

Theory of turbulence augmentation across hypersonic shock waves

74th Annual Meeting of the APS Division of Fluid Dynamics

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Huete: Spanish MCINN and BBVA Foundation (Leonardo Grant)

Urzay: US AFOSR and US DoE/NNSA



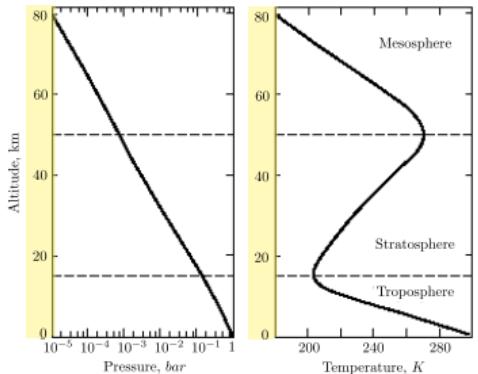
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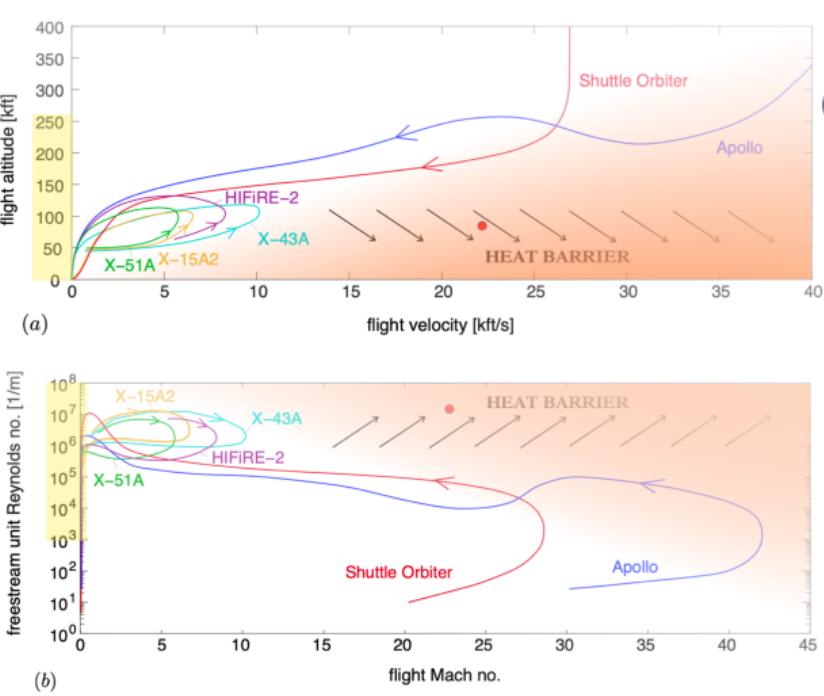
Motivation



$$\frac{\Delta p}{p} \sim \frac{\Delta \rho}{\rho} = O(10^5), \quad \frac{\Delta T}{T} = O(1).$$

In hypersonic flight near the ground, the Reynolds number becomes large because of the comparatively larger densities

$$\frac{\Delta Re}{Re} = O(10^5).$$



Urzay, J., & Di Renzo, M. (2021). Annual Research Briefs, Center for Turbulence Research, 7-32.

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1 Motivation

Effects of dissociation and vibrational excitation on the mean post-shock quantities

LIA of turbulence interacting with hypersonic shocks

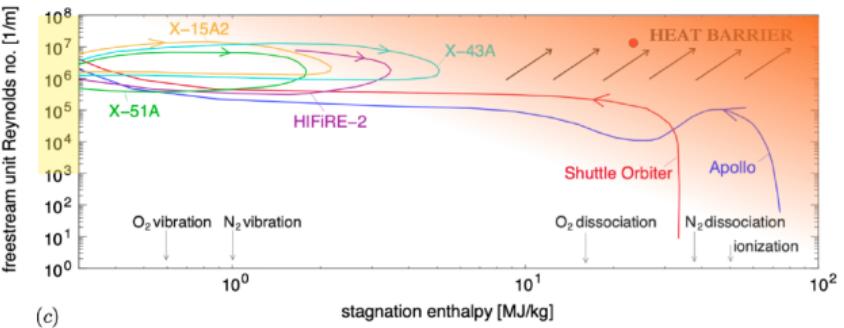
Conclusion



2 Motivation

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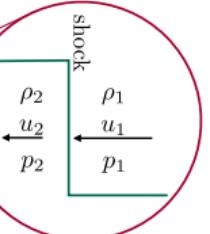
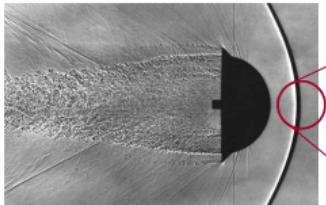


Urzay, J., & Di Renzo, M. (2021). Annual Research Briefs, Center for Turbulence Research, 7-32.

Hypersonic flight **at low altitudes** is characterized by:

- High free-stream Mach numbers $Ma \geq 5$
- High free-stream and post-shock unit Reynolds numbers $Re \sim 10^7 - 10^9 \text{ m}^{-1}$
- High stagnation enthalpies $h_0 \sim 5 - 30 \text{ MJ/kg}$
- Small mean free paths $\lambda \sim 0.1 \mu\text{m}$
- large normal Mach numbers
- turbulent boundary layers
- much higher than the vibrational specific energies of O_2 and N_2
- short vibrational relaxation distances





Integral conservation equations across shock waves in **dissociating** gases

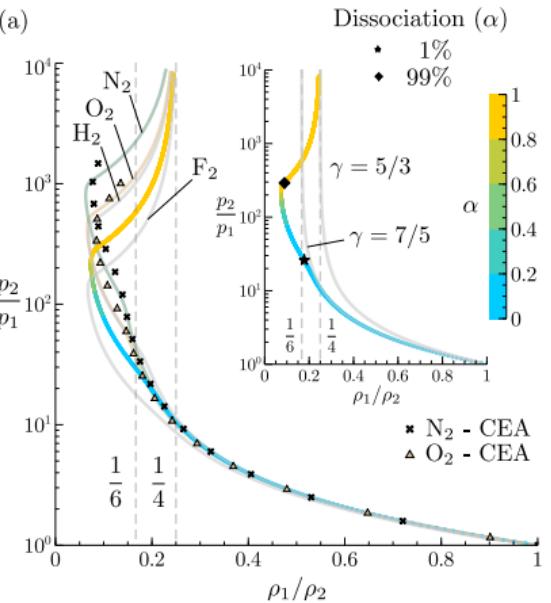
$$\rho_1 u_1 = \rho_2 u_2,$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2,$$

$$e_1 + p_1/\rho_1 + u_1^2/2 = e_2 + p_2/\rho_2 + u_2^2/2 + q_d,$$

Upstream flow is sufficiently cold to be approximated as

$$e_1 = (5/2)R_g T_1, \quad p_1 = \rho_1 R_g T_1.$$



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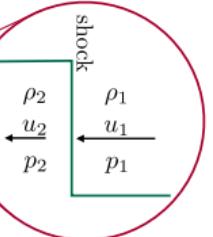
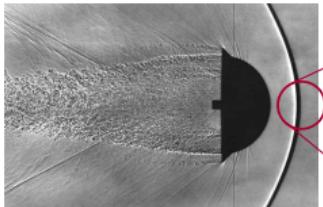
- 3 Effects of dissociation and vibrational excitation on the mean post-shock quantities

LIA of turbulence interacting with hypersonic shocks

Conclusion

Get the code





Integral conservation equations accross shock waves in **dissociating** gases

$$\rho_1 u_1 = \rho_2 u_2,$$

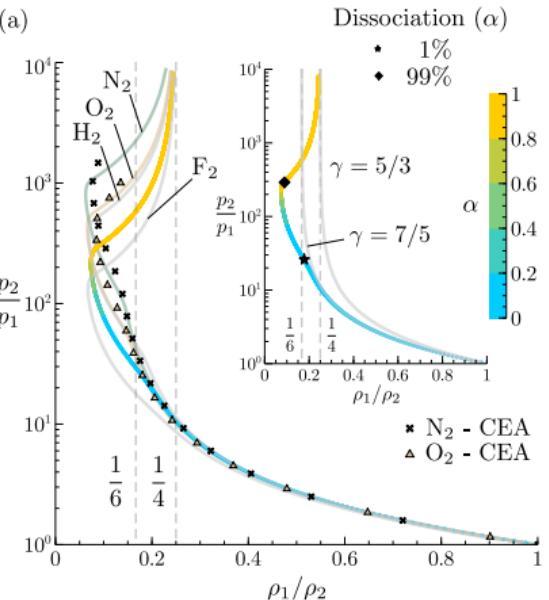
$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2,$$

$$e_1 + p_1/\rho_1 + u_1^2/2 = e_2 + p_2/\rho_2 + u_2^2/2 + q_d,$$

$$p_2 = \rho_2 R_g T_2 (1 + \alpha), \quad q_d = \alpha R_g \Theta_d,$$

$$e_2 = R_{g,A_2} T_2 \left[3\alpha + (1 - \alpha) \left(\frac{5}{2} + \frac{\Theta_v/T_2}{e^{\Theta_v/T_2} - 1} \right) \right],$$

$$\frac{\alpha^2}{1 - \alpha} = Gm\Theta_r \left(\frac{\pi m k_B}{\hbar^2} \right)^{3/2} \frac{\sqrt{T_2}}{\rho_2} e^{-\frac{\Theta_d}{T_2}} \left(1 - e^{-\frac{\Theta_v}{T_2}} \right),$$



where α is the **degree of dissociation**, defined as the mass fraction of A atoms in the reaction $A_2 \rightleftharpoons A + A$, that must be solved with the aid of the chemical equilibrium condition.

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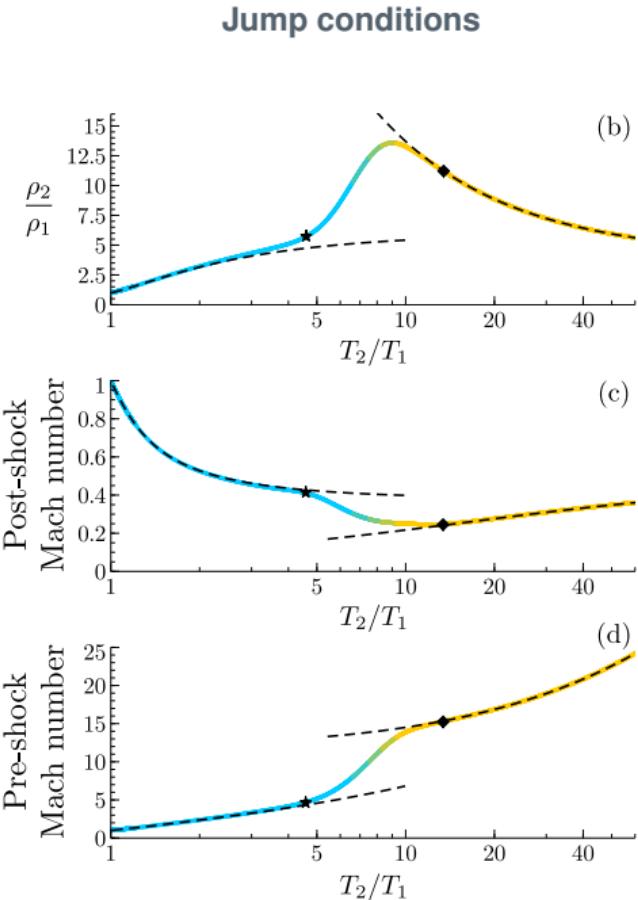
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Endothermicity due to dissociation and vibrational excitation does the following:

- increases the mean post-shock density
- decreases the mean post-shock velocity
- decreases the mean post-shock Mach number
- decreases the mean post-shock temperature



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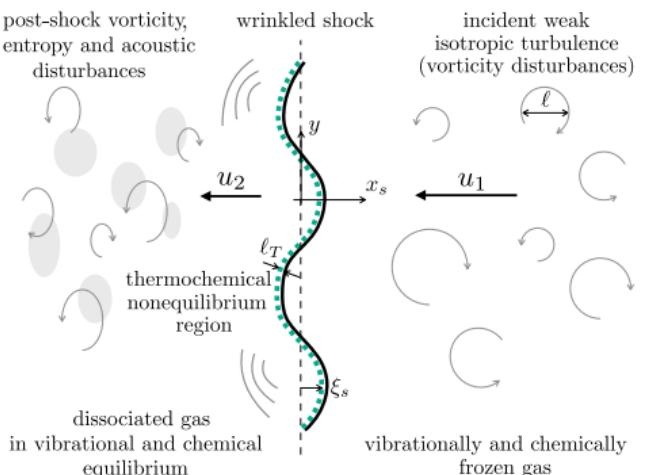


Considering:

- turbulence is comprised of small fluctuations,
- Kovasznay decomposition into vortical, entropic and acoustic modes,

we can solve this problem analytically by using

- linearized Rankine-Hugoniot relations,
- linearized Euler equations in the post-shock gas.



Limits of validity

Assumptions standard LIA:

- $\text{rms}(u_\ell) \ll a_1$ and a_2 ,
- $\xi_s \ll \ell$,
- $\ell/u_\ell \ll \ell^2/\nu$.

With thermochemical effects:

- $\ell_T \ll \ell$

For a given ℓ , this condition becomes increasingly more accurate as the altitude decreases.

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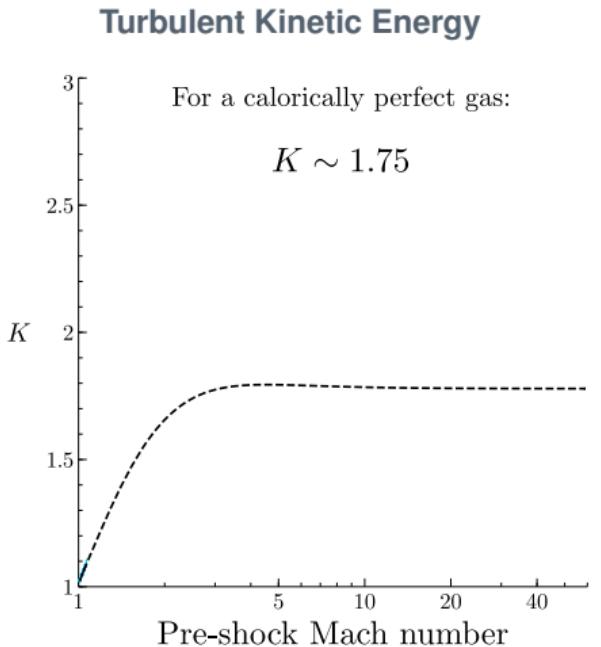
Motivation

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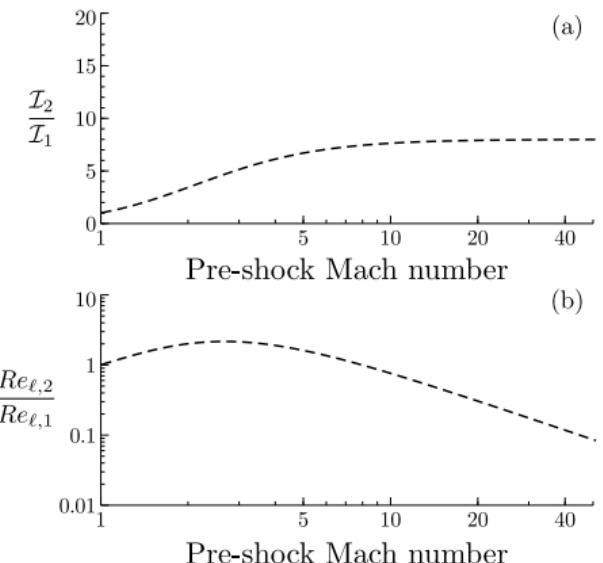
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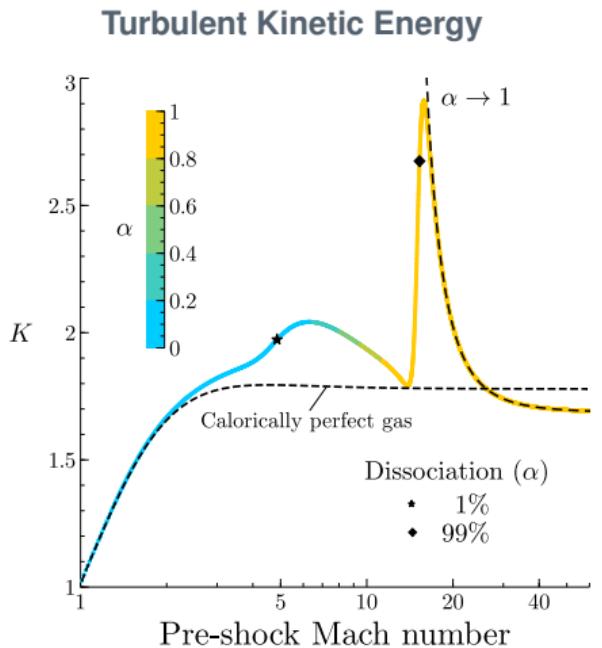


Turbulence intensity and Turbulent Reynolds number

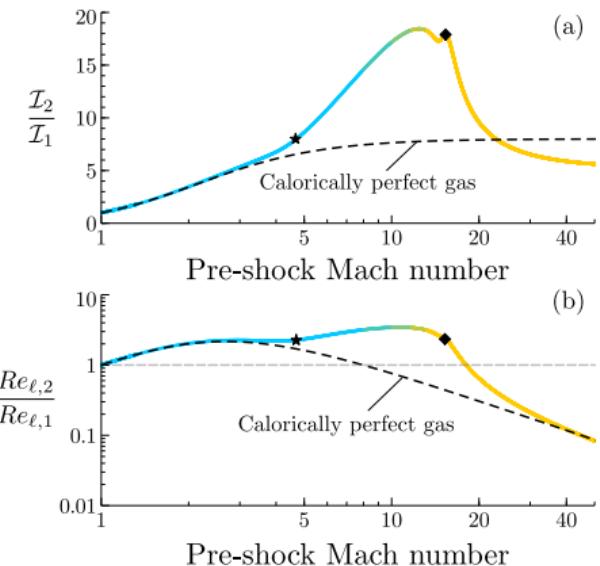


At hypervelocities ($Ma \gtrsim 10$), the calorically perfect gas approximation predicts a saturation in the amplification of kinetic energy and turbulence intensity, along with a decrease in the turbulent Reynolds number across the shock.





Turbulence intensity and Turbulent Reynolds number



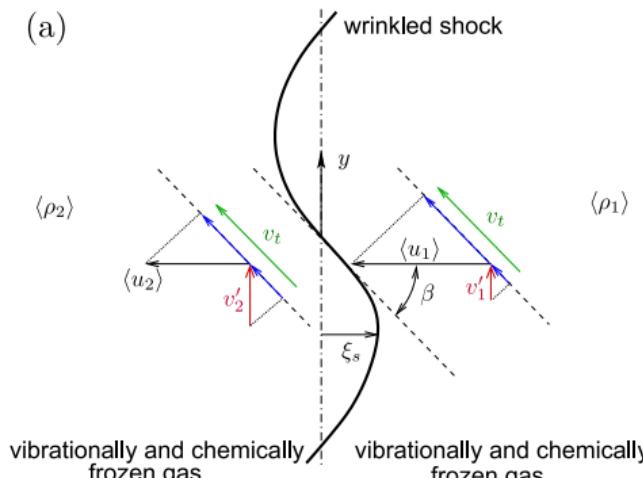
In contrast, the incorporation of dissociation and vibrational excitation predicts larger kinetic energy and turbulence intensity amplification rates, along with an increase in the turbulent Reynolds number accross the shock.



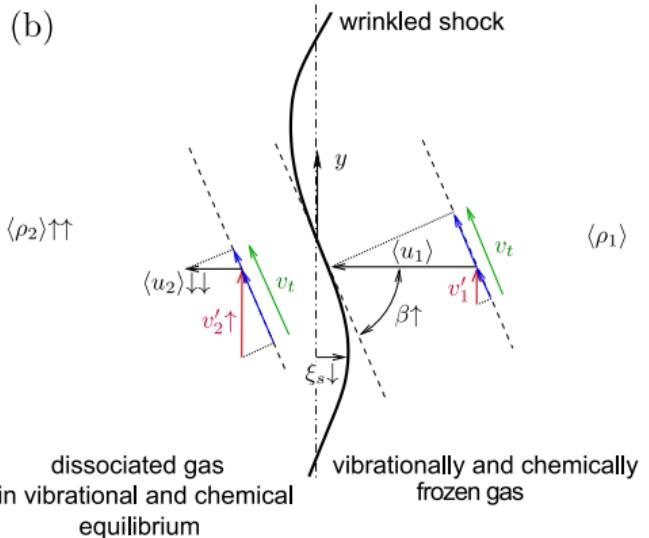
Calorically perfect gas

Vibrationally Excited, Dissociating Gas

(a)



(b)



Conservation of tangential momentum dictates that the transverse velocity fluctuations should increase across the shock – these are larger at hypersonic velocities because of the associated larger post-shock densities induced by endothermic thermochemical effects.



Key takeaways

- Significant departures from calorically perfect gas behaviour can be observed in the solution even at modest degrees of dissociation of 1%.
- A turning point in the Hugoniot curve is observed at approximately Mach 13 and 70% degree of dissociation.
- The amplification of TKE doubles that observed in calorically perfect gases, with most of the content of TKE downstream in form of vortical modes.
- The turbulent Reynolds number is amplified across the shock at hypersonic Mach numbers in the presence of dissociation and vibrational excitation, as opposed to the attenuation observed in the calorically perfect case.
- Thermochemical effects arising at hypersonic velocities appear to enhance turbulent fluctuations in the post-shock gas.



Huete, C. et al. (2021). Thermochemical effects on hypersonic shock waves interacting with weak turbulence. *Physics of Fluids*, 33(8), 086111