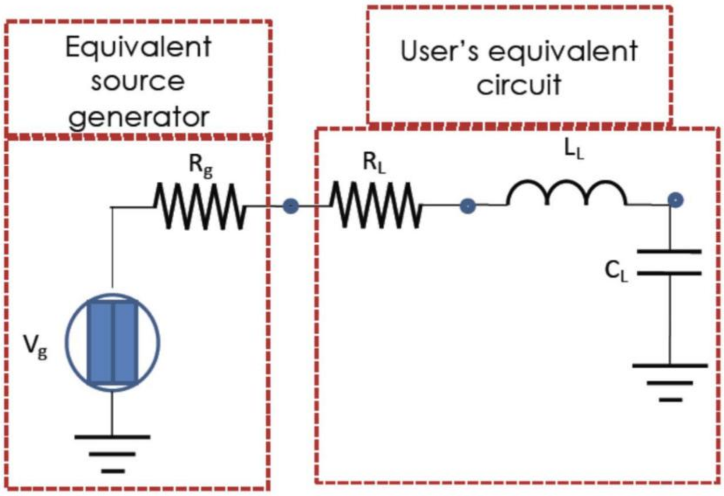
# Exercise on Power Balance in a Resonant Circuit




$$\boxed{\omega = \frac{1}{\sqrt{L_L C_L}}}$$

è l'unica ***pulsazione di risonanza*** del circuito

Calculate the power balance of the system, both for the resonant and non-resonant cases. Consider:

$$R_G = R_L = 50\Omega \quad V_G = 3V \quad \omega = 2\pi \cdot 5 \cdot 10^9 \quad L = 10nH$$

To solve the exercise, we proceed step by step to determine the system's power flow, both under resonance and off-resonance conditions.

We start with the resonance case, where the circuit frequency equals the resonance frequency $\omega_0$. Resonance occurs when $Z_L = Z_G^*$. In our case, this means the inductive and capacitive reactances of the load cancel each other out, since $Z_G^* = R_G$ (purely real).

$$X_L = \omega L_L$$

$$X_C = \frac{1}{\omega C_L}$$

$$X_L = X_C$$

The value of $C_L$ can be derived using the resonance frequency formula:

$$\omega_0 = \frac{1}{\sqrt{L_L C_L}}$$

$$C_L = \frac{1}{{\omega_0}^2 L_L} = \frac{1}{(2\pi \cdot 5 \cdot 10^9)^2 \cdot 10nH} = 1.013 \cdot 10^{-13} F$$

After determining $C_L$, the total impedance $Z$ of the circuit at resonance equals $R_L$, since the reactances cancel out. Consequently, the absorbed power is:

$$Z = R_L$$

$$P = \frac{{V_s}^2}{8Z} = \frac{3V^2}{8 \cdot 50\Omega} = 0.0225W$$

In the non-resonant case, the absorbed power is computed as:

$$P = \frac{{V_s}^2}{8R_G} \cdot \frac{4R_G R_A}{|Z_G + Z_A|^2}$$

Here, $\rho_D = \frac{4R_G R_A}{|Z_G+Z_A|^2}$ is the matching factor, which equals 1 under resonance. Substituting $Z_G = Z_A = R_A = 50\Omega$, we obtain:

$$\rho_D = \frac{4 \cdot 50 \cdot 50}{100^2 + {X_L}^2}$$

$${X_L}^2 = \left(\frac{\omega L}{1 - \omega^2 LC}\right)^2$$

Assuming a capacitance of $C = 10^{-12}F$ (off-resonance), we obtain ) ${X_L}^2 = 1255$ and $\rho_D = 0.888$ , leading to a power of: $P = 0.1998$

## Numerical Calculation of Radiated Power, Radiation Resistance, and Gain for an Electromagnetic Dipole

Using the Matlab script *plot_ddr_dipole_EM.m*, compute the radiated power (for unit current), the radiation resistance, and the directivity gain of an electromagnetic dipole in the following cases:

$$2l = 0.25\lambda$$

$$2l = 1.2\lambda$$

By modifying the Matlab code, the required values were obtained. Considering $\beta = \frac{2\pi}{\lambda}$, the results are:

Case $2l = 0.25\lambda$

$$Radiation\ resistance = 13.4405\Omega$$

$$Radiated\ power = 3.3601W$$

$$Directivity\ gain = 1.5318$$

Case $2l = 1.2\lambda$

$$Radiation\ resistance = 360.1938\Omega$$

$$Radiated\ power = 62.222W$$

$$Directivity\ gain = 3.1557$$

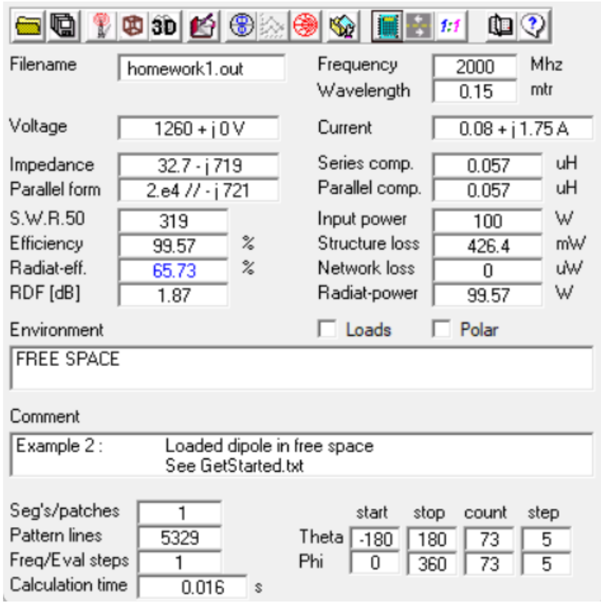
## Simulation and Analysis Using 4NEC

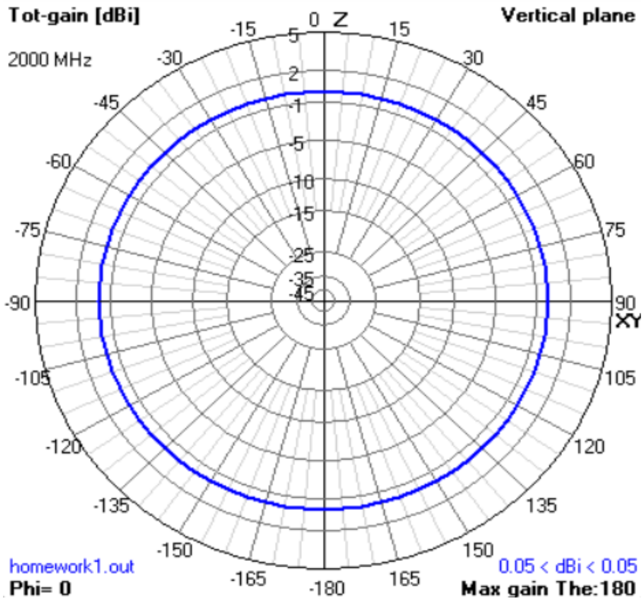
Using 4NEC, simulate the two dipoles at 2 GHz with a thickness of 0.5 mm. Obtain the radiation pattern, gain, SWR, and reflection coefficient around the operating frequency. For the $0.25\lambda$ dipole, determine two inductive loads that allow resonance.

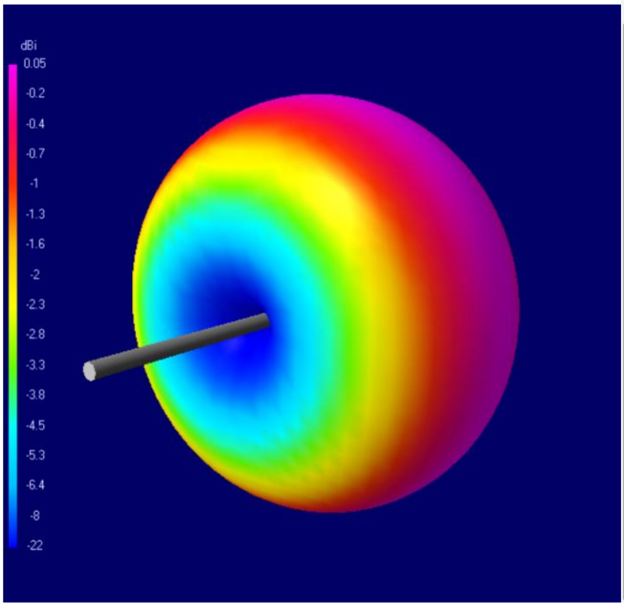
Since the antenna operates in air, we assume the wavelength as $\lambda = \lambda_0 = \frac{c}{f}$. Therefore, the dipole lengths for the two cases are:

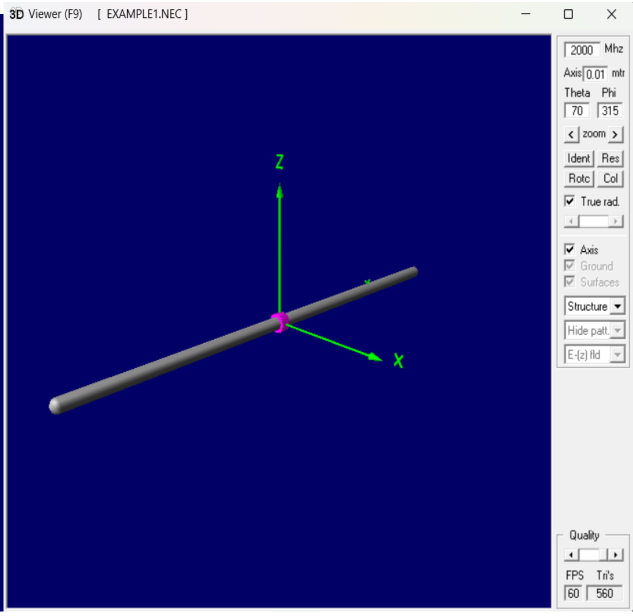
Case $2l = 0.25\lambda$

$$l = \frac{0.25}{2}\lambda = 0.01875\text{m}$$

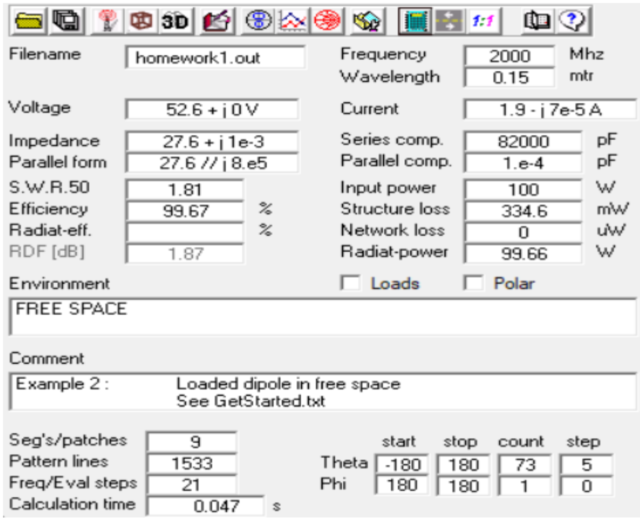
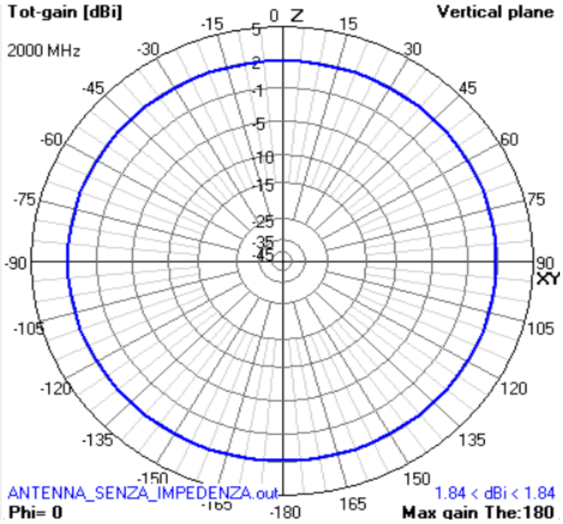
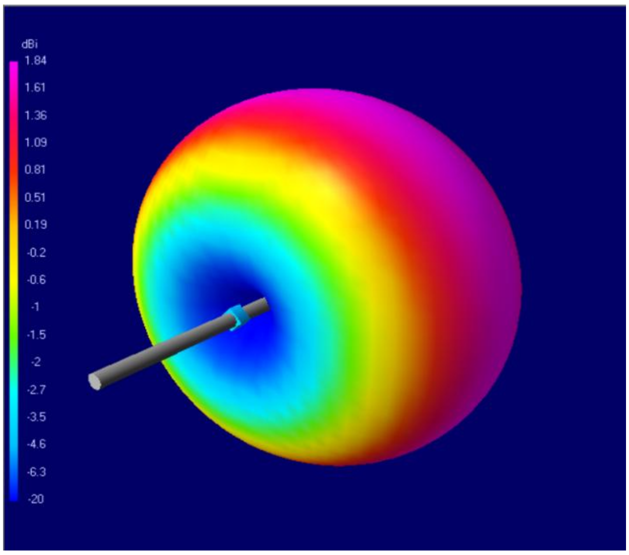
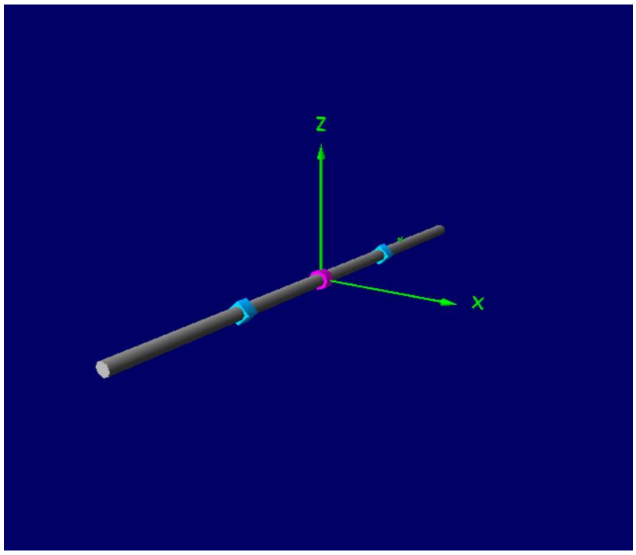



Inductive loads were introduced to achieve resonance:


Filename homework1.out
Frequency 2000 Mhz
Wavelength 0.15 mtr
Voltage 52.6 + j 0 V
Current 1.9 - j 7e-5 A
Impedance 27.6 + j 1e-3
Series comp. 82000 pF
Parallel form 27.6 // j 8.e5
Parallel comp. 1.e-4 pF
S.W.R.50 1.81
Input power 100 W
Efficiency 99.67 %
Structure loss 334.6 mW
Radiat-eff. %
Network loss 0 uW
RDF [dB] 1.87
Radiat-power 99.66 W
Environment
Loads
Polar
FREE SPACE
Comment
Example 2 : Loaded dipole in free space
See GetStarted.txt
Seg's/patches 9
Pattern lines 1533
Freq/Eval steps 21
Calculation time 0.047 s
start stop count step
Theta -180 180 73 5
Phi 180 180 1 0

Tot-gain [dBi]
Vertical plane
2000 MHz
Z
XY
ANTENNA_SENZA_IMPEDENZA.out
1.84 < dBi < 1.84
Phi= 0
Max gain The:180

dBi
1.84
1.61
1.36
1.09
0.81
0.51
0.19
-0.2
-0.6
-1
-1.5
-2
-2.7
-3.5
-4.6
-6.3
-20

Z
X

Case $2l = 1.2\lambda$

$$l = \frac{1.2}{2}\lambda = 0.09\text{m}$$