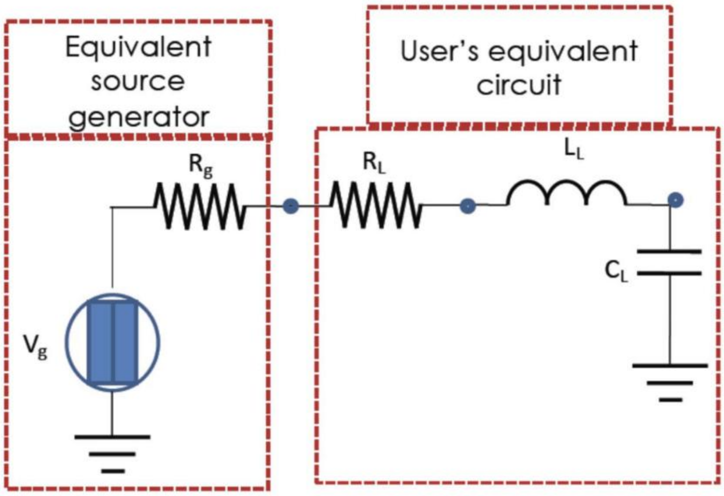
Homework 1

Exercise on Power Balance in a Resonant Circuit



$$\boxed{\omega = \frac{1}{\sqrt{L_L C_L}}}$$

è l'unica ***pulsazione di risonanza*** del circuito

Calculate the power balance of the system, both for the resonant and non-resonant cases. Consider:

$$R_G = R_L = 50\Omega \quad V_G = 3V \quad \omega = 2\pi \cdot 5 \cdot 10^9 \quad L = 10nH$$

To solve the exercise, we proceed step by step to determine the system's power flow, both under resonance and off-resonance conditions.

We start with the resonance case, where the circuit frequency equals the resonance frequency ω_0. Resonance occurs when $Z_L = Z_G^*$. In our case, this means the inductive and capacitive reactances of the load cancel each other out, since $Z_G^* = R_G$ (purely real).

$$X_L = \omega L_L$$

$$X_C = \frac{1}{\omega C_L}$$

$$X_L = X_C$$

The value of C_L can be derived using the resonance frequency formula:

$$\omega_0 = \frac{1}{\sqrt{L_L C_L}}$$

$$C_L = \frac{1}{{\omega_0}^2 L_L} = \frac{1}{(2\pi \cdot 5 \cdot 10^9)^2 \cdot 10nH} = 1.013 \cdot 10^{-13} F$$

After determining C_L, the total impedance Z of the circuit at resonance equals R_L, since the reactances cancel out. Consequently, the absorbed power is:

$$Z = R_L$$

$$P = \frac{{V_s}^2}{8Z} = \frac{3V^2}{8 \cdot 50\Omega} = 0.0225W$$

In the non-resonant case, the absorbed power is computed as:

$$P = \frac{{V_s}^2}{8R_G} \cdot \frac{4R_G R_A}{|Z_G + Z_A|^2}$$

Here, $\rho_D = \frac{4R_G R_A}{|Z_G+Z_A|^2}$ is the matching factor, which equals 1 under resonance. Substituting $Z_G = Z_A = R_A = 50\Omega$, we obtain:

$$\rho_D = \frac{4 \cdot 50 \cdot 50}{100^2 + {X_L}^2}$$

$${X_L}^2 = \left(\frac{\omega L}{1 - \omega^2 LC}\right)^2$$

Assuming a capacitance of $C = 10^{-12}F$ (off-resonance), we obtain) ${X_L}^2 = 1255$ and $\rho_D = 0.888$, leading to a power of: $P = 0.1998$

Numerical Calculation of Radiated Power, Radiation Resistance, and Gain for an Electromagnetic Dipole

Using the Matlab script *plot_ddr_dipole_EM.m*, compute the radiated power (for unit current), the radiation resistance, and the directivity gain of an electromagnetic dipole in the following cases:

$$2l = 0.25\lambda$$

$$2l = 1.2\lambda$$

By modifying the Matlab code, the required values were obtained. Considering $\beta = \frac{2\pi}{\lambda}$, the results are:

Case $2l = 0.25\lambda$

$$Radiation\ resistance = 13.4405\Omega$$

$$Radiated\ power = 3.3601W$$

$$Directivity\ gain = 1.5318$$

Case $2l = 1.2\lambda$

$$Radiation\ resistance = 360.1938\Omega$$

$$Radiated\ power = 62.222W$$

$$Directivity\ gain = 3.1557$$

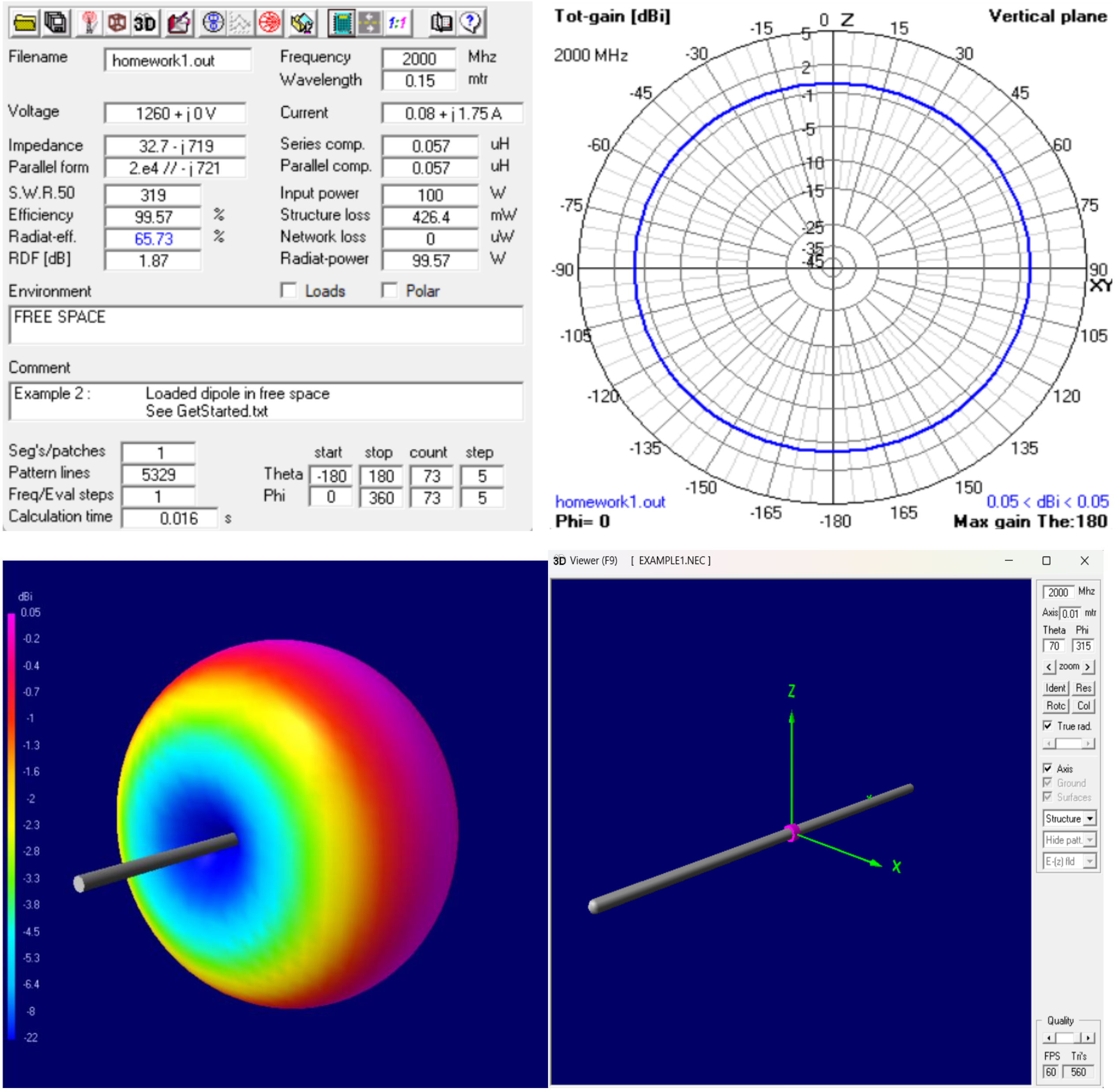
Simulation and Analysis Using 4NEC

Using 4NEC, simulate the two dipoles at 2 GHz with a thickness of 0.5 mm. Obtain the radiation pattern, gain, SWR, and reflection coefficient around the operating frequency. For the 0.25λ dipole, determine two inductive loads that allow resonance.

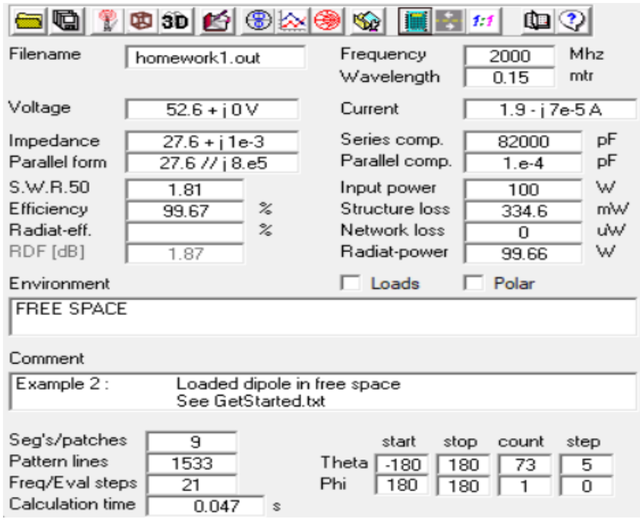
Since the antenna operates in air, we assume the wavelength as $\lambda = \lambda_0 = \frac{c}{f}$. Therefore, the dipole lengths for the two cases are:

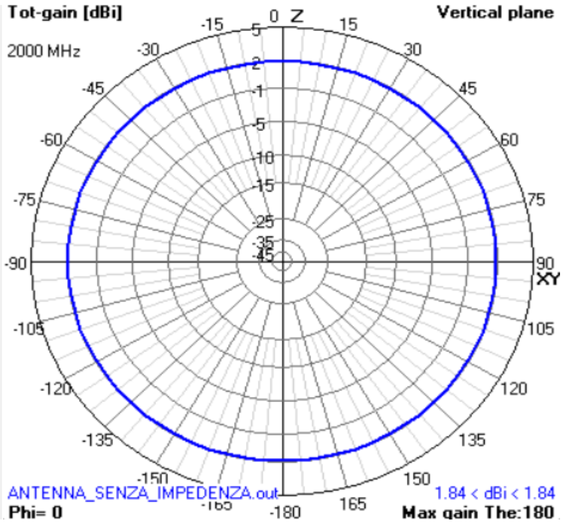
Case $2l = 0.25\lambda$

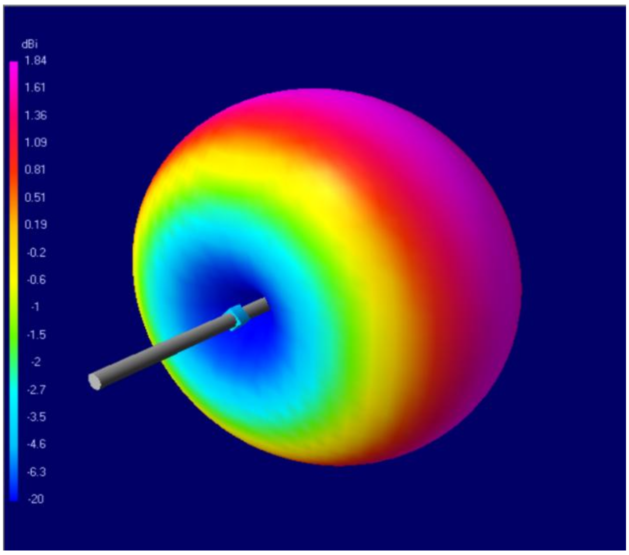
$$l = \frac{0.25}{2}\lambda = 0.01875\text{m}$$

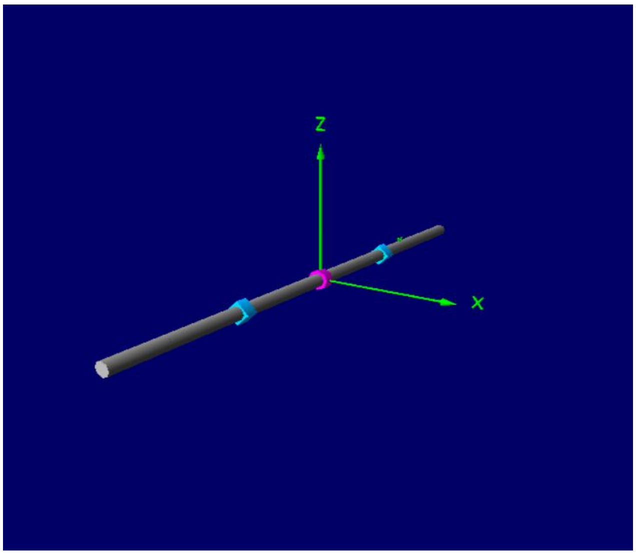


Inductive loads were introduced to achieve resonance:


Filename homework1.out
Frequency 2000 Mhz
Wavelength 0.15 mtr
Voltage 52.6 + j 0 V
Current 1.9 - j 7e-5 A
Impedance 27.6 + j 1e-3
Series comp. 82000 pF
Parallel form 27.6 // j 8.e5
Parallel comp. 1.e-4 pF
S.W.R.50 1.81
Input power 100 W
Efficiency 99.67 %
Structure loss 334.6 mW
Radiat-eff. %
Network loss 0 uW
RDF [dB] 1.87
Radiat-power 99.66 W
Environment
Loads
Polar
FREE SPACE
Comment
Example 2 : Loaded dipole in free space
See GetStarted.txt
Seg's/patches 9
Pattern lines 1533
Freq/Eval steps 21
Calculation time 0.047 s
start stop count step
Theta -180 180 73 5
Phi 180 180 1 0


Tot-gain [dBi]
Vertical plane
2000 MHz
Z
XY
ANTENNA_SENZA_IMPEDENZA.out
1.84 < dBi < 1.84
Phi= 0
Max gain The:180


dBi
1.84
1.61
1.36
1.09
0.81
0.51
0.19
-0.2
-0.6
-1
-1.5
-2
-2.7
-3.5
-4.6
-6.3
-20


Z
X

Case $2l = 1.2\lambda$

$$l = \frac{1.2}{2}\lambda = 0.09\text{m}$$