / i)  { mul *= i; tmp += n / mul; }
  ans = min(ans, tmp / cnt);
 }
  return ans;
}
int gcd(int a, int b, int& x, int& y)  {
  if (b == 0)  {
    x = 1;
    y = 0;
    return a;
   }
  int x1, y1;
  int d = gcd(b, a % b, x1, y1);
  x = y1;
  y = x1 - y1 * (a / b);
  return d;
}
bool find_any_solution(int a, int b, int c, int &x0, int &y0, int &g)  {
  g = gcd(abs(a), abs(b), x0, y0);
  if (c % g)  {
    return false;
   }
  x0 *= c / g;
  y0 *= c / g;
  if (a < 0) x0 = -x0;
  if (b < 0) y0 = -y0;
  return true;
}
void shift_solution(int & x, int & y, int a, int b, int cnt)  {
  x += cnt * b;
  y -= cnt * a;
}
int find_all_solutions(int a, int b, int c, int minx, int maxx, int miny, int maxy)  {
  int x, y, g;
  if (!find_any_solution(a, b, c, x, y, g))
    return -1;
  a /= g;
  b /= g;
  int sign_a = a > 0 ? +1 : -1;
  int sign_b = b > 0 ? +1 : -1;
  shift_solution(x, y, a, b, (minx - x) / b);
  if (x < minx)
    shift_solution(x, y, a, b, sign_b);
  if (x > maxx)
    return -1;
  int lx1 = x;
  shift_solution(x, y, a, b, (maxx - x) / b);
  if (x > maxx)
    shift_solution(x, y, a, b, -sign_b);
  int rx1 = x;
  shift_solution(x, y, a, b, -(miny - y) / a);
  if (y < miny)
    shift_solution(x, y, a, b, -sign_a);
  if (y > maxy)
    return -1;
  int lx2 = x;
  shift_solution(x, y, a, b, -(maxy - y) / a);
  if (y > maxy)
    shift_solution(x, y, a, b, sign_a);
  int rx2 = x;
  if (lx2 > rx2)
    swap(lx2, rx2);
  int lx = max(lx1, lx2);
  int rx = min(rx1, rx2);
  if (lx > rx)
    return -1;
  return lx;
}
int gauss(int n) {
  return (n*(n+1))/2;
}
// sum of floor((p*i)/q), 1<=i<=n
int f(int p, int q, int n)  {
  int ans = gauss(n) * (p/q);
  cout<<p<<" "<<q<<" "<<n<<" "<<ans<<endl;
  p %= q;
  if (p != 0)  {
    int N = (p*n)/q;
    cout<<N<<endl;
    ans += n * N - f(q, p, N) + N/p;
   }
  return ans;
}
// sum of floor((a*i+b)/c), 1<=i<=n
lli f(lli a, lli b, lli c, lli n) {
 lli m = (a*n + b)/c;
 if(n==0 || m==0) return b/c;
 if(n==1) return b/c + (a+b)/c;
 if(a<c && b<c) return m*n - f(c, c-b-1, a, m-1);
 else return (a/c)*n*(n+1)/2 + (b/c)*(n+1) + f(a%c, b%c, c, n);
}
int main() {
 // Intsote a = 1ll;
  int n ,k, a;
 // for( int i = 0; i < 100; i++ )
 //  a <<= 1;
 // cout << a << endl;
 // cout<<mul_inv(42, 2017)<<endl;
  cin>>n>>k>>a;
  discrete_root(n,k,a);
 return 0;
}
```

4.21 PollarRho.cpp

135 lines

```cpp
#include <bits/stdc++.h>
using namespace std;
#define endl "\n"
#define __ ios_base::sync_with_stdio(false);cin.tie(NULL);
typedef unsigned long long int ull;
typedef long long int lli;
ull mulMod( ull a,ull b,ull m) {
  ull res = 0, tmp = a % m;
  while (b) {
    if (b & 1) {
      res = res + tmp;
      res = (res >= m ? res - m : res);
     }
    b >>= 1;
    tmp <<= 1;
    tmp = (tmp >= m ? tmp - m : tmp);
   }
  return res;
}
lli powMod(lli a,lli b,lli m) {
  lli res = 1 % m, tmp = a % m;
  while (b) {
    if (b & 1)
      res = mulMod(res, tmp, m);
    tmp = mulMod(tmp, tmp, m);
    b >>= 1;
   }
```

```cpp
    return res;
}
bool millerRabin(lli n) {
    int a[5] = { 2, 3, 7, 61, 24251 };
    if (n == 2) return true;
    if (n == 1 || (n & 1) == 0) return false;
    lli b = n - 1;
    for (int i = 0; i < 5; i++) {
        if (a[i] >= n) break;
        while ((b & 1) == 0) b >>= 1;
        lli t = powMod(a[i], b, n);
        while (b != n - 1 && t != 1 && t != n - 1) {
            t = mulMod(t, t, n);
            b <<= 1;
        }
        if (t == n - 1 || (b & 1)) continue;
        else return false;
    }
    return true;
}
lli gcd(lli a, lli b) {
    while (b > 0) {
        lli t = a % b;
        a = b;
        b = t;
    }
    return a;
}
lli pollard_rho(lli n) {
    lli x = 2 % n, y = x, k = 2, i = 1;
    lli d = 1;
    while (true) {
        i++;
        x = (mulMod(x, x, n) + 1) % n;
        d = gcd((y - x + n) % n, n);
        if (d > 1 && d < n)
            return d;
        if (y == x) {
            lli d = 2;
            while (n % d) d++;
            return d;
        }
        if (i == k) {
            y = x;
            k <<= 1;
        }
    }
}
lli factors[1000], fCount;
void _factorize(lli n) {
    if (n <= 1) return;
    if (millerRabin(n)) {
        factors[ fCount++ ] = n;
        return;
    }
    lli d = pollard_rho(n);
    _factorize(d);
    _factorize(n/d);
}
void factorize(lli n) {
    fCount = 0;
    _factorize(n);
    sort(factors, factors + fCount);
}
vector<lli> getDivs(const vector<pair<lli,lli>> &factors)
{
    int n = SZ(factors);
    int factors_count = 1;
    for (int i = 0; i < n; ++i)
    {
        factors_count *= 1+factors[i].second;
    }
    vector<lli> divs(factors_count); divs[0] = 1;
    int count = 1;
    for (int stack_level = 0; stack_level < n; ++stack_level)
    {
        int count_so_far = count;
        int prime = factors[stack_level].first;
        int exponent = factors[stack_level].second;
        int multiplier = 1;
        for (int j = 0; j < exponent; ++j)
        {
            multiplier *= prime;
            for (int i = 0; i < count_so_far; ++i)
            {
                divs[count++] = divs[i] * multiplier;
            }
        }
    }
    return divs;
}
int main() {
    int t;
    cin >> t;
    while (t--) {
        lli n;
        cin >> n;
        factorize(n);
        for (int i = 0; i < fCount; i++) {
            if(i==fCount-1)
                cout<<factors[i];
            else
                cout<<factors[i]<<" ";
        }
        cout<<endl;
    }
    return 0;
}
```

4.22 PrimeBasis.cpp

Description: A prime basis for a set of numbers , it defines an array basis such that every number can written as $\Pi basis[i]^{f_i}$ for some f and all operations like *,/, gcd, lcm works same as if you have the real prime factorization.

111 lines

```cpp
template<typename T>
struct PrimeBasis {
    void reduce_pair(T& x, T& y) {
        bool to_swap = 0;
        if(x > y) {
            to_swap ^= 1;
            swap(x, y);
        }
        while(x > 1 && y % x == 0) {
            y /= x;
            if(x > y) {
                to_swap ^= 1;
                swap(x, y);
            }
        }
        if(to_swap) swap(x, y);
    }
    vector<T> basis;
    void solve_inner(int pos, T &val) {
        while(basis[pos] % val == 0) basis[pos] /= val;
        vector<T> curr_basis = { basis[pos], val };
        int c_ptr = 1;
        while(c_ptr < curr_basis.size()) {
            for(int i=0;i<c_ptr;i++) {
                reduce_pair(curr_basis[i], curr_basis[c_ptr]);
                if(curr_basis[c_ptr] == 1) break;
                if(curr_basis[i] == 1) continue;
                T g = gcd(curr_basis[c_ptr], curr_basis[i]);
                if(g > 1) {
                    curr_basis[c_ptr] /= g;
                    curr_basis[i] /= g;
                    curr_basis.push_back(g);
                }
            }
            ++c_ptr;
        }
        basis[pos] = curr_basis[0];
        val = curr_basis[1];
```

```cpp
    for(int i=2;i<curr_basis.size();i++) if(curr_basis[i] > 1) basis.
        ↪ push_back(curr_basis[i]);
    if(basis[pos] == 1) {
      swap(basis[pos], basis.back());
      basis.pop_back();
    }
  }
  void add_element(T val) {
    for(int i=0;i<basis.size();i++) {
      reduce_pair(val, basis[i]);
      if(basis[i] == 1) {
        swap(basis[i], basis.back());
        basis.pop_back();
        continue;
      }
      if(val == 1) return;
      if(gcd(basis[i], val) > 1) solve_inner(i, val);
    }
    if(val > 1) basis.push_back(val);
  }
  void verify_basis() {
    for(int i=0;i<basis.size();i++) {
      for(int j=i+1;j<basis.size();j++) {
        assert(gcd(basis[i], basis[j]) == 1);
      }
    }
  }
  bool verify_element(T ele) {
    for(auto &x : basis) {
      while(ele % x == 0) ele /= x;
    }
    return (ele == 1);
  }
  auto factorization(T ele) {
    vector<int> factors(basis.size());
    for(int i=0;i<basis.size();i++) {
      while(ele % basis[i] == 0) {
        factors[i]++;
        ele /= basis[i];
      }
    }
    return factors;
  }
  auto lcm(T a,T b) {
    vector<int> lcm(basis.size());
    if(!verify_element(a))
      add_element(a);
    if(!verify_element(b))
      add_element(b);
    vector<int> fa = factorization(a);
    vector<int> fb = factorization(b);
    return lcm(fa,fb);
  }
  auto lcm(vector<int> fa,vector<int> fb) {
    vector<int> lcm(basis.size());
    for(int i = 0;i<basis.size();i++) {
      lcm[i] = max(fa[i],fb[i]);
    }
    return lcm;
  }
  auto gcd(T a,T b) {
    vector<int> gcd(basis.size());
    if(!verify_element(a))
      add_element(a);
    if(!verify_element(b))
      add_element(b);
    vector<int> fa = factorization(a);
    vector<int> fb = factorization(b);
    for(int i = 0;i<basis.size();i++) {
      gcd[i] = min(fa[i],fb[i]);
    }
    return gcd;
  }
};
```

4.23 Primes.cpp

620 lines

```cpp
#include <bits/stdc++.h>
using namespace std;
typedef long long int lli;
//______Neds__________//
lli mod_mult(lli a, lli b, lli mod) {
 lli x = 0;
 while(b) {
  if(b & 1) x = (x + a) % mod;
  a = (a << 1) % mod;
  b >>= 1;
 }
 return x;
}
lli mod_pow(lli a, lli n, lli mod) {
 lli x = 1;
 while(n) {
  if(n & 1) x = mod_mult(x, a, mod);
  a = mod_mult(a, a, mod);
  n >>= 1;
 }
 return x;
}
//_________Criba de Eratostenes______________//
vector<int> Criba(int n)  {
  int raiz = sqrt(n);
  vector<int> criba(n + 1);
  for (int i = 4; i <= n; i += 2)
    criba[i] = 2;
  for (int i = 3; i <= raiz; i += 2)
    if (!criba[i])
      for (int j = i * i; j <= n; j += i)
        if (!criba[j]) criba[j] = i;
  return criba;
}
//________Criba lineal________________//
/*
  Save primes up to 10^7
*/
const int N = 10000000;
int lp[N+1];
vector<int> primes;
void criba() {
  for (int i=2; i<=N; ++i)  {
    if (lp[i] == 0)  {
      lp[i] = i;
      primes.push_back (i);
    }
    for (int j=0; j<(int)primes.size() && primes[j]<=lp[i] && i*primes
        ↪ [j]<=N; ++j) {
      lp[i * primes[j]] = primes[j];
      if(i%primes[j]==0)break;
    }
  }
}
// __________Bitwise prime sieve________
/*
  get primes up to 1e8 in ~ 3s
*/
const int NMAX = 100000000;
signed main()  {
  bitset<NMAX / 2> bits;
  bits.set();
 auto sum = 2LL;
 int cnt = 1;
  for (int i = 3; i / 2 < bits.size(); i = 2 * bits._Find_next(i / 2) +
      ↪ 1)  {
    sum += i;
    ++cnt;
    for (auto j = (int64_t) i * i / 2; j < bits.size(); j += i)
      bits[j] = 0;
  }
 cout << "sum = " << sum << endl;
 cout << "cnt = " << cnt << endl;
 return 0;
}
```

```cpp
//________Criba de factor primo mas pequeño______//
   vector<int> lowestPrime;
void lowestPrimeSieve(int n) {
   lowestPrime.resize(n + 1, 1);
   lowestPrime[0] = lowestPrime[1] = 0;
   for(int i = 2; i <= n; ++i)
      lowestPrime[i] = (i & 1 ? i : 2);
   int limit = sqrt(n);
   for(int i = 3; i <= limit; i += 2)
      if(lowestPrime[i] == i)
         for(int j = i * i; j <= n; j += 2 * i)
         if(lowestPrime[j] == j) lowestPrime[j] = i;
   }
//_______Criba de factor primo mas grande_________//
vector<int> greatestPrime;
void greatestPrimeSieve(int n) {
   greatestPrime.resize(n + 1, 1);
   greatestPrime[0] = greatestPrime[1] = 0;
   for(int i = 2; i <= n; ++i) greatestPrime[i] = i;
   for(int i = 2; i <= n; i++)
      if(greatestPrime[i] == i)
         for(int j = i; j <= n; j += i)
         greatestPrime[j] = i;
   }
//_________Criba de la funcion phi de euler__________//
bool is_composite[MAXN];
int phi[MAXN];
void sieve (int n)  {
  std::fill (is_composite, is_composite + n, false);
  phi[1] = 1;
  for (int i = 2; i < n; ++i)  {
    if (!is_composite[i])  {
     prime.push_back (i);
     phi[i] = i - 1;      //i is prime
    }
   for (int j = 0; j < prime.size () && i * prime[j] < n; ++j)  {
     is_composite[i * prime[j]] = true;
     if (i % prime[j] == 0)  {
      phi[i * prime[j]] = phi[i] * prime[j]; //prime[j] divides i
      break;
     } else  {
      phi[i * prime[j]] = phi[i] * phi[prime[j]]; //prime[j] does not divide i
     }
    }
  }
}
/*Criba para hallar todos los valores de 1-n de una
función multiplicativa*/
int N = 10000007;
vector<int> F(N+1); //funcion multiplicativa evaluada en i
int g(int p, int a) {
  //Ejemplo para la phi de euler
  return power(p, a - 1) * (p - 1);
 }
vector<int> sieve(int N) {
  vector<int> primes;
  vector<int> lp(N+1); //factor primo mas pequeño de i
  vector<int> cnt(N+1); //exponente del primo mas pequeño de i
  vector<int> pot(N+1); //pow(lp[i], cnt[i])
  F[1] = 1;
  for(int i = 2; i <= N; ++i) {
    if(lp[i] == 0) {
     primes.push_back(i);
     lp[i] = i;
     F[i] = g(i, 1);
     cnt[i] = 1;
     pot[i] = i;
    }
    for(int p : primes) {
     int d = i * p;
     if(d > N) break;
     lp[d] = p;
     if(p == lp[i]) {
      F[d] = F[i / pot[i]] * g(p, cnt[i]+1);
      cnt[d] = cnt[i] + 1;
      pot[d] = pot[i] * p;
      break;
     } else {
      F[d] = F[i] * F[p];
      cnt[d] = 1;
      pot[d] = p;
     }
    }
   }
  return f;
 }
vector<int> Phi;
void phiSieve(int n) {
   Phi.resize(n + 1);
   for(int i = 1; i <= n; ++i)
      Phi[i] = i;
   for(int i = 2; i <= n; ++i)
      if(Phi[i] == i)
         for(int j = i; j <= n; j += i)
            Phi[j] -= Phi[j] / i;
 }
//____________*****************************________//
vector<int> Mu;
void muSieve(int n) {
   Mu.resize(n + 1, -1);
   Mu[0] = 0, Mu[1] = 1;
   for(int i = 2; i <= n; ++i)
      if(Mu[i])
         for(int j = 2*i; j <= n; j += i)
            Mu[j] -= Mu[i];
 }
//_________Triangulo de pascal__________//
vector<vector<lli>> Ncr;
void ncrSieve(lli n) {
   Ncr.resize(n + 1);
   Ncr[0] = { 1 };
   for(lli i = 1; i <= n; ++i) {
      Ncr[i].resize(i + 1);
      Ncr[i][0] = Ncr[i][i] = 1;
      for(lli j = 1; j <= i / 2; j++)
         Ncr[i][i - j] = Ncr[i][j] = Ncr[i - 1][j - 1] + Ncr[i +1][j];
      }
   }
//________GOLDBACH CONJETURE TO 10^7______________/
int findPrimes(int n)  {
   for (int i=0 ; primes[i] <= n/2; i++)  {
      int diff = n - primes[i];
      if (binary_search(primes.begin(), primes.end(), diff)) {
         // Express as a sum of primes
         cout << primes[i]<<" "<<diff<<endl;
         return 1;
      }
   }
   return 0;
 }
//________bLOCK SIEVE___________//
int count_primes(int n)  {
   const int S = 10000;
   vector<int> primes;
   int nsqrt = sqrt(n);
   vector<char> is_prime(nsqrt + 1, true);
   for (int i = 2; i <= nsqrt; i++)  {
      if (is_prime[i])  {
         primes.push_back(i);
         for (int j = i * i; j <= nsqrt; j += i)
            is_prime[j] = false;
      }
   }
   int result = 0;
   vector<char> block(S);
   for (int k = 0; k * S <= n; k++)  {
      fill(block.begin(), block.end(), true);
      int start = k * S;
      for (int p : primes)  {
         int start_idx = (start + p - 1) / p;
         int j = max(start_idx, p) * p - start;
         for (; j < S; j += p)
```

```
      block[j] = false;
    }
    if (k == 0)
      block[0] = block[1] = false;
    for (int i = 0; i < S && start + i <= n; i++) {
      if (block[i])
        result++;
    }
  }
  return result;
}
//_________________PHI DE EULER_____________________//
//numero de numeros menores a n coprimos con n
int phi (int n) {
 int result = n;
 for (int i=2; i*i<=n; ++i)
  if(n %i==0) {
   while(n %i==0)
   n /= i;
   result -= result / i;
  }
 if (n > 1)
 result -= result / n;
 return result;
}
//___________Miller Rabin___________//
/*
  need mod_pow,mod_mult
  Miller-Rabin-Test, s = # iterations
  probability for an error <= 2^(-s)
  [method is only called with n > 10^6
  and n is not divisible by primes < 10^6]
  Primaly test works up to 10^18
  [Tested SPOJ PON]
*/
lli random(lli a, lli b) {
  lli intervallLength = b - a + 1;
  int neededSteps = 0;
  lli base = RAND_MAX + 1LL;
  while(intervallLength > 0) {
   intervallLength /= base;
   neededSteps++;
  }
  intervallLength = b - a + 1;
  lli result = 0;
  for(int stepsDone = 0; stepsDone < neededSteps; stepsDone++) {
    result = (result * base + rand());
  }
  result %= intervallLength;
  if(result < 0) result += intervallLength;
  return result + a;
}
bool witness(lli a, lli n) {
  lli u = n-1;
  int t = 0;
  while (u % 2 == 0) {
    t++;
    u /= 2;
  }
  lli next = mod_pow(a, u, n);
  if(next == 1)return false;
  lli last;
  for(int i = 0; i < t; i++) {
   last = next;
    next = mod_mult(last, last, n);//(last * last) % n;
    if (next == 1) {
     return last != n - 1;
    }
  }
  return next != 1;
}
bool isPrime(lli n, int s) {
  if (n <= 1) return false;
  if (n == 2) return true;
  if (n % 2 == 0) return false;
  for(int i = 0; i < s; i++) {
    lli a = random(1, n-1);
    if (witness(a, n)) return false;
  }
  return true;
}
//___________DETERMINIST___________//
bool check_compsite(lli n, lli a,lli d, int s) {
  lli x = mod_pow(a, d, n);
  if(x == 1 || x == n - 1)
    return false;
  for(int r = 1; r < s; r++) {
    x = (lli)x * x % n;
    if(x == n - 1)
      return false;
  }
  return true;
}
bool MillerRabin(lli n) {
  if(n < 2)
    return false;
  int s = 0;
  lli d = n - 1;
  while((d & 1) == 0) {
    d >>= 1;
    s++;
  }
  for (int a : {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37}) {
    if (n == a)
      return true;
    if (check_compsite(n, a, d, s))
      return false;
  }
  return true;
}
//___________Count divisors in n^1/3___________//
/*
  Need miller rabin,criba(),primes[],lp[],N= 10^6
  [Tested Codeforces GYM GCPC-15 F-Divisions]
*/
bool isSquare(lli val) {
 lli lo = 1, hi = val;
 while(lo <= hi) {
  lli mid = lo + (hi - lo) / 2;
  lli tmp = (val / mid) / mid; // be careful with overflows!!
  if(tmp == 0)hi = mid - 1;
  else if(mid * mid == val)return true;
  else if(mid * mid < val)lo = mid + 1;
 }
 return false;
}
lli countDivisors(lli n) {
  lli ans = 1;
 for(int i = 0; i < primes.size(); i++) {
  if(n == 1)break;
  int p = primes[i];
  if(n % p == 0) { // checks whether p is a divisor of n
   int num = 0;
   while(n % p == 0) {
    n /= p;
    ++num;
   }
   // p^num divides initial n but p^(num + 1) does not divide initial val
   // => p can be taken 0 to num times => num + 1 possibilities!!
   ans *= num + 1;
  }
 }
  if(n == 1)return ans; // first case
 else if(isPrime(n,20))return ans * 2; // second case
 else if(isSquare(n))return ans * 3; // third case but with p == q
 else return ans * 4; // third case with p != q
}
// arr is a primeFact of n with a pair <prime,exponent>
void generateDivisors(int curIndex, int curDivisor, vector<pair<int,int>>& arr) {
  if (curIndex == arr.size()) {
    cout << curDivisor << ' ';
```

```
        return;
    }
    for (int i = 0; i <= arr[curIndex].y; ++i) {
        generateDivisors(curIndex + 1, curDivisor, arr);
        curDivisor *= arr[curIndex].x;
    }
}
//_________Prime Factors__________//
map<lli,lli> fact;
void trial_division4(lli n) {
    for (lli d : primes) {
        if (d * d > n)
            break;
        while (n % d == 0) {
            fact[d]++;
            n /= d;
        }
    }
}
void trial_division2(lli n) {
    while (n % 2 == 0) {
        fact[2]++;
        n /= 2;
    }
    for (long long d = 3; d * d <= n; d += 2) {
        while (n % d == 0) {
            fact[d]++;
            n /= d;
        }
    }
    if (n > 1)
        fact[n]++;
}
/*
    Pollard Method p-1
*/
lli pollard_p_1(lli n) {
  int b = 13;
  int q[] = { 2, 3, 5, 7, 11, 13 } ;
  lli a = 5% n;
  for (int j = 0; j <10; j ++) {
    while (__gcd(a, n)!= 1) {
      mod_mult (a, a, n);
      a+= 3;
      a%= n;
    }
    for (int i = 0; i<6; i ++) {
      int qq = q [i];
      int e = floor(log((double) b) / log((double) qq));
      lli aa = mod_pow(a, mod_pow (qq, e, n), n);
      if (aa == 0)
        continue;
        lli g = __gcd (aa-1, n);
      if (1 <g && g <n)
        return g;
    }
  }
  return 1;
}
/*
    Pollard rho
*/
lli pollard_rho (lli n, unsigned iterations_count = 100000) {
  lli b0 = rand ()% n,b1 = b0,g;
  mod_mult (b1, b1, n);
  if (++b1 == n)
    b1 = 0;
  g = __gcd(abs(b1 - b0), n);
  for (unsigned count = 0; count <iterations_count && (g == 1 || g == n);
      count ++) {
    mod_mult (b0, b0, n);
    if (++ b0 == n)
      b0 = 0;
    mod_mult (b1, b1, n);
    ++ b1;
    mod_mult (b1, b1, n);
    if (++ b1 == n)
      b1 = 0;
    g = __gcd(abs(b1 - b0), n);
  }
  return g;
}
lli pollard_bent (lli n, unsigned iterations_count = 19) {
  lli b0 = rand ()% n,
    b1 = (b0 * b0 + 2)% n,
    a = b1;
  for (unsigned iteration = 0, series_len = 1; iteration <
      iterations_count; iteration ++, series_len *= 2) {
    lli g = __gcd(b1-b0, n);
    for (unsigned len = 0; len <series_len && (g == 1 && g == n); len ++)
        {
      b1 = (b1 * b1 + 2)% n;
      g = __gcd(abs (b1-b0), n);
    }
    b0 = a;
    a = b1;
    if (g != 1 && g != n)
      return g;
  }
  return 1;
}
/*
    Pollard monte Carlo
*/
lli pollard_monte_carlo (lli n, unsigned m = 100) {
  lli b = rand ()% (m-2) + 2;
  lli g = 1;
  for (int i = 0; i <100 && g == 1; i++) {
    lli cur = primes[i];
    while (cur <= n)
      cur *= primes[i];
    cur /= primes[i];
    b = mod_pow (b, cur, n);
    g = __gcd(abs (b-1), n);
    if (g == n)
      g = 1;
  }
  return g;
}
lli prime_div_trivial (lli n) {
  if (n == 2 || n == 3)
    return 1;
  if (n <2)
    return 0;
  if (!n&1)
    return 2;
  lli pi;
  for (auto p:primes) {
    if (p*p >n)
      break;
    else
      if (n% p == 0)
        return p;
  }
  if (n <1000*10000)
    return 1;
  return 0;
}
lli ferma (lli n) {
  lli x = floor(sqrt((double)n)),y = 0,r = x * x - y * y - n;
  for (;;)
    if (r == 0)
      return x != y? x*y: x + y;
    else
      if (r> 0) {
        r-= y + y + 1;
        ++y;
      }
      else {
        r+= x + x + 1;
        ++x;
      }
}
```

```cpp
lli mult(lli a, lli b, lli mod) {
  return (lli)a * b % mod;
}
lli f(lli x, lli c, lli mod) {
  return (mult(x, x, mod) + c) % mod;
}
lli brent(lli n, lli x0=2, lli c=1) {
  lli x = x0;
  lli g = 1;
  lli q = 1;
  lli xs, y;
  int m = 128;
  int l = 1;
  while (g == 1) {
    y = x;
    for (int i = 1; i < l; i++)
      x = f(x, c, n);
    int k = 0;
    while (k < l && g == 1) {
      xs = x;
      for (int i = 0; i < m && i < l - k; i++) {
        x = f(x, c, n);
        q = mult(q, abs(y - x), n);
      }
      g = __gcd(q, n);
      k += m;
    }
    l *= 2;
  }
  if (g == n) {
    do {
      xs = f(xs, c, n);
      g = __gcd(abs(xs - y), n);
    } while (g == 1);
  }
  return g;
}
void factorize (lli n) {
  if (isPrime(n,20))
    fact[n]++;
  else {
    if (n <1000 * 1000) {
      lli div = prime_div_trivial(n);
      fact[div]++;
      factorize(n / div);
    }
    else {
      lli div;
      // 'Pollards fast algorithms come first
      div = pollard_monte_carlo(n);
      if (div == 1)
        div = brent(n);
      if (div == 1)
        div = pollard_rho (n),cout<<"USE POLLAR_RHO\n";
      if (div == 1)
        div = pollard_p_1 (n),cout<<"USE POLLARD_P_1\n";
      if (div == 1)
        div = pollard_bent (n),cout<<"USE POLLARD_BENT\n";
      if (div == 1)
        div = ferma (n);
      // recursively process the found factors
      factorize (div);
      factorize (n / div);
    }
  }
}
// Get prime factors of a number in time O(log(n)) with precompute array
    ↪ of lowest factor of a number up to 10^7 complexity of precalc
    ↪ is O(nlognlogn)
// uses lowestprime
vector<int> facts;
void factorizeLog(lli n) {
  while(n>1) {
    facts.push_back(lowestPrime[n]);
    n/=lowestPrime[n];
  }
}
int main() {
 lli n,i,j;
  cin>>n;
 criba();
  // for(int i = 0;i<N;i++) {
  //   cout<<primes[i]<<" ";
  // }
  // cin>>n;
  // if(!findPrimes(n))cout<<-1<<endl;
  // cout<<countDivisors(n);
  // factorize(n);
  lowestPrimeSieve(10000000);
  factorize(n);
  factorizeLog(n);
  for(auto c:facts)cout<<c<<" ";
  cout<<endl;
  // cout<<fact.size();
  for(auto c: fact)cout<<c.first<<"^"<<c.second<<" ";
}
```

4.24 SOSConvolutions.cpp

110 lines

```cpp
#include<bits/stdc++.h>
using namespace std;
const int N = 3e5 + 9, mod = 998244353;
// s' $ s defines all subsets of s
namespace SOS {
const int B = 20; // Every input vector must need to be of size 1<<B
// $z(f(s))=\sum_{s' \subseteq s}{f(s')}$
// $O(B * 2^B)$
// zeta transform is actually SOS DP
vector<int> zeta_transform(vector<int> f) {
 for (int i = 0; i < B; i++) {
  for (int mask = 0; mask < (1 << B); mask++) {
   if ((mask & (1 << i)) != 0) {
    f[mask] += f[mask ^(1 << i)];// you can change the operator from +
        ↪ to min/gcd to find min/gcd of all f[submasks]
   }
  }
 }
 return f;
}
// $mu(f(s))=\sum_{s' $ s}{(-1)^|s\s'| * f(s')}$
// O(B * 2 ^B)
vector<int> mobius_transform(vector<int> f) {
 for (int i = 0; i < B; i++) {
  for (int mask = 0; mask < (1 << B); mask++) {
   if ((mask & (1 << i)) != 0) {
    f[mask] -= f[mask ^(1 << i)];
   }
  }
 }
 return f;
}
vector<int> inverse_zeta_transform(vector<int> f) {
 return mobius_transform(f);
}
vector<int> inverse_mobius_transform(vector<int> f) {
 return zeta_transform(f);
}
// z(f(s))=\sum_{s' is supermask of s}{f(s')}
// O(B * 2 ^B)
// zeta transform is actually SOS DP
vector<int> zeta_transform_for_supermasks(vector<int> f) {
 for (int i = 0; i < B; i++) {
  for (int mask = (1 << B) - 1 ; mask >= 0 ; mask--) {
   if ((mask & (1 << i)) == 0) f[mask] += f[mask ^(1 << i)] ;
  }
 }
 return f;
}
// f*g(s)=sum_{s' $ s} {f(s')*g(s\s')}
// O(B * B * 2 ^B)
vector<int> subset_sum_convolution(vector<int> f, vector<int> g) {
 vector< vector<int> > fhat(B + 1, vector<int> (1 << B, 0));
```

```cpp
  vector< vector<int> > ghat(B + 1, vector<int> (1 << B, 0));
  // Make fhat[][] = {0} and ghat[][] = {0}
  for (int mask = 0; mask < (1 << B); mask++) {
   fhat[__builtin_popcount(mask)][mask] = f[mask];
   ghat[__builtin_popcount(mask)][mask] = g[mask];
  }
  // Apply zeta transform on fhat[][] and ghat[][]
  for (int i = 0; i <= B; i++) {
   for (int j = 0; j <= B; j++) {
    for (int mask = 0; mask < (1 << B); mask++) {
     if ((mask & (1 << j)) != 0) {
      fhat[i][mask] += fhat[i][mask ^(1 << j)];
      if (fhat[i][mask] >= mod) fhat[i][mask] -= mod;
      ghat[i][mask] += ghat[i][mask ^(1 << j)];
      if (ghat[i][mask] >= mod) ghat[i][mask] -= mod;
     }
    }
   }
  }
  vector< vector<int> > h(B + 1, vector<int> (1 << B, 0));
  // Do the convolution and store into h[][] = {0}
  for (int mask = 0; mask < (1 << B); mask++) {
   for (int i = 0; i <= B; i++) {
    for (int j = 0; j <= i; j++) {
     h[i][mask] += 1LL * fhat[j][mask] * ghat[i - j][mask] % mod;
     if (h[i][mask] >= mod) h[i][mask] -= mod;
    }
   }
  }
  // Apply inverse SOS dp on h[][]
  for (int i = 0; i <= B; i++) {
   for (int j = 0; j <= B; j++) {
    for (int mask = 0; mask < (1 << B); mask++) {
     if ((mask & (1 << j)) != 0) {
      h[i][mask] -= h[i][mask ^(1 << j)];
      if (h[i][mask] < 0) h[i][mask] += mod;
     }
    }
   }
  }
  vector<int> fog(1 << B, 0);
  for (int mask = 0; mask < (1 << B); mask++) fog[mask] = h[
       __builtin_popcount(mask)][mask];
  return fog;
 }
};
int32_t main() {
  ios_base::sync_with_stdio(0);
  cin.tie(0);
  int n;
  cin >> n;
  vector<int> a(1 << 20, 0), b(1 << 20, 0);
  for (int i = 0; i < (1 << n); i++) cin >> a[i];
  for (int i = 0; i < (1 << n); i++) cin >> b[i];
  auto ans = SOS::subset_sum_convolution(a, b);
  for (int i = 0; i < (1 << n); i++) cout << ans[i] << ' ';
  cout << '\n';
  return 0;
}
```

4.25 divisorSigma.cpp

34 lines

```cpp
typedef long long ll;
ll divisor_sigma(ll n)
{
  ll sigma = 0, d = 1;
  for (; d * d < n; ++d)
   if (n % d == 0)
    sigma += d + n / d;
  if (d * d == n)
   sigma += d;
  return sigma;
}
// sigma(n) for all n in [lo, hi)
vector<ll> divisor_sigma(ll lo, ll hi)
{
  vector<ll> ps = primes(sqrt(hi) + 1);
  vector<ll> res(hi - lo), sigma(hi - lo, 1);
  iota(res.begin(), res.end(), lo);
  for (ll p : ps)
   for (ll k = ((lo + (p - 1)) / p) * p; k < hi; k += p)
   {
    ll b = 1;
    while (res[k - lo] > 1 && res[k - lo] % p == 0)
    {
     res[k - lo] /= p;
     b = 1 + b * p;
    }
    sigma[k - lo] *= b;
   }
  for (ll k = lo; k < hi; ++k)
   if (res[k - lo] > 1)
    sigma[k - lo] *= (1 + res[k - lo]);
  return sigma; // sigma[k-lo] = sigma(k)
}
```

4.26 euclid.h

Description: Finds two integers x and y, such that $ax + by = \gcd(a, b)$. If you just need gcd, use the built in `__gcd` instead. If a and b are coprime, then x is the inverse of $a \pmod b$.

5 lines

```cpp
int euclid(int a, int b, int &x, int &y) {
  if (!b) return x = 1, y = 0, a;
  int d = euclid(b, a % b, y, x);
  return y -= a/b * x, d;
}
```

4.27 eulerPhi.cpp

46 lines

```cpp
/*
  Euler Phi (Totient Function)
  Tested: SPOJ ETFS, AIZU NTL_1_D
*/
typedef long long ll;
ll euler_phi(ll n)
{
  if (n == 0)
   return 0;
  ll ans = n;
  for (ll x = 2; x * x <= n; ++x)
   if (n % x == 0)
   {
    ans -= ans / x;
    while (n % x == 0)
     n /= x;
   }
  if (n > 1)
   ans -= ans / n;
  return ans;
}
// phi(n) for all n in [lo, hi)
vector<ll> euler_phi(ll lo, ll hi)
{
  vector<ll> ps = primes(sqrt(hi) + 1);
  vector<ll> res(hi - lo), phi(hi - lo, 1);
  iota(res.begin(), res.end(), lo);
  for (ll p : ps)
   for (ll k = ((lo + (p - 1)) / p) * p; k < hi; k += p)
   {
    if (res[k - lo] < p)
     continue;
    phi[k - lo] *= (p - 1);
    res[k - lo] /= p;
    while (res[k - lo] > 1 && res[k - lo] % p == 0)
    {
     phi[k - lo] *= p;
     res[k - lo] /= p;
    }
   }
  for (ll k = lo; k < hi; ++k)
```

```cpp
  if (res[k - lo] > 1)
    phi[k - lo] *= (res[k - lo] - 1);
 return phi; // phi[k-lo] = phi(k)
}
```

4.28 fastPrimeCount

Description: Count the number of primes up to n^{12}

Usage: `k = 0; gen(); lehmer(n) // k = 0 is count of primes`
`for sum of primes -> k = 1; count_primes(n);`

Time: $\mathcal{O}\left(n^{\frac{2}{3}}\right)$ count of primes for $n^{12} \sim 5.15s$ for sum of primes $n^{11} \sim 4.39s$

123 lines

```cpp
// If sum of primes is needed use int128
#define int __int128
#define MAXN 100
#define MAXM 100007
#define MAXP 10000007
int prime_cnt[MAXP];
int prime_sum[MAXP];
long long dp[MAXN][MAXM];
//Function to print __int128
std::ostream&
operator<<( std::ostream& dest, __int128_t value ) {
  std::ostream::sentry s( dest );
  if ( s )  {
    __uint128_t tmp = value < 0 ? -value : value;
    char buffer[ 128 ];
    char* d = std::end( buffer );
    do
     {
      -- d;
      *d = "0123456789"[ tmp % 10 ];
      tmp /= 10;
     } while ( tmp != 0 );
    if ( value < 0 )  {
      -- d;
      *d = '-';
     }
    int len = std::end( buffer ) - d;
    if ( dest.rdbuf()->sputn( d, len ) != len )  {
      dest.setstate( std::ios_base::badbit );
     }
   }
  return dest;
}
vector<int> primes;
bitset<MAXP> is_prime;
// modify k to calc sum of primes p^k with p^k <= n
int k = 0;
int F(int n) {
  return pow(n,k);
};
int pref(int n) {
  if(k == 0)return n;
  if(k == 1)return (n*(n+1))/2;
  if(k == 2)return (n*(n+1)*(2*n+1))/6;
  return 1;
}
vector<int> lp(MAXP+1);
void gen() {
  lp.assign(MAXP,0);
  primes.clear();
  for(int i= 2;i<=MAXP;i++) {
    if(lp[i]==0)lp[i] = i,primes.push_back(i);
    for(int j = 0;j<primes.size() && primes[j]<=lp[i] && primes[j]*i<=MAXP;j++)
      lp[primes[j]*i] = primes[j];
  }
  for (int i = 2; i < MAXP; i++) {
    prime_cnt[i] = prime_cnt[i-1] + (lp[i]==i);
    prime_sum[i] = prime_sum[i-1] + (lp[i]==i?F(i):0);
  }
  for (int m = 0; m < MAXM; m++) dp[0][m] = pref(m);
  for (int n = 1; n < MAXN; n++) {
    for (int m = 0; m < MAXM; m++) {
      dp[n][m] = dp[n - 1][m] - (dp[n - 1][m/primes[n - 1]]*F(primes[n-1]));
    }
  }
}
int phi(int m, int n) {
  if (n == 0) return pref(m);
  if (m < MAXM && n < MAXN) return dp[n][m];
  if (primes[n - 1] * primes[n - 1] >= m && m < MAXP) return prime_sum[m] - pref(n) + 1;
  return phi(m, n - 1) - (phi(m/primes[n - 1], n - 1)*F(primes[n-1]));
}
/*- for some reason this not work for sum of power primes or for k>=1 use count primes instead */
int lehmer(int m) {
  if (m < MAXP) return prime_sum[m];
  int s = sqrt(0.5 + m), y = cbrt(0.5 + m);
  int a = prime_cnt[y];
  int res = phi(m, a);
  for (int i = a; primes[i] <= s; i++) {
    int x = lehmer(m/primes[i]);
    int y = lehmer(primes[i-1]);
    res = res - ((lehmer(m / primes[i]) - lehmer(primes[i-1]))*F(primes[i])) - F(primes[i]);
  }
  a = prime_sum[s];
  return (res+a)-1;
}
/** Use this function if k>=1 */
int count_primes(int n)  {
  vector<int> v;
  v.reserve((int)sqrt(n) * 2 + 20);
  int sq; {
    int k = 1;
    for (; k * k <= n; ++k)  {
      v.push_back(k);
     }
    --k;
    sq = k;
    if (k * k == n) --k;
    for (; k >= 1; --k)  {
      v.push_back(n / k);
     }
   }
  vector<int> s(v.size());
  for (int i = 0; i < s.size(); ++i)
    s[i] = pref(v[i]) - 1;
  auto geti = [&](int x)  {
    if (x <= sq) return (int)x - 1;
    else return (int)(v.size() - (n / x));
  };
  for (int p = 2; p * p <= n; ++p)  {
    if (s[p - 1] != s[p - 2])  {
      int sp = s[p - 2];
      int p2 = p * p;
      for (int i = (int)v.size() - 1; i >= 0; --i)  {
        if (v[i] < p2)  {
          break;
         }
        s[i] -= (s[geti(v[i] / p)] - sp) * F(p);
       }
     }
   }
  return s.back();
}
```

4.29 fastPrimeCount2.cpp

240 lines

```cpp
#include <bits/stdtr1c++.h>
// O(n^2/3)
struct _count_primes_struct_t_  {
  vector<int> primes;
  vector<int> mnprimes;
  int ans;
  int y;
  vector<pair<pair<int, int>, char>> queries;
  int count_primes(int n)  {
    // this y is actually n/y
```

```cpp
// also no logarithms, welcome to reality, this y is the best for n=10^12 or n=10^13
y = pow(n, 0.64);
if (n < 100) y = n;
// linear sieve
primes.clear();
mnprimes.assign(y + 1, -1);
ans = 0;
for (int i = 2; i ≤ y; ++i) {
   if (mnprimes[i] == -1) {
      mnprimes[i] = primes.size();
      primes.push_back(i);
   }
   for (int k = 0; k < primes.size(); ++k) {
      int j = primes[k];
      if (i * j > y) break;
      mnprimes[i * j] = k;
      if (i % j == 0) break;
   }
}
if (n < 100) return primes.size();
int s = n / y;
for (int p : primes) {
   if (p > s) break;
   ans++;
}
// pi(n / y)
int ssz = ans;
// F with two pointers
int ptr = primes.size() - 1;
for (int i = ssz; i < primes.size(); ++i) {
   while (ptr ≥ i && (int)primes[i] * primes[ptr] > n)
      --ptr;
   if (ptr < i) break;
   ans -= ptr - i + 1;
}
// phi, store all queries
phi(n, ssz - 1);
sort(queries.begin(), queries.end());
int ind = 2;
int sz = primes.size();
// the order in fenwick will be reversed, because prefix sum in a fenwick is just one query
fenwick fw(sz);
for (auto [na, sign] : queries) {
   auto [n, a] = na;
   while (ind ≤ n)
      fw.add(sz - 1 - mnprimes[ind++], 1);
   ans += (fw.ask(sz - a - 2) + 1) * sign;
}
queries.clear();
    return ans - 1;
  }
  void phi(int n, int a, int sign = 1) {
    if (n == 0) return;
    if (a == -1) {
      ans += n * sign;
      return;
    }
    if (n ≤ y) {
      queries.emplace_back(make_pair(n, a), sign);
      return;
    }
    phi(n, a - 1, sign);
    phi(n / primes[a], a - 1, -sign);
  }
  struct fenwick {
    vector<int> tree;
    int n;
    fenwick(int n = 0) : n(n) {
      tree.assign(n, 0);
    }
    void add(int i, int k) {
      for (; i < n; i = (i | (i + 1)))
        tree[i] += k;
    }
    int ask(int r) {
      int res = 0;
      for (; r ≥ 0; r = (r & (r + 1)) - 1)
        res += tree[r];
      return res;
    }
  };
} _count_primes_struct_;
// O(n^3/4)
int count_primes(int n) {
  auto f = [&](int n) {
    return n;
  };
  auto pref = [&](int n) {
    return (n*(n+1))/2;
  };
  vector<int> v;
  v.reserve((int)sqrt(n) * 2 + 20);
  int sq; {
    int k = 1;
    for (; k * k ≤ n; ++k) {
      v.push_back(k);
    }
    --k;
    sq = k;
    if (k * k == n) --k;
    for (; k ≥ 1; --k) {
      v.push_back(n / k);
    }
  }
  for(auto c:v)cout<<c<<" ";
  cout<<endl;
  vector<int> s(v.size());
  for (int i = 0; i < s.size(); ++i)
    s[i] = pref(v[i]) - 1;
  for(auto c:s)cout<<c<<" ";
  cout<<endl;
  auto geti = [&](int x) {
    if (x ≤ sq) return (int)x - 1;
    else      return (int)(v.size() - (n / x));
  };
  cout<<s[geti(37)]<<endl;
  for (int p = 2; p * p ≤ n; ++p) {
    cout<<p<<endl;
    if (s[p - 1] ≠ s[p - 2]) {
      int sp = s[p - 2];
      int p2 = p * p;
      cout<<sp<<" "<<p2<<endl;
      for (int i = (int)v.size() - 1; i ≥ 0; --i) {
        if (v[i] < p2) {
          break;
        }
        s[i] -= (s[geti(v[i] / p)] - sp) * f(p);
        cout<<"I: "<<i<<" "<<v[i]/p<<" "<<geti(v[i]/p)<<" "<<f(p)<< endl;
      }
    }
  }
  for(auto c:s)cout<<c<<" ";
  cout<<endl;
  return s.back();
}
int count_primes2(int n) {
  vector<int> v;
  for (int k = 1; k * k ≤ n; ++k) {
    v.push_back(n / k);
    v.push_back(k);
  }
  sort(v.begin(), v.end());
  v.erase(unique(v.begin(), v.end()), v.end());
  // for(auto c:v)cout<<c<<" ";
  // cout<<endl;
  int sq = sqrt(n);
  auto geti = [&](int x) {
    if (x ≤ sq) return (int)x - 1;
    else      return (int)(v.size() - (n / x));
  };
```

```cpp
  vector<int> dp(v.size());
  for (int i = 0; i < v.size(); ++i)
    dp[i] = v[i];
  int a = 0;
  for (int p = 2; p * p <= n; ++p)  {
    if (dp[geti(p)] != dp[geti(p - 1)])  {
      ++a;
      for (int i = (int)v.size() - 1; i >= 0; --i)  {
        if (v[i] < p * p) break;
        dp[i] -= dp[geti(v[i] / p)] - a;
      }
    }
  }
  return dp[geti(n)] - 1;
}
#define MAXN 100
#define MAXM 100010
#define MAXP 10000010
using namespace std;
int prime_cnt[MAXP];
long long dp[MAXN][MAXM];
vector<int> primes;
bitset<MAXP> is_prime;
// void sieve() {
//   is_prime[2] = true;
//   for (int i = 3; i < MAXP; i += 2) is_prime[i] = true;
//   for (int i = 3; i * i < MAXP; i += 2) {
//     for (int j = i * i; is_prime[i] && j < MAXP; j += (i << 1)) {
//       is_prime[j] = false;
//     }
//   }
//   for (int i = 1; i < MAXP; i++) {
//     prime_cnt[i] = prime_cnt[i - 1] + is_prime[i];
//     if (is_prime[i]) primes.push_back(i);
//   }
// }
void sieve() {
  vector<int> lp(MAXP+1);
  for(int i= 2;i<=MAXP;i++) {
    if(lp[i]==0)lp[i] = i,primes.push_back(i);
    for(int j = 0;j<primes.size() && primes[j]<=lp[i] && primes[j]*i<=MAXP;j++)
      lp[primes[j]*i] = primes[j];
  }
  for (int i = 1; i < MAXP; i++)
    prime_cnt[i] = prime_cnt[i - 1] + (lp[i]==i);
}
void gen() {
  sieve();
  for (int m = 0; m < MAXM; m++) dp[0][m] = m;
  for (int n = 1; n < MAXN; n++) {
    for (int m = 0; m < MAXM; m++) {
      dp[n][m] = dp[n - 1][m] - dp[n - 1][m / primes[n - 1]];
    }
  }
}
long long phi(long long m, int n) {
  if (n == 0) return m;
  if (m < MAXM && n < MAXN) return dp[n][m];
  if ((long long)primes[n - 1] * primes[n - 1] >= m && m < MAXP) return prime_cnt[m] - n + 1;
  return phi(m, n - 1) - phi(m / primes[n - 1], n - 1);
}
long long lehmer(long long m) {
  if (m < MAXP) return prime_cnt[m];
  int s = sqrt(0.5 + m), y = cbrt(0.5 + m);
  int a = prime_cnt[y];
  long long res = phi(m, a) + a - 1;
  for (int i = a; primes[i] <= s; i++) {
    res = res - lehmer(m / primes[i]) + lehmer(primes[i]) - 1;
  }
  return res;
}
int main() {
  auto start = clock();
  gen();
  printf("Time taken to generate = %0.6f\n\n", (clock() - start) / (double)CLOCKS_PER_SEC);
  cout << lehmer(1e12) << endl;
  printf("\nTime taken = %0.6f\n", (clock() - start) / (double)CLOCKS_PER_SEC);
  return 0;
}
```

4.30 hash.cpp

32 lines

```cpp
#include <bits/stdc++.h>
using namespace std;
lli mod_mult(lli a, lli b) {
 lli x = 0;
 while(b) {
  if(b & 1) x = (x + a);
  a = (a << 1);
  b >>= 1;
 }
 return x;
}
unsigned int hashh(unsigned int x)  {
  x = mod_mult(((x >> 16) ^x) ,0x45d9f3b);
  x = mod_mult(((x >> 16) ^x),0x45d9f3b);
  x = (x >> 16) ^x;
  return x;
}
unsigned int unhash(unsigned int x)  {
  x = mod_mult((x >> 16) ^x, 0x119de1f3);
  x = mod_mult((x >> 16) ^x , 0x119de1f3);
  x = (x >> 16) ^x;
  return x;
}
int main() {
  int n;
  set<int> hashes;
  for(int i = 0;i<10000000;i++) {
    hashes.insert(hashh(i));
  }
  cout<<hashes.size();
  return 0;
}
```

4.31 isPrime.cpp

78 lines

```cpp
#include <bits/stdc++.h>
using namespace std;
typedef unsigned long long int lli;
lli mod_mult(lli a, lli b, lli mod) {
 lli x = 0;
 while(b) {
  if(b & 1) x = (x + a) % mod;
  a = (a << 1) % mod;
  b >>= 1;
 }
 return x;
}
lli mod_pow(lli a, lli n, lli mod) {
 lli x = 1;
 while(n) {
  if(n & 1) x = mod_mult(x, a, mod);
  a = mod_mult(a, a, mod);
  n >>= 1;
 }
 return x;
}
lli random(lli a, lli b)  {
  lli intervallLength = b - a + 1;
  int neededSteps = 0;
  lli base = RAND_MAX + 1LL;
  while(intervallLength > 0) {
   intervallLength /= base;
   neededSteps++;
  }
```

```
  intervallLength = b - a + 1;
  lli result = 0;
  for(int stepsDone = 0; stepsDone < neededSteps; stepsDone++) {
    result = (result * base + rand());
  }
  result %= intervallLength;
  if(result < 0) result += intervallLength;
  return result + a;
}
bool witness(lli a, lli n)  {
 // check as in Miller Rabin Primality Test described
  lli u = n-1;
  int t = 0;
  while (u % 2 == 0)  {
    t++;
    u /= 2;
  }
  lli next = mod_pow(a, u, n);
  if(next == 1)return false;
  lli last;
  for(int i = 0; i < t; i++)  {
   last = next;
    next = mod_mult(last, last, n);//(last * last) % n;
    if (next == 1) {
     return last != n - 1;
    }
  }
  return next != 1;
}
bool isPrime(lli n, int s)  {
  if (n <= 1) return false;
  if (n == 2) return true;
  if (n % 2 == 0) return false;
  for(int i = 0; i < s; i++)  {
    lli a = random(1, n-1);
    if (witness(a, n)) return false;
  }
  return true;
}
int main() {
  long long int n,t;
  cin>>t;
  while(t--) {
    cin>>n;
    if(isPrime(n,20))cout<<"YES"<<endl;
    else cout<<"NO"<<endl;
  }
  return 0;
}
```

4.32 nthFibonacci.cpp

55 lines

```
#include <bits/stdc++.h>
using namespace std;
typedef long long int lli;
const lli M = 1000000000000000000; // modulo
void multiply(lli F[2][2], lli M[2][2])  {
 lli x = F[0][0]*M[0][0] + F[0][1]*M[1][0];
 lli y = F[0][0]*M[0][1] + F[0][1]*M[1][1];
 lli z = F[1][0]*M[0][0] + F[1][1]*M[1][0];
 lli w = F[1][0]*M[0][1] + F[1][1]*M[1][1];
 F[0][0] = x;
 F[0][1] = y;
 F[1][0] = z;
 F[1][1] = w;
}
void power(lli F[2][2], lli n)
{
 if( n == 0 || n == 1)
   return;
 lli M[2][2] = { {1,1} , {1,0} } ;
 power(F, n/2);
 multiply(F, F);
 if (n%2 != 0)
   multiply(F, M);
}
lli fibF(lli n)  {
 double phi = (1 + sqrt(5)) / 2;
 return round(pow(phi, n) / sqrt(5));
}
lli fib(lli n) {
 lli F[2][2] = { {1,1} , {1,0} } ;
 if (n == 0)
  return 0;
 power(F, n-1);
 return F[0][0];
}
map<lli, lli> F;
lli f(lli n)  {
 if (F.count(n)) return F[n];
 lli k=n/2;
 if (n%2==0)  { // n=2*k
  return F[n] = (f(k)*f(k) + f(k-1)*f(k-1)) % M;
 } else  { // n=2*k+1
  return F[n] = (f(k)*f(k+1) + f(k-1)*f(k)) % M;
 }
}
a
int main() {
  lli n;
  F[0]=F[1]=1;
  cin>>n;
  // cout<<fibF(n)<<endl;
  // cout<<fib(n)<<endl;
  cout<<f(n-1)<<endl;
  return 0;
}
```

4.33 otros.cpp

25 lines

```
//_______________________String to int_____________________//
int str_int(string st) {
  int n;
  stringstream ss(st);
  ss>>n;
  return n;
}
//________________________pow of large
    ↪ numbers_____________________________//
long long powerStrings(string sa, string sb) {
  // Convert strings to number
  long long a = 0, b = 0;
  for (int i = 0; i < sa.length(); i++)
    a = (a * 10 + (sa[i] - '0')) % MOD;
  for (int i = 0; i < sb.length(); i++)
    b = (b * 10 + (sb[i] - '0')) % (MOD - 1);
  return Mod(a, b);
}
//_____________pow of string_______________//
string b = "100000000000000000000000000";
long long remainderB = 0;
long long MOD = 1000000007;
  // using Fermat Little
for (int i = 0; i < b.length(); i++)
 remainderB = (remainderB * 10 + b[i] - '0') % (MOD - 1);
```

4.34 phiFunction.h

Description: *Euler's* ϕ function is defined as $\phi(n) := \#$ of positive integers $\leq n$ that are coprime with n. $\phi(1) = 1$, p prime $\Rightarrow \phi(p^k) = (p-1)p^{k-1}$, m, n coprime $\Rightarrow \phi(mn) = \phi(m)\phi(n)$. If $n = p_1^{k_1} p_2^{k_2} ... p_r^{k_r}$ then $\phi(n) = (p_1 - 1)p_1^{k_1-1} ... (p_r - 1)p_r^{k_r-1}$. $\phi(n) = n \cdot \prod_{p|n}(1 - 1/p)$. $\sum_{d|n} \phi(d) = n$, $\sum_{1 \leq k \leq n, \gcd(k,n)=1} k = n\phi(n)/2, n > 1$
Euler's thm: a, n coprime $\Rightarrow a^{\phi(n)} \equiv 1 \pmod{n}$.
Fermat's little thm: p prime $\Rightarrow a^{p-1} \equiv 1 \pmod{p}\ \forall a$.

10 lines

```
vector<int> Phi;
void phiSieve(int n) {
  Phi.resize(n + 1);
  for(int i = 1; i <= n; ++i)
    Phi[i] = i;
```

```
  for(int i = 2; i <= n; ++i)
    if(Phi[i] == i)
      for(int j = i; j <= n; j += i)
        Phi[j] -= Phi[j] / i;
}
```

4.35 segmentedSieve.cpp

Description: Segmented sieve , get primes in range[L,R] with complexity O(max(sqrt(R)log(sqrt(R)),R-L)) ??? ; Also is one of the fastest sieve get all primes in range [1-n] with n = 1e9 in 8.62s and for n = 1e8 in 0.76s [TESTED:] [SPOJ: PRIME1 - Prime Generator]

67 lines

```
vector<int> PrimesInRange;
void calcPrimes(int l ,int r) {
  auto sum = l<=2?2:0;
  if(l<=2)PrimesInRange.push_back(2);
  int cnt = 1;
  const int S = round(sqrt(r));
  vector<char> sieve(S + 1, true);
  vector<array<int, 2>> cp;
  for (int i = 3; i <= S; i += 2) {
    if (!sieve[i])
      continue;
    cp.push_back( { i, (i * i - 1) / 2 } );
    for (int j = i * i; j <= S; j += 2 * i) {
      sieve[j] = false;
    }
  }
  vector<char> block(S);
  int high = (r - 1) / 2;
  int x = l/S;
  int L = (x/2)*S;
  for(auto &i:cp) {
    int p = i[0],idx = i[1];
    if(idx>L) {
      i[1]-=L;
    }
    else {
      int X = (L-idx)/p;
      if((L-idx)%p)X++;
      if(X>=1 && idx<=L)
        i[1] = (idx+(p*X))-L;
    }
  }
  for (int low =(x/2)*S; low <= high; low += S) {
    fill(block.begin(), block.end(), true);
    for (auto &i : cp) {
      int p = i[0], idx = i[1];
      for (; idx < S; idx += p) {
        block[idx] = false;
      }
      i[1] = idx - S;
    }
    if (low == 0)
      block[0] = false;
    for (int i = 0; i < S && low + i <= high; i++) {
      if (block[i] && (((low+i)*2)+1)>=l) {
        // push the primes here if needed
        ++cnt, sum += (low + i) * 2 + 1;
      }
    }
  } ;
  // cout << "sum = " << sum << endl;
  // cout << "cnt = " << cnt << endl;
}
signed main() { __
  int l,r,t,id = 1;
  cin>>t;
  while(t--) {
    if(id>1)cout<<endl;
    PrimesInRange.clear();
    cin>>l>>r;
    calcPrimes(l,r);
    // for(auto c:primes)
    //   cout<<c<<endl;
    id++;
  }
  return 0;
}
```

numerical (5)

5.1 BasisXor

Description: Structure to form a basis in Z2 that alows compute things around xor because xor is a sum in Z2 like the maxxor possible , minimum , number of diferent xor´s, kth possible xor
Time: $\mathcal{O}(\log N)$

117 lines

```
struct Basis {
  vector<int> a;
  void insert(int x) {
    for (auto &i: a) x = min(x, x ^i);
    if (!x) return;
    for (auto &i: a) if ((i ^x) < i) i ^= x;
    a.push_back(x);
    sort(a.begin(), a.end());
  }
  bool can(int x) {
    for (auto &i: a) x = min(x, x ^i);
    return !x;
  }
  int maxxor(int x = 0) {
    for (auto &i: a) x = max(x, x ^i);
    return x;
  }
  int minxor(int x = 0) {
    for (auto &i: a) x = min(x, x ^i);
    return x;
  }
  int kth(int k) { // 1st is 0
    int sz = (int)a.size();
    if (k > (1LL << sz)) return -1;
    k--; int ans = 0;
    for (int i = 0; i < sz; i++) if (k >> i & 1) ans ^= a[i];
    return ans;
  }
} t;
// Arbitrary size
const int sz = 500;
struct basisxor {
  bitset<sz> bs[sz];
  bitset<sz> index[sz];
  bitset<sz> zero;
  basisxor() {
    for(int i=0;i<sz;i++) bs[i]=zero;
  }
  bitset<sz> chk(bitset<sz> &b) {
    bitset<sz> ans;
    int i;
    for(i = sz-1;i>=0;i--) {
      if(b[i]== 0)continue;
      if(b.count()== 0)break;
      if(bs[i].count()==0)break;
      b^=bs[i];
      ans^=index[i];
    }
    if(b.count()==0) return ans;
    return zero;
  }
  bool add(bitset<sz> &b,int idx) {
    int i;
    bitset<sz> x;
    x[idx]= 1;
    for(i = sz-1;i>=0;i--) {
      if(b[i]== 0)continue;
      if(b.count()== 0)break;
      if(bs[i].count()==0)break;
      b^=bs[i];
      x^=index[i];
    }
    if(i ==-1)return false;
    bs[i] = b;
```

```
    index[i] = x;
    return true;
  }
  void print() {
    for(int i =sz-1;i>=0;i--) {
      cout<<"I: "<<i<<" "<<bs[i]<<" idx: "<<endl;
    }
  }
};
int bin_pow(int a,int b) {
  int x = 1;
  while(b) {
    if(b&1) x*=a;
    a*=a;
    b>>=1;
  }
  return x;
}
// another
struct basisxor {
  int base[32];
  int sz = 0;
  basisxor() {
    for(int i = 0;i<32;i++)
      base[i] =0;
  }
  void add(int x) {
    while(x != 0 && base[31-__builtin_clz(x)]!= 0) {
      x^= base[31-__builtin_clz(x)];
    }
    if(!x)return;
    base[31-__builtin_clz(x)] = x;
    sz++;
  }
  int kth(int k) {
    int total = bin_pow(2,sz);
    int val = 0;
    for(int i = 31;i>=0;i--) {
      if(base[i] == 0)continue;
      if(k<total/2) {
        if((val>>i)&1)
          val^=base[i];
      }
      else {
        if(!((val>>i)&1))
          val^=base[i];
        k-=total/2;
      }
      total>>=1;
    }
    return val;
  }
};
```

5.2 BerlekampMassey

Description: Recovers any n-order linear recurrence relation from the first $2n$ terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size $\leq n$.
Usage: `berlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}`
Time: $\mathcal{O}(N^2)$

20 lines

```
#include "ModPow()"
vector<int> berlekampMassey(vector<int> s) {
 int n = sz(s), L = 0, m = 0;
 vector<int> C(n), B(n), T;
 C[0] = B[0] = 1;
 int b = 1;
 for(int i = 0;i<n;i++) {
  ++m;
  int d = s[i] % mod;
  for(int j = 1;j<L+1) d = (d + C[j] * s[i - j]) % mod;
  if (!d) continue;
  T = C; int coef = d * modpow(b, mod-2) % mod;
  for(int j = m;j<n;j++) C[j] = (C[j] - coef * B[j - m]) % mod;
  if (2 * L > i) continue;
  L = i + 1 - L; B = T; b = d; m = 0;
 }
 C.resize(L + 1); C.erase(C.begin());
 for (int& x : C) x = (mod - x) % mod;
 return C;
}
```

5.3 Determinant

Description: Calculates determinant of a matrix. Destroys the matrix.
Time: $\mathcal{O}(N^3)$

15 lines

```
double det(vector<vector<double>>& a) {
 int n = sz(a); double res = 1;
 rep(i,0,n) {
  int b = i;
  rep(j,i+1,n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
  if (i != b) swap(a[i], a[b]), res *= -1;
  res *= a[i][i];
  if (res == 0) return 0;
  rep(j,i+1,n) {
   double v = a[j][i] / a[i][i];
   if (v != 0) rep(k,i+1,n) a[j][k] -= v * a[i][k];
  }
 }
 return res;
}
```

5.4 ExtendedPolynomial.cpp

Description: A complete structure for polynomial and his operations

229 lines

```
const int N = 3e5 + 9, mod = 998244353;
struct base {
 double x, y;
 base() { x = y = 0; }
 base(double x, double y): x(x), y(y) { }
};
inline base operator + (base a, base b) { return base(a.x + b.x, a.y + b
    ↪ .y); }
inline base operator - (base a, base b) { return base(a.x - b.x, a.y - b
    ↪ .y); }
inline base operator * (base a, base b) { return base(a.x * b.x - a.y *
    ↪ b.y, a.x * b.y + a.y * b.x); }
inline base conj(base a) { return base(a.x, -a.y); }
int lim = 1;
vector<base> roots = { {0, 0}, {1, 0} };
vector<int> rev = {0, 1};
const double PI = acosl(- 1.0);
void ensure_base(int p) {
 if(p <= lim) return;
 rev.resize(1 << p);
 for(int i = 0; i < (1 << p); i++) rev[i] = (rev[i >> 1] >> 1) + ((i &
    ↪ 1) << (p - 1));
 roots.resize(1 << p);
 while(lim < p) {
  double angle = 2 * PI / (1 << (lim + 1));
  for(int i = 1 << (lim - 1); i < (1 << lim); i++) {
   roots[i << 1] = roots[i];
   double angle_i = angle * (2 * i + 1 - (1 << lim));
   roots[(i << 1) + 1] = base(cos(angle_i), sin(angle_i));
  }
  lim++;
 }
}
void fft(vector<base> &a, int n = -1) {
 if(n == -1) n = a.size();
 assert((n & (n - 1)) == 0);
 int zeros = __builtin_ctz(n);
 ensure_base(zeros);
 int shift = lim - zeros;
 for(int i = 0; i < n; i++) if(i < (rev[i] >> shift)) swap(a[i], a[rev[i
    ↪ ] >> shift]);
 for(int k = 1; k < n; k <<= 1) {
  for(int i = 0; i < n; i += 2 * k) {
   for(int j = 0; j < k; j++) {
    base z = a[i + j + k] * roots[j + k];
    a[i + j + k] = a[i + j] - z;
    a[i + j] = a[i + j] + z;
   }
```

```
    }
  }
}
//eq = 0: 4 FFTs in total
//eq = 1: 3 FFTs in total
vector<int> multiply(vector<int> &a, vector<int> &b, int eq = 0) {
  int need = a.size() + b.size() - 1;
  int p = 0;
  while((1 << p) < need) p++;
  ensure_base(p);
  int sz = 1 << p;
  vector<base> A, B;
  if(sz > (int)A.size()) A.resize(sz);
  for(int i = 0; i < (int)a.size(); i++) {
    int x = (a[i] % mod + mod) % mod;
    A[i] = base(x & ((1 << 15) - 1), x >> 15);
  }
  fill(A.begin() + a.size(), A.begin() + sz, base{0, 0});
  fft(A, sz);
  if(sz > (int)B.size()) B.resize(sz);
  if(eq) copy(A.begin(), A.begin() + sz, B.begin());
  else {
    for(int i = 0; i < (int)b.size(); i++) {
      int x = (b[i] % mod + mod) % mod;
      B[i] = base(x & ((1 << 15) - 1), x >> 15);
    }
    fill(B.begin() + b.size(), B.begin() + sz, base{0, 0});
    fft(B, sz);
  }
  double ratio = 0.25 / sz;
  base r2(0, -1), r3(ratio, 0), r4(0, -ratio), r5(0, 1);
  for(int i = 0; i <= (sz >> 1); i++) {
    int j = (sz - i) & (sz - 1);
    base a1 = (A[i] + conj(A[j])), a2 = (A[i] - conj(A[j])) * r2;
    base b1 = (B[i] + conj(B[j])) * r3, b2 = (B[i] - conj(B[j])) * r4;
    if(i != j) {
      base c1 = (A[j] + conj(A[i])), c2 = (A[j] - conj(A[i])) * r2;
      base d1 = (B[j] + conj(B[i])) * r3, d2 = (B[j] - conj(B[i])) * r4;
      A[i] = c1 * d1 + c2 * d2 * r5;
      B[i] = c1 * d2 + c2 * d1;
    }
    A[j] = a1 * b1 + a2 * b2 * r5;
    B[j] = a1 * b2 + a2 * b1;
  }
  fft(A, sz); fft(B, sz);
  vector<int> res(need);
  for(int i = 0; i < need; i++) {
    long long aa = A[i].x + 0.5;
    long long bb = B[i].x + 0.5;
    long long cc = A[i].y + 0.5;
    res[i] = (aa + ((bb % mod) << 15) + ((cc % mod) << 30))%mod;
```

```
  }
  return res;
}
template <int32_t MOD>
struct modint {
  int32_t value;
  modint() = default;
  modint(int32_t value_) : value(value_) { }
  inline modint<MOD> operator + (modint<MOD> other) const { int32_t c = this->value + other.value; return modint<MOD>(c >= MOD ? c - MOD : c); }
  inline modint<MOD> operator - (modint<MOD> other) const { int32_t c = this->value - other.value; return modint<MOD>(c < 0 ? c + MOD : c); }
  inline modint<MOD> operator * (modint<MOD> other) const { int32_t c = (int64_t)this->value * other.value % MOD; return modint<MOD>(c < 0 ? c + MOD : c); }
  inline modint<MOD> & operator += (modint<MOD> other) { this->value += other.value; if (this->value >= MOD) this->value -= MOD; return *this; }
  inline modint<MOD> & operator -= (modint<MOD> other) { this->value -= other.value; if (this->value < 0) this->value += MOD; return *this; }
  inline modint<MOD> & operator *= (modint<MOD> other) { this->value = (int64_t)this->value * other.value % MOD; if (this->value < 0) this->value += MOD; return *this; }
  inline modint<MOD> operator - () const { return modint<MOD>(this->value ? MOD - this->value : 0); }
  modint<MOD> pow(uint64_t k) const {
    modint<MOD> x = *this, y = 1;
    for (; k; k >>= 1) {
      if (k & 1) y *= x;
      x *= x;
    }
    return y;
  }
  modint<MOD> inv() const { return pow(MOD - 2); } // MOD must be a prime
  inline modint<MOD> operator / (modint<MOD> other) const { return *this * other.inv(); }
  inline modint<MOD> operator /= (modint<MOD> other) { return *this *= other.inv(); }
  inline bool operator == (modint<MOD> other) const { return value == other.value; }
  inline bool operator != (modint<MOD> other) const { return value != other.value; }
  inline bool operator < (modint<MOD> other) const { return value < other.value; }
  inline bool operator > (modint<MOD> other) const { return value > other.value; }
};
```

```
template <int32_t MOD> modint<MOD> operator * (int64_t value, modint<MOD> n) { return modint<MOD>(value) * n; }
template <int32_t MOD> modint<MOD> operator * (int32_t value, modint<MOD> n) { return modint<MOD>(value % MOD) * n; }
template <int32_t MOD> ostream & operator << (ostream & out, modint<MOD> n) { return out << n.value; }
using mint = modint<mod>;
struct poly {
  vector<mint> a;
  inline void normalize() {
    while((int)a.size() && a.back() == 0) a.pop_back();
  }
  template<class... Args> poly(Args... args): a(args...) { }
  poly(const initializer_list<mint> &x): a(x.begin(), x.end()) { }
  int size() const { return (int)a.size(); }
  inline mint coef(const int i) const { return (i < a.size() && i >= 0) ? a[i]: mint(0); }
  mint operator[](const int i) const { return (i < a.size() && i >= 0) ? a[i]: mint(0); } //Beware!! p[i] = k won't change the value of p.a[i]
  bool is_zero() const {
    for (int i = 0; i < size(); i++) if (a[i] != 0) return 0;
    return 1;
  }
  poly operator + (const poly &x) const {
    int n = max(size(), x.size());
    vector<mint> ans(n);
    for(int i = 0; i < n; i++) ans[i] = coef(i) + x.coef(i);
    while ((int)ans.size() && ans.back() == 0) ans.pop_back();
    return ans;
  }
  poly operator - (const poly &x) const {
    int n = max(size(), x.size());
    vector<mint> ans(n);
    for(int i = 0; i < n; i++) ans[i] = coef(i) - x.coef(i);
    while ((int)ans.size() && ans.back() == 0) ans.pop_back();
    return ans;
  }
  poly operator * (const poly& b) const {
    if(is_zero() || b.is_zero()) return {};
    vector<int> A, B;
    for(auto x: a) A.push_back(x.value);
    for(auto x: b.a) B.push_back(x.value);
    auto res = multiply(A, B, (A == B));
    vector<mint> ans;
    for(auto x: res) ans.push_back(mint(x));
    while ((int)ans.size() && ans.back() == 0) ans.pop_back();
    return ans;
  }
  poly operator * (const mint& x) const {
    int n = size();
```

```
  vector<mint> ans(n);
  for(int i = 0; i < n; i++) ans[i] = a[i] * x;
  return ans;
 }
 poly operator / (const mint &x) const { return (*this) * x.inv(); }
 poly& operator += (const poly &x) { return *this = (*this) + x; }
 poly& operator -= (const poly &x) { return *this = (*this) - x; }
 poly& operator *= (const poly &x) { return *this = (*this) * x; }
 poly& operator *= (const mint &x) { return *this = (*this) * x; }
 poly& operator /= (const mint &x) { return *this = (*this) / x; }
 poly mod_xk(int k) const { return { a.begin(), a.begin() + min(k, size()) }; } //modulo by x^k
 poly mul_xk(int k) const { // multiply by x^k
  poly ans(*this);
  ans.a.insert(ans.a.begin(), k, 0);
  return ans;
 }
 poly div_xk(int k) const { // divide by x^k
  return vector<mint>(a.begin() + min(k, (int)a.size()), a.end());
 }
 poly substr(int l, int r) const { // return mod_xk(r).div_xk(l)
  l = min(l, size());
  r = min(r, size());
  return vector<mint>(a.begin() + l, a.begin() + r);
 }
 poly differentiate() const {
  int n = size(); vector<mint> ans(n);
  for(int i = 1; i < size(); i++) ans[i - 1] = coef(i) * i;
  return ans;
 }
 poly integrate() const {
  int n = size(); vector<mint> ans(n + 1);
  for(int i = 0; i < size(); i++) ans[i + 1] = coef(i) / (i + 1);
  return ans;
 }
 poly inverse(int n) const { // 1 / p(x) % x^n, O(nlogn)
  assert(!is_zero()); assert(a[0] != 0);
  poly ans{ mint(1) / a[0] };
  for(int i = 1; i < n; i *= 2) {
   ans = (ans * mint(2) - ans * ans * mod_xk(2 * i)).mod_xk(2 * i);
  }
  return ans.mod_xk(n);
 }
 poly log(int n) const { //ln p(x) mod x^n
  assert(a[0] == 1);
  return (differentiate().mod_xk(n) * inverse(n)).integrate().mod_xk(n);
 }
 poly exp(int n) const { //e ^p(x) mod x^n
  if(is_zero()) return {1};
  assert(a[0] == 0);
  poly ans({1});
  int i = 1;
  while(i < n) {
   poly C = ans.log(2 * i).div_xk(i) - substr(i, 2 * i);
   ans -= (ans * C).mod_xk(i).mul_xk(i);
   i *= 2;
  }
  return ans.mod_xk(n);
 }
};
```

5.5 FWHT.cpp

Description: Gets an xor convolution. if you have two arrays like (1,2,2) and (3,4,5) ,all possible xor results in (1,1,2,4,5,6,6,7,7)

71 lines

```
const int N = 3e5 + 9, mod = 1e9 + 7;
int POW(long long n, long long k) {
 int ans = 1 % mod; n %= mod; if (n < 0) n += mod;
 while (k) {
  if (k & 1) ans = (long long) ans * n % mod;
  n = (long long) n * n % mod;
  k >>= 1;
 }
 return ans;
}
const int inv2 = (mod + 1) >> 1;
#define M (1 << 20)
#define OR 0
#define AND 1
#define XOR 2
struct FWHT{
 int P1[M], P2[M];
 void wt(int *a, int n, int flag = XOR) {
  if (n == 0) return;
  int m = n / 2;
  wt(a, m, flag); wt(a + m, m, flag);
  for (int i = 0; i < m; i++){
   int x = a[i], y = a[i + m];
   if (flag == OR) a[i] = x, a[i + m] = (x + y) % mod;
   if (flag == AND) a[i] = (x + y) % mod, a[i + m] = y;
   if (flag == XOR) a[i] = (x + y) % mod, a[i + m] = (x - y + mod) % mod;
  }
 }
 void iwt(int* a, int n, int flag = XOR) {
  if (n == 0) return;
  int m = n / 2;
  iwt(a, m, flag); iwt(a + m, m, flag);
  for (int i = 0; i < m; i++){
   int x = a[i], y = a[i + m];
   if (flag == OR) a[i] = x, a[i + m] = (y - x + mod) % mod;
   if (flag == AND) a[i] = (x - y + mod) % mod, a[i + m] = y;
   if (flag == XOR) a[i] = 1LL * (x + y) * inv2 % mod, a[i + m] = 1LL * (x - y + mod) * inv2 % mod; // replace inv2 by >>1 if not required
  }
 }
 vector<int> multiply(int n, vector<int> A, vector<int> B, int flag = XOR) {
  assert(__builtin_popcount(n) == 1);
  A.resize(n); B.resize(n);
  for (int i = 0; i < n; i++) P1[i] = A[i];
  for (int i = 0; i < n; i++) P2[i] = B[i];
  wt(P1, n, flag); wt(P2, n, flag);
  for (int i = 0; i < n; i++) P1[i] = 1LL * P1[i] * P2[i] % mod;
  iwt(P1, n, flag);
  return vector<int> (P1, P1 + n);
 }
 vector<int> pow(int n, vector<int> A, long long k, int flag = XOR) {
  assert(__builtin_popcount(n) == 1);
  A.resize(n);
  for (int i = 0; i < n; i++) P1[i] = A[i];
  wt(P1, n, flag);
  for(int i = 0; i < n; i++) P1[i] = POW(P1[i], k);
  iwt(P1, n, flag);
  return vector<int> (P1, P1 + n);
 }
}t;
int32_t main() {
 int n; cin >> n;
 vector<int> a(M, 0);
 for(int i = 0; i < n; i++) {
  int k; cin >> k; a[k]++;
 }
 vector<int> v = t.pow(M, a, n, AND);
 int ans = 1;
 for(int i = 1; i < M; i++) ans += v[i] > 0;
 cout << ans << '\n';;
 return 0;
}
```

5.6 FastFourierTransform

Description: fft(a) computes $\hat{f}(k) = \sum_x a[x]\exp(2\pi i \cdot kx/N)$ for all k. N must be a power of 2. Useful for convolution: `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For convolution of complex numbers or more than two vectors: FFT, multiply pointwise, divide by n, reverse(start+1, end), FFT back. Rounding is safe if $(\sum a_i^2 + \sum b_i^2)\log_2 N < 9\cdot 10^{14}$ (in practice 10^{16}; higher for random inputs). Otherwise, use NTT/FFTMod.
Time: $\mathcal{O}(N\log N)$ with $N = |A| + |B|$ ($\tilde{1}s$ for $N = 2^{22}$)

35 lines

```
typedef complex<double> C;
```

```
typedef vector<double> vd;
void fft(vector<C>& a) {
 int n = sz(a), L = 31 - __builtin_clz(n);
 static vector<complex<long double>> R(2, 1);
 static vector<C> rt(2, 1); // (^ 10% faster if double)
 for (static int k = 2; k < n; k *= 2) {
  R.resize(n); rt.resize(n);
  auto x = polar(1.0L, acos(-1.0L) / k);
  rep(i,k,2*k) rt[i] = R[i] = i&1 ? R[i/2] * x : R[i/2];
 }
 vi rev(n);
 rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
 rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
 for (int k = 1; k < n; k *= 2)
  for (int i = 0; i < n; i += 2 * k) rep(j,0,k) {
   C z = rt[j+k] * a[i+j+k]; // (25% faster if hand-rolled) /// include
       ↪ -line
   a[i + j + k] = a[i + j] - z;
   a[i + j] += z;
  }
}
vd conv(const vd& a, const vd& b) {
 if (a.empty() || b.empty()) return {};
 vd res(sz(a) + sz(b) - 1);
 int L = 32 - __builtin_clz(sz(res)), n = 1 << L;
 vector<C> in(n), out(n);
 copy(all(a), begin(in));
 rep(i,0,sz(b)) in[i].imag(b[i]);
 fft(in);
 for (C& x : in) x *= x;
 rep(i,0,n) out[i] = in[-i & (n - 1)] - conj(in[i]);
 fft(out);
 rep(i,0,sz(res)) res[i] = imag(out[i]) / (4 * n);
 return res;
}
```

5.7 FastFourierTransformMod

Description: Higher precision FFT, can be used for convolutions modulo arbitrary integers as long as $N \log_2 N \cdot \text{mod} < 8.6 \cdot 10^{14}$ (in practice 10^{16} or higher). Inputs must be in $[0, \text{mod})$.
Time: $\mathcal{O}(N \log N)$, where $N = |A| + |B|$ (twice as slow as NTT or FFT)

23 lines

```
#include "FastFourierTransform.h"
typedef vector<ll> vl;
template<int M> vl convMod(const vl &a, const vl &b) {
 if (a.empty() || b.empty()) return {};
 vl res(sz(a) + sz(b) - 1);
 int B=32-__builtin_clz(sz(res)), n=1<<B, cut=int(sqrt(M));
 vector<C> L(n), R(n), outs(n), outl(n);
 rep(i,0,sz(a)) L[i] = C((int)a[i] / cut, (int)a[i] % cut);
 rep(i,0,sz(b)) R[i] = C((int)b[i] / cut, (int)b[i] % cut);
 fft(L), fft(R);
 rep(i,0,n) {
  int j = -i & (n - 1);
  outl[j] = (L[i] + conj(L[j])) * R[i] / (2.0 * n);
  outs[j] = (L[i] - conj(L[j])) * R[i] / (2.0 * n) / 1i;
 }
 fft(outl), fft(outs);
 rep(i,0,sz(res)) {
  ll av = ll(real(outl[i])+.5), cv = ll(imag(outs[i])+.5);
  ll bv = ll(imag(outl[i])+.5) + ll(real(outs[i])+.5);
  res[i] = ((av % M * cut + bv) % M * cut + cv) % M;
 }
 return res;
}
```

5.8 FastSubsetTransform

Description: Transform to a basis with fast convolutions of the form $c[z] = \sum_{z=x\oplus y} a[x] \cdot b[y]$, where $\oplus$ is one of AND, OR, XOR. The size of a must be a power of two.
Time: $\mathcal{O}(N \log N)$

16 lines

```
void FST(vi& a, bool inv) {
 for (int n = sz(a), step = 1; step < n; step *= 2) {
  for (int i = 0; i < n; i += 2 * step) rep(j,i,i+step) {
   int &u = a[j], &v = a[j + step]; tie(u, v) =
    inv ? pii(v - u, u) : pii(v, u + v); // AND
    inv ? pii(v, u - v) : pii(u + v, u); // OR /// include-line
    pii(u + v, u - v);           // XOR /// include-line
  }
 }
 if (inv) for (int& x : a) x /= sz(a); // XOR only /// include-line
}
vi conv(vi a, vi b) {
 FST(a, 0); FST(b, 0);
 rep(i,0,sz(a)) a[i] *= b[i];
 FST(a, 1); return a;
}
```

5.9 GoldenSectionSearch

Description: Finds the argument minimizing the function f in the interval $[a, b]$ assuming f is unimodal on the interval, i.e. has only one local minimum. The maximum error in the result is *eps*. Works equally well for maximization with a small change in the code. See TernarySearch.h in the Various chapter for a discrete version.
Usage: `double func(double x) { return 4+x+.3*x*x; }`
`double xmin = gss(-1000,1000,func);`
Time: $\mathcal{O}(\log((b-a)/\epsilon))$

15 lines

```
/// It is important for r to be precise, otherwise we don't necessarily
    ↪ maintain the inequality a < x1 < x2 < b.
double gss(double a, double b, double (*f)(double)) {
 double r = (sqrt(5)-1)/2, eps = 1e-7;
 double x1 = b - r*(b-a), x2 = a + r*(b-a);
 double f1 = f(x1), f2 = f(x2);
 while (b-a > eps)
  if (f1 < f2) { //change to > to find maximum
   b = x2; x2 = x1; f2 = f1;
   x1 = b - r*(b-a); f1 = f(x1);
  } else {
   a = x1; x1 = x2; f1 = f2;
   x2 = a + r*(b-a); f2 = f(x2);
  }
 return a;
}
```

5.10 HillClimbing.h

Description: Poor man's optimization for unimodal functions.

13 lines

```
typedef array<double, 2> P;
template<class F> pair<double, P> hillClimb(P start, F f) {
 pair<double, P> cur(f(start), start);
 for (double jmp = 1e9; jmp > 1e-20; jmp /= 2) {
  rep(j,0,100) rep(dx,-1,2) rep(dy,-1,2) {
   P p = cur.second;
   p[0] += dx*jmp;
   p[1] += dy*jmp;
   cur = min(cur, make_pair(f(p), p));
  }
 }
 return cur;
}
```

5.11 IntDeterminant

Description: Calculates determinant using modular arithmetics. Modulos can also be removed to get a pure-integer version.
Time: $\mathcal{O}(N^3)$

18 lines

```
const ll mod = 12345;
ll det(vector<vector<ll>>& a) {
 int n = sz(a); ll ans = 1;
 rep(i,0,n) {
  rep(j,i+1,n) {
   while (a[j][i] != 0) { // gcd step
    ll t = a[i][i] / a[j][i];
    if (t) rep(k,i,n)
     a[i][k] = (a[i][k] - a[j][k] * t) % mod;
    swap(a[i], a[j]);
    ans *= -1;
   }
  }
```

```
  ans = ans * a[i][i] % mod;
  if (!ans) return 0;
 }
 return (ans + mod) % mod;
}
```

5.12 Integrate.h

Description: Simple integration of a function over an interval using Simpson's rule. The error should be proportional to h^4, although in practice you will want to verify that the result is stable to desired precision when epsilon changes.

7 lines

```
template<class F>
double quad(double a, double b, F f, const int n = 1000) {
 double h = (b - a) / 2 / n, v = f(a) + f(b);
 rep(i,1,n*2)
  v += f(a + i*h) * (i&1 ? 4 : 2);
 return v * h / 3;
}
```

5.13 IntegrateAdaptive.h

Description: Fast integration using an adaptive Simpson's rule.
Usage: `double sphereVolume = quad(-1, 1, [](double x) {`
`return quad(-1, 1, [&](double y) {`
`return quad(-1, 1, [&](double z) {`
`return x*x + y*y + z*z < 1; });});});`

14 lines

```
typedef double d;
#define S(a,b) (f(a) + 4*f((a+b) / 2) + f(b)) * (b-a) / 6
template <class F>
d rec(F& f, d a, d b, d eps, d S) {
 d c = (a + b) / 2;
 d S1 = S(a, c), S2 = S(c, b), T = S1 + S2;
 if (abs(T - S) <= 15 * eps || b - a < 1e-10)
  return T + (T - S) / 15;
 return rec(f, a, c, eps / 2, S1) + rec(f, c, b, eps / 2, S2);
}
template<class F>
d quad(d a, d b, F f, d eps = 1e-8) {
 return rec(f, a, b, eps, S(a, b));
}
```

5.14 LinearRecurrence

Description: Generates the k'th term of an n-order linear recurrence $S[i] = \sum_j S[i-j-1]tr[j]$, given $S[0 \ldots \geq n-1]$ and $tr[0 \ldots n-1]$. Faster than matrix multiplication. Useful together with Berlekamp–Massey.
Usage: `linearRec({0, 1}, {1, 1}, k) // k'th Fibonacci number`
Time: $\mathcal{O}\left(n^2 \log k\right)$

22 lines

```
typedef vector<ll> Poly;
ll linearRec(Poly S, Poly tr, ll k) {
 int n = sz(tr);
 auto combine = [&](Poly a, Poly b) {
  Poly res(n * 2 + 1);
  rep(i,0,n+1) rep(j,0,n+1)
   res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
  for (int i = 2 * n; i > n; --i) rep(j,0,n)
   res[i - 1 - j] = (res[i - 1 - j] + res[i] * tr[j]) % mod;
  res.resize(n + 1);
  return res;
 };
 Poly pol(n + 1), e(pol);
 pol[0] = e[1] = 1;
 for (++k; k; k /= 2) {
  if (k % 2) pol = combine(pol, e);
  e = combine(e, e);
 }
 ll res = 0;
 rep(i,0,n) res = (res + pol[i + 1] * S[i]) % mod;
 return res;
}
```

5.15 MatrixInverse-mod

Description: Invert matrix A modulo a prime. Returns rank; result is stored in A unless singular (rank < n). For prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$ where A^{-1} starts as the inverse of A mod p, and k is doubled in each step.
Time: $\mathcal{O}\left(n^3\right)$

34 lines

```
#include "../number-theory/ModPow.h"
int matInv(vector<vector<ll>>& A) {
 int n = sz(A); vi col(n);
 vector<vector<ll>> tmp(n, vector<ll>(n));
 rep(i,0,n) tmp[i][i] = 1, col[i] = i;
 rep(i,0,n) {
  int r = i, c = i;
  rep(j,i,n) rep(k,i,n) if (A[j][k]) {
   r = j; c = k; goto found;
  }
  return i;
found:
  A[i].swap(A[r]); tmp[i].swap(tmp[r]);
  rep(j,0,n) swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
  swap(col[i], col[c]);
  ll v = modpow(A[i][i], mod - 2);
  rep(j,i+1,n) {
   ll f = A[j][i] * v % mod;
   A[j][i] = 0;
   rep(k,i+1,n) A[j][k] = (A[j][k] - f*A[i][k]) % mod;
   rep(k,0,n) tmp[j][k] = (tmp[j][k] - f*tmp[i][k]) % mod;
  }
  rep(j,i+1,n) A[i][j] = A[i][j] * v % mod;
  rep(j,0,n) tmp[i][j] = tmp[i][j] * v % mod;
  A[i][i] = 1;
 }
 for (int i = n-1; i > 0; --i) rep(j,0,i) {
  ll v = A[j][i];
  rep(k,0,n) tmp[j][k] = (tmp[j][k] - v*tmp[i][k]) % mod;
 }
 rep(i,0,n) rep(j,0,n)
  A[col[i]][col[j]] = tmp[i][j] % mod + (tmp[i][j] < 0 ? mod : 0);
 return n;
}
```

5.16 MatrixInverse

Description: Invert matrix A. Returns rank; result is stored in A unless singular (rank < n). Can easily be extended to prime moduli; for prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$ where A^{-1} starts as the inverse of A mod p, and k is doubled in each step.
Time: $\mathcal{O}\left(n^3\right)$

33 lines

```
int matInv(vector<vector<double>>& A) {
 int n = sz(A); vi col(n);
 vector<vector<double>> tmp(n, vector<double>(n));
 rep(i,0,n) tmp[i][i] = 1, col[i] = i;
 rep(i,0,n) {
  int r = i, c = i;
  rep(j,i,n) rep(k,i,n)
   if (fabs(A[j][k]) > fabs(A[r][c]))
    r = j, c = k;
  if (fabs(A[r][c]) < 1e-12) return i;
  A[i].swap(A[r]); tmp[i].swap(tmp[r]);
  rep(j,0,n)
   swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
  swap(col[i], col[c]);
  double v = A[i][i];
  rep(j,i+1,n) {
   double f = A[j][i] / v;
   A[j][i] = 0;
   rep(k,i+1,n) A[j][k] -= f*A[i][k];
   rep(k,0,n) tmp[j][k] -= f*tmp[i][k];
  }
  rep(j,i+1,n) A[i][j] /= v;
  rep(j,0,n) tmp[i][j] /= v;
  A[i][i] = 1;
 }
 /// forget A at this point, just eliminate tmp backward
 for (int i = n-1; i > 0; --i) rep(j,0,i) {
  double v = A[j][i];
  rep(k,0,n) tmp[j][k] -= v*tmp[i][k];
 }
 rep(i,0,n) rep(j,0,n) A[col[i]][col[j]] = tmp[i][j];
 return n;
```

```
}
```

5.17 NumberTheoreticTransform

Description: ntt(a) computes $\hat{f}(k) = \sum_x a[x]g^{xk}$ for all k, where $g = \text{root}^{(mod-1)/N}$. N must be a power of 2. Useful for convolution modulo specific nice primes of the form $2^a b + 1$, where the convolution result has size at most 2^a. For arbitrary modulo, see FFTMod. `conv(a, b) = c`, where $c[x] = \sum a[i]b[x-i]$. For manual convolution: NTT the inputs, multiply pointwise, divide by n, reverse(start+1, end), NTT back. Inputs must be in [0, mod).

Usage:
```
vector<int> X(n), Y(m);
for(int i = 0; i < n; i++) s[i]== c?X[i] = 1:X[i] = 0;
for(int i = 0; i < m; i++) t[i]== c?Y[i] = 1:Y[i] = 0;
reverse(Y.begin(),Y.end());
mult<998244353, 3>(X, Y);
```
Time: $\mathcal{O}(N \log N)$

61 lines

```
const double PI = acos(-1.0L);
using lli = int64_t;
using comp = complex<long double>;
#define print(A)for(auto c:A)cout<<c<<" ";cout<<endl;
#define printc(A)for(auto c:A)cout<<c.real()<<" ";cout<<endl;
#define __ ios_base::sync_with_stdio(false);cin.tie(NULL);
#define endl '\n'
typedef vector<comp> vec;
int nearestPowerOfTwo(int n){
 int ans = 1;
 while(ans < n) ans <<= 1;
 return ans;
}
lli powerMod(lli b, lli e, lli m){
 lli ans = 1;
 e %= m-1;
 if(e < 0) e += m-1;
 while(e){
  if(e & 1) ans = ans * b % m;
  e >>= 1;
  b = b * b % m;
 }
 return ans;
}
template<int p, int g>
void ntt(vector<int> & X, int inv){
 int n = X.size();
 for(int i = 1, j = 0; i < n - 1; ++i){
  for(int k = n >> 1; (j ^= k) < k; k >>= 1);
  if(i < j) swap(X[i], X[j]);
 }
 vector<lli> wp(n>>1, 1);
 for(int k = 1; k < n; k <<= 1){
  lli wk = powerMod(g, inv * (p - 1) / (k<<1), p);
  for(int j = 1; j < k; ++j)
   wp[j] = wp[j - 1] * wk % p;
  for(int i = 0; i < n; i += k << 1){
   for(int j = 0; j < k; ++j){
    int u = X[i + j], v = X[i + j + k] * wp[j] % p;
    X[i + j] = u + v < p ? u + v : u + v - p;
    X[i + j + k] = u - v < 0 ? u - v + p : u - v;
   }
  }
 }
 if(inv == -1){
  lli nrev = powerMod(n, p - 2, p);
  for(int i = 0; i < n; ++i)
   X[i] = X[i] * nrev % p;
 }
}
template<int p, int g>
void mult(vector<int> &A, vector<int> &B){
 int sz = A.size() + B.size() - 1;
 int size = nearestPowerOfTwo(sz);
 A.resize(size), B.resize(size);
 ntt<p, g>(A, 1), ntt<p, g>(B, 1);
 for(int i = 0; i < size; i++)
  A[i] = (lli)A[i] * B[i] % p;
 ntt<p, g>(A, -1);
 A.resize(sz);
}
```

5.18 PolyInterpolate

Description: Given n points (x[i], y[i]), computes an n-1-degree polynomial p that passes through them: $p(x) = a[0] * x^0 + ... + a[n-1] * x^{n-1}$. For numerical precision, pick $x[k] = c * \cos(k/(n-1) * \pi), k = 0 \dots n-1$.
Time: $\mathcal{O}(n^2)$

13 lines

```
typedef vector<double> vd;
vd interpolate(vd x, vd y, int n) {
 vd res(n), temp(n);
 rep(k,0,n-1) rep(i,k+1,n)
  y[i] = (y[i] - y[k]) / (x[i] - x[k]);
 double last = 0; temp[0] = 1;
 rep(k,0,n) rep(i,0,n) {
  res[i] += y[k] * temp[i];
  swap(last, temp[i]);
  temp[i] -= last * x[k];
 }
 return res;
}
```

5.19 PolyRoots

Description: Finds the real roots to a polynomial.
Usage: `polyRoots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0`
Time: $\mathcal{O}\left(n^2 \log(1/\epsilon)\right)$

24 lines

```
// #include "Polynomial.h"
vector<double> polyRoots(Poly p, double xmin, double xmax) {
 if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
 vector<double> ret;
 Poly der = p;
 der.diff();
 auto dr = polyRoots(der, xmin, xmax);
 dr.push_back(xmin-1);
 dr.push_back(xmax+1);
 sort(all(dr));
 rep(i,0,sz(dr)-1) {
  double l = dr[i], h = dr[i+1];
  bool sign = p(l) > 0;
  if (sign ^(p(h) > 0)) {
   rep(it,0,60) { // while (h - l > 1e-8)
    double m = (l + h) / 2, f = p(m);
    if ((f <= 0) ^sign) l = m;
    else h = m;
   }
   ret.push_back((l + h) / 2);
  }
 }
 return ret;
}
```

5.20 Polynomial.h

17 lines

```
struct Poly {
 vector<double> a;
 double operator()(double x) const {
  double val = 0;
  for (int i = sz(a); i--;) (val *= x) += a[i];
  return val;
 }
 void diff() {
  rep(i,1,sz(a)) a[i-1] = i*a[i];
  a.pop_back();
 }
 void divroot(double x0) {
  double b = a.back(), c; a.back() = 0;
  for(int i=sz(a)-1; i--;) c = a[i], a[i] = a[i+1]*x0+b, b=c;
  a.pop_back();
 }
};
```

5.21 PolynomialInt.cpp

489 lines

```
//
// Polynomial with integer coefficient (mod M)
//
// Implemented routines:
//  1) addition
//  2) subtraction
//  3) multiplication (naive O(n^2), Karatsuba O(n^1.5..), FFT O(n log n))
//  4) division (naive O(n^2), Newton O(M(n)))
//  5) gcd
//  6) multipoint evaluation (divide conquer: O(M(n) log |X|))
//  7) interpolation (naive O(n^2), divide conquer O(M(n) log n))
//  8) polynomial shift (naive, fast)
//
//  *) n! mod M in O(n^{1/2} log n) time
//
//
typedef long long ll;
ll add(ll a, ll b, ll M)
{
  return (a += b) >= M ? a - M : a;
}
ll sub(ll a, ll b, ll M)
{
  return (a -= b) < 0 ? a + M : a;
}
ll mul(ll a, ll b, ll M)
{
  ll q = (long double) a * (long double) b / (long double) M;
  ll r = a * b - q * M;
  return (r + 5 * M) % M;
}
// solve b x == a (mod M)
ll div(ll a, ll b, ll M)
{
  ll u = 1, x = 0, s = b, t = M;
  while (s)
  {
    ll q = t / s;
    swap(x -= u * q, u);
    swap(t -= s * q, s);
  }
  if (a % t)
    return -1; // infeasible
  return mul(x < 0 ? x + M : x, a / t, M); // b (xa/t) == a (mod M)
}
ll pow(ll a, ll b, ll M)
{
  ll x = 1;
  for (; b > 0; b >>= 1)
  {
    if (b & 1)
      x = (a * x) % M;
    a = (a * a) % M;
  }
  return x;
}
// p(x) = p[0] + p[1] x + ... + p[n-1] x^n-1
// assertion: p.back() != 0
typedef vector<ll> poly;
ostream& operator<<(ostream &os, const poly &p)
{
  bool head = true;
  for (int i = 0; i < p.size(); ++i)
  {
    if (p[i] == 0)
      continue;
    if (!head)
      os << " + ";
    os << p[i];
    head = false;
    if (i >= 1)
      os << " x";
    if (i >= 2)
      os << "^" << i;
  }
  return os;
}
poly add(poly p, const poly &q, ll M)
{
  if (p.size() < q.size())
    p.resize(q.size());
  for (int i = 0; i < q.size(); ++i)
    p[i] = add(p[i], q[i], M);
  while (!p.empty() && !p.back())
    p.pop_back();
  return p;
}
poly sub(poly p, const poly &q, ll M)
{
  if (p.size() < q.size())
    p.resize(q.size());
  for (int i = 0; i < q.size(); ++i)
    p[i] = sub(p[i], q[i], M);
  while (!p.empty() && !p.back())
    p.pop_back();
  return p;
}
// naive multiplication in O(n^2)
poly mul_n(const poly &p, const poly &q, ll M)
{
  if (p.empty() || q.empty())
    return { };
  poly r(p.size() + q.size() - 1);
  for (int i = 0; i < p.size(); ++i)
    for (int j = 0; j < q.size(); ++j)
      r[i + j] = add(r[i + j], mul(p[i], q[j], M), M);
  while (!r.empty() && !r.back())
    r.pop_back();
  return r;
}
// naive division (long division) in O(n^2)
pair<poly, poly> divmod_n(poly p, poly q, ll M)
{
  poly u(p.size() - q.size() + 1);
  ll inv = div(1, q.back(), M);
  for (int i = u.size() - 1; i >= 0; --i)
  {
    u[i] = mul(p.back(), inv, M);
    for (int j = 0; j < q.size(); ++j)
      p[j + p.size() - q.size()] = sub(p[j + p.size() - q.size()],
        mul(q[j], u[i], M), M);
    p.pop_back();
  }
  return {u, p};
}
// Karatsuba multiplication; this works correctly for M in [long long]
poly mul_k(poly p, poly q, ll M)
{
  int n = max(p.size(), q.size()), m = p.size() + q.size() - 1;
  for (int k : {1, 2, 4, 8, 16 })
    n |= (n >> k);
  ++n; // n is power of two
  p.resize(n);
  q.resize(n);
  poly r(6 * n);
  function<void(ll*, ll*, int, ll*)> rec = [&](ll *p0, ll *q0, int n, ll *r0)
  {
    if (n <= 4)
    {
      // 4 is the best threshold
      fill_n(r0, 2*n, 0);
      for (int i = 0; i < n; ++i)
        for (int j = 0; j < n; ++j)
          r0[i+j] = add(r0[i+j], mul(p0[i], q0[j], M), M);
      return;
    }
    ll *p1=p0+n/2,*q1=q0+n/2,*r1=r0+n/2,*r2=r0+n,*u=r0+5*n,*v=u+n/2,*w=r0+2*n;
    for (int i = 0; i < n/2; ++i)
    {
```

```
    u[i] = add(p0[i], p1[i], M);
    v[i] = add(q0[i], q1[i], M);
   }
  rec(p0, q0, n/2, r0);
  rec(p1, q1, n/2, r2);
  rec( u, v, n/2, w);
  for (int i = 0; i < n; ++i) w[i] = sub(w[i], add(r0[i], r2[i], M), M)
      ↪ ;
  for (int i = 0; i < n; ++i) r1[i] = add(r1[i], w[i], M);
 };
 rec(&p[0], &q[0], n, &r[0]);
 r.resize(m);
 return r;
}
// FFT-based multiplication: this works correctly for M in [int]
// assume: size of a/b is power of two, mod is predetermined
template<int mod, int sign>
void fmt(vector<ll>& x)
{
 const int n = x.size();
 int h = pow(3, (mod - 1) / n, mod);
 if (sign < 0)
  h = div(1, h, mod);
 for (int i = 0, j = 1; j < n - 1; ++j)
 {
  for (int k = n >> 1; k > (i ^= k); k >≥ 1);
  if (j < i) swap(x[i], x[j]);
 }
 for (int m = 1; m < n; m *= 2)
 {
  ll w = 1, wk = pow(h, n / (2 * m), mod);
  for (int i = 0; i < m; ++i)
  {
   for (int s = i; s < n; s += 2 * m)
   {
    ll u = x[s], d = x[s + m] * w % mod;
    if ((x[s] = u + d) ≥ mod)
     x[s] -= mod;
    if ((x[s + m] = u - d) < 0)
     x[s + m] += mod;
   }
   w = w * wk % mod;
  }
 }
 if (sign < 0)
 {
  ll inv = div(1, n, mod);
  for (auto &a : x)
   a = a * inv % mod;
 }
}
// assume: size of a/b is power of two, mod is predetermined
template<int mod>
vector<ll> conv(vector<ll> a, vector<ll> b)
{
 fmt<mod, +1>(a);
 fmt<mod, +1>(b);
 for (int i = 0; i < a.size(); ++i)
  a[i] = a[i] * b[i] % mod;
 fmt<mod, -1>(a);
 return a;
}
// general convolution where mod < 2^31.
vector<ll> conv(vector<ll> a, vector<ll> b, ll mod)
{
 int n = a.size() + b.size() - 1;
 for (int k : {1, 2, 4, 8, 16 })
  n |= (n >> k);
 ++n;
 a.resize(n);
 b.resize(n);
 const int A = 167772161, B = 469762049, C = 1224736769, D = (ll) (A) *
      ↪ B % mod;
 vector<ll> x = conv<A>(a, b), y = conv<B>(a, b), z = conv<C>(a, b);
 for (int i = 0; i < x.size(); ++i)
 {
  ll X = (y[i] - x[i]) * 104391568;
  if ((X %= B) < 0)
   X += B;
  ll Y = (z[i] - (x[i] + A * X) % C) * 721017874;
  if ((Y %= C) < 0)
   Y += C;
  x[i] += A * X + D * Y;
  if ((x[i] %= mod) < 0)
   x[i] += mod;
 }
 x.resize(n);
 return x;
}
poly mul(poly p, poly q, ll M)
{
 poly pq = conv(p, q, M);
 pq.resize(p.size() + q.size() - 1);
 while (!pq.empty() && !pq.back())
  pq.pop_back();
 return pq;
}
// Newton division: O(M(n)); M is the complexity of multiplication
// fast when FFT multiplication is used
//
// Note: complexity = M(n) + M(n/2) + M(n/4) + ... ≤ 2 M(n).
pair<poly, poly> divmod(poly p, poly q, ll M)
{
 if (p.size() < q.size())
  return {{}, p};
 reverse(p.begin(), p.end());
 reverse(q.begin(), q.end());
 poly t = {div(1, q[0], M) };
 if (t[0] < 0)
  return {{}, {}}; // infeasible
 for (int k = 1; k ≤ 2 * (p.size() - q.size() + 1); k *= 2)
 {
  poly s = mul(mul(t, q, M), t, M);
  t.resize(k);
  for (int i = 0; i < k; ++i)
   t[i] = sub(2 * t[i], s[i], M);
 }
 t.resize(p.size() - q.size() + 1);
 t = mul(t, p, M);
 t.resize(p.size() - q.size() + 1);
 reverse(t.begin(), t.end());
 reverse(p.begin(), p.end());
 reverse(q.begin(), q.end());
 while (!t.empty() && !t.back())
  t.pop_back();
 return {t, sub(p, mul(q, t, M), M)};
}
// polynomial GCD: O(M(n) log n);
poly gcd(poly p, poly q, ll M)
{
 for (; !p.empty(); swap(p, q = divmod(q, p, M).second));
 return p;
}
// value of p(x)
ll eval(poly p, ll x, ll M)
{
 ll ans = 0;
 for (int i = p.size() - 1; i ≥ 0; --i)
  ans = add(mul(ans, x, M), p[i], M);
 return ans;
}
//
// faster multipoint evaluation
// fast if |x| ≥ 10000.
//
// algo:
//  evaluate(p, {x[0], ..., x[n-1]})
//  = evaluate(p mod (X-x[0])...(X-x[n/2-1]), {x[0], ..., x[n/2-1]}),
//  + evaluate(p mod (X-x[n/2])...(X-x[n-1]), {x[n/2], ..., x[n-1]}),
//
// f(n) = 2 f(n/2) + M(n) ==> O(M(n) log n)
//
vector<ll> evaluate(poly p, vector<ll> x, ll M)
```

```
{
 vector<poly> prod(8 * x.size()); // segment tree
 function<poly(int, int, int)> run = [&](int i, int j, int k)
 {
  if (i == j) return prod[k] = (poly) { 1 };
  if (i+1 == j) return prod[k] = (poly) { M-x[i], 1 };
  return prod[k] = mul(run(i,(i+j)/2,2*k+1), run((i+j)/2,j,2*k+2), M);
 };
 run(0, x.size(), 0);
 vector<ll> y(x.size());
 function<void(int, int, int, poly)> rec = [&](int i, int j, int k, poly p)
 {
  if (j - i <= 8)
  {
   for (; i < j; ++i) y[i] = eval(p, x[i], M);
  }
  else
  {
   rec(i, (i+j)/2, 2*k+1, divmod(p, prod[2*k+1], M).second);
   rec((i+j)/2, j, 2*k+2, divmod(p, prod[2*k+2], M).second);
  }
 };
 rec(0, x.size(), 0, p);
 return y;
}
poly interpolate_n(vector<ll> x, vector<ll> y, ll M)
{
 int n = x.size();
 vector<ll> dp(n + 1);
 dp[0] = 1;
 for (int i = 0; i < n; ++i)
 {
  for (int j = i; j >= 0; --j)
  {
   dp[j + 1] = add(dp[j + 1], dp[j], M);
   dp[j] = mul(dp[j], M - x[i], M);
  }
 }
 poly r(n);
 for (int i = 0; i < n; ++i)
 {
  ll den = 1, res = 0;
  for (int j = 0; j < n; ++j)
   if (i != j)
    den = mul(den, sub(x[i], x[j], M), M);
  den = div(1, den, M);
  for (int j = n - 1; j >= 0; --j)
  {
   res = add(dp[j + 1], mul(res, x[i], M), M);
   r[j] = add(r[j], mul(res, mul(den, y[i], M), M), M);
  }
 }
 while (!r.empty() && !r.back())
  r.pop_back();
 return r;
}
//
// faster algo to find a poly p such that
//  p(x[i]) = y[i] for each i
//
// see http://people.mpi-inf.mpg.de/~csaha/lectures/lec6.pdf
//
poly interpolate(vector<ll> x, vector<ll> y, ll M)
{
 vector<poly> prod(8 * x.size()); // segment tree
 function<poly(int, int, int)> run = [&](int i, int j, int k)
 {
  if (i == j) return prod[k] = (poly) { 1 };
  if (i+1 == j) return prod[k] = (poly) { M-x[i], 1 };
  return prod[k] = mul(run(i,(i+j)/2,2*k+1), run((i+j)/2,j,2*k+2), M);
 };
 run(0, x.size(), 0); // preprocessing in O(n log n) time
 poly H = prod[0]; // newton polynomial
 for (int i = 1; i < H.size(); ++i)
  H[i - 1] = mul(H[i], i, M);
 do
  H.pop_back();
 while (!H.empty() && !H.back());
 vector<ll> u(x.size());
 function<void(int, int, int, poly)> rec = [&](int i, int j, int k, poly p)
 {
  if (j - i <= 8)
  {
   for (; i < j; ++i) u[i] = eval(p, x[i], M);
  }
  else
  {
   rec(i, (i+j)/2, 2*k+1, divmod(p, prod[2*k+1], M).second);
   rec((i+j)/2, j, 2*k+2, divmod(p, prod[2*k+2], M).second);
  }
 };
 rec(0, x.size(), 0, H); // multipoint evaluation
 for (int i = 0; i < x.size(); ++i)
  u[i] = div(y[i], u[i], M);
 function<poly(int, int, int)> f = [&](int i, int j, int k)
 {
  if (i >= j) return poly();
  if (i+1 == j) return (poly) { u[i] };
  return add(mul(f(i,(i+j)/2,2*k+1), prod[2*k+2], M),
    mul(f((i+j)/2,j,2*k+2), prod[2*k+1], M), M);
 };
 return f(0, x.size(), 0);
}
//
// return p(x+a)
//
poly shift_n(poly p, ll a, ll M)
{
 poly q(p.size());
 for (int i = p.size() - 1; i >= 0; --i)
 {
  for (int j = p.size() - i - 1; j >= 1; --j)
   q[j] = add(mul(q[j], a, M), q[j - 1], M);
  q[0] = add(mul(q[0], a, M), p[i], M);
 }
 return q;
}
//
// faster algorithm for computing p(x + a)
//
// fast if n >= 4096
// algo: p(x+a) = p_h(x) (x+a)^m + q_h(x)
// cplx: preproc: O(M(n))
//      div-con: O(M(n) log n)
//
poly shift(poly p, ll a, ll M)
{
 vector<poly> pow(p.size());
 pow[0] = { 1 };
 pow[1] = { a,1 };
 int m = 2;
 for (; m < p.size(); m *= 2)
  pow[m] = mul(pow[m / 2], pow[m / 2], M);
 function<poly(poly, int)> rec = [&](poly p, int m)
 {
  if (p.size() <= 1) return p;
  while (m >= p.size()) m /= 2;
  poly q(p.begin() + m, p.end());
  p.resize(m);
  return add(mul(rec(q, m), pow[m], M), rec(p, m), M);
 };
 return rec(p, m);
}
//
// overpeform when n >= 134217728 lol
//
ll factmod(ll n, ll M)
{
 if (n <= 1)
  return 1;
 ll m = sqrt(n);
```

```
  function<poly(int, int)> get = [&](int i, int j)
  {
    if (i == j) return poly();
    if (i+1 == j) return (poly) { i,1 };
    return mul(get(i, (i+j)/2), get((i+j)/2, j), M);
  };
  poly p = get(0, m); // = x (x+1) (x+2) ... (x+(m-1))
  vector<ll> x(m);
  for (int i = 0; i < m; ++i)
    x[i] = 1 + i * m;
  vector<ll> y = evaluate(p, x, M);
  ll fac = 1;
  for (int i = 0; i < m; ++i)
    fac = mul(fac, y[i], M);
  for (ll i = m * m + 1; i <= n; ++i)
    fac = mul(fac, i, M);
  return fac;
}
ll factmod_n(ll n, ll M)
{
  ll fac = 1;
  for (ll k = 1; k <= n; ++k)
    fac = mul(k, fac, M);
  return fac;
}
ll factmod_p(ll n, ll M)
{
  // only works for prime M
  ll fac = 1;
  for (; n > 1; n /= M)
  {
    fac = mul(fac, (n / M) % 2 ? M - 1 : 1, M);
    for (ll i = 2; i <= n % M; ++i)
      fac = mul(fac, i, M);
  }
  return fac;
}
```

5.22 Simplex

Description: Solves a general linear maximization problem: maximize $c^T x$ subject to $Ax \le b$, $x \ge 0$. Returns -inf if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of $c^T x$ otherwise. The input vector is set to an optimal x (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that $x = 0$ is viable.

Usage: `vvd A = {{1,-1}, {-1,1}, {-1,-2}};`
`vd b = {1,1,-4}, c = {-1,-1}, x;`
`T val = LPSolver(A, b, c).solve(x);`

Time: $\mathcal{O}(NM * \#pivots)$, where a pivot may be e.g. an edge relaxation. $\mathcal{O}(2^n)$ in the general case.

62 lines

```
typedef double T; // long double, Rational, double + mod<P>...
typedef vector<T> vd;
typedef vector<vd> vvd;
const T eps = 1e-8, inf = 1/.0;
#define MP make_pair
#define ltj(X) if(s == -1 || MP(X[j],N[j]) < MP(X[s],N[s])) s=j
struct LPSolver {
  int m, n;
  vi N, B;
  vvd D;
  LPSolver(const vvd& A, const vd& b, const vd& c) :
    m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
      rep(i,0,m) rep(j,0,n) D[i][j] = A[i][j];
      rep(i,0,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[i]; }
      rep(j,0,n) { N[j] = j; D[m][j] = -c[j]; }
      N[n] = -1; D[m+1][n] = 1;
    }
  void pivot(int r, int s) {
    T *a = D[r].data(), inv = 1 / a[s];
    rep(i,0,m+2) if (i != r && abs(D[i][s]) > eps) {
      T *b = D[i].data(), inv2 = b[s] * inv;
      rep(j,0,n+2) b[j] -= a[j] * inv2;
      b[s] = a[s] * inv2;
    }
    rep(j,0,n+2) if (j != s) D[r][j] *= inv;
    rep(i,0,m+2) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv;
    swap(B[r], N[s]);
  }
  bool simplex(int phase) {
    int x = m + phase - 1;
    for (;;) {
      int s = -1;
      rep(j,0,n+1) if (N[j] != -phase) ltj(D[x]);
      if (D[x][s] >= -eps) return true;
      int r = -1;
      rep(i,0,m) {
        if (D[i][s] <= eps) continue;
        if (r == -1 || MP(D[i][n+1] / D[i][s], B[i])
                     < MP(D[r][n+1] / D[r][s], B[r])) r = i;
      }
      if (r == -1) return false;
      pivot(r, s);
    }
  }
  T solve(vd &x) {
    int r = 0;
    rep(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
    if (D[r][n+1] < -eps) {
      pivot(r, n);
      if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
      rep(i,0,m) if (B[i] == -1) {
        int s = 0;
        rep(j,1,n+1) ltj(D[i]);
        pivot(i, s);
      }
    }
    bool ok = simplex(1); x = vd(n);
    rep(i,0,m) if (B[i] < n) x[B[i]] = D[i][n+1];
    return ok ? D[m][n+1] : inf;
  }
};
```

5.23 SolveLinear

Description: Solves $A * x = b$. If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in A and b is lost.
Time: $\mathcal{O}(n^2 m)$

35 lines

```
typedef vector<double> vd;
const double eps = 1e-12;
int solveLinear(vector<vd>& A, vd& b, vd& x) {
  int n = sz(A), m = sz(x), rank = 0, br, bc;
  if (n) assert(sz(A[0]) == m);
  vi col(m); iota(all(col), 0);
  rep(i,0,n) {
    double v, bv = 0;
    rep(r,i,n) rep(c,i,m)
      if ((v = fabs(A[r][c])) > bv)
        br = r, bc = c, bv = v;
    if (bv <= eps) {
      rep(j,i,n) if (fabs(b[j]) > eps) return -1;
      break;
    }
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) swap(A[j][i], A[j][bc]);
    bv = 1/A[i][i];
    rep(j,i+1,n) {
      double fac = A[j][i] * bv;
      b[j] -= fac * b[i];
      rep(k,i+1,m) A[j][k] -= fac*A[i][k];
    }
    rank++;
  }
  x.assign(m, 0);
  for (int i = rank; i--;) {
    b[i] /= A[i][i];
    x[col[i]] = b[i];
    rep(j,0,i) b[j] -= A[j][i] * b[i];
  }
  return rank; // (multiple solutions if rank < m)
}
```

5.24 SolveLinear2.h

Description: To get all uniquely determined values of x back from SolveLinear, make the following changes:

8 lines

```
#include "SolveLinear.h"
rep(j,0,n) if (j != i) // instead of rep(j,i+1,n)
// ... then at the end:
x.assign(m, undefined);
rep(i,0,rank) {
 rep(j,rank,m) if (fabs(A[i][j]) > eps) goto fail;
 x[col[i]] = b[i] / A[i][i];
fail:; }
```

5.25 SolveLinearBinary

Description: Solves $Ax = b$ over $\mathbb{F}_2$. If there are multiple solutions, one is returned arbitrarily. Returns rank, or -1 if no solutions. Destroys A and b.
Time: $\mathcal{O}\left(n^2 m\right)$

32 lines

```
typedef bitset<1000> bs;
int solveLinear(vector<bs>& A, vi& b, bs& x, int m) {
 int n = sz(A), rank = 0, br;
 assert(m <= sz(x));
 vi col(m); iota(all(col), 0);
 rep(i,0,n) {
  for (br=i; br<n; ++br) if (A[br].any()) break;
  if (br == n) {
   rep(j,i,n) if(b[j]) return -1;
   break;
  }
  int bc = (int)A[br]._Find_next(i-1);
  swap(A[i], A[br]);
  swap(b[i], b[br]);
  swap(col[i], col[bc]);
  rep(j,0,n) if (A[j][i] != A[j][bc]) {
   A[j].flip(i); A[j].flip(bc);
  }
  rep(j,i+1,n) if (A[j][i]) {
   b[j] ^= b[i];
   A[j] ^= A[i];
  }
  rank++;
 }
 x = bs();
 for (int i = rank; i--;) {
  if (!b[i]) continue;
  x[col[i]] = 1;
  rep(j,0,i) b[j] ^= A[j][i];
 }
 return rank; // (multiple solutions if rank < m)
}
```

5.26 SubsetSum.cpp

Description: Some aplications of functions of polynomial

80 lines

```
// #include "ExtendedPolynomial"
// number of subsets of an array of n elements having sum equal to k for
    each k from 1 to m
int main() {
 int n, m; cin >> n >> m;
 vector<int> a(m + 1, 0);
 for (int i = 0; i < n; i++) {
  int k; cin >> k; // k >= 1, handle [k = 0] separately
  if (k <= m) a[k]++;
 }
 poly p(m + 1, 0);
 for (int i = 1; i <= m; i++) {
  for (int j = 1; i * j <= m; j++) {
   if (j & 1) p.a[i * j] += mint(a[i]) / j;
   else p.a[i * j] -= mint(a[i]) / j;
  }
 }
 p = p.exp(m + 1);
 for (int i = 1; i <= m; i++) cout << p[i] << ' '; cout << '\n'; //
    check for m = 0
 return 0;
}
// Calc bell numbers
vector<mint> bell(int n) { // e^(e^x - 1)
 poly p(n + 1);
 mint f = 1;
 for (int i = 0; i <= n; i++) {
  p.a[i] = mint(1) / f;
  f *= i + 1;
 }
 p.a[0] -= 1;
 p = p.exp(n + 1);
 vector<mint> ans(n + 1);
 f = 1;
 for (int i = 0; i <= n; i++) {
  ans[i] = p[i] * f;
  f *= i + 1;
 }
 return ans;
}
int32_t main() {
 ios_base::sync_with_stdio(0);
 cin.tie(0);
 int n; cin >> n;
 auto ans = bell(n);
 cout << ans[n] << '\n';
 return 0;
}
//$O((MAXK^2)/32)$
//MAXK is the maximum possible sum.
bitset<MAXK> dp;
dp[0] = 1;
for (int i = 1; i <= n; i++) {
 for (int x = 0; (1<<x) <= m[i]; x++) {
  dp |= (dp << (a[i]*(1<<x)));
  m[i] -= (1<<x);
 }
 dp |= (dp << (a[i]*m[i]));
}
long long kSum(vector<int>& nums, int k) {
 ll sum=0,n=nums.size();
   vector<ll> ans;
   for(ll i=0;i<nums.size();i++){
      if(nums[i]>0)
         sum+=nums[i];
      nums[i]=abs(nums[i]);
   }
   ans.push_back(sum);
   priority_queue<pair<ll,ll>> pq;
   sort(nums.begin(),nums.end());
   pq.push({sum-nums[0],0});
   while(ans.size()<k){
      auto [sum,ind]=pq.top();
      pq.pop();
      if(ind+1<n){
         pq.push({sum+nums[ind]-nums[ind+1],ind+1});
         pq.push({sum-nums[ind+1],ind+1});
      }
      ans.push_back(sum);
   }
   return ans.back();
 }
```

5.27 Template.cpp

Description: A template for Matrix structure with all basic operations like exponensation ,sum , rest, determinant ,reduction by gauss etc
Usage: `Matrix<type> M`

379 lines

```
#define mod 1e9+7
#define INF INT_MAX
const double EPS = 1e-9;
typedef long long int lli;
template <typename T>
struct Matrix {
   vector < vector <T> > A;
   int r,c;
   Matrix() {
      this->r = 0;
```

```cpp
		this->c = 0;
	}
	Matrix(int r,int c) {
		this->r = r;
		this->c = c;
		A.assign(r , vector <T> (c));
	}
	Matrix(int r,int c,const T &val) {
		this->r = r;
		this->c = c;
		A.assign(r , vector <T> (c , val));
	}
	Matrix(int n) {
		this->r = this->c = n;
		A.assign(n , vector <T> (n));
		for(int i=0;i<n;i++)
			A[i][i] = (T)1;
	}
	Matrix operator * (const Matrix<T> &B) {
//		Matrix <T> C(r,B.c,0);
//		for(int i=0 ; i<r ; i++)
//			for(int j=0 ; j<B.c ; j++)
//				for(int k=0 ; k<c ; k++)
//					C[i][j] = (C[i][j] + ( (long long )A[i][k] * (long long)B[k][j] ));
//		return C;
		Matrix<T> C(r,B.c,0);
		for(int i = 0;i<r;i++) {
			for(int j = 0;j<B.c;j++) {
				for(int k = 0;k<c;k++) {
					C[i][j] = (C[i][j] + ((lli)A[i][k] * (lli)B[k][j]));
					if(C[i][j]>= 8ll*mod*mod)
						C[i][j]%=mod;
				}
			}
		}
		for(int i = 0;i<r;i++)for(int j = 0;j<c;j++)C[i][j]%=mod;
		return C;
	}
	Matrix operator + (const Matrix<T> &B) {
		assert(r == B.r);
		assert(c == B.c);
		Matrix <T> C(r,c,0);
		int i,j;
		for(i=0;i<r;i++)
			for(j=0;j<c;j++)
				C[i][j] = ((A[i][j] + B[i][j]));
		return C;
	}
	Matrix operator*(int & c)  {
		Matrix<T> C(r, c);
		for(int i = 0; i < r; i++)
			for(int j = 0; j < c; j++)
				C[i][j] = A[i][j] * c;
		return C;
	}
	Matrix operator - () {
		Matrix <T> C(r,c,0);
		int i,j;
		for(i=0;i<r;i++)
			for(j=0;j<c;j++)
				C[i][j] = -A[i][j];
		return C;
	}
	Matrix operator - (const Matrix<T> &B) {
		assert(r == B.r);
		assert(c == B.c);
		Matrix <T> C(r,c,0);
		int i,j;
		for(i=0;i<r;i++)
			for(j=0;j<c;j++)
				C[i][j] = A[i][j] - B[i][j];
		return C;
	}
	Matrix operator ^(long long n) {
		assert(r == c);
		int i,j;
		Matrix <T> C(r);
		Matrix <T> X(r,c,0);
		for(i=0;i<r;i++)
			for(j=0;j<c;j++)
				X[i][j] = A[i][j];
		while(n) {
			if(n&1)
				C *= X;
			X *= X;
			n /= 2;
		}
		return C;
	}
	vector<T>& operator [] (int i) {
		assert(i < r);
		assert(i >= 0);
		return A[i];
	}
	const vector<T>& operator [] (int i) const {
		assert(i < r);
		assert(i >= 0);
		return A[i];
	}
	friend ostream& operator << (ostream &out,const Matrix<T> &M) {
		for (int i = 0; i < M.r; ++i)  {
			for (int j = 0; j < M.c; ++j)  {
				out << M[i][j] << " ";
			}
			out << '\n';
		}
		return out;
	}
	void operator *= (const Matrix<T> &B) {
		(*this) = (*this)*B;
	}
	void operator += (const Matrix<T> &B) {
		(*this) = (*this)+B;
	}
	void operator -= (const Matrix<T> &B) {
		(*this) = (*this)-B;
	}
	void operator ^= (long long n) {
		(*this) =(*this)^n;
	}
	//Inverse
	bool Inverse(Matrix<double> &inverse) {
		if(this->detGauss() == 0)return false;
		int n = A[0].size();
		Matrix<double> temp(n,2*n);
		for(int i = 0;i<n;i++)
			for(int j = 0;j<n;j++)temp[i][j] = A[i][j];
		Matrix<double> ident(n);
		for(int i = 0;i<n;i++)
			for(int j = n;j<2*n;j++)temp[i][j] = ident[i][j-n];
		int m = n*2;
		vector<int> where (m, -1);
		for (int col=0, row=0; col<m && row<n; ++col)  {
			int sel = row;
			for (int i=row; i<n; ++i)
				if (abs (temp[i][col]) > abs (temp[sel][col]))
					sel = i;
			if (abs (temp[sel][col]) < EPS)
				continue;
			for (int i=col; i<m; ++i)
				swap (temp[sel][i], temp[row][i]);
			where[col] = row;
			double div = temp[row][col];
			for(int i = 0;i<m;i++)
				if(fabs(temp[row][i])>EPS)temp[row][i] /=div;
			for (int i=0; i<n; ++i)
				if (i != row)  {
					double c = temp[i][col] / temp[row][col];
					for (int j=col; j<m; ++j)
						temp[i][j] -= temp[row][j] * c;
				}
			++row;
```

```
    }
    for(int i = 0;i<n;i++)
      for(int j = 0;j<n;j++)
        inverse[i][j] = temp[i][j+n];
    return true;
  }
  //Adjoint
  Matrix<T> minor(int x, int y) {
    Matrix<T> M(r-1, c-1);
    for(int i = 0; i < c-1; ++i)
      for(int j = 0; j < r-1; ++j)
        M[i][j] = A[i < x ? i : i+1][j < y ? j : j+1];
    return M;
  }
  T cofactor(int x, int y) {
    T ans = minor(x, y).detGauss();
    if((x + y) % 2 == 1) ans *= -1;
    return ans;
  }
  Matrix<T> cofactorMatrix() {
    Matrix<T> C(r, c);
    for(int i = 0; i < c; i++)
      for(int j = 0; j < r; j++)
        C[i][j] = cofactor(i, j);
    return C;
  }
  Matrix<T> Adjunta() {
    int n = A[0].size();
    Matrix<int> adjoint(n);
    Matrix<double> inverse(n);
    this->Inverse(inverse);
    int determinante = this->detGauss();
    if(determinante) {
      for(int i = 0;i<n;i++)
        for(int j = 0;j<n;j++)
          adjoint[i][j] = (T)round((inverse[i][j]*determinante));
    }
    else {
      adjoint = this->cofactorMatrix().transpose();
    }
    return adjoint;
  }
  //Transpuesta
  Matrix transpose() {
    Matrix <T> C(c,r);
    int i,j;
    for(i=0;i<r;i++)
      for(j=0;j<c;j++)
        C[j][i] = A[i][j];
    return C;
  }
  //Traza
  T trace() {
    T sum = 0;
    for(int i = 0; i < min(r, c); i++)
      sum += A[i][i];
    return sum;
  }
  //Determinante
  int determinant() {
    int n = r;
    Matrix<T> temp(n);
    temp.A = A;
    for (int i = 0; i < n; i++)
      for (int j = 0; j < n; j++)
        temp[i][j] %= mod;
    lli res = 1;
    for (int i = 0; i < n; i++) {
      for (int j = i + 1; j < n; j++) {
        for (; temp[j][i]; res = -res) {
          long long t = temp[i][i] / temp[j][i];
          for (int k = i; k < n; k++) {
            temp[i][k] = (temp[i][k] - temp[j][k] * t) % mod;
            std::swap(temp[j][k], temp[i][k]);
          }
        }
      }
      if (temp[i][i] == 0)
        return 0;
      res = res * temp[i][i] % mod;
    }
    if (res < 0)
      res += mod;
    return static_cast<int>(res);
  }
  int detGauss() {
    assert(r == c);
    double det = 1;
    Matrix<double> temp(r);
    temp.r = r;
    temp.c = c;
    int n = r;
    for(int i = 0;i<n;i++)
      for(int j = 0;j<n;j++)
        temp[i][j] = (double)A[i][j];
    for (int i=0; i<n; ++i) {
      int k = i;
      for (int j=i+1; j<n; ++j)
        if (fabs (temp[j][i]) > fabs (temp[k][i]))
          k = j;
      if (abs (temp[k][i]) < EPS) {
        det = 0;
        break;
      }
      swap (temp[i], temp[k]);
      if (i != k)
        det = -det;
      det *= temp[i][i];
      for (int j=i+1; j<n; ++j)
        temp[i][j] /= temp[i][i];
      for (int j=0; j<n; ++j)
        if (j != i && abs (temp[j][i]) > EPS)
          for (int k=i+1; k<n; ++k)
            temp[j][k] -= temp[i][k] * temp[j][i];
    }
    return (int)det;
  }
  int gauss (vector<double> & ans) {
    Matrix<double> Temp(this->r,this->c);
    int n = (int) Temp.A.size();
    int m = (int) Temp[0].size() - 1;
    for(int i = 0;i<n;i++)
      for(int j = 0;j<n;j++)
        Temp[i][j] = (double)A[i][j];
    vector<int> where (m, -1);
    for (int col=0, row=0; col<m && row<n; ++col) {
      int sel = row;
      for (int i=row; i<n; ++i)
        if (fabs (Temp[i][col]) > fabs (Temp[sel][col]))
          sel = i;
      if (fabs (Temp[sel][col]) < EPS)
        continue;
      for (int i=col; i<m; ++i)
        swap (Temp[sel][i], Temp[row][i]);
      where[col] = row;
      for (int i=0; i<n; ++i)
        if (i != row) {
          double c = Temp[i][col] / Temp[row][col];
          for (int j=col; j<m; ++j)
            Temp[i][j] -= Temp[row][j] * c;
        }
      ++row;
    }
    ans.assign (m, 0);
    for (int i=0; i<m; ++i)
      if (where[i] != -1)
        ans[i] = Temp[where[i]][m] / Temp[where[i]][i];
    for (int i=0; i<n; ++i) {
      double sum = 0;
      for (int j=0; j<m; ++j)
        sum += ans[j] * Temp[i][j];
      if (fabs (sum - Temp[i][m]) > EPS)
        return 0;
```

```
    }
    for (int i=0; i<m; ++i)
      if (where[i] == -1)
        return INF;
    return 1;
  }
  /** Kirchhoff Matrix Tree Theorem Describe in Graphs -> Math */
  int Kirchof() {
    cin>>n>>m>>k;
    Matrix<lli> Kirchof(n);
    for(int i = 0;i<m;i++) {
      cin>>a>>b;
      a--;
      b--;
      Kirchof[a][b] = Kirchof[b][a] = 1;
      Kirchof[a][a]++;
      Kirchof[b][b]++;
    }
    for(int i =0;i<n;i++)
      Kirchof[i][i] = (1ll*n*k%mod-Kirchof[i][i]+mod)%mod;
    lli ans = 1;
    ans = ans*(mod_pow(1ll*k*n%mod*k%mod*n%mod,mod-2));
    lli determinante =Kirchof.det();
    ans = ans*(mod_pow(determinante,k))%mod;
    cout<<ans<<endl;
  }
};
/**   [f(n)]     [1 1 1 1 1 1]  [f(5)]
      [f(n-1)]   [1 0 0 0 0 0]  [f(4)]
      [f(n-2)]   [0 1 0 0 0 0]  [f(3)]
      [f(n-3)]   [0 0 1 0 0 0]  [f(2)]
      [f(n-4)]   [0 0 0 1 0 0]  [f(1)]
      [(e)]      [0 0 0 0 1 0]  [ e ]
*/
lli Linear_recurrence(vector<lli> C, vector<lli> init,lli n,bool
    ↪ constante) {
  int k = C.size();
  Matrix<lli> T(k,k);
  Matrix<lli> first(k,1);
  for(int i = 0;i<k;i++)T[0][i] = C[i];
  for(int i = 0,col=1;i<k && col<k;i++,col++)
    T[col][i]=1;
  if(constante) {
    for(int i = 0;i<k;i++)first[i][0]=init[(k-2)-i];
    first[k-1][0]=init[k-1];
  }
  else
    for(int i = 0;i<k;i++)first[i][0]=init[(k-1)-i];
  if(constante)
    T^=((n-k)+1);
  else
    T^=(n-k);
  Matrix<lli> sol = T*first;
  return sol[0][0];
  //Example Tribonacci F(i) = 1*F(i-1) + 1*F(i-2) + 1*F(i-3) + (c= 0)
  // vector<lli>C(3);
  // C[0] =1;
  // C[1] =1;
  // C[2] =1;
  // vector<lli> ini(3);
  // ini[0] =1;
  // ini[1] =1;
  // ini[2] =2;
  // cout<<Linear_recurrence(C,ini,nth,false)<<endl;
}
```

5.28 Tridiagonal

Description: $x = \text{tridiagonal}(d, p, q, b)$ solves the equation system

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} d_0 & p_0 & 0 & 0 & \cdots & 0 \\ q_0 & d_1 & p_1 & 0 & \cdots & 0 \\ 0 & q_1 & d_2 & p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & q_{n-3} & d_{n-2} & p_{n-2} \\ 0 & 0 & \cdots & 0 & q_{n-2} & d_{n-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \end{pmatrix}.$$

This is useful for solving problems on the type

$$a_i = b_i a_{i-1} + c_i a_{i+1} + d_i,\ 1 \le i \le n,$$

where a_0, a_{n+1}, b_i, c_i and d_i are known. a can then be obtained from

$$\{a_i\} = \text{tridiagonal}(\{1, -1, -1, ..., -1, 1\}, \{0, c_1, c_2, \ldots, c_n\},$$
$$\{b_1, b_2, \ldots, b_n, 0\}, \{a_0, d_1, d_2, \ldots, d_n, a_{n+1}\}).$$

Fails if the solution is not unique. If $|d_i| > |p_i| + |q_{i-1}|$ for all i, or $|d_i| > |p_{i-1}| + |q_i|$, or the matrix is positive definite, the algorithm is numerically stable and neither `tr` nor the check for `diag[i] == 0` is needed.
Time: $\mathcal{O}(N)$

26 lines

```
typedef double T;
vector<T> tridiagonal(vector<T> diag, const vector<T>& super,
    const vector<T>& sub, vector<T> b) {
  int n = sz(b); vi tr(n);
  rep(i,0,n-1) {
    if (abs(diag[i]) < 1e-9 * abs(super[i])) { // diag[i] == 0
      b[i+1] -= b[i] * diag[i+1] / super[i];
      if (i+2 < n) b[i+2] -= b[i] * sub[i+1] / super[i];
      diag[i+1] = sub[i]; tr[++i] = 1;
    } else {
      diag[i+1] -= super[i]*sub[i]/diag[i];
      b[i+1] -= b[i]*sub[i]/diag[i];
    }
  }
  for (int i = n; i--;) {
    if (tr[i]) {
      swap(b[i], b[i-1]);
      diag[i-1] = diag[i];
      b[i] /= super[i-1];
    } else {
      b[i] /= diag[i];
      if (i) b[i-1] -= b[i]*super[i-1];
    }
  }
  return b;
}
```

5.29 gauss.cpp

49 lines

```
/*
  Tested: SPOJ GS
  Complexity: O(n^3)
*/
const int oo = 0x3f3f3f3f;
const double eps = 1e-9;
int gauss(vector<vector<double>> a, vector<double> &ans)
{
  int n = (int) a.size();
  int m = (int) a[0].size() - 1;
  vector<int> where(m, -1);
  for (int col = 0, row = 0; col < m && row < n; ++col)
  {
    int sel = row;
    for (int i = row; i < n; ++i)
      if (abs(a[i][col]) > abs(a[sel][col]))
        sel = i;
    if (abs(a[sel][col]) < eps)
      continue;
    for (int i = col; i <= m; ++i)
      swap(a[sel][i], a[row][i]);
    where[col] = row;
    for (int i = 0; i < n; ++i)
      if (i != row)
      {
        double c = a[i][col] / a[row][col];
        for (int j = col; j <= m; ++j)
          a[i][j] -= a[row][j] * c;
      }
    ++row;
  }
  ans.assign(m, 0);
  for (int i = 0; i < m; ++i)
    if (where[i] != -1)
      ans[i] = a[where[i]][m] / a[where[i]][i];
  for (int i = 0; i < n; ++i)
  {
    double sum = 0;
    for (int j = 0; j < m; ++j)
```

```
    sum += ans[j] * a[i][j];
  if (abs(sum - a[i][m]) > eps)
    return 0;
 }
 for (int i = 0; i < m; ++i)
  if (where[i] == -1)
    return oo;
 return 1;
}
```

5.30 simplex.cpp

87 lines

```
/*
 Parametric Self-Dual Simplex method
 Description:
 - Solve a canonical LP:
    min. c x
   s.t. A x <= b
     x >= 0
 Complexity: O(n+m) iterations on average
 Tested: http://codeforces.com/contest/375/problem/E
*/
const double eps = 1e-9, oo = numeric_limits<double>::infinity();
typedef vector<double> vec;
typedef vector<vec> mat;
double simplexMethodPD(mat &A, vec &b, vec &c)
 {
 int n = c.size(), m = b.size();
 mat T(m + 1, vec(n + m + 1));
 vector<int> base(n + m), row(m);
 for(int j = 0; j < m; ++j)
  {
  for (int i = 0; i < n; ++i)
    T[j][i] = A[j][i];
  T[j][n + j] = 1;
  base[row[j] = n + j] = 1;
  T[j][n + m] = b[j];
  }
 for (int i = 0; i < n; ++i)
  T[m][i] = c[i];
 while (1)
  {
  int p = 0, q = 0;
  for (int i = 0; i < n + m; ++i)
    if (T[m][i] <= T[m][p])
     p = i;
  for (int j = 0; j < m; ++j)
    if (T[j][n + m] <= T[q][n + m])
     q = j;
  double t = min(T[m][p], T[q][n + m]);
  if (t >= -eps)
   {
    vec x(n);
    for (int i = 0; i < m; ++i)
     if (row[i] < n) x[row[i]] = T[i][n + m];
    // x is the solution
    return -T[m][n + m]; // optimal
   }
  if (t < T[q][n + m])
   {
    // tight on c -> primal update
    for (int j = 0; j < m; ++j)
     if (T[j][p] >= eps)
      if (T[j][p] * (T[q][n + m] - t) >=
        T[q][p] * (T[j][n + m] - t))
        q = j;
    if (T[q][p] <= eps)
     return oo; // primal infeasible
   }
  else
   {
    // tight on b -> dual update
    for (int i = 0; i < n + m + 1; ++i)
     T[q][i] = -T[q][i];
    for (int i = 0; i < n + m; ++i)
     if (T[q][i] >= eps)
      if (T[q][i] * (T[m][p] - t) >=
        T[q][p] * (T[m][i] - t))
        p = i;
    if (T[q][p] <= eps)
     return -oo; // dual infeasible
   }
  for (int i = 0; i < m + n + 1; ++i)
   if (i != p) T[q][i] /= T[q][p];
  T[q][p] = 1; // pivot(q, p)
  base[p] = 1;
  base[row[q]] = 0;
  row[q] = p;
  for (int j = 0; j < m + 1; ++j)
   if (j != q)
    {
     double alpha = T[j][p];
     for (int i = 0; i < n + m + 1; ++i)
      T[j][i] -= T[q][i] * alpha;
    }
  }
 return oo;
}
```

5.31 simpson.cpp

14 lines

```
template<class F>
double simpson(F f, double a, double b, int n = 2000)
 {
 double h = (b - a) / (2 * n), fa = f(a), nfa, res = 0;
 for (int i = 0; i < n; ++i, fa = nfa)
  {
   nfa = f(a + 2 * h);
   res += (fa + 4 * f(a + h) + nfa);
   a += 2 * h;
  }
 res = res * h / 3;
 return res;
 }
```

data-structures (6)

6.1 FenwickTree

Description: Fenwick tree is an structures that allows compute an assosiative but not invertible functon (Group) in a range [l,r] efficiently
Usage: `bit.resize(n);`
`for(auto &c:nums){cin>>c;add(i++,c);}`
Time: $\mathcal{O}(\log N)$ per query or $\mathcal{O}(\log N^2)$ for bit2D.

50 lines

```
//Usefull define to print vectors
#define print(A)for(auto c:A)cout<<c<<" ";cout<<endl;
#define printM(A)for(auto c:A){print(c);}
vector<int> bit;
vector<vector<int>> bit2D;
int n,m;
int sum(int idx) {
  int ret = 0;
  for (++idx; idx > 0; idx -= idx & -idx)ret += bit[idx];
  return ret;
 }
int sum(int l, int r) {
  return sum(r) - sum(l - 1);
 }
void add(int idx, int delta) {
  for (++idx; idx < n; idx += idx & -idx) bit[idx] += delta;
 }
/** This only can accept querys in a point */
void range_add(int l, int r, int val) {
  add(l, val);
  add(r + 1, -val);
 }
// Search for first position such \sum_{0}^{pos} a[i] \geq s;
int bit_search(int s) {
  int sum = 0;
```

```
  int pos = 0;
  for(int i = ceil(log2(n));i≥0;i--) {
    if((pos+(1<<i))<n && (sum+bit[pos+(1<<i)])<s) {
      sum+=bit[pos+(1<<i)];
      pos+=(1<<i);
    }
  }
  return pos;
}
// Return sum over submatrix with corners (0,0), (x,y)
int sum2D(int x, int y) {
  int ret = 0;
  for (int i = x; i ≥ 0; i = (i & (i + 1)) - 1)
    for (int j = y; j ≥ 0; j = (j & (j + 1)) - 1)
      ret += bit2D[i][j];
  return ret;
}
int sum2D(int x0,int y0,int x,int y){
  return sum2D(x,y)-sum2D(x,y0-1)-sum2D(x0-1,y)+sum2D(x0-1,y0-1);
}
void add2D(int x, int y, int delta) {
  for (int i = x; i < n; i = i | (i + 1))
    for (int j = y; j < m; j = j | (j + 1))
      bit2D[i][j] += delta;
}
```

6.2 FenwickTree

Description: Computes partial sums a[0] + a[1] + ... + a[pos - 1], and updates single elements a[i], taking the difference between the old and new value.
Time: Both operations are $\mathcal{O}(\log N)$.

22 lines

```
struct FT {
 vector<ll> s;
 FT(int n) : s(n) { }
 void update(int pos, ll dif) { // a[pos] += dif
  for (; pos < sz(s); pos |= pos + 1) s[pos] += dif;
 }
 ll query(int pos) { // sum of values in [0, pos)
  ll res = 0;
  for (; pos > 0; pos &= pos - 1) res += s[pos-1];
  return res;
 }
 int lower_bound(ll sum) { // min pos st sum of [0, pos] ≥ sum
  // Returns n if no sum is ≥ sum, or -1 if empty sum is.
  if (sum ≤ 0) return -1;
  int pos = 0;
  for (int pw = 1 << 25; pw; pw >≥ 1) {
   if (pos + pw ≤ sz(s) && s[pos + pw-1] < sum)
    pos += pw, sum -= s[pos-1];
  }
  return pos;
 }
};
```

6.3 HashMap.cpp

Description: Hash map with mostly the same API as unordered_map, but ~ 3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).

18 lines

```
#include <bits/stdc++.h>
using namespace std;
typedef ll long long
#include <bits/extc++.h> /** keep-include */
// To use most bits rather than just the lowest ones:
struct chash { // large odd number for C
 const uint64_t C = ll(4e18 * acos(0)) | 71;
 ll operator()(ll x) const { return __builtin_bswap64(x*C); }
};
__gnu_pbds::gp_hash_table<ll,int,chash> h({},{},{},{},{1<<16
  ↪ });
/** For CodeForces, or other places where hacking might be a problem:
const int RANDOM = chrono::high_resolution_clock::now().time_since_epoch
  ↪ ().count();
struct chash { // To use most bits rather than just the lowest ones:
 const uint64_t C = ll(4e18 * acos(0)) | 71; // large odd number
 ll operator()(ll x) const { return __builtin_bswap64((x^RANDOM)*C); }
};
__gnu_pbds::gp_hash_table<ll, int, chash> h({},{},{},{}, {1
  ↪ << 16});
*/
```

6.4 ImplicitTreap

Description: A powerfull dynamic array that allows operations like: Insert/erase in every position, Range sum/minimum/max, Reverse/Rotate a sub array
Usage: `Root is global and not need modifications`
`Only erase need root -> erase(root,pos)`
`All operations are 0 indexed`
Time: $\mathcal{O}(\log N)$

162 lines

```
mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());
uniform_int_distribution<> dis(numeric_limits<int>::min(),
  ↪ numeric_limits<int>::max()) ;
struct Treap{
  Treap *l = NULL,*r = NULL;
  int p,sz = 1,val,sum = 0,mn = 1e9;
  int rev = 0, lazySum = 0,lazyReplace = 0;
  int sumPend = 0,ReplacePend = 0;
  Treap(int v ,int prior = dis(rng)):val(v),p(prior){ }
};
Treap *root = NULL;
void update(Treap *T){
  if (!T) return;
  T->sz = 1;
  T->sum = T->val;
  T->mn = T->val;
  if (T->l) {
   T->sz += T->l->sz;
   T->sum += T->l->sum;
   T->mn = min(T->mn,T->l->mn);
  }
  if (T->r) {
   T->sz += T->r->sz;
   T->sum += T->r->sum;
   T->mn = min(T->mn,T->r->mn);
  }
}
void applyRev(Treap *T){
  if(!T)return;
  T->rev^=1;
  swap(T->l,T->r);
}
void applySum(Treap *T,int x){
  if(!T)return;
  T->val+=x;
  T->mn+=x;
  T->sumPend+=x;
  T->lazySum = 1;
  T->sum+=x*T->sz;
}
void applyReplace(Treap *T,int x){
  if(!T)return;
  T->val=x;
  T->mn=x;
  T->ReplacePend=x;
  T->lazyReplace = 1;
  T->sum=x*T->sz;
}
void lazy(Treap *T){
  if(!T)return;
  if(T->rev){
    applyRev(T->l);
    applyRev(T->r);
    T->rev = 0;
  }
  if(T->lazySum){
    applySum(T->l,T->sumPend);
    applySum(T->r,T->sumPend);
    T->lazySum = 0;
    T->sumPend = 0;
  }
  if(T->lazyReplace){
```

```
        applyReplace(T->l,T->ReplacePend);
        applyReplace(T->r,T->ReplacePend);
        T->lazyReplace = 0;
        T->ReplacePend = 0;
    }
}
pair<Treap*,Treap*> split(Treap *T,int idx,int cont = 0){
    if(!T)return {NULL,NULL};
    lazy(T);
    Treap *L, *R;
    int idxt = cont + (T->l?T->l->sz:0);
    if(idx<idxt)
        tie(L,T->l) = split(T->l,idx,cont),R = T;
    else
        tie(T->r,R) = split(T->r,idx,idxt+1),L = T;
    update(T);
    return {L,R};
}
void insert(Treap *&T,Treap *v,int x, int cnt) {
    lazy(T);
    int idxt = T ? cnt + (T->l ? T->l->sz : 0) : 0;
    if (!T) T = v;
    else if (v->p > T->p)
        tie(v->l, v->r) = split(T, x, cnt), T = v;
    else if (x < idxt) insert(T->l, v, x, cnt);
    else insert(T->r, v, x, idxt + 1);
    update(T);
}
void insert(int e, int i) {
    insert(root, new Treap(e), i-1, 0);
}
Treap *merge(Treap *a,Treap *b){
    lazy(a),lazy(b);
    Treap *T;
    if(!a || !b)T = a?a:b;
    else if(a->p > b->p)
        a->r = merge(a->r,b),T = a;
    else b->l = merge(a,b->l),T = b;
    update(T);
    return T;
}
void erase(Treap *&T,int x ,int cnt = 0){
    if(!T)return;
    lazy(T);
    int left = cnt+(T->l? T->l->sz:0);
    if(left == x)T = merge(T->l,T->r);
    else if(x<left)erase(T->l,x,cnt);
    else erase(T->r,x,left+1);
    update(T);
}
void print(Treap *t) {
    if (!t) return;
    lazy(t);
    print(t->l);
    print(t->r);
}
void push_back(int e) {
    root = merge(root, new Treap(e));
}
void op(int l,int r, function<void(Treap *T)> f){
    Treap *a,*b,*c;
    tie(a,b) = split(root,l-1);
    tie(b,c) = split(b,r-l);
    f(b);
    root = merge(a, merge(b,c));
}
void reverse(int l,int r){
    op(l,r,[&](Treap *T){ applyRev(T); });
}
void rotate(int l,int r,int k){
    op(l,r,[&](Treap *T){
        Treap *l,*r;
        k%=T->sz;
        tie(l,r) = split(T,T->sz-k-1);
        T = merge(r,l);
    });
}
void add(int l,int r,int x){
    op(l,r,[&](Treap *T){
        applySum(T,x);
    });
}
void replace(int l,int r,int x){
    op(l,r,[&](Treap *T){
        applyReplace(T,x);
    });
}
int get_sum(int l,int r){
    int ans;
    op(l,r,[&](Treap *T){
        ans = T->sum;
    });
    return ans;
}
int get_min(int l,int r){
    int mn;
    op(l,r,[&](Treap *T){
        mn = T->mn;
    });
    return mn;
}
```

6.5 IntervalTree

Description: Interval tree is a structure that stores segments in a efficient way, allows to get all intervals that intersects with another interval

Usage: `root = build_interval_tree(vector<recta>);` `query(root,R); R is an instance of recta`

Time: $\mathcal{O}(ans)$ where ans is the number of intervals that intersects. 105 lines

```
typedef long long int lli;
typedef long double ld;
struct recta {
    ld x1,x2;
    int id;
    friend ostream& operator << (ostream &out, const recta&p ){
        out<<"("<<p.x1<<","<<p.x2<<", "<<p.id<<")";
        return out;
    }
};
struct central{
    ld x;
    vector<recta> x1order;
    vector<recta> x2order;
    friend ostream& operator <<(ostream &out, const central&p){
        out<<"[ ";
        for(int i = 0;i<p.x1order.size();i++){
            out<<p.x1order[i]<<" ";
        }
        out<<"]";
        return out;
    }
};
struct node {
  node *l, *r;
    central C;
  node(node *l, node *r, central C) :
    l(l), r(r),C(C) { }
};
inline bool leaf(node *x){
    return !x->l && !x->r;
}
node* build_interval_tree(vector<recta> &R){
    if(R.size() == 0)return NULL;
    int n = R.size();
    int mid = (n-1)>>1;
  vector<recta> r1,r2;
    central c;
    ld x = (R[mid].x1+R[mid].x2)/2.0;
    c.x = x;
    for(int i = 0;i<n;i++){
        if(islessequal(R[i].x1,x) && islessequal(x,R[i].x2)){
            c.x1order.push_back(R[i]);
            c.x2order.push_back(R[i]);
        }
```

```
      else if(R[i].x2<x)
        r1.push_back(R[i]);
      else
        r2.push_back(R[i]);
    }
    sort(c.x1order.begin(),c.x1order.end(),[&](recta a,recta b) {
      return islessequal(a.x1,b.x1);
    });
    sort(c.x2order.begin(),c.x2order.end(),[&](recta a,recta b) {
      return islessequal(a.x2,b.x2);
    });
  node *left = build_interval_tree(r1);
  node *right = build_interval_tree(r2);
  return new node(left,right,c);
}
set<lli> ids;
void findI(central C,recta R,bool dir) {
  if(dir) {
    int l = -1, r = C.x2order.size();
    while(l+1<r) {
      int m = (l+r)>>1;
      if(isgreaterequal(C.x2order[m].x2,R.x1))
        r = m;
      else
        l = m;
    }
    int n = C.x2order.size();
    for(int i = r;i<n;i++)
      ids.insert(C.x2order[i].id);
  }
  else {
    int l = -1, r = C.x2order.size();
    while(l+1<r) {
      int m = (l+r)>>1;
      if(islessequal(C.x1order[m].x1,R.x2))
        l = m;
      else
        r = m;
    }
    for(int i = l;i>=0;i--)
      ids.insert(C.x1order[i].id);
  }
}
void query(node *t, const recta& R) {
  if(!t)return;
  if(isgreaterequal(t->C.x,R.x1) && islessequal(t->C.x,R.x2)) {
    for(int i = 0;i<t->C.x1order.size();i++)
      ids.insert(t->C.x1order[i].id);
    query(t->l,R);
    query(t->r,R);
  }
  else if(isless(R.x2,t->C.x)) {
    findI(t->C,R,0);
    query(t->l,R);
  }
  else {
    findI(t->C,R,1);
    query(t->r,R);
  }
}
```

6.6 LineContainer

Description: Container where you can add lines of the form kx+m, and query maximum values at points x. Useful for dynamic programming ("convex hull trick").
Time: $\mathcal{O}(\log N)$

29 lines

```
struct Line {
  mutable int k, m, p;
  bool operator<(const Line& o) const { return k < o.k; }
  bool operator<(int x) const { return p < x; }
};
struct LineContainer : multiset<Line, less<>> {
  // (for doubles, use inf = 1/.0, div(a,b) = a/b)
  static const int inf = intONG_MAX;
  int div(int a, int b) { // floored division
    return a / b - ((a ^b) < 0 && a % b); }
  bool isect(iterator x, iterator y) {
    if (y == end()) return x->p = inf, 0;
    if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
    else x->p = div(y->m - x->m, x->k - y->k);
    return x->p >= y->p;
  }
  void add(int k, int m) {
    auto z = insert({k, m, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p)
      isect(x, erase(y));
  }
  int query(int x) {
    assert(!empty());
    auto l = *lower_bound(x);
    return l.k * x + l.m;
  }
};
```

6.7 MoQueries

Description: Answer interval or tree path queries by finding an approximate TSP through the queries, and moving from one query to the next by adding/removing points at the ends. If values are on tree edges, change `step` to add/remove the edge (a, c) and remove the initial `add` call (but keep `in`).
Time: $\mathcal{O}(N\sqrt{Q})$

47 lines

```
void add(int ind, int end) { ... } // add a[ind] (end = 0 or 1)
void del(int ind, int end) { ... } // remove a[ind]
int calc() { ... } // compute current answer
vi mo(vector<pii> Q) {
  int L = 0, R = 0, blk = 350; // $\sim \frac{N}{\sqrt Q}$
  vi s(sz(Q)), res = s;
#define K(x) pii(x.first/blk, x.second ^-(x.first/blk & 1))
  iota(all(s), 0);
  sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]); });
  for (int qi : s) {
    pii q = Q[qi];
    while (L > q.first) add(--L, 0);
    while (R < q.second) add(R++, 1);
    while (L < q.first) del(L++, 0);
    while (R > q.second) del(--R, 1);
    res[qi] = calc();
  }
  return res;
}
vi moTree(vector<array<int, 2>> Q, vector<vi>& ed, int root=0){
  int N = sz(ed), pos[2] = {}, blk = 350; // $\sim \frac{N}{\sqrt Q}$
  vi s(sz(Q)), res = s, I(N), L(N), R(N), in(N), par(N);
  add(0, 0), in[0] = 1;
  auto dfs = [&](int x, int p, int dep, auto& f) -> void {
    par[x] = p;
    L[x] = N;
    if (dep) I[x] = N++;
    for (int y : ed[x]) if (y != p) f(y, x, !dep, f);
    if (!dep) I[x] = N++;
    R[x] = N;
  };
  dfs(root, -1, 0, dfs);
#define K(x) pii(I[x[0]] / blk, I[x[1]] ^-(I[x[0]] / blk & 1))
  iota(all(s), 0);
  sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]); });
  for (int qi : s) rep(end,0,2) {
    int &a = pos[end], b = Q[qi][end], i = 0;
#define step(c) { if (in[c]) { del(a, end); in[a] = 0; } \
                  else { add(c, end); in[c] = 1; } a = c; }
    while (!(L[b] <= L[a] && R[a] <= R[b]))
      I[i++] = b, b = par[b];
    while (a != b) step(par[a]);
    while (i--) step(I[i]);
    if (end) res[qi] = calc();
  }
  return res;
}
```

6.8 moHilbert.cpp

106 lines

```
#include <bits/stdc++.h>
using namespace std;
#define endl '\n'
#define int long long
#define __ ios_base::sync_with_stdio(false);cin.tie(NULL);
inline int gilbertOrder(int x, int y, int pow, int rotate) {
 if (pow == 0) {
  return 0;
 }
 int hpow = 1 << (pow-1);
 int seg = (x < hpow) ? (
  (y < hpow) ? 0 : 3
 ) : (
  (y < hpow) ? 1 : 2
 );
 seg = (seg + rotate) & 3;
 const int rotateDelta[4] = {3, 0, 0, 1};
 int nx = x & (x ^hpow), ny = y & (y ^hpow);
 int nrot = (rotate + rotateDelta[seg]) & 3;
 int subSquareSize = 1ll << (2*pow - 2);
 int ans = seg * subSquareSize;
 int add = gilbertOrder(nx, ny, pow-1, nrot);
 ans += (seg == 1 || seg == 2) ? add : (subSquareSize - add - 1);
 return ans;
}
struct query{
 int l,r,id;int ord;
 inline void calcOrder() {
  ord = gilbertOrder(l, r, 21, 0);
 }
};
inline bool operator<(const query &a, const query &b) {
 return a.ord < b.ord;
}
// Without hilbert order.
// Around 4 times slower , depend of number of queries
// int block = 316;
// struct query {
//  int l, r, id;
//  inline pair<int, int> toPair() const {
//   return make_pair(l / block, ((l / block) & 1) ? -r : +r);
//  }
// };
// inline bool operator<(const query &a, const query &b) {
//  return a.toPair() < b.toPair();
// }
int k = 0;
int total = 0;
int mp[1<<20];
void add(int x) {
   total+=mp[x^k];
   mp[x]++;
}
void rem(int x) {
   mp[x]--;
   total-=mp[x^k];
}
signed main() {__
   int T = 1,n,m,l,r;
   while(T--) {
      cin>>n>>m>>k;
      vector<int> nums(n);
      for(auto &c:nums)cin>>c;
      vector<int> pref(n+1);
      for(int i = 1;i<=n;i++) {
         pref[i] = pref[i-1]^nums[i-1];
      }
      vector<query> Q;
      for(int id = 0;id<m;id++) {
         cin>>l>>r;
         l--;
         // Consider manage a range (l,r] if the problem work with
             ↪ prefix , this to be able to delete the contribution of
             ↪ prefix_l
         Q.push_back( { l,r,id } );
         // if you use hilbert order
         Q.back().calcOrder();
      }
      sort(Q.begin(),Q.end());
      int L = 0,R = -1;
      vector<int> ans(m);
      for(int i = 0;i<m;i++) {
         int l = Q[i].l;
         int r = Q[i].r;
         while(L>l) {
            L--;
            add(pref[L]);
         }
         while(R<r) {
            R++;
            add(pref[R]);
         }
         while(L<l) {
            rem(pref[L]);
            L++;
         }
         while(R>r) {
            rem(pref[R]);
            R--;
         }
         ans[Q[i].id] = total;
      }
      for(int i = 0;i<m;i++)
         cout<<ans[i]<<endl;
      cout<<endl;
   }
   return 0;
}
```

6.9 OrderStatisticTree

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element. To get a map, change `null_type`.
Time: $\mathcal{O}(\log N)$

19 lines

```
#include <bits/extc++.h> /** keep-include */
using namespace __gnu_pbds;
template<class T>
using Tree = tree<T, null_type, less<T>, rb_tree_tag,
   tree_order_statistics_node_update>;
void example() {
 Tree<int> t, t2; t.insert(8);
 auto it = t.insert(10).first;
 assert(it == t.lower_bound(9));
 assert(t.order_of_key(10) == 1);
 assert(t.order_of_key(11) == 2);
 assert(*t.find_by_order(0) == 8);
 t.join(t2); // assuming T < T2 or T > T2, merge t2 into t
}
// Multiset
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
typedef tree<int,null_type,less_equal<int>,rb_tree_tag,
     ↪ tree_order_statistics_node_update> ordered_set;
```

6.10 QuadTree

Description: A divide and conquer structure that divides a plane in fou sections to answer efficiently queries like how many points are inside of a rectangle
Usage: `node* head = new node(range(0,MAXN,0,MAXN));`
`count(range(S[i].x,MAXN-S[i].x,S[i].y,MAXN-S[i].y),head); Range is an structure for a rectangle`
Time: $\mathcal{O}(n \log N)$.

87 lines

```
struct point{
   int x,y;
   point(int _x,int _y):x(_x),y(_y){ }
};
int capacity = 4;
struct range{
   int x,y,w,h; // w is width and h is height,x and y are the left upper
       ↪ corner
```

```
  range(int _x,int _w,int _y,int _h):x(_x),y(_y),w(_w),h(_h) { }
  bool contains(point p) {
    if( p.x >= x &&
      p.x <= x + w &&
      p.y >= y &&
      p.y <= y + h)
      return true;
    return false;
  }
  bool intersects(range R) {
    return !(R.x > x+w ||
         R.x+R.w < x ||
         R.y > y+h ||
         R.y+R.h < y
      );
  }
};
struct node {
  range boundary;
  node(range bound):boundary(bound) { }
  bool divided = false;
  vector<point> P;
  node *nw = NULL,*ne = NULL,*sw = NULL,*se = NULL;
  void divide() {
    divided = true;
    nw = new node(range(0,boundary.w/2,boundary.h/2,boundary.h/2));
    ne = new node(range(boundary.w/2,boundary.w/2,boundary.h/2, boundary.h/2));
    sw = new node(range(0,boundary.w/2,0,boundary.h/2));
    se = new node(range(boundary.w,boundary.w/2,0,boundary.h/2));
  }
};
int MAXN = 65536/4;
bool insert(point p,node *N) {
  if(!N->boundary.contains(p))return false;
  if(!N->P.size()<capacity) {
    N->P.push_back(p);
    return true;
  }
  else {
    if(!N->divided)N->divide();
    if(insert(p,N->nw))return true;
    if(insert(p,N->ne))return true;
    if(insert(p,N->sw))return true;
    if(insert(p,N->se))return true;
  }
  return true;
}
int count (range R,node *N) {
  int ans = 0;
  if(!N->boundary.intersects(R))return 0;
  for(auto p:N->P) {
    if(R.contains(p))ans++;
  }
  if(N->divided) {
    ans+=count(R,N->nw);
    ans+=count(R,N->ne);
    ans+=count(R,N->sw);
    ans+=count(R,N->se);
  }
  return ans;
}
int main() {
  int n,x,y;
  cin>>n;
  vector<point> S;
  node* head = new node(range(0,MAXN,0,MAXN));
  for(int i = 0;i<n;i++) {
    cin>>x>>y;
    S.push_back( { x,y } );
    insert(point(x,y),head);
  }
  vector<int> ans;
  cout<<endl;
  for(int i = 0;i<n;i++) {
    if(count(range(S[i].x,MAXN-S[i].x,S[i].y,MAXN-S[i].y),head)-count( range(S[i].x,0,S[i].y,0),head)==0)ans.push_back(i+1);
  }
  for(auto c:ans)cout<<c<<" ";
  cout<<endl;
  return 0;
}
```

6.11 SegmentTree

Description: A Segment Tree is a data structure that allows answering range queries over an array effectively, This includes finding an asossiative function of consecutive array elements

Usage: `for(int i = 0;i<n;i++)update(i,i,vector[i]);`
`STmin ST(N); fill like the recursive one`

Time: build: $\mathcal{O}(n \log N)$ query: $\mathcal{O}(\log N)$. 56 lines

```
/** ---- Recursive segment tree with lazy propagation ---- */
vector<int> st;
vector<int> lazy;
void propagate(int v,int l ,int r) {
  if(!lazy[v])return ;
 // For asigments replace += to =
  st[v] += ((r-l)+1)*lazy[v];
  if(l!= r) {
    lazy[v<<1] += lazy[v];
    lazy[v<<1|1]+= lazy[v];
  }
  lazy[v] = 0;
}
int n; /** n is global for use default values and send less parameters */
void update(int l,int r,int val,int v = 1,int sl = 0,int sr = n-1) {
  propagate(v,sl,sr);
  if(r<sl || l>sr || sl>sr)return ;
  if(sl>= l && sr<=r) {
    lazy[v] += val;
    propagate(v,sl,sr);
    return;
  }
  int m = (sl+sr)>>1;
  update(l,r,val,v<<1,sl,m);
  update(l,r,val,v<<1|1,m+1,sr);
  st[v] = st[v<<1]+st[v<<1|1];
}
int query(int l,int r,int v = 1,int sl = 0,int sr = n-1) {
  propagate(v,sl,sr);
  if(r<sl || l>sr || sl>sr)return 0;
  if(sl>= l && sr<=r)return st[v];
  int m = (sl+sr)>>1;
  return query(l,r,v<<1,sl,m)+query(l,r,v<<1|1,m+1,sr);
}
/** ---- Iterative segment tree much faster, setted to return min in a range ---- */
struct STmin {
  int n;
  vector<int> st;
  STmin(int n):n(n) {
    st.resize(2*n,inf);
  }
 inline void update(int x, int val)  {
  x += n;
  st[x] = val;
  for (; x >=1 ; st[x] = min(st[x<<1], st[x<<1|1]));
 }
 inline int query(int l, int r)  {
  int ans = inf;
    if(r<l)return 0;
  for (l += n, r += n; l <= r; l = (l + 1) / 2, r = (r - 1) / 2)  {
   if (l & 1) ans = min(ans, st[l]);
   if (~r & 1) ans = min(ans, st[r]);
  }
  return ans;
 }
};
```

6.12 SegmentTreeBeats

Description: A segment tree with special queries, allows to update For all i in [l,r), change Ai to max/min(Ai, x) Query for the sum of Ai in [l, r]
Usage: `build(); //check that array is global`
`update_max/min(--l,r,x); // update is in a range [l,r) and 0 indexed`
Time: $\mathcal{O}(\log N)$ per query.

126 lines

```
const int INF = 1e15;
struct info {
 int mx1, mx2, mx_cnt, mn1, mn2, mn_cnt;
 info(int a = -INF, int b = -INF, int c = 0, int d = INF, int e = INF, int f = 0) {
  mx1 = a, mx2 = b, mx_cnt = c, mn1 = d, mn2 = e, mn_cnt = f;
 }
 friend inline info merge(info u, info v) {
  if (u.mx1 < v.mx1) {
   u.mx_cnt = 0;
   u.mx2 = u.mx1;
   u.mx1 = v.mx1;
  }
  if (u.mx1 == v.mx1) {
   u.mx_cnt += v.mx_cnt;
   u.mx2 = max(u.mx2, v.mx2);
  }
  else
   u.mx2 = max(u.mx2, v.mx1);
  if (u.mn1 > v.mn1) {
   u.mn_cnt = 0;
   u.mn2 = u.mn1;
   u.mn1 = v.mn1;
  }
  if (u.mn1 == v.mn1) {
   u.mn_cnt += v.mn_cnt;
   u.mn2 = min(u.mn2, v.mn2);
  }
  else
   u.mn2 = min(u.mn2, v.mn1);
  return u;
 }
};
struct node {
 info p;
 long long sum;
 int mx_lazy, mn_lazy;
 node(info a = info(), long long b = 0) {
  p = a, sum = b, reset();
 }
 inline void reset() {
  mx_lazy = -INF, mn_lazy = INF;
 }
 inline void set_max(int x) {
  if (x <= p.mn1)
   return;
  sum += 1LL * (x - p.mn1) * p.mn_cnt;
  if (p.mx1 == p.mn1)
   p.mx1 = x;
  if (p.mx2 == p.mn1)
   p.mx2 = x;
  p.mn1 = mx_lazy = x;
 }
 inline void set_min(int x) {
  if (x >= p.mx1)
   return;
  sum += 1LL * (x - p.mx1) * p.mx_cnt;
  if (p.mn1 == p.mx1)
   p.mn1 = x;
  if (p.mn2 == p.mx1)
   p.mn2 = x;
  p.mx1 = mn_lazy = x;
 }
 friend inline node merge(node u, node v) {
  u.p = merge(u.p, v.p);
  u.sum += v.sum;
  u.reset();
  return u;
 }
};
const int N = 2e5;
node seg[N << 2];
int n, q, a[N]; /** Global variables are important for default values in functions */
inline void find(int id) {
 seg[id] = merge(seg[id << 1], seg[id << 1 | 1]);
}
inline void shift(int id) {
 for (auto p: {id << 1, id << 1 | 1}) {
  seg[p].set_max(seg[id].mx_lazy);
  seg[p].set_min(seg[id].mn_lazy);
 }
 seg[id].reset();
}
void build(int id = 1, int st = 0, int en = n) {
 if (en - st == 1) {
  info x;
  x.mx1 = a[st], x.mx_cnt = 1;
  x.mn1 = a[st], x.mn_cnt = 1;
  seg[id] = {x, a[st]};
  return;
 }
 int mid = st + en >> 1;
 build(id << 1, st, mid);
 build(id << 1 | 1, mid, en);
 find(id);
}
void update_max(int l, int r, int x, int id = 1, int st = 0, int en = n) {
 if (r <= st || en <= l || seg[id].p.mn1 >= x)
  return;
 if (l <= st && en <= r && seg[id].p.mn2 > x)
  return seg[id].set_max(x);
 shift(id);
 int mid = st + en >> 1;
 update_max(l, r, x, id << 1, st, mid);
 update_max(l, r, x, id << 1 | 1, mid, en);
 find(id);
}
void update_min(int l, int r, int x, int id = 1, int st = 0, int en = n) {
 if (r <= st || en <= l || seg[id].p.mx1 <= x)
  return;
 if (l <= st && en <= r && seg[id].p.mx2 < x)
  return seg[id].set_min(x);
 shift(id);
 int mid = st + en >> 1;
 update_min(l, r, x, id << 1, st, mid);
 update_min(l, r, x, id << 1 | 1, mid, en);
 find(id);
}
long long get(int l, int r, int id = 1, int st = 0, int en = n) {
 if (r <= st || en <= l)
  return 0;
 if (l <= st && en <= r)
  return seg[id].sum;
 shift(id);
 int mid = st + en >> 1;
 return get(l, r, id << 1, st, mid) + get(l, r, id << 1 | 1, mid, en);
}
```

6.13 SegmentTreeDynamic

Description: A Segment Tree that stores data only if is needed or asked , that allows to manage bigger "arrays" more than 10^7
Usage: `Node st(0,maximum_size);`
Time: $\mathcal{O}(\log N)$ per query.

49 lines

```
struct Node {
    int sum,greater,l,r,lazy;
    bool prop;
    vector<Node> sons;
    Node(int _l,int _r):l(_l),r(_r),lazy(0),greater(0),sum(0),prop(false) { }
    void propagate() {
        if(sons.empty() && l!=r) {
            int m = (l+r)>>1;
            sons.push_back(Node(l,m));
            sons.push_back(Node(m+1,r));
```

```
    }
    if(prop) {
      sum = greater = lazy*((r-l)+1);
      if(l!=r) {
        sons[0].prop = true;
        sons[1].prop = true;
        sons[1].lazy = lazy;
        sons[0].lazy = lazy;
      }
      prop = false;
    }
  }
  // Update in a range [a,b]
  void update(int a,int b ,int v) {
    propagate();
    if(a>r || b<l)return ;
    if(l>=a && r<=b) {
      lazy = v;
      prop = true;
      propagate();
      return;
    }
    int m = (l+r)>>1;
    sons[0].update(a,b,v);
    sons[1].update(a,b,v);
    sum = sons[0].sum+sons[1].sum;
    greater=max(sons[0].greater,sons[0].sum+sons[1].greater);
  }
  int query(int k) {
    propagate();
    if(l == r) { return greater>k?l-1:l; }
    sons[0].propagate();
    // sons[1].propagate();
    if(sons[0].greater>k)
      return sons[0].query(k);
    else
      return sons[1].query(k-sons[0].sum);
  }
};
```

6.14 SegmentTreeDynamic2D

Description: A 2D segment tree with updates in a point

Usage: `ST t(0, n - 1);`

`t.upd(x, y, v); // update in a point(x,y) a value v (asigment)`

`t.query(x1, x2, y1, y2); // Returns an assosiative function in a submatrix`

Time: $\mathcal{O}(\log N)$ per query.

143 lines

```
mt19937 rnd(chrono::steady_clock::now().time_since_epoch().count());
const int N = 3e5 + 9;
struct node {
  node *l, *r;
  int pos, key, mn, mx;
  long long val, g;
  node(int position, long long value) {
    l = r = nullptr;
    mn = mx = pos = position;
    key = rnd();
    val = g = value;
  }
  void pull() {
    g = val;
    if(l) g = __gcd(g, l->g);
    if(r) g = __gcd(g, r->g);
    mn = (l ? l->mn : pos);
    mx = (r ? r->mx : pos);
  }
};
//memory O(n)
struct treap {
  node *root;
  treap() {
    root = nullptr;
  }
  void split(node *t, int pos, node *&l, node *&r) {
    if (t == nullptr) {
      l = r = nullptr;
      return;
    }
    if (t->pos < pos) {
      split(t->r, pos, l, r);
      t->r = l;
      l = t;
    } else {
      split(t->l, pos, l, r);
      t->l = r;
      r = t;
    }
    t->pull();
  }
  node* merge(node *l, node *r) {
    if (!l || !r) return l ? l : r;
    if (l->key < r->key) {
      l->r = merge(l->r, r);
      l->pull();
      return l;
    }
    r->l = merge(l, r->l);
    r->pull();
    return r;
  }
  bool find(int pos) {
    node *t = root;
    while (t) {
      if (t->pos == pos) return true;
      if (t->pos > pos) t = t->l;
      else t = t->r;
    }
    return false;
  }
  void upd(node *t, int pos, long long val) {
    if (t->pos == pos) {
      t->val = val;
      t->pull();
      return;
    }
    if (t->pos > pos) upd(t->l, pos, val);
    else upd(t->r, pos, val);
    t->pull();
  }
  void insert(int pos, long long val) { //set a_pos = val
    if (find(pos)) upd(root, pos, val);
    else {
      node *l, *r;
      split(root, pos, l, r);
      root = merge(merge(l, new node(pos, val)), r);
    }
  }
  long long query(node *t, int st, int en) {
    if (t->mx < st || en < t->mn) return 0;
    if (st <= t->mn && t->mx <= en) return t->g;
    long long ans = (st <= t->pos && t->pos <= en ? t->val : 0);
    if (t->l) ans = __gcd(ans, query(t->l, st, en));
    if (t->r) ans = __gcd(ans, query(t->r, st, en));
    return ans;
  }
  long long query(int l, int r) { //gcd of a_i such that l <= i <= r
    if (!root) return 0;
    return query(root, l, r);
  }
  void print(node *t) {
    if (!t) return;
    print(t->l);
    cout << t->val << " ";
    print(t->r);
  }
};
//total memory along with treap = nlogn
struct ST {
  ST *l, *r;
  treap t;
  int b, e;
  ST() {
    l = r = nullptr;
```

```
  }
 ST(int st, int en) {
  l = r = nullptr;
  b = st, e = en;
 }
 void fix(int pos) {
  long long val = 0;
  if (l) val = __gcd(val, l->t.query(pos, pos));
  if (r) val = __gcd(val, r->t.query(pos, pos));
  t.insert(pos, val);
 }
 void upd(int x, int y, long long val) { //set a[x][y] = val
  if (e < x || x < b) return;
  if (b == e) {
   t.insert(y, val);
   return;
  }
  if (b != e) {
   if (x <= (b + e) / 2) {
    if (!l) l = new ST(b, (b + e) / 2);
    l->upd(x, y, val);
   } else {
    if (!r) r = new ST((b + e) / 2 + 1, e);
    r->upd(x, y, val);
   }
  }
  fix(y);
 }
 long long query(int i, int j, int st, int en) { //gcd of a[x][y] such
     ↪ that i <= x <= j && st <= y <= en
  if (e < i || j < b) return 0;
  if (i <= b && e <= j) return t.query(st, en);
  long long ans = 0;
  if (l) ans = __gcd(ans, l->query(i, j, st, en));
  if (r) ans = __gcd(ans, r->query(i, j, st, en));
  return ans;
 }
};
```

6.15 SegmentTreeDynamicOpt

Description: An optimized in memory dynamic segment tree
Time: $\mathcal{O}(\log N)$ per query.

61 lines

```
struct node {
  int sum,idl,idr;
  node(int l,int r,int _idl = -1,int _idr = -1):sum((r-l)+1),idl(_idl),
      ↪ idr(_idr) { }
};
struct SegmentTree {
  const int l,r;
  vector<node> tree;
  SegmentTree(int low,int h):l(low),r(h) {
    tree.reserve((1u << 21) + (1u << 20));
    tree.push_back(node(l,r));
  }
  void extend(int id,int a,int b) {
    if(tree[id].idl == -1) {
      int m = (a+b)/2;
      tree.push_back(node(a,m));
      tree[id].idl = tree.size()-1;
      tree.push_back(node(m+1,b));
      tree[id].idr = tree.size()-1;
    }
  }
  void update(int id ,int pos,int val,int a ,int b ) {
    if(pos<a || pos >b)return ;
    if(a == b) {
      tree[id].sum += val;
      return;
    }
    extend(id,a,b);
    int m = (a+b)/2;
    const int idl = tree[id].idl; assert(idl != -1);
    const int idr = tree[id].idr; assert(idr != -1);
    update(idl,pos,val,a,m);
    update(idr,pos,val,m+1,b);
    tree[id].sum = tree[idl].sum+tree[idr].sum;
  }
  int query(int id,int k,int a ,int b ) {
    if(a == b)return a;
    extend(id,a,b);
    const int idl = tree[id].idl; assert(idl != -1);
    const int idr = tree[id].idr; assert(idr != -1);
    int m = (a+b)>>1;
    if(tree[idl].sum>= k)
      return query(idl,k,a,m);
    else
      return query(idr,k-tree[idl].sum,m+1,b);
  }
};
int main() {
  int n, m,q;
  char t;
  cin>>n>>m;
  SegmentTree st(1,n);
  for(int i = 0;i<m;i++) {
    cin>>t>>q;
    int x = st.query(0,q,1,n);
    if (t == 'L')
      cout<<x<<endl;
    else
      st.update(0,x, -1,1,n);
  }
  return 0;
}
```

6.16 SegmentTreePersistent

Description: A persistent data structure is a data structure that remembers it previous state for each modification. This allows to access any version of this data structure that interest us and execute a query on it.
Time: $\mathcal{O}(\log N)$ per query.

52 lines

```
const int maxn = 800007;
int L[maxn],R[maxn],st[maxn],lazy[maxn],N;
int n; /** Must be global for default values in functions */
int newLeaf(int val) {
  int p = ++N;
  L[p] = R[p] = 0;
  st[p] = val;
  return p;
}
int newParent(int l,int r) {
  int p = ++N;
  L[p] = l;
  R[p] = r;
  st[p] = st[l]+st[r];
  return p;
}
int newLazy(int v,int val,int l,int r) {
  int p = ++N;
  L[p] = L[v];
  R[p] = R[v];
  lazy[p] += val;
  st[p] = st[v]+((r-l)+1)*val;
  return p;
}
int build(vector<int> &A,int l = 0,int r = n-1) {
  if(l== r)return newLeaf(A[l]);
  int mid = (l+r)>>1;
  return newParent(build(A,l,mid),build(A,mid+1,r));
}
void propagate(int p,int l,int r) {
  if(lazy[p]==0)return;
  if(l!= r) {
    int mid = (l+r)>>1;
    L[p] = newLazy(L[p],lazy[p],l,mid);
    R[p] = newLazy(R[p],lazy[p],mid+1,r);
  }
  lazy[p] = 0;
}
int update(int l,int r,int val,int p,int sl = 0 ,int sr = n-1) {
  if(sr<l || sl>r)return p;
```

```
  if(sl>=l && sr<=r)return newLazy(p,val,sl,sr);
  propagate(p,sl,sr);
  int mid = (sl+sr)>>1;
  return newParent(update(l,r,val,L[p],sl,mid),update(l,r,val,R[p],mid+1,sr));
}
int query(int l,int r,int p,int sl = 0,int sr = n-1){
  if(sr<l || sl> r)return 0;
  if(sl>=l && sr<=r)return st[p];
  int mid = (sl+sr)>>1;
  propagate(p,sl,sr);
  return query(l,r,L[p],sl,mid)+query(l,r,R[p],mid+1,r);
}
```

6.17 SparseTable

Description: Sparse table is similar to segment tree but don´t allows updates
Time: $\mathcal{O}(1)$ per query, $\mathcal{O}(N \log N)$ build

38 lines

```
#define MAXN 1000000
#define MAXPOWN 1048576  // 2^(ceil(log_2(MAXN)))
#define MAXLEV 21   // ceil(log_2(MAXN)) + 1
int n, P, Q;
int A[MAXPOWN];
int table[MAXLEV][MAXPOWN];
int maxlev, siz
size = n;
maxlev = __builtin_clz(n) ^31;  // floor(log_2(n))
if( (1<<maxlev) != n)
  size = 1<<++maxlev;
void build(int level=0,int l=0, int r=size){
 int m = (l+r)/2;
 table[level][m] = A[m]%P;
 for(int i=m-1;i>=l;i--)
  table[level][i] = (long long)table[level][i+1] * A[i] % P;
 if(m+1 < r)  {
  table[level][m+1] = A[m+1]%P;
  for(int i=m+2;i<r;i++)
   table[level][i] = (long long)table[level][i-1] * A[i] % P;
 }
 if(l + 1 != r)  // r - l > 1
 {
  build(level+1, l, m);
  build(level+1, m, r);
 }
}
int query(int x, int y)
{
 if(x == y)
  return A[x]%P;
 int k2 = __builtin_clz(x^y) ^31;
 int lev = maxlev - 1 - k2;
 int ans = table[lev][x];
 if(y & ((1<<k2) - 1)) // y % (1<<k2)
  ans = (long long)ans * table[lev][y] % P;
 return ans;
}
```

6.18 SQRTDecomposition.cpp

Description: Find de fist and last ocurrence of x in an array with lazy updates
Time: $\mathcal{O}(sqrt(n))$ per query

95 lines

```
#include <bits/stdc++.h>
using namespace std;
#define print(A) for(auto c:A)cout<<c<<" ";cout<<endl;
#define printM(A) for(auto c:A){print(c);}
#define x first
#define y second
#define printP(A)for(auto c:A)cout<<"("<<c.x<<","<<c.y<<") ";cout<<endl;
#define printMP(A)for(auto c:A){printP(c);}
#define endl '\n'
#define MOD(n,k) ( ( ((n) % (k)) + (k) ) % (k))
#define ALL(A) A.begin(),A.end()
#define error(args...) { string _s = #args; replace(_s.begin(), _s.end(), ',', ' ');\
stringstream _ss(_s); istream_iterator<string> _it(_ss); err(_it, args); }
#define rep(i, begin, end) for (__typeof(end) i = (begin) - ((begin) > (end));\
i != (end) - ((begin) > (end)); i += 1 - 2 * ((begin) > (end)))
#define cerr(s) cerr << "\033[48;5;196m\033[38;5;15m" << s << "\033[0m"
void err(istream_iterator<string> it) { }
template<typename T, typename... Args>
void err(istream_iterator<string> it, T a, Args... args) {
 cerr << *it << " = " << a << endl;
 err(++it, args...);
}
typedef long long int lli;
const lli inf = 1000000000;
struct block{
  int l,r;
  int size;
  vector<pair<lli,lli>>A;
  lli plus = 0;
  block(int _l,int _r,int _size): l(_l), r(_r),size(_size){A.resize(size,{-1,-1});}
  void st(){
    sort(A.begin(),A.end());
  }
  void add(int a,int b,lli v){
    if(a>b || b<l ||a>r)return;
    if(a<l)a = l;
    if(b>r)b = r;
    // error(l,r,a,b)
    if(a == l&& b == r){plus+=v;return; }
    for(int i = 0;i<size;i++)
      if(A[i].y>=a && A[i].y<=b)A[i].first+=v;
    st();
  }
  pair<lli,lli> find(lli z){
    z-=plus;
    int index = lower_bound(A.begin(),A.end(),make_pair(z,-inf))-A.begin();
    if(A[index].x!= z)return {inf,-1};
    int index2 = lower_bound(A.begin(),A.end(),make_pair(z+1,-inf))-A.begin();
    index2--;
    // error(index,index2)
    return {A[index].second,A[index2].second};
  }
};
int main(){
  int n,m,t,v,l,r,z;
  cin>>n>>m;
  vector<int> nums(n);
  for(auto &c:nums)cin>>c;
  int raiz = ceil(sqrt(n*1.0));
  vector<block> bloques(raiz,block(0,0,0));
  vector<vector<pair<int,int>>> sq(raiz);
  for(int i = 0;i<raiz;i++){
    bloques[i] = block(i*raiz,((i*raiz)+raiz)-1,raiz);
    for(int j = 0;j<raiz;j++){
      if(i*raiz+j<n)
        bloques[i].A[j] = {nums[(i*raiz)+j],(i*raiz)+j};
    }
    bloques[i].st();
  }
  for(int i = 0;i<m;i++){
    cin>>t;
    if(t ==1){
      cin>>l>>r>>v;
      l--;
      r--;
      for(int i = 0;i<raiz;i++)
        bloques[i].add(l,r,v);
    }
    else{
      cin>>z;
      lli mn = n+2,mx = -1;
      for(int j = 0;j<raiz;j++){
```

```
            auto c = bloques[j].find(z);
            mn = min(c.x,mn);
            mx = max(c.y,mx);
            // error(c.x,c.y);
        }
        // error(mx,mn)
        if(mx == -1)cout<<-1<<endl;
        else cout<<mx-mn<<endl;
    }
  }
  // for(int i = 0;i<raiz;i++) {printP(bloques[i].A);cout<<endl; }
  return 0;
}
```

6.19 SubMatrix

Description: Calculate submatrix sums quickly, given upper-left and lower-right corners (half-open).
Usage: `SubMatrix<int> m(matrix);`
`m.sum(0, 0, 2, 2); // top left 4 elements`
Time: $\mathcal{O}\left(N^2+Q\right)$

13 lines

```
template<class T>
struct SubMatrix {
  vector<vector<T>> p;
  SubMatrix(vector<vector<T>>& v) {
    int R = sz(v), C = sz(v[0]);
    p.assign(R+1, vector<T>(C+1));
    rep(r,0,R) rep(c,0,C)
      p[r+1][c+1] = v[r][c] + p[r][c+1] + p[r+1][c] - p[r][c];
  }
  T sum(int u, int l, int d, int r) {
    return p[d][r] - p[d][l] - p[u][r] + p[u][l];
  }
};
```

6.20 randomizedKdTree.cpp

193 lines

```
typedef complex<double> point;
struct randomized_kd_tree
{
  struct node
  {
    point p;
    int d, s;
    node *l, *r;
    bool is_left_of(node *x)
    {
      if (x->d)
        return real(p) < real(x->p);
      else
        return imag(p) < imag(x->p);
    }
  } *root;
  randomized_kd_tree() : root(0) { }
  int size(node *t)
  {
    return t ? t->s : 0;
  }
  node *update(node *t)
  {
    t->s = 1 + size(t->l) + size(t->r);
    return t;
  }
  pair<node*, node*> split(node *t, node *x)
  {
    if (!t)
      return {0, 0};
    if (t->d == x->d)
    {
      if (t->is_left_of(x))
      {
        auto p = split(t->r, x);
        t->r = p.first;
        return {update(t), p.second};
      }
      else
      {
        auto p = split(t->l, x);
        t->l = p.second;
        return {p.first, update(t)};
      }
    }
    else
    {
      auto l = split(t->l, x);
      auto r = split(t->r, x);
      if (t->is_left_of(x))
      {
        t->l = l.first;
        t->r = r.first;
        return {update(t), join(l.second, r.second, t->d)};
      }
      else
      {
        t->l = l.second;
        t->r = r.second;
        return {join(l.first, r.first, t->d), update(t)};
      }
    }
  }
  node *join(node *l, node *r, int d)
  {
    if (!l)
      return r;
    if (!r)
      return l;
    if (rand() % (size(l) + size(r)) < size(l))
    {
      if (l->d == d)
      {
        l->r = join(l->r, r, d);
        return update(l);
      }
      else
      {
        auto p = split(r, l);
        l->l = join(l->l, p.first, d);
        l->r = join(l->r, p.second, d);
        return update(l);
      }
    }
    else
    {
      if (r->d == d)
      {
        r->l = join(l, r->l, d);
        return update(r);
      }
      else
      {
        auto p = split(l, r);
        r->l = join(p.first, r->l, d);
        r->r = join(p.second, r->r, d);
        return update(r);
      }
    }
  }
  node *insert(node *t, node *x)
  {
    if (rand() % (size(t) + 1) == 0)
    {
      auto p = split(t, x);
      x->l = p.first;
      x->r = p.second;
      return update(x);
    }
    else
    {
      if (x->is_left_of(t))
        t->l = insert(t->l, x);
      else
        t->r = insert(t->r, x);
```

```
      return update(t);
    }
  }
  void insert(point p)
  {
    root = insert(root, new node( { p, rand() % 2 } ));
  }
  node *remove(node *t, node *x)
  {
    if (!t)
      return t;
    if (t->p == x->p)
      return join(t->l, t->r, t->d);
    if (x->is_left_of(t))
      t->l = remove(t->l, x);
    else
      t->r = remove(t->r, x);
    return update(t);
  }
  void remove(point p)
  {
    node n = { p };
    root = remove(root, &n);
  }
  void closest(node *t, point p, pair<double, node*> &ub)
  {
    if (!t)
      return;
    double r = norm(t->p - p);
    if (r < ub.first)
      ub = { r, t };
    node *first = t->r, *second = t->l;
    double w = t->d ? real(p - t->p) : imag(p - t->p);
    if (w < 0)
      swap(first, second);
    closest(first, p, ub);
    if (ub.first > w * w)
      closest(second, p, ub);
  }
  point closest(point p)
  {
    pair<double, node*> ub(1.0 / 0.0, 0);
    closest(root, p, ub);
    return ub.second->p;
  }
  // verification
  int height(node *n)
  {
    return n ? 1 + max(height(n->l), height(n->r)) : 0;
  }
  int height()
  {
    return height(root);
  }
  int size_rec(node *n)
  {
    return n ? 1 + size_rec(n->l) + size_rec(n->r) : 0;
  }
  int size_rec()
  {
    return size_rec(root);
  }
  void display(node *n, int tab = 0)
  {
    if (!n)
      return;
    display(n->l, tab + 2);
    for (int i = 0; i < tab; ++i)
      cout << " ";
    cout << n->p << " (" << n->d << ")" << endl;
    display(n->r, tab + 2);
  }
  void display()
  {
    display(root);
  }
};
```

6.21 vantagePointTree.cpp

91 lines

```
/*
  Vantage Point Tree (vp tree)
  Description:
  Vantage point tree is a metric tree.
  Each tree node has a point, radius, and two childs.
  The points of left descendants are contained in the ball B(p,r)
  and the points of right descendants are excluded from the ball.
  We can find k-nearest neighbors of a given point p efficiently
  by pruning search.
  Complexity:
  Construction: O(n log n)
  Search: O(log n)
*/
typedef complex<double> point;
namespace std
{
  bool operator <(point p, point q)
  {
    if (real(p) != real(q))
      return real(p) < real(q);
    return imag(p) < imag(q);
  }
}
struct vantage_point_tree
{
  struct node
  {
    point p;
    double th;
    node *l, *r;
  } *root;
  vector<pair<double, point>> aux;
  vantage_point_tree(vector<point> ps)
  {
    for (int i = 0; i < ps.size(); ++i)
      aux.push_back( { 0, ps[i] } );
    root = build(0, ps.size());
  }
  node *build(int l, int r)
  {
    if (l == r)
      return 0;
    swap(aux[l], aux[l + rand() % (r - l)]);
    point p = aux[l++].second;
    if (l == r)
      return new node( { p } );
    for (int i = l; i < r; ++i)
      aux[i].first = norm(p - aux[i].second);
    int m = (l + r) / 2;
    nth_element(aux.begin() + l, aux.begin() + m, aux.begin() + r);
    return new node( { p, sqrt(aux[m].first), build(l, m), build(m, r) } );
  }
  priority_queue<pair<double, node*>> que;
  void k_nn(node *t, point p, int k)
  {
    if (!t)
      return;
    double d = abs(p - t->p);
    if (que.size() < k)
      que.push( { d, t } );
    else if (que.top().first > d)
    {
      que.pop();
      que.push( { d, t } );
    }
    if (!t->l && !t->r)
      return;
    if (d < t->th)
    {
      k_nn(t->l, p, k);
      if (t->th - d <= que.top().first)
```

```
      k_nn(t->r, p, k);
    }
   else
    {
     k_nn(t->r, p, k);
     if (d - t->th <= que.top().first)
      k_nn(t->l, p, k);
    }
  }
 vector<point> k_nn(point p, int k)
  {
   k_nn(root, p, k);
   vector<point> ans;
   for (; !que.empty(); que.pop())
    ans.push_back(que.top().second->p);
   reverse(ans.begin(), ans.end());
   return ans;
  }
};
```

6.22 waveletTree

Description: Binary tree based in values instead of ranges like segment tree, thah alows compute queries in a range like , kth smallest element in a range [l,r], other queries in the code.
Considerations:

- compression if the elements are to big.
- Array passed is modified

Usage: `wavelet wt(Array,Max_element+1);`
Time: $\mathcal{O}(\log N)$.

135 lines

```
typedef vector<int>::iterator it;
struct wavelet{
   vector<vector<int>> mapLeft;
   int mx;
   wavelet(vector<int> &A,int mx):mapLeft(mx*2),mx(mx){
      build(A.begin(),A.end(),0,mx-1,1);
   }
   void build(it s,it e,int l,int r,int v){
      if(l== r)return;
      int m = (l+r)>>1;
      mapLeft[v].reserve(e-s+1);
      mapLeft[v].push_back(0);
      auto f = [m](int x){
         return x<=m;
      };
      for(it iter = s; iter!= e;iter++)
         mapLeft[v].push_back(mapLeft[v].back() + (*iter<=m));
      it p = stable_partition(s,e,f);
      build(s,p,l,m,v<<1);
      build(p,e,m+1,r,v<<1|1);
   }
   //counts the number of elements equal to c in range [1,i]
   //IF you want in the range [i,j] only calls rank(j)- rank(i-1)
   int rank(int c,int i){
      i++;
      int l = 0,r = mx-1,u = 1,m,left;
      while(l!= r){
         m = (l+r)>>1;
         left = mapLeft[u][i];
         u<<=1;
         if(c<=m)
            i = left,r = m;
         else
            i-=left,l = m+1,u|=1;
      }
      return i;
   }
   // return the kth smallest element in a range [i,j]
   // k=1 is the smallest
   // 0 indexed this is indexes are in [0,n-1]
   int kth(int i,int j,int k){
      j++;
      int l = 0,r = mx-1,u = 1,li,lj;
      while(l!=r){
         int m = (l+r)>>1;
         li = mapLeft[u][i],lj = mapLeft[u][j];
         u<<=1;
         if(k<= lj-li)
            i = li,j = lj, r = m;
         else
            i-=li,j-=lj,l = m+1,u|=1,k-=(lj-li);
      }
      return r;
   }
   int kthSum(int i,int j,int k){
      j++;
      int l = 0,r = mx,li,lj;
      int si,sj;
      int ans = 0;
      int u = 1;
      while(l!=r){
         int m = (l+r)>>1;
         li = mapLeft[u][i],lj = mapLeft[u][j];
         si = sumLeft[u][i],sj = sumLeft[u][j];
         u<<=1;
         if(k<= lj-li){
            i = li,j = lj, r = m;
         }
         else {
            ans+=(sj-si);
            u|=1;
            i-=li,j-=lj,l = m+1,k-=(lj-li);
         }
      }
      ans+=rev[r]*k;
      return ans;
   }
   int l,r;
   // count the ocurrences of numbers in the range [a,b]
   // and only in the secuende [i,j]
   // can be seen as how many points are in a specified rectangle with corns i,a and j,b
   int range(int i ,int j ,int a,int b){
      if( b<a || j<i)return 0;
      l = a,r = b;
      return range(i,j+1,0,mx-1,1);
   }
   int range(int i, int j,int a,int b,int v){
      if(b<l || a>r)return 0;
      if(a>=l && b<=r)return j-i;
      int m = (a+b)>>1;
      int li = mapLeft[v][i],lj = mapLeft[v][j];
      return range(li,lj,a,m,v<<1)+range(i-li,j-lj,m+1,b,v<<1|1);
   }
   /*
      Return the minimun number that their frecuence in the range [i,j] is at least k
      complexity depends of k, if k is small and j-i is large maybe go up to o(n)
      the problem tested has a k up to $(j-i)/5$ and the complexity has $O(5 \log n)$
   */
   int minimun_of_ocurrences(int i,int j,int k){
      return minimun_of_ocurrences(i,j+1,k,1,0,mx-1);
   }
   int minimun_of_ocurrences(int i,int j,int k,int v ,int l,int r ){
      if(l == r)return j-i>= k?l:mx+2;
      if(j-i<k)return mx+2;
      int m = (l+r)>>1;
      int li = mapLeft[v][i],lj = mapLeft[v][j];
      int c = lj-li;
      int ans= mx+2;
      if(c >=k)
         ans = min(ans,minimun_of_ocurrences(li,lj,k,v<<1,l,m));
      if((j-i)-c>=k)
         ans = min(ans,minimun_of_ocurrences(i-li,j-lj,k,v<<1|1,m+1,r));
      if(c <k && (j-i)-c<k)return mx+2;
      return ans;
   }
   /* swap element arr[i] and arr[i+1] */
   /*- No tested */
   void swapadjacent(int i){
      swapadjacent(i,0,mx-1,1);
```

```
  }
  void swapadjacent(int i,int l,int r,int v) {
    if(l == r)
      return ;
    mapLeft[v][i]= mapLeft[v][i-1] + mapLeft[v][i+1] - mapLeft[v][i];
    // c[i] = c[i-1] + c[i+1] - c[i];
    if(mapLeft[v][i+1]-mapLeft[v][i] == mapLeft[v][i] - mapLeft[v][i
      ↪ -1]) {
      if(mapLeft[v][i]-mapLeft[v][i-1])
        return swapadjacent(mapleft[v][i],l,mid,v<<1);
      else
        return swapadjacent(i-mapLeft[v][i],mid+1,r,v<<1|1);
    }
    else
      return ;
  }
};
```

graph (7)

7.1 2sat.h

Description: Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem, Here is an example of such a 2-SAT problem. Find an assignment of a,b,c such that the following formula is true: so that an expression of the type $(a \vee \neg b) \wedge (\neg a \vee b) \wedge (\neg a \vee \neg b) \wedge \ldots$ becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions (~x).

70 lines

```
int N,timeD ;
vector<vector<int> > gr(1007);
vector<int> val, comp, z;
vector<int> values; // 0 = false, 1 = true
void addCondition(int u, int v,int Nu,int Nv)  {
 (u*=2)^=Nu;
 (v*=2)^=Nv;
 gr[u^1].push_back(v);
 gr[v^1].push_back(u);
}
// 0 -> 00
// 1 -> 01
// 2 -> 10
// 3 -> 11
// For must be same mask = 9 -> 1001
// for must be diferent mask = 6 -> 0110 ./
void canBe(int u,int v,int mask) {
 if(!(mask&1)) {
  addCondition(u,v,1,1);
 }
 if(!((mask>>1)&1)) {
  addCondition(u,v,1,0);
 }
 if(!((mask>>2)&1)) {
  addCondition(u,v,0,1);
 }
 if(!((mask>>3)&1)) {
  addCondition(u,v,0,0);
 }
}
void cannotBe(int u,int v,int mask) {
 if((mask&1)) {
  addCondition(u,v,0,0);
 }
 if(((mask>>1)&1)) {
  addCondition(u,v,0,1);
 }
 if(((mask>>2)&1)) {
  addCondition(u,v,1,0);
 }
 if(((mask>>3)&1)) {
  addCondition(u,v,1,1);
 }
}
int dfs(int i)  {
 int low = val[i] = ++timeD, x; z.push_back(i);
 for(int j = 0;j<gr[i].size();j++) {
  int e = gr[i][j];
  if (!comp[e])
   low = min(low, val[e] ?: dfs(e));
 }
 ++timeD;
 if (low == val[i]) do  {
  x = z.back(); z.pop_back();
  comp[x] = timeD;
  if (values[x>>1] == -1)
   values[x>>1] = !(x&1);
 } while (x != i);
 return val[i] = low;
}
bool solve()  {
 values.assign(N, -1);
 val.assign(2*N, 0); comp = val;
 timeD = 0;
 for(int i = 0;i<2*N;i++)
     if(!comp[i])
       dfs(i);
 for(int i = 0;i<N;i++) if (comp[2*i] == comp[2*i+1]) return 0;
 return 1;
}
```

7.2 Arborescence2.cpp

73 lines

```
/*
 Minimum Arborescence (Chu-Liu/Edmonds)
 Tested: UVA 11183
 Complexity: O(mn)
*/
template<typename T>
struct minimum_aborescense
{
 struct edge
  {
   int src, dst;
   T weight;
  } ;
 vector<edge> edges;
 void add_edge(int u, int v, T w)
  {
   edges.push_back( {  u, v, w  } );
  }
 T solve(int r)
  {
   int n = 0;
   for (auto e : edges)
     n = max(n, max(e.src, e.dst) + 1);
   int N = n;
   for (T res = 0;;)
    {
     vector<edge> in(N, { -1, -1, numeric_limits<T>::max()  } );
     vector<int> C(N, -1);
     for (auto e : edges) // cheapest comming edges
       if (in[e.dst].weight > e.weight)
         in[e.dst] = e;
     in[r] =  { r, r, 0 } ;
     for (int u = 0; u < N; ++u)
      { // no comming edge ==> no aborescense
       if (in[u].src < 0)
         return numeric_limits<T>::max();
       res += in[u].weight;
      }
     vector<int> mark(N, -1); // contract cycles
     int index = 0;
     for (int i = 0; i < N; ++i)
      {
       if (mark[i] != -1)
         continue;
       int u = i;
       while (mark[u] == -1)
        {
         mark[u] = i;
         u = in[u].src;
        }
```

```
      if (mark[u] != i || u == r)
        continue;
      for (int v = in[u].src; u != v; v = in[v].src)
        C[v] = index;
      C[u] = index++;
     }
    if (index == 0)
      return res; // found arborescence
    for (int i = 0; i < N; ++i) // contract
      if (C[i] == -1)
        C[i] = index++;
    vector<edge> next;
    for (auto &e : edges)
      if (C[e.src] != C[e.dst] && C[e.dst] != C[r])
        next.push_back( { C[e.src], C[e.dst],
          e.weight - in[e.dst].weight } );
    edges.swap(next);
    N = index;
    r = C[r];
   }
  }
};
```

7.3 BellmanFord

Description: Calculates shortest paths from s in a graph that might have negative edge weights. Unreachable nodes get dist = inf; nodes reachable through negative-weight cycles get dist = -inf. Assumes $V^2 \max |w_i| < 2^{63}$.
Time: $\mathcal{O}(VE)$

21 lines

```
const ll inf = LLONG_MAX;
struct Ed { int a, b, w, s() { return a < b ? a : -a; } };
struct Node { ll dist = inf; int prev = -1; };
void bellmanFord(vector<Node>& nodes, vector<Ed>& eds, int s) {
 nodes[s].dist = 0;
 sort(all(eds), [](Ed a, Ed b) { return a.s() < b.s(); });
 int lim = sz(nodes) / 2 + 2; // /3+100 with shuffled vertices
 rep(i,0,lim) for (Ed ed : eds) {
  Node cur = nodes[ed.a], &dest = nodes[ed.b];
  if (abs(cur.dist) == inf) continue;
  ll d = cur.dist + ed.w;
  if (d < dest.dist) {
   dest.prev = ed.a;
   dest.dist = (i < lim-1 ? d : -inf);
  }
 }
 rep(i,0,lim) for (Ed e : eds) {
  if (nodes[e.a].dist == -inf)
   nodes[e.b].dist = -inf;
 }
}
```

7.4 BinaryLifting

Description: Calculate power of two jumps in a tree, to support fast upward jumps and LCAs. Assumes the root node points to itself.
Time: construction $\mathcal{O}(N \log N)$, queries $\mathcal{O}(\log N)$

23 lines

```
vector<vi> treeJump(vi& P) {
 int on = 1, d = 1;
 while(on < sz(P)) on *= 2, d++;
 vector<vi> jmp(d, P);
 rep(i,1,d) rep(j,0,sz(P))
  jmp[i][j] = jmp[i-1][jmp[i-1][j]];
 return jmp;
}
int jmp(vector<vi>& tbl, int nod, int steps){
 rep(i,0,sz(tbl))
  if(steps&(1<<i)) nod = tbl[i][nod];
 return nod;
}
int lca(vector<vi>& tbl, vi& depth, int a, int b) {
 if (depth[a] < depth[b]) swap(a, b);
 a = jmp(tbl, a, depth[a] - depth[b]);
 if (a == b) return a;
 for (int i = sz(tbl); i--;) {
  int c = tbl[i][a], d = tbl[i][b];
  if (c != d) a = c, b = d;
 }
 return tbl[0][a];
}
```

7.5 Bridges.cpp

48 lines

```
/*
 Tested: AIZU(judge.u-aizu.ac.jp) GRL_3_B
 Complexity: O(n + m)
*/
struct graph
{
 int n;
 vector<vector<int>> adj;
 graph(int n) : n(n), adj(n) { }
 void add_edge(int src, int dst)
  {
   adj[src].push_back(dst);
   adj[dst].push_back(src);
  }
};
void bridge_components(const graph &g)
{
 vector<int> num(g.n), low(g.n), stk;
 vector<vector<int>> comps;
 function<void(int, int, int&)> dfs = [&](int u, int p, int &t)
  {
   num[u] = low[u] = ++t;
   stk.push_back(u);
   for (int v : g.adj[u])
    {
     if (v == p) continue;
     if (num[v] == 0)
      {
       dfs(v, u, t);
       low[u] = min(low[u], low[v]);
      }
     else low[u] = min(low[u], num[v]);
    }
   if (low[u] == num[u])
    {
     comps.push_back( { } );
     do
      {
       comps.back().push_back(stk.back());
       stk.pop_back();
      }
     while (comps.back().back() != u);
    }
  };
 for (int u = 0, t; u < g.n; ++u)
   if (num[u] == 0) dfs(u, -1, t = 0);
}
```

7.6 CentroidDecomposition

Description: Decomposes a tree into centroids and create a new tree with the centroids
Time: $\mathcal{O}(n \log n)$

65 lines

```
const int maxn = 100005;
vector<int> graph[maxn];
vector<int> parent(maxn,-1);
vector<int> depth(maxn,-1);
vector<int> best(maxn,1e16);
int P[maxn][25];
bitset<maxn> cent;
int sz[maxn];
void dfs (int u, int p = -1,int d = 0) {
    sz[u] = 1;
    P[u][0] = p;
    depth[u] = d;
    for (int v : graph[u]) {
        if(v ==p)continue;
        dfs(v,u,d+1);
        sz[u] += sz[v];
```

```
  }
}
void build(int n){
  for(int i = 0;i<n;i++)
    for(int j = 0;j<25;j++)
      P[i][j] = -1;
  dfs(0);
  for(int i = 1;i<25;i++)
    for(int u = 0;u<n;u++)
      if(P[u][i-1]!=-1)
        P[u][i] = P[P[u][i-1]][i-1];
}
int lca(int u,int v){
  if(depth[u]<depth[v])swap(u,v);
  int diff = depth[u]-depth[v];
  for(int i = 24;i>=0;i--){
    if((diff>>i)&1){
      u = P[u][i];
    }
  }
  if(u==v)return u;
  for(int i= 24;i>=0;i--){
    if(P[u][i]!=P[v][i]){
      u = P[u][i];
      v = P[v][i];
    }
  }
  return P[u][0];
}
int descomp (int u) {
  int tam = 1;
  for (int v : graph[u])
    if (!cent[v])
      tam += sz[v];
  while (1) {
    int idx = -1;
    for (int v : graph[u])
      if (!cent[v] && 2 * sz[v] > tam)
        idx = v;
    if (idx == -1)break;<
    sz[u] = tam - sz[idx];
    u = idx;
  }
  cent[u] = 1;
 for (int v : graph[u])
    if (!cent[v])
      parent[descomp(v)] = u;
  return u;
}
```

7.7 CompressTree

Description: Given a rooted tree and a subset S of nodes, compute the minimal subtree that contains all the nodes by adding all (at most $|S|-1$) pairwise LCA's and compressing edges. Returns a list of (par, orig_index) representing a tree rooted at 0. The root points to itself.
Time: $\mathcal{O}(|S|\log|S|)$

22 lines

```
#include "LCA.h"
typedef vector<pair<int, int>> vpi;
vpi compressTree(LCA& lca, const vi& subset) {
 static vi rev; rev.resize(sz(lca.time));
 vi li = subset, &T = lca.time;
 auto cmp = [&](int a, int b) { return T[a] < T[b]; };
 sort(all(li), cmp);
 int m = sz(li)-1;
 rep(i,0,m) {
  int a = li[i], b = li[i+1];
  li.push_back(lca.lca(a, b));
 }
 sort(all(li), cmp);
 li.erase(unique(all(li)), li.end());
 rep(i,0,sz(li)) rev[li[i]] = i;
 vpi ret = {pii(0, li[0])};
 rep(i,0,sz(li)-1) {
  int a = li[i], b = li[i+1];
  ret.emplace_back(rev[lca.lca(a, b)], b);
 }
 return ret;
}
```

7.8 cycle.cpp

44 lines

```
int n;
vector<vector<int>> adj;
vector<char> color;
vector<int> parent;
int cycle_start, cycle_end;
bool dfs(int v) {
  color[v] = 1;
  for (int u : adj[v]) {
    if (color[u] == 0) {
      parent[u] = v;
      if (dfs(u))
        return true;
    } else if (color[u] == 1) {
      cycle_end = v;
      cycle_start = u;
      return true;
    }
  }
  color[v] = 2;
  return false;
}
void find_cycle() {
  color.assign(n, 0);
  parent.assign(n, -1);
  cycle_start = -1;
  for (int v = 0; v < n; v++) {
    if (color[v] == 0 && dfs(v))
      break;
  }
  if (cycle_start == -1) {
    cout << "Acyclic" << endl;
  } else {
    vector<int> cycle;
    cycle.push_back(cycle_start);
    for (int v = cycle_end; v != cycle_start; v = parent[v])
      cycle.push_back(v);
    cycle.push_back(cycle_start);
    reverse(cycle.begin(), cycle.end());
    cout << "Cycle found: ";
    for (int v : cycle)
      cout << v << " ";
    cout << endl;
  }
}
```

7.9 DFSMatching

Description: Simple bipartite matching algorithm. Graph g should be a list of neighbors of the left partition, and $btoa$ should be a vector full of -1's of the same size as the right partition. Returns the size of the matching. $btoa[i]$ will be the match for vertex i on the right side, or -1 if it's not matched.
Usage: `vi btoa(m, -1); dfsMatching(g, btoa);`
Time: $\mathcal{O}(VE)$

63 lines

```
bool find(int j, vector<vector<int>>& g, vector<int>& btoa, vector<int>&
    ↪ vis) {
 if (btoa[j] == -1) return 1;
 vis[j] = 1; int di = btoa[j];
 for (int e : g[di])
  if (!vis[e] && find(e, g, btoa, vis)) {
   btoa[e] = di;
   return 1;
  }
 return 0;
}
int dfsMatching(vector<vector<int>>& g, vector<int>& btoa) {
 vector<int> vis;
 for(int i = 0;i<g.size();i++) {
  vis.assign(btoa.size(), 0);
  for (int j : g[i])
   if (find(j, g, btoa, vis)) {
```

```
            btoa[j] = i;
            break;
        }
    }
    return btoa.size() - (int)count(btoa.begin(),btoa.end(), -1);
}
const int maxn = 100007;
vector<int> graph[maxn];
// If you graph is not dividen in proper way (not divided in two sets L
    ↪ and R) call this function
vector<int> ConverLR(int n) {
    vector<bool> vis(n);
    vector<int> color(n);
    auto bfsColor = [&](int s) {
        vector<int> q;
        q.push_back(s);
        while(q.size()) {
            int u = q.back();
            vis[u] = true;
            q.pop_back();
            for(auto v:graph[u]) {
                if(!vis[v]) {
                    q.push_back(v);
                    color[v] = color[u]^1;
                }
            }
        }
    };
    for(int i = 0;i<n;i++)
        if(!vis[i])
            bfsColor(i);
    map<int,int> mpR;
    int m= 0,key = 0;
    vector<vector<int>> g;
    for(int i = 0;i<n;i++) {
        if(color[i]) {
            g.push_back(vector<int>());
            for(auto d:graph[i]) {
                if(!mpR.count(d))
                    mpR[d] = key++;
                g.back().push_back(mpR[d]);
            }
        }
        else m++;
    }
    vector<int> match(m, -1);
    return dfsMatching(g,match);
}
```

7.10 DSUTree.cpp

Description: A trick to get information about subtree , like how many nodes in my subtree has color c

37 lines

```
void dfs_size(int v, int p) {
 sz[v] = 1;
 for (auto u : adj[v]) {
  if (u != p) {
   dfs_size(u, v);
   sz[v] += sz[u];
  }
 }
}
vector<int> *vec[maxn];
int cnt[maxn];
void dfs(int v, int p, bool keep) {
    int mx = -1, bigChild = -1;
    for(auto u : g[v])
       if(u != p && sz[u] > mx)
           mx = sz[u], bigChild = u;
    for(auto u : g[v])
       if(u != p && u != bigChild)
           dfs(u, v, 0);
    if(bigChild != -1)
        dfs(bigChild, v, 1), vec[v] = vec[bigChild];
    else
        vec[v] = new vector<int> ();
    vec[v]->push_back(v);
    cnt[ col[v] ]++;
    for(auto u : g[v])
       if(u != p && u != bigChild)
           for(auto x : *vec[u]) {
               cnt[ col[x] ]++;
               vec[v] -> push_back(x);
           }
    //now cnt[c] is the number of vertices in subtree of vertex v that
        ↪ has color c.
    // note that in this step *vec[v] contains all of the subtree of
        ↪ vertex v.
    if(keep == 0)
        for(auto u : *vec[v])
            cnt[ col[u] ]--;
}
```

7.11 DSURollback.cpp

78 lines

```
#include <bits/stdc++.h>
using namespace std;
#define endl '\n'
#define int long long
#define __ ios_base::sync_with_stdio(false);cin.tie(NULL);
const int N = 3e5 + 7;
int p[N], sz[N], ans;
stack<int> st;
int n, k, u[N << 1], v[N << 1], o[N << 1];
char op[N << 1];
map<pair<int,int>, int> mp;
int find(int u) {
    while(p[u] != u) u = p[u]; // Notice: No path compression. Path
        ↪ Compression will make the algorithm O(n^2)
    return u;
}
void Union(int u, int v) {
    u = find(u); v = find(v);
    if(u == v) return;
    if(sz[u] > sz[v]) swap(u, v);
    p[u] = v;
    sz[v] += sz[u];
    ans--;
    st.push(u);
}
void rollbax(int t) {
    while(st.size() > t) {
        int u = st.top(); st.pop();
        sz[p[u]] -= sz[u];
        p[u] = u; ans++;
    }
}
void solve(int l, int r) {
    if(l == r) {
        if(op[l] == '?') cout<<ans<<endl;
        return;
    }
    int m = l + r >> 1, now = st.size();
    for(int i = m + 1; i <= r; i++)
        if(o[i] < l) Union(u[i], v[i]);
    solve(l, m);
    rollbax(now);
    for(int i = l; i <= m; i++)
        if(o[i] > r) Union(u[i], v[i]);
    solve(m + 1, r);
    rollbax(now);
}
signed main() {
    freopen("connect.in", "r", stdin);
    freopen("connect.out", "w", stdout);
    cin>>n>>k;
    for(int i = 1; i <= n; i++)
        p[i] = i, sz[i] = 1;
    if(!k) return 0;
    for(int i = 1; i <= k; i++) {
```

```
    cin>>op[i];
    if(op[i] == '?') continue;
    cin>>u[i]>>v[i];
    if(u[i] > v[i])
      swap(u[i], v[i]);
    pair<int,int> x(u[i], v[i]);
    if(mp.count(x)) {
      o[i] = mp[x];
      o[o[i]] = i;
      mp.erase(x);
    } else {
      mp[x] = i;
    }
  }
  int idx = k;
  for(auto it : mp) {
    o[it.second] = ++idx;
    o[idx] = it.second;
    op[idx] = '-';
    tie(u[idx], v[idx]) = it.first;
  }
  ans = n;
  solve(1, idx);
}
```

7.12 Dijkstra

Description: Calculates shortest paths from s in a graph
Time: $\mathcal{O}(V \log E)$

62 lines

```
const int INF = 1e9;
const int MAX = 1440007;
int D[MAX];
int P[MAX];
int N;
vector<pair<int,int>> graph[MAX];
void add_edge(int u,int v,int cost) {
  graph[u].push_back( { v,cost } );
  graph[v].push_back( { u,cost } );
}
vector<int> restore_path(int s, int t, vector<int> const& p) {
  vector<int> path;
  for (int v = t; v != s; v = p[v])
    path.push_back(v);
  path.push_back(s);
  reverse(path.begin(), path.end());
  return path;
}
void dijkstra(int n,int Source) {
  set<pair<int,int> > s;
  for(int i = 0; i < n; ++i)
    D[i] = INF;
  D[Source] = 0;
  s.insert(make_pair(D[0], Source));
  while (!s.empty()) {
    int v = s.begin()->second;
    s.erase(s.begin());
    for(auto c:e[v]) {
      int u = c.first;
      int w = c.second;
      if (D[v] + w < D[u]) {
        s.erase(make_pair(D[u], u));
        D[u] = D[v]+ w;
        p[u] = v;
        s.insert(make_pair(D[u], u));
      }
    }
  }
}
// A bit faster
int dijkstra(int ini, int fin, int n) {
  vector<int> dist(n,INF);
  dist[ini] = 0;
  priority_queue<pair<int, int>, vector<pair<int,int>>, greater<pair<int, int>> > pq;
  pq.push( { 0, ini } );
  while (pq.size() != 0) {
    int minVal = pq.top().first;
    int idx = pq.top().second;
    pq.pop();
    if(dist[idx] < minVal)continue;
    for(auto arista: graph[idx]) {
      int nuevaDist = dist[idx] + arista.second ;
      if(nuevaDist < dist[arista.first]) {
        dist[arista.first] = nuevaDist;
        pq.push( { nuevaDist, arista.first } );
      }
    }
    // uncomment this if you only need the distance to one node
    //if(idx == fin)return dist[fin]+inc;
  }
  return -1;
}
```

7.13 Dinic.h

Description: Flow algorithm with complexity $O(VE\log U)$ where $U = \max|\text{cap}|$. $O(\min(E^{1/2}, V^{2/3})E)$ if $U = 1$; $O(\sqrt{V}E)$ for bipartite matching.

56 lines

```
template<typename flow_type>
struct dinic {
  struct edge {
    size_t src, dst, rev;
    flow_type flow, cap;
  };
  int n;
  vector<vector<edge>> adj;
  dinic(int n) : n(n), adj(n), level(n), q(n), it(n) { }
  void add_edge(size_t src, size_t dst, flow_type cap, flow_type rcap = 0) {
    adj[src].push_back( { src, dst, adj[dst].size(), 0, cap } );
    if (src == dst) adj[src].back().rev++;
    adj[dst].push_back( { dst, src, adj[src].size() - 1, 0, rcap } );
  }
  vector<int> level, q, it;
  bool bfs(int source, int sink) {
    fill(level.begin(), level.end(), -1);
    for (int qf = level[q[0] = sink] = 0, qb = 1; qf < qb; ++qf) {
      sink = q[qf];
      for (edge &e : adj[sink]) {
        edge &r = adj[e.dst][e.rev];
        if (r.flow < r.cap && level[e.dst] == -1)
          level[q[qb++] = e.dst] = 1 + level[sink];
      }
    }
    return level[source] != -1;
  }
  flow_type augment(int source, int sink, flow_type flow) {
    if (source == sink) return flow;
    for (; it[source] != adj[source].size(); ++it[source]) {
      edge &e = adj[source][it[source]];
      if (e.flow < e.cap && level[e.dst] + 1 == level[source]) {
        flow_type delta = augment(e.dst, sink,
              min(flow, e.cap - e.flow));
        if (delta > 0) {
          e.flow += delta;
          adj[e.dst][e.rev].flow -= delta;
          return delta;
        }
      }
    }
    return 0;
  }
  flow_type max_flow(int source, int sink) {
    for (int u = 0; u < n; ++u)
      for (edge &e : adj[u]) e.flow = 0;
    flow_type flow = 0;
    flow_type oo = numeric_limits<flow_type>::max();
    while (bfs(source, sink)) {
      fill(it.begin(), it.end(), 0);
      for (flow_type f; (f = augment(source, sink, oo)) > 0;)
        flow += f;
```

```
    }
   return flow;
  }
};
```

7.14 DirectedMST

Description: Finds a minimum spanning tree/arborescence of a directed graph, given a root node. If no MST exists, returns -1.
Time: $\mathcal{O}(E \log V)$

59 lines

```
#include "../data-structures/UnionFindRollback.h"
struct Edge { int a, b; ll w; };
struct Node { /// lazy skew heap node
 Edge key;
 Node *l, *r;
 ll delta;
 void prop() {
  key.w += delta;
  if (l) l->delta += delta;
  if (r) r->delta += delta;
  delta = 0;
 }
 Edge top() { prop(); return key; }
};
Node *merge(Node *a, Node *b) {
 if (!a || !b) return a ?: b;
 a->prop(), b->prop();
 if (a->key.w > b->key.w) swap(a, b);
 swap(a->l, (a->r = merge(b, a->r)));
 return a;
}
void pop(Node*& a) { a->prop(); a = merge(a->l, a->r); }
pair<ll, vi> dmst(int n, int r, vector<Edge>& g) {
 RollbackUF uf(n);
 vector<Node*> heap(n);
 for (Edge e : g) heap[e.b] = merge(heap[e.b], new Node{e});
 ll res = 0;
 vi seen(n, -1), path(n), par(n);
 seen[r] = r;
 vector<Edge> Q(n), in(n, {-1,-1}), comp;
 deque<tuple<int, int, vector<Edge>>> cycs;
 rep(s,0,n) {
  int u = s, qi = 0, w;
  while (seen[u] < 0) {
   if (!heap[u]) return {-1, {}};
   Edge e = heap[u]->top();
   heap[u]->delta -= e.w, pop(heap[u]);
   Q[qi] = e, path[qi++] = u, seen[u] = s;
   res += e.w, u = uf.find(e.a);
   if (seen[u] == s) { /// found cycle, contract
    Node* cyc = 0;
    int end = qi, time = uf.time();
    do cyc = merge(cyc, heap[w = path[--qi]]);
    while (uf.join(u, w));
    u = uf.find(u), heap[u] = cyc, seen[u] = -1;
    cycs.push_front({u, time, {&Q[qi], &Q[end]}});
   }
  }
  rep(i,0,qi) in[uf.find(Q[i].b)] = Q[i];
 }
 for (auto& [u,t,comp] : cycs) { // restore sol (optional)
  uf.rollback(t);
  Edge inEdge = in[u];
  for (auto& e : comp) in[uf.find(e.b)] = e;
  in[uf.find(inEdge.b)] = inEdge;
 }
 rep(i,0,n) par[i] = in[i].a;
 return {res, par};
}
```

7.15 DominatorTree.cpp

86 lines

```
#include<bits/stdc++.h>
using namespace std;
const int N = 2e5 + 9;
vector<int> g[N];
vector<int> t[N], rg[N], bucket[N]; //t = dominator tree of the nodes reachable from root
int sdom[N], par[N], idom[N], dsu[N], label[N];
int id[N], rev[N], T;
int find_(int u, int x = 0) {
 if(u == dsu[u]) return x ? -1 : u;
 int v = find_(dsu[u], x+1);
 if(v < 0)return u;
 if(sdom[label[dsu[u]]] < sdom[label[u]]) label[u] = label[dsu[u]];
 dsu[u] = v;
 return x ? v : label[u];
}
void dfs(int u) {
 T++; id[u] = T;
 rev[T] = u; label[T] = T;
 sdom[T] = T; dsu[T] = T;
 for(int i = 0; i < g[u].size(); i++) {
  int w = g[u][i];
  if(!id[w]) dfs(w), par[id[w]] = id[u];
  rg[id[w]].push_back(id[u]);
 }
}
void build(int r, int n) {
 dfs(r);
 n = T;
 for(int i = n; i >= 1; i--) {
  for(int j = 0; j < rg[i].size(); j++) sdom[i] = min(sdom[i], sdom[find_(rg[i][j])]);
  if(i > 1) bucket[sdom[i]].push_back(i);
  for(int j = 0; j < bucket[i].size(); j++) {
   int w = bucket[i][j];
   int v = find_(w);
   if(sdom[v] == sdom[w]) idom[w] = sdom[w];
   else idom[w] = v;
  }
  if(i > 1) dsu[i] = par[i];
 }
 for(int i = 2; i <= n; i++) {
  if(idom[i] != sdom[i]) idom[i]=idom[idom[i]];
  t[rev[i]].push_back(rev[idom[i]]);
  t[rev[idom[i]]].push_back(rev[i]);
 }
}
int st[N], en[N];
void yo(int u, int pre = 0) {
 st[u] = ++T;
 for(auto v: t[u]) {
  if(v == pre) continue;
  yo(v, u);
 }
 en[u] = T;
}
int main() {
 ios_base::sync_with_stdio(0);
 cin.tie(0);
 int n, m;
 while(cin >> n >> m) {
  vector<pair<int, int>> ed;
  for(int i = 0; i < m; i++) {
   int u, v;
   cin >> u >> v;
   g[u].push_back(v);
   ed.push_back({u, v});
  }
  build(1, n);
  T = 0;
  yo(1);
  vector<int> ans;
  for(int i = 0; i < m; i++) {
   int u = ed[i].first, v = ed[i].second;
   if(st[u] && !(st[v] <= st[u] && en[u] <= en[v])) ans.push_back(i);
  }
  yo(1);
  cout << ans.size() << '\n';
  for(auto x: ans) cout << x + 1 << ' ';
```

```
  cout << '\n';
  T = 0;
  for(int i = 0; i <= n; i++) {
   t[i].clear(), g[i].clear(), rg[i].clear(), bucket[i].clear();
   sdom[i] = par[i] = idom[i] = dsu[i] = label[i] = id[i] = rev[i] = st[i] = en[i] = 0;
  }
 }
 return 0;
}
```

7.16 EdgeColoring

Description: Given a simple, undirected graph with max degree D, computes a $(D+1)$-coloring of the edges such that no neighboring edges share a color. (D-coloring is NP-hard, but can be done for bipartite graphs by repeated matchings of max-degree nodes.)
Time: $\mathcal{O}(NM)$

31 lines

```
vector<int> edgeColoring(int N, vector<pair<int,int>> edges) {
 vector<int> cc(N + 1), ret(sz(edges)), fan(N), free(N), loc;
 for (pii e : edges) ++cc[e.first], ++cc[e.second];
 int u, v, ncols = *max_element(all(cc)) + 1;
 vector<vector<int>> adj(N, vector<int>(ncols, -1));
 for (pii e : edges) {
  tie(u, v) = e;
  fan[0] = v;
  loc.assign(ncols, 0);
  int at = u, end = u, d, c = free[u], ind = 0, i = 0;
  while (d = free[v], !loc[d] && (v = adj[u][d]) != -1)
   loc[d] = ++ind, cc[ind] = d, fan[ind] = v;
  cc[loc[d]] = c;
  for (int cd = d; at != -1; cd ^= c ^d, at = adj[at][cd])
   swap(adj[at][cd], adj[end = at][cd ^c ^d]);
  while (adj[fan[i]][d] != -1) {
   int left = fan[i], right = fan[++i], e = cc[i];
   adj[u][e] = left;
   adj[left][e] = u;
   adj[right][e] = -1;
   free[right] = e;
  }
  adj[u][d] = fan[i];
  adj[fan[i]][d] = u;
  for (int y : {fan[0], u, end})
   for (int& z = free[y] = 0; adj[y][z] != -1; z++);
 }
 for(int i = 0;i<edges.size();i++)
  for (tie(u, v) = edges[i]; adj[u][ret[i]] != v;) ++ret[i];
 return ret;
}
```

7.17 EdmondsKarp.h

Description: Flow algorithm with guaranteed complexity $O(VE^2)$. To get edge flow values, compare capacities before and after, and take the positive values only.
Usage:

```
graph.clear();graph.resize(n);capacity.resize(n,vector<int> (n,0));
capacity[u][v] = x;
graph[u].push_back(v);
```

71 lines

```
vector<vector<int>> capacity;
vector<vector<int>> graph;
const int INF = 1e9;
int bfs(int s, int t, vector<int>& parent) {
   fill(parent.begin(), parent.end(), -1);
   parent[s] = -2;
   queue<pair<int, int>> q;
   q.push({s, INF});
   while (!q.empty()) {
      int cur = q.front().first;
      int flow = q.front().second;
      q.pop();
      for (int next : graph[cur]) {
         if (parent[next] == -1 && capacity[cur][next]) {
            parent[next] = cur;
            int new_flow = min(flow, capacity[cur][next]);
            if (next == t)
               return new_flow;
            q.push({next, new_flow});
         }
      }
   }
   return 0;
}
int maxflow(int s, int t,int n) {
   int flow = 0;
   vector<int> parent(n);
   int new_flow;
   while (new_flow = bfs(s, t, parent)) {
      flow += new_flow;
      int cur = t;
      while (cur != s) {
         int prev = parent[cur];
         capacity[prev][cur] -= new_flow;
         capacity[cur][prev] += new_flow;
         cur = prev;
      }
   }
   return flow;
}
struct edge{
   int u,v,c;
};
void get_min_cut(int S,int n){
   vector<bool> cut(n);
   int idx = 0;
   cut[S] = 1;
   queue<int> q;
   q.push(S);
   while(!q.empty()){
      int u = q.front();
      q.pop();
      for(auto v:graph[u]){
         if(capacity[u][v])
            q.push(v),cut[v] = 1;
      }
   }
   for(auto c:cut)cout<<c<<" ";
   cout<<endl;
   vector<edge> edges;
   for(int i = 0;i<n;i++){
      if(cut[i]){
         for(auto v:graph[i]){
            if(!cut[v])
               edges.push_back({i,v,capacity[v][i]});
         }
      }
   }
   for(auto c:edges)
      cout<<c.u<<" "<<c.v<<" "<<c.c<<endl;
}
```

7.18 EulerWalk

Description: Eulerian undirected/directed path/cycle algorithm. Input should be a vector of (dest, global edge index), where for undirected graphs, forward/backward edges have the same index. Returns a list of nodes in the Eulerian path/cycle with src at both start and end, or empty list if no cycle/path exists. To get edge indices back, add .second to s and ret.
Time: $\mathcal{O}(V+E)$

15 lines

```
vi eulerWalk(vector<vector<pii>>& gr, int nedges, int src=0) {
 int n = sz(gr);
 vi D(n), its(n), eu(nedges), ret, s = {src};
 D[src]++; // to allow Euler paths, not just cycles
 while (!s.empty()) {
  int x = s.back(), y, e, &it = its[x], end = sz(gr[x]);
  if (it == end){ ret.push_back(x); s.pop_back(); continue; }
  tie(y, e) = gr[x][it++];
  if (!eu[e]) {
   D[x]--, D[y]++;
   eu[e] = 1; s.push_back(y);
  }}
 for (int x : D) if (x < 0 || sz(ret) != nedges+1) return {};
 return {ret.rbegin(), ret.rend()};
}
```

7.19 FloydWarshall

Description: Calculates all-pairs shortest path in a directed graph that might have negative edge weights. Input is an distance matrix m, where $m[i][j] =$ `inf` if i and j are not adjacent. As output, $m[i][j]$ is set to the shortest distance between i and j, `inf` if no path, or `-inf` if the path goes through a negative-weight cycle.
Time: $\mathcal{O}(N^3)$

17 lines

```
const int inf = 1LL << 62;
void floydWarshall(vector<vector<int>>& m) {
 int n = sz(m);
 for(int i = 0;i<n;i++) m[i][i] = min(m[i][i], 0LL);
 for(int k = 0;k<n;k++)
  for(int i = 0;i<n;i++)
   for(int j = 0;j<n;j++)
    if (m[i][k] != inf && m[k][j] != inf) {
     auto newDist = max(m[i][k] + m[k][j], -inf);
     m[i][j] = min(m[i][j], newDist);
    }
 for(int k = 0;k<n;k++)
  if(m[k][k] < 0)
   for(int i = 0;i<n;i++)
    for(int j =0;j<n;j++)
     if (m[i][k] != inf && m[k][j] != inf) m[i][j] = -inf;
}
```

7.20 GeneralMatching

Description: Matching for general graphs.
Time: $\mathcal{O}(NM)$

57 lines

```
const int maxn = 507;
vector<int> graph[maxn];
vector<int> Blossom(int n) {
  int timer = -1;
  vector<int> mate(n, -1), label(n), parent(n), orig(n), aux(n, -1), q;
  auto lca = [&](int x, int y) {
    for (timer++; ; swap(x, y)) {
      if (x == -1) continue;
      if (aux[x] == timer) return x;
      aux[x] = timer;
      x = (mate[x] == -1 ? -1 : orig[parent[mate[x]]]);
    }
  };
  auto blossom = [&](int v, int w, int a) {
    while (orig[v] != a) {
      parent[v] = w; w = mate[v];
      if (label[w] == 1) label[w] = 0, q.push_back(w);
      orig[v] = orig[w] = a; v = parent[w];
    }
  };
  auto augment = [&](int v) {
    while (v != -1) {
      int pv = parent[v], nv = mate[pv];
      mate[v] = pv;
      mate[pv] = v;
      v = nv;
    }
  };
  auto bfs = [&](int root) {
    fill(label.begin(), label.end(), -1);
    iota(orig.begin(), orig.end(), 0);
    q.clear();
    label[root] = 0; q.push_back(root);
    for (int i = 0; i < (int)q.size(); ++i) {
      int v = q[i];
      for (auto x : graph[v]) {
        if (label[x] == -1) {
          label[x] = 1;
          parent[x] = v;
          if (mate[x] == -1)
            return augment(x), 1;
          label[mate[x]] = 0; q.push_back(mate[x]);
        }
        else if (label[x] == 0 && orig[v] != orig[x]) {
          int a = lca(orig[v], orig[x]);
          blossom(x, v, a);
          blossom(v, x, a);
        }
      }
    }
    return 0;
  };
  for (int i = 0; i < n; i++)
    if (mate[i] == -1)
      bfs(i);
  return mate;
}
```

7.21 GlobalMinCut

Description: Find a global minimum cut in an undirected graph, as represented by an adjacency matrix.
Time: $\mathcal{O}(V^3)$

21 lines

```
pair<int, vector<int>> globalMinCut(vector<vector<int>> mat) {
 pair<int, vector<int>> best = {INT_MAX, {}};
 int n = sz(mat);
 vector<vector<int>> co(n);
 for(int i = 0;i<n;i++) co[i] = {i};
 for(int ph = 1;ph<n;ph++) {
  vi w = mat[0];
  size_t s = 0, t = 0;
  for(int it = 0;it<n-ph;it++) { // O(V^2) -> O(E log V) with prio.
      ↪ queue
   w[t] = INT_MIN;
   s = t, t = max_element(all(w)) - w.begin();
   for(int i = 0;i<n;i++) w[i] += mat[t][i];
  }
  best = min(best, {w[t] - mat[t][t], co[t]});
  co[s].insert(co[s].end(), co[t].begin(),co[t].end());
  for(int i = 0;i<n;i++)mat[s][i] += mat[t][i];
  for(int i = 0;i<n;i++)mat[i][s] = mat[s][i];
  mat[0][t] = INT_MIN;
 }
 return best;
}
```

7.22 GomoryHu

Description: Given a list of edges representing an undirected flow graph, returns edges of the Gomory-Hu tree. The max flow between any pair of vertices is given by minimum edge weight along the Gomory-Hu tree path.
Time: $\mathcal{O}(V)$ Flow Computations

14 lines

```
// #include "PushRelabel.h"
typedef array<ll, 3> Edge;
vector<Edge> gomoryHu(int N, vector<Edge> ed) {
 vector<Edge> tree;
 vi par(N);
 rep(i,1,N) {
  PushRelabel D(N); // Dinic also works
  for (Edge t : ed) D.addEdge(t[0], t[1], t[2], t[2]);
  tree.push_back({i, par[i], D.calc(i, par[i])});
  rep(j,i+1,N)
   if (par[j] == par[i] && D.leftOfMinCut(j)) par[j] = i;
 }
 return tree;
}
```

7.23 HLD

Description: Decomposes a tree into vertex disjoint heavy paths and light edges such that the path from any leaf to the root contains at most log(n) light edges.
Time: $\mathcal{O}((\log N)^2)$ per query in a path

62 lines

```
// #include "SegmentTree.cpp"
const int maxn = 100007;
vector<pair<int,int>> graph[maxn];
void add_edge(int u,int v,int c){
  graph[u].push_back({v,c});
  graph[v].push_back({u,c});
}
vector<int> p(maxn),head(maxn),stpos(maxn),lvl(maxn),sz(maxn),val(maxn);
vector<int> heavy(maxn,-1);
int cn = 0;
```

```cpp
void dfs(int u ,int pr = -1,int lev = 0) {
   lvl[u] = lev;
   sz[u]= 1;
   int mx= 0;
   p[u] = pr;
   for(auto v:graph[u]) {
      if(v.x == pr)continue;
      val[v.x] = v.y;
      dfs(v.x,u,lev+1);
      if(sz[v.x]>mx) {
         mx = sz[v.x];
         heavy[u] = v.x;
       }
      sz[u]+=sz[v.x];
    }
 }
void HLD(int u,int ch,int n) {
   head[u] = ch;
   stpos[u] = cn++;
   for(int i=0, currpos = 0; i < n; ++i)
   if(p[i] == -1 || heavy[p[i]] != i)
    for(int j = i; j != -1; j = heavy[j])
     {
      head[j] = i;
      stpos[j] = currpos;
      currpos++;
         }
 }
int query(int a,int b,int n) {
   int res = 0;
   while(head[a]!= head[b]) {
      if(lvl[head[a]]< lvl[head[b]])
         swap(a,b);
      res += query(1,0,n-1,stpos[head[a]],stpos[a]);
      a = p[head[a]];
    }
   if(lvl[a]> lvl[b])
      swap(a,b);
   res+=query(1,0,n-1,stpos[a],stpos[b]);
   return res;
 }
int update(int a,int b,int val, int n) {
   while(head[a] != head[b]) {
      if(lvl[head[a]] < lvl[head[b]])
         swap(a,b);
      update(1,0,n-1,stpos[head[a]],stpos[a],val);
      a = p[head[a]];
    }
   if(lvl[a]> lvl[b])
      swap(a,b);
   update(1,0,n-1,stpos[a],stpos[b],val);
 }
```

7.24 Hungarian.cpp

47 lines

```cpp
/*
 Tested: TIMUS 1833
 Complexity: O(n^3)
*/
// max weight matching
template<typename T>
T hungarian(const vector<vector<T>> &a)
 {
  int n = a.size(), p, q;
  vector<T> fx(n, numeric_limits<T>::min()), fy(n, 0);
  vector<int> x(n, -1), y(n, -1);
  for (int i = 0; i < n; ++i)
   for (int j = 0; j < n; ++j)
    fx[i] = max(fx[i], a[i][j]);
  for (int i = 0; i < n;)
   {
   vector<int> t(n, -1), s(n + 1, i);
   for (p = q = 0; p <= q && x[i] < 0; ++p)
    for (int k = s[p], j = 0; j < n && x[i] < 0; ++j)
     if (fx[k] + fy[j] == a[k][j] && t[j] < 0)
      {
       s[++q] = y[j], t[j] = k;
       if (s[q] < 0)
        for (p = j; p >= 0; j = p)
         y[j] = k = t[j], p = x[k], x[k] = j;
      }
   if (x[i] < 0)
    {
     T d = numeric_limits<T>::max();
     for (int k = 0; k <= q; ++k)
      for (int j = 0; j < n; ++j)
       if (t[j] < 0)
        d = min(d, fx[s[k]] + fy[j] - a[s[k]][j]);
     for (int j = 0; j < n; ++j)
      fy[j] += (t[j] < 0 ? 0 : d);
     for (int k = 0; k <= q; ++k)
      fx[s[k]] -= d;
    }
   else
     ++i;
  }
  T ret = 0;
  for (int i = 0; i < n; ++i)
   ret += a[i][x[i]];
  return ret;
 }
```

7.25 LCA

Description: Data structure for computing lowest common ancestors in a tree (with 0 as root). C should be an adjacency list of the tree, either directed or undirected.
Time: $\mathcal{O}(N \log N + Q)$

47 lines

```cpp
const int maxn = 100007;
const int mxlog = 25;
vector<int> graph[maxn];
int parent[mxlog][maxn];
vector<int> deep(maxn);
int N;
void add_edge(int u,int v) {
   graph[u].push_back(v);
   graph[v].push_back(u);
 }
void dfs(int u,int p = -1,int d = 0) {
   deep[u] = d;
   parent[0][u] = p;
   for(auto v:graph[u]) {
      if(v== p)continue;
      dfs(v,u,d+1);
    }
 }
void build() {
   for(int i = 0;i<N;i++)for(int j = 0;j<mxlog;j++)parent[j][i] = -1;
   for(int i = 0;i<N;i++)deep[i]= -1;
   dfs(0);
   for(int i = 0;i<N;i++)
      if(deep[i]== -1)dfs(i);
   for(int i = 1;i<mxlog;i++) {
      for(int u = 0;u<N;u++) {
         if(parent[i-1 ][u]!= -1)
         parent[i][u] = parent[i-1][parent[i-1][u]];
       }
    }
 }
int lca(int u ,int v) {
   if(deep[u]>deep[v])swap(u,v);
   int diff = deep[v]-deep[u];
   for(int i = mxlog-1;i>=0;i--) {
      if(diff & (1<<i))
         v = parent[i][v];
    }
   if(u == v)return u;
   for(int i = mxlog-1;i>=0;i--) {
      if(parent[i][u]!= parent[i][v]) {
         u = parent[i][u];
         v = parent[i][v];
       }
    }
   return parent[0][u];
```

```
}
```

7.26 LinkCutTree

Description: Represents a forest of unrooted trees. You can add and remove edges (as long as the result is still a forest), and check whether two nodes are in the same tree.
Time: All operations take amortized $\mathcal{O}(\log N)$.

89 lines

```
struct SplayTree {
  struct Node {
    int ch[2] = {0, 0}, p = 0;
    long long self = 0, path = 0; // Path aggregates
    long long sub = 0, vir = 0; // Subtree aggregates
    bool flip = 0;              // Lazy tags
  };
  vector<Node> T;
  SplayTree(int n) : T(n + 1) {}
  void push(int x) {
    if (!x || !T[x].flip) return;
    int l = T[x].ch[0], r = T[x].ch[1];
    T[l].flip ^= 1, T[r].flip ^= 1;
    swap(T[x].ch[0], T[x].ch[1]);
    T[x].flip = 0;
  }
  void pull(int x) {
    int l = T[x].ch[0], r = T[x].ch[1]; push(l); push(r);
    T[x].path = T[l].path + T[x].self + T[r].path;
    T[x].sub = T[x].vir + T[l].sub + T[r].sub + T[x].self;
  }
  void set(int x, int d, int y) {
    T[x].ch[d] = y; T[y].p = x; pull(x);
  }
  void splay(int x) {
    auto dir = [&](int x) {
      int p = T[x].p; if (!p) return -1;
      return T[p].ch[0] == x ? 0 : T[p].ch[1] == x ? 1 : -1;
    };
    auto rotate = [&](int x) {
      int y = T[x].p, z = T[y].p, dx = dir(x), dy = dir(y);
      set(y, dx, T[x].ch[!dx]);
      set(x, !dx, y);
      if (~dy) set(z, dy, x);
      T[x].p = z;
    };
    for (push(x); ~dir(x); ) {
      int y = T[x].p, z = T[y].p;
      push(z); push(y); push(x);
      int dx = dir(x), dy = dir(y);
      if (~dy) rotate(dx != dy ? x : y);
      rotate(x);
    }
  }
};
struct LinkCut : SplayTree {
  LinkCut(int n) : SplayTree(n) {}
  int access(int x) {
    int u = x, v = 0;
    for (; u; v = u, u = T[u].p) {
      splay(u);
      int& ov = T[u].ch[1];
      T[u].vir += T[ov].sub;
      T[u].vir -= T[v].sub;
      ov = v; pull(u);
    }
    return splay(x), v;
  }
  void reroot(int x) {
    access(x); T[x].flip ^= 1; push(x);
  }
  void Link(int u, int v) {
    reroot(u); access(v);
    T[v].vir += T[u].sub;
    T[u].p = v; pull(v);
  }
  void Cut(int u, int v) {
    reroot(u); access(v);
    T[v].ch[0] = T[u].p = 0; pull(v);
  }
  // Rooted tree LCA. Returns 0 if u and v arent connected.
  int LCA(int u, int v) {
    if (u == v) return u;
    access(u); int ret = access(v);
    return T[u].p ? ret : 0;
  }
  // Query subtree of u where v is outside the subtree.
  long long Subtree(int u, int v) {
    reroot(v); access(u); return T[u].vir + T[u].self;
  }
  // Query path [u..v]
  long long Path(int u, int v) {
    reroot(u); access(v); return T[v].path;
  }
  // Update vertex u with value v
  void Update(int u, long long v) {
    access(u); T[u].self = v; pull(u);
  }
};
```

7.27 MaximalCliques

Description: A clique is a subset of vertices of an undirected graph such that every two distinct vertices in the clique are adjacent.Runs a callback for all maximal cliques in a graph (given as a symmetric bitset matrix; self-edges not allowed). Callback is given a bitset representing the maximal clique.
Time: $\mathcal{O}\left(3^{n/3}\right)$, much faster for sparse graphs

16 lines

```
/// Possible optimization: on the top-most
/// recursion level, ignore 'cands', and go through nodes in order of increasing
/// degree, where degrees go down as nodes are removed.
/// (mostly irrelevant given MaximumClique)
typedef bitset<128> B;
template<class F>
void cliques(vector<B>& eds, F f, B P = ~B(), B X={}, B R={}) {
  if (!P.any()) { if (!X.any()) f(R); return; }
  auto q = (P | X)._Find_first();
  auto cands = P & ~eds[q];
  rep(i,0,sz(eds)) if (cands[i]) {
    R[i] = 1;
    cliques(eds, f, P & eds[i], X & eds[i], R);
    R[i] = P[i] = 0; X[i] = 1;
  }
}
```

7.28 MaximumClique

Description: a cliqeu is a subsets of vertices, all adjacent to each other, also called complete subgraphs, Quickly finds a maximum clique of a graph (given as symmetric bitset matrix; self-edges not allowed). Can be used to find a maximum independent set by finding a clique of the complement graph.
Time: Runs in about 1s for n=155 and worst case random graphs (p=.90). Runs faster for sparse graphs.

49 lines

```
typedef vector<bitset<200>> vb;
struct Maxclique {
  double limit=0.025, pk=0;
  struct Vertex { int i, d=0; };
  typedef vector<Vertex> vv;
  vb e;
  vv V;
  vector<vi> C;
  vi qmax, q, S, old;
  void init(vv& r) {
    for (auto& v : r) v.d = 0;
    for (auto& v : r) for (auto j : r) v.d += e[v.i][j.i];
    sort(all(r), [](auto a, auto b) { return a.d > b.d; });
    int mxD = r[0].d;
    rep(i,0,sz(r)) r[i].d = min(i, mxD) + 1;
  }
  void expand(vv& R, int lev = 1) {
    S[lev] += S[lev - 1] - old[lev];
    old[lev] = S[lev - 1];
    while (sz(R)) {
```

```
      if (sz(q) + R.back().d <= sz(qmax)) return;
      q.push_back(R.back().i);
      vv T;
      for(auto v:R) if (e[R.back().i][v.i]) T.push_back({v.i});
      if (sz(T)) {
        if (S[lev]++ / ++pk < limit) init(T);
        int j = 0, mxk = 1, mnk = max(sz(qmax) - sz(q) + 1, 1);
        C[1].clear(), C[2].clear();
        for (auto v : T) {
          int k = 1;
          auto f = [&](int i) { return e[v.i][i]; };
          while (any_of(all(C[k]), f)) k++;
          if (k > mxk) mxk = k, C[mxk + 1].clear();
          if (k < mnk) T[j++].i = v.i;
          C[k].push_back(v.i);
        }
        if (j > 0) T[j - 1].d = 0;
        rep(k,mnk,mxk + 1) for (int i : C[k])
          T[j].i = i, T[j++].d = k;
        expand(T, lev + 1);
      } else if (sz(q) > sz(qmax)) qmax = q;
      q.pop_back(), R.pop_back();
    }
  }
  vi maxClique() { init(V), expand(V); return qmax; }
  Maxclique(vb conn) : e(conn), C(sz(e)+1), S(sz(C)), old(S) {
    rep(i,0,sz(e)) V.push_back({i});
  }
};
```

7.29 MaximumIndependentSet.h

Description: An independent vertex set of a graph G is a subset of the vertices such that no two vertices in the subset represent an edge of G To obtain a maximum independent set of a graph, find a max clique of the complement. If the graph is bipartite, see MinimumVertexCover.

7.30 MinCostMaxFlow

Description: Min-cost max-flow. cap[i][j] != cap[j][i] is allowed; double edges are not. If costs can be negative, call setpi before maxflow, but note that negative cost cycles are not supported. To obtain the actual flow, look at positive values only.
Time: Approximately $\mathcal{O}(E^2)$

69 lines

```
struct Edge {
  int from, to, capacity, cost;
};
vector<vector<int>> adj, cost, capacity;
const int INF = 1e9;
void shortest_paths(int n, int v0, vector<int>& d, vector<int>& p) {
  d.assign(n, INF);
  d[v0] = 0;
  vector<bool> inq(n, false);
  queue<int> q;
  q.push(v0);
  p.assign(n, -1);
  while (!q.empty()) {
    int u = q.front();
    q.pop();
    inq[u] = false;
    for (int v : adj[u]) {
      if (capacity[u][v] > 0 && d[v] > d[u] + cost[u][v]) {
        d[v] = d[u] + cost[u][v];
        p[v] = u;
        if (!inq[v]) {
          inq[v] = true;
          q.push(v);
        }
      }
    }
  }
}
int min_cost_flow(int N, vector<Edge> edges, int K, int s, int t) {
  N+=7;
  adj.assign(N, vector<int>());
  cost.assign(N, vector<int>(N, 0));
  capacity.assign(N, vector<int>(N, 0));
  for (Edge e : edges) {
    adj[e.from].push_back(e.to);
    adj[e.to].push_back(e.from);
    cost[e.from][e.to] = e.cost;
    cost[e.to][e.from] = -e.cost;
    capacity[e.from][e.to] = e.capacity;
  }
  int flow = 0;
  int cost = 0;
  vector<int> d, p;
  while (flow < K) {
    shortest_paths(N, s, d, p);
    if (d[t] == INF)
      break;
    // find max flow on that path
    int f = K - flow;
    int cur = t;
    while (cur != s) {
      f = min(f, capacity[p[cur]][cur]);
      cur = p[cur];
    }
    // apply flow
    flow += f;
    cost += f * d[t];
    cur = t;
    while (cur != s) {
      capacity[p[cur]][cur] -= f;
      capacity[cur][p[cur]] += f;
      cur = p[cur];
    }
  }
  if (flow < K)
    return -1;
  else
    return cost;
}
```

7.31 MinCut.h

Description: After running max-flow, the left side of a min-cut from s to t is given by all vertices reachable from s, only traversing edges with positive residual capacity. Check EdmonsKarp for an implementation.

7.32 MinimumVertexCover.h

Description: Finds a minimum vertex cover in a bipartite graph. The size is the same as the size of a maximum matching, and the complement is a maximum independent set. A vertex cover is a set of vertex such that every edge has an endpoint to one of the vertex in the set

24 lines

```
#include "hopcroftKarp.h"
vector<int> cover(vector<vector<int>>& g, int n, int m) {
  //From maximum matrching de minimun vertex cover is the nodes not
      ↪ matched for the left side
   // and the nodes visited in the right part if we run a dfs from no
      ↪ matched nodes in the left j
    Bipartite_Matching BM(n,m,g);
    int res = BM.bipartite_matching();
    vector<int> match = BM.match;
  vector<bool> lfound(n, true), seen(m);
  for (int it : match) if (it != -1) lfound[it] = false;
  vector<int> q, cover;
  for(int i = 0;i<n;i++) if (lfound[i]) q.push_back(i);
  while (!q.empty()) {
    int i = q.back(); q.pop_back();
    lfound[i] = 1;
    for (int e : g[i]) if (!seen[e] && match[e] != -1) {
      seen[e] = true;
      q.push_back(match[e]);
    }
  }
  for(int i = 0;i<n;i++) if (!lfound[i]) cover.push_back(i);
  for(int i = 0;i<m;i++) if (seen[i]) cover.push_back(n+i);
  assert(cover.size() == res);
  return cover;
}
```

7.33 PushRelabel

Description: Push-relabel using the highest label selection rule and the gap heuristic. Quite fast in practice. To obtain the actual flow, look at positive values only.

Time: $\mathcal{O}\left(V^2\sqrt{E}\right)$

45 lines

```
struct PushRelabel {
 struct Edge {
  int dest, back;
  ll f, c;
 };
 vector<vector<Edge>> g;
 vector<ll> ec;
 vector<Edge*> cur;
 vector<vi> hs; vi H;
 PushRelabel(int n) : g(n), ec(n), cur(n), hs(2*n), H(n) {}
 void addEdge(int s, int t, ll cap, ll rcap=0) {
  if (s == t) return;
  g[s].push_back({t, sz(g[t]), 0, cap});
  g[t].push_back({s, sz(g[s])-1, 0, rcap});
 }
 void addFlow(Edge& e, ll f) {
  Edge &back = g[e.dest][e.back];
  if (!ec[e.dest] && f) hs[H[e.dest]].push_back(e.dest);
  e.f += f; e.c -= f; ec[e.dest] += f;
  back.f -= f; back.c += f; ec[back.dest] -= f;
 }
 ll calc(int s, int t) {
  int v = sz(g); H[s] = v; ec[t] = 1;
  vi co(2*v); co[0] = v-1;
  rep(i,0,v) cur[i] = g[i].data();
  for (Edge& e : g[s]) addFlow(e, e.c);
  for (int hi = 0;;) {
   while (hs[hi].empty()) if (!hi--) return -ec[s];
   int u = hs[hi].back(); hs[hi].pop_back();
   while (ec[u] > 0) // discharge u
    if (cur[u] == g[u].data() + sz(g[u])) {
     H[u] = 1e9;
     for (Edge& e : g[u]) if (e.c && H[u] > H[e.dest]+1)
      H[u] = H[e.dest]+1, cur[u] = &e;
     if (++co[H[u]], !--co[hi] && hi < v)
      rep(i,0,v) if (hi < H[i] && H[i] < v)
       --co[H[i]], H[i] = v + 1;
     hi = H[u];
    } else if (cur[u]->c && H[u] == H[cur[u]->dest]+1)
     addFlow(*cur[u], min(ec[u], cur[u]->c));
    else ++cur[u];
  }
 }
 bool leftOfMinCut(int a) { return H[a] >= sz(g); }
};
```

7.34 SCCTarjan.cpp

53 lines

```
#include <bits/stdc++.h>
using namespace std;
#define __ ios_base::sync_with_stdio(false),cin.tie(NULL);
#define endl '\n'
const int maxn = 100007;
vector<int> graph[maxn], graph_rev[maxn];
vector<bool> used;
vector<int> order, component;
void dfs1(int v) {
    used[v] = true;
    for (auto u : graph[v])
        if (!used[u])
            dfs1(u);
    order.push_back(v);
}
void dfs2(int v) {
    used[v] = true;
    component.push_back(v);
    for (auto u : graph_rev[v])
        if (!used[u])
            dfs2(u);
}
signed main(){__
  int n,m,u,v;
  cin>>n>>m;
  vector<int> in(n);
  vector<int> out(n);
  for(int i = 0;i<m;i++){
    cin>>u>>v;
    u--,v--;
    graph[u].push_back(v);
    graph_rev[v].push_back(u);
   }
  used.assign(n, false);
  for (int i = 0; i < n; i++)
    if (!used[i])
      dfs1(i);
  used.assign(n, false);
  reverse(order.begin(), order.end());
  vector<int> roots(n, 0);
  vector<int> root_nodes;
  vector<vector<int>> graphC(n);
  for (auto v : order){
    if (!used[v]) {
        dfs2(v);
        int root = component.front();
        for (auto u : component) roots[u] = root;
        root_nodes.push_back(root);
        component.clear();
      }
    }
  }
```

7.35 TopoSort

Description: Topological sorting. Given is an oriented graph. Output is an ordering of vertices, such that there are edges only from left to right. If there are cycles, the returned list will have size smaller than n – nodes reachable from cycles will not be returned.

Time: $\mathcal{O}(|V| + |E|)$

71 lines

```
const int maxn = 100007;
vector<int> graph[maxn];
set<int> graphL[maxn];
vector<int> inDegree(maxn,0);
void add_edge(int u,int v){
 inDegree[v]++;
 graph[u].push_back(v);
}
int n;
vector<int> topoSort(){
 vector<int> ans;
 priority_queue<int,vector<int>,greater<int>> q; // priority queue if you need a small lexicografic order
 // queue<int> q;
 for(int i = 0;i<n;i++)
  if(inDegree[i] == 0)
   q.push(i);
 while(!q.empty()){
  int u = q.top();
  // int u = q.front(); For a normal queue
  q.pop();
  ans.push_back(u);
  for(auto v:graph[u]){
   inDegree[v]--;
   if(inDegree[v] == 0){
    q.push(v);
   }
  }
 }
 return ans;
}
// Function to get all topological sorts
int ALLTPS(stack<int>& s,int *recStack,vector<int>& res,int& c){
  int flag = 0;
 for(int i = 0;i<NODOS; i++){
  if(vis[i] == -1&& indegree[i] == 0){
   for(int u:grafo[i]){
     indegree[u]--;
    }
  vis[i] = 1;
```

```
    recStack[i] = 1;
    res.push_back(i);
    if(ALLTPS(s,recStack,res,c)==1)
      return 1;
    if(c ==1)
      return 2;
    vis[i] = 0;
    res.erase(res.end()-1);
      for(int u:grafo[i]) {
          indegree[u]++;
      }
    flag =1 ;
    }
  }
  if(flag == 0) {
    if(res.size() <NODOS)
      return 1;
    for (int i = 0; i < res.size(); i++)
    cout << res[i]+1 << " ";
    c++;
  }
    return 0;
}
int AlltopoSort(vector<int> graph[], int N) {
    stack<int> s;
  int recS[N];
  vector<int> ATP;
  int c = 0;
    if(ALLTPS(s,recS,ATP,c) == 2 )
      return 1;
    return 0;
}
```

7.36 WeightedMatching

Description: Given a weighted bipartite graph, matches every node on the left with a node on the right such that no nodes are in two matchings and the sum of the edge weights is minimal. Takes cost[N][M], where cost[i][j] = cost for L[i] to be matched with R[j] and returns (min cost, match), where L[i] is matched with R[match[i]]. Negate costs for max cost.
Time: $\mathcal{O}\left(N^2 M\right)$

31 lines

```
pair<int, vector<int>> hungarian(const vector<vector<int>> &a) {
  if (a.empty()) return {0, {}};
  int n = sz(a) + 1, m = sz(a[0]) + 1;
  vector<int> u(n), v(m), p(m), ans(n - 1);
  for(int i = 1;i<n;i++) {
    p[0] = i;
    int j0 = 0; // add "dummy" worker 0
    vector<int> dist(m, INT_MAX), pre(m, -1);
    vector<bool> done(m + 1);
    do { // dijkstra
      done[j0] = true;
      int i0 = p[j0], j1, delta = INT_MAX;
      for(int j = 1;j<m;j++) if (!done[j]) {
        auto cur = a[i0 - 1][j - 1] - u[i0] - v[j];
        if (cur < dist[j]) dist[j] = cur, pre[j] = j0;
        if (dist[j] < delta) delta = dist[j], j1 = j;
      }
      for(int j = 0;j<m;j++) {
        if (done[j]) u[p[j]] += delta, v[j] -= delta;
        else dist[j] -= delta;
      }
      j0 = j1;
    } while (p[j0]);
    while (j0) { // update alternating path
      int j1 = pre[j0];
      p[j0] = p[j1], j0 = j1;
    }
  }
  for(int j = 1;j<m;j++)if (p[j]) ans[p[j] - 1] = j - 1;
  return {-v[0], ans}; // min cost
}
```

7.37 articulationPoints.cpp

Description: - Let G = (V, E). If G-v is disconnected, v in V is said to be an articulation point. If G has no articulation points, it is said to be biconnected. - A biconnected component is a maximal biconnected subgraph. The algorithm finds all articulation points and biconnected components. Time Complexity: O(n + m) Tested: - http://www.spoj.com/problems/SUBMERGE/ - http://codeforces.com/problemset/problem/487/E

64 lines

```
struct graph
{
  int n;
  vector<vector<int>> adj;
  graph(int n) : n(n), adj(n) {}
  void add_edge(int u, int v)
  {
    adj[u].push_back(v);
    adj[v].push_back(u);
  }
  int add_node()
  {
    adj.push_back({});
    return n++;
  }
  vector<int>& operator[](int u) { return adj[u]; }
};
vector<vector<int>> biconnected_components(graph &adj)
{
  int n = adj.n;
  vector<int> num(n), low(n), art(n), stk;
  vector<vector<int>> comps;
  function<void(int, int, int&)> dfs = [&](int u, int p, int &t)
  {
    num[u] = low[u] = ++t;
    stk.push_back(u);
    for (int v : adj[u]) if (v != p)
    {
      if (!num[v])
      {
        dfs(v, u, t);
        low[u] = min(low[u], low[v]);
        if (low[v] >= num[u])
        {
          art[u] = (num[u] > 1 || num[v] > 2);
          comps.push_back({u});
          while (comps.back().back() != v)
            comps.back().push_back(stk.back()),
            stk.pop_back();
        }
      }
      else low[u] = min(low[u], num[v]);
    }
  };
  for (int u = 0, t; u < n; ++u)
    if (!num[u]) dfs(u, -1, t = 0);
  // build the block cut tree
  function<graph()> build_tree = [&]()
  {
    graph tree(0);
    vector<int> id(n);
    for (int u = 0; u < n; ++u)
      if (art[u]) id[u] = tree.add_node();
    for (auto &comp : comps)
    {
      int node = tree.add_node();
      for (int u : comp)
        if (!art[u]) id[u] = node;
        else tree.add_edge(node, id[u]);
    }
    return tree;
  };
  return comps;
}
```

7.38 blockCutTree.cpp

Description: Decompose the tree aroun articilation points

79 lines

```
const int N = 4e5 + 9;
int T, low[N], dis[N], art[N], sz;
vector<int> g[N], bcc[N], st;
```

```cpp
void dfs(int u, int pre = 0) {
  low[u] = dis[u] = ++T;
  st.push_back(u);
  for(auto v: g[u]) {
    if(!dis[v]) {
      dfs(v, u);
      low[u] = min(low[u], low[v]);
      if(low[v] >= dis[u]) {
        sz ++;
        int x;
        do {
          x = st.back();
          st.pop_back();
          bcc[x].push_back(sz);
        } while(x ^v);
        bcc[u].push_back(sz);
      }
    } else if(v != pre) low[u] = min(low[u], dis[v]);
  }
}
int dep[N], par[N][20], cnt[N], id[N];
vector<int> bt[N];
void dfs1(int u, int pre = 0) {
  dep[u] = dep[pre] + 1;
  cnt[u] = cnt[pre] + art[u];
  par[u][0] = pre;
  for(int k = 1; k <= 18; k++) par[u][k] = par[par[u][k - 1]][k - 1];
  for(auto v: bt[u]) if(v != pre) dfs1(v, u);
}
int lca(int u, int v) {
  if(dep[u] < dep[v]) swap(u, v);
  for(int k = 18; k >= 0; k--) if(dep[par[u][k]] >= dep[v]) u = par[u][k
    ];
  if(u == v) return u;
  for(int k = 18; k >= 0; k--) if(par[u][k] != par[v][k]) u = par[u][k],
    v = par[v][k];
  return par[u][0];
}
int dist(int u, int v) {
  int lc = lca(u, v);
  return cnt[u] + cnt[v] - 2 * cnt[lc] + art[lc];
}
int32_t main() {
  ios_base::sync_with_stdio(0);
  cin.tie(0);
  int n, m;
  cin >> n >> m;
  while(m--) {
    int u, v;
    cin >> u >> v;
    g[u].push_back(v);
    g[v].push_back(u);
  }
  dfs(1);
  for(int u = 1; u <= n; u++) {
    if(bcc[u].size() > 1) { //AP
      id[u] = ++sz;
      art[id[u]] = 1; //if id in BCT is an AP on real graph or not
      for(auto v: bcc[u]) {
        bt[id[u]].push_back(v);
        bt[v].push_back(id[u]);
      }
    } else if(bcc[u].size() == 1) id[u] = bcc[u][0];
  }
  dfs1(1);
  int q;
  cin >> q;
  while(q--) {
    int u, v;
    cin >> u >> v;
    int ans;
    if(u == v) ans = 0;
    else ans = dist(id[u], id[v]) - art[id[u]] - art[id[v]];
    cout << ans << '\n';; //number of articulation points in the path from
      u to v except u and v
    //u and v are in the same bcc if ans == 0
  }
  return 0;
}
```

7.39 bridgeTree.cpp

Description: Get biconnected components and compress and compress in a node each component, form a graph with the new compressed nodes 110 lines

```cpp
const lli maxn = 200007;
set<lli> graph[maxn];
set<lli> graph2[maxn];
vector<lli> low(maxn),d(maxn),label(maxn),bridge(maxn),vis(maxn),parent(
  maxn),sz(maxn);
lli idx;
void dfs(lli u,lli p = -1){
    d[u] =idx++;
    low[u] = d[u];
    vis[u] = true;
    sz[u]= 1;
    parent[u] = p;
    for(auto v:graph[u]){
        if(v == p)continue;
        if(!vis[v]){
            dfs(v,u);
            sz[u]+=sz[v];
            if(low[v]>d[u]) bridge[v] = true;
        }
        low[u] = min(low[u],low[v]);
    }
}
void dfs_label(lli u){
    vis[u] = 1;
    label[u] = idx;
    for(auto v : graph[u])
        if(!vis[v])
            dfs_label(v);
}
int main(){ __
    int t= 1,n,u,v;
    cin>>t;
    while(t--){
        cin>>n;
        for(lli i = 0;i<n;i++)graph[i].clear(),graph2[i].clear(),bridge[i]
          = false,vis[i] = false;
        for(lli i = 0;i<n;i++){
            cin>>u>>v;
            u--,v--;
            graph[u].insert(v);
            graph[v].insert(u);
            graph2[u].insert(v);
            graph2[v].insert(u);
        }
        dfs(0);
        lli root;
        set<lli> cycle;
        vector<pair<lli,lli>> bridges;
        for(lli i = 0;i<n;i++){
            vis[i] = false;
            if(bridge[i]){
                graph[i].erase(parent[i]);
                graph[parent[i]].erase(i);
                if(i)bridges.push_back( { i,parent[i] } );
            }
            else{
                if(i){
                    cycle.insert(parent[i]);
                    cycle.insert(i);
                    root = i;
                }
            }
        }
        idx = 0;
        for(lli i = 0;i<n;i++){
            if(!vis[i]){
                dfs_label(i);
                idx++;
```

```
            }
        }
        lli y = cycle.size();
        lli x = n-cycle.size();
        lli ans = (y*x*2)-x;
        ans += (y*(y-1));
        for(lli i = 0;i<n;i++)
            graph[i].clear(),vis[i] = false;
        for(auto v:bridges) {
            lli a = label[v.first],b = label[v.second];
            graph[a].insert(b);
            graph[b].insert(a);
        }
        dfs(label[root]);
        lli total = 0;
        for(auto c:graph[label[root]]) {
            if(sz[c]>1) {
                ans+= (sz[c]* (sz[c]-1))/2;
            }
            total+=sz[c];
        }
        lli m = 0;
        for(auto c:graph[label[root]]) {
            m+= sz[c]*(total-sz[c])*2;
        }
        m/=2;
        ans+=m;
        for(auto c:cycle) {
            lli cont = 0,childs= 0,cont2 = 0;
            for(auto d:graph2[c]) {
                if(label[d]!= label[root]) {
                    cont+=sz[label[d]];
                    if(sz[label[d]]>1)
                        cont2+=(sz[label[d]]* (sz[label[d]]-1))/2;
                    childs++;
                }
            }
            if(childs>1)
                ans-=((cont*(cont-1))/2)-cont2;
        }
        cout<<ans<<endl;
    }
    return 0;
}
```

7.40 dominatorTree.cpp

86 lines

```
/*
 Dominator Tree (Lengauer-Tarjan)
 Tested: SPOJ EN
 Complexity: O(m log n)
*/
struct graph
{
 int n;
 vector<vector<int>> adj, radj;
 graph(int n) : n(n), adj(n), radj(n) { }
 void add_edge(int src, int dst)
 {
  adj[src].push_back(dst);
  radj[dst].push_back(src);
 }
 vector<int> rank, semi, low, anc;
 int eval(int v)
 {
  if (anc[v] < n && anc[anc[v]] < n)
  {
   int x = eval(anc[v]);
   if (rank[semi[low[v]]] > rank[semi[x]])
    low[v] = x;
   anc[v] = anc[anc[v]];
  }
  return low[v];
 }
 vector<int> prev, ord;
 void dfs(int u)
 {
  rank[u] = ord.size();
  ord.push_back(u);
  for (auto v : adj[u])
  {
   if (rank[v] < n)
    continue;
   dfs(v);
   prev[v] = u;
  }
 }
 vector<int> idom; // idom[u] is an immediate dominator of u
 void dominator_tree(int r)
 {
  idom.assign(n, n);
  prev = rank = anc = idom;
  semi.resize(n);
  iota(semi.begin(), semi.end(), 0);
  low = semi;
  ord.clear();
  dfs(r);
  vector<vector<int>> dom(n);
  for (int i = (int) ord.size() - 1; i >= 1; --i)
  {
   int w = ord[i];
   for (auto v : radj[w])
   {
    int u = eval(v);
    if (rank[semi[w]] > rank[semi[u]])
     semi[w] = semi[u];
   }
   dom[semi[w]].push_back(w);
   anc[w] = prev[w];
   for (int v : dom[prev[w]])
   {
    int u = eval(v);
    idom[v] = (rank[prev[w]] > rank[semi[u]]
     ? u : prev[w]);
   }
   dom[prev[w]].clear();
  }
  for (int i = 1; i < (int) ord.size(); ++i)
  {
   int w = ord[i];
   if (idom[w] != semi[w])
    idom[w] = idom[idom[w]];
  }
 }
 vector<int> dominators(int u)
 {
  vector<int> S;
  for (; u < n; u = idom[u])
   S.push_back(u);
  return S;
 }
};
```

7.41 flowWithLowerBound

Description: Solves max flow problem with lower bound for capacities
Time: Approximately $\mathcal{O}(v^2 E)$

117 lines

```
template<typename T>
struct dinic
{
 struct edge
 {
  int src, dst;
  T low, cap, flow;
  int rev;
 };
 int n;
 vector<vector<edge>> adj;
 dinic(int n) : n(n), adj(n + 2) { }
 void add_edge(int src, int dst, T low, T cap)
 {
```

```
adj[src].push_back( { src, dst, low, cap, 0, (int) adj[dst].size() }
    ↪ );
if (src == dst)
  adj[src].back().rev++;
adj[dst].push_back( { dst, src, 0, 0, 0, (int) adj[src].size() - 1 }
    ↪ );
}
vector<int> level, iter;
T augment(int u, int t, T cur)
{
 if (u == t)
  return cur;
 for (int &i = iter[u]; i < (int) adj[u].size(); ++i)
 {
  edge &e = adj[u][i];
  if (e.cap - e.flow > 0 && level[u] > level[e.dst])
  {
   T f = augment(e.dst, t, min(cur, e.cap - e.flow));
   if (f > 0)
   {
    e.flow += f;
    adj[e.dst][e.rev].flow -= f;
    return f;
   }
  }
 }
 return 0;
}
int bfs(int s, int t)
{
 level.assign(n + 2, n + 2);
 level[t] = 0;
 queue<int> Q;
 for (Q.push(t); !Q.empty(); Q.pop())
 {
  int u = Q.front();
  if (u == s)
   break;
  for (edge &e : adj[u])
  {
   edge &erev = adj[e.dst][e.rev];
   if (erev.cap - erev.flow > 0
    && level[e.dst] > level[u] + 1)
   {
    Q.push(e.dst);
    level[e.dst] = level[u] + 1;
   }
  }
 }
 return level[s];
}
const T oo = numeric_limits<T>::max();
T max_flow(int source, int sink)
{
 vector<T> delta(n + 2);
 for (int u = 0; u < n; ++u) // initialize
  for (auto &e : adj[u])
  {
   delta[e.src] -= e.low;
   delta[e.dst] += e.low;
   e.cap -= e.low;
   e.flow = 0;
  }
 T sum = 0;
 int s = n, t = n + 1;
 for (int u = 0; u < n; ++u)
 {
  if (delta[u] > 0)
  {
   add_edge(s, u, 0, delta[u]);
   sum += delta[u];
  }
  else if (delta[u] < 0)
   add_edge(u, t, 0, -delta[u]);
 }
 add_edge(sink, source, 0, oo);
 T flow = 0;
 while (bfs(s, t) < n + 2)
 {
  iter.assign(n + 2, 0);
  for (T f; (f = augment(s, t, oo)) > 0;)
   flow += f;
 }
 if (flow != sum)
  return -1; // no solution
 for (int u = 0; u < n; ++u)
  for (auto &e : adj[u])
  {
   e.cap += e.low;
   e.flow += e.low;
   edge &erev = adj[e.dst][e.rev];
   erev.cap -= e.low;
   erev.flow -= e.low;
  }
 adj[sink].pop_back();
 adj[source].pop_back();
 while (bfs(source, sink) < n + 2)
 {
  iter.assign(n + 2, 0);
  for (T f; (f = augment(source, sink, oo)) > 0;)
   flow += f;
 } // level[u] == n + 2 ==> s-side
  return flow;
 }
};
```

7.42 gabowEdmonds.cpp

91 lines

```
/*
 Tested: Timus 1099
 Complexity: O(n^3)
*/
struct graph
{
 int n;
 vector<vector<int>> adj;
 graph(int n) : n(n), adj(n) { }
 void add_edge(int u, int v)
 {
  adj[u].push_back(v);
  adj[v].push_back(u);
 }
 queue<int> q;
 vector<int> label, mate, cycle;
 void rematch(int x, int y)
 {
  int m = mate[x];
  mate[x] = y;
  if (mate[m] == x)
  {
   if (label[x] < n)
    rematch(mate[m] = label[x], m);
   else
   {
    int s = (label[x] - n) / n, t = (label[x] - n) % n;
    rematch(s, t);
    rematch(t, s);
   }
  }
 }
 void traverse(int x)
 {
  vector<int> save = mate;
  rematch(x, x);
  for (int u = 0; u < n; ++u)
   if (mate[u] != save[u])
    cycle[u] ^= 1;
  save.swap(mate);
 }
 void relabel(int x, int y)
 {
  cycle = vector<int>(n, 0);
```

```
  traverse(x);
  traverse(y);
  for (int u = 0; u < n; ++u)
   {
    if (!cycle[u] || label[u] ≥ 0)
      continue;
    label[u] = n + x + y * n;
    q.push(u);
   }
  }
 int augment(int r)
  {
   label.assign(n, -2);
   label[r] = -1;
   q = queue<int>();
   for (q.push(r); !q.empty(); q.pop())
    {
     int x = q.front();
     for (int y : adj[x])
      {
       if (mate[y] < 0 && r ≠ y)
        {
         rematch(mate[y] = x, y);
         return 1;
        }
       else if (label[y] ≥ -1)
         relabel(x, y);
       else if (label[mate[y]] < -1)
        {
         label[mate[y]] = x;
         q.push(mate[y]);
        }
      }
    }
   return 0;
  }
 int maximum_matching()
  {
   mate.assign(n, -2);
   int matching = 0;
   for (int u = 0; u < n; ++u)
    if (mate[u] < 0)
      matching += augment(u);
   return matching;
  }
};
```

7.43 gomoryHuTree.cpp

26 lines

```
/*
 Gomory-Hu tree
 Tested: SPOj MCQUERY
 Complexity: O(n-1) max-flow call
*/
template<typename flow_type>
struct edge
 {
  int src, dst;
  flow_type cap;
 };
template<typename flow_type>
vector<edge<flow_type>> gomory_hu(dinic<flow_type> &adj)
 {
  int n = adj.n;
  vector<edge<flow_type>> tree;
  vector<int> parent(n);
  for (int u = 1; u < n; ++u)
   {
    tree.push_back( { u, parent[u], adj.max_flow(u, parent[u]) } );
    for (int v = u + 1; v < n; ++v)
      if (adj.level[v] == -1 && parent[v] == parent[u])
        parent[v] = u;
   }
  return tree;
 }
```

7.44 hopcroftKarp

Description: Fast bipartite matching algorithm. Graph g should be a list of neighbors of the left partition, and $btoa$ should be a vector full of -1's of the same size as the right partition. Returns the size of the matching. $btoa[i]$ will be the match for vertex i on the right side, or -1 if it's not matched.
Usage: `vi btoa(m, -1); hopcroftKarp(g, btoa);`
Time: $\mathcal{O}\left(\sqrt{V}E\right)$

91 lines

```
struct Bipartite_Matching {
    vector<vector<int>> es;
    vector<int> d, match;
    vector<bool> used, used2;
    const int n, m;
    Bipartite_Matching(int n, int m) : es(n), d(n), match(m), used(n),
        ↪ used2(n), n(n), m(m)  { }
    void add_edge(int u, int v) {
        es[u].push_back(v);
     }
    void bfs() {
        fill(begin(d), end(d), -1);
        queue<int> que;
        for(int i = 0; i < n; i++) {
            if(!used[i]) que.emplace(i), d[i] = 0;
         }
        while(!que.empty()) {
            int i = que.front(); que.pop();
            for(auto &e: es[i]) {
                int j = match[e];
                if(j ≠ -1 && d[j] == -1) {
                    que.emplace(j), d[j] = d[i]+1;
                 }
             }
         }
     }
    bool dfs(int now) {
        used2[now] = true;
        for(auto &e: es[now]) {
            int u = match[e];
            if(u == -1 || (!used2[u] && d[u] == d[now]+1 && dfs(u))) {
                match[e] = now, used[now] = true;
                return true;
             }
         }
        return false;
     }
    int bipartite_matching() {
        fill(begin(match), end(match), -1), fill(begin(used), end(used),
            ↪ false);
        int ret = 0;
        while(true) {
            bfs();
            fill(begin(used2), end(used2), false);
            int flow = 0;
            for(int i = 0; i < n; i++) {
                if(!used[i] && dfs(i)) flow++;
             }
            if(flow == 0) break;
            ret += flow;
         }
        return ret;
     }
 };
// If you graph is not dividen in proper way (not divided in two sets L
    ↪ and R) call this function
int ConverLR(int n) {
    vector<bool> vis(n);
    vector<int> color(n);
    auto bfsColor = [&](int s) {
        vector<int> q;
        q.push_back(s);
        while(q.size()) {
            int u = q.back();
            vis[u] = true;
            q.pop_back();
            for(auto v:graph[u]) {
                if(!vis[v]) {
```

```
                q.push_back(v);
                color[v] = color[u]^1;
            }
        }
    }
  };
  for(int i = 0;i<n;i++)
    if(!vis[i])
      bfsColor(i);
  map<int,int> mpR;
  int m= 0,key = 0;
  vector<vector<int>> g;
  for(int i = 0;i<n;i++){
    if(color[i]){
      g.push_back(vector<int>());
      for(auto d:graph[i]){
        if(!mpR.count(d))
          mpR[d] = key++;
        g.back().push_back(mpR[d]);
      }
    }
    else m++;
  }
  Bipartite_matching BM(n-m,m,g);
  return BM.bipartite_matching();
}
```

7.45 kShortestPaths

Description: Calculates shortest paths from s in a graph
Usage: adj.resize(n + 1);rev.resize(n + 1); adj[u].pb(new Edge(v, w));
rev[v].pb(new Edge(u, w));
adj[u].back()->rev = rev[v].back();
rev[v].back()->rev = adj[u].back();
vector<int> res = k_shortest_paths(1, n, k); 1 indexed
Time: ??

74 lines

```
const int inf = 1e18;
struct Edge {
 int to, w;
 Edge *rev;
 Edge (int to, int w) : to(to), w(w) { }
};
pair<vector<int>, vector<Edge*>> dijkstra (vector<vector<Edge*>> gra,
    ↪ int s) {
 vector<int> dis(gra.size(), inf);
 vector<Edge*> par(gra.size(), nullptr);
 priority_queue<pair<int,int>, vector<pair<int,int>>, greater<pair<int,
    ↪ int>>> pq;
 pq.emplace(0, s);
 dis[s] = 0;
 while (pq.size()) {
  auto [d, u] = pq.top();
  pq.pop();
  if (dis[u] < d) continue;
  for (auto *e : gra[u]) {
   ll w = d + e->w;
   if (w < dis[e->to]) {
    par[e->to] = e->rev;
    pq.emplace(dis[e->to] = w, e->to);
   }
  }
 }
 return {dis, par};
}
vector<vector<Edge*>> adj, rev;
vector<int> k_shortest_paths (int s, int t, int k) {
 auto [dis, par] = dijkstra(rev, t);
 vector<int> res;
 priority_queue<pair<int,int>, vector<pair<int,int>>, greater<pair<int,
    ↪ int>>> pq;
 pq.emplace(dis[s], s);
 while (k && pq.size()) {
  auto [d, u] = pq.top();
  pq.pop();
  res.push_back(d);
  k--;
  while (1) {
   for (Edge *e : adj[u]) {
    int v = e->to;
    if (e != par[u]) {
     ll w = d - dis[u] + e->w + dis[v];
     pq.emplace(w, v);
    }
   }
   if (!par[u])
    break;
   u = par[u]->to;
  }
 }
 while (k) {
  res.push_back(-1);
  k--;
 }
 return res;
}
void main_() {
 int n, m, k;
 cin >> n >> m >> k;
 adj.resize(n + 1);
 rev.resize(n + 1);
 for(int i = 0;i<m;i++){
  int u, v, w;
  cin >> u >> v >> w;
  adj[u].pb(new Edge(v, w));
  rev[v].pb(new Edge(u, w));
  adj[u].back()->rev = rev[v].back();
  rev[v].back()->rev = adj[u].back();
 }
 vector<int> res = k_shortest_paths(1, n, k);
 for (ll r : res)
  cout << r << " ";
 cout << endl;
}
```

7.46 kuhn-munkras.cpp

59 lines

```
template<typename T>
struct KuhnMunkras { // n for left, m for right
 int n, m, match[maxM];
 T g[maxN][maxM], lx[maxN], ly[maxM], slack[maxM];
 bool vx[maxN], vy[maxM];
 void init(int n_, int m_) {
  mset0(g); n = n_, m = m_;
 }
 void add(int u, int v, T w) {
  g[u][v] = w;
 }
 bool find(int x) {
  vx[x] = true;
  for (int y = 1; y ≤ m; ++y) {
   if (!vy[y]) {
    T delta = lx[x] + ly[y] - g[x][y];
    if (equalT(delta, T(0))) {
     vy[y] = true;
     if (match[y] == 0 || find(match[y])) {
      match[y] = x;
      return true;
     }
    } else slack[y] = min(slack[y], delta);
   }
  }
  return false;
 }
 T matching() { // maximum weight matching
  fill(lx + 1, lx + 1 + n, numeric_limits<T>::lowest());
  mset0(ly); mset0(match);
  for (int i = 1; i ≤ n; ++i) {
   for (int j = 1; j ≤ m; ++j) lx[i] = max(lx[i], g[i][j]);
  }
  for (int k = 1; k ≤ n; ++k) {
   fill(slack + 1, slack + 1 + m, numeric_limits<T>::max());
   while (true) {
    mset0(vx); mset0(vy);
```

```cpp
        if (find(k)) break;
        else {
          T delta = numeric_limits<T>::max();
          for (int i = 1; i <= m; ++i) {
            if (!vy[i]) delta = min(delta, slack[i]);
          }
          for (int i = 1; i <= n; ++i) {
            if (vx[i]) lx[i] -= delta;
          }
          for (int i = 1; i <= m; ++i) {
            if (vy[i]) ly[i] += delta;
            if (!vy[i]) slack[i] -= delta;
          }
        }
      }
    }
    T result = 0;
    for (int i = 1; i <= n; ++i) result += lx[i];
    for (int i = 1; i <= m; ++i) result += ly[i];
    return result;
  }
};
```

7.47 maxFlowDinic

Description: Solves max flow problem
Time: Approximately $\mathcal{O}\left(v^2 E\right)$ but fast in practice , also for bipartite matching complexity is :$\mathcal{O}\left(E\sqrt{V}\right)$

68 lines

```cpp
template<typename flow_type>
struct dinic
{
  struct edge
  {
    size_t src, dst, rev;
    flow_type flow, cap;
  };
  int n;
  vector<vector<edge>> adj;
  dinic(int n) : n(n), adj(n), level(n), q(n), it(n) { }
  void add_edge(size_t src, size_t dst, flow_type cap, flow_type rcap = 0)
  {
    adj[src].push_back( { src, dst, adj[dst].size(), 0, cap } );
    if (src == dst) adj[src].back().rev++;
    adj[dst].push_back( { dst, src, adj[src].size() - 1, 0, rcap } );
  }
  vector<int> level, q, it;
  bool bfs(int source, int sink)
  {
    fill(level.begin(), level.end(), -1);
    for (int qf = level[q[0] = sink] = 0, qb = 1; qf < qb; ++qf)
    {
      sink = q[qf];
      for (edge &e : adj[sink])
      {
        edge &r = adj[e.dst][e.rev];
        if (r.flow < r.cap && level[e.dst] == -1)
          level[q[qb++] = e.dst] = 1 + level[sink];
      }
    }
    return level[source] != -1;
  }
  flow_type augment(int source, int sink, flow_type flow)
  {
    if (source == sink) return flow;
    for (; it[source] != adj[source].size(); ++it[source])
    {
      edge &e = adj[source][it[source]];
      if (e.flow < e.cap && level[e.dst] + 1 == level[source])
      {
        flow_type delta = augment(e.dst, sink,
                min(flow, e.cap - e.flow));
        if (delta > 0)
        {
          e.flow += delta;
          adj[e.dst][e.rev].flow -= delta;
          return delta;
        }
      }
    }
    return 0;
  }
  flow_type max_flow(int source, int sink)
  {
    for (int u = 0; u < n; ++u)
      for (edge &e : adj[u]) e.flow = 0;
    flow_type flow = 0;
    flow_type oo = numeric_limits<flow_type>::max();
    while (bfs(source, sink))
    {
      fill(it.begin(), it.end(), 0);
      for (flow_type f; (f = augment(source, sink, oo)) > 0;)
        flow += f;
    } // level[u] = -1 => source side of min cut
    return flow;
  }
};
```

7.48 maxFlowPushRelabel.cpp

94 lines

```cpp
/*
  Maximum Flow (Goldberg-Tarjan)
  Complexity: O(n^3) faster than Dinic in most cases
  Tested: http://www.spoj.com/problems/FASTFLOW/
*/
template<typename flow_type>
struct goldberg_tarjan
{
  struct edge
  {
    size_t src, dst, rev;
    flow_type flow, cap;
  };
  int n;
  vector<vector<edge>> adj;
  goldberg_tarjan(int n) : n(n), adj(n) { }
  void add_edge(size_t src, size_t dst, flow_type cap, flow_type rcap = 0)
  {
    adj[src].push_back( { src, dst, adj[dst].size(), 0, cap } );
    if (src == dst) adj[src].back().rev++;
    adj[dst].push_back( { dst, src, adj[src].size() - 1, 0, rcap } );
  }
  flow_type max_flow(int source, int sink)
  {
    vector<flow_type> excess(n);
    vector<int> dist(n), active(n), count(2 * n);
    queue<int> q;
    auto enqueue = [&](int v)
    {
      if (!active[v] && excess[v] > 0)
      {
        active[v] = true;
        q.push(v);
      }
    };
    auto push = [&](edge &e)
    {
      flow_type f = min(excess[e.src], e.cap - e.flow);
      if (dist[e.src] <= dist[e.dst] || f == 0) return;
      e.flow += f;
      adj[e.dst][e.rev].flow -= f;
      excess[e.dst] += f;
      excess[e.src] -= f;
      enqueue(e.dst);
    };
    dist[source] = n;
    active[source] = active[sink] = true;
    count[0] = n - 1;
    count[n] = 1;
```

```
    for (int u = 0; u < n; ++u)
      for (edge &e : adj[u]) e.flow = 0;
    for (edge &e : adj[source])
    {
      excess[source] += e.cap;
      push(e);
    }
    for (int u; !q.empty(); q.pop())
    {
      active[u = q.front()] = false;
      for (auto &e : adj[u]) push(e);
      if (excess[u] > 0)
      {
        if (count[dist[u]] == 1)
        {
          int k = dist[u]; // Gap Heuristics
          for (int v = 0; v < n; v++)
          {
            if (dist[v] < k)
              continue;
            count[dist[v]]--;
            dist[v] = max(dist[v], n + 1);
            count[dist[v]]++;
            enqueue(v);
          }
        }
        else
        {
          count[dist[u]]--; // Relabel
          dist[u] = 2 * n;
          for (edge &e : adj[u])
            if (e.cap > e.flow)
              dist[u] = min(dist[u], dist[e.dst] + 1);
          count[dist[u]]++;
          enqueue(u);
        }
      }
    }
    flow_type flow = 0;
    for (edge e : adj[source])
      flow += e.flow;
    return flow;
  }
};
```

7.49 minCostMaxFlow.cpp

87 lines

```
/*
  Minimum Cost Flow (Tomizawa, Edmonds-Karp)
  Complexity: O(F m log n), where F is the amount of maximum flow
  Tested: Codeforces [http://codeforces.com/problemset/problem/717/G]
*/
template<typename flow_type, typename cost_type>
struct min_cost_max_flow
{
  struct edge
  {
    size_t src, dst, rev;
    flow_type flow, cap;
    cost_type cost;
  };
  int n;
  vector<vector<edge>> adj;
  min_cost_max_flow(int n) : n(n), adj(n), potential(n), dist(n), back(n)
      { }
  void add_edge(size_t src, size_t dst, flow_type cap, cost_type cost)
  {
    adj[src].push_back( { src, dst, adj[dst].size(), 0, cap, cost } );
    if (src == dst)
      adj[src].back().rev++;
    adj[dst].push_back( { dst, src, adj[src].size() - 1, 0, 0, -cost } );
  }
  vector<cost_type> potential;
  inline cost_type rcost(const edge &e)
  {
    return e.cost + potential[e.src] - potential[e.dst];
  }
  void bellman_ford(int source)
  {
    for (int k = 0; k < n; ++k)
      for (int u = 0; u < n; ++u)
        for (edge &e : adj[u])
          if (e.cap > 0 && rcost(e) < 0)
            potential[e.dst] += rcost(e);
  }
  const cost_type oo = numeric_limits<cost_type>::max();
  vector<cost_type> dist;
  vector<edge*> back;
  cost_type dijkstra(int source, int sink)
  {
    fill(dist.begin(), dist.end(), oo);
    typedef pair<cost_type, int> node;
    priority_queue<node, vector<node>, greater<node>> pq;
    for (pq.push( { dist[source] = 0, source } ); !pq.empty();)
    {
      node p = pq.top(); pq.pop();
      if (dist[p.second] < p.first) continue;
      if (p.second == sink) break;
      for (edge &e : adj[p.second])
        if (e.flow < e.cap &&
          dist[e.dst] > dist[e.src] + rcost(e))
        {
          back[e.dst] = &e;
          pq.push( { dist[e.dst] = dist[e.src] + rcost(e),
               e.dst } );
        }
    }
    return dist[sink];
  }
  pair<flow_type, cost_type> max_flow(int source, int sink)
  {
    flow_type flow = 0;
    cost_type cost = 0;
    for (int u = 0; u < n; ++u)
      for (edge &e : adj[u]) e.flow = 0;
    potential.assign(n, 0);
    dist.assign(n, 0);
    back.assign(n, nullptr);
    bellman_ford(source); // remove negative costs
    while (dijkstra(source, sink) < oo)
    {
      for (int u = 0; u < n; ++u)
        if (dist[u] < dist[sink])
          potential[u] += dist[u] - dist[sink];
      flow_type f = numeric_limits<flow_type>::max();
      for (edge *e = back[sink]; e; e = back[e->src])
        f = min(f, e->cap - e->flow);
      for (edge *e = back[sink]; e; e = back[e->src])
        e->flow += f, adj[e->dst][e->rev].flow -= f;
      flow += f;
      cost += f * (potential[sink] - potential[source]);
    }
    return { flow, cost } ;
  }
};
```

7.50 stoerWagner.cpp

47 lines

```
/*
  Tested: ZOJ 2753
  Complexity: O(n^3)
*/
template<typename T>
pair<T, vector<int>> stoer_wagner(vector<vector<T>> &weights)
{
  int n = weights.size();
  vector<int> used(n), cut, best_cut;
  T best_weight = -1;
  for (int phase = n - 1; phase >= 0; --phase)
  {
    vector<T> w = weights[0];
```

```
  vector<int> added = used;
  int prev, last = 0;
  for (int i = 0; i < phase; ++i)
   {
    prev = last;
    last = -1;
    for (int j = 1; j < n; ++j)
      if (!added[j] && (last == -1 || w[j] > w[last]))
        last = j;
    if (i == phase - 1)
     {
      for (int j = 0; j < n; ++j)
        weights[prev][j] += weights[last][j];
      for (int j = 0; j < n; ++j)
        weights[j][prev] = weights[prev][j];
      used[last] = true;
      cut.push_back(last);
      if (best_weight == -1 || w[last] < best_weight)
       {
        best_cut = cut;
        best_weight = w[last];
       }
     }
    else
     {
      for (int j = 0; j < n; j++)
        w[j] += weights[last][j];
      added[last] = true;
     }
   }
  }
 return make_pair(best_weight, best_cut);
}
```

7.51 treeIsomorphism.cpp

116 lines

```
/*
 Tested: SPOJ TREEISO
 Complexity: O(n log n)
*/
#define all(c) (c).begin(), (c).end()
struct tree
 {
 int n;
 vector<vector<int>> adj;
 tree(int n) : n(n), adj(n) { }
 void add_edge(int src, int dst)
  {
   adj[src].push_back(dst);
   adj[dst].push_back(src);
  }
 vector<int> centers()
  {
   vector<int> prev;
   int u = 0;
   for (int k = 0; k < 2; ++k)
    {
     queue<int> q;
     prev.assign(n, -1);
     for (q.push(prev[u] = u); !q.empty(); q.pop())
      {
       u = q.front();
       for (auto v : adj[u])
        {
         if (prev[v] >= 0)
           continue;
         q.push(v);
         prev[v] = u;
        }
      }
    }
   vector<int> path = { u } ;
   while (u != prev[u])
     path.push_back(u = prev[u]);
   int m = path.size();
   if (m % 2 == 0)
     return { path[m/2-1], path[m/2] } ;
   else
     return { path[m/2] } ;
  }
 vector<vector<int>> layer;
 vector<int> prev;
 int levelize(int r)
  {
   prev.assign(n, -1);
   prev[r] = n;
   layer = { { r } } ;
   while (1)
    {
     vector<int> next;
     for (int u : layer.back())
       for (int v : adj[u])
        {
         if (prev[v] >= 0)
           continue;
         prev[v] = u;
         next.push_back(v);
        }
     if (next.empty())
       break;
     layer.push_back(next);
    }
   return layer.size();
  }
 } ;
bool isomorphic(tree S, int s, tree T, int t)
 {
  if (S.n != T.n)
    return false;
  if (S.levelize(s) != T.levelize(t))
    return false;
  vector<vector<int>> longcodeS(S.n + 1), longcodeT(T.n + 1);
  vector<int> codeS(S.n), codeT(T.n);
  for (int h = (int) S.layer.size() - 1; h >= 0; --h)
   {
    map<vector<int>, int> bucket;
    for (int u : S.layer[h])
     {
      sort(all(longcodeS[u]));
      bucket[longcodeS[u]] = 0;
     }
    for (int u : T.layer[h])
     {
      sort(all(longcodeT[u]));
      bucket[longcodeT[u]] = 0;
     }
    int id = 0;
    for (auto &p : bucket)
      p.second = id++;
    for (int u : S.layer[h])
     {
      codeS[u] = bucket[longcodeS[u]];
      longcodeS[S.prev[u]].push_back(codeS[u]);
     }
    for (int u : T.layer[h])
     {
      codeT[u] = bucket[longcodeT[u]];
      longcodeT[T.prev[u]].push_back(codeT[u]);
     }
   }
  return codeS[s] == codeT[t];
 }
bool isomorphic(tree S, tree T)
 {
  auto x = S.centers(), y = T.centers();
  if (x.size() != y.size())
    return false;
  if (isomorphic(S, x[0], T, y[0]))
    return true;
  return x.size() > 1 && isomorphic(S, x[1], T, y[0]);
 }
```

strings (8)

8.1 AhoCorasick

Description: Aho-Corasick automaton, used for multiple pattern matching. Initialize with AhoCorasick ac(patterns); the automaton start node will be at index 0. find(word) returns for each position the index of the longest word that ends there, or -1 if none. findAll(−, word) finds all words (up to $N\sqrt{N}$ many if no duplicate patterns) that start at each position (shortest first). Duplicate patterns are allowed; empty patterns are not. To find the longest words that start at each position, reverse all input. For large alphabets, split each symbol into chunks, with sentinel bits for symbol boundaries.
Considerations: The exit links are a compression of links , with exit links go directly to the next node that is a end of some word
Usage: `AHO.insert(`s_i`) //for all `s_i`;AHO.pushLinks();`
Time: construction takes $\mathcal{O}(kN)$, where N = sum of length of patterns and K is the size of alphabet. find(x) is $\mathcal{O}(N)$, where N = length of x. findAll is $\mathcal{O}(NM)$.

66 lines

```
struct AhoCorasick {
  struct Node : map<char, int> {
    int link = 0;
    int cnt = 0;
    int tin = 0, tout = 0;
  };
  vector<Node> trie;
  vector<vi> graph;
  AhoCorasick() { newNode(); }
  int newNode() {
    trie.pb({ });
    graph.pb({ });
    return sz(trie) - 1;
  }
  int insert(string &s, int u = 0) {
    for (char c : s) {
      if (!trie[u][c])
        trie[u][c] = newNode();
      u = trie[u][c];
    }
    trie[u].cnt++;
    return u;
  }
  int go(int u, char c) {
    while (u && !trie[u].count(c))
      u = trie[u].link;
    return trie[u][c];
  }
  void pushLinks() {
    queue<int> Q;
    Q.push(0);
    while (!Q.empty()) {
      int u = Q.front();
      Q.pop();
      for (auto &[c, v] : trie[u]) {
        int l = (trie[v].link = u ? go(trie[u].link, c) : 0);
        trie[v].cnt += trie[l].cnt;
        Q.push(v);
      }
    }
  }
  void buildTree() {
    fore (u, 0, sz(trie))
      graph[trie[u].link].pb(u);
    int timer = 0;
    function<void(int)> dfs = [&](int u) {
      trie[u].tin = ++timer;
      for (int v : graph[u])
        if (!trie[v].tin)
          dfs(v);
      trie[u].tout = timer;
    };
    dfs(0);
  }
  int match(string &s, int u = 0) {
    lli ans = 0;
    for (char c : s) {
      u = go(u, c);
      ans += trie[u].cnt;
    }
    return ans;
  }
  Node& operator [](int u) {
    return trie[u];
  }
} AHO;
```

8.2 Hashing-codeforces.h

Description: Various self-explanatory methods for string hashing. Use on Codeforces, which lacks 64-bit support and where solutions can be hacked.

46 lines

```
typedef uint64_t ull;
static int C; // initialized below
// Arithmetic mod two primes and 2^32 simultaneously.
// "typedef uint64_t H;" instead if Thue-Morse does not apply.
template<int M, class B>
struct A {
  int x; B b; A(int x=0) : x(x), b(x) { }
  A(int x, B b) : x(x), b(b) { }
  A operator+(A o){ int y = x+o.x; return{ y - (y≥M)*M, b+o.b }; }
  A operator-(A o){ int y = x-o.x; return{ y + (y< 0)*M, b-o.b }; }
  A operator*(A o) { return { (int)(1LL*x*o.x % M), b*o.b }; }
  explicit operator ull() { return x ^(ull) b << 21; }
};
typedef A<1000000007, A<1000000009, unsigned>> H;
struct HashInterval {
  vector<H> ha, pw;
  HashInterval(string& str) : ha(sz(str)+1), pw(ha) {
    pw[0] = 1;
    rep(i,0,sz(str))
      ha[i+1] = ha[i] * C + str[i],
      pw[i+1] = pw[i] * C;
  }
  H hashInterval(int a, int b) { // hash [a, b)
    return ha[b] - ha[a] * pw[b - a];
  }
};
vector<H> getHashes(string& str, int length) {
  if (sz(str) < length) return { };
  H h = 0, pw = 1;
  rep(i,0,length)
    h = h * C + str[i], pw = pw * C;
  vector<H> ret = { h };
  rep(i,length,sz(str)) {
    ret.push_back(h = h * C + str[i] - pw * str[i-length]);
  }
  return ret;
}
H hashString(string& s){H h{ }; for(char c:s) h=h*C+c;return h;}
#include <sys/time.h>
int main() {
  timeval tp;
  gettimeofday(&tp, 0);
  C = (int)tp.tv_usec; // (less than modulo)
  assert((ull)(H(1)*2+1-3) == 0);
  // ...
}
```

8.3 Hashing.h

Description: Self-explanatory methods for string hashing.

40 lines

```
// Arithmetic mod 2^64-1. 2x slower than mod 2^64 and more
// code, but works on evil test data (e.g. Thue-Morse, where
// ABBA... and BAAB... of length 2^10 hash the same mod 2^64).
// "typedef ull H;" instead if you think test data is random,
// or work mod 10^9+7 if the Birthday paradox is not a problem.
struct H {
  typedef uint64_t ull;
  ull x; H(ull x=0) : x(x) { }
  #define OP(O,A,B) H operator O(H o) { ull r = x; asm (A "addq %%rdx,
      ↪ %0\n adcq $0,%0" : "+a"(r) : B); return r; }
  OP(+,,"d"(o.x)) OP(*,"mul %1\n", "r"(o.x) : "rdx")
  H operator-(H o) { return *this + ~o.x; }
  ull get() const { return x + !~x; }
  bool operator==(H o) const { return get() == o.get(); }
```

```
 bool operator<(H o) const { return get() < o.get(); }
};
static const H C = (ll)1e11+3; // (order ~ 3e9; random also ok)
struct HashInterval {
 vector<H> ha, pw;
 HashInterval(string& str) : ha(sz(str)+1), pw(ha) {
  pw[0] = 1;
  rep(i,0,sz(str))
   ha[i+1] = ha[i] * C + str[i],
   pw[i+1] = pw[i] * C;
 }
 H hashInterval(int a, int b) { // hash [a, b)
  return ha[b] - ha[a] * pw[b - a];
 }
};
vector<H> getHashes(string& str, int length) {
 if (sz(str) < length) return {};
 H h = 0, pw = 1;
 rep(i,0,length)
  h = h * C + str[i], pw = pw * C;
 vector<H> ret = {h};
 rep(i,length,sz(str)) {
  ret.push_back(h = h * C + str[i] - pw * str[i-length]);
 }
 return ret;
}
H hashString(string& s){H h{}; for(char c:s) h=h*C+c;return h;}
```

8.4 KMP

Description: pi[x] computes the length of the longest prefix of s that ends at x, other than s

$0 \ldots x$

itself (abacaba -> 0010123). Can be used to find all occurrences of a string.
Time: $\mathcal{O}(n)$

30 lines

```
vector<int> p_function(const string& v){
   int n = v.size();
   vector<int> p(n);
   for(int i = 1; i < n; i++) {
      int j = p[i - 1];
      while(j > 0 && v[j] != v[i]) {
         j = p[j - 1];
      }
      if(v[j] == v[i])
         j++;
      p[i] = j;
   }
   return p;
}
bool match(const string& s, const string& pat) {
 int n = pat.size();
 int m = s.size();
 if(m<n){
  cout<<endl;
  return false;
 }
 string match = pat+"#"+s;
 vector<int> kmp =p_function(match);
 for(int i = 0; i < m - n + 1; i++){
  if(kmp[2 * n + i] == n){
      return true;
    }
  }
  return false;
}
```

8.5 Manacher

Description: For each position in a string, computes p[0][i] = half length of longest even palindrome around pos i, p[1][i] = longest odd (half rounded down).
Time: $\mathcal{O}(N)$

15 lines

```
vector<vector<int>> manacher(const string& s) {
 n = s.size();
 vector<vector<int>> p(2,vector<int>(n,0));
  for(int z=1,l=0,r=0;z<2;z++,l=0,r=0)
    for(int i=0;i<n;i++)
    {
      if(i<r) p[z][i]=min(r-i+!z,p[z][l+r-i+!z]);
      int L=i-p[z][i], R=i+p[z][i]-!z;
      cout<<L<<" "<<R<<" "<<(L-1>=0) <<" "<<(R+1<n)<<endl;
      while(L-1>=0 && R+1<n && s[L-1]==s[R+1]) p[z][i]++,L--,R++;
      if(R>r) l=L,r=R;
    }
  for(int i = 0;i<n;i++)cout<<p[0][i]<<" "<<p[1][i]<<endl;
 return p;
}
```

8.6 MinRotation

Description: Finds the lexicographically smallest rotation of a string.
Usage: `rotate(v.begin(), v.begin()+minRotation(v), v.end());`
Time: $\mathcal{O}(N)$

9 lines

```
int minRotation(string s) {
 int a=0, N=sz(s); s += s;
 for(int b = 0;b<N;b++)
  for(int k = 0;k<N;k++){
   if (a+k == b || s[a+k] < s[b+k]) {b += max(0, k-1); break;}
   if (s[a+k] > s[b+k]) {a = b; break; }
  }
 return a;
}
```

8.7 SuffixArray

Description: Builds suffix array for a string. `sa[i]` is the starting index of the suffix which is i'th in the sorted suffix array. The returned vector is of size $n+1$, and `sa[0] = n`. The `lcp` array contains longest common prefixes for neighbouring strings in the suffix array: `lcp[i] = lcp(sa[i], sa[i-1])`, `lcp[0] = 0`. The input string must not contain any zero bytes.
Time: $\mathcal{O}(n \log n)$

70 lines

```
void radix_sort(vector<int> &P,vector<int> &c){
   int n = P.size();
   vector<int> cnt(n);
   for(auto d:c)
      cnt[d]++;
   vector<int> pos(n);
   vector<int> nP(n);
   pos[0]= 0;
   for(int i = 1;i<n;i++)
      pos[i] = pos[i-1]+cnt[i-1];
   for(auto d:P){
      int i = c[d];
      nP[pos[i]] = d;
      pos[i]++;
   }
   P = nP;
}
// SuffixArray and LCP (Longest common preffix)
void suffixArray(string s){
 s+=char(31);
 int N;
 cin>>N;
 vector<int> nums(N);
 for(auto &c:nums)cin>>c;
 int n = s.size();
 vector<int>c(n);
 vector<int>p(n);
 vector<pair<char,int>> a(n);
 for(int i = 0;i<n;i++)a[i] = {s[i],i};
 sort(a.begin(),a.end());
 for(int i = 0;i<n;i++)
  p[i] = a[i].second;
 c[p[0]] = 0;
 for(int i = 1;i<n;i++){
  if(a[i].first == a[i-1].first)
   c[p[i]] = c[p[i-1]];
  else c[p[i]] = c[p[i-1]]+1;
 }
 int k = 0;
 while((1<<k)<n){
  for(int i = 0 ;i<n;i++)
   p[i] = ((p[i]-(1<<k))+n)%n;
  radix_sort(p,c);
  vector<int> nC(n);
```

```
  nC[p[0]] = 0;
  for(int i = 1;i<n;i++) {
    pair<int,int> prev = { c[p[i-1]],c[(p[i-1]+ (1<<k))%n] } ;
    pair<int,int> now = { c[p[i]],c[(p[i]+ (1<<k))%n] } ;
    if(prev == now)
      nC[p[i]] = nC[p[i-1]];
    else nC[p[i]] = nC[p[i-1]]+1;
  }
  c = nC;
  k++;
 }
 // LCP O(n)
 k = 0;
 vector<int> lcp(n);
 for(int i = 0;i<n-1;i++) {
  int x = c[i];
  int j = p[x-1];
  while(s[i+k] == s[j+k])k++;
  lcp[x] = k;
  k = max(k-1,0ll);
 }
 for(int i = 0;i<N;i++)cout<<p[nums[i]+1]<<" ";
 cout<<endl;
 for(int i = 0;i<n;i++)cout<<lcp[i]<<" "<<p[i]<<" "<<s.substr(p[i],n-p[i
     ↪ ])<<endl;
 cout<<endl;
}
```

8.8 SuffixTree

Description: Ukkonen's algorithm for online suffix tree construction. Each node contains indices [l, r) into the string, and a list of child nodes. Suffixes are given by traversals of this tree, joining [l, r) substrings. The root is 0 (has l = -1, r = 0), non-existent children are -1. To get a complete tree, append a dummy symbol – otherwise it may contain an incomplete path (still useful for substring matching, though).
Time: $\mathcal{O}(26N)$

59 lines

```
const int inf = 1e9;
const int maxn = 1e6 ;
char s[maxn];
map<int, int> to[maxn];
int len[maxn], start[maxn], link[maxn];
int node, remaind;
int sz = 1, n = 0;
int make_node(int _pos, int _len) {
   start[sz] = _pos;
   len [sz] = _len;
   return sz++;
}
void go_edge() {
   while(remaind > len[to[node][s[n - remaind]]]) {
      node = to[node][s[n - remaind]];
      remaind -= len[node];
   }
}
void add_letter(int c) {
   s[n++] = c;
   remaind++;
   int last = 0;
   while(remaind > 0) {
      go_edge();
      int edge = s[n - remaind];
      int &v = to[node][edge];
      int t = s[start[v] + remaind - 1];
      if(v == 0) {
         v = make_node(n - remaind, inf);
         link[last] = node;
         last = 0;
      }
      else if(t == c) {
         link[last] = node;
         return;
      }
      else {
         int u = make_node(start[v], remaind - 1);
         to[u][c] = make_node(n - 1, inf);
         to[u][t] = v;
         start[v] += remaind - 1;
         len [v] -= remaind - 1;
         v = u;
         link[last] = u;
         last = u;
      }
      if(node == 0)
         remaind--;
      else
         node = link[node];
   }
}
bool dfsForPrint(int node,char edge) {
   if(node != 0)
      cout<<edge<<" "<<node<<" "<<len[node]<<" "<<start[node]<<endl;
   for(auto c:to[node])
      dfsForPrint(c.second,c.first);
   return 0 ;
}
```

8.9 Zfunc

Description: z[x] computes the length of the longest common prefix of s[i:] and s, except z[0] = 0. (abacaba -> 0010301)
Time: $\mathcal{O}(n)$

13 lines

```
vector<int> zf (string s) {
  int n = s.size();
  vector<int> z (n);
  for (int i = 1, l = 0, r = 0; i < n; i++) {
    if (i <= r)
      z[i] = min (r - i + 1, z[i - l]);
    while (i + z[i] < n && s[z[i]] == s[i + z[i]])
      z[i]++;
    if (i + z[i] - 1 > r)
      l = i, r = i + z[i] - 1;
  }
  return z;
}
```

8.10 palindromicTree.h

Description: A nice data structure allowing to solve some interesting problems involving palindromes.

95 lines

```
const int MAXN = 105000;
struct node {
   int next[26];
   int len;
   int sufflink;
   int num;
};
int len;
char s[MAXN];
node tree[MAXN];
int num;       // node 1 - root with len -1, node 2 - root with len 0
int suff;      // max suffix palindrome
long long ans;
bool addLetter(int pos) {
   int cur = suff, curlen = 0;
   int let = s[pos] - 'a';
   while (true) {
      curlen = tree[cur].len;
      if (pos - 1 - curlen >= 0 && s[pos - 1 - curlen] == s[pos])
         break;
      cur = tree[cur].sufflink;
   }
   if (tree[cur].next[let]) {
      suff = tree[cur].next[let];
      return false;
   }
   num++;
   suff = num;
   tree[num].len = tree[cur].len + 2;
   tree[cur].next[let] = num;
   if (tree[num].len == 1) {
      tree[num].sufflink = 2;
```

```
      tree[num].num = 1;
      return true;
    }
  while (true) {
      cur = tree[cur].sufflink;
      curlen = tree[cur].len;
      if (pos - 1 - curlen >= 0 && s[pos - 1 - curlen] == s[pos]) {
        tree[num].sufflink = tree[cur].next[let];
        break;
      }
    }
  tree[num].num = 1 + tree[tree[num].sufflink].num;
  return true;
}
void initTree() {
  num = 2; suff = 2;
  tree[1].len = -1; tree[1].sufflink = 1;
  tree[2].len = 0; tree[2].sufflink = 1;
}
/*
 Palindromic Tree
 Complexity: O(n)
 Tested: ??
*/
template<size_t maxlen, size_t alpha>
struct PalindromicTree
 {
 int go[maxlen + 2][alpha], slink[maxlen + 2], length[maxlen + 2];
 int s[maxlen], slength, size, last;
 int new_node()
  {
  memset(go[size], 0, sizeof go[size]);
  slink[size] = length[size] = 0;
  return size++;
  }
 PalindromicTree() { reset(); }
 void reset()
  {
  size = slength = 0;
  length[new_node()] = -1;
  last = new_node();
  }
 int get_link(int p)
  {
  for (int i = slength - 1;
    i - 1 - length[p] < 0 || s[i - 1 - length[p]] != s[i];)
    p = slink[p];
  return p;
  }
 int _extend(int c)
  {
  s[slength++] = c;
  int p = get_link(last), np;
  if (go[p][c]) return go[p][c];
  length[np = new_node()] = 2 + length[p];
  go[p][c] = np;
  if (length[np] == 1) return slink[np] = 1, np;
  p = slink[p];
  slink[np] = go[get_link(p)][c];
  return np;
  }
 void extend(int c) { last = _extend(c); }
};
```

8.11 positionInLexicograficOrder.cpp

35 lines

```
#include <bits/stdc++.h>
using namespace std;
int fact(int n) {
  return (n<=1)?1:n*fact(n-1);
}
void cnt(int * count,string s) {
  for (int i = 0; s[i]; ++i)
    ++count[s[i]];
  for (int i = 1; i < 256; ++i)
    count[i] += count[i - 1];
}
void update(int * count,char s) {
  int i;
  for (i = s; i < 256; ++i)
    --count[i];
}
int findRank(string s) {
  int n = s.size();
  int mul = fact(n);
  int position = 1, i;
  int count[256] = { 0 };
  cnt(count,s);
  for (i = 0; i < n; ++i) {
    mul /= n - i;
    position += count[s[i] - 1] * mul;
    update(count,s[i]);
   }
  return position;
}
int main() {
  int n;
  string s;
  cin>>s;
  cout<<findRank(s);
}
```

8.12 suffixAutomaton.cpp

88 lines

```
#include <bits/stdc++.h>
using namespace std;
#define endl '\n'
#define __ ios_base::sync_with_stdio(false),cin.tie(NULL);
struct SuffixAutomaton {
  vector<map<int,int>> to;
  vector<int> link;
  vector<int> len;
  int last;
  SuffixAutomaton(string s) {
    to.push_back(map<int,int>());
    link.push_back(-1);
    len.push_back(0);
    last = 0;
    for(int i=0;i<s.size();i++) {
      int c = int(s[i]);
      to.push_back(map<int,int>());
      len.push_back(i+1);
      link.push_back(0);
      int cur = to.size() - 1;
      int p = last;
      while(p >= 0 && !to[p].count(c)) {
        to[p][c] = cur;
        p = link[p];
      }
      if(p != -1) {
        int q = to[p][c];
        if(len[p] + 1 == len[q])
          link[cur] = q;
        else {
          to.push_back(to[q]);
          len.push_back(len[p] + 1);
          link.push_back(link[q]);
          int clone = to.size()-1;
          link[q] = link[cur] = clone;
          while(p >= 0 && to[p].count(c) && to[p][c] == q) {
            to[p][c] = clone;
            p = link[p];
          }
        }
      }
      last = cur;
    }
  }
};
//Aditonial implementations.
//Get de kth lexicografic string
vector<int> dp;
void getSize(SuffixAutomaton SA) {
```

```
    int n = SA.to.size();
    vector<int> order(n);
    dp.resize(n,0);
    iota(order.begin(),order.end(),0);
    sort(order.begin(),order.end(),[&](int a,int b) {
        return SA.len[a]>SA.len[b];
    });
    for(int i = 0;i<n;i++) {
        dp[order[i]] = 1;
        for(auto c:SA.to[order[i]])
            dp[order[i]]+=dp[c.second];
    }
};
int main() {
    int n = 1,k;
    string s;
    cin>>s;
    cin>>n;
    SuffixAutomaton SA(s);
    getSize(SA);
    for(int i = 0;i<n;i++) {
        cin>>k;
        int u = 0;
        while(k) {
            for(auto c:SA.to[u]) {
                if(dp[c.second]>=k) {
                    k--;
                    u = c.second;
                    cout<<char(c.first);
                    break;
                }
                else
                    k-=dp[c.second];
            }
        }
        cout<<endl;
    }
    return 0;
}
```

8.13 Trie.cpp

94 lines

```
#include <bits/stdc++.h>
using namespace std;
struct Trie {
    unordered_map<char, Trie*> child;
    int prefix;
    bool end;
};
struct Trie *getNode() {
    struct Trie *p = new Trie;
    //Must initialize values
    p->end = false;
    p->prefix = 0;
    return p;
}
void insert(struct Trie *root, string key) {
    struct Trie *S = root;
    for(int i = 0 ; i < key.length(); i++) {
        if(S->child.find(key[i]) == S->child.end())
            S->child[key[i]] = getNode();
        S = S->child[key[i]];
        S->prefix++;
    }
    S-> end =true;
}
bool search(struct Trie *root, string key) {
    struct Trie *S = root;
    int n = key.size();
    for(int i = 0;i<n;i++) {
        if(S->child.find(key[i]) == S->child.end())
            return false;
        S = S->child[key[i]];
    }
    if(S->end)return true;
    else
        return false;
}
int countprefixes(Trie* root,string s) {
    int n= s.size();
    Trie* mov = root;
    for(int i=0;i<n;i++) {
        if(mov->child.find(s[i]) == mov->child.end())
            return 0;
        mov=mov->child[s[i]];
    }
    return mov->prefix;
}
Trie* remove(Trie* root, string word, int depth = 0) {
    if(!root)
        return NULL;
    if (depth == word.size())  {
        if (root->end)
            root->end = false;
        if (root->child.size())  {
            delete (root);
            root = NULL;
        }
        return root;
    }
    root->child[word[depth]] = remove(root->child[word[depth]], word,
        depth + 1);
    if (root->child.size()== 0 && root->end == false)  {
        delete(root);
        root=NULL;
    }
    return root;
}
void print(Trie* root,char str[],int level) {
    if(root->end) {
        str[level] = '\0';
        cout<<str<<endl;
    }
    for(auto c:root->child) {
        str[level] = c.first;
        print(c.second,str,level+1);
    }
}
int main() {
    int t,n,m;
    string s;
    cin >> t;
    Trie *root = getNode();
    while(t--) {
        cin >> s;
        insert(root,s);
    }
    char str[100];
    print(root,str,0);
    cin>>m;
    for(int i = 0;i<m;i++) {
        cin>>s;
        remove(root,s);
    }
    print(root,str,0);
}
```

8.14 TrieBinary.cpp

60 lines

```
#include <bits/stdc++.h>
using namespace std;
const int N = 20000043;
int t[N];
int nxt[N][2];
int szt = 1;
int new_vertex() {
  nxt[szt][0] = nxt[szt][1] = -1;
  t[szt] = 0;
  return szt++;
}
```

```
int go(int x, int y) {
 if (nxt[x][y] == -1)
  nxt[x][y] = new_vertex();
 return nxt[x][y];
}
void add_number(int x, int val) {
 int cur = 0;
 t[cur] += val;
 for(int i = 29; i >= 0; i--) {
  int z = ((x >> i) & 1);
  cur = go(cur, z);
  t[cur] += val;
 }
}
int descend(int x) {
 int ans = 0;
 int cur = 0;
 for(int i = 29; i >= 0; i--) {
  int z = ((x >> i) & 1);
  int k;
  if(nxt[cur][z] != -1 && t[nxt[cur][z]] > 0)
   k = z;
  else
   k = z ^1;
  ans ^= (k << i);
  cur = go(cur, k);
 }
 return ans;
}
int n;
const int M = 300043;
int a[M];
int p[M];
int main()  {
  int n;
  cin>>n;
  vector<int> a(n);
  vector<int> p(n);
  nxt[0][0] = nxt[0][1] = -1;
  for(auto &c:a)cin>>c;
  for(auto &c:p)cin>>c;
 for(auto c:p) add_number(c,1);
 for(int i = 0;i<n;i++) {
  int z = descend(a[i]);
  cout<<a[i]^z<<" ";
  add_number(z, -1);
 }
 return 0;
}
```

geometry (9)

9.1 AllGeometry.cpp

821 lines

```
#include <bits/stdc++.h>
using namespace std;
using ld = long double;
const ld eps = 1e-9, inf = numeric_limits<ld>::max(), pi = acos(-1);
// For use with integers, just set eps=0 and everything remains the same
bool geq(ld a, ld b) { return a-b >= -eps; }  //a >= b
bool leq(ld a, ld b) { return b-a >= -eps; }  //a <= b
bool ge(ld a, ld b) { return a-b > eps; }  //a > b
bool le(ld a, ld b) { return b-a > eps; }  //a < b
bool eq(ld a, ld b) { return abs(a-b) <= eps; }  //a == b
bool neq(ld a, ld b) { return abs(a-b) > eps; }  //a != b
struct point {
 ld x, y;
 point(): x(0), y(0) { }
 point(ld x, ld y): x(x), y(y) { }
 point operator+(const point & p) const { return point(x + p.x, y + p.y); }
 point operator-(const point & p) const { return point(x - p.x, y - p.y); }
 point operator*(const ld & k) const { return point(x * k, y * k); }
 point operator/(const ld & k) const { return point(x / k, y / k); }
 point operator+=(const point & p) { *this = *this + p; return *this; }
 point operator-=(const point & p) { *this = *this - p; return *this; }
 point operator*=(const ld & p) { *this = *this * p; return *this; }
 point operator/=(const ld & p) { *this = *this / p; return *this; }
 point rotate(const ld & a) const { return point(x*cos(a) - y*sin(a), x*sin(a) + y*cos(a)); }
 point perp() const { return point(-y, x); }
 ld ang() const {
  ld a = atan2l(y, x); a += le(a, 0) ? 2*pi : 0; return a;
 }
 ld dot(const point & p) const { return x * p.x + y * p.y; }
 ld cross(const point & p) const { return x * p.y - y * p.x; }
 ld norm() const { return x * x + y * y; }
 ld length() const { return sqrtl(x * x + y * y); }
 point unit() const { return (*this) / length(); }
 bool operator==(const point & p) const { return eq(x, p.x) && eq(y, p.y); }
 bool operator!=(const point & p) const { return !(*this == p); }
 bool operator<(const point & p) const { return le(x, p.x) || (eq(x, p.x) && le(y, p.y)); }
 bool operator>(const point & p) const { return ge(x, p.x) || (eq(x, p.x) && ge(y, p.y)); }
 bool half(const point & p) const { return le(p.cross(*this), 0) || (eq(p.cross(*this), 0) && le(p.dot(*this), 0)); }
};
istream &operator>>(istream &is, point & p) { return is >> p.x >> p.y; }
ostream &operator<<(ostream &os, const point & p) { return os << "(" << p.x << ", " << p.y << ")"; }
int sgn(ld x) {
 if(ge(x, 0)) return 1;
 if(le(x, 0)) return -1;
 return 0;
}
void polarSort(vector<point> & P, const point & o, const point & v) {
 //sort points in P around o, taking the direction of v as first angle
 sort(P.begin(), P.end(), [&](const point & a, const point & b) {
  return point((a - o).half(v), 0) < point((b - o).half(v), (a - o).cross(b - o));
 });
}
bool pointInLine(const point & a, const point & v, const point & p) {
 //line a+tv, point p
 return eq((p - a).cross(v), 0);
}
bool pointInSegment(const point & a, const point & b, const point & p) {
 //segment ab, point p
 return pointInLine(a, b - a, p) && leq((a - p).dot(b - p), 0);
}
int intersectLinesInfo(const point & a1, const point & v1, const point & a2, const point & v2) {
 //lines a1+tv1 and a2+tv2
 ld det = v1.cross(v2);
 if(eq(det, 0)) {
  if(eq((a2 - a1).cross(v1), 0)) {
   return -1; //infinity points
  } else {
   return 0; //no points
  }
 } else {
  return 1; //single point
 }
}
point intersectLines(const point & a1, const point & v1, const point & a2, const point & v2) {
 //lines a1+tv1, a2+tv2
 //assuming that they intersect
 ld det = v1.cross(v2);
 return a1 + v1 * ((a2 - a1).cross(v2) / det);
}
int intersectLineSegmentInfo(const point & a, const point & v, const point & c, const point & d) {
 //line a+tv, segment cd
 point v2 = d - c;
 ld det = v.cross(v2);
 if(eq(det, 0)) {
```

```
    if(eq((c - a).cross(v), 0)) {
      return -1; //infinity points
    } else {
      return 0; //no point
    }
  } else {
    return sgn(v.cross(c - a)) != sgn(v.cross(d - a)); //1: single point, 0: no point
  }
}
int intersectSegmentsInfo(const point & a, const point & b, const point & c, const point & d) {
  //segment ab, segment cd
  point v1 = b - a, v2 = d - c;
  int t = sgn(v1.cross(c - a)), u = sgn(v1.cross(d - a));
  if(t == u) {
    if(t == 0) {
      if(pointInSegment(a, b, c) || pointInSegment(a, b, d) || pointInSegment(c, d, a) || pointInSegment(c, d, b)) {
        return -1; //infinity points
      } else {
        return 0; //no point
      }
    } else {
      return 0; //no point
    }
  } else {
    return sgn(v2.cross(a - c)) != sgn(v2.cross(b - c)); //1: single point, 0: no point
  }
}
ld distancePointLine(const point & a, const point & v, const point & p) {
  //line: a + tv, point p
  return abs(v.cross(p - a)) / v.length();
}
ld perimeter(vector<point> & P) {
  int n = P.size();
  ld ans = 0;
  for(int i = 0; i < n; i++) {
    ans += (P[i] - P[(i + 1) % n]).length();
  }
  return ans;
}
ld area(vector<point> & P) {
  int n = P.size();
  ld ans = 0;
  for(int i = 0; i < n; i++) {
    ans += P[i].cross(P[(i + 1) % n]);
  }
  return abs(ans / 2);
}
vector<point> convexHull(vector<point> P) {
  sort(P.begin(), P.end());
  vector<point> L, U;
  for(int i = 0; i < P.size(); i++) {
    while(L.size() >= 2 && leq((L[L.size() - 2] - P[i]).cross(L[L.size() - 1] - P[i]), 0)) {
      L.pop_back();
    }
    L.push_back(P[i]);
  }
  for(int i = P.size() - 1; i >= 0; i--) {
    while(U.size() >= 2 && leq((U[U.size() - 2] - P[i]).cross(U[U.size() - 1] - P[i]), 0)) {
      U.pop_back();
    }
    U.push_back(P[i]);
  }
  L.pop_back();
  U.pop_back();
  L.insert(L.end(), U.begin(), U.end());
  return L;
}
bool pointInPerimeter(const vector<point> & P, const point & p) {
  int n = P.size();
  for(int i = 0; i < n; i++) {
    if(pointInSegment(P[i], P[(i + 1) % n], p)) {
      return true;
    }
  }
  return false;
}
bool crossesRay(const point & a, const point & b, const point & p) {
  return (geq(b.y, p.y) - geq(a.y, p.y)) * sgn((a - p).cross(b - p)) > 0;
}
int pointInPolygon(const vector<point> & P, const point & p) {
  if(pointInPerimeter(P, p)) {
    return -1; //point in the perimeter
  }
  int n = P.size();
  int rays = 0;
  for(int i = 0; i < n; i++) {
    rays += crossesRay(P[i], P[(i + 1) % n], p);
  }
  return rays & 1; //0: point outside, 1: point inside
}
//point in convex polygon in O(log n)
//make sure that P is convex and in ccw
//before the queries, do the preprocess on P:
// rotate(P.begin(), min_element(P.begin(), P.end()), P.end());
// int right = max_element(P.begin(), P.end()) - P.begin();
//returns 0 if p is outside, 1 if p is inside, -1 if p is in the perimeter
int pointInConvexPolygon(const vector<point> & P, const point & p, int right) {
  if(p < P[0] || P[right] < p) return 0;
  int orientation = sgn((P[right] - P[0]).cross(p - P[0]));
  if(orientation == 0) {
    if(p == P[0] || p == P[right]) return -1;
    return (right == 1 || right + 1 == P.size()) ? -1 : 1;
  } else if(orientation < 0) {
    auto r = lower_bound(P.begin() + 1, P.begin() + right, p);
    int det = sgn((p - r[-1]).cross(r[0] - r[-1])) - 1;
    if(det == -2) det = 1;
    return det;
  } else {
    auto l = upper_bound(P.rbegin(), P.rend() - right - 1, p);
    int det = sgn((p - l[0]).cross((l == P.rbegin() ? P[0] : l[-1]) - l[0])) - 1;
    if(det == -2) det = 1;
    return det;
  }
}
vector<point> cutPolygon(const vector<point> & P, const point & a, const point & v) {
  //returns the part of the convex polygon P on the left side of line a+tv
  int n = P.size();
  vector<point> lhs;
  for(int i = 0; i < n; ++i) {
    if(geq(v.cross(P[i] - a), 0)) {
      lhs.push_back(P[i]);
    }
    if(intersectLineSegmentInfo(a, v, P[i], P[(i+1)%n]) == 1) {
      point p = intersectLines(a, v, P[i], P[(i+1)%n] - P[i]);
      if(p != P[i] && p != P[(i+1)%n]) {
        lhs.push_back(p);
      }
    }
  }
  return lhs;
}
point centroid(vector<point> & P) {
  point num;
  ld den = 0;
  int n = P.size();
  for(int i = 0; i < n; ++i) {
    ld cross = P[i].cross(P[(i + 1) % n]);
    num += (P[i] + P[(i + 1) % n]) * cross;
    den += cross;
  }
  return num / (3 * den);
```

```
}
vector<pair<int, int>> antipodalPairs(vector<point> & P){
	vector<pair<int, int>> ans;
	int n = P.size(), k = 1;
	auto f = [&](int u, int v, int w){return abs((P[v%n]-P[u%n]).cross(P[w%n]-P[u%n]));};
	while(ge(f(n-1, 0, k+1), f(n-1, 0, k))) ++k;
	for(int i = 0, j = k; i <= k && j < n; ++i){
		ans.emplace_back(i, j);
		while(j < n-1 && ge(f(i, i+1, j+1), f(i, i+1, j)))
			ans.emplace_back(i, ++j);
	}
	return ans;
}
pair<ld, ld> diameterAndWidth(vector<point> & P){
	int n = P.size(), k = 0;
	auto dot = [&](int a, int b){return (P[(a+1)%n]-P[a]).dot(P[(b+1)%n]-P[b]);};
	auto cross = [&](int a, int b){return (P[(a+1)%n]-P[a]).cross(P[(b+1)%n]-P[b]);};
	ld diameter = 0;
	ld width = inf;
	while(ge(dot(0, k), 0)) k = (k+1) % n;
	for(int i = 0; i < n; ++i){
		while(ge(cross(i, k), 0)) k = (k+1) % n;
		//pair: (i, k)
		diameter = max(diameter, (P[k] - P[i]).length());
		width = min(width, distancePointLine(P[i], P[(i+1)%n] - P[i], P[k]));
	}
	return {diameter, width};
}
pair<ld, ld> smallestEnclosingRectangle(vector<point> & P){
	int n = P.size();
	auto dot = [&](int a, int b){return (P[(a+1)%n]-P[a]).dot(P[(b+1)%n]-P[b]);};
	auto cross = [&](int a, int b){return (P[(a+1)%n]-P[a]).cross(P[(b+1)%n]-P[b]);};
	ld perimeter = inf, area = inf;
	for(int i = 0, j = 0, k = 0, m = 0; i < n; ++i){
		while(ge(dot(i, j), 0)) j = (j+1) % n;
		if(!i) k = j;
		while(ge(cross(i, k), 0)) k = (k+1) % n;
		if(!i) m = k;
		while(le(dot(i, m), 0)) m = (m+1) % n;
		//pairs: (i, k) , (j, m)
		point v = P[(i+1)%n] - P[i];
		ld h = distancePointLine(P[i], v, P[k]);
		ld w = distancePointLine(P[j], v.perp(), P[m]);
		perimeter = min(perimeter, 2 * (h + w));
		area = min(area, h * w);
	}
	return {area, perimeter};
}
ld distancePointCircle(const point & c, ld r, const point & p){
	//point p, circle with center c and radius r
	return max((ld)0, (p - c).length() - r);
}
point projectionPointCircle(const point & c, ld r, const point & p){
	//point p (outside the circle), circle with center c and radius r
	return c + (p - c).unit() * r;
}
pair<point, point> pointsOfTangency(const point & c, ld r, const point & p){
	//point p (outside the circle), circle with center c and radius r
	point v = (p - c).unit() * r;
	ld d2 = (p - c).norm(), d = sqrt(d2);
	point v1 = v * (r / d), v2 = v.perp() * (sqrt(d2 - r*r) / d);
	return {c + v1 - v2, c + v1 + v2};
}
vector<point> intersectLineCircle(const point & a, const point & v, const point & c, ld r){
	//line a+tv, circle with center c and radius r
	ld h2 = r*r - v.cross(c - a) * v.cross(c - a) / v.norm();
	point p = a + v * v.dot(c - a) / v.norm();
	if(eq(h2, 0)) return {p}; //line tangent to circle
	else if(le(h2, 0)) return {}; //no intersection
	else{
		point u = v.unit() * sqrt(h2);
		return {p - u, p + u}; //two points of intersection (chord)
	}
}
vector<point> intersectSegmentCircle(const point & a, const point & b, const point & c, ld r){
	//segment ab, circle with center c and radius r
	vector<point> P = intersectLineCircle(a, b - a, c, r), ans;
	for(const point & p : P){
		if(pointInSegment(a, b, p)) ans.push_back(p);
	}
	return ans;
}
pair<point, ld> getCircle(const point & m, const point & n, const point & p){
	//find circle that passes through points p, q, r
	point c = intersectLines((n + m) / 2, (n - m).perp(), (p + n) / 2, (p - n).perp());
	ld r = (c - m).length();
	return {c, r};
}
vector<point> intersectionCircles(const point & c1, ld r1, const point & c2, ld r2){
	//circle 1 with center c1 and radius r1
	//circle 2 with center c2 and radius r2
	point d = c2 - c1;
	ld d2 = d.norm();
	if(eq(d2, 0)) return {}; //concentric circles
	ld pd = (d2 + r1*r1 - r2*r2) / 2;
	ld h2 = r1*r1 - pd*pd/d2;
	point p = c1 + d*pd/d2;
	if(eq(h2, 0)) return {p}; //circles touch at one point
	else if(le(h2, 0)) return {}; //circles don't intersect
	else{
		point u = d.perp() * sqrt(h2/d2);
		return {p - u, p + u};
	}
}
int circleInsideCircle(const point & c1, ld r1, const point & c2, ld r2){
	//test if circle 2 is inside circle 1
	//returns "-1" if 2 touches internally 1, "1" if 2 is inside 1, "0" if they overlap
	ld l = r1 - r2 - (c1 - c2).length();
	return (ge(l, 0) ? 1 : (eq(l, 0) ? -1 : 0));
}
int circleOutsideCircle(const point & c1, ld r1, const point & c2, ld r2){
	//test if circle 2 is outside circle 1
	//returns "-1" if they touch externally, "1" if 2 is outside 1, "0" if they overlap
	ld l = (c1 - c2).length() - (r1 + r2);
	return (ge(l, 0) ? 1 : (eq(l, 0) ? -1 : 0));
}
int pointInCircle(const point & c, ld r, const point & p){
	//test if point p is inside the circle with center c and radius r
	//returns "0" if it's outside, "-1" if it's in the perimeter, "1" if it's inside
	ld l = (p - c).length() - r;
	return (le(l, 0) ? 1 : (eq(l, 0) ? -1 : 0));
}
vector<vector<point>> tangents(const point & c1, ld r1, const point & c2, ld r2, bool inner){
	//returns a vector of segments or a single point
	if(inner) r2 = -r2;
	point d = c2 - c1;
	ld dr = r1 - r2, d2 = d.norm(), h2 = d2 - dr*dr;
	if(eq(d2, 0) || le(h2, 0)) return {};
	point v = d*dr/d2;
	if(eq(h2, 0)) return {{c1 + v*r1}};
	else{
		point u = d.perp()*sqrt(h2)/d2;
		return {{c1 + (v - u)*r1, c2 + (v - u)*r2}, {c1 + (v + u)*r1, c2 + (v + u)*r2}};
	}
}
```

```cpp
ld signed_angle(const point & a, const point & b){
  return sgn(a.cross(b)) * acosl(a.dot(b) / (a.length() * b.length()));
}
ld intersectPolygonCircle(const vector<point> & P, const point & c, ld r){
  //Gets the area of the intersection of the polygon with the circle
  int n = P.size();
  ld ans = 0;
  for(int i = 0; i < n; ++i){
    point p = P[i], q = P[(i+1)%n];
    bool p_inside = (pointInCircle(c, r, p) != 0);
    bool q_inside = (pointInCircle(c, r, q) != 0);
    if(p_inside && q_inside){
      ans += (p - c).cross(q - c);
    }else if(p_inside && !q_inside){
      point s1 = intersectSegmentCircle(p, q, c, r)[0];
      point s2 = intersectSegmentCircle(c, q, c, r)[0];
      ans += (p - c).cross(s1 - c) + r*r * signed_angle(s1 - c, s2 - c);
    }else if(!p_inside && q_inside){
      point s1 = intersectSegmentCircle(c, p, c, r)[0];
      point s2 = intersectSegmentCircle(p, q, c, r)[0];
      ans += (s2 - c).cross(q - c) + r*r * signed_angle(s1 - c, s2 - c);
    }else{
      auto info = intersectSegmentCircle(p, q, c, r);
      if(info.size() <= 1){
        ans += r*r * signed_angle(p - c, q - c);
      }else{
        point s2 = info[0], s3 = info[1];
        point s1 = intersectSegmentCircle(c, p, c, r)[0];
        point s4 = intersectSegmentCircle(c, q, c, r)[0];
        ans += (s2 - c).cross(s3 - c) + r*r * (signed_angle(s1 - c, s2 - c) + signed_angle(s3 - c, s4 - c));
      }
    }
  }
  return abs(ans)/2;
}
pair<point, ld> mec2(vector<point> & S, const point & a, const point & b, int n){
  ld hi = inf, lo = -hi;
  for(int i = 0; i < n; ++i){
    ld si = (b - a).cross(S[i] - a);
    if(eq(si, 0)) continue;
    point m = getCircle(a, b, S[i]).first;
    ld cr = (b - a).cross(m - a);
    if(le(si, 0)) hi = min(hi, cr);
    else lo = max(lo, cr);
  }
  ld v = (ge(lo, 0) ? lo : le(hi, 0) ? hi : 0);
  point c = (a + b) / 2 + (b - a).perp() * v / (b - a).norm();
  return {c, (a - c).norm()};
}
pair<point, ld> mec(vector<point> & S, const point & a, int n){
  random_shuffle(S.begin(), S.begin() + n);
  point b = S[0], c = (a + b) / 2;
  ld r = (a - c).norm();
  for(int i = 1; i < n; ++i){
    if(ge((S[i] - c).norm(), r)){
      tie(c, r) = (n == S.size() ? mec(S, S[i], i) : mec2(S, a, S[i], i));
    }
  }
  return {c, r};
}
pair<point, ld> smallestEnclosingCircle(vector<point> S){
  assert(!S.empty());
  auto r = mec(S, S[0], S.size());
  return {r.first, sqrt(r.second)};
}
bool comp1(const point & a, const point & b){
  return le(a.y, b.y);
}
pair<point, point> closestPairOfPoints(vector<point> P){
  sort(P.begin(), P.end(), comp1);
  set<point> S;
  ld ans = inf;
  point p, q;
  int pos = 0;
  for(int i = 0; i < P.size(); ++i){
    while(pos < i && geq(P[i].y - P[pos].y, ans)){
      S.erase(P[pos++]);
    }
    auto lower = S.lower_bound({P[i].x - ans - eps, -inf});
    auto upper = S.upper_bound({P[i].x + ans + eps, -inf});
    for(auto it = lower; it != upper; ++it){
      ld d = (P[i] - *it).length();
      if(le(d, ans)){
        ans = d;
        p = P[i];
        q = *it;
      }
    }
    S.insert(P[i]);
  }
  return {p, q};
}
struct vantage_point_tree{
  struct node
  {
    point p;
    ld th;
    node *l, *r;
  }*root;
  vector<pair<ld, point>> aux;
  vantage_point_tree(vector<point> &ps){
    for(int i = 0; i < ps.size(); ++i)
      aux.push_back({ 0, ps[i] });
    root = build(0, ps.size());
  }
  node *build(int l, int r){
    if(l == r)
      return 0;
    swap(aux[l], aux[l + rand() % (r - l)]);
    point p = aux[l++].second;
    if(l == r)
      return new node({ p });
    for(int i = l; i < r; ++i)
      aux[i].first = (p - aux[i].second).dot(p - aux[i].second);
    int m = (l + r) / 2;
    nth_element(aux.begin() + l, aux.begin() + m, aux.begin() + r);
    return new node({ p, sqrt(aux[m].first), build(l, m), build(m, r) });
  }
  priority_queue<pair<ld, node*>> que;
  void k_nn(node *t, point p, int k){
    if(!t)
      return;
    ld d = (p - t->p).length();
    if(que.size() < k)
      que.push({ d, t });
    else if(ge(que.top().first, d)){
      que.pop();
      que.push({ d, t });
    }
    if(!t->l && !t->r)
      return;
    if(le(d, t->th)){
      k_nn(t->l, p, k);
      if(leq(t->th - d, que.top().first))
        k_nn(t->r, p, k);
    }else{
      k_nn(t->r, p, k);
      if(leq(d - t->th, que.top().first))
        k_nn(t->l, p, k);
    }
  }
  vector<point> k_nn(point p, int k){
    k_nn(root, p, k);
    vector<point> ans;
    for(; !que.empty(); que.pop())
      ans.push_back(que.top().second->p);
    reverse(ans.begin(), ans.end());
    return ans;
  }
```

```
};
vector<point> minkowskiSum(vector<point> A, vector<point> B) {
 int na = (int)A.size(), nb = (int)B.size();
 if(A.empty() || B.empty()) return {};
 rotate(A.begin(), min_element(A.begin(), A.end()), A.end());
 rotate(B.begin(), min_element(B.begin(), B.end()), B.end());
 int pa = 0, pb = 0;
 vector<point> M;
 while(pa < na && pb < nb) {
  M.push_back(A[pa] + B[pb]);
  ld x = (A[(pa + 1) % na] - A[pa]).cross(B[(pb + 1) % nb] - B[pb]);
  if(leq(x, 0)) pb++;
  if(geq(x, 0)) pa++;
 }
 while(pa < na) M.push_back(A[pa++] + B[0]);
 while(pb < nb) M.push_back(B[pb++] + A[0]);
 return M;
}
//Delaunay triangulation in O(n log n)
const point inf_pt(inf, inf);
struct QuadEdge {
 point origin;
 QuadEdge* rot = nullptr;
 QuadEdge* onext = nullptr;
 bool used = false;
 QuadEdge* rev() const { return rot->rot; }
 QuadEdge* lnext() const { return rot->rev()->onext->rot; }
 QuadEdge* oprev() const { return rot->onext->rot; }
 point dest() const { return rev()->origin; }
};
QuadEdge* make_edge(const point & from, const point & to) {
 QuadEdge* e1 = new QuadEdge;
 QuadEdge* e2 = new QuadEdge;
 QuadEdge* e3 = new QuadEdge;
 QuadEdge* e4 = new QuadEdge;
 e1->origin = from;
 e2->origin = to;
 e3->origin = e4->origin = inf_pt;
 e1->rot = e3;
 e2->rot = e4;
 e3->rot = e2;
 e4->rot = e1;
 e1->onext = e1;
 e2->onext = e2;
 e3->onext = e4;
 e4->onext = e3;
 return e1;
}
void splice(QuadEdge* a, QuadEdge* b) {
 swap(a->onext->rot->onext, b->onext->rot->onext);
 swap(a->onext, b->onext);
}
void delete_edge(QuadEdge* e) {
 splice(e, e->oprev());
 splice(e->rev(), e->rev()->oprev());
 delete e->rot;
 delete e->rev()->rot;
 delete e;
 delete e->rev();
}
QuadEdge* connect(QuadEdge* a, QuadEdge* b) {
 QuadEdge* e = make_edge(a->dest(), b->origin);
 splice(e, a->lnext());
 splice(e->rev(), b);
 return e;
}
bool left_of(const point & p, QuadEdge* e) {
 return ge((e->origin - p).cross(e->dest() - p), 0);
}
bool right_of(const point & p, QuadEdge* e) {
 return le((e->origin - p).cross(e->dest() - p), 0);
}
ld det3(ld a1, ld a2, ld a3, ld b1, ld b2, ld b3, ld c1, ld c2, ld c3) {
 return a1 * (b2 * c3 - c2 * b3) - a2 * (b1 * c3 - c1 * b3) + a3 * (b1 * c2 - c1 * b2);
}
bool in_circle(const point & a, const point & b, const point & c, const point & d) {
 ld det = -det3(b.x, b.y, b.norm(), c.x, c.y, c.norm(), d.x, d.y, d.norm());
 det += det3(a.x, a.y, a.norm(), c.x, c.y, c.norm(), d.x, d.y, d.norm());
 det -= det3(a.x, a.y, a.norm(), b.x, b.y, b.norm(), d.x, d.y, d.norm());
 det += det3(a.x, a.y, a.norm(), b.x, b.y, b.norm(), c.x, c.y, c.norm());
 return ge(det, 0);
}
pair<QuadEdge*, QuadEdge*> build_tr(int l, int r, vector<point> & P) {
 if(r - l + 1 == 2) {
  QuadEdge* res = make_edge(P[l], P[r]);
  return {res, res->rev()};
 }
 if(r - l + 1 == 3) {
  QuadEdge *a = make_edge(P[l], P[l + 1]), *b = make_edge(P[l + 1], P[r]);
  splice(a->rev(), b);
  int sg = sgn((P[l + 1] - P[l]).cross(P[r] - P[l]));
  if(sg == 0)
   return {a, b->rev()};
  QuadEdge* c = connect(b, a);
  if(sg == 1)
   return {a, b->rev()};
  else
   return {c->rev(), c};
 }
 int mid = (l + r) / 2;
 QuadEdge *ldo, *ldi, *rdo, *rdi;
 tie(ldo, ldi) = build_tr(l, mid, P);
 tie(rdi, rdo) = build_tr(mid + 1, r, P);
 while(true) {
  if(left_of(rdi->origin, ldi)) {
   ldi = ldi->lnext();
   continue;
  }
  if(right_of(ldi->origin, rdi)) {
   rdi = rdi->rev()->onext;
   continue;
  }
  break;
 }
 QuadEdge* basel = connect(rdi->rev(), ldi);
 auto valid = [&basel](QuadEdge* e) { return right_of(e->dest(), basel); };
 if(ldi->origin == ldo->origin)
  ldo = basel->rev();
 if(rdi->origin == rdo->origin)
  rdo = basel;
 while(true) {
  QuadEdge* lcand = basel->rev()->onext;
  if(valid(lcand)) {
   while(in_circle(basel->dest(), basel->origin, lcand->dest(), lcand->onext->dest())) {
    QuadEdge* t = lcand->onext;
    delete_edge(lcand);
    lcand = t;
   }
  }
  QuadEdge* rcand = basel->oprev();
  if(valid(rcand)) {
   while(in_circle(basel->dest(), basel->origin, rcand->dest(), rcand->oprev()->dest())) {
    QuadEdge* t = rcand->oprev();
    delete_edge(rcand);
    rcand = t;
   }
  }
  if(!valid(lcand) && !valid(rcand))
   break;
  if(!valid(lcand) || (valid(rcand) && in_circle(lcand->dest(), lcand->origin, rcand->origin, rcand->dest())))
   basel = connect(rcand, basel->rev());
```

```
    else
      basel = connect(basel->rev(), lcand->rev());
  }
  return { ldo, rdo } ;
}
vector<tuple<point, point, point>> delaunay(vector<point> & P) {
  sort(P.begin(), P.end());
  auto res = build_tr(0, (int)P.size() - 1, P);
  QuadEdge* e = res.first;
  vector<QuadEdge*> edges = { e } ;
  while(le((e->dest() - e->onext->dest()).cross(e->origin - e->onext->dest()), 0))
    e = e->onext;
  auto add = [&P, &e, &edges]() {
    QuadEdge* curr = e;
    do {
      curr->used = true;
      P.push_back(curr->origin);
      edges.push_back(curr->rev());
      curr = curr->lnext();
    } while(curr != e);
  } ;
  add();
  P.clear();
  int kek = 0;
  while(kek < (int)edges.size())
    if(!(e = edges[kek++])->used)
      add();
  vector<tuple<point, point, point>> ans;
  for(int i = 0; i < (int)P.size(); i += 3) {
    ans.emplace_back(P[i], P[i + 1], P[i + 2]);
  }
  return ans;
}
struct circ {
  point c;
  ld r;
  circ() { }
  circ(const point & c, ld r): c(c), r(r) { }
  set<pair<ld, ld>> ranges;
  void disable(ld l, ld r) {
    ranges.emplace(l, r);
  }
  auto getActive() const {
    vector<pair<ld, ld>> ans;
    ld maxi = 0;
    for(const auto & dis : ranges) {
      ld l, r;
      tie(l, r) = dis;
      if(l > maxi) {
        ans.emplace_back(maxi, l);
      }
      maxi = max(maxi, r);
    }
    if(!eq(maxi, 2*pi)) {
      ans.emplace_back(maxi, 2*pi);
    }
    return ans;
  }
} ;
ld areaUnionCircles(const vector<circ> & circs) {
  vector<circ> valid;
  for(const circ & curr : circs) {
    if(eq(curr.r, 0)) continue;
    circ nuevo = curr;
    for(circ & prev : valid) {
      if(circleInsideCircle(prev.c, prev.r, nuevo.c, nuevo.r)) {
        nuevo.disable(0, 2*pi);
      } else if(circleInsideCircle(nuevo.c, nuevo.r, prev.c, prev.r)) {
        prev.disable(0, 2*pi);
      } else {
        auto cruce = intersectionCircles(prev.c, prev.r, nuevo.c, nuevo.r);
        if(cruce.size() == 2) {
          ld a1 = (cruce[0] - prev.c).ang();
          ld a2 = (cruce[1] - prev.c).ang();
          ld b1 = (cruce[1] - nuevo.c).ang();
          ld b2 = (cruce[0] - nuevo.c).ang();
          if(a1 < a2) {
            prev.disable(a1, a2);
          } else {
            prev.disable(a1, 2*pi);
            prev.disable(0, a2);
          }
          if(b1 < b2) {
            nuevo.disable(b1, b2);
          } else {
            nuevo.disable(b1, 2*pi);
            nuevo.disable(0, b2);
          }
        }
      }
    }
    valid.push_back(nuevo);
  }
  ld ans = 0;
  for(const circ & curr : valid) {
    for(const auto & range : curr.getActive()) {
      ld l, r;
      tie(l, r) = range;
      ans += curr.r*(curr.c.x * (sin(r) - sin(l)) - curr.c.y * (cos(r) - cos(l))) + curr.r*curr.r*(r-l);
    }
  }
  return ans/2;
} ;
struct plane {
  point a, v;
  plane(): a(), v() { }
  plane(const point& a, const point& v): a(a), v(v) { }
  point intersect(const plane& p) const {
    ld t = (p.a - a).cross(p.v) / v.cross(p.v);
    return a + v*t;
  }
  bool outside(const point& p) const { // test if point p is strictly outside
    return le(v.cross(p - a), 0);
  }
  bool inside(const point& p) const { // test if point p is inside or in the boundary
    return geq(v.cross(p - a), 0);
  }
  bool operator<(const plane& p) const { // sort by angle
    auto lhs = make_tuple(v.half( { 1, 0 } ), ld(0), v.cross(p.a - a));
    auto rhs = make_tuple(p.v.half( { 1, 0 } ), v.cross(p.v), ld(0));
    return lhs < rhs;
  }
  bool operator==(const plane& p) const { // paralell and same directions, not really equal
    return eq(v.cross(p.v), 0) && ge(v.dot(p.v), 0);
  }
} ;
vector<point> halfPlaneIntersection(vector<plane> planes) {
  planes.push_back( { { 0, -inf } , { 1, 0 } } );
  planes.push_back( { { inf, 0 } , { 0, 1 } } );
  planes.push_back( { { 0, inf } , { -1, 0 } } );
  planes.push_back( { { -inf, 0 } , { 0, -1 } } );
  sort(planes.begin(), planes.end());
  planes.erase(unique(planes.begin(), planes.end()), planes.end());
  deque<plane> ch;
  deque<point> poly;
  for(const plane& p : planes) {
    while(ch.size() >= 2 && p.outside(poly.back())) ch.pop_back(), poly.pop_back();
    while(ch.size() >= 2 && p.outside(poly.front())) ch.pop_front(), poly.pop_front();
    if(p.v.half( { 1, 0 } ) && poly.empty()) return { } ;
    ch.push_back(p);
    if(ch.size() >= 2) poly.push_back(ch[ch.size()-2].intersect(ch[ch.size()-1]));
  }
  while(ch.size() >= 3 && ch.front().outside(poly.back())) ch.pop_back(), poly.pop_back();
```

```cpp
  while(ch.size() >= 3 && ch.back().outside(poly.front())) ch.pop_front()
      , poly.pop_front();
  poly.push_back(ch.back().intersect(ch.front()));
  return vector<point>(poly.begin(), poly.end());
 }
vector<point> halfPlaneIntersectionRandomized(vector<plane> planes) {
  point p = planes[0].a;
  int n = planes.size();
  random_shuffle(planes.begin(), planes.end());
  for(int i = 0; i < n; ++i) {
   if(planes[i].inside(p)) continue;
   ld lo = -inf, hi = inf;
   for(int j = 0; j < i; ++j) {
    ld A = planes[j].v.cross(planes[i].v);
    ld B = planes[j].v.cross(planes[j].a - planes[i].a);
    if(ge(A, 0)) {
     lo = max(lo, B/A);
    } else if(le(A, 0)) {
     hi = min(hi, B/A);
    } else {
     if(ge(B, 0)) return {};
    }
    if(ge(lo, hi)) return {};
   }
   p = planes[i].a + planes[i].v*lo;
  }
  return {p};
 }
```

9.2 Geometry with complex c++

9.3 geometryComplex.cpp

245 lines

```cpp
#include <bits/stdc++.h>
using namespace std;
#define __ ios_base::sync_with_stdio(false),cin.tie(NULL);
#define endl '\n'
#define x real()
#define y imag()
typedef double T;
typedef complex<T> pt;
//Translations
pt translate(pt v,pt p){return p+v;}
pt scale(pt c ,double factor,pt p){return c + (p-c)*factor;}
pr rotate(pt p,double a){return p * polar(1.0,a);}
pt perp(pt p){return {-p.y,p.x};} //rotate 90°
pt linearTransfo(pt p, pt q, pt r, pt fp, pt fq) {
   return fp + (r-p) * (fq-fp) / (q-p);
 }
T dot(pt v, pt w) {return (conj(v)*w).x;}
T cross(pt v, pt w) {return (conj(v)*w).y;}
T sq(pt p) {return p.x*p.x + p.y*p.y;}
double abs(pt p) {return sqrt(sq(p));}
// 0 if is colinear (or are in the line AB), >0 if is left turn , <0 if
    is a right turn
T orient(pt a, pt b, pt c) {return cross(b-a,c-a);}
bool isPerp(pt v, pt w) {return dot(v,w) == 0;}
double angle(pt v, pt w) {
   double cosTheta = dot(v,w) / abs(v) / abs(w);
   return acos(max(-1.0, min(1.0, cosTheta)));
 }
// Angle betwen the lines AB and AC in oriented way
double orientedAngle(pt a, pt b, pt c) {
   if (orient(a,b,c) >= 0)
   return angle(b-a, c-a);
   else
   return 2*M_PI - angle(b-a, c-a);
 }
// Check if a point is betwen the angle formed by the lines AB ,AC
bool inAngle(pt a, pt b, pt c, pt p) {
   assert(orient(a,b,c) != 0);
   if (orient(a,b,c) < 0) swap(b,c);
   return orient(a,b,p) >= 0 && orient(a,c,p) <= 0;
 }
// If we want some vector v to be the first angle
pt v = {1e9,0}; // diferent from {0,0}
bool half(pt p) {
   return cross(v,p) < 0 || (cross(v,p) == 0 && dot(v,p) <0);
 }
// bool half(pt p) {// true if in blue half
//   assert(p.x != 0 || p.y != 0); // the argument of (0,0) is undefined
//   return p.y > 0 || (p.y == 0 && p.x < 0);
// }
void polarSortAround(pt o, vector<pt> &v) {
   sort(v.begin(), v.end(), [](pt v, pt w) {
      return make_tuple(half(v-o), 0) <make_tuple(half(w-o), cross(v-o,
          w-o));
    });
 }
struct line {
   pt v; T c;
   pt p,q;
   // T a,b,c;
   // From direction vector v and offset c
   line(pt v, T c) : v(v), c(c) {}
   // From equation ax+by=c
   line(T a, T b, T c) : v({b,-a}), c(c) {}
   // From points P and Q
   line(pt p, pt q) : v(q-p), c(cross(v,p)) ,p(p),q(q){}
   // Will be defined later:
   // - these work with T = int
   T side(pt p) {return cross(v,p)-c;}
   double dist(pt p) {return abs(side(p)) / abs(v);}
   double sqDist(pt p) {return side(p)*side(p) / (double)sq(v);}
   line perpThrough(pt p) {return {p, p + perp(v)};}
   bool cmpProj(pt p, pt q) {
      return dot(v,p) < dot(v,q);
    }
   line translate(pt t) {return {v, c + cross(v,t)};}
   // - these require T = double
   line shiftLeft(double dist) {return {v, c + dist*abs(v)};}
   pt proj(pt p) {return p - perp(v)*side(p)/sq(v);}
   pt refl(pt p) {return p - perp(v)*2*side(p)/sq(v);}
 }
//Mapping pendients
map<pair<int,int>,map<int,set<int>> mp;
void add(pt a,pt b,int ida,int idb){
   int A = a.y-b.y;
   int B = a.x-b.x;
   int gcd = __gcd(A,B);
   A/=gcd;
   B/=gcd;
   C = (a.x*A) - (a.y*B);
   mp[{A,B}][C].insert(ida);
   mp[{A,B}][C].insert(idb);
 }
bool intersect(line l1, line l2, pt &out) {
   T d = cross(l1.v, l2.v);
   if (d == 0) return false;
   out = (l2.v*l1.c - l1.v*l2.c) / d; // requires floating-point
   coordinates
   return true;
 }
// Is a line that forms equal angles with l1 and l2
line bisector(line l1, line l2, bool interior) {
   assert(cross(l1.v, l2.v) != 0); // l1 and l2 cannot be parallel!
   double sign = interior ? 1 : -1;
   return {l2.v/abs(l2.v) + l1.v/abs(l1.v) * sign,
   l2.c/abs(l2.v) + l1.c/abs(l1.v) * sign};
 }
// Segments
// Check if a point are int the circle of radius AB , if the angle is
    less of 90° is inside
bool pointInDisk(pt a, pt b, pt p) {
   return dot(a-p, b-p) <= 0;
 }
bool pointInSegment(pt a, pt b, pt p) {
   return orient(a,b,p) == 0 && pointInDisk(a,b,p);
 }
bool properInter(pt a, pt b, pt c, pt d, pt &out) {
   double oa = orient(c,d,a),
```

```
    ob = orient(c,d,b),
    oc = orient(a,b,c),
    od = orient(a,b,d);
    if (oa*ob < 0 && oc*od < 0) {
    out = (a*ob - b*oa) / (ob-oa);
    return true;
    }
    return false;
}
struct cmpX {
    bool operator()(pt a, pt b) {
        return make_pair(a.x, a.y) < make_pair(b.x, b.y);
    }
};
set<pt,cmpX> inters(pt a, pt b, pt c, pt d) {
    pt out;
    if (properInter(a,b,c,d,out)) return {out};
    set<pt,cmpX> s;
    if (onSegment(c,d,a)) s.insert(a);
    if (onSegment(c,d,b)) s.insert(b);
    if (onSegment(a,b,c)) s.insert(c);
    if (onSegment(a,b,d)) s.insert(d);
    return s;
}
double closestDistanceSegmentPoint(pt a, pt b, pt p) {
    if (a != b) {
        line l(a,b);
        if (l.cmpProj(a,p) && l.cmpProj(p,b)) // if closest to projection
        return l.dist(p); // output distance to line
    }
    return min(abs(p-a), abs(p-b)); // otherwise distance to A or B
}
double closestDistanceSegmentSegment(pt a, pt b, pt c, pt d) {
    pt dummy;
    if (properInter(a,b,c,d,dummy))
    return 0;
    return min({segPoint(a,b,c), segPoint(a,b,d),
    segPoint(c,d,a), segPoint(c,d,b)});
}
/** Polygons */
double areaTriangle(pt a, pt b, pt c) {
    return abs(cross(b-a, c-a)) / 2.0;
}
double areaPolygon(vector<pt> p) {
    double area = 0.0;
    for (int i = 0, n = p.size(); i < n; i++) {
    area += cross(p[i], p[(i+1)%n]); // wrap back to 0 if i == n-1
    }
    return abs(area) / 2.0;
}
bool above(pt a, pt p) {
    return p.y >= a.y;
}
// check if [PQ] crosses ray from A
bool crossesRay(pt a, pt p, pt q) {
    return (above(a,q) - above(a,p)) * orient(a,p,q) > 0;
}
// if strict, returns false when A is on the boundary
bool inPolygon(vector<pt> p, pt a, bool strict = true) {
    int numCrossings = 0;
    for (int i = 0, n = p.size(); i < n; i++) {
        if (onSegment(p[i], p[(i+1)%n], a))
            return !strict;
        numCrossings += crossesRay(a, p[i], p[(i+1)%n]);
    }
    return numCrossings & 1; // inside if odd number of crossings
}
// amplitude travelled around point A, from P to Q
double angleTravelled(pt a, pt p, pt q) {
    double ampli = angle(p-a, q-a);
    if (orient(a,p,q) > 0) return ampli;
    else return -ampli;
}
double angleTravelled(pt a, pt p, pt q) {
    // remainder ensures the value is in [-pi,pi]
    return remainder(arg(q-a) - arg(p-a), 2*M_PI);
}
int windingNumber(vector<pt> p, pt a) {
    double ampli = 0;
    for (int i = 0, n = p.size(); i < n; i++)
    ampli += angleTravelled(a, p[i], p[(i+1)%n]);
    return round(ampli / (2*M_PI));
}
/** Circles */
pt circumCenter(pt a, pt b, pt c) {
    b = b-a, c = c-a; // consider coordinates relative to A
    assert(cross(b,c) != 0); // no circumcircle if A,B,C aligned
    return a + perp(b*sq(c) - c*sq(b))/cross(b,c)/2;
}
int circleLine(pt o, double r, line l, pair<pt,pt> &out) {
    double h2 = r*r - l.sqDist(o);
    if (h2 >= 0) { // the line touches the circle
    pt p = l.proj(o); // point P
    pt h = l.v*sqrt(h2)/abs(l.v); // vector parallel to l, of
    length h
    out = {p-h, p+h};
    }
    return 1 + sgn(h2);
}
int circleCircle(pt o1, double r1, pt o2, double r2, pair<pt,pt> &out) {
    pt d=o2-o1; double d2=sq(d);
    if (d2 == 0) {assert(r1 != r2); return 0;} // concentric circles
    double pd = (d2 + r1*r1 - r2*r2)/2; // = |O_1P| * d
    double h2 = r1*r1 - pd*pd/d2; // = ^h2
    if (h2 >= 0) {
    pt p = o1 + d*pd/d2, h = perp(d)*sqrt(h2/d2);
    out = {p-h, p+h};
    }
    return 1 + sgn(h2);
}
int tangents(pt o1, double r1, pt o2, double r2, bool inner, vector<pair<pt,pt>> &out) {
    if (inner) r2 = -r2;
    pt d = o2-o1;
    double dr = r1-r2, d2 = sq(d), h2 = d2-dr*dr;
    if (d2 == 0 || h2 < 0) {assert(h2 != 0); return 0;}
    for (double sign : {-1,1}) {
    pt v = (d*dr + perp(d)*sqrt(h2)*sign)/d2;
    out.push_back({o1 + v*r1, o2 + v*r2});
    }
    return 1 + (h2 > 0);
}
int main() {
    pt p{3,-4};
    cout<<p.x<<" "<<p.y<<endl;
    cout<<p<<endl;
    pt a{3,1}, b{1,-2};
    a += 2.0*b;
    cout<<a<<endl;
    return 0;
}
```

9.4 Other geometry template

9.5 2d-base.cpp

118 lines

```
struct point_t {
 double x, y;
 point_t() { }
 point_t(double tx, double ty) : x(tx), y(ty) { }
 point_t operator-(const point_t &r) const { return point_t(x - r.x, y - r.y); }
 point_t operator+(const point_t &r) const { return point_t(x + r.x, y + r.y); }
 point_t operator*(double r) const { return point_t(x * r, y * r); }
 point_t operator/(double r) const { return point_t(x / r, y / r); }
 point_t rot90() const { return point_t(-y, x); }
 double l() const { return sqrt(x * x + y * y); }
 void read() { scanf("%lf%lf", &x, &y); }
};
```

```
int dblcmp(double x) {
  return (x < -eps ? -1 : x > eps);
}
double dist(point_t p1, point_t p2) {
  return (p2 - p1).l();
}
double cross(point_t p1, point_t p2) {
  return p1.x * p2.y - p2.x * p1.y;
}
double dot(point_t p1, point_t p2) {
  return p1.x * p2.x + p1.y * p2.y;
}
// count-clock wise is positive direction
double angle(point_t p1, point_t p2) {
  double x1 = p1.x, y1 = p1.y, x2 = p2.x, y2 = p2.y;
  double a1 = atan2(y1, x1), a2 = atan2(y2, x2);
  double a = a2 - a1;
  while (a < -pi) a += 2 * pi;
  while (a >= pi) a -= 2 * pi;
  return a;
}
bool onSeg(point_t p, point_t a, point_t b) {
  return dblcmp(cross(a - p, b - p)) == 0 && dblcmp(dot(a - p, b - p)) <=
    0;
}
// 1 normal intersected, -1 denormal intersected, 0 not intersected
int testSS(point_t a, point_t b, point_t c, point_t d) {
  if (dblcmp(max(a.x, b.x) - min(c.x, d.x)) < 0) return 0;
  if (dblcmp(max(c.x, d.x) - min(a.x, b.x)) < 0) return 0;
  if (dblcmp(max(a.y, b.y) - min(c.y, d.y)) < 0) return 0;
  if (dblcmp(max(c.y, d.y) - min(a.y, b.y)) < 0) return 0;
  int d1 = dblcmp(cross(c - a, b - a));
  int d2 = dblcmp(cross(d - a, b - a));
  int d3 = dblcmp(cross(a - c, d - c));
  int d4 = dblcmp(cross(b - c, d - c));
  if ((d1 * d2 < 0) && (d3 * d4 < 0)) return 1;
  if ((d1 * d2 <= 0 && d3 * d4 == 0) || (d1 * d2 == 0 && d3 * d4 <= 0))
    return -1;
  return 0;
}
vector<point_t> isLL(point_t a, point_t b, point_t c, point_t d) {
  point_t p1 = b - a, p2 = d - c;
  vector<point_t> ret;
  double a1 = p1.y, b1 = -p1.x, c1;
  double a2 = p2.y, b2 = -p2.x, c2;
  if (dblcmp(a1 * b2 - a2 * b1) == 0) return ret; // colined <=> a1*c2-a2
    *c1=0 && b1*c2-b2*c1=0
  else {
    c1 = a1 * a.x + b1 * a.y;
    c2 = a2 * c.x + b2 * c.y;
    ret.push_back(point_t((c1 * b2 - c2 * b1) / (a1 * b2 - a2 * b1), (c1
      * a2 - c2 * a1) / (b1 * a2 - b2 * a1)));
    return ret;
  }
}
point_t angle_bisector(point_t p0, point_t p1, point_t p2) {
  point_t v1 = p1 - p0, v2 = p2 - p0;
  v1 = v1 / dist(v1) * dist(v2);
  return v1 + v2 + p0;
}
point_t perpendicular_bisector(point_t p1, point_t p2) {
  point_t v = p2 - p1;
  swap(v.x, v.y);
  v.x = -v.x;
  return v + (p1 + p2) / 2;
}
point_t circumcenter(point_t p0, point_t p1, point_t p2) {
  point_t v1 = perpendicular_bisector(p0, p1);
  point_t v2 = perpendicular_bisector(p1, p2);
  return isLL((p0 + p1) / 2, v1, (p1 + p2) / 2, v2);
}
point_t incenter(point_t p0, point_t p1, point_t p2) {
  point_t v1 = angle_bisector(p0, p1, p2);
  point_t v2 = angle_bisector(p1, p2, p0);
  return isLL(p0, v1, p1, v2);
}
point_t orthocenter(point_t p0, point_t p1, point_t p2) {
  return p0 + p1 + p2 - circumcenter(p0, p1, p2) * 2;
}
// count-clock wise is positive direction
point_t rotate(point_t p, double a) {
  double s = sin(a), c = cos(a);
  return point_t(p.x * c - p.y * s, p.y * c + p.x * s);
}
bool insidePoly(point_t *p, int n, point_t t) {
  p[0] = p[n];
  for (int i = 0; i < n; ++i) if (onSeg(t, p[i], p[i + 1])) return true;
  point_t r = point_t(2353456.663, 5326546.243); // random point
  int cnt = 0;
  for (int i = 0; i < n; ++i) {
    if (testSS(t, r, p[i], p[i + 1]) != 0) ++cnt;
  }
  return cnt & 1;
}
bool insideConvex(point_t *convex, int n, point_t t) { // O(logN),
    convex polygen, cross(p[2] - p[1], p[3] - p[1]) > 0
  if (n == 2) return onSeg(t, convex[1], convex[2]);
  int l = 2, r = n;
  while (l < r) {
    int mid = (l + r) / 2 + 1;
    int side = dblcmp(cross(convex[mid] - convex[1], t - convex[1]));
    if (side == 1) l = mid;
    else r = mid - 1;
  }
  int s = dblcmp(cross(convex[l] - convex[1], t - convex[1]));
  if (s == -1 || l == n) return false;
  point_t v = convex[l + 1] - convex[l];
  if (dblcmp(cross(v, t - convex[l])) >= 0) return true;
  return false;
}
```

9.6 3dHull

Description: Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.
Time: $\mathcal{O}\left(n^2\right)$

46 lines

```
#include "Point3D.h"
typedef Point3D<double> P3;
struct PR {
  void ins(int x) { (a == -1 ? a : b) = x; }
  void rem(int x) { (a == x ? a : b) = -1; }
  int cnt() { return (a != -1) + (b != -1); }
  int a, b;
};
struct F { P3 q; int a, b, c; };
vector<F> hull3d(const vector<P3>& A) {
  assert(sz(A) >= 4);
  vector<vector<PR>> E(sz(A), vector<PR>(sz(A), {-1, -1}));
#define E(x,y) E[f.x][f.y]
  vector<F> FS;
  auto mf = [&](int i, int j, int k, int l) {
    P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
    if (q.dot(A[l]) > q.dot(A[i]))
      q = q * -1;
    F f{q, i, j, k};
    E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
    FS.push_back(f);
  };
  rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
    mf(i, j, k, 6 - i - j - k);
  rep(i,4,sz(A)) {
    rep(j,0,sz(FS)) {
      F f = FS[j];
      if(f.q.dot(A[i]) > f.q.dot(A[f.a])) {
        E(a,b).rem(f.c);
        E(a,c).rem(f.b);
        E(b,c).rem(f.a);
        swap(FS[j--], FS.back());
        FS.pop_back();
      }
    }
```

```
  int nw = sz(FS);
  rep(j,0,nw) {
   F f = FS[j];
#define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i, f.c);
   C(a, b, c); C(a, c, b); C(b, c, a);
  }
 }
 for (F& it : FS) if ((A[it.b] - A[it.a]).cross(
  A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
 return FS;
};
```

9.7 Angle.h

Description: A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.

Usage:
```
vector<Angle> v = {w[0], w[0].t360() ...}; // sorted
int j = 0; rep(i,0,n) { while (v[j] < v[i].t180()) ++j; }
// sweeps j such that (j-i) represents the number of positively oriented triangles with
vertices at 0 and i
```

34 lines

```
struct Angle {
 int x, y;
 int t;
 Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
 Angle operator-(Angle b) const { return {x-b.x, y-b.y, t}; }
 int half() const {
  assert(x || y);
  return y < 0 || (y == 0 && x < 0);
 }
 Angle t90() const { return {-y, x, t + (half() && x >= 0)}; }
 Angle t180() const { return {-x, -y, t + half()}; }
 Angle t360() const { return {x, y, t + 1}; }
};
bool operator<(Angle a, Angle b) {
 // add a.dist2() and b.dist2() to also compare distances
 return make_tuple(a.t, a.half(), a.y * (ll)b.x) <
       make_tuple(b.t, b.half(), a.x * (ll)b.y);
}
// Given two points, this calculates the smallest angle between
// them, i.e., the angle that covers the defined line segment.
pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
 if (b < a) swap(a, b);
 return (b < a.t180() ?
        make_pair(a, b) : make_pair(b, a.t360()));
}
Angle operator+(Angle a, Angle b) { // point a + vector b
 Angle r(a.x + b.x, a.y + b.y, a.t);
 if (a.t180() < r) r.t--;
 return r.t180() < a ? r.t360() : r;
}
Angle angleDiff(Angle a, Angle b) { // angle b - angle a
 int tu = b.t - a.t; a.t = b.t;
 return {a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b < a)};
}
```

9.8 CircleIntersection.h

Description: Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

12 lines

```
#include "Point.h"
typedef Point<double> P;
bool circleInter(P a,P b,double r1,double r2,pair<P, P>* out) {
 if (a == b) { assert(r1 != r2); return false; }
 P vec = b - a;
 double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2,
       p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*d2;
 if (sum*sum < d2 || dif*dif > d2) return false;
 P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) / d2);
 *out = {mid + per, mid - per};
 return true;
}
```

9.9 CircleLine.h

Description: Finds the intersection between a circle and a line. Returns a vector of either 0, 1, or 2 intersection points. P is intended to be Point<double>.

10 lines

```
#include "Point.h"
template<class P>
vector<P> circleLine(P c, double r, P a, P b) {
 P ab = b - a, p = a + ab * (c-a).dot(ab) / ab.dist2();
 double s = a.cross(b, c), h2 = r*r - s*s / ab.dist2();
 if (h2 < 0) return {};
 if (h2 == 0) return {p};
 P h = ab.unit() * sqrt(h2);
 return {p - h, p + h};
}
```

9.10 CirclePolygonIntersection

Description: Returns the area of the intersection of a circle with a ccw polygon.
Time: $\mathcal{O}(n)$

20 lines

```
#include "../../content/geometry/Point.h"
typedef Point<double> P;
#define arg(p, q) atan2(p.cross(q), p.dot(q))
double circlePoly(P c, double r, vector<P> ps) {
 auto tri = [&](P p, P q) {
  auto r2 = r * r / 2;
  P d = q - p;
  auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.dist2();
  auto det = a * a - b;
  if (det <= 0) return arg(p, q) * r2;
  auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det));
  if (t < 0 || 1 <= s) return arg(p, q) * r2;
  P u = p + d * s, v = p + d * t;
  return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;
 };
 auto sum = 0.0;
 rep(i,0,sz(ps))
  sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c);
 return sum;
}
```

9.11 CircleTangents.h

Description: Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

14 lines

```
#include "Point.h"
template<class P>
vector<pair<P, P>> tangents(P c1, double r1, P c2, double r2) {
 P d = c2 - c1;
 double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
 if (d2 == 0 || h2 < 0) return {};
 vector<pair<P, P>> out;
 for (double sign : {-1, 1}) {
  P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
  out.push_back({c1 + v * r1, c2 + v * r2});
 }
 if (h2 == 0) out.pop_back();
 return out;
}
```

9.12 ClosestPair

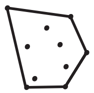
Description: Finds the closest pair of points.
Time: $\mathcal{O}(n \log n)$

18 lines

```
#include "Point.h"
typedef Point<ll> P;
pair<P, P> closest(vector<P> v) {
 assert(sz(v) > 1);
 set<P> S;
 sort(all(v), [](P a, P b) { return a.y < b.y; });
 pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
 int j = 0;
 for (P p : v) {
```

```
  P d{1 + (ll)sqrt(ret.first), 0};
  while (v[j].y <= p.y - d.x) S.erase(v[j++]);
  auto lo = S.lower_bound(p - d), hi = S.upper_bound(p + d);
  for (; lo != hi; ++lo)
   ret = min(ret, {(*lo - p).dist2(), {*lo, p}});
  S.insert(p);
 }
 return ret.second;
}
```

9.13 ConvexHull



Description:
Returns a vector of the points of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull.
Time: $\mathcal{O}(n \log n)$

14 lines

```
#include "Point.h"
typedef Point<ll> P;
vector<P> convexHull(vector<P> pts) {
 if (sz(pts) <= 1) return pts;
 sort(all(pts));
 vector<P> h(sz(pts)+1);
 int s = 0, t = 0;
 for (int it = 2; it--; s = --t, reverse(all(pts)))
  for (P p : pts) {
   while (t >= s + 2 && h[t-2].cross(h[t-1], p) <= 0) t--;
   h[t++] = p;
  }
 return {h.begin(), h.begin() + t - (t == 2 && h[0] == h[1])};
}
```

9.14 DelaunayTriangulation

Description: Computes the Delaunay triangulation of a set of points. Each circumcircle contains none of the input points. If any three points are collinear or any four are on the same circle, behavior is undefined.
Time: $\mathcal{O}(n^2)$

12 lines

```
#include "Point.h"
#include "3dHull.h"
template<class P, class F>
void delaunay(vector<P>& ps, F trifun) {
 if (sz(ps) == 3) { int d = (ps[0].cross(ps[1], ps[2]) < 0);
  trifun(0,1+d,2-d); }
 vector<P3> p3;
 for (P p : ps) p3.emplace_back(p.x, p.y, p.dist2());
 if (sz(ps) > 3) for(auto t:hull3d(p3)) if ((p3[t.b]-p3[t.a]).
   cross(p3[t.c]-p3[t.a]).dot(P3(0,0,1)) < 0)
  trifun(t.a, t.c, t.b);
}
```

9.15 FastDelaunay

Description: Fast Delaunay triangulation. Each circumcircle contains none of the input points. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order {t[0][0], t[0][1], t[0][2], t[1][0], ...}, all counter-clockwise.
Time: $\mathcal{O}(n \log n)$

83 lines

```
#include "Point.h"
typedef Point<ll> P;
typedef struct Quad* Q;
typedef __int128_t lll; // (can be ll if coords are < 2e4)
P arb(LLONG_MAX,LLONG_MAX); // not equal to any other point
struct Quad {
 Q rot, o; P p = arb; bool mark;
 P& F() { return r()->p; }
 Q& r() { return rot->rot; }
 Q prev() { return rot->o->rot; }
 Q next() { return r()->prev(); }
} *H;
bool circ(P p, P a, P b, P c) { // is p in the circumcircle?
 lll p2 = p.dist2(), A = a.dist2()-p2,
     B = b.dist2()-p2, C = c.dist2()-p2;
 return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B > 0;
}
Q makeEdge(P orig, P dest) {
 Q r = H ? H : new Quad{new Quad{new Quad{new Quad{0}}}};
 H = r->o; r->r()->r() = r;
 rep(i,0,4) r = r->rot, r->p = arb, r->o = i & 1 ? r : r->r();
 r->p = orig; r->F() = dest;
 return r;
}
void splice(Q a, Q b) {
 swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
}
Q connect(Q a, Q b) {
 Q q = makeEdge(a->F(), b->p);
 splice(q, a->next());
 splice(q->r(), b);
 return q;
}
pair<Q,Q> rec(const vector<P>& s) {
 if (sz(s) <= 3) {
  Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back());
  if (sz(s) == 2) return { a, a->r() };
  splice(a->r(), b);
  auto side = s[0].cross(s[1], s[2]);
  Q c = side ? connect(b, a) : 0;
  return {side < 0 ? c->r() : a, side < 0 ? c : b->r() };
 }
#define H(e) e->F(), e->p
#define valid(e) (e->F().cross(H(base)) > 0)
 Q A, B, ra, rb;
 int half = sz(s) / 2;
 tie(ra, A) = rec({all(s) - half});
 tie(B, rb) = rec({sz(s) - half + all(s)});
 while ((B->p.cross(H(A)) < 0 && (A = A->next())) ||
        (A->p.cross(H(B)) > 0 && (B = B->r()->o)));
 Q base = connect(B->r(), A);
 if (A->p == ra->p) ra = base->r();
 if (B->p == rb->p) rb = base;
#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
  while (circ(e->dir->F(), H(base), e->F())) { \
   Q t = e->dir; \
   splice(e, e->prev()); \
   splice(e->r(), e->r()->prev()); \
   e->o = H; H = e; e = t; \
  }
 for (;;) {
  DEL(LC, base->r(), o); DEL(RC, base, prev());
  if (!valid(LC) && !valid(RC)) break;
  if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
   base = connect(RC, base->r());
  else
   base = connect(base->r(), LC->r());
 }
 return { ra, rb };
}
vector<P> triangulate(vector<P> pts) {
 sort(all(pts)); assert(unique(all(pts)) == pts.end());
 if (sz(pts) < 2) return {};
 Q e = rec(pts).first;
 vector<Q> q = {e};
 int qi = 0;
 while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->p); \
 q.push_back(c->r()); c = c->next(); } while (c != e); }
 ADD; pts.clear();
 while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
 return pts;
}
```

9.16 HullDiameter.h

Description: Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points).

13 lines

```
#include "Point.h"
typedef Point<ll> P;
array<P, 2> hullDiameter(vector<P> S) {
 int n = sz(S), j = n < 2 ? 0 : 1;
 pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
 rep(i,0,j)
```

```
    for (;; j = (j + 1) % n) {
      res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j]}});
      if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >= 0)
        break;
    }
  return res.second;
}
```

9.17 InsidePolygon

Description: Returns true if p lies within the polygon. If strict is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow.
Usage: `vector<P> v = {P{4,4}, P{1,2}, P{2,1}};`
`bool in = inPolygon(v, P{3, 3}, false);`
Time: $\mathcal{O}(n)$

14 lines

```
#include "Point.h"
#include "OnSegment.h"
#include "SegmentDistance.h"
template<class P>
bool inPolygon(vector<P> &p, P a, bool strict = true) {
  int cnt = 0, n = sz(p);
  rep(i,0,n) {
    P q = p[(i + 1) % n];
    if (onSegment(p[i], q, a)) return !strict;
    //or: if (segDist(p[i], q, a) <= eps) return !strict;
    cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q) > 0;
  }
  return cnt;
}
```

9.18 LineHullIntersection

Description: Line-convex polygon intersection. The polygon must be ccw and have no collinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: • $(-1,-1)$ if no collision, • $(i,-1)$ if touching the corner i, • (i,i) if along side $(i,i+1)$, • (i,j) if crossing sides $(i,i+1)$ and $(j,j+1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i,i+1)$. The points are returned in the same order as the line hits the polygon. `extrVertex` returns the point of a hull with the max projection onto a line.
Time: $\mathcal{O}(\log n)$

39 lines

```
#include "Point.h"
#define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%n]))
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
template <class P> int extrVertex(vector<P>& poly, P dir) {
  int n = sz(poly), lo = 0, hi = n;
  if (extr(0)) return 0;
  while (lo + 1 < hi) {
    int m = (lo + hi) / 2;
    if (extr(m)) return m;
    int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
    (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) = m;
  }
  return lo;
}
#define cmpL(i) sgn(a.cross(poly[i], b))
template <class P>
array<int, 2> lineHull(P a, P b, vector<P>& poly) {
  int endA = extrVertex(poly, (a - b).perp());
  int endB = extrVertex(poly, (b - a).perp());
  if (cmpL(endA) < 0 || cmpL(endB) > 0)
    return {-1, -1};
  array<int, 2> res;
  rep(i,0,2) {
    int lo = endB, hi = endA, n = sz(poly);
    while ((lo + 1) % n != hi) {
      int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n;
      (cmpL(m) == cmpL(endB) ? lo : hi) = m;
    }
    res[i] = (lo + !cmpL(hi)) % n;
    swap(endA, endB);
  }
  if (res[0] == res[1]) return {res[0], -1};
  if (!cmpL(res[0]) && !cmpL(res[1]))
    switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
      case 0: return {res[0], res[0]};
      case 2: return {res[1], res[1]};
    }
  return res;
}
```

9.19 LineProjectionReflection.h

Description: Projects point p onto line ab. Set refl=true to get reflection of point p across line ab insted. The wrong point will be returned if P is an integer point and the desired point doesn't have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow.

6 lines

```
#include "Point.h"
template<class P>
P lineProj(P a, P b, P p, bool refl=false) {
  P v = b - a;
  return p - v.perp()*(1+refl)*v.cross(p-a)/v.dist2();
}
```

9.20 ManhattanMST

Description: Given N points, returns up to 4*N edges, which are guaranteed to contain a minimum spanning tree for the graph with edge weights w(p, q) = |p.x - q.x| + |p.y - q.y|. Edges are in the form (distance, src, dst). Use a standard MST algorithm on the result to find the final MST.
Time: $\mathcal{O}(N \log N)$

24 lines

```
#include "Point.h"
typedef Point<int> P;
vector<array<int, 3>> manhattanMST(vector<P> ps) {
  vi id(sz(ps));
  iota(all(id), 0);
  vector<array<int, 3>> edges;
  rep(k,0,4) {
    sort(all(id), [&](int i, int j) {
        return (ps[i]-ps[j]).x < (ps[j]-ps[i]).y;});
    map<int, int> sweep;
    for (int i : id) {
      for (auto it = sweep.lower_bound(-ps[i].y);
                it != sweep.end(); sweep.erase(it++)) {
        int j = it->second;
        P d = ps[i] - ps[j];
        if (d.y > d.x) break;
        edges.push_back({d.y + d.x, i, j});
      }
      sweep[-ps[i].y] = i;
    }
    for (P& p : ps) if (k & 1) p.x = -p.x; else swap(p.x, p.y);
  }
  return edges;
}
```

9.21 MinimumEnclosingCircle

Description: Computes the minimum circle that encloses a set of points.
Time: expected $\mathcal{O}(n)$

18 lines

```
#include "circumcircle.h"
pair<P, double> mec(vector<P> ps) {
  shuffle(all(ps), mt19937(time(0)));
  P o = ps[0];
  double r = 0, EPS = 1 + 1e-8;
  rep(i,0,sz(ps)) if ((o - ps[i]).dist() > r * EPS) {
    o = ps[i], r = 0;
    rep(j,0,i) if ((o - ps[j]).dist() > r * EPS) {
      o = (ps[i] + ps[j]) / 2;
      r = (o - ps[i]).dist();
      rep(k,0,j) if ((o - ps[k]).dist() > r * EPS) {
        o = ccCenter(ps[i], ps[j], ps[k]);
        r = (o - ps[i]).dist();
      }
    }
  }
  return {o, r};
}
```

9.22 OnSegment.h

Description: Returns true iff p lies on the line segment from s to e. Use `(segDist(s,e,p)<=epsilon)` instead when using Point<double>.

4 lines

```
#include "Point.h"
template<class P> bool onSegment(P s, P e, P p) {
  return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
}
```

9.23 Point.h

Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

28 lines

```
template <class T> int sgn(T x) { return (x > 0) - (x < 0); }
template<class T>
struct Point {
  typedef Point P;
  T x, y;
  explicit Point(T x=0, T y=0) : x(x), y(y) {}
  bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y); }
  bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); }
  P operator+(P p) const { return P(x+p.x, y+p.y); }
  P operator-(P p) const { return P(x-p.x, y-p.y); }
  P operator*(T d) const { return P(x*d, y*d); }
  P operator/(T d) const { return P(x/d, y/d); }
  T dot(P p) const { return x*p.x + y*p.y; }
  T cross(P p) const { return x*p.y - y*p.x; }
  T cross(P a, P b) const { return (a-*this).cross(b-*this); }
  T dist2() const { return x*x + y*y; }
  double dist() const { return sqrt((double)dist2()); }
  // angle to x-axis in interval [-pi, pi]
  double angle() const { return atan2(y, x); }
  P unit() const { return *this/dist(); } // makes dist()=1
  P perp() const { return P(-y, x); } // rotates +90 degrees
  P normal() const { return perp().unit(); }
  // returns point rotated 'a' radians ccw around the origin
  P rotate(double a) const {
    return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
  friend ostream& operator<<(ostream& os, P p) {
    return os << "(" << p.x << "," << p.y << ")"; }
};
```

9.24 Point3D.h

Description: Class to handle points in 3D space. T can be e.g. double or long long.

32 lines

```
template<class T> struct Point3D {
  typedef Point3D P;
  typedef const P& R;
  T x, y, z;
  explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z) {}
  bool operator<(R p) const {
    return tie(x, y, z) < tie(p.x, p.y, p.z); }
  bool operator==(R p) const {
    return tie(x, y, z) == tie(p.x, p.y, p.z); }
  P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
  P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
  P operator*(T d) const { return P(x*d, y*d, z*d); }
  P operator/(T d) const { return P(x/d, y/d, z/d); }
  T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
  P cross(R p) const {
    return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
  }
  T dist2() const { return x*x + y*y + z*z; }
  double dist() const { return sqrt((double)dist2()); }
  //Azimuthal angle (longitude) to x-axis in interval [-pi, pi]
  double phi() const { return atan2(y, x); }
  //Zenith angle (latitude) to the z-axis in interval [0, pi]
  double theta() const { return atan2(sqrt(x*x+y*y),z); }
  P unit() const { return *this/(T)dist(); } //makes dist()=1
  //returns unit vector normal to *this and p
  P normal(P p) const { return cross(p).unit(); }
  //returns point rotated 'angle' radians ccw around axis
  P rotate(double angle, P axis) const {
    double s = sin(angle), c = cos(angle); P u = axis.unit();
    return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
  }
};
```

9.25 PointInsideHull

Description: Determine whether a point t lies inside a convex hull (CCW order, with no collinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.
Time: $\mathcal{O}(\log N)$

16 lines

```
#include "Point.h"
#include "sideOf.h"
#include "OnSegment.h"
typedef Point<ll> P;
bool inHull(const vector<P>& l, P p, bool strict = true) {
  int a = 1, b = sz(l) - 1, r = !strict;
  if (sz(l) < 3) return r && onSegment(l[0], l.back(), p);
  if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
  if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p)<= -r)
    return false;
  while (abs(a - b) > 1) {
    int c = (a + b) / 2;
    (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
  }
  return sgn(l[a].cross(l[b], p)) < r;
}
```

9.26 PolygonArea.h

Description: Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

7 lines

```
#include "Point.h"
template<class T>
T polygonArea2(vector<Point<T>>& v) {
  T a = v.back().cross(v[0]);
  rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
  return a;
}
```

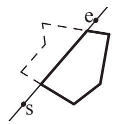
9.27 PolygonCenter

Description: Returns the center of mass for a polygon.
Time: $\mathcal{O}(n)$

10 lines

```
#include "Point.h"
typedef Point<double> P;
P polygonCenter(const vector<P>& v) {
  P res(0, 0); double A = 0;
  for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) {
    res = res + (v[i] + v[j]) * v[j].cross(v[i]);
    A += v[j].cross(v[i]);
  }
  return res / A / 3;
}
```

9.28 PolygonCut.h



Description:
Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.
Usage: `vector<P> p = ...;`
`p = polygonCut(p, P(0,0), P(1,0));`

15 lines

```
#include "Point.h"
#include "lineIntersection.h"
typedef Point<double> P;
vector<P> polygonCut(const vector<P>& poly, P s, P e) {
  vector<P> res;
  rep(i,0,sz(poly)) {
    P cur = poly[i], prev = i ? poly[i-1] : poly.back();
    bool side = s.cross(e, cur) < 0;
    if (side != (s.cross(e, prev) < 0))
      res.push_back(lineInter(s, e, cur, prev).second);
    if (side)
      res.push_back(cur);
  }
  return res;
}
```

9.29 PolygonUnion

Description: Calculates the area of the union of n polygons (not necessarily convex). The points within each polygon must be given in CCW order. (Epsilon checks may optionally be added to sideOf/sgn, but shouldn't be needed.)
Time: $\mathcal{O}\left(N^2\right)$, where N is the total number of points

35 lines

```
#include "Point.h"
#include "sideOf.h"
typedef Point<double> P;
double rat(P a, P b) { return sgn(b.x) ? a.x/b.x : a.y/b.y; }
double polyUnion(vector<vector<P>>& poly) {
 double ret = 0;
 rep(i,0,sz(poly)) rep(v,0,sz(poly[i])) {
  P A = poly[i][v], B = poly[i][(v + 1) % sz(poly[i])];
  vector<pair<double, int>> segs = {{0, 0}, {1, 0}};
  rep(j,0,sz(poly)) if (i != j) {
   rep(u,0,sz(poly[j])) {
    P C = poly[j][u], D = poly[j][(u + 1) % sz(poly[j])];
    int sc = sideOf(A, B, C), sd = sideOf(A, B, D);
    if (sc != sd) {
     double sa = C.cross(D, A), sb = C.cross(D, B);
     if (min(sc, sd) < 0)
      segs.emplace_back(sa / (sa - sb), sgn(sc - sd));
    } else if (!sc && !sd && j<i && sgn((B-A).dot(D-C))>0){
     segs.emplace_back(rat(C - A, B - A), 1);
     segs.emplace_back(rat(D - A, B - A), -1);
    }
   }
  }
  sort(all(segs));
  for (auto& s : segs) s.first = min(max(s.first, 0.0), 1.0);
  double sum = 0;
  int cnt = segs[0].second;
  rep(j,1,sz(segs)) {
   if (!cnt) sum += segs[j].first - segs[j - 1].first;
   cnt += segs[j].second;
  }
  ret += A.cross(B) * sum;
 }
 return ret / 2;
}
```

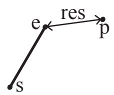
9.30 PolyhedronVolume.h

Description: Magic formula for the volume of a polyhedron. Faces should point outwards.

6 lines

```
template<class V, class L>
double signedPolyVolume(const V& p, const L& trilist) {
 double v = 0;
 for (auto i : trilist) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
 return v / 6;
}
```

9.31 SegmentDistance.h



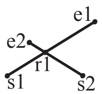
Description:
Returns the shortest distance between point p and the line segment from point s to e.
Usage: `Point<double> a, b(2,2), p(1,1);`
`bool onSegment = segDist(a,b,p) < 1e-10;`

7 lines

```
#include "Point.h"
typedef Point<double> P;
double segDist(P& s, P& e, P& p) {
 if (s==e) return (p-s).dist();
 auto d = (e-s).dist2(), t = min(d,max(.0,(p-s).dot(e-s)));
 return ((p-s)*d-(e-s)*t).dist()/d;
}
```

9.32 SegmentIntersection.h

Description:
If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.



Usage: `vector<P> inter = segInter(s1,e1,s2,e2);`
`if (sz(inter)==1)`
`cout << "segments intersect at " << inter[0] << endl;`

15 lines

```
#include "Point.h"
#include "OnSegment.h"
template<class P> vector<P> segInter(P a, P b, P c, P d) {
 auto oa = c.cross(d, a), ob = c.cross(d, b),
      oc = a.cross(b, c), od = a.cross(b, d);
 // Checks if intersection is single non-endpoint point.
 if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
  return {(a * ob - b * oa) / (ob - oa)};
 set<P> s;
 if (onSegment(c, d, a)) s.insert(a);
 if (onSegment(c, d, b)) s.insert(b);
 if (onSegment(a, b, c)) s.insert(c);
 if (onSegment(a, b, d)) s.insert(d);
 return {all(s)};
}
```

9.33 antipodalPoints.cpp

41 lines

```
/*
 Antipodal points
 Tested: AIZU(judge.u-aizu.ac.jp) CGL.4B
 Complexity: O(n)
*/
vector<pair<int, int>> antipodal(const polygon &P)
{
 vector<pair<int, int>> ans;
 int n = P.size();
 if (P.size() == 2)
  ans.push_back({ 0, 1 });
 if (P.size() < 3)
  return ans;
 int q0 = 0;
 while (abs(area2(P[n - 1], P[0], P[NEXT(q0)]))
    > abs(area2(P[n - 1], P[0], P[q0])))
   ++q0;
 for (int q = q0, p = 0; q != 0 && p <= q0; ++p)
 {
  ans.push_back({ p, q });
  while (abs(area2(P[p], P[NEXT(p)], P[NEXT(q)]))
     > abs(area2(P[p], P[NEXT(p)], P[q])))
  {
   q = NEXT(q);
   if (p != q0 || q != 0)
    ans.push_back({ p, q });
   else
    return ans;
  }
  if (abs(area2(P[p], P[NEXT(p)], P[NEXT(q)]))
     == abs(area2(P[p], P[NEXT(p)], P[q])))
  {
   if (p != q0 || q != n - 1)
    ans.push_back({ p, NEXT(q) });
   else
    ans.push_back({ NEXT(p), q });
  }
 }
 return ans;
}
```

9.34 basics.cpp

32 lines

```
typedef complex<double> point;
typedef vector<point> polygon;
#define NEXT(i) (((i) + 1) % n)
struct circle { point p; double r; };
struct line { point p, q; };
using segment = line;
const double eps = 1e-9;
// fix comparations on doubles with this two functions
int sign(double x) { return x < -eps ? -1 : x > eps; }
int dblcmp(double x, double y) { return sign(x - y); }
double dot(point a, point b) { return real(conj(a) * b); }
double cross(point a, point b) { return imag(conj(a) * b); }
```

```
double area2(point a, point b, point c) { return cross(b - a, c - a); }
int ccw(point a, point b, point c)
{
  b -= a; c -= a;
  if (cross(b, c) > 0) return +1; // counter clockwise
  if (cross(b, c) < 0) return -1; // clockwise
  if (dot(b, c) < 0) return +2; // c--a--b on line
  if (dot(b, b) < dot(c, c)) return -2; // a--b--c on line
  return 0;
}
namespace std
{
  bool operator<(point a, point b)
  {
    if (a.real() != b.real())
      return a.real() < b.real();
    return a.imag() < b.imag();
  }
}
```

9.35 circle.cpp

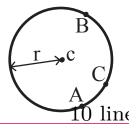
120 lines

```
/*
  Circles
  Tested: AIZU
*/
// circle-circle intersection
vector<point> intersect(circle C, circle D)
{
  double d = abs(C.p - D.p);
  if (sign(d - C.r - D.r) > 0) return { }; // too far
  if (sign(d - abs(C.r - D.r)) < 0) return { }; // too close
  double a = (C.r*C.r - D.r*D.r + d*d) / (2*d);
  double h = sqrt(C.r*C.r - a*a);
  point v = (D.p - C.p) / d;
  if (sign(h) == 0) return {C.p + v*a}; // touch
  return {C.p + v*a + point(0,1)*v*h, // intersect
    C.p + v*a - point(0,1)*v*h};
}
// circle-line intersection
vector<point> intersect(line L, circle C)
{
  point u = L.p - L.q, v = L.p - C.p;
  double a = dot(u, u), b = dot(u, v), c = dot(v, v) - C.r*C.r;
  double det = b*b - a*c;
  if (sign(det) < 0) return { }; // no solution
  if (sign(det) == 0) return {L.p - b/a*u}; // touch
  return {L.p + (-b + sqrt(det))/a*u,
    L.p + (-b - sqrt(det))/a*u};
}
// circle tangents through point
vector<point> tangent(point p, circle C)
{
  // not tested enough
  double D = abs(p - C.p);
  if (D + eps < C.r) return { };
  point t = C.p - p;
  double theta = asin( C.r / D );
  double d = cos(theta) * D;
  t = t / abs(t) * d;
  if ( abs(D - C.r) < eps ) return {p + t};
  point rot( cos(theta), sin(theta) );
  return {p + t * rot, p + t * conj(rot)};
}
bool incircle(point a, point b, point c, point p)
{
  a -= p; b -= p; c -= p;
  return norm(a) * cross(b, c)
      + norm(b) * cross(c, a)
      + norm(c) * cross(a, b) >= 0;
      // < : inside, = cocircular, > outside
}
point three_point_circle(point a, point b, point c)
{
  point x = 1.0 / conj(b - a), y = 1.0 / conj(c - a);
  return (y - x) / (conj(x) * y - x * conj(y)) + a;
}
/*
   Get the center of the circles that pass through p0 and p1
   and has ratio r.
   Be careful with epsilon.
*/
vector<point> two_point_ratio_circle(point p0, point p1, double r) {
   if (abs(p1 - p0) > 2 * r + eps) // Points are too far.
      return {};
   point pm = (p1 + p0) / 2.0l;
   point pv = p1 - p0;
   pv = point(-pv.imag(), pv.real());
   double x1 = p1.real(), y1 = p1.imag();
   double xm = pm.real(), ym = pm.imag();
   double xv = pv.real(), yv = pv.imag();
   double A = (sqr(xv) + sqr(yv));
   double C = sqr(xm - x1) + sqr(ym - y1) - sqr(r);
   double D = sqrt( - 4 * A * C );
   double t = D / 2.0 / A;
   if (abs(t) <= eps)
      return {pm};
   return {c1, c2};
}
/*
  Area of the intersection of a circle with a polygon
  Circle's center lies in (0, 0)
  Polygon must be given counterclockwise
  Tested: LightOJ 1358
  Complexity: O(n)
*/
#define x(_t) (xa + (_t) * a)
#define y(_t) (ya + (_t) * b)
double radian(double xa, double ya, double xb, double yb)
{
  return atan2(xa * yb - xb * ya, xa * xb + ya * yb);
}
double part(double xa, double ya, double xb, double yb, double r)
{
  double l = sqrt((xa - xb) * (xa - xb) + (ya - yb) * (ya - yb));
  double a = (xb - xa) / l, b = (yb - ya) / l, c = a * xa + b * ya;
  double d = 4.0 * (c * c - xa * xa - ya * ya + r * r);
  if (d < eps)
    return radian(xa, ya, xb, yb) * r * r * 0.5;
  else
  {
    d = sqrt(d) * 0.5;
    double s = -c - d, t = -c + d;
    if (s < 0.0) s = 0.0;
    else if (s > l) s = l;
    if (t < 0.0) t = 0.0;
    else if (t > l) t = l;
    return (x(s) * y(t) - x(t) * y(s)
        + (radian(xa, ya, x(s), y(s))
        + radian(x(t), y(t), xb, yb)) * r * r) * 0.5;
  }
}
double intersection_circle_polygon(const polygon &P, double r)
{
  double s = 0.0;
  int n = P.size();;
  for (int i = 0; i < n; i++)
    s += part(P[i].real(), P[i].imag(),
      P[NEXT(i)].real(), P[NEXT(i)].imag(), r);
  return fabs(s);
}
```

9.36 circumcircle.h

Description:

The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.



10 lines

```
#include "Point.h"
typedef Point<double> P;
double ccRadius(const P& A, const P& B, const P& C) {
```

```
  return (B-A).dist()*(C-B).dist()*(A-C).dist()/
    abs((B-A).cross(C-A))/2;
}
P ccCenter(const P& A, const P& B, const P& C) {
  P b = C-A, c = B-A;
  return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
}
```

9.37 kdTree.h

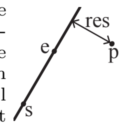
Description: KD-tree (2d, can be extended to 3d)

55 lines

```
#include "Point.h"
typedef long long T;
typedef Point<T> P;
const T INF = numeric_limits<T>::max();
bool on_x(const P& a, const P& b) { return a.x < b.x; }
bool on_y(const P& a, const P& b) { return a.y < b.y; }
struct Node {
  P pt; // if this is a leaf, the single point in it
  T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
  Node *first = 0, *second = 0;
  T distance(const P& p) { // min squared distance to a point
    T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
    T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
    return (P(x,y) - p).dist2();
  }
  Node(vector<P>&& vp) : pt(vp[0]) {
    for (P p : vp) {
      x0 = min(x0, p.x); x1 = max(x1, p.x);
      y0 = min(y0, p.y); y1 = max(y1, p.y);
    }
    if (vp.size() > 1) {
      // split on x if width ≥ height (not ideal ...)
      sort(all(vp), x1 - x0 ≥ y1 - y0 ? on_x : on_y);
      // divide by taking half the array for each child (not
      // best performance with many duplicates in the middle)
      int half = sz(vp)/2;
      first = new Node({vp.begin(), vp.begin() + half});
      second = new Node({vp.begin() + half, vp.end()});
    }
  }
};
struct KDTree {
  Node* root;
  KDTree(const vector<P>& vp) : root(new Node({all(vp)})) {}
  pair<T, P> search(Node *node, const P& p) {
    if (!node->first) {
      // uncomment if we should not find the point itself:
      // if (p == node->pt) return {INF, P()};
      return make_pair((p - node->pt).dist2(), node->pt);
    }
    Node *f = node->first, *s = node->second;
    T bfirst = f->distance(p), bsec = s->distance(p);
    if (bfirst > bsec) swap(bsec, bfirst), swap(f, s);
    // search closest side first, other side if needed
    auto best = search(f, p);
    if (bsec < best.first)
      best = min(best, search(s, p));
    return best;
  }
  // find nearest point to a point, and its squared distance
  // (requires an arbitrary operator< for Point)
  pair<T, P> nearest(const P& p) {
    return search(root, p);
  }
};
```

9.38 lineDistance.h

Description:
Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.

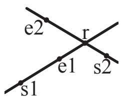


5 lines

```
#include "Point.h"
template<class P>
double lineDist(const P& a, const P& b, const P& p) {
  return (double)(b-a).cross(p-a)/(b-a).dist();
}
```

9.39 lineIntersection.h

Description:
If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.



Usage: `auto res = lineInter(s1,e1,s2,e2);`

```
if (res.first == 1)
cout << "intersection point at " << res.second << endl;
```

9 lines

```
#include "Point.h"
template<class P>
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
  auto d = (e1 - s1).cross(e2 - s2);
  if (d == 0) // if parallel
    return {-(s1.cross(e1, s2) == 0), P(0, 0)};
  auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
  return {1, (s1 * p + e1 * q) / d};
}
```

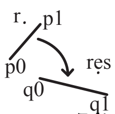
9.40 lineSegmentSntersections.cpp

78 lines

```
/*
 Line and segments predicates
 Tested: AIZU(judge.u-aizu.ac.jp) CGL
*/
bool intersectLL(const line &l, const line &m)
{
  return abs(cross(l.q - l.p, m.q - m.p)) > eps || // non-parallel
    abs(cross(l.q - l.p, m.p - l.p)) < eps; // same line
}
bool intersectLS(const line &l, const segment &s)
{
  return cross(l.q - l.p, s.p - l.p) * // s[0] is left of l
    cross(l.q - l.p, s.q - l.p) < eps; // s[1] is right of l
}
bool intersectLP(const line &l, const point &p)
{
  return abs(cross(l.q - p, l.p - p)) < eps;
}
bool intersectSS(const segment &s, const segment &t)
{
  return ccw(s.p, s.q, t.p) * ccw(s.p, s.q, t.q) ≤ 0
    && ccw(t.p, t.q, s.p) * ccw(t.p, t.q, s.q) ≤ 0;
}
bool intersectSP(const segment &s, const point &p)
{
  return abs(s.p - p) + abs(s.q - p) - abs(s.q - s.p) < eps;
  // triangle inequality
  return min(real(s.p), real(s.q)) ≤ real(p)
    && real(p) ≤ max(real(s.p), real(s.q))
    && min(imag(s.p), imag(s.q)) ≤ imag(p)
    && imag(p) ≤ max(imag(s.p), imag(s.q))
    && cross(s.p - p, s.q - p) == 0;
}
point projection(const line &l, const point &p)
{
  double t = dot(p - l.p, l.p - l.q) / norm(l.p - l.q);
  return l.p + t * (l.p - l.q);
}
point reflection(const line &l, const point &p)
{
  return p + 2.0 * (projection(l, p) - p);
}
double distanceLP(const line &l, const point &p)
{
  return abs(p - projection(l, p));
```

```
}
double distanceLL(const line &l, const line &m)
{
 return intersectLL(l, m) ? 0 : distanceLP(l, m.p);
}
double distanceLS(const line &l, const line &s)
{
 if (intersectLS(l, s)) return 0;
 return min(distanceLP(l, s.p), distanceLP(l, s.q));
}
double distanceSP(const segment &s, const point &p)
{
 const point r = projection(s, p);
 if (intersectSP(s, r)) return abs(r - p);
 return min(abs(s.p - p), abs(s.q - p));
}
double distanceSS(const segment &s, const segment &t)
{
 if (intersectSS(s, t)) return 0;
 return min(min(distanceSP(s, t.p), distanceSP(s, t.q)),
    min(distanceSP(t, s.p), distanceSP(t, s.q)));
}
point crosspoint(const line &l, const line &m)
{
 double A = cross(l.q - l.p, m.q - m.p);
 double B = cross(l.q - l.p, l.q - m.p);
 if (abs(A) < eps && abs(B) < eps)
  return m.p; // same line
 if (abs(A) < eps)
  assert(false); // !!!PRECONDITION NOT SATISFIED!!!
 return m.p + B / A * (m.q - m.p);
}
```

9.41 linearTransformation.h



Description:
Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.

7 lines

```
#include "Point.h"
typedef Point<double> P;
P linearTransformation(const P& p0, const P& p1,
   const P& q0, const P& q1, const P& r) {
 P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
 return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.dist2();
}
```

9.42 polygon-area-union.cpp

130 lines

```
// modified from syntax_error's code
bool operator<(const point_t &a, const point_t &b) {
 if (dblcmp(a.x - b.x) == 0) return a.y < b.y;
 return a.x < b.x;
}
bool operator==(const point_t &a, const point_t &b) {
 return dblcmp(a.x - b.x) == 0 && dblcmp(a.y - b.y) == 0;
}
struct segment_t {
 point_t a, b;
 segment_t() { a = b = point_t(); }
 segment_t(point_t ta, point_t tb) : a(ta), b(tb) { }
 double len() const { return dist(a, b); }
 double k() const { return (a.y - b.y) / (a.x - b.x); }
 double l() const { return a.y - k() * a.x; }
};
struct line_t {
 double a, b, c;
 line_t(point_t p) { a = p.x, b = -1.0, c = -p.y; }
 line_t(point_t p, point_t q) {
  a = p.y - q.y;
  b = q.x - p.x;
  c = a * p.x + b * p.y;
 }
};
bool ccutl(line_t p, line_t q) {
 if (dblcmp(p.a * q.b - q.a * p.b) == 0) return false;
 return true;
}
point_t cutl(line_t p, line_t q) {
 double x = (p.c * q.b - q.c * p.b) / (p.a * q.b - q.a * p.b);
 double y = (p.c * q.a - q.c * p.a) / (p.b * q.a - q.b * p.a);
 return point_t(x, y);
}
bool onseg(point_t p, segment_t s) {
 if (dblcmp(p.x - min(s.a.x, s.b.x)) < 0 || dblcmp(p.x - max(s.a.x, s.b.
     ↪ x)) > 0) return false;
 if (dblcmp(p.y - min(s.a.y, s.b.y)) < 0 || dblcmp(p.y - max(s.a.y, s.b.
     ↪ y)) > 0) return false;
 return true;
}
bool ccut(segment_t p, segment_t q) {
 if (!ccutl(line_t(p.a, p.b), line_t(q.a, q.b))) return false;
 point_t r = cutl(line_t(p.a, p.b), line_t(q.a, q.b));
 if (!onseg(r, p) || !onseg(r, q)) return false;
 return true;
}
point_t cut(segment_t p, segment_t q) {
 return cutl(line_t(p.a, p.b), line_t(q.a, q.b));
}
struct event_t {
 double x;
 int type;
 event_t() { x = 0, type = 0; }
 event_t(double _x, int _t) : x(_x), type(_t) { }
 bool operator<(const event_t &r) const {
  return x < r.x;
 }
};
vector<segment_t> s;
double solve(const vector<segment_t> &v, const vector<int> &sl) {
 double ret = 0;
 vector<point_t> lines;
 for (int i = 0; i < v.size(); ++i) lines.push_back(point_t(v[i].k(), v[
     ↪ i].l()));
 sort(lines.begin(), lines.end());
 lines.erase(unique(lines.begin(), lines.end()), lines.end());
 for(int i = 0; i < lines.size(); ++i) {
  vector<event_t> e;
  vector<int>::const_iterator it = sl.begin();
  for(int j = 0; j < s.size(); j += *it++) {
   bool touch = false;
   for (int k = 0; k < *it; ++k) if (lines[i] == point_t(s[j + k].k(),
       ↪ s[j + k].l())) touch = true;
   if (touch) continue;
   vector<point_t> cuts;
   for (int k = 0; k < *it; ++k) {
    if (!ccutl(line_t(lines[i]), line_t(s[j + k].a, s[j + k].b)))
        ↪ continue;
    point_t r = cutl(line_t(lines[i]), line_t(s[j + k].a, s[j + k].b))
        ↪ ;
    if (onseg(r, s[j + k])) cuts.push_back(r);
   }
   sort(cuts.begin(), cuts.end());
   cuts.erase(unique(cuts.begin(), cuts.end()), cuts.end());
   if (cuts.size() == 2) {
    e.push_back(event_t(cuts[0].x, 0));
    e.push_back(event_t(cuts[1].x, 1));
   }
  }
  for (int j = 0; j < v.size(); ++j) {
   if (lines[i] == point_t(v[j].k(), v[j].l())) {
    e.push_back(event_t(min(v[j].a.x, v[j].b.x), 2));
    e.push_back(event_t(max(v[j].a.x, v[j].b.x), 3));
   }
  }
  sort(e.begin(), e.end());
  double last = e[0].x;
  int cntg = 0, cntb = 0;
  for (int j = 0; j < e.size(); ++j) {
   double y0 = lines[i].x * last + lines[i].y;
   double y1 = lines[i].x * e[j].x + lines[i].y;
   if (cntb == 0 && cntg) ret += (y0 + y1) * (e[j].x - last) / 2;
```

```cpp
      last = e[j].x;
      if (e[j].type == 0) ++cntb;
      if (e[j].type == 1) --cntb;
      if (e[j].type == 2) ++cntg;
      if (e[j].type == 3) --cntg;
    }
  }
  return ret;
}
double polyUnion(vector<vector<point_t>> polys) {
  s.clear();
  vector<segment_t> A, B;
  vector<int> sl;
  for (int i = 0; i < polys.size(); ++i) {
    double area = 0;
    int tot = polys[i].size();
    for (int j = 0; j < tot; ++j) {
      area += cross(polys[i][j], polys[i][(j + 1) % tot]);
    }
    if (dblcmp(area) > 0) reverse(polys[i].begin(), polys[i].end());
    if (dblcmp(area) != 0) {
      sl.push_back(tot);
      for (int j = 0; j < tot; ++j) s.push_back(segment_t(polys[i][j],
          ↪ polys[i][(j + 1) % tot]));
    }
  }
  for (int i = 0; i < s.size(); ++i) {
    int sgn = dblcmp(s[i].a.x - s[i].b.x);
    if (sgn == 0) continue;
    else if (sgn < 0) A.push_back(s[i]);
    else B.push_back(s[i]);
  }
  return solve(A, sl) - solve(B, sl);
}
```

9.43 rectangleUnion.cpp

59 lines

```cpp
/*
  Tested: MIT 2008 Team Contest 1 (Rectangles)
  Complexity: O(n log n)
*/
typedef long long ll;
struct rectangle
{
  ll xl, yl, xh, yh;
};
ll rectangle_area(vector<rectangle> &rs)
{
  vector<ll> ys; // coordinate compression
  for (auto r : rs)
  {
    ys.push_back(r.yl);
    ys.push_back(r.yh);
  }
  sort(ys.begin(), ys.end());
  ys.erase(unique(ys.begin(), ys.end()), ys.end());
  int n = ys.size(); // measure tree
  vector<ll> C(8 * n), A(8 * n);
  function<void(int, int, int, int, int, int)> aux =
    [&](int a, int b, int c, int l, int r, int k)
    {
      if ((a = max(a,l)) >= (b = min(b,r))) return;
      if (a == l && b == r) C[k] += c;
      else
      {
        aux(a, b, c, l, (l+r)/2, 2*k+1);
        aux(a, b, c, (l+r)/2, r, 2*k+2);
      }
      if (C[k]) A[k] = ys[r] - ys[l];
      else A[k] = A[2*k+1] + A[2*k+2];
    };
  struct event
  {
    ll x, l, h, c;
  };
  // plane sweep
  vector<event> es;
  for (auto r : rs)
  {
    int l = lower_bound(ys.begin(), ys.end(), r.yl) - ys.begin();
    int h = lower_bound(ys.begin(), ys.end(), r.yh) - ys.begin();
    es.push_back( { r.xl, l, h, +1 } );
    es.push_back( { r.xh, l, h, -1 } );
  }
  sort(es.begin(), es.end(), [](event a, event b)
    { return a.x != b.x ? a.x < b.x : a.c > b.c; } );
  ll area = 0, prev = 0;
  for (auto &e : es)
  {
    area += (e.x - prev) * A[0];
    prev = e.x;
    aux(e.l, e.h, e.c, 0, n, 0);
  }
  return area;
}
```

9.44 rectilinearMst.cpp

57 lines

```cpp
/*
  Tested: USACO OPEN08 (Cow Neighborhoods)
  Complexity: O(n log n)
*/
typedef long long ll;
typedef complex<ll> point;
ll rectilinear_mst(vector<point> ps)
{
  vector<int> id(ps.size());
  iota(id.begin(), id.end(), 0);
  struct edge
  {
    int src, dst;
    ll weight;
  };
  vector<edge> edges;
  for (int s = 0; s < 2; ++s)
  {
    for (int t = 0; t < 2; ++t)
    {
      sort(id.begin(), id.end(), [&](int i, int j)
      {
        return real(ps[i] - ps[j]) < imag(ps[j] - ps[i]);
      });
      map<ll, int> sweep;
      for (int i : id)
      {
        for (auto it = sweep.lower_bound(-imag(ps[i]));
            it != sweep.end(); sweep.erase(it++))
        {
          int j = it->second;
          if (imag(ps[j] - ps[i]) < real(ps[j] - ps[i]))
            break;
          ll d = abs(real(ps[i] - ps[j]))
              + abs(imag(ps[i] - ps[j]));
          edges.push_back( { i, j, d } );
        }
        sweep[-imag(ps[i])] = i;
      }
      for (auto &p : ps)
        p = point(imag(p), real(p));
    }
    for (auto &p : ps)
      p = point(-real(p), imag(p));
  }
  ll cost = 0;
  sort(edges.begin(), edges.end(), [](edge a, edge b)
  {
    return a.weight < b.weight;
  });
  union_find uf(ps.size());
  for (edge e : edges)
    if (uf.join(e.src, e.dst))
```

```
    cost += e.weight;
  return cost;
}
```

9.45 semiplaneIntersection.cpp

55 lines

```
/*
  Check wether there is a point in the intersection of
  several semi-planes. if p lies in the border of some
  semiplane it is considered to belong to the semiplane.
  Expected Running time: linear
  Tested on Triathlon [Cuban Campament Contest]
*/
bool intersect( vector<line> semiplane ) {
  function<bool(line&,point&)> side = [](line &l, point &p) {
    // IMPORTANT: point p belongs to semiplane defined by l
    // iff p it's clockwise respect to segment < l.p, l.q >
    // i.e. (non negative cross product)
    return cross( l.q - l.p, p - l.p ) >= 0;
  };
  function<bool(line&, line&, point&)> crosspoint = [](const line &l, const line &m, point &x) {
    double A = cross(l.q - l.p, m.q - m.p);
    double B = cross(l.q - l.p, l.q - m.p);
    if (abs(A) < eps) return false;
    x = m.p + B / A * (m.q - m.p);
    return true;
  };
  int n = (int)semiplane.size();
  random_shuffle( semiplane.begin(), semiplane.end() );
  point cent(0, 1e9);
  for (int i = 0; i < n; ++i) {
    line &S = semiplane[ i ];
    if (side(S, cent)) continue;
    point d = S.q - S.p; d /= abs( d );
    point A = S.p - d * 1e8, B = S.p + d * 1e8;
    for (int j = 0; j < i; ++j) {
      point x;
      line &T = semiplane[j];
      if ( crosspoint(T, S, x) ) {
        int cnt = 0;
        if (!side(T, A)) {
          A = x;
          cnt++;
        }
        if (!side(T, B)) {
          B = x;
          cnt++;
        }
        if (cnt == 2)
          return false;
      }
      else {
        if (!side(T, A)) return false;
      }
    }
    if (imag(B) > imag(A)) swap(A, B);
    cent = A;
  }
  return true;
}
```

9.46 sideOf.h

Description: Returns where p is as seen from s towards e. 1/0/-1 $\Leftrightarrow$ left/on line/right. If the optional argument eps is given 0 is returned if p is within distance eps from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.
Usage: `bool left = sideOf(p1,p2,q)==1;`

9 lines

```
#include "Point.h"
template<class P>
int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }
template<class P>
int sideOf(const P& s, const P& e, const P& p, double eps) {
  auto a = (e-s).cross(p-s);
  double l = (e-s).dist()*eps;
  return (a > l) - (a < -l);
}
```

9.47 sphericalDistance.h

Description: Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 (ϕ_1) and f2 (ϕ_2) from x axis and zenith angles (latitude) t1 (θ_1) and t2 (θ_2) from z axis (0 = north pole). All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx*radius is then the difference between the two points in the x direction and d*radius is the total distance between the points.

8 lines

```
double sphericalDistance(double f1, double t1,
    double f2, double t2, double radius) {
  double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
  double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
  double dz = cos(t2) - cos(t1);
  double d = sqrt(dx*dx + dy*dy + dz*dz);
  return radius*2*asin(d/2);
}
```

9.48 triangle-circle-area-intersection.cpp

33 lines

```
double areaTC(point_t ct, double r, point_t p1, point_t p2) { // intersected area
  double a, b, c, x, y, s = cross(p1 - ct, p2 - ct) / 2;
  a = dist(ct, p2), b = dist(ct, p1), c = dist(p1, p2);
  double hr2 = r * r / 2, cr2 = c * c * r * r;
  if (a <= r && b <= r) {
    return s;
  } else if (a < r && b >= r) {
    x = (dot(p1 - p2, ct - p2) + sqrt(cr2 - sqr(cross(p1 - p2, ct - p2)))) / c;
    return asin(s * (c - x) * 2 / c / b / r) * hr2 + s * x / c;
  } else if (a >= r && b < r) {
    y = (dot(p2 - p1, ct - p1) + sqrt(cr2 - sqr(cross(p2 - p1, ct - p1)))) / c;
    return asin(s * (c - y) * 2 / c / a / r) * hr2 + s * y / c;
  } else {
    if (fabs(2 * s) >= r * c || dot(p2 - p1, ct - p1) <= 0 || dot(p1 - p2, ct - p2) <= 0) {
      if (dot(p1 - ct, p2 - ct) < 0) {
        if (cross(p1 - ct, p2 - ct) < 0) {
          return (-pi - asin(s * 2 / a / b)) * hr2;
        } else {
          return (pi - asin(s * 2 / a / b)) * hr2;
        }
      } else {
        return asin(s * 2 / a / b) * hr2;
      }
    } else {
      x = (dot(p1 - p2, ct - p2) + sqrt(cr2 - sqr(cross(p1 - p2, ct - p2)))) / c;
      y = (dot(p2 - p1, ct - p1) + sqrt(cr2 - sqr(cross(p2 - p1, ct - p1)))) / c;
      return (asin(s * (1 - x / c) * 2 / r / b) + asin(s * (1 - y / c) * 2 / r / a)) * hr2 + s * ((y + x) / c - 1);
    }
  }
}
double areaTC(point_t ct, double r, point_t p1, point_t p2, point_t p3) {
  return areaTC(ct, r, p1, p2) + areaTC(ct, r, p2, p3) + areaTC(ct, r, p3, p1);
}
```

Various (10)

10.1 Intervals

10.2 IntervalContainer

Description: Add and remove intervals from a set of disjoint intervals. Will merge the added interval with any overlapping intervals in the set when adding. Intervals are [inclusive, exclusive).
Time: $\mathcal{O}(\log N)$

22 lines

```
set<pii>::iterator addInterval(set<pii>& is, int L, int R) {
  if (L == R) return is.end();
  auto it = is.lower_bound({L, R}), before = it;
  while (it != is.end() && it->first <= R) {
    R = max(R, it->second);
    before = it = is.erase(it);
  }
  if (it != is.begin() && (--it)->second >= L) {
    L = min(L, it->first);
    R = max(R, it->second);
    is.erase(it);
  }
  return is.insert(before, {L,R});
}
void removeInterval(set<pii>& is, int L, int R) {
  if (L == R) return;
  auto it = addInterval(is, L, R);
  auto r2 = it->second;
  if (it->first == L) is.erase(it);
  else (int&)it->second = L;
  if (R != r2) is.emplace(R, r2);
}
```

10.3 IntervalCover

Description: Compute indices of smallest set of intervals covering another interval. Intervals should be [inclusive, exclusive). To support [inclusive, inclusive], change (A) to add `|| R.empty()`. Returns empty set on failure (or if G is empty).
Time: $\mathcal{O}(N \log N)$

19 lines

```
template<class T>
vi cover(pair<T, T> G, vector<pair<T, T>> I) {
  vi S(sz(I)), R;
  iota(all(S), 0);
  sort(all(S), [&](int a, int b) { return I[a] < I[b]; });
  T cur = G.first;
  int at = 0;
  while (cur < G.second) { // (A)
    pair<T, int> mx = make_pair(cur, -1);
    while (at < sz(I) && I[S[at]].first <= cur) {
      mx = max(mx, make_pair(I[S[at]].second, S[at]));
      at++;
    }
    if (mx.second == -1) return {};
    cur = mx.first;
    R.push_back(mx.second);
  }
  return R;
}
```

10.4 ConstantIntervals

Description: Split a monotone function on [from, to) into a minimal set of half-open intervals on which it has the same value. Runs a callback g for each such interval.
Usage: `constantIntervals(0, sz(v), [&](int x){return v[x];}, [&](int lo, int hi, T val){...});`
Time: $\mathcal{O}\left(k \log \frac{n}{k}\right)$

19 lines

```
template<class F, class G, class T>
void rec(int from, int to, F& f, G& g, int& i, T& p, T q) {
  if (p == q) return;
  if (from == to) {
    g(i, to, p);
    i = to; p = q;
  } else {
    int mid = (from + to) >> 1;
    rec(from, mid, f, g, i, p, f(mid));
    rec(mid+1, to, f, g, i, p, q);
  }
}
template<class F, class G>
void constantIntervals(int from, int to, F f, G g) {
  if (to <= from) return;
  int i = from; auto p = f(i), q = f(to-1);
  rec(from, to-1, f, g, i, p, q);
  g(i, to, q);
}
```

10.5 Misc. algorithms

10.6 TernarySearch

Description: Find the smallest i in $[a, b]$ that maximizes $f(i)$, assuming that $f(a) < \dots < f(i) \geq \dots \geq f(b)$. To reverse which of the sides allows non-strict inequalities, change the $<$ marked with (A) to $<=$, and reverse the loop at (B). To minimize f, change it to $>$, also at (B).
Usage: `int ind = ternSearch(0,n-1,[&](int i){return a[i];});`
Time: $\mathcal{O}(\log(b-a))$

11 lines

```
template<class F>
int ternSearch(int a, int b, F f) {
  assert(a <= b);
  while (b - a >= 5) {
    int mid = (a + b) / 2;
    if (f(mid) < f(mid+1)) a = mid; // (A)
    else b = mid+1;
  }
  rep(i,a+1,b+1) if (f(a) < f(i)) a = i; // (B)
  return a;
}
```

10.7 LIS.cpp

18 lines

```
#include <bits/stdc++.h>
using namespace std;
// Longest increasing secuence in o(nlogn)
vector<int> d;
int main() {
    int ans, n;
    cin>>n;
    vector<int> nums(n);
    for(auto &c:nums)cin>>c;
    for (int i = 0; i<n;i++) {
        auto it = upper_bound(d.begin(), d.end(), nums[i]);
        if (it == d.end()) d.push_back(nums[i]);
        else *it = nums[i];
    }
    cout<<"LIS ="<<d.size<<endl;
    return 0;
}
```

10.8 farPairOfPointsManhatan.cpp

22 lines

```
#include <bits/stdc++.h>
using namespace std;
#define endl '\n'
typedef long long int lli;
#define __ ios_base::sync_with_stdio(false);cin.tie(NULL);
int main() {__
    lli t = 1,n,x,y;
    while(t--) {
        cin>>n;
        lli minz= 2e9,maxz = -2e9,miny = 2e9,maxy = -2e9;
        for(int i = 0;i<n;i++) {
            cin>>x>>y;
            minz = min(minz,x+y);
            maxz = max(maxz,x+y);
            miny = min(miny,x-y);
            maxy = max(maxy,x-y);
        }
        cout<<max(maxz-minz,maxy-miny)<<endl;
    }
    return 0;
}
```

10.9 MaximumProductKElemets.cpp

43 lines

```
#include <bits/std++.h>
using namespace std;
int maxProductSubarrayOfSizeK(vector<int>&A, int n, int k) {
  sort(A.begin(), A.end());
  int product = 1;
  if (A[n - 1] == 0 && (k & 1))
   return 0;
  if (A[n - 1] <= 0 && (k & 1))  {
   for (int i = n - 1; i >= n - k; i--)
    product *= A[i];
   return product;
  }
  int i = 0;
  int j = n - 1;
  if (k & 1)  {
   product *= A[j];
   j--;
   k--;
  }
  k >>= 1;
  for (int itr = 0; itr < k; itr++)  {
   int left_product = A[i] * A[i + 1];
   int right_product = A[j] * A[j - 1];
   if (left_product > right_product)  {
    product *= left_product;
    i += 2;
   }
   else  {
    product *= right_product;
    j -= 2;
   }
  }
  return product;
 }
int main() {
   int n,k;
   cin>>n;
   vector<int> nums(n);
   for(auto &c:nums)cin>>c;
  cout << maxProductSubarrayOfSizeK(A, n, 5 b );
  return 0;
 }
```

10.10 QuickPermutation.cpp

37 lines

```
#include <bits/stdc++.h>
using namespace std;
int N = 4;
void display(unsigned int *a) {
   for(int x = 0; x < N; x++)
      cout<<a[x]<<" ";
   cout<<endl;
 }
void QuickPerm(void) {
   unsigned int a[N], p[N+1];
   register unsigned int i, j, tmp;
   for(i = 0; i < N; i++) {
      a[i] = i ;
      p[i] = i;
    }
   p[N] = N;
   display(a);
   i = 1;
   while(i < N) {
      p[i]--;
      j = i % 2 * p[i];
      tmp = a[j];
      a[j] = a[i];
      a[i] = tmp;
      display(a);
      i = 1;
      while (!p[i]) {
         p[i] = i;
         i++;
       }
    }
 }
int main() {
   int n;
   QuickPerm();
   return 0;
 }
```

10.11 permutation.cpp

64 lines

```
#include <bits/stdc++.h>
using namespace std;
vector<int> bit;
int n;
int sum(int idx) {
   int ans = 0;
   for(++idx;idx>0 ;idx-= idx&-idx)ans+=bit[idx];
   return ans;
 }
void add(int idx,int val) {
   for(++idx;idx<n;idx+= idx&-idx)bit[idx]+=val;
 }
int bit_search(int s) {
   int sum = 0;
   int pos = 0;
   for(int i = ceil(log2(n));i>=0;i--) {
      if((pos+(1<<i))<n && (sum+bit[pos+(1<<i)])<s) {
         sum+=bit[pos+(1<<i)];
         pos+=(1<<i);
       }
    }
   return pos;
 }
int main() {
   int x;
   cin>>n;
   vector<int> factoradicA(n);
   vector<int> factoradicB(n);
   bit.resize(n);
   for(int i = 0;i<n;i++)
      add(i,1);
   for(int i = 0;i<n;i++) {
      cin>>x;
      factoradicA[i] = sum(x-1);
      add(x,-1);
    }
   bit.assign(n,0);
   for(int i = 0;i<n;i++)
      add(i,1);
   for(int i = 0;i<n;i++) {
      cin>>x;
      factoradicB[i] = sum(x-1);
      add(x,-1);
    }
   vector<int> final(n);
   int carry= 0;
   for(int i = n-1;i>=0;i--) {
      int fact = (n-1)-i;
      final[i] = (factoradicA[i]+factoradicB[i])+carry;
      if(final[i]>=fact+1) {
         final[i]-=fact+1;
         carry = 1;
       }
      else carry = 0;
    }
   for(int i = 0;i<n;i++)add(i,1);
   for(int i = 0;i<n;i++) {
      x = bit_search(final[i]+1);
      cout<<x<<" ";
      add(x,-1);
    }
   cout<<endl;
   return 0;
 }
```

10.12 shuntingYard.cpp

87 lines

```cpp
enum type { op, value, obracket, cbracket } ; //types
struct token
{
  string text;
  type ttype;
} ;
template <typename T>
struct operation
{
  int precedence;
  function<void(stack<T> &s)> operate;
} ;
void mul(stack<string> &s); //operator
void pluss(stack<string> &s);
void poww(stack<string> &s);
unordered_map<string, operation<string>> operations;
bool rpn(const vector<token> &tokens, queue<token> &rpn)
{
  stack<token> operators;
  for (auto &token : tokens)
  {
    if (token.ttype == value)
      rpn.push(token);
    else if (token.ttype == op)
    {
      while (operators.size() > 0 &&
          operators.top().ttype != obracket && operations[token.text].precedence > operations[operators.top().text].precedence)
      {
        rpn.push(operators.top());
        operators.pop();
      }
      operators.push(token);
    }
    else if (token.ttype == obracket)
      rpn.push(token);
    else if (token.ttype == cbracket)
    {
      while (operators.top().ttype != obracket)
      {
        rpn.push(operators.top());
        operators.pop();
        if (operators.size() == 0)
          return false;
      }
      operators.pop();
    }
  }
  while (operators.size() > 0)
  {
    if (operators.top().ttype == obracket)
      return false;
    rpn.push(operators.top());
    operators.pop();
  }
  return true;
}
template <typename T>
T eval(queue<token> &rpn, bool &ok)
{
  stack<T> result;
  while (rpn.size() > 0)
  {
    auto t = rpn.front();
    rpn.pop();
    if (t.ttype == value)
      result.push(t.text); //parsear t.text
    if (t.ttype == op)
      operations[t.text].operate(result);
  }
  ok = result.size() == 1;
  return result.top();
}
vector<token> lex(const string &str); //lexer
int main()
{
  operations["*"] = { 1, poww } ;
  operations["."] = { 2, mul } ;
  operations["|"] = { 3, pluss } ;
  string str;
  auto toks = lex(str);
  queue<token> q;
  rpn(toks, q);
  bool ok;
  auto result = eval<string>(q, ok);
  cout << result << '\n';
  return 0;
}
```

10.13 subsetSum.cpp

74 lines

```cpp
#include <bits/stdc++.h>
using namespace std;
#define endl '\n'
#define int long long
#define __ ios_base::sync_with_stdio(false);cin.tie(NULL);
// Find all possibles values of subset sum
signed main() { __
  int N;
  cin >> N;
  // Sum of elements <= N implies that every element is <= N
  vector<int> freq(N + 1, 0);
  for (int i = 0; i < N; i++) {
    int x;
    cin >> x;
    freq[x]++;
  }
  vector<pair<int, int>> compressed;
  for (int i = 1; i <= N; i++) {
    if (freq[i] > 0) compressed.emplace_back(i, freq[i]);
  }
  vector<int> dp(N + 1, 0);
  dp[0] = 1;
  for (const auto &[w, k] : compressed) {
    vector<int> ndp = dp;
    for (int p = 0; p < w; p++) {
      int sum = 0;
      for (int multiple = p, count = 0; multiple <= N; multiple += w, count++) {
        if (count > k) {
          sum -= dp[multiple - w * count];
          count--;
        }
        if (sum > 0) ndp[multiple] = 1;
        sum += dp[multiple];
      }
    }
    swap(dp, ndp);
  }
  cout << "Possible subset sums are:\n";
  for (int i = 0; i <= N; i++) {
    if (dp[i] > 0) cout << i << " ";
  }
  // Subset sum with bitset, complexity is k²/32 if you try to find if the sum k is obtenible,
  // and you compress the array propertly for example only take a[i]<=k and m[i]*a[i]<=k where m[i]
  // represent the frecuency of the number a[i] in our array,
  // Otherwise if the total sum is bounded to N equal to the number of elements , the complexity is
  // O(n/32 sqrt(n))
  bitset<MAXK> dp;
  dp[0] = 1;
  for (int i = 1; i <= n; i++) {
    for (int x = 0; (1<<x) <= m[i]; x++) {
      dp |= (dp << (a[i]*(1<<x)));
      m[i] -= (1<<x);
    }
    dp |= (dp << (a[i]*m[i]));
```

```
  }
  // DP if you need the minimum elements needed to form the sum x
  vector<int> dp(N + 1, INF);
  dp[0] = 0;
  for (const auto &[w, k] : components) {
    vector<int> ndp = dp;
    for (int i = 0; i < w; i++) {
      deque<pair<int, int>> window;
      for (int j = i, mul = 0; j <= N; j += w, mul++) {
        while (!window.empty() && window.front().second < mul - k)
          window.pop_front();
        if (!window.empty()) smin(ndp[j], window.front().first + mul);
        while (!window.empty() && window.back().first >= dp[j] - mul)
          window.pop_back();
        window.emplace_back(dp[j] - mul, mul);
      }
    }
    swap(ndp, dp);
  }
}
```

10.14 SubsetSumDPMOD.cpp

45 lines

```
#include <bits/stdc++.h>
using namespace std;
#define endl '\n'
#define int long long
#define __ ios_base::sync_with_stdio(false);cin.tie(NULL);
// Find a subset of a vector of integers such that sum%M is i for all i <=M
signed main() { __
 int t= 1,n,m;
  while(t--) {
    cin>>n>>m;
    vector<int> nums(n);
    for(auto &c:nums)cin>>c,c%=m;
    sort(nums.begin(),nums.end());
    vector<bool> DP(m+1);
    vector<int> a(m+1);
    sort(nums.begin(),nums.end());
    for(int i = 0;i<n;i++) {
      int l = i;
      while(l<n && nums[i]== nums[l]) {
        l++;
      }
      a.assign(m+1,0);
      int x = nums[i];
      int last = 0;
      for(int j = 0;j<min(m,l-i);j++) {
        if(!DP[x%m])
          a[x%m]=a[last%m]+1;
        DP[x%m] = true;
        last = x;
        x+=nums[i];
      }
      =======
      for(int j = 0;j<=m;j++) {
        int nw = (((j-nums[i])%m)+m)%m;
        if(!DP[j]&& DP[nw] && a[nw]<l-i) {
          a[j] = a[nw]+1;
          DP[j] = true;
        }
      }
      i = l-1;
    }
    cout<<(DP[0]||DP[m]?"YES":"NO")<<endl;
  }
  return 0;
}
```

10.15 Dynamic programming

10.16 KnuthDP

Description: When doing DP on intervals: $a[i][j] = \min_{i<k<j}(a[i][k] + a[k][j]) + f(i,j)$, where the (minimal) optimal k increases with both i and j, one can solve intervals in increasing order of length, and search $k = p[i][j]$ for $a[i][j]$ only between $p[i][j-1]$ and $p[i+1][j]$. This is known as Knuth DP. Sufficient criteria for this are if $f(b,c) \leq f(a,d)$ and $f(a,c) + f(b,d) \leq f(a,d) + f(b,c)$ for all $a \leq b \leq c \leq d$. Consider also: LineContainer (ch. Data structures), monotone queues, ternary search.
Time: $\mathcal{O}\left(N^2\right)$

10.17 DivideAndConquerDP

Description: Given $a[i] = \min_{lo(i)\leq k<hi(i)}(f(i,k))$ where the (minimal) optimal k increases with i, computes $a[i]$ for $i = L..R-1$.
Time: $\mathcal{O}((N + (hi - lo)) \log N)$

17 lines

```
struct DP { // Modify at will:
  int lo(int ind) { return 0; }
  int hi(int ind) { return ind; }
  ll f(int ind, int k) { return dp[ind][k]; }
  void store(int ind, int k, ll v) { res[ind] = pii(k, v); }
  void rec(int L, int R, int LO, int HI) {
    if (L >= R) return;
    int mid = (L + R) >> 1;
    pair<ll, int> best(LLONG_MAX, LO);
    rep(k, max(LO,lo(mid)), min(HI,hi(mid)))
      best = min(best, make_pair(f(mid, k), k));
    store(mid, best.second, best.first);
    rec(L, mid, LO, best.second+1);
    rec(mid+1, R, best.second, HI);
  }
  void solve(int L, int R) { rec(L, R, INT_MIN, INT_MAX); }
};
```

10.18 brokenProfile.cpp

36 lines

```
#include <bits/stdc++.h>
using namespace std;
#define __ ios_base::sync_with_stdio(false),cin.tie(NULL);
int n,m;
int dp[1<<11][1007];
bool occupied(int i, int q) {
  return q&(1 << (i-1));
}
void search(int i, int j, int p, int q) {
  if(i == n+1) {
    dp[p][j+1] += dp[q][j];
    return;
  }
  if(occupied(i, q)) {
    search(i+1, j, p, q);
    return;
  }
  if(i+1 <= n && !occupied(i+1, q)) {
    search(i+2, j, p, q);
  }
  if(j+1 <= m) {
    search(i+1, j, p^(1 << (i-1)), q);
  }
}
int main() { __
  cin>>n>>m;
  memset(dp, 0, sizeof(dp));
  dp[0][0] = 1; //Initial condition
  for(int j = 1; j < m; j++) {
    for(int q = 0; q < (1<<n); q++) {
      search(1, j, 0, q);
    }
  }
  cout<<dp[0][m]<<endl;
}
```

10.19 convexHullTrick.cpp

81 lines

```cpp
/*
   Dynamic hull for max dot queries
   Complexity:
      - Add: O(log n)
      - Query: O(log^2 n) but very fast in practice
   Tested:
      - http://codeforces.com/gym/100377/problem/L
*/
typedef long long ll;
typedef complex<ll> point;
ll cross(point a, point b) { return imag(conj(a) * b); }
ll dot(point a, point b) { return real(conj(a) * b); }
ll area2(point a, point b, point c) { return cross(b - a, c - a); }
namespace std
{
   bool operator<(const point &a, const point &b)
   {
      return real(a) < real(b) || (real(a) == real(b) && imag(a) < imag(b));
   }
}
const ll oo = 0x3f3f3f3f3f3f3f3f;
struct dynamic_hull
{
   dynamic_hull() : hulls() { }
   void add_point(point p)
   {
      hull h;
      h.add_point(p);
      for (hull &_h : hulls)
         if (_h.empty())
         {
            h.swap(_h);
            break;
         }
         else h = merge(h, _h), _h.clear();
      if (!h.empty()) hulls.emplace_back(h);
   }
   ll max_dot(point p)
   {
      ll best = -oo;
      for (hull &h : hulls)
         if (!h.empty()) best = max(best, h.max_dot(p));
      return best;
   }
private:
   struct hull : vector<point>
   {
      void add_point(point p)
      {
         for (int s = size(); s > 1; --s)
            if (area2(at(s - 2), at(s - 1), p) < 0) break;
            else pop_back();
         push_back(p);
      }
      ll max_dot(point p)
      {
         int lo = 0, hi = (int) size() - 1, mid;
         while (lo < hi)
         {
            mid = (lo + hi) / 2;
            if (dot(at(mid), p) <= dot(at(mid + 1), p))
               lo = mid + 1;
            else hi = mid;
         }
         return dot(at(lo), p);
      }
   };
   static hull merge(const hull &a, const hull &b)
   {
      hull h;
      size_t i = 0, j = 0;
      while (i < a.size() && j < b.size())
         if (a[i] < b[j]) h.add_point(a[i++]);
         else h.add_point(b[j++]);
      while (i < a.size()) h.add_point(a[i++]);
      while (j < b.size()) h.add_point(b[j++]);
      return h;
   }
   vector<hull> hulls;
};
```

10.20 DigitDP.cpp

44 lines

```cpp
#include <bits/stdc++.h>
using namespace std;
int a,b,k;
int DP[20][2][200][200];
vector<int> Num;
int go(int pos,int f,int sum,int rem) {
   if(pos == Num.size()) {
      if(sum%k == 0 && rem%k == 0)
         return 1;
      return 0;
   }
   int &x = DP[pos][f][sum][rem];
   if(x!= -1)return x;
   int res = 0;
   int LIM ;
   if(!f)LIM = Num[pos];
   else LIM = 9;
   for(int i =0;i<=LIM;i++) {
      int nf = f;
      if(i<LIM)nf = 1;
      res +=go(pos+1,nf,(sum+i)%k,(rem*10+i)%k);
   }
   return x = res;
}
int solve(int n) {
   Num.clear();
   while(n) {
      Num.push_back(n%10);
      n/=10;
   }
   memset(DP,-1,sizeof(DP));
   reverse(Num.begin(),Num.end());
   return go(0,0,0,0);
}
int main() {
   int t;
   cin>>t;
   int cont = 1;
   while(t--) {
      cin>>a>>b>>k;
      cout<<"Case "<<cont++<<": "<<solve(b)-solve(a-1)<<endl;
   }
   return 0;
}
```

10.21 DPCorte.cpp

50 lines

```cpp
#include <bits/stdc++.h>
using namespace std;
const int maxn = 2007;
const int maxk = 22;
int DP[maxn][maxk];
const int inf = 1000000000;
int n;
vector<int> ac(maxn);
int cost(int i,int j) {
   int sum;
   if(i)
      sum = ac[j]-ac[i-1];
   else
      sum = ac[j];
   if(sum%10<5)sum-=sum%10;
   else sum+=10-sum%10;
   return sum;
}
// return min or max of proffit cutting an array in exactly k parts
// Complexity O(n^2 *k * cost)
```

```cpp
int solve(int idx,int k) {
   if(k==0 && idx==n)return 0;
   else if(k == 0 && idx<n)return inf+1;
   int &x = DP[idx][k];
   if(x!= inf)return x;
   for(int i = idx;i<n;i++)
      x = min(x,cost(idx,i)+solve(i+1,k-1));
   return x;
}
int main() {
   int k;
   cin>>n>>k;
   vector<int> nums(n);
   for(auto &c:nums)cin>>c;
   for(int i = 0;i<n;i++)
      for(int j = 0;j<=k+1;j++)DP[i][j] = inf;
   ac[0] = nums[0];
   for(int i = 1;i<n;i++)
      ac[i] = ac[i-1]+nums[i];
   for(int i = 0;i<n;i++)
      DP[i][1] = cost(i,n-1);
   for(int i = 0;i<=k;i++)
      DP[n][i] = 0;
   solve(0,k+1);
   // for(int i = 0;i<=n;i++)
      // for(int j = 0;j<=k+1;j++)
         // cout<<DP[i][j]<<" \n"[j == k+1];
   cout<<DP[0][k+1]<<endl;
   return 0;
}
```

10.22 Misc

10.23 various.cpp

89 lines

```cpp
//cout for __int128
ostream &operator<<(ostream &os, const __int128 & value) {
 char buffer[64];
 char *pos = end(buffer) - 1;
 *pos = '\0';
 __int128 tmp = value < 0 ? -value : value;
 do {
  --pos;
  *pos = tmp % 10 + '0';
  tmp /= 10;
 } while(tmp != 0);
 if(value < 0) {
  --pos;
  *pos = '-';
 }
 return os << pos;
}
//cin for __int128
istream &operator>>(istream &is, __int128 & value) {
 char buffer[64];
 is >> buffer;
 char *pos = begin(buffer);
 int sgn = 1;
 value = 0;
 if(*pos == '-') {
  sgn = -1;
  ++pos;
 } else if(*pos == '+') {
  ++pos;
 }
 while(*pos != '\0') {
  value = (value << 3) + (value << 1) + (*pos - '0');
  ++pos;
 }
 value *= sgn;
 return is;
}
//Random number generation in C++11
mt19937_64 rng(chrono::steady_clock::now().time_since_epoch().count());
//Generate a random integer in [a, b], you can also use long long int
int aleatorio_int(int a, int b) {
 uniform_int_distribution<int> dist(a, b);
 return dist(rng);
}
//Generate a random double in [a, b], you can also use long double
double aleatorio_double(double a, double b) {
 uniform_real_distribution<double> dist(a, b);
 return dist(rng);
}
//Read from a file
freopen("problemname.in", "r", stdin);
// the following line creates/overwrites the output file
freopen("problemname.out", "w", stdout);
int lcs(string & a, string & b) {
 int m = a.size(), n = b.size();
 vector<vector<int>> aux(m + 1, vector<int>(n + 1));
 for(int i = 1; i <= m; ++i) {
  for(int j = 1; j <= n; ++j) {
   if(a[i - 1] == b[j - 1])
    aux[i][j] = 1 + aux[i - 1][j - 1];
   else
    aux[i][j] = max(aux[i - 1][j], aux[i][j - 1]);
  }
 }
 return aux[m][n];
}
//0:saturday, 1:sunday, ..., 6:friday
int dayOfWeek(int d, int m, lli y) {
 if(m == 1 || m == 2) {
  m += 12;
  --y;
 }
 int k = y % 100;
 lli j = y / 100;
 return (d + 13*(m+1)/5 + k + k/4 + j/4 + 5*j) % 7;
}
int LevenshteinDistance(string & a, string & b) {
 int m = a.size(), n = b.size();
 vector<vector<int>> aux(m + 1, vector<int>(n + 1));
 for(int i = 1; i <= m; ++i)
  aux[i][0] = i;
 for(int j = 1; j <= n; ++j)
  aux[0][j] = j;
 for(int j = 1; j <= n; ++j)
  for(int i = 1; i <= m; ++i)
   aux[i][j] = min( { aux[i-1][j] + 1, aux[i][j-1] + 1, aux[i-1][j-1] +
    (a[i-1] != b[j-1]) } );
 return aux[m][n];
}
```

10.24 Optimization tricks

10.24.1 Bit hacks

- `x & -x` is the least bit in `x`.
- `for (int x = m; x; ) { --x &= m; ... }` loops over all subset masks of `m` (except `m` itself).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- `rep(b,0,K) rep(i,0,(1 << K))`
  `if (i & 1 << b) D[i] += D[i^(1 << b)];` computes all sums of subsets.

10.24.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize loops and optimizes floating points better.
- `#pragma GCC target ("avx2")` can double performance of vectorized code, but causes crashes on old machines.
- `#pragma GCC optimize ("trapv")` kills the program on integer overflows (but is really slow).

10.25 FastMod.h

Description: Compute $a\%b$ about 5 times faster than usual, where b is constant but not known at compile time. Returns a value congruent to $a \pmod{b}$ in the range $[0, 2b)$.

8 lines

```
typedef unsigned long long ull;
struct FastMod {
 ull b, m;
 FastMod(ull b) : b(b), m(-1ULL / b) { }
 ull reduce(ull a) { // a % b + (0 or b)
  return a - (ull)((__uint128_t(m) * a) >> 64) * b;
 }
};
```

10.26 FastInput

Description: Read an integer from stdin. Usage requires your program to pipe in input from file.
Usage: ./a.out < input.txt
Time: About 5x as fast as cin/scanf.

16 lines

```
inline char gc() { // like getchar()
 static char buf[1 << 16];
 static size_t bc, be;
 if (bc >= be) {
  buf[0] = 0, bc = 0;
  be = fread(buf, 1, sizeof(buf), stdin);
 }
 return buf[bc++]; // returns 0 on EOF
}
int readInt() {
 int a, c;
 while ((a = gc()) < 40);
 if (a == '-') return -readInt();
 while ((c = gc()) >= 48) a = a * 10 + c - 480;
 return a - 48;
}
```

BitMask (11)

11.1 SubsetSubset.cpp

35 lines

```
/*
  Computing all subset of subset.
  Time: 3^n
*/
#include <bits/stdc++.h>
using namespace std;
int main() {
  int N = 4;
  for(int i=0; i<(1<<N); ++i) {
    bitset<8> n(i);
    cout<<"MASK: "<<n<<endl;
    cout<<"SUBMASK:"<<endl;
    for(int j = i; j; j = (j-1) & i) {
      bitset<8> p(j);
      cout<<p<<endl;
    }
    cout<<endl;
  }
  return 0;
}
int f(int mask, int k) {
 if (dp[mask] != -1) return dp[mask];
 if (k == -1) return a[mask];
 dp[mask] = f(mask, k - 1);
 if (mask & (1 << k))
  dp[mask] += f(mask ^(1 << k), k - 1);
 return dp[mask];
}
for (int i = 0; i < (1<<m); i++) f(i, m-1);
for(int i = 0; i<(1<<N); ++i)
 F[i] = A[i];
for(int i = 0;i < N; ++i) for(int mask = 0; mask < (1<<N); ++mask) {
 if(mask & (1<<i))
  F[mask] += F[mask^(1<<i)];
```

11.2 amountOfHamiltonianWalks.cpp

39 lines

```
/*
 task:       Finding the number of Hamiltonian walks in the
             unweighted and directed graph G = (V, E).
 complexity: O(2^n * n^2)
 notes:      Let dp[msk][v] be the amount of Hamiltonian walks
             on the subgraph generated by vertices in msk that
             end in the vertex v.
*/
#define BIT(n) (1 << n)
const int MAXN = 20;
int n, m, u, v, g[MAXN], dp[BIT(MAXN)][MAXN], ans;
int main()
{
 cin >> n >> m;
 for (int i = 0; i < m; ++i)
 {
  cin >> u >> v;
  g[u] |= BIT(v);
 }
 for (int i = 0; i < n; ++i)
   dp[BIT(i)][i] = 1;
 for (int msk = 1; msk < BIT(n); ++msk)
 {
  for (int i = 0; i < n; ++i) if (msk & BIT(i))
  {
   int tmsk = msk ^BIT(i);
   for (int j = 0; tmsk && j < n; ++j)
   {
    if (g[j] & BIT(i))
       dp[msk][i] += dp[tmsk][j];
   }
  }
 }
 for (int i = 0; i < n; ++i)
   ans += dp[BIT(n) - 1][i];
 cout << ans << endl;
 return 0;
}
```

11.3 existenceOfHamiltonianCycle.cpp

33 lines

```
/*
 task:       Check for existence of Hamiltonian cycle in a
        directed graph G = <V, E>.
 complexity: O(2^n * n)
 notes:      Let dp[msk] be the mask of the subset consisting
       of those vertices j such that exist a Hamiltonian
       walk over the subset msk beginning in vertex 0 and
       ending in j.
*/
#define  BIT(n) (1 << n)
const int MAXN = 20;
int n, m, u, v, g[MAXN], dp[BIT(MAXN)];
int main()
{
 cin >> n >> m;
 for (int i = 0; i < m; ++i)
 {
  cin >> u >> v;
  g[v] |= BIT(u);
 }
 dp[1] = 1;
 for (int msk = 2; msk < BIT(n); ++msk)
 {
  for (int i = 0; i < n; ++i)
  {
   if ((msk & BIT(i)) && (dp[msk ^BIT(i)] & g[i]))
    dp[msk] |= BIT(i);
  }
 }
 cout << ((dp[BIT(n) - 1] & g[0]) != 0) << endl;
 return 0;
```

```
}
```

11.4 existenceOfHamiltonianWalk.cpp

33 lines

```
/*
 task:       Check for existence of Hamiltonian walk in the
       directed graph G = <V, E>.
 complexity: O(2^n * n)
 notes:      Let dp[msk] be the mask of the subset consisting
       of those vertices v for which exist a Hamiltonian
       walk over the subset msk ending in v.
*/
#define  BIT(n) (1 << n)
const int MAXN = 20;
int n, m, u, v, g[MAXN], dp[BIT(MAXN)];
int main()
 {
  cin >> n >> m;
  for (int i = 0; i < m; ++i)
   {
    cin >> u >> v;
    g[v] |= BIT(u);
   }
  for (int i = 0; i < n; ++i)
    dp[BIT(i)] = BIT(i);
  for (int msk = 1; msk < BIT(n); ++msk)
   {
    for (int i = 0; i < n; ++i)
     {
      if ((msk & BIT(i)) && (dp[msk ^BIT(i)] & g[i]))
        dp[msk] |= BIT(i);
     }
   }
  cout << (dp[BIT(n) - 1] != 0) << endl;
  return 0;
 }
```

11.5 findingTheNumberOfSimplePaths.cpp

35 lines

```
/*
 task:       Finding the number of simple paths in the
       directed graph G = <V, E>.
 complexity: O(f)
 notes:      Let dp[msk][v] be the number of Hamiltonian
       walks in the subgraph generated by vertices
       in msk that end in v.
*/
#define  BIT(n) (1 << n)
const int MAXN = 20;
int n, m, u, v, ans, g[MAXN], dp[BIT(MAXN)][MAXN];
int main()
 {
  cin >> n >> m;
  for (int i = 0; i < m; ++i)
   {
    cin >> u >> v;
    g[u] |= BIT(v);
   }
  for (int i = 0; i < n; ++i)
    dp[BIT(i)][i] = 1;
  for (int msk = 1; msk < BIT(n); ++msk)
   {
    for (int i = 0; i < n; ++i) if (BIT(i) & msk)
     {
      int tmsk = msk ^BIT(i);
      for (int j = 0; tmsk && j < n; ++j) if (g[j] & BIT(i))
        dp[msk][i] += dp[tmsk][j];
      ans += dp[msk][i];
     }
   }
  cout << ans - n << endl;
  return 0;
 }
```

11.6 findingTheShortestHamiltonianCycle.cpp

51 lines

```
/*
 task:       Search for the shortest Hamiltonian cycle.
       Let the directed graph G = (V, E) have
       n vertices, and each
       edge have weight d(i, j). We want to find a Hamiltonian
       cycle for which the sum of weights of its edges
       is minimal.
 complexity: O(2^n * n^2)
 notes:      Let dp[msk][v] be the length of the shortest Hamiltonian
       walk on the subgraph generated by vertices in msk
       beginning in verex 0 and ending in vertex v.
*/
#define  BIT(n) (1 << n)
using namespace std;
const int MAXN = 20,
    INF  = 0x1fffffff;
int n, m, u, v, w, g[MAXN][MAXN], dp[BIT(MAXN)][MAXN], ans = INF;
int main()
 {
  cin >> n >> m;
  for (int i = 0; i < n; ++i)
   {
    for (int j = 0; j < n; ++j)
      g[i][j] = INF;
   }
  for (int i = 0; i < BIT(n); ++i)
   {
    for (int j = 0; j < n; ++j)
      dp[i][j] = INF;
   }
  for (int i = 0; i < m; ++i)
   {
    cin >> u >> v;
    cin >> g[u][v];
   }
  dp[1][0] = 0;
  for (int msk = 2; msk < BIT(n); ++msk)
   {
    for (int i = 0; i < n; ++i) if (msk & BIT(i))
     {
      int tmsk = msk ^BIT(i);
      for (int j = 0; tmsk && j < n; ++j)
       dp[msk][i] = min(dp[msk][i], dp[tmsk][j] + g[j][i]);
     }
   }
  for (int i = 1; i < n; ++i)
    ans = min(ans, dp[BIT(n) - 1][i] + g[i][0]);
  cout << ans << endl;
  return 0;
}
```

11.7 numberOfHamiltonianCycles.cpp

35 lines

```
/*
 task:       Finding the number of Hamiltonian cycles in
       the unweighted and directed graph G = (V, E).
 complexity: O(2^n * n^2)
 notes:      Let dp[msk][v] be the amount of Hamiltonian walks
       on the subgraph generated by vertices in msk
       that begin in vertex 0 and end in vertex v.
*/
#define  BIT(n) (1 << n)
const int MAXN = 20;
int n, m, u, v, ans, g[MAXN], dp[BIT(MAXN)][MAXN];
int main()
 {
  cin >> n >> m;
  for (int i = 0; i < m; ++i)
   {
    cin >> u >> v;
    g[u] |= (1 << v);
   }
```

```
dp[1][0] = 1;
for (int msk = 2; msk < BIT(n); ++msk)
 {
  for (int i = 0; i < n; ++i) if (msk & BIT(i))
   {
    int tmsk = msk ^BIT(i);
    for (int j = 0; tmsk && j < n; ++j) if (g[j] & BIT(i))
      dp[msk][i] += dp[tmsk][j];
   }
 }
for (int i = 1; i < n; ++i) if (g[i] & 1)
    ans += dp[BIT(n) - 1][i];
cout << ans << endl;
return 0;
}
```

11.8 numberOfSimpleCycles.cpp

42 lines

```
/*
 task:       Finding the number of simple cycles in a
        directed graph G = <V, E>.
 complexity: O(2^n * n^2)
 notes:      Let dp[msk][v] be the number of Hamiltonian
        walks in the subgraph generated by vertices
        in msk that begin in the lowest vertex in msk
        and end in vertex v.
*/
#define  BIT(n) (1 << n)
#define  ONES(n) __builtin_popcount(n)
const int MAXN =  20;
int n, m, u, v, g[MAXN];
long long dp[BIT(MAXN)][MAXN], ans;
int main()
 {
 cin >> n >> m;
 for (int i = 0; i < m; ++i)
  {
   cin >> u >> v;
   g[u] |= BIT(v);
  }
 for (int i = 0; i < n; ++i)
    dp[BIT(i)][i] = 1;
 for (int msk = 1; msk < BIT(n); ++msk)
  {
   for (int i = 0; i < n; ++i)
    {
     if ((msk & BIT(i)) && !(msk & -msk & BIT(i)))
      {
       int tmsk = msk ^BIT(i);
       for (int j = 0; tmsk && j < n; ++j) if (g[j] & BIT(i))
         dp[msk][i] += dp[tmsk][j];
       if (ONES(msk) > 2 && (g[i] & msk & -msk))
         ans += dp[msk][i];
      }
    }
  }
 cout << ans << endl;
 return 0;
}
```

11.9 shortestHamiltonianWalk.cpp

51 lines

```
/*
 task:       Search for the shortest Hamiltonian walk.
        Let the directed graph G = (V, E) have n
        vertices, and each edge have weight d(i, j).
        We want to find a Hamiltonian walk for which
        the sum of weights of its edges is minimal.
 complexity: O(2^n * n^2)
 notes:      Let dp[msk][v] be the length of the shortest
        Hamiltonian walk on the subgraph generated by
        vertices in msk that end in vertex v.
*/
#define MAXN 20
#define INF 0x1fffffff
#define BIT(n) (1 << n)
using namespace std;
int n, m, ans = INF, d[MAXN][MAXN], u, v, w, dp[1 << MAXN][MAXN];
int main()
 {
 cin >> n >> m;
 for (int i = 0; i < n; ++i)
  {
   for (int j = 0; j < n; ++j)
     d[i][j] = INF;
  }
 for (int i = 0; i < BIT(n); ++i)
  {
   for (int j = 0; j < n; ++j)
     dp[i][j] = INF;
   }
 for (int i = 0; i < m; ++i)
  {
   cin >> u >> v >> w;
   d[u][v] = w;
  }
 for (int i = 0; i < n; ++i)
    dp[1 << i][i] = 0;
 for (int msk = 1; msk < (1 << n); ++msk)
  {
   for (int i = 0; i < n; ++i) if (msk & BIT(i))
    {
     int tmsk = msk ^BIT(i);
     for (int j = 0; tmsk && j < n; ++j)
       dp[msk][i] = min(dp[tmsk][j] + d[j][i], dp[msk][i]);
    }
  }
 for (int i = 0; i < n; ++i)
    ans = min(ans, dp[BIT(n) - 1][i]);
 cout << ans << endl;
 return 0;
}
```

test (12)

12.1 gen.cpp

101 lines

```
#include <bits/stdc++.h>
using namespace std;
#define int long long
#define accuracy chrono::steady_clock::now().time_since_epoch().count()
#define rep(i,a,n) for (int i = a; i <= n; ++i)
const int N = 1e6 + 4;
int32_t permutation[N];
mt19937 rng(accuracy);
int rand(int l, int r) {
 uniform_int_distribution<int> ludo(l, r); return ludo(rng);
}
const int inf = 1LL << 31;
using pii = pair<int,int>;
namespace generator  {
 string gen_string(int len = 0, bool upperCase = false, int l = 1, int r
     ↪  = 26)  {
   assert(len >= 0 && len <= 5e6);
   string str(len, (upperCase ? 'A' : 'a'));
   for (char &ch: str)  {
     ch += rand(l, r) - 1;
    }
   return str;
  }
 vector<int> gen_array(int len = 0, int minRange = 0, int maxRange = inf
     ↪ ) {
  assert(len >= 0 and len <= 5e6);
   vector<int> a(len);
   for (int &x: a) x = rand(minRange, maxRange);
   return a;
  }
 vector<pair<int, int>> gen_tree(int n = 0) {
   assert(n >= 0);
```

```cpp
    vector<pii> res(n ? n - 1 : 0);
    // if you like to have bamboo like tree or star like tree uncomment below 8 lines
    /*if (rng() % 5 == 0) { // bamboo like tree
     for (int i = 1; i < n; ++i) res[i-1] = {i, i + 1};
     return res;
    }
    if (rng() % 7 == 0) { // star tree
     for (int i = 2; i <= n; ++i) res[i-2] = {1, i};
     return res;
    }*/
    iota(permutation, permutation + 1 + n, 0);
    shuffle(permutation + 1, permutation + 1 + n, rng);
    for(int i = 2; i <= n; ++i){
     int u = i, v = rand(1 , i-1);
     u = permutation[u], v = permutation[v];
     res[i-2] = minmax(u, v); // u < v, just for convenience while debugging
    }
    shuffle(res.begin() , res.end() , rng);
    return res;
   }
  vector<pair<int, int>> simple_graph(int n = 0, int m = 0) {
    assert(n > 0 && m >= n);
    int max_edges = n * (n - 1) / 2;
    assert(m <= max_edges);
    vector<pii> res = gen_tree(n);
    set<pii> edge(res.begin(), res.end());
    for (int i = n; i <= m; ++i) {
     while (true) {
      int u = rand(1, n), v = rand(1, n);
      if (u == v) continue;
      auto it = edge.insert(minmax(u, v));
      if (it.second) break;
     }
    }
    res.assign(edge.begin(), edge.end());
    return res;
   }
 }
using namespace generator;
template<typename T = int>
ostream& operator<< (ostream &other, const vector<T> &v) {
   for (const T &x: v) other << x << ' ';
   other << '\n';
   return other;
 }
ostream& operator<< (ostream &other, const vector<pair<int,int>> &v) {
   for (const auto &x: v) other << x.first << ' ' << x.second << '\n';
   return other;
 }
vector<string> D= {"Mon","Tue","Wed","Thu","Fri","Sat","Sun"};
// comment the just below line if test cases required
#define SINGLE_TEST
const int max_tests = 10;
// complete this function according to the requirements
void generate_test() {
 int n = rand(0, 6);
 cout<<D[n];
 n = rand(1,1000);
 cout << n << '\n';
 cout << gen_array(n, 0, 10000);
}
signed main() {
 srand(accuracy);
 int t = 1;
 #ifndef SINGLE_TEST
  t = rand(1, max_tests), cout << t << '\n';
 #endif
 while (t--) {
  generate_test();
 }
}
```

12.2 s.sh

53 lines

```bash
green=$(tput setaf 71);
red=$(tput setaf 12);
blue=$(tput setaf 32);
orange=$(tput setaf 178);
bold=$(tput bold);
reset=$(tput sgr0);
g++ -std=c++17 gen.cpp -o generator || {echo ${bold}${orange}Compilation Error in ${reset} gen.cpp; exit 1; }
g++ -std=c++17 $1.cpp -o original || {echo ${bold}${orange}Compilation Error${reset} in $1.cpp; exit 1; }
g++ -std=c++17 $2.cpp -o brute || {echo ${bold}${orange}Compilation Error${reset} in $2.cpp; exit 1; }
if [ $# -eq 2 ]
  then
    max_tests=10
  else
    max_tests=$3
fi
diff_found=0
i=1
while [ $i -le $max_tests ]
do
  ./generator > input1
  ./original < input1 > original_output #|| {echo failed; exit 1;}
  ./brute < input1 > brute_output
  if diff --tabsize=1 -F --label --side-by-side --ignore-space-change original_output brute_output > dont_show_on_terminal; then
    echo "${orange}test_case #$i: ${bold}${green}AC${reset}"
  else
    echo "${orange}test_case #$i: ${bold}${red}WA${reset}"
    diff_found=1
    break
  fi
  i=$((i+1))
done
if [ $diff_found -eq 1 ]
then
  echo "${blue}Input: ${reset}"
  cat input1
  echo ""
  echo "${blue}Output: ${reset}"
  cat original_output
  echo ""
  echo "${blue}Expected: ${reset}"
  cat brute_output
  echo ""
  notify-send "Wrong Answer"
else
  notify-send "Accepted"
  rm input1
fi
rm generator
rm original
rm brute
rm original_output
rm brute_output
rm dont_show_on_terminal
```

Techniques (A)

A.1 techniques.txt

207 lines

```
#Do stress test with a brute solution
#Recursion
#Divide and conquer
  - Finding interesting points in N log N
#Techniques/ things to thing
  * Meet in the middle
  * Two pointers
  * Sweep line
  * Prefix function
  * Try different complexity for different sizes SQRT for sizes ≤ SQRT
      ↪ and other solution
  * How many different values are?
  * Greedy
  * See in reverse way
  * Duplicate array
  * How fast grow
  * Does have a binary behavior
  * Divide and conquer
  * xor hashing
  * Mo's algorithm , SQRT decomposition etc.
#Algorithm analysis
  - Master theorem
  - Amortized time complexity
#Greedy algorithm
  - Scheduling
  - Max contiguous subvector sum
  - Invariants
  - Huffman encoding
#Graph theory
  - Dynamic graphs (extra book-keeping)
  - Breadth first search
  - Depth first search
  - * Normal trees / DFS trees
  - Dijkstra's algorithm
  - MST: Prim's algorithm
  - Bellman-Ford
  - Konig's theorem and vertex cover
  - Min-cost max flow
  - Lovasz toggle
  - Matrix tree theorem / Number Spaning trees in a graph
  - Maximal matching, general graphs
  - Hopcroft-Karp
  - Hall's marriage theorem { \displaystyle |W|\leq |N_ { G } (W)|. }
  - Graphical sequences
  - Floyd-Warshall
  - Euler cycles
  - Flow networks
  - * Augmenting paths
  - * Edmonds-Karp
  - * Min cost max flow
  - * Min cut
  - Bipartite matching
  - Min. path cover
  - Topological sorting
  - Strongly connected components
  - 2-SAT
  - Cut vertices, cut-edges and biconnected components
  - Edge coloring
  - * Trees
  - Vertex coloring
  - * Bipartite graphs (=> trees)
  - * 3^n (special case of set cover)
  - Diameter and centroid
  - DSU on tree.
  - Small to large.
  - K'th shortest path
  - Shortest cycle
  - Euler tour, can manage path to root and subtree queries
  - Euler tour tree
  - link cut tree
#Dynamic programming
  - Knapsack
  - Coin change
  - Longest common subsequence
  - Longest increasing subsequence
  - Number of paths in a dag
  - Shortest path in a dag
  - Dynprog over intervals
  - Dynprog over subsets
  - Dynprog over probabilities
  - Dynprog over trees
  - 3^n set cover
  - SOS DP (n*2^n)
  - Divide and conquer
  - Knuth optimization
  - Convex hull optimizations
  - Alien trick
  - RMQ (sparse table a.k.a 2^k-jumps)
  - Bitonic cycle
  - Log partitioning (loop over most restricted)
Combinatorics
  - Computation of binomial coefficients
  - Pigeon-hole principle
  - Inclusion/exclusion
  - Catalan number
  - Stirling
  - Bell numbers
  - Pick's theorem
Number theory
  - Integer parts
  - Divisibility
  - Euclidean algorithm
  - Modular arithmetic
  - Linear Congruence Equation -> a . x == b (mod n) -> x = b . a^-1 b
  - Linear diophantine Equation -> ax +by = c
  - Discrete log -> find x such a^x == b (mod n)
  - Discrete root -> find x such x^k == a (mod n)
  - * Modular multiplication
  - * Modular inverses
  - * Modular exponentiation by squaring
  - Chinese remainder theorem
  - Fermat's little theorem
  - Euler's theorem
  - Phi function
  - Frobenius number
  - Quadratic reciprocity
  - Pollard-Rho
  - Miller-Rabin
  - Hensel lifting
  - Vieta root jumping
Game theory
  - Combinatorial games
  - Game trees
  - Mini-max
  - Nim
  - Games on graphs
  - Games on graphs with loops
  - Grundy numbers
  - Bipartite games without repetition
  - General games without repetition
  - Alpha-beta pruning
Probability theory
Optimization
  - Binary search
  - Ternary search
  - Unimodality and convex functions
  - Binary search on derivative
Numerical methods
  - Numeric integration
  - Newton's method
  - Root-finding with binary/ternary search
  - Golden section search
Matrices
  - Gaussian elimination
  - Exponentiation by squaring
Sorting
  - Radix sort
```

Geometry
- Coordinates and vectors
- * Cross product
- * Scalar product
- Convex hull
- Polygon cut
- Closest pair
- Coordinate-compression
- Quadtrees
- KD-trees
- All segment-segment intersection

Sweeping
- Discretization (convert to events and sweep)
- Angle sweeping
- Line sweeping
- Discrete second derivatives

Strings
- Longest common substring
- Palindrome subsequences
- Knuth-Morris-Pratt
- Tries
- Rolling polynomial hashes
- Suffix array
- Suffix tree
- Aho-Corasick
- Manacher's algorithm
- Letter position lists

Combinatorial search
- Meet in the middle
- Brute-force with pruning
- Best-first (A*)
- Bidirectional search
- Iterative deepening DFS / A*

Data structures
- LCA (2^k-jumps in trees in general)
- Pull/push-technique on trees
- Heavy-light decomposition
- Centroid decomposition
- Lazy propagation
- Self-balancing trees
- Convex hull trick
- Monotone queues / monotone stacks / sliding queues
- Sliding queue using 2 stacks
- Persistent segment tree
- Treap
- O(1) queries with disjoint sparse table

General
- If problem is check for all multiples of a number in [1,n] and this
  ↪ multiples don´t exed maxn complexity is (maxn log(maxn))
- Sum of n/1+ n/2 + n/3 + n/4 + ... is nlogn
- Merge many sets can be do it in n logn if we insert elements of the
  ↪ minor set to the mayor set
- Strings? Do you made a suffix array or suffix tree
- Graphs? shortes Path with two variables or many types of edges? try
  ↪ to clone graph
- try to decompose the formula
- TLE? and modulos , try to do less % operations if you have long long
  ↪ do modulo only when a≥(mod*8) where modulo is something like
  ↪ 1e9+7
- Best of all posibilities whit small n like 30-40 try meet in the
  ↪ middle
- need a subset whit some features and at least n/2 elements? try
  ↪ randomize
- Boolean assigments? 2-sat? basisxor? SLAE?
- Queries about paths with some specific value like sum xor? try to
  ↪ decompose with centroid decomposition and solve for each tree
  ↪ root in each centroid
- Check parity
- querys on path of tree? if it´s only to root is enough to do an euler
  ↪ traversal and flat the tree in other case use HLD