

Notebook - Maratona de Programação

Tiago de Souza Fernandes

Contents			7		8
1	Algoritmos	2			8
-	1.1 Recursive-BS	2		1 0	8
2	Grafos	2	8	Strings	8
	2.1 BFS	2			
	2.2 Dijkstra	2			
	2.3 Floyd-Warshall	2			
	2.4 Kruskal	2			
	2.5 DFS	3			
	2.6 Represent	3			
3	Geometria	3			
	3.1 Inter-Retas	3			
4	$\mathbf{E}\mathbf{D}$	3			
	4.1 Iterative-SegTree	3			
	4.2 Recursive-SegTree	4			
	4.3 Delta-Encoding	4			
	4.4 BIT-2D	5			
	4.5 BIT	5			
	4.6 Union-Find	5			
5	STL	5			
	5.1 Pair	5			
	5.2 Set	6			
	5.3 Stack	6			
	5.4 Queue	6			
	5.5 Priority-Queue	6			
	5.6 Map	6			
	5.7 Vector	7			
6	Math	7			
	6.1 Verif-primo	7			
	6.2 Crivo	7			
	6.3 Kamenetsky	7			

1 Algoritmos

1.1 Recursive-BS

```
1 // Recursive binary search
3 int bs(int x, int ini, int fim)
       if(fim>=ini)
           int meio = (ini+fim)/2;
           if (vetor[mid] == x)
1.0
               return x:
           if (vetor[meio] < x)</pre>
               return bs(x, ini, meio-1);
13
               return bs(x, meio+1, fim);
15
16
17
18
       return -1;
19 }
```

2 Grafos

2.1 BFS

```
1 //BFS (Breadth First Search) O(V+A)
3 void BFS(int x)
       int atual, v, u;
5
       queue < int > fila;
       fila.push(x);
       componente[x] = valor;
       atual = 0;
10
       while (!fila.empty())
           v = fila.front();
13
14
           fila.pop();
15
           for(int i = 0;i < (int)vizinhos[v].size();i</pre>
16
               u = vizinhos[v][i];
18
                if (componente[u] == -1)
20
                    componente[u] = componente[v];
21
                    fila.push(u);
               }
23
24
           }
       }
25
26 }
```

2.2 Dijkstra

```
1 // Dijkstra - Shortest Path
2
3 #define pii pair<int, int>
4 #define vi vector<int>
5 #define vii vector< pair<int, int> >
6 #define INF 0x3f3f3f3f
7
8 vector<vii>y grafo;
9 vi distancia;
10 priority_queue< pii, vii, greater<pii>> fila;
11
12 void dijkstra(int k)
```

```
13
14
       int dist, vert, aux;
       distancia[k]=0;
15
16
       fila.push(mp(k, 0));
18
       while(!fila.empty())
19
2.0
            aux=fila.top().f;
21
           fila.pop();
23
            for(int i=0; i<grafo[aux].size(); i++)</pre>
25
                vert=grafo[aux][i].f;
26
                dist=grafo[aux][i].s;
27
                if (distancia[vert]>distancia[aux]+dist)
28
                     distancia[vert] = distancia[aux] + dist;
3.0
                     fila.push(mp(vert, distancia[vert]));
                }
32
           }
33
3.4
35 }
36
37 int main()
38 {
       dist.assign(N+1, INF);
39
40
       grafo.assign(N+1, vii());
4.1
       for(int i=0; i<M; i++)</pre>
42
43
            cin >> a >> b >> p;
44
45
            grafo[a].pb(mp(b, p));
            grafo[b].pb(mp(a, p));
46
47
48 }
```

2.3 Floyd-Warshall

2.4 Kruskal

```
1 // Kruskal - Minimum Spanning Tree
3 typedef struct
4 {
5
      int A, B;
      int dist;
7 } vertice;
9 vertice grafo[MAX];
10 int pai[MAX];
12 int find(int X) // Union-Find
      if (pai[X] == X)
14
          return X;
16
           return pai[X]=find(pai[X]);
17
```

```
18 }
                                                               1.7
19
                                                               18
20 void join(int X, int Y)
                                                               19
21 {
                                                               20
       int paix = find(X);
       int paiy = find(Y);
23
       pai[paix]=paiy;
24
                                                                 3
25
26
                                                                 3.1
27 bool comp(vertice A, vertice B)
28 €
29
       return A.dist < B.dist;</pre>
30 }
31
                                                               3 typedef struct
32 void kruskal()
                                                               4 {
33 {
                                                                      int x, y;
34
       for (int i=1;i<=N;i++)</pre>
                                                               6 } pnt;
           pai[i]=i;
3.5
       for(int i=1;i<=M;i++)</pre>
37
                                                               9 {
           cin >> grafo[i].A >> grafo[i].B >> grafo[i].
38
       dist:
39
       sort(grafo+1, grafo+M+1, comp);
41
                                                              13
       for(int i=1;i<M;i++)</pre>
42
                                                              14 }
43
                                                               15
            if(find(grafo[i].A)!=find(grafo[i].B))
44
45
                                                              17 {
                join(grafo[i].A, grafo[i].B);
46
                                                              18
                soma += grafo[i]. dist;
                                                               19
           }
48
                                                              20
                                                                      if(val ==0)
49
                                                              21
50
                                                              22
       cout << soma << endl:
51
                                                              23
52 }
                                                              24
                                                                      else
                                                              25
  2.5 DFS
                                                              26 }
                                                              27
1 //DFS (Depth First Search) O(V+A)
                                                              29 ┫
3 void DFS(int x)
                                                              3.0
4 {
                                                               31
       for(int i=0; i<(int)vizinhos[x].size(); i++)</pre>
                                                               32
6
                                                               33
            int v = vizinhos[x][i];
           if (componente[v] == -1)
                                                              3.5
                componente[v] = componente[x];
1.0
                                                              37
                DFS(v);
                                                               38
12
           }
                                                               39
       }
13
                                                               40
14 }
                                                               41
                                                               42
  2.6 Represent
                                                               43
                                                               44
                                                               45
1 // Grafos
                                                               47
3 // List of edges
                                                               48
                                                               49
       vector< pair<int, int> > arestas;
                                                              50
       arestas.push_back(make_pair(1, 2));
                                                              51
       arestas.push_back(make_pair(1, 3));
                                                              52 }
9 // Adjacency Matrix
                                                                       ED
                                                                 4
1.0
       int grafo[10][10];
11
                                                                 4.1
       grafo[1][2] = grafo[2][1] = 1;
1.3
```

grafo[1][3] = grafo[3][1] = 2;

16 // Adjacency List

```
vector < int > vizinhos[10];
vizinhos[1].push_back(2);
vizinhos[1].push_back(2);
```

Geometria

Inter-Retas

```
1 // Intersection between lines
8 bool collinear(pnt p, pnt q, pnt r)
      if(q.x<=max(p.x,r.x) && q.x>=min(p.x,r.x) && q.y
      <=max(p.y,r.y) && q.y>=min(p.y,r.y))
          return true;
      return false;
int orientation(pnt p, pnt q, pnt r)
       int val = (q.y-p.y)*(r.x-q.x)-(q.x-p.x)*(r.y-q.y);
          return 0;
       else if(val>0)
          return 1;
          return 2;
28 bool intersect(pnt p1, pnt q1, pnt p2, pnt q2)
      int o1 = orientation(p1, q1, p2);
      int o2 = orientation(p1, q1, q2);
      int o3 = orientation(p2, q2, p1);
      int o4 = orientation(p2, q2, q1);
      if (o1!=o2 and o3!=o4)
          return true;
      if(o1 == 0 && collinear(p1, p2, q1))
           return true;
       if(o2 == 0 && collinear(p1, q2, q1))
          return true;
      if(o3 == 0 && collinear(p2, p1, q2))
          return true;
       if(o4==0 && collinear(p2, q1, q2))
          return true:
      return false;
```

Iterative-SegTree

```
1 // Segment Tree Iterativa - Range maximum query
```

```
3 #define N 100010
                                                                   val[no]=max(val[esq], val[dir]);
                                                            2.1
                                                            22 }
5 struct Segtree
                                                            2.3
                                                            24 void atualiza(int no, int i, int j, int pos, int
                                                                   novo_valor)
       int t[2*N]={0};
                                                            25
       void build()
                                                                   if(i==j)
9
                                                            26
1.0
                                                            2.7
           for(int i=N-1; i>0; i--)
                                                                        val[no] = no vo_valor;
11
                                                            28
               t[i]=max(t[i<<1], t[1<<1|1]);
                                                            29
                                                                   }else
                                                                   {
13
                                                            30
14
                                                            31
                                                                        int esq = 2*no;
      int query(int 1, int r)
                                                                        int dir = 2*no+1;
15
                                                            32
                                                                       int meio = (i+j)/2;
                                                            33
16
17
           int ans=0;
                                                            34
           for (i+=N, r+=N; 1< r; 1>>=1, r>>=1)
                                                                        if (pos<=meio)</pre>
18
                                                            35
19
                                                            36
                                                                            atualiza(esq, i, meio, pos, novo_valor);
               if(1&1)
20
                                                            3.7
                   ans=max(ans, t[1++]);
                                                                            atualiza(dir, meio+1, j, pos, novo_valor)
               if (r&1)
22
                   ans=max(ans, t[--r]);
                                                            39
           }
                                                                        if(val[esq]>val[dir])
                                                            40
24
                                                                           val[no]=val[esq];
2.5
                                                            41
           return ans;
                                                            42
      }
                                                                            val[no]=val[dir];
27
                                                            43
                                                            44
28
       void update(int p, int value)
                                                            45 }
29
30
                                                            46
           for(t[p+=n]=value; p>1; p>>=1)
                                                            47 int consulta(int no, int i, int j, int A, int B)
31
               t[p>>1] = max(t[p], t[p^1]);
                                                            48 €
32
                                                                   if(i>B || j<A)</pre>
33
                                                            49
34
                                                            5.0
                                                                       return -1;
35 };
                                                            51
                                                                    if(i>=A and j<=B)
                                                            52
                                                                       return val[no];
37 int main()
                                                            53
                                                                   int esq = 2*no;
38 {
                                                            54
                                                                   int dir = 2*no+1;
       Segtree st;
3.9
                                                            5.5
                                                                   int meio = (i+j)/2;
40
                                                            56
       for (int i=0; i < n; i++)</pre>
                                                            57
41
                                                                   int resp_esq = consulta(esq, i, meio, A, B);
                                                            58
42
                                                                   int resp_dir = consulta(dir, meio+1, j, A, B);
43
           cin >> aux;
                                                            59
          st.t[N+i]=aux; //Leaves are stored in
44
                                                            60
       continuous nodes with indices starting with {\tt N}
                                                                    if(resp_dir==-1)
                                                            61
45
                                                            62
                                                                       return resp_esq;
                                                                    if(resp_esq==-1)
                                                            63
46
      st.build();
                                                            64
                                                                       return resp_dir;
47
      x = st.query(inicio, fim);
48
                                                            6.5
       st.update(ind, value);
                                                            66
                                                                   if(resp_esq>resp_dir)
50
                                                            6.7
                                                                       return resp_esq;
51 }
                                                            68
                                                            69
                                                                       return resp_dir;
  4.2 Recursive-SegTree
                                                            70 }
                                                            72 int main()
1 // Segment Tree Recursiva - Range maximum query
                                                            73 -{
                                                                   monta(1, N, 1);
3 vector < int > val(MAX, 0);
                                                            74
                                                                   atualiza(1, 1, N, pos, valor);
4 vector < int > vet(N);
                                                            7.5
                                                                   x = consulta(1, 1, N, inicio, fim);
                                                            76
6 void monta(int i, int j, int no)
                                                            78 }
       if(i==j)
                                                                    Delta-Encoding
                                                               4.3
9
           val[no]=vet[i];
10
           return;
                                                             1 // Delta encoding
      }
12
                                                             3 for(int i=0;i<q;i++)</pre>
13
14
      int esq = 2*no;
                                                             4 {
      int dir = 2*no+1;
15
                                                                   int 1,r,x;
      int meio = (i+j)/2;
                                                                   cin >> 1 >> r >> x;
                                                                   delta[1] += x;
                                                             7
1.7
       monta(i, meio, esq);
                                                                   delta[r+1] -= x;
      monta(meio+1, j, dir);
                                                             9 }
19
```

10

20

```
11 int atual = 0;
                                                                       bit[x] += v;
                                                            3.3
                                                            34
                                                                       x += (x & -x);
13 for(int i=0;i<n;i++)
                                                                   }
                                                            35
14 {
                                                           36 }
       atual += delta[i];
                                                             4.6 Union-Find
      v[i] += atual;
16
17 }
                                                            1 // Union-Find Functions
 4.4 BIT-2D
                                                            3 int pai[MAX], peso[MAX];
1 // BIT 2D
                                                            5 int find(int aux)
3 int bit[MAX][MAX];
                                                            6 {
                                                                   if(pai[aux] == aux)
5 int sum(int x, int y)
                                                                      return aux;
                                                                   else
6 {
                                                            9
       int resp=0;
                                                                       return pai[aux]=find(pai[aux], pai);
                                                            10
                                                            11 }
      for (int i = x; i > 0; i - = i & - i)
                                                            12
                                                            13 void join(int x, int y)
10
           for(int j=y;j>0;j-=j&-j)
               resp+=bit[i][j];
                                                           14 {
                                                                   x = find(x);
12
                                                            15
                                                                   y = find(y);
13
      return resp;
                                                            16
14 }
                                                            17
                                                                   if(pesos[x]<pesos[y])</pre>
15
                                                            18
16 void update(int x, int y, int delta)
                                                            19
                                                                      pai[x] = y;
17 {
                                                            20
                                                                   else if(pesos[x]>pesos[y])
                                                                      pai[y] = x;
       for (int i = x; i < MAX; i += i & - i)</pre>
18
                                                           21
          for(int j=y;j<MAX;j+=j&-j)</pre>
                                                                   else if(pesos[x]==pesos[y])
                                                            22
               bit[i][j]+=delta;
                                                                   {
20
                                                            23
                                                                       pai[x] = y;
21 }
                                                            24
                                                            25
                                                                       pesos[y]++;
23 int query(int x1, y1, x2, y2)
                                                            26
       return sum(x2,y2) - sum(x2,y1) - sum(x1,y2) + sum 28
25
      (x1,y1);
                                                            29 int main()
26 }
                                                            30 {
                                                                   for(int i=1;i<=N;i++)</pre>
                                                           31
  4.5 BIT
                                                            32
                                                                       pai[i]=i;
                                                           33 }
1 // (BIT) Fenwick Tree
                                                                   STL
                                                              5
3 int bit[MAX];
                                                              5.1 Pair
5 int soma(int x)
6 {
                                                            pair < string, int > P;
      int resp=0;
                                                            3 cin>>P.first>>P.second;
      // for(int i=x;i>0;i-=i&-i)
9
      // resp+=bit[i];
10
                                                            5 // Pair of pair
      while (x > 0)
12
                                                                  pair < string , pair < double , double >> P;
13
           resp += bit[x];
1.4
                                                                   P.first = "Joao";
15
           x -= (x & -x);
                                                            9
                                                                  P.second.first = 8.2;
16
                                                            10
                                                                   P.second.second = 10;
17
                                                            11
      return resp;
                                                            12
                                                            13 // Vector of pair
19 }
20
                                                            14
                                                                   vector<pair<int, string> > V;
21 int query(int L, R)
                                                            15
                                                                   sort(V.begin(), V.end());
22 {
                                                            16
       return soma(R)-soma(L);
                                                            17
24 }
                                                            18 //make.pair()
                                                            19
26 void update(int x, int v)
                                                                   P = make_pair("Joao", 10);
                                                            20
27
                                                            21
       // for(;x<=n;x+=x&-x)
                                                                  for(int i=1;i<10;i++)
                                                            22
      // bit[x] += v;
                                                                   {
29
                                                            23
                                                                       cin >> a >> b;
                                                            24
      while(x <= N)</pre>
                                                                       V.push_back(make_pair(a,b));
3.1
                                                            2.5
32
                                                            26
```

```
5.2 Set.
                                                                   if(fila.empty() == true/false)
                                                            12
1 // Set - Red-Black Trees - O(logn)
                                                            13 //fila.pop()
s set < int > S;
                                                                   fila.pop();
                                                            15
5 //S.insert()
                                                            17 //fila.front()
      S.insert(10); // O(logN)
                                                                   p = fila.front();
                                                            19
9 //S.find()
                                                               5.5 Priority-Queue
       if(S.find(3) != S.end())// O(logN)
12
13 //S.erase
                                                             1 // Priority Queue - O(logn)
14
      S.erase(10);
                                                             3 priority_queue < int > plista;
16
       //Outros
17
                                                             5 //plista.push()
      S.clear();
18
      S.size();
19
                                                                   plista.push(N);
      S.begin();
      S.end();
21
                                                             9 //plista.empty()
      p = S.lower_bound(n); // Retorna um ponteiro para 11
23
                                                                   if(plista.empty() == true/false)
       . o primeiro elemento maior ou igual a n (not less_{
m 12}
                                                            13 //plista.pop()
       {\tt p = S.upper\_bound(n); // Retorna \ um \ ponteiro \ para}_{14}
24
        o primeiro elemento maior que n (greater than n) _{15}\,
                                                                   plista.pop();
                                                            17 //plista.front()
27 // (set <int>::iterator)
                                                                   p = plista.top();
       for(set<int>::iterator it=S.begin(); it!=S.end();
       it.++)
                                                               5.6 Map
           cout << *it << " ";
31
                                                             1 // Map - Red-Black Trees
  5.3 Stack
                                                             3 map < string , int > M;
                                                             5 //S.insert()
1 // Stack
                                                                   M.insert(make_pair("Tiago", 18));
s stack<int> pilha;
                                                                   M["Tiago"]=18; // O(logN)
5 //pilha.push()
                                                            11 //S.find()
      pilha.push(N);
                                                                   if(M.find("Tiago") != M.end()) // O(logN)
9 //pilha.empty()
                                                            14
                                                                   cout << M["Tiago"] << endl;</pre>
      if(pilha.empty() == true/false)
                                                            16
                                                            17 //S.erase
13 //pilha.pop()
                                                            1.8
                                                                   M.erase("Tiago"); // O(logN)
                                                            19
15
      pilha.pop();
                                                            21
17 //pilha.front()
                                                            22 //S.count()
                                                            23
      p = pilha.top();
                                                                   if (S.count(N))
                                                            24
  5.4 Queue
                                                            _{26} //Outher
1 // Queue
                                                                   M.clear();
                                                            28
                                                                   M.size();
                                                                   M.begin();
3 queue < int > fila;
                                                            3.0
                                                                   M.end();
                                                            31
5 //fila.push()
                                                            33 // (map<int>::iterator)
      fila.push(N);
                                                                   for(map < string, int >::iterator it = M.begin(); it! = M
                                                            3.5
9 //fila.empty()
                                                                   .end(); it++)
```

```
cout << "(" << it->first << ", " << it->
                                                             65
                                                                    int func(int x, int y)
       second << ") ";
                                                             66
                                                             67
                                                                         //return x*y;
                                                             68
                                                                         return x+y;
  5.7 Vector
                                                             69
                                                             71 //assign()
1 // Vector - Vetor
                                                                    //Diferente do resize() por mudar o valor de
                                                             72
                                                                    todos os elementos do vector
3 vector < int > V;
4 vector < tipo > nome;
                                                             73
5 vector < tipo > V(n, value);
                                                             74
                                                                    vector < int > vet;
                                                             7.5
                                                                    vet.assign(N, x);
                                                             76
7 //push_back()
                                                                    vector< vector<int> > vet;
                                                             7.7
                                                             78
                                                                    vet.assign(N, vector<int>());
       V.push_back(2);
      V.push_front(2);
                                                             79
10
                                                             80 //sort()
12 // front() back()
                                                                    sort(vet, vet+N, func);
                                                             82
       cout << V.front() << endl;</pre>
                                                             83
                                                                    bool func(Aluno a, Aluno b)
       cout << V.back() << endl;</pre>
                                                             84
15
                                                             85
                                                                         return a.nota < b.nota; // True caso a venha
17 //size()
                                                                    antes de b, False caso contrario
      tamanho = V.size();
19
20
21 //resize()
                                                                     Math
                                                                6
       V.resize(10);
                                                                     Verif-primo
                                                                6.1
      V.resize(n, k);
24
25
26 //pop_back()
                                                              1 // prime verrification sqrt(N)
      V . pop_back();
                                                              3 long long eh_primo(long long N)
                                                              4 {
30 //clear()
                                                                    if(N==2)
3.1
                                                                    {
                                                              6
       V.clear();
                                                                         return true;
       sort(V.begin(), V.end());
34
                                                                    else if (N == 1 or N %2 == 0)
                                                              9
35 //upper_bound() e lower_bound()
                                                             10
                                                                    {
36
                                                                         return false;
                                                             11
       vector < int > :: iterator low, up;
3.7
3.8
       low=lower_bound(v.begin(), v.end(), 20);
                                                                    for(long long i=3;i*i<=N;i+=2)</pre>
                                                             1.3
      up=upper_bound(v.begin(), v.end(), 20);
cout << "lower_bound at position " << (low- v.
39
                                                                        if(N\%i==0)
                                                             14
40
                                                                             return false;
                                                             15
       begin()) << '\n';
                                                             16
41
       cout << "upper_bound at position " << (up - v.
                                                             17
                                                                    return true;
       begin()) << '\n';
                                                             18 }
42
43 //binary_search()
                                                                6.2 Crivo
44
       if(binary_search(vet.begin(), vet.end(), 15))
                                                              1 // Sieve of Eratosthenes
46
47 //accumulate()
                                                              3 int N:
                                                              4 vector < bool > primos (100010, true);
       cout << accumulate(first, last, sum, func) <<</pre>
49
                                                              5 cin >> N;
50
       //first - pointer to the first element
                                                              7 primos[0]=false;
       //last - last element
51
                                                              8 primos[1]=false;
       //sum - inicial value
52
       //func
53
                                                             10 for(int i=2;i<=N;i++)
54
                                                             11
                                                                    if(primos[i])
       int func(int x, int y)
5.5
                                                                         for(int j=i+i; j \le N; j+=i)
                                                             12
                                                                             primos[j]=false;
                                                             13
5.7
           //return x*v;
58
           return x+y;
                                                                     Kamenetsky
                                                                6.3
59
6.0
61 //partial_sum()
                                                              1 // Number of digits in n! O(1)
62
                                                              3 #define Pi 3.14159265358979311599796346854
       partial_sum(first, last, vet, func);
```

36

```
4 #define Eul 2.71828182845904509079559829842
6 long long findDigits(int n)
      double x;
9
      if (n < 0)
10
          return 0;
      if (n == 1)
12
          return 1;
13
14
      x = ((n * log10(n / euler) + log10(2 * Pi * n))
15
      /2.0));
      return floor(x) + 1;
17
18 }
```

7 Misc

7.1 Bitwise

```
1 // Bitwise
      unsigned char a = 5, b = 9; // a = (00000101), b
3
      = (00001001)
                           // The result is 00000001
      AND -
                      a&b
      (1)
      0R -
                      a | b
                             // The result is 00001101
      (13)
                            // The result is 00001100
      XOR. -
                      a^b
       (12)
                             // The result is 11111010
      NOT -
                      ~a
      (250)
      Left shift - b <<1 // The result is 00010010
      Right shift - b>>1 // The result is 00000100
10
      (4)
      // Exchange two int variables
12
13
           a ^ = b :
14
          b^=a;
15
           a^=b;
1.7
      // Even or Odd
19
           (x & 1)? printf("Odd"): printf("Even");
20
21
      // Turn on the j-th bit
22
           int S = 34; //(100010)
24
           int j = 3;
25
26
           S = S | (1 << j);
27
      // Turn off the j-th bit
29
           int S = 42; //(101010)
3.1
          int j = 1;
3.2
           S &= ~(1<<j)
34
           S == 40 //(101000)
36
```

```
// Check the j-th element
    int S = 42; //(101010)
    int j = 3;
    T = S & (1 << j); // T = 0
// Exchange o j-th element
    S ^= (1<<j)
// Position of the first bit on
    T = (S & (-S))
    T -> 4 bit ligado //(1000)
// Most significant digit of N
    double K = log10(N);
    K = K - floor(K);
    int X = pow(10, K);
// Number of digits in N
    X =floor(log10(N)) + 1;
// Power of two
    bool isPowerOfTwo(int x)
    {
        return x && (!(x&(x-1)));
    }
```

7.2 Complexity

37 38

3.9

40

41

42

44

45

47

49 50

51

52 53

54 55 56

57

58

59

6.1

63 64

65

66

67

68

69

```
1 // Complexity
2
3     If n <= 12, the time complexity can be O(n!).
4     If n <= 25, the time complexity can be O(2^n).
5     If n <= 100, the time complexity can be O(n^4).
6     If n <= 500, the time complexity can be O(n^3).
7     If n <= 10^4, the time complexity can be O(n^2).
8     If n <= 10^6, the time complexity can be O(n log n).
9     If n <= 10^8, the time complexity can be O(n).
10     If n > 10^8, the time complexity can be O(n).
11     If n > 10^8, the time complexity can be O(log n)
12     or O(1).
```

7.3 Aprox

```
1 // Approximation
3 value - round() floor() ceil() trunc()
                  ----
5 +2.3
         +2.0
                 +2.0
                          +3.0
                                  +2.0
6 +3.8
         +4.0
                 +3.0
                          +4.0
                                  +3.0
7 +5.5
         +6.0
                 +5.0
                         +6.0
                                  +5.0
8 -2.3
         -2.0
                 -3.0
                          -2.0
                                  -2.0
9 -3.8
         -4.0
                 -4.0
                          -3.0
                                  -3.0
10 -5.5
         -6.0
                 -6.0
                          -5.0
                                  -5.0
```

8 Strings