



# Notebook - Maratona de Programação

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# 1 Algoritmos

## 1.1 Mochila

```
1 int val[MAXN], peso[MAXN], dp[MAXN][MAXS]
2
3 int knapsack(int N, int M) // Objetos | Peso max
4 {
5     for(i=0; i<=N; i++)
6     {
7         for(j=0; j<=M; j++)
8         {
9             if (i==0 || j==0)
10                dp[i][j] = 0;
11             else if (peso[i-1] <= j)
12                dp[i][j] = max(val[i-1]+dp[i-1][j-
13                    peso[i-1]], dp[i-1][j]);
14             else
15                dp[i][j] = dp[i-1][j];
16         }
17     }
18     return dp[N][M];
19 }
```

## 1.2 Iterative-BS

```
1 int main()
2 {
3     int l=1, r=N;
4     int res=-1;
5
6     while(l <= r)
7     {
8         int m = (l + r)/2;
9         if(!ver(m))
10         {
11             l = m+1;
12         }
13         else
14         {
15             res = m;
16             r = m-1;
17         }
18     }
19     cout << res << endl;
20
21     return 0;
22 }
```

# 2 Grafos

## 2.1 BFS

```
1 //BFS (Breadth First Search) O(V+A)
2
3 vector<vector<int>>> adj; // adjacency list
4 // representation
5 int n; // number of nodes
6 int s; // source vertex
7
8 queue<int> q;
9 vector<int> d(n, INF), p(n);
10
11 q.push(s);
12 used[s] = true;
13 p[s] = -1;
14 while (!q.empty()) {
15     int v = q.front();
16     q.pop();
17     for (int u : adj[v]) {
```

```
17         if (d[u] > d[v] + 1) {
18             q.push(u);
19             d[u] = d[v] + 1;
20             //p[u] = v;
21         }
22     }
23 }
24 // Restore Path
25 if (!used[u]) {
26     cout << "No path!";
27 } else {
28     vector<int> path;
29     for (int v = u; v != -1; v = p[v])
30         path.push_back(v);
31     reverse(path.begin(), path.end());
32     cout << "Path: ";
33     for (int v : path)
34         cout << v << " ";
35 }
```

## 2.2 Find-bridges

```
1 #define vi vector<int>
2
3 vector< vector<int>> > grafo;
4 vector<bool> visited;
5 vi t, low;
6 int timer=0;
7
8 void find_bridges(int v, int p=-1)
9 {
10     visited[v] = true;
11     t[v] = low[v] = timer++;
12     for(int i=0; i<(int)grafo[v].size(); i++)
13     {
14         int vert = grafo[v][i];
15         if(vert == p)
16             continue;
17         if(visited[vert])
18             low[v] = min(low[v], t[vert]);
19         else
20         {
21             find_bridges(vert, v);
22             low[v] = min(low[v], low[vert]);
23             if(low[tv] > t[v])
24                 IS_BRIDGE(v, vert);
25         }
26     }
27 }
28
29 int main()
30 {
31     timer = 0;
32     visited.assign(N+1, false);
33     t.assign(N+1, 0);
34     low.assign(N+1, 0);
35
36     for(int i=0; i<N; i++)
37         if(!visited[i])
38             find_bridges(i);
39
40     return 0;
41 }
```

## 2.3 Dijkstra

```
1 // Dijkstra - Shortest Path
2
3 #define pii pair<int, int>
4 #define vi vector<int>
5 #define vii vector< pair<int,int>>
6 #define INF 0x3f3f3f3f
```

```

7
8 vector<vii> grafo;
9 vi distancia;
10 priority_queue< pii, vii, greater<pii> > fila;
11
12 void dijkstra(int k)
13 {
14     int dist, vert, aux;
15     distancia[k]=0;
16
17     fila.push(mp(k, 0));
18
19     while(!fila.empty())
20     {
21         aux=fila.top().f;
22         fila.pop();
23
24         for(int i=0; i<grafo[aux].size(); i++)
25         {
26             vert=grafo[aux][i].f;
27             dist=grafo[aux][i].s;
28             if(distancia[vert]>distancia[aux]+dist)
29             {
30                 distancia[vert]=distancia[aux]+dist;
31                 fila.push(mp(vert, distancia[vert]));
32             }
33         }
34     }
35 }
36
37 int main()
38 {
39     dist.assign(N+1, INF);
40     grafo.assign(N+1, vii());
41
42     for(int i=0; i<M; i++)
43     {
44         cin >> a >> b >> p;
45         grafo[a].pb(mp(b, p));
46         grafo[b].pb(mp(a, p));
47     }
48 }

```

## 2.4 Floyd-Warshall

```

1 // Floyd Warshall
2
3 int dist[MAX][MAX];
4
5 void Floydwarshall()
6 {
7     for(int k = 1; k <= n; k++)
8         for(int i = 1; i <= n; i++)
9             for(int j = 1; j <= n; j++)
10                 dist[i][j] = min(dist[i][j], dist[i][k] + dist[k][j]);
11 }

```

## 2.5 Kruskal

```

1 // Kruskal - Minimum Spanning Tree
2
3 typedef struct
4 {
5     int A, B;
6     int dist;
7 } vertice;
8
9 vertice grafo[MAX];
10 int pai[MAX];
11

```

```

12 int find(int X) // Union-Find
13 {
14     if(pai[X]==X)
15         return X;
16     else
17         return pai[X]=find(pai[X]);
18 }
19
20 void join(int X, int Y)
21 {
22     int paix = find(X);
23     int paiy = find(Y);
24     pai[paix]=paiy;
25 }
26
27 bool comp(vertice A, vertice B)
28 {
29     return A.dist < B.dist;
30 }
31
32 void kruskal()
33 {
34     for(int i=1; i<=N; i++)
35         pai[i]=i;
36
37     for(int i=1; i<=M; i++)
38         cin >> grafo[i].A >> grafo[i].B >> grafo[i].dist;
39
40     sort(grafo+1, grafo+M+1, comp);
41
42     for(int i=1; i<M; i++)
43     {
44         if(find(grafo[i].A)!=find(grafo[i].B))
45         {
46             join(grafo[i].A, grafo[i].B);
47             soma+=grafo[i].dist;
48         }
49     }
50
51     cout << soma << endl;
52 }

```

## 2.6 DFS

```

1 //DFS (Depth First Search) O(V+A)
2
3 void DFS(int x)
4 {
5     for(int i=0; i<(int)vizinhos[x].size(); i++)
6     {
7         int v = vizinhos[x][i];
8         if(componente[v] == -1)
9         {
10             componente[v] = componente[x];
11             DFS(v);
12         }
13     }
14 }

```

## 2.7 Represent

```

1 // Grafos
2
3 // List of edges
4
5 vector< pair<int, int> > arestas;
6 arestas.push_back(make_pair(1, 2));
7 arestas.push_back(make_pair(1, 3));
8
9 // Adjacency Matrix
10

```

```

11     int grafo[10][10];
12
13     grafo[1][2] = grafo[2][1] = 1;
14     grafo[1][3] = grafo[3][1] = 2;
15
16     // Adjacency List
17
18     vector<int> vizinhos[10];
19
20     vizinhos[1].push_back(2);
21     vizinhos[1].push_back(3);

```

## 2.8 Prim

```

1  // Prim Algorithm
2  #define MAXN 10100
3  #define INFINITO 999999999
4
5  int n, m;
6  int distancia[MAXN];
7  int processado[MAXN];
8  vector<pii> vizinhos[MAXN];
9
10 int Prim()
11 {
12     for(int i = 2; i <= n; i++) distancia[i] = INFINITO;
13     distancia[1] = 0;
14
15     priority_queue< pii, vector<pii>, greater<pii> >
16     fila;
17     fila.push( pii(distancia[1], 1) );
18
19     while(1)
20     {
21         int davez = -1;
22
23         while(!fila.empty())
24         {
25             int atual = fila.top().second;
26             fila.pop();
27
28             if(!processado[atual])
29             {
30                 davez = atual;
31                 break;
32             }
33
34             if(davez == -1)
35                 break;
36
37             processado[davez] = true;
38
39             for(int i = 0; i < (int)vizinhos[davez].size()
40             ;i++)
41             {
42                 int dist = vizinhos[davez][i].first;
43                 int atual = vizinhos[davez][i].second;
44
45                 if( distancia[atual] > dist && !
46                 processado[atual])
47                 {
48                     distancia[atual] = dist;
49                     fila.push( pii(distancia[atual],
50                     atual) );
51                 }
52             }
53
54             int custo_arvore = 0;
55             for(int i = 1; i <= n; i++)

```

```

55         custo_arvore += distancia[i];
56
57         return custo_arvore;
58     }
59
60     int main(){
61
62         cin >> n >> m;
63
64         for(int i = 1; i <= m; i++){
65
66             int x, y, tempo;
67             cin >> x >> y >> tempo;
68
69             vizinhos[x].pb( pii(tempo, y) );
70             vizinhos[y].pb( pii(tempo, x) );
71         }
72
73         cout << Prim() << endl;
74
75         return 0;
76     }

```

## 3 Geometria

### 3.1 Inter-Retas

```

1  // Intersection between lines
2
3  typedef struct
4  {
5      int x, y;
6  } pnt;
7
8  bool collinear(pnt p, pnt q, pnt r)
9  {
10     if(q.x<=max(p.x,r.x) && q.x>=min(p.x,r.x) && q.y
11     <=max(p.y,r.y) && q.y>=min(p.y,r.y))
12         return true;
13
14     return false;
15 }
16
17 int orientation(pnt p, pnt q, pnt r)
18 {
19     int val=(q.y-p.y)*(r.x-q.x)-(q.x-p.x)*(r.y-q.y);
20
21     if(val==0)
22         return 0;
23     else if(val>0)
24         return 1;
25     else
26         return 2;
27
28 bool intersect(pnt p1, pnt q1, pnt p2, pnt q2)
29 {
30     int o1 = orientation(p1, q1, p2);
31     int o2 = orientation(p1, q1, q2);
32     int o3 = orientation(p2, q2, p1);
33     int o4 = orientation(p2, q2, q1);
34
35     if(o1!=o2 and o3!=o4)
36         return true;
37
38     if(o1==0 && collinear(p1, p2, q1))
39         return true;
40
41     if(o2==0 && collinear(p1, q2, q1))
42         return true;
43
44     if(o3==0 && collinear(p2, p1, q2))

```

```

45         return true;
46
47     if(o4==0 && collinear(p2, q1, q2))
48         return true;
49
50     return false;
51 }
52 }

```

## 4 ED

### 4.1 Range-query-bigger-than-k-BIT

```

1  // C++ program to print the number of elements
2  // greater than k in a subarray of range L-R.
3  #include <bits/stdc++.h>
4  using namespace std;
5
6  // Structure which will store both
7  // array elements and queries.
8  struct node {
9      int pos;
10     int l;
11     int r;
12     int val;
13 };
14
15 // Boolean comparator that will be used
16 // for sorting the structural array.
17 bool comp(node a, node b)
18 {
19     // If 2 values are equal the query will
20     // occur first then array element
21     if (a.val == b.val)
22         return a.l > b.l;
23
24     // Otherwise sorted in descending order.
25     return a.val > b.val;
26 }
27
28 // Updates the node of BIT array by adding
29 // 1 to it and its ancestors.
30 void update(int* BIT, int n, int idx)
31 {
32     while (idx <= n) {
33         BIT[idx]++;
34         idx += idx & (-idx);
35     }
36 }
37
38 // Returns the count of numbers of elements
39 // present from starting till idx.
40 int query(int* BIT, int idx)
41 {
42     int ans = 0;
43     while (idx) {
44         ans += BIT[idx];
45         idx -= idx & (-idx);
46     }
47     return ans;
48 }
49
50 // Function to solve the queries offline
51 void solveQuery(int arr[], int n, int QueryL[],
52                 int QueryR[], int QueryK[], int q)
53 {
54     // create node to store the elements
55     // and the queries
56     node a[n + q + 1];
57     // 1-based indexing.
58
59     // traverse for all array numbers

```

```

60     for (int i = 1; i <= n; ++i) {
61         a[i].val = arr[i - 1];
62         a[i].pos = 0;
63         a[i].l = 0;
64         a[i].r = i;
65     }
66
67     // iterate for all queries
68     for (int i = n + 1; i <= n + q; ++i) {
69         a[i].pos = i - n;
70         a[i].val = QueryK[i - n - 1];
71         a[i].l = QueryL[i - n - 1];
72         a[i].r = QueryR[i - n - 1];
73     }
74
75     // In-built sort function used to
76     // sort node array using comp function.
77     sort(a + 1, a + n + q + 1, comp);
78
79     // Binary Indexed tree with
80     // initially 0 at all places.
81     int BIT[n + 1];
82
83     // initially 0
84     memset(BIT, 0, sizeof(BIT));
85
86     // For storing answers for each query( 1-based
87     // indexing).
88     int ans[q + 1];
89
90     // traverse for numbers and query
91     for (int i = 1; i <= n + q; ++i) {
92         if (a[i].pos != 0) {
93             // call function to returns answer for
94             // each query
95             int cnt = query(BIT, a[i].r) - query(BIT,
96                 a[i].l - 1);
97
98             // This will ensure that answer of each
99             // query
100             // are stored in order it was initially
101             // asked.
102             ans[a[i].pos] = cnt;
103         }
104         else {
105             // a[i].r contains the position of the
106             // element in the original array.
107             update(BIT, n, a[i].r);
108         }
109     }
110
111     // Output the answer array
112     for (int i = 1; i <= q; ++i) {
113         cout << ans[i] << endl;
114     }
115 }
116
117 // Driver Code
118 int main()
119 {
120     int arr[] = { 7, 3, 9, 13, 5, 4 };
121     int n = sizeof(arr) / sizeof(arr[0]);
122
123     // 1-based indexing
124     int QueryL[] = { 1, 2 };
125     int QueryR[] = { 4, 6 };
126
127     // k for each query
128     int QueryK[] = { 6, 8 };
129
130     // number of queries
131     int q = sizeof(QueryL) / sizeof(QueryL[0]);

```

```

128 // Function call to get
129 solveQuery(arr, n, QueryL, QueryR, QueryK, q);
130
131 return 0;
132 }

```

## 4.2 Iterative-SegTree

```

1 // Segment Tree Iterativa - Range maximum query
2
3 #define N 100010
4
5 struct Segtree
6 {
7     int t[2*N]={0};
8
9     void build()
10    {
11        for(int i=N-1; i>0; i--)
12            t[i]=max(t[i<<1], t[1<<1|1]);
13    }
14
15    int query(int l, int r)
16    {
17        int ans=0;
18        for(i+=N, r+=N; l<r; l>>=1, r>>=1)
19        {
20            if(l&1)
21                ans=max(ans, t[l++]);
22            if(r&1)
23                ans=max(ans, t[--r]);
24        }
25
26        return ans;
27    }
28
29    void update(int p, int value)
30    {
31        for(t[p+=N]=value; p>1; p>>=1)
32            t[p>>1]= max(t[p], t[p^1]);
33    }
34 };
35
36
37 int main()
38 {
39     Segtree st;
40
41     for(int i=0;i<n;i++)
42     {
43         cin >> aux;
44         st.t[N+i]=aux; //Leaves are stored in
45         continuous nodes with indices starting with N
46     }
47
48     st.build();
49     x = st.query(inicio, fim);
50     st.update(ind, value);
51 }

```

## 4.3 Recursive-SegTree

```

1 // Segment Tree Recursiva - Range maximum query
2
3 vector<int> val(MAX, 0);
4 vector<int> vet(N);
5
6 void monta(int i, int j, int no)
7 {
8     if(i==j)
9     {

```

```

10         val[no]=vet[i];
11         return;
12     }
13
14     int esq = 2*no;
15     int dir = 2*no+1;
16     int meio = (i+j)/2;
17
18     monta(i, meio, esq);
19     monta(meio+1, j, dir);
20
21     val[no]=max(val[esq], val[dir]);
22 }
23
24 void atualiza(int no, int i, int j, int pos, int
25     novo_valor)
26 {
27     if(i==j)
28     {
29         val[no]=novo_valor;
30     }else
31     {
32         int esq = 2*no;
33         int dir = 2*no+1;
34         int meio = (i+j)/2;
35
36         if(pos<=meio)
37             atualiza(esq, i, meio, pos, novo_valor);
38         else
39             atualiza(dir, meio+1, j, pos, novo_valor)
40
41         ;
42
43         if(val[esq]>val[dir])
44             val[no]=val[esq];
45         else
46             val[no]=val[dir];
47     }
48 }
49
50 int consulta(int no, int i, int j, int A, int B)
51 {
52     if(i>B || j<A)
53         return -1;
54     if(i>=A and j<=B)
55         return val[no];
56
57     int esq = 2*no;
58     int dir = 2*no+1;
59     int meio = (i+j)/2;
60
61     int resp_esq = consulta(esq, i, meio, A, B);
62     int resp_dir = consulta(dir, meio+1, j, A, B);
63
64     if(resp_dir==-1)
65         return resp_esq;
66     if(resp_esq==-1)
67         return resp_dir;
68
69     if(resp_esq>resp_dir)
70         return resp_esq;
71     else
72         return resp_dir;
73 }
74
75 int main()
76 {
77     monta(1, N, 1);
78     atualiza(1, 1, N, pos, valor);
79     x = consulta(1, 1, N, inicio, fim);
80 }

```

## 4.4 Delta-Encoding

```
1 // Delta encoding
2
3 for(int i=0;i<q;i++)
4 {
5     int l,r,x;
6     cin >> l >> r >> x;
7     delta[l] += x;
8     delta[r+1] -= x;
9 }
10
11 int atual = 0;
12
13 for(int i=0;i<n;i++)
14 {
15     atual += delta[i];
16     v[i] += atual;
17 }
```

## 4.5 Seg-Tree-Farao

```
1 typedef struct
2 {
3     pii prefix, sufix, total, maximo;
4 } no;
5
6 int noleft[MAX], noright[MAX]; //Guarda os valores
7                                //dos nos para que nao sejam calculados novamente
8                                //nas queries
9 int v[MAX];
10 no arvore[MAX];
11
12 pii somar(pii a, pii b) // une pairs
13 {
14     return mp(a.f+b.f, a.s+b.s);
15 }
16
17 no une(no l, no r)
18 {
19     if(l.total.s==0)
20         return r;
21     if(r.total.s==0)
22         return l;
23
24     no m;
25
26     m.prefix = max(l.prefix, somar(l.total, r.prefix));
27     m.sufix = max(r.sufix, somar(r.total, l.sufix));
28     m.total = somar(l.total, r.total); //Soma de
29     m.maximo = max(max(l.maximo, r.maximo), somar(l.sufix, r.prefix)); //Resultado para cada
30     m.sufix = somar(l.sufix, r.sufix);
31     return m;
32 }
33
34 no makenozero()
35 {
36     no m;
37     m.prefix=m.sufix=m.total=m.maximo=mp(0,0);
38     return m;
39 }
40
41 no makeno(int k)
42 {
43     no m;
44     m.prefix=m.sufix=m.total=m.maximo=mp(k,1);
45     return m;
46 }
```

```
44 }
45
46 void monta(int n)
47 {
48     if(noleft[n]==noright[n])
49     {
50         arvore[n]=makeno(v[noleft[n]]);
51         return;
52     }
53
54     int mid = (noleft[n]+noright[n])/2;
55     noleft[2*n]=noleft[n]; noright[2*n]=mid;
56     noleft[2*n+1]=mid+1; noright[2*n+1]=noright[n];
57
58     monta(2*n);
59     monta(2*n+1);
60
61     arvore[n]=une(arvore[2*n], arvore[2*n+1]);
62 }
63
64 no busca(int n, int esq, int dir)
65 {
66     if(noleft[n]>=esq and noright[n]<=dir)
67         return arvore[n];
68     if(noright[n]<esq or noleft[n]>dir)
69         return makenozero();
70
71     return une(busca(2*n, esq, dir), busca(2*n+1, esq, dir));
72 }
73
74 int main()
75 {
76     int T, N, Q, A, B;
77     no aux;
78
79     scanf("%d", &T);
80
81     while(T--)
82     {
83         scanf("%d", &N);
84         for(int i=1;i<=N;i++)
85             scanf("%d", &v[i]); //Elementos da arvore
86
87         noleft[1]=1;noright[1]=N;
88         monta(1);
89
90         cin >> Q;
91         while(Q--)
92         {
93             scanf("%d%d", &A, &B); //Intervalo da
94             query
95             aux = busca(1, A, B);
96             printf("%d %d\n", aux.maximo.f, aux.maximo.s);
97         }
98
99     }
100
101     return 0;
102 }
```

## 4.6 BIT-2D

```
1 // BIT 2D
2
3 int bit[MAX][MAX];
4
5 int sum(int x, int y)
6 {
7     int resp=0;
8
9     for(int i=x;i>0;i-=i&-i)
```

```

10         for(int j=y;j>0;j-=j&-j)
11             resp+=bit[i][j];
12     }
13     return resp;
14 }
15
16 void update(int x, int y, int delta)
17 {
18     for(int i=x;i<MAX;i+=i&-i)
19         for(int j=y;j<MAX;j+=j&-j)
20             bit[i][j]+=delta;
21 }
22
23 int query(int x1, y1, x2, y2)
24 {
25     return sum(x2,y2) - sum(x2,y1) - sum(x1,y2) + sum
26         (x1,y1);
27 }

```

## 4.7 BIT

```

1 // (BIT) Fenwick Tree
2
3 int N, bit[MAX];
4
5 int soma(int x)
6 {
7     int resp=0;
8
9     // for(int i=x;i>0;i-=i&-i)
10    //     resp+=bit[i];
11
12    while(x > 0)
13    {
14        resp += bit[x];
15        x -= (x & -x);
16    }
17
18    return resp;
19 }
20
21 int query(int L, int R)
22 {
23     return soma(R)-soma(L-1);
24 }
25
26 void update(int x, int v) // add v in x
27 {
28     // for(;x<=n;x+=x&-x)
29     //     bit[x] += v;
30
31    while(x <= N)
32    {
33        bit[x] += v;
34        x += (x & -x);
35    }
36 }

```

## 4.8 Sparse-Table

```

1 // Precompute log2
2 int logv[MAXN+1];
3 logv[1] = 0;
4 for (int i = 2; i <= MAXN; i++)
5     logv[i] = logv[i/2] + 1;
6
7 int st[MAXN][K];
8
9 void precompute(int N)
10 {
11     for (int i = 0; i < N; i++)
12         st[i][0] = array[i];

```

```

13
14     int k = logv[N];
15     for (int j = 1; j <= k; j++)
16         for (int i = 0; i + (1 << j) <= N; i++)
17             st[i][j] = max(st[i][j-1], st[i + (1 <<
18         j - 1)][j - 1]);
19 }
20
21 int query(int L, int R)
22 {
23     int j = logv[R - L + 1];
24     int minimum = min(st[L][j], st[R - (1 << j) + 1][
25         j]);
26 }

```

## 4.9 Union-Find

```

1 // Union-Find Functions
2
3 int pai[MAX], peso[MAX];
4
5 int find(int aux)
6 {
7     if(pai[aux]==aux)
8         return aux;
9     else
10        return pai[aux]=find(pai[aux], pai);
11 }
12
13 void join(int x, int y)
14 {
15     x = find(x);
16     y = find(y);
17
18     if(pesos[x]<pesos[y])
19         pai[x] = y;
20     else if(pesos[x]>pesos[y])
21         pai[y] = x;
22     else if(pesos[x]==pesos[y])
23     {
24         pai[x] = y;
25         pesos[y]++;
26     }
27 }
28
29 int main()
30 {
31     for(int i=1;i<=N;i++)
32         pai[i]=i;
33 }

```

## 5 Math

### 5.1 Linear-Diophantine-Equation

```

1 // Linear Diophantine Equation
2 int gcd(int a, int b, int &x, int &y)
3 {
4     if (a == 0)
5     {
6         x = 0; y = 1;
7         return b;
8     }
9     int x1, y1;
10    int d = gcd(b%a, a, x1, y1);
11    x = y1 - (b / a) * x1;
12    y = x1;
13    return d;
14 }
15
16 bool find_any_solution(int a, int b, int c, int &x0,
17     int &y0, int &g)

```



```

17 {
18     g = gcd(abs(a), abs(b), x0, y0);
19     if (c % g)
20         return false;
21
22     x0 *= c / g;
23     y0 *= c / g;
24     if (a < 0) x0 = -x0;
25     if (b < 0) y0 = -y0;
26     return true;
27 }
28
29 // All solutions
30 // x = x0 + k*b/g
31 // y = y0 - k*a/g

```

## 5.2 Factorization-sqrt

```

1 // Factorization of a number in sqrt(n)
2
3 int main()
4 {
5     ll N;
6     vector<int> div;
7
8     cin >> N;
9
10    for(ll i=2; i*i<=N; i++)
11    {
12        if(N%i==0)
13        {
14            vet.pb(i);
15            while(N%i==0)
16                N/=i;
17        }
18    }
19    if(N!=1)
20        vet.pb(N);
21
22    return 0;
23 }

```

## 5.3 Modular-Exponentiation

```

1 // Modular exponentiaion - (x^y)%mod in O(log y)
2 ll power(ll x, ll y, ll mod)
3 {
4     ll res = 1;
5     x%=mod;
6
7     while(y)
8     {
9         if(y&1)
10            res=(res*x)%mod;
11
12            y=y>>1;
13            x=(x*x)%mod;
14        }
15    return res;
16 }

```

## 5.4 Miller-Habin

```

1 #include <bits/stdc++.h>
2 #define mod 1000000007
3 #define Pi 3.14159265358979311599796346854
4 #define INF 0x3f3f3f3f
5 #define MAX 1000010
6 #define f first
7 #define s second
8 #define ll long long
9 #define pb push_back

```

```

10 #define mp make_pair
11 #define pii pair<int, int>
12 #define vi vector<int>
13 #define vii vector< pii >
14 #define sws ios_base::sync_with_stdio(false); cin.tie(
15     NULL)
16 #define forn(i, n) for(int i=0; i<(int)(n); i++)
17 #define mdc(a, b) (__gcd((a), (b)))
18 #define mmc(a, b) (((a)/__gcd(a, b)) * b)
19 #define endl '\n'
20 #define teto(a, b) (a+b-1)/b
21
22 using namespace std;
23
24 ll llrand()
25 {
26     ll tmp = rand();
27     return (tmp << 31) | rand();
28 }
29
30 ll add(ll a, ll b, ll c)
31 {
32     return (a + b)%c;
33 }
34
35 ll mul(ll a, ll b, ll c)
36 {
37     ll ans = 0;
38     while(b)
39     {
40         if(b & 1)
41             ans = add(ans, a, c);
42         a = add(a, a, c);
43         b /= 2;
44     }
45     return ans;
46 }
47
48 ll rho(ll n)
49 {
50     ll x, c, y, d, k;
51     int i;
52     do{
53         i = 1;
54         x = llrand()%n;
55         c = llrand()%n;
56         y = x, k = 4;
57         do{
58             if(++i == k)
59             {
60                 y = x;
61                 k *= 2;
62             }
63             x = add(mul(x, x, n), c, n);
64             d = __gcd(abs(x - y), n);
65             while(d == 1);
66         }
67         while(d == n);
68     }
69     return d;
70 }
71
72 ll fexp(ll a, ll b, ll c)
73 {
74     ll ans = 1;
75     while(b)
76     {
77         if(b & 1)
78             ans = mul(ans, a, c);
79         a = mul(a, a, c);
80         b /= 2;
81     }

```

```

82     return ans;
83 }
84
85 bool rabin(ll n)
86 {
87     if(n <= 1)
88         return 1;
89     if(n <= 3)
90         return 1;
91
92     ll s=0, d=n-1;
93     while(d%2==0)
94     {
95         d/=2;
96         s++;
97     }
98
99     for(int k = 0; k < 64*4; k++)
100     {
101         ll a = (llrand()%(n - 3)) + 2;
102         ll x = fexp(a, d, n);
103         if(x != 1 and x != n-1)
104         {
105             for(int r = 1; r < s; r++)
106             {
107                 x = mul(x, x, n);
108                 if(x == 1)
109                     return 0;
110                 if(x == n-1)
111                     break;
112             }
113             if(x != n-1)
114                 return 0;
115         }
116     }
117
118     return 1;
119 }
120
121 int main()
122 {
123     //sws;
124     //freopen("input.txt", "r", stdin);
125     //freopen("output.txt", "w", stdout);
126
127     ll N, resp;
128     vector<ll> div;
129
130     cin >> N;
131     resp = N;
132
133     while(N>1 and !rabin(N))
134     {
135         ll d = rho(N);
136         if(!rabin(d))
137             continue;
138         div.pb(d);
139         while(N%d==0)
140             N/=d;
141     }
142     if(N!=resp and N!=1)
143         div.pb(N);
144
145     if(div.empty())
146         cout << resp << endl;
147     else
148     {
149         for(int i=0; i<(int)div.size(); i++)
150             resp = __gcd(resp, div[i]);
151
152         cout << resp << endl;
153
154     }

```

```

155     }
156
157     return 0;
158
159 }

```

## 5.5 Inverso-Mult

```

1 // ax + my = 1, e gcd(a, m) = 1 para existir solucao
2 // outra forma de escrever: a*x = 1 (mod m)
3 int x, y;
4 int g = gcd(a, m, x, y);
5 if (g != 1)
6     cout << "No solution!";
7 else
8 {
9     x = (x%m + m) % m;
10    cout << x << endl;
11 }

```

## 5.6 Pollard-Rho

```

1 // Pollard Rho Algorithm
2
3 #include <bits/stdc++.h>
4 #define ll long long
5
6 using namespace std;
7
8 ll llrand()
9 {
10     ll tmp = rand();
11     return (tmp << 31) | rand();
12 }
13
14 ll add(ll a, ll b, ll c)
15 {
16     return (a + b)%c;
17 }
18
19 ll mul(ll a, ll b, ll c)
20 {
21     ll ans = 0;
22     while(b)
23     {
24         if(b & 1)
25             ans = add(ans, a, c);
26         a = add(a, a, c);
27         b /= 2;
28     }
29     return ans;
30 }
31
32 ll rho(ll n)
33 {
34     ll x, c, y, d, k;
35     int i;
36     do{
37         i = 1;
38         x = llrand()%n;
39         c = llrand()%n;
40         y = x, k = 4;
41         do{
42             if(++i == k)
43             {
44                 y = x;
45                 k *= 2;
46             }
47             x = add(mul(x, x, n), c, n);
48             d = __gcd(abs(x - y), n);
49         }
50         while(d == 1);

```

```

51     }
52     while(d == n);
53
54     return d;
55 }
56
57 int main()
58 {
59     srand(time(0));
60
61     ll N;
62     cin >> N;
63
64     ll div = rho(N);
65     cout << div << " " << N/div << endl;
66
67     // Finding all divisors
68
69     vector<ll> div;
70
71     while(N>1 and !rabin(N))
72     {
73         ll d = rho(N);
74         if(!rabin(d))
75             continue;
76         div.pb(d);
77         while(N%d==0)
78             N/=d;
79     }
80     if(N!=resp and N!=1)
81         div.pb(N);
82
83     return 0;
84 }
85
86 }

```

## 5.7 Verif-primo

```

1 // prime verification sqrt(N)
2
3 bool eh_primo(long long N)
4 {
5     if(N==2)
6         return true;
7     else if(N==1 or N%2==0)
8         return false;
9     for(long long i=3;i*i<=N;i+=2)
10         if(N%i==0)
11             return false;
12     return true;
13 }

```

## 5.8 Crivo

```

1 // Sieve of Eratosthenes
2
3 int N;
4 vector<bool> primos(100010, true);
5 cin >> N;
6
7 primos[0]=false;
8 primos[1]=false;
9
10 for(int i=2;i<=N;i++)
11     if(primos[i])
12         for(int j=i+i; j<=N; j+=i)
13             primos[j]=false;

```

## 5.9 Formulas

```

1 int sum_x2(ll N)

```

```

2 {
3     return (2*N*N*N + 3*N*N + N)/6;
4 }

```

## 5.10 FFT-golffbot

```

1 #include <bits/stdc++.h>
2
3 using namespace std;
4
5 const int N = (1<<19);
6 const double two_pi = 4 * acos(0);
7
8 struct cpx
9 {
10     cpx(){}
11     cpx(double aa): a(aa){}
12     cpx(double aa,double bb):a(aa),b(bb){}
13     double a;
14     double b;
15     double modsq(void) const
16     {
17         return a*a+b*b;
18     }
19     cpx bar(void) const
20     {
21         return cpx(a,-b);
22     }
23 };
24
25 cpx b[N+100];
26 cpx c[N+100];
27 cpx B[N+100];
28 cpx C[N+100];
29 int a[N+100];
30 int x[N+100];
31 double coss[N+100], sins[N+100];
32 int n,m,p;
33
34 cpx operator +(cpx a,cpx b)
35 {
36     return cpx(a.a+b.a,a.b+b.b);
37 }
38
39 cpx operator *(cpx a,cpx b)
40 {
41     return cpx(a.a*b.a-a.b*b.b,a.a*b.b+a.b*b.a);
42 }
43
44 cpx operator /(cpx a,cpx b)
45 {
46     cpx r = a*b.bar();
47     return cpx(r.a/b.modsq(),r.b/b.modsq());
48 }
49
50 cpx EXP(int i,int dir)
51 {
52     return cpx(coss[i],sins[i]*dir);
53 }
54
55 void FFT(cpx *in,cpx *out,int step,int size,int dir)
56 {
57     if(size<1) return;
58     if(size==1)
59     {
60         out[0]=in[0];
61         return;
62     }
63     FFT(in,out,step*2,size/2,dir);
64     FFT(in+step,out+size/2,step*2,size/2,dir);
65     for(int i=0;i<size/2;++i)
66     {
67         cpx even=out[i];

```

```

68         cpx odd=out[i+size/2];
69         out[i] = even+EXP(i*step,dir)*odd;
70         out[i+size/2]=even+EXP((i+size/2)*step,dir)*
odd;
71     }
72 }
73
74 int main()
75 {
76     for(int i=0;i<=N;++i)
77     {
78         coss[i]=cos(two_pi*i/N);
79         sins[i]=sin(two_pi*i/N);
80     }
81     while(cin >> n) // Numero de tacadas possiveis
82     {
83         fill(x,x+N+100,0);
84         fill(a,a+N+100,0);
85         for(int i=0;i<n;++i)
86         {
87             cin >> p; // Distancia das tacadas
88             x[p]=1;
89         }
90         for(int i=0;i<N+100;++i)
91         {
92             b[i]=cpx(x[i],0);
93         }
94         cin >> m; // Querys
95         for(int i=0;i<m;++i)
96         {
97             cin >> a[i]; // Distancia da query
98         }
99         FFT(b,B,1,N,1);
100        for(int i=0;i<N;++i)
101            C[i]=B[i]*B[i];
102        FFT(C,c,1,N,-1);
103        for(int i=0;i<N;++i)
104            c[i]=c[i]/N;
105        int cnt=0;
106        for(int i=0;i<m;++i)
107            if(c[a[i]].a>0.5 || x[a[i]])
108                cnt++;
109        cout << cnt << endl;
110    }
111    return 0;
112 }

```

## 5.11 Modular-Factorial

```

1 // C++ program to compute n! % p using Wilson's
Theorem
2 #include <bits/stdc++.h>
3 using namespace std;
4
5 int power(int x, unsigned int y, int p)
6 {
7     int res = 1;
8     x = x % p;
9
10    while(y > 0)
11    {
12        if(y & 1)
13            res = (res * x) % p;
14
15        y = y >> 1;
16        x = (x * x) % p;
17    }
18    return res;
19 }
20
21 int modInverse(int a, int p)
22 {
23     return power(a, p-2, p);

```

```

24 }
25
26 int modFact(int n, int p)
27 {
28     if (p <= n)
29         return 0;
30
31     int res = (p - 1);
32
33     for(int i = n + 1; i < p; i++)
34         res = (res * modInverse(i, p)) % p;
35     return res;
36 }
37
38 int main()
39 {
40     int n = 25, p = 29;
41     cout << modFact(n, p);
42     return 0;
43 }

```

## 5.12 Kamenetsky

```

1 // Number of digits in n! O(1)
2
3 #define Pi 3.14159265358979311599796346854
4 #define Eul 2.71828182845904509079559829842
5
6 long long findDigits(int n)
7 {
8     double x;
9
10    if (n < 0)
11        return 0;
12    if (n == 1)
13        return 1;
14
15    x = ((n * log10(n / euler) + log10(2 * Pi * n)
/2.0));
16
17    return floor(x) + 1;
18 }

```

## 6 Misc

### 6.1 LIS

```

1 multiset<int> S;
2 for(int i = 0; i < n; i++){
3     auto it = S.upper_bound(vet[i]); // low for inc
4     if(it != S.end())
5         S.erase(it);
6     S.insert(vet[i]);
7 }
8 // size of the lis
9 int ans = S.size();

```

### 6.2 Bitwise

```

1 // Bitwise
2
3 unsigned char a = 5, b = 9; // a = (00000101), b
= (00001001)
4
5 AND -          a&b    // The result is 00000001
(1)
6 OR -           a|b    // The result is 00001101
(13)
7 XOR -          a^b    // The result is 00001100
(12)
8 NOT -          ~a     // The result is 11111010
(250)

```

```

9 Left shift - b<<1 // The result is 00010010
(18)
10 Right shift - b>>1 // The result is 00000100
(4)
11
12 // Exchange two int variables
13
14 a^=b;
15 b^=a;
16 a^=b;
17
18 // Even or Odd
19
20 (x & 1)? printf("Odd"): printf("Even");
21
22 // Turn on the j-th bit
23
24 int S = 34; //(100010)
25 int j = 3;
26
27 S = S | (1<<j);
28
29 // Turn off the j-th bit
30
31 int S = 42; //(101010)
32 int j = 1;
33
34 S &= ~(1<<j)
35
36 S == 40 //(101000)
37
38 // Check the j-th element
39
40 int S = 42; //(101010)
41 int j = 3;
42
43 T = S & (1<<j); // T = 0
44
45 // Exchange o j-th element
46
47 S ^= (1<<j)
48
49 // Position of the first bit on
50
51 T = (S & (-S))
52 T -> 4 bit ligado //(1000)
53
54 // Most significant digit of N
55
56
57 double K = log10(N);
58 K = K - floor(K);
59 int X = pow(10, K);
60
61 // Number of digits in N
62
63 X =floor(log10(N)) + 1;
64
65 // Power of two
66
67 bool isPowerOfTwo(int x)
68 {
69     return x && (!(x&(x-1)));
70 }

```

## 7 Strings

### 7.1 KMP

```

1 vector<int> prefix_function(const string &s){

```

```

2     int n = s.size(); vector<int> b(n+1);
3     b[0] = -1; int i = 0, j = -1;
4     while(i < n){
5         while(j >= 0 && s[i] != s[j]) j = b[j];
6         b[++i] = ++j;
7     }
8     return b;
9 }
10 void kmp(const string &t, const string &p){
11     vector<int> b = prefix_function(p);
12     int n = t.size(), m = p.size();
13     int j = 0;
14     for(int i = 0; i < n; i++){
15         while(j >= 0 && t[i] != p[j]) j = b[j];
16         j++;
17         if(j == m){
18             j = b[j];
19         }
20     }
21 }
22 }

```

### 7.2 Pal-int

```

1 bool ehpalindromo(ll n)
2 {
3     if(n<0)
4         return false;
5
6     int divisor = 1;
7     while(n/divisor >= 10)
8         divisor *= 10;
9
10    while(n != 0)
11    {
12        int leading = n / divisor;
13        int trailing = n % 10;
14
15        if(leading != trailing)
16            return false;
17
18        n = (n % divisor)/10;
19
20        divisor = divisor/100;
21    }
22
23    return true;
24 }

```

### 7.3 Z-Func

```

1 vector<int> z_algo(const string &s)
2 {
3     int n = s.size();
4     int L = 0, R = 0;
5     vector<int> z(n, 0);
6     for(int i = 1; i < n; i++)
7     {
8         if(i <= R)
9             z[i] = min(z[i-L], R - i + 1);
10        while(z[i]+i < n && s[ z[i]+i ] == s[ z[i] ])
11            z[i]++;
12        if(i+z[i]-1 > R)
13        {
14            L = i;
15            R = i + z[i] - 1;
16        }
17    }
18    return z;
19 }

```