

# Frank-Wolfe for White Box Adversarial Attacks

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Adversarial image: element drawn from data distribution that is perturbed with some noise

- Misclassification: distorted image is not properly recognized by the DNN
- Transferability: different DNNs misclassify in the same way

daisy : 85.09% Confidence  
distortion = 0.0000



→ adding a  
small noise,  
the  
adversarial  
image fools  
the DNN →

bee : 71.64% Confidence  
distortion = 0.0015



We can decide if we want control on the output target of the adversarial image

- **Untargeted** attacks: just interested in the misclassification
- **Targeted** attacks: want a specific class as output

According to the information we can retrieve from the DNNs, we can have:

- **White box** attacks: access to all information, also gradients
- **Black box** attacks: access only to input and output  
→ techniques for gradient estimation

## Notation:

- $x_{\text{ori}}$  : original image
- $\ell(\cdot)$ : loss function

One of the simplest adversarial method and one of first implemented:

- One-step
- Gradient-based

$$x = x_{\text{ori}} + \epsilon \text{sign}(\nabla_x \ell(x_{\text{ori}})) \quad (\text{untargeted});$$

$$x = x_{\text{ori}} - \epsilon \text{sign}(\nabla_x \ell(x_{\text{ori}})) \quad (\text{untargeted});$$

- Projection-based approach
- Slow method

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**Algorithm 1** PGM

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```
1: for  $k = 1 \dots$  do  
2:   Set  $\bar{x}_k = \rho_C(x_k + s_k \nabla f(x_k))$            ▷ if untargeted,  $s_k > 0$   
3:   Set  $\bar{x}_k = \rho_C(x_k - s_k \nabla f(x_k))$            ▷ if targeted,  $s_k > 0$   
4:   If  $\bar{x}_k$  satisfies some specific condition, then STOP  
5:   Set  $x_{k+1} = x_k + \gamma_k(\bar{x}_k - x_k)$            ▷ with  $\gamma_k \in (0, 1]$   
6: end for
```

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- Iterative version of *FGSM*, adding a momentum term
- High distortion values

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## Algorithm 2 MI-FGSM

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1: Fix  $g_0 = 0$  and  $x_0^*$ 
2: for  $t = 0$  to  $T - 1$  do
3:   Input  $x_t$  and obtain the gradient  $\nabla_x f(x_t)$ 
4:    $g_{t+1} = \beta \cdot g_t + \frac{\nabla_x f(x_t)}{\|\nabla_x f(x_t)\|_1}$ 
5:    $x_{t+1} = x_t + \gamma \cdot \text{sign}(g_{t+1})$  ▷ if untargeted
6:    $x_{t+1} = x_t - \gamma \cdot \text{sign}(g_{t+1})$  ▷ if targeted
7: end for
```

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- Projection-free method
- Good trade-off between success-distortion

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### Algorithm 3 FW-White

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- 1: Set  $x_0 = x_{\text{ori}}$ ,  $m_{-1} = -\nabla_x f(x_0)$  if untargeted attack,  $m_{-1} = \nabla_x f(x_0)$  if targeted attack
  - 2: **for**  $t = 0$  to  $T - 1$  **do**
  - 3:      $m_t = \beta \cdot m_{t-1} - (1 - \beta) \cdot \nabla f(x_t)$  ▷ if untargeted
  - 4:      $m_t = \beta \cdot m_{t-1} + (1 - \beta) \cdot \nabla f(x_t)$  ▷ if targeted
  - 5:      $v_t = \operatorname{argmin}_{x \in C} \langle x, m_t \rangle = -\epsilon \cdot \operatorname{sign}(m_t) + x_{\text{ori}}$
  - 6:      $d_t = v_t - x_t$
  - 7:      $x_{t+1} = x_t + \gamma d_t$
  - 8: **end for**
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Now we hand over the word to Alberto so that we can see a demo of our project.



