Effect of fuel mass fraction heterogeneity on the detonation propagation speed

25th International Congress of Theoretical and Applied Mechanics

Alberto Cuadra *, César Huete & Marcos Vera

*acuadra@ing.uc3m.es

Grupo de Mecánica de Fluidos
Universidad Carlos III de Madrid

Milano | August 22-27, 2021

ICTAM Milano 2020+1

uc3m | Universidad Carlos III de Madrid

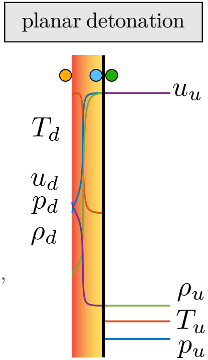
Reactive Rankine-Hugoniot equations

1. Fresh homogeneous mixture with $\mathcal{M}_u \geq \mathcal{M}_{\rm cj} = (1+Q)^{1/2} + Q^{1/2}$.
2. Obtain the Rankine-Hugoniot jump conditions from the conservation equations and assuming a calorically perfect gas, namely

$$\mathcal{R}_d = \frac{\rho_d}{\rho_u} = \frac{(\gamma+1)\,\mathcal{M}_u^2}{\gamma\mathcal{M}_u^2 + 1 - [(\mathcal{M}_u^2 - 1)^2 - 4Q\mathcal{M}_u^2]^{1/2}},$$

$$\mathcal{P}_d = \frac{p_d}{p_u} = \frac{\gamma\mathcal{M}_u^2 + 1 + \gamma\left[(\mathcal{M}_u^2 - 1)^2 - 4Q\mathcal{M}_u^2\right]^{1/2}}{\gamma+1},$$

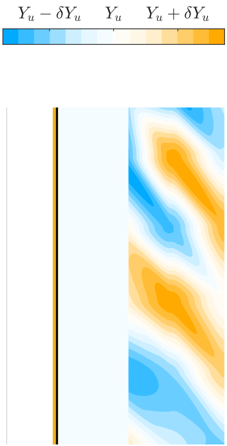
for density and pressure, respectively.



How will heterogeneities in the gas mixture affect the detonation properties?

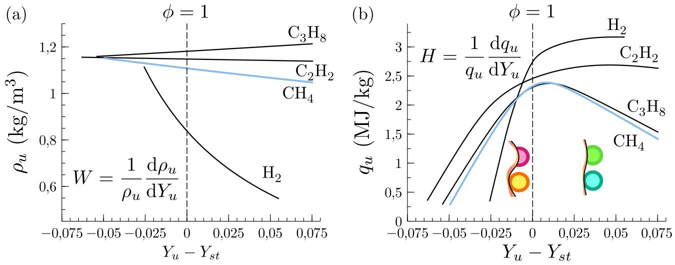
Faster detonation speed!

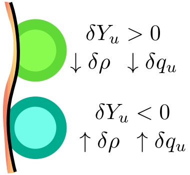
- The deviation of the fuel mass fraction δY_u introduce two source of perturbations:
  1. $\delta\rho$,
  2. δq_u.
- New contributions:
  - acoustic,
  - rotational,
  - entropic.
- Linear analysis: weak deviations $|\delta Y_u| = |Y_u - \langle Y_u \rangle| \ll 1$.



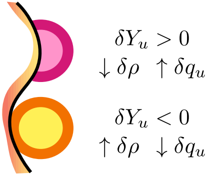
Detonation in heterogeneous gaseous mixture

The slopes W and H play a pivotal role in the turbulence generation.





destructive interference



constructive interference

Influences on the propagation properties

- Average density and pressure are lower while average velocity and Mach are higher.
- **Do they affect the detonation propagation speed?**

$$\frac{\langle \rho_d \rangle}{\langle \rho_u \rangle} = \mathcal{R}_d \left(1 + \bar{\epsilon}^2 \delta \mathcal{R}\right), \quad \frac{\langle p_d \rangle}{p_u} = \mathcal{P}_d \left(1 + \bar{\epsilon}^2 \delta \mathcal{P}\right), \quad \frac{\langle u_d \rangle}{u_u} = \frac{1}{\mathcal{R}_d} \left(1 + \bar{\epsilon}^2 \delta \mathcal{U}\right).$$

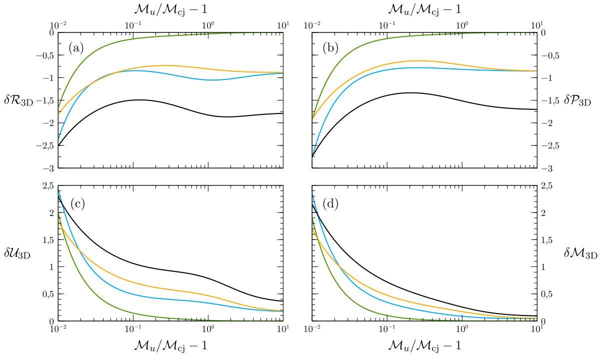
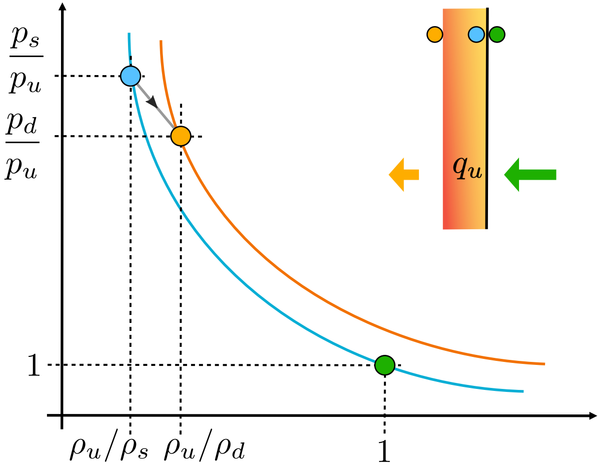


Figure 1: Three-dimensional second-order correction to the RH jump conditions (a) $\delta\mathcal{R}$, (b) $\delta\mathcal{P}$, (c) $\delta\mathcal{U}$, and (d) $\delta\mathcal{M}$ as a function of the overdrive parameter $\mathcal{M}_u/\mathcal{M}_{cj} - 1$. Computations correspond to $\gamma = 1.2$, $Q = 1$, and $|W| \gg |H|$ (black line), $|H| \gg |W|$ (green line), $W = H$ (orange line) and $W = -H$ (blue line).

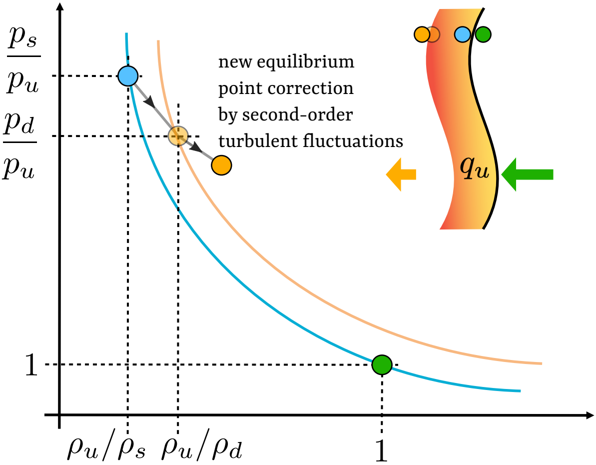
2nd-order variations

$$\delta\mathcal{S}|_{\delta\langle\mathcal{M}_d\rangle=0} = -\frac{1}{\sqrt{\mathcal{R}_d\mathcal{P}_d}}\left(\frac{\mathrm{d}\mathcal{M}_d}{\mathrm{d}\mathcal{M}_u}\right)^{-1}\left(\delta\mathcal{U}+\frac{1}{2}\delta\mathcal{R}-\frac{1}{2}\delta\mathcal{P}\right). \quad (1)$$



2nd-order variations

$$\delta\mathcal{S}|_{\delta\langle\mathcal{M}_d\rangle=0} = -\frac{1}{\sqrt{\mathcal{R}_d\mathcal{P}_d}}\left(\frac{\mathrm{d}\mathcal{M}_d}{\mathrm{d}\mathcal{M}_u}\right)^{-1}\left(\delta\mathcal{U}+\frac{1}{2}\delta\mathcal{R}-\frac{1}{2}\delta\mathcal{P}\right). \quad (1)$$



Influences on the propagation properties

2nd-order variations

$$\delta\mathcal{S}|_{\delta\langle\mathcal{M}_d\rangle=0} = -\frac{1}{\sqrt{\mathcal{R}_d\mathcal{P}_d}}\left(\frac{\mathrm{d}\mathcal{M}_d}{\mathrm{d}\mathcal{M}_u}\right)^{-1}\left(\delta\mathcal{U}+\frac{1}{2}\delta\mathcal{R}-\frac{1}{2}\delta\mathcal{P}\right). \tag{1}$$

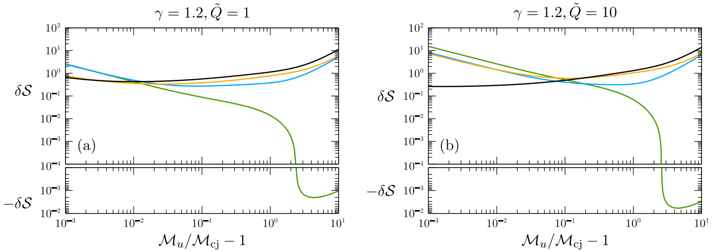


Figure 2: Three-dimensional second-order correction of the detonation propagation velocity $\delta\mathcal{S}$, according to (1), as a function of the overdrive parameter $\mathcal{M}_u/\mathcal{M}_{\mathrm{cj}} - 1$. Computations are provided for $\gamma = 1.2$, $\tilde{Q} = 1$ (a) and $\tilde{Q} = 10$ (b) evaluated for $|W| \gg |H|$ (black line), $|H| \gg |W|$ (green line), $W = H$ (orange line) and $W = -H$ (blue line).

Key takeaways

- The **thin-detonation limit** allows the description of the transient evolution in **analytical** form.
- Turbulence generation provides **second-order corrections** to the averaged Rankine–Hugoniot jump conditions.
- In almost most of the cases of interest, these (purely) hydro-averaged perturbations translate into **faster propagation speeds**.

ICTAM Milano 2020+1

uc3m | Universidad Carlos III de Madrid