



# Numerical solution of PDEs using the Finite Element Method

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16 March - 20 March 2020

# Course schedule

Time	Duration	Content	
		MONDAY 16.03.2020	TUESDAY 17.03.2020
09:30	1.25 hours	Introduction First steps	Local refinement Hanging nodes - Part 1
		COFFEE / TEA	
11:15	1.25 hours	Introduction to FEM	Exercises, Q&A
		LUNCH	
14:00	1.25 hours	Solving Poisson's equation	Local refinement Hanging nodes - Part 2
		COFFEE / TEA	
15:45	1.25 hours	Exercises, Q&A	Exercises, Q&A
		WEDNESDAY 18.03.2020	THURSDAY 19.03.2020
09:30	1.25 hours	Shared memory parallelisation: Part 1	MPI parallelisation: Part 1
		COFFEE / TEA	
11:15	1.25 hours	Exercises, Q&A	Exercises, Q&A
		LUNCH	
14:00	1 hour	Shared memory parallelisation: Part 2	MPI parallelisation: Part 2
		COFFEE / TEA	
15:45	1.25 hours		Exercises, Q&A
		FRIDAY 20.03.2020	Project

# Course goals

- Learn the fundamentals of deal.II
  - Commonly used data structures, their interface
  - Structure of finite element problems
  - Good implementation practices
  - Navigate the documentation
- Implement PDE solver (prototypical problem)
  - On complex Geometries
  - Using local refinement
  - Running in parallel (on both shared and distributed memory systems)

# How the course will be run

- Each module will have a lecture
  - Present salient information
  - Put what we'll learn into context
- Then we'll walk through aspects of the tutorials/examples together
  - Discuss important functionality
    - What it does
    - How it works
    - Caveats and tips
- Remainder of the lecture will be spent doing some exercises/assignments
  - Suggestion: Work in groups of two/three
  - Continued at in the last session of the day

# Resources

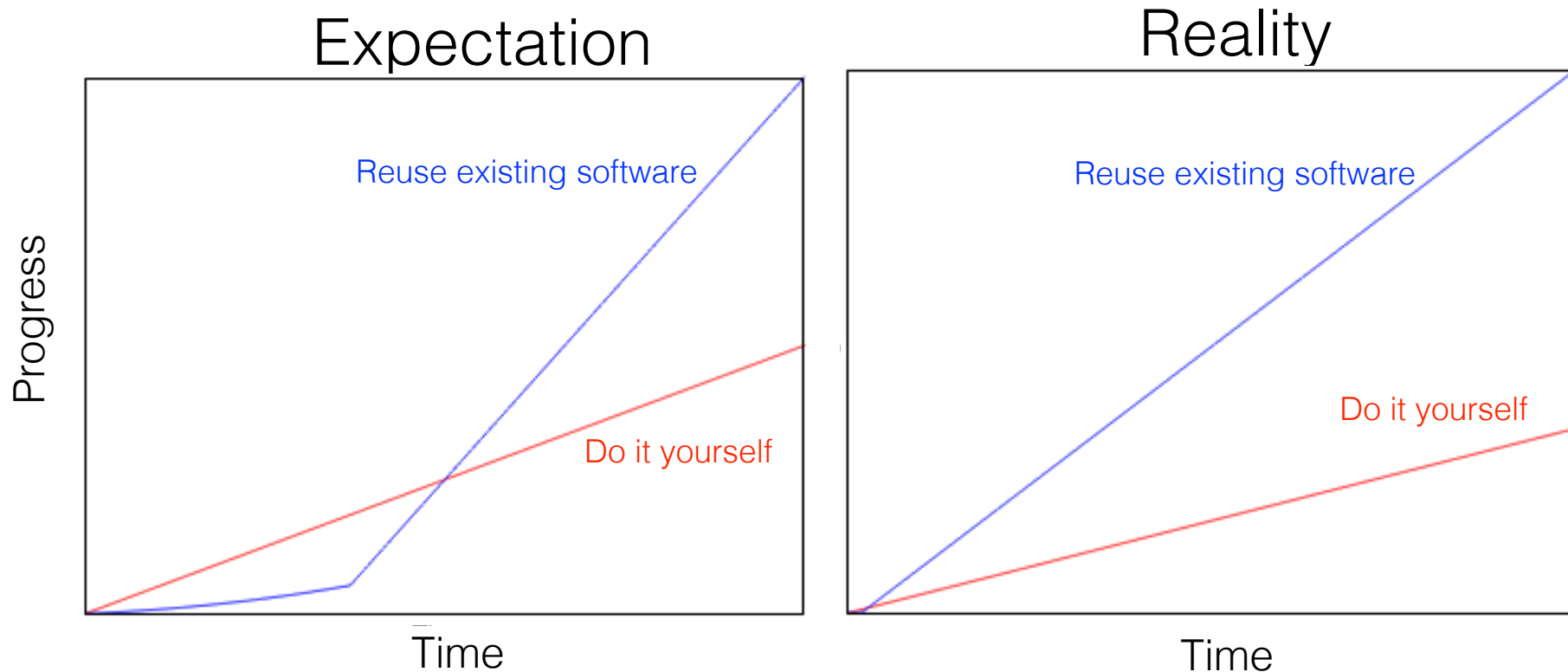
- deal.II user manual
  - <https://www.dealii.org/current/doxygen/deal.II/index.html>
  - <https://www.dealii.org/current/doxygen/deal.II/modules.html>
  - <https://www.dealii.org/current/doxygen/deal.II/DEALGlossary.html>
- deal.II tutorials and code gallery
  - <https://www.dealii.org/current/doxygen/deal.II/Tutorial.html>
  - <https://www.dealii.org/current/doxygen/deal.II/CodeGallery.html>
- Me :-)
  - Don't hesitate to ask questions



# Lecture 1: Introduction to deal.II (tools configuration)

Luca Heltai ([luca.heltai@sissa.it](mailto:luca.heltai@sissa.it))

# Why use deal.II (or any other PDE toolbox)?

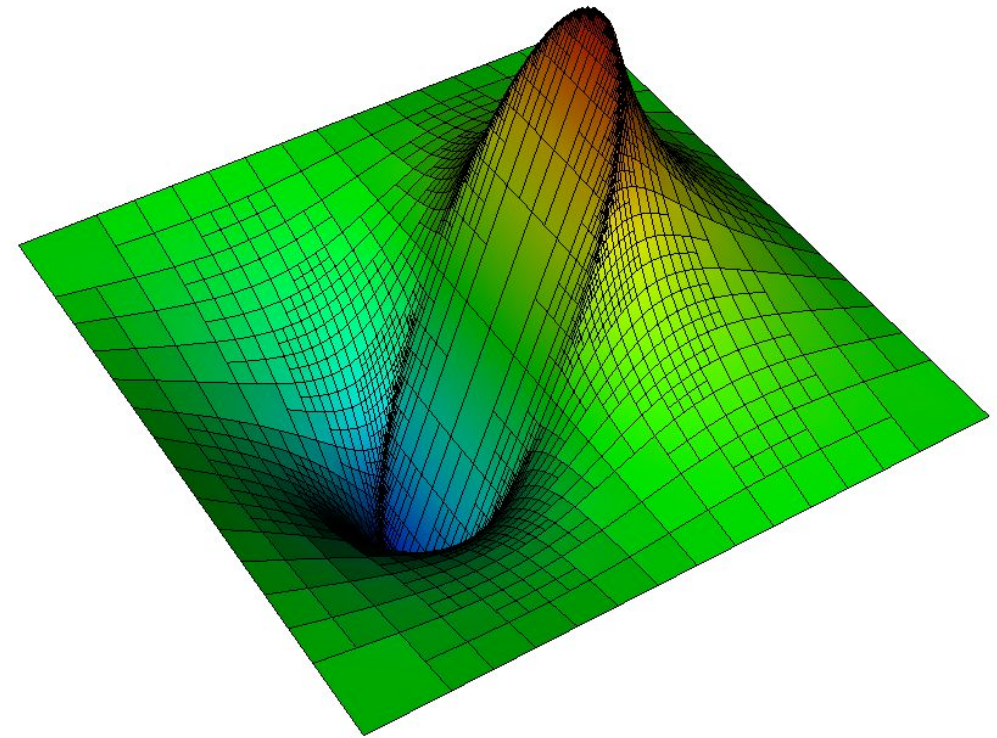


- Applies to:
  - Users
  - Developers
- “The secret to good scientific software is (re)using existing libraries”

# Why deal.II?

## Differential Equation Analysis Library

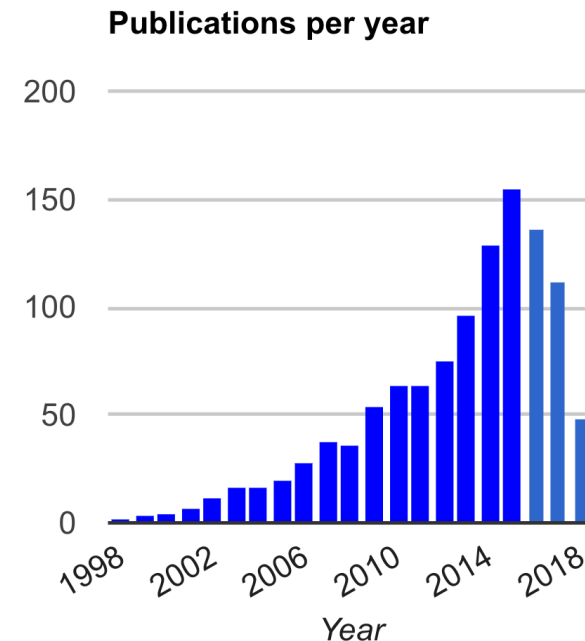
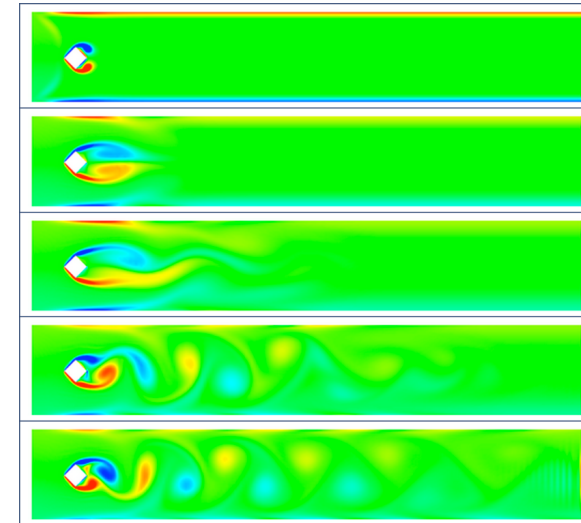
- Flexible open-source finite element toolkit
  - All the support functionality required to describe and solve a FE problem (PDEs)
  - Optimized for speed
  - Heavily tested
    - Many error checks (debug mode)
    - +10,000 regression tests run continuously
  - Part of SPEC CPU 2017 benchmark
- Templated C++ library (Object Orientated)
  - Dimension independent programming
- Portable
  - OS, architecture, compiler
- Origins
  - Study mesh adaptivity and error estimation
  - Now used in many other applications, frameworks





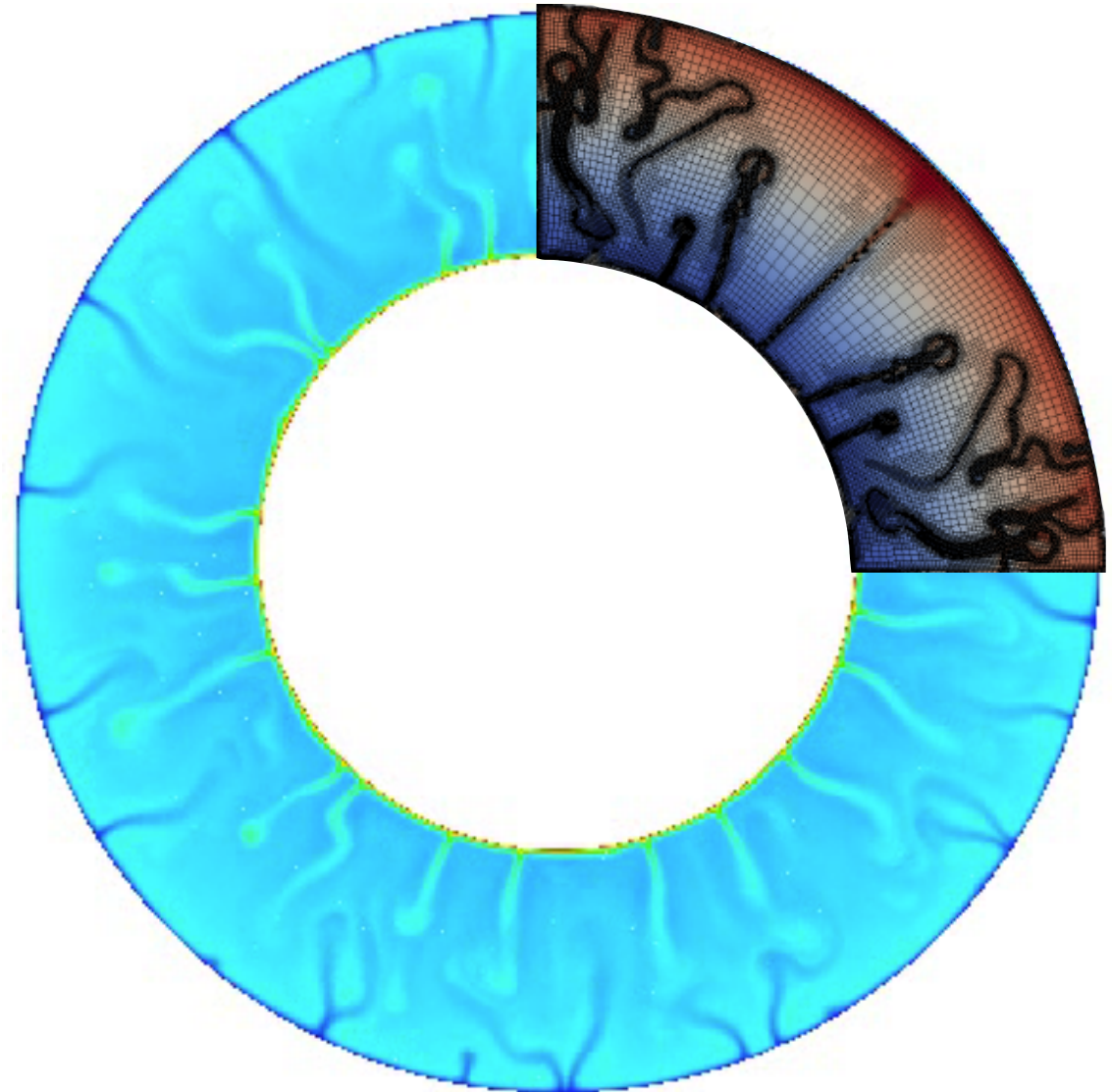
# Why deal.II?

- Heavily documented
  - Over 10000 pages of interface documentation
  - Numerous tutorials
    - Illustrate functionality
    - Present methods to solve problems
- Quite widely used, and growing
- **Active** community
  - Approachable developers
  - Helpful online forum



# Classes of problems solved using deal.II

- Geomechanics
- Fluid and gas dynamics
- Porous media
- Fluid-structure interaction
- Boundary element method
- Topology optimization
- Medical image reconstruction
- Structural mechanics
- Biomechanics
- Crystal growth
- Gradient and crystal plasticity
- Generalized continua
- Contact mechanics
- Atomistic-to-Continuum coupling
- Quantum mechanics
- Magneto- and electro-elasticity
- Thermo-plasticity

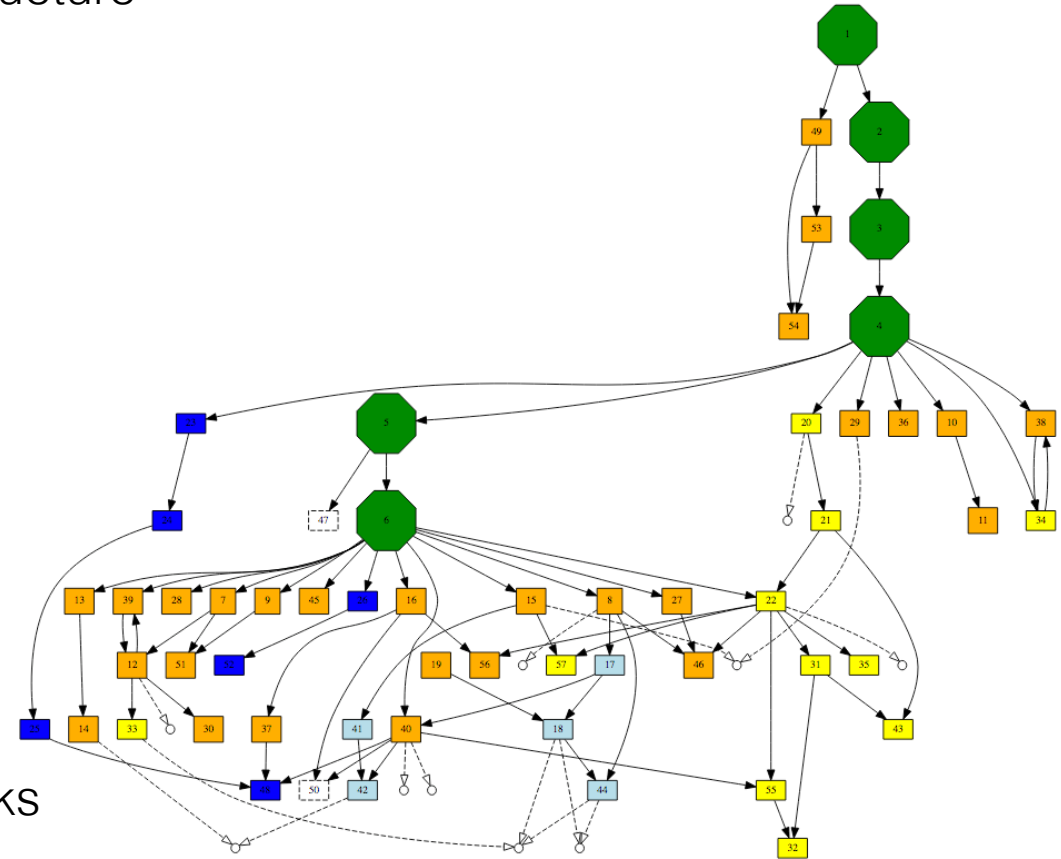


# What deal.II is not

- A black box
  - You can't throw a problem at it, and hope it will solve it (like, say, comsol)
  - Won't do anything more than you ask it to
- deal.II knows little about
  - Numerical methods
  - Problem-specific details, i.e.
    - Preconditioners
    - Constitutive equations

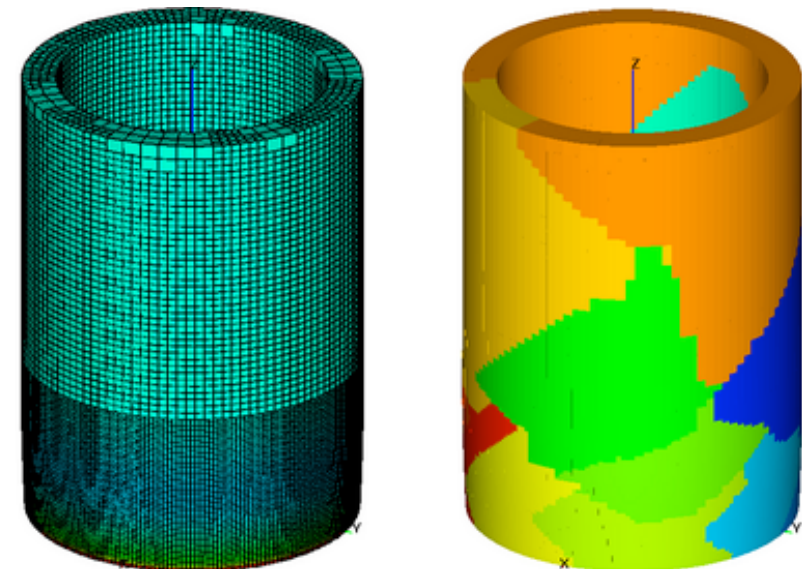
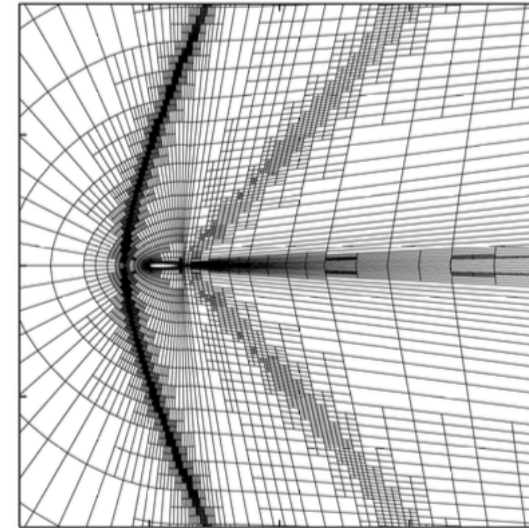
# How deal.II will help you

- Unified and well thought out data structure
  - Problem implementation
- Many tutorials
  - Baseline from which to build on
  - Demonstrate how to use features
- Comprehensive debugging support
  - Error messages everywhere!
- Some built in numerical tools
- Integration with advanced frameworks
  - Nonlinear solvers
  - Time integrators
  - Parallel sparse and dense linear algebra



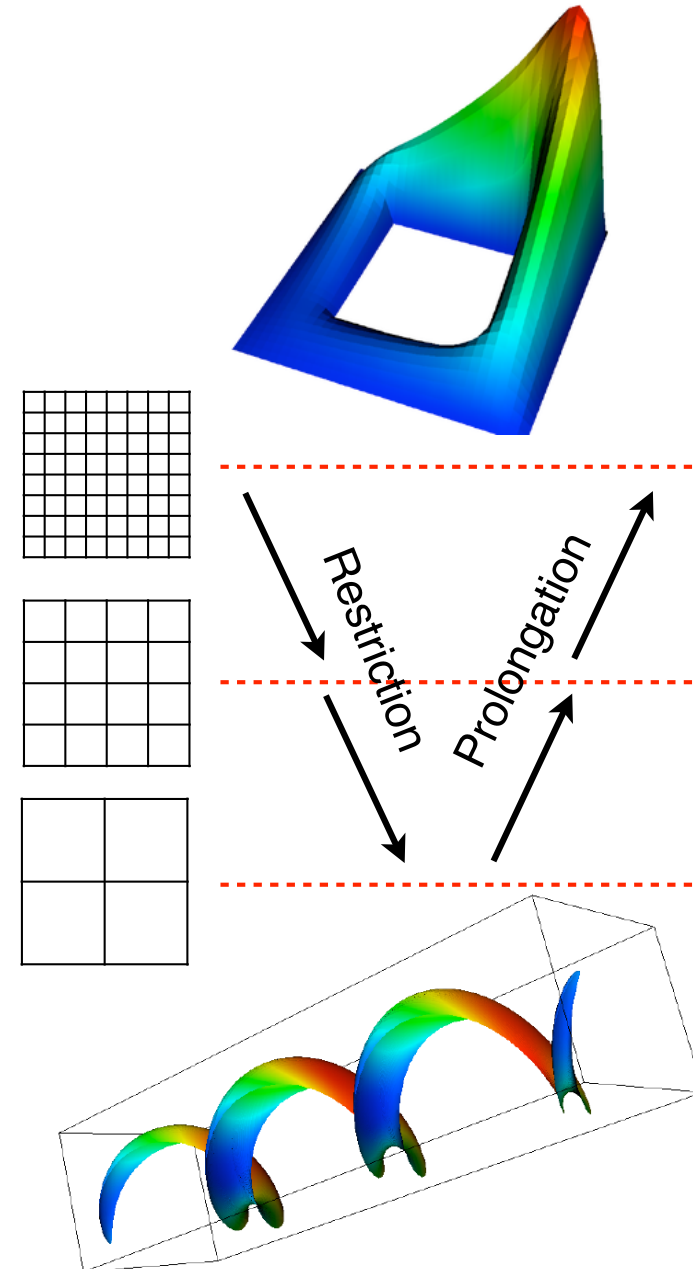
# Fundamental capabilities and frameworks

- Mesh adaptivity
- Dense and sparse linear algebra
  - Built in tensor, dense matrix/vector classes
  - BLAS and LAPACK integration; GSL
  - Built in linear solvers and preconditioners
  - Eigenvalue solvers
- Parallelization
  - MPI
    - Linear algebra libraries (PETSc, Trilinos)
    - Distributed meshes → Billion DoFs
  - Threading (Intel TBB)
  - Vectorized numbers (AVX extensions)
- Pre/post-processing



# Advanced capabilities and frameworks

- hp-finite element support
- Meshworker
  - Assembly assistance
  - Functions to perform assembly for specific problem classes
- Geometric multi-grid
  - Using coarse grid as preconditioner to solution for finer grid
- Matrix-free
  - No explicit storing of matrix elements
  - Exchange memory transfer for computations
- Charts and manifolds
  - Accurate description of topologically complex objects



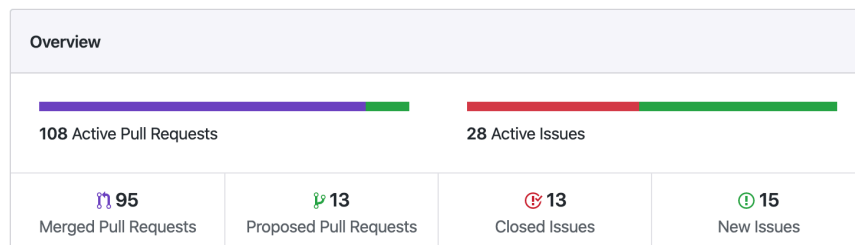


# How deal.II is developed

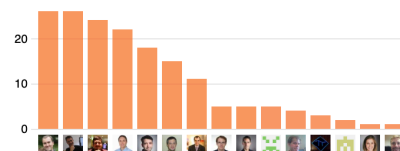
- Open repository on GitHub
  - <https://github.com/dealii/dealii>
- Anyone can contribute!
  - We encourage all to participate
- On average: 3/4 pull requests/day

February 14, 2020 – March 14, 2020

Period: 1 month ▾



Excluding merges, **17 authors** have pushed **168 commits** to master and **168 commits** to all branches. On master, **1,950 files** have changed and there have been **14,063 additions** and **6,159 deletions**.



95 Pull requests merged by 16 people

dealii / dealii

Unwatch 78 Unstar 594 Fork 438

Code Issues 414 Pull requests 49 ZenHub Actions Projects 15 Wiki Security Insights

Filters is:pr is:open Labels 50 Milestones 2 New pull request

49 Open	7,493 Closed	Author ▾	Label ▾	Projects ▾	Milestones ▾	Reviews ▾	Assignee ▾	Sort ▾
<input type="checkbox"/>	<input checked="" type="checkbox"/>	Enable std::max_element for VectorizedArray	✗					
#9660 opened 4 hours ago by peterrum • Review required								
<input type="checkbox"/>	<input checked="" type="checkbox"/>	Replace small numbers in test suite output by zero.	✓	ready to test				5
#9659 opened 13 hours ago by bangerth • Review required								
<input type="checkbox"/>	<input checked="" type="checkbox"/>	disallow h2 in tutorial headers	•					5
#9658 opened 2 days ago by tjhei • Changes requested								
<input type="checkbox"/>	<input checked="" type="checkbox"/>	Implement matrix-free evaluation of geometry for MF::MappingInfo	✓	Matrix-free				20
#9656 opened 2 days ago by kronbichler • Review required 2 of 2								
<input type="checkbox"/>	<input checked="" type="checkbox"/>	WIP refactoring MatrixFreeFunctions::ShapeInfo	•	Matrix-free	WIP			15
#9655 opened 2 days ago by jwitte08 • Review required								
<input type="checkbox"/>	<input checked="" type="checkbox"/>	PreconditionChebyshev: report stats with output_details	•	Linear Algebra	WIP			9
#9645 opened 5 days ago by tjhei • Review required								
<input type="checkbox"/>	<input checked="" type="checkbox"/>	Simplify internal::DoFHandlerImplementation::Policy::ParallelDistributed::distribute_dofs()	✓	ready to test				24
#9644 opened 5 days ago by peterrum • Approved								
<input type="checkbox"/>	<input checked="" type="checkbox"/>	Enable Utilities::MPI::NoncontiguousPartitioner to handle padding	[WIP]					2
#9639 opened 6 days ago by peterrum • Review required								
<input type="checkbox"/>	<input checked="" type="checkbox"/>	Step-69: Introduce asynchronous IO mechanism	•	Discussion	Documentation	Tutorials		7
#9636 opened 7 days ago by tamiko • Approved								
<input type="checkbox"/>	<input checked="" type="checkbox"/>	new free function LinearAlgebra::set_zero_mean_value()	✓	Linear Algebra	ready to test			14
#9631 opened 9 days ago by nfehn • Review required								
<input type="checkbox"/>	<input checked="" type="checkbox"/>	Deprecate another function in DoFTools.	✗	ready to test				8
#9566 opened 18 days ago by bangerth • Approved								
<input type="checkbox"/>	<input checked="" type="checkbox"/>	step-47: Add derivation for the simply supported plates to result section.	•					11
#9558 opened 20 days ago by sophy1029 • Review required								

# Configuration and Setup

- Course repository: [https://github.com/sissa/p2.3\\_seed](https://github.com/sissa/p2.3_seed)
- Docker image: <https://hub.docker.com/r/dealii/dealii>
- .dmg file (for mac users): <https://github.com/dealii/dealii/releases>
- Manual installation with spack: **spack install dealii**
- Other means of installation: <https://dealii.org/download.html>
- Notice: build from scratch, with all dependencies ~ 6hours...
- We'll use an IDE (**Qt Creator** for me, but you can choose the one you prefer)
- We'll use a VTK viewer (**Paraview** for me, but **Visit** is also possible)