

Notebook - Maratona de Programação

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1 Algoritmos

1.1 Iterative-BS

```
1 int main()
2 {
       int l=1, r=N;
       int res=-1;
       while(1 <= r)</pre>
6
            int m = (1 + r)/2;
            if(!ver(m))
            {
10
                 1 = m+1;
11
            }
12
            else
13
            {
                 res = m:
15
16
                 r = m-1:
            }
17
18
       cout << res << endl;</pre>
19
20
       return 0;
22 }
```

2 Grafos

2.1 BFS

```
1 //BFS (Breadth First Search) O(V+A)
3 void BFS(int x)
       int atual, v, u;
       queue < int > fila;
       fila.push(x);
       componente[x] = valor;
9
       atual = 0;
10
       while(!fila.empty())
11
12
13
           v = fila.front();
           fila.pop();
14
           for(int i = 0;i < (int)vizinhos[v].size();i</pre>
16
       ++)
           {
               u = vizinhos[v][i];
18
                if (componente[u] == -1)
20
                    componente[u] = componente[v];
21
22
                    fila.push(u);
23
           }
24
       }
25
```

2.2 Find-bridges

```
#define vi vector<int>

vector< vector<int> > grafo;
vector<bool> visited;
vi t, low;
int timer=0;

void find_bridges(int v, int p=-1)
{
```

```
visited[v] = true;
10
11
       t[v] = low[v] = timer++;
       for(int i=0;i<(int)grafo[v].size();i++)</pre>
12
14
            int vert = grafo[v][i];
           if (vert == p)
15
                continue;
16
           if (visited[vert])
17
                low[v] = min(low[v], t[vert]);
18
19
           else
           {
20
21
                find_bridges(vert, v);
                low[v] = min(low[v], low[vert]);
22
                if(low[to] > t[v])
23
                    IS_BRIDGE(v, vert);
24
25
26
       }
27 }
29 int main()
30 {
       timer = 0;
31
       visited.assign(N+1, false);
32
       t.assign(N+1, 0);
       low.assign(N+1, 0);
34
35
       for(int i=0;i<N;i++)</pre>
36
           if(!visited[i])
37
38
                find_bridges(1);
39
40
       return 0;
41 }
```

2.3 Dijkstra

```
1 // Dijkstra - Shortest Path
3 #define pii pair<int, int>
4 #define vi vector<int>
5 #define vii vector< pair<int,int> >
6 #define INF 0x3f3f3f3f
8 vector < vii > grafo;
9 vi distancia;
10 priority_queue < pii, vii, greater <pii>> fila;
12 void dijkstra(int k)
13 {
14
       int dist, vert, aux;
       distancia[k]=0;
16
       fila.push(mp(k, 0));
18
       while(!fila.empty())
19
       {
20
21
           aux=fila.top().f;
22
           fila.pop();
23
           for(int i=0; i<grafo[aux].size(); i++)</pre>
25
                vert=grafo[aux][i].f;
26
27
               dist=grafo[aux][i].s;
28
                if (distancia[vert]>distancia[aux]+dist)
                    distancia[vert] = distancia[aux] + dist;
30
                    fila.push(mp(vert, distancia[vert]));
32
33
           }
       }
34
35 }
37 int main()
```

```
38 €
39
       dist.assign(N+1, INF);
      grafo.assign(N+1, vii());
40
41
      for(int i=0; i<M; i++)</pre>
43
           cin >> a >> b >> p;
           grafo[a].pb(mp(b, p));
45
           grafo[b].pb(mp(a, p));
46
47
48 }
       Floyd-Warshall
1 // Floyd Warshall
```

2.5 Kruskal

```
1 // Kruskal - Minimum Spanning Tree
3 typedef struct
       int A, B;
       int dist;
7 } vertice;
9 vertice grafo[MAX];
10 int pai[MAX];
12 int find(int X) // Union-Find
13 €
       if(pai[X] == X)
14
15
           return X;
16
17
           return pai[X]=find(pai[X]);
18 }
19
20 void join(int X, int Y)
21 {
       int paix = find(X);
22
       int paiy = find(Y);
23
       pai[paix]=paiy;
25 }
26
27 bool comp(vertice A, vertice B)
28 {
       return A.dist < B.dist;</pre>
29
30 }
31
32 void kruskal()
33 {
       for(int i=1;i<=N;i++)</pre>
           pai[i]=i;
35
37
       for(int i=1;i<=M;i++)</pre>
          cin >> grafo[i].A >> grafo[i].B >> grafo[i].
38
39
       sort(grafo+1, grafo+M+1, comp);
41
       for(int i=1;i<M;i++)</pre>
42
```

```
43
44
            if (find(grafo[i].A)!=find(grafo[i].B))
            {
45
                 join(grafo[i].A, grafo[i].B);
46
47
                 soma+=grafo[i].dist;
            }
48
       }
49
50
       cout << soma << endl;</pre>
51
52 }
```

2.6 DFS

2.7 Represent

```
_1 // Grafos
3 // List of edges
      vector < pair < int , int > > arestas;
      arestas.push_back(make_pair(1, 2));
      arestas.push_back(make_pair(1, 3));
9 // Adjacency Matrix
10
      int grafo[10][10];
11
12
       grafo[1][2] = grafo[2][1] = 1;
13
      grafo[1][3] = grafo[3][1] = 2;
14
15
16 // Adjacency List
17
18
      vector < int > vizinhos[10];
19
       vizinhos[1].push_back(2);
       vizinhos[1].push_back(2);
```

2.8 Prim

```
int x, y;
       while (1)
                                                            6 } pnt;
18
19
           int davez = -1;
                                                            8 bool collinear(pnt p, pnt q, pnt r)
20
                                                            9 {
           while(!fila.empty())
                                                                   if(q.x<=max(p.x,r.x) && q.x>=min(p.x,r.x) && q.y
22
                                                            10
                                                                   <=max(p.y,r.y) && q.y>=min(p.y,r.y))
23
               int atual = fila.top().second;
                                                                       return true;
24
               fila.pop();
25
                                                            12
                                                                   return false;
                                                            13
               if (!processado[atual])
                                                            14 }
27
                    davez = atual;
29
                                                            int orientation(pnt p, pnt q, pnt r)
                   break;
                                                            17 {
30
               }
31
                                                            18
                                                                   int val=(q.y-p.y)*(r.x-q.x)-(q.x-p.x)*(r.y-q.y);
           }
32
                                                            19
                                                            20
                                                                   if(val==0)
           if(davez == -1)
                                                                       return 0;
34
                                                            21
               break:
                                                                   else if(val>0)
36
                                                            23
                                                                       return 1;
           processado[davez] = true;
                                                                   else
                                                            24
37
                                                                       return 2;
           for(int i = 0;i < (int)vizinhos[davez].size() 26 }</pre>
39
       ;i++)
           {
                                                            28 bool intersect(pnt p1, pnt q1, pnt p2, pnt q2)
40
                                                            29 {
41
               int dist = vizinhos[davez][i].first;
                                                                   int o1 = orientation(p1, q1, p2);
42
                                                            30
                                                                   int o2 = orientation(p1, q1, q2);
               int atual = vizinhos[davez][i].second;
43
                                                            31
                                                                   int o3 = orientation(p2, q2, p1);
                                                            32
               if( distancia[atual] > dist && !
                                                                   int o4 = orientation(p2, q2, q1);
45
                                                            33
      processado[atual])
                                                            34
                                                                   if(o1!=o2 \text{ and } o3!=o4)
46
               {
                                                            35
                    distancia[atual] = dist;
                                                                       return true;
47
                                                            36
                   fila.push( pii(distancia[atual],
      atual));
                                                                   if(o1==0 && collinear(p1, p2, q1))
                                                            38
                                                                       return true;
           }
50
                                                            40
                                                                   if (o2==0 && collinear(p1, q2, q1))
51
                                                            41
                                                            42
                                                                       return true;
      int custo_arvore = 0;
53
                                                            43
54
       for(int i = 1; i <= n; i++)</pre>
                                                            44
                                                                   if(o3==0 && collinear(p2, p1, q2))
           custo_arvore += distancia[i];
                                                                       return true;
55
                                                            45
56
                                                            46
57
      return custo_arvore;
                                                            47
                                                                   if(o4==0 && collinear(p2, q1, q2))
58 }
                                                                       return true;
                                                            48
59
                                                            49
60 int main(){
                                                                   return false;
                                                            50
                                                            51
                                                            52 }
      cin >> n >> m;
62
63
      for(int i = 1;i <= m;i++){</pre>
                                                                    ED
64
                                                              4
65
           int x, y, tempo;
                                                                     Iterative-SegTree
           cin >> x >> y >> tempo;
                                                              4.1
67
68
           vizinhos[x].pb( pii(tempo, y) );
                                                            1 // Segment Tree Iterativa - Range maximum query
69
           vizinhos[y].pb( pii(tempo, x) );
70
71
                                                             3 #define N 100010
72
      cout << Prim() << endl;</pre>
                                                             5 struct Segtree
74
                                                             6 {
75
      return 0;
                                                                   int t[2*N]={0};
76 }
                                                             9
                                                                   void build()
  3
       Geometria
                                                                   {
                                                            10
                                                                       for(int i=N-1; i>0; i--)
                                                            11
                                                                           t[i]=max(t[i<<1], t[1<<1|1]);
                                                            12
  3.1 Inter-Retas
                                                            13
                                                            14
1 // Intersection between lines
                                                                   int query(int 1, int r)
                                                            15
                                                            16
```

17

18

int ans=0:

for (i+=N, r+=N; l< r; l>>=1, r>>=1)

3 typedef struct

```
{
                                                                           atualiza(esq, i, meio, pos, novo_valor);
19
                                                            36
20
               if (1&1)
                                                            37
                                                                       else
                   ans=max(ans, t[1++]);
                                                                           atualiza(dir, meio+1, j, pos, novo_valor)
21
                                                            38
               if (r&1)
                   ans=max(ans, t[--r]);
           }
                                                                       if (val[esq]>val[dir])
24
                                                            40
                                                                           val[no]=val[esq];
                                                            41
           return ans:
26
                                                            42
                                                                           val[no]=val[dir];
27
                                                            43
                                                           44
                                                                   }
       void update(int p, int value)
                                                           45 }
29
30
                                                           47 int consulta(int no, int i, int j, int A, int B)
           for(t[p+=n]=value; p>1; p>>=1)
31
               t[p>>1] = max(t[p], t[p^1]);
                                                           48 {
32
       7
33
                                                           49
                                                                   if(i>B || j<A)</pre>
                                                                       return -1;
34
                                                            50
35 };
                                                            51
                                                                   if(i>=A and j<=B)
                                                                       return val[no];
36
                                                            52
37 int main()
                                                            53
                                                                   int esq = 2*no;
38 {
                                                            54
       Segtree st;
                                                                   int dir = 2*no+1;
39
                                                            55
40
                                                            56
                                                                   int meio = (i+j)/2;
       for(int i=0;i<n;i++)</pre>
                                                            57
41
                                                                   int resp_esq = consulta(esq, i, meio, A, B);
                                                                   int resp_dir = consulta(dir, meio+1, j, A, B);
           cin >> aux:
43
                                                            59
          st.t[N+i] = aux; //Leaves are stored in
44
                                                            60
       continuous nodes with indices starting with N
                                                            61
                                                                   if (resp_dir == -1)
                                                                       return resp_esq;
45
                                                            62
                                                                   if(resp_esq==-1)
46
                                                            63
      st.build();
                                                                      return resp_dir;
47
                                                            64
      x = st.query(inicio, fim);
                                                            65
48
       st.update(ind, value);
                                                                   if (resp_esq>resp_dir)
49
                                                            66
                                                            67
                                                                      return resp_esq;
50
51 }
                                                            68
                                                                       return resp_dir;
                                                            69
  4.2 Recursive-SegTree
                                                            70 }
                                                            71
1 // Segment Tree Recursiva - Range maximum query
                                                            72 int main()
                                                            73 {
                                                                   monta(1, N, 1);
                                                            74
3 vector<int> val(MAX, 0);
4 vector < int > vet(N);
                                                            75
                                                                   atualiza(1, 1, N, pos, valor);
                                                                   x = consulta(1, 1, N, inicio, fim);
                                                            76
                                                            77
6 void monta(int i, int j, int no)
                                                           78 }
       if(i==j)
                                                                     Delta-Encoding
9
       {
10
           val[no]=vet[i];
           return;
                                                            1 // Delta encoding
12
13
                                                            3 for(int i=0;i<q;i++)</pre>
      int esq = 2*no;
14
                                                            4 {
       int dir = 2*no+1;
                                                                   int 1, r, x;
      int meio = (i+j)/2;
16
                                                                   cin >> 1 >> r >> x;
17
                                                                   delta[1] += x;
                                                            7
      monta(i, meio, esq);
18
                                                                   delta[r+1] -= x;
                                                            8
      monta(meio+1, j, dir);
19
                                                            9 }
20
                                                            10
       val[no]=max(val[esq], val[dir]);
21
                                                            11 int atual = 0;
22 }
23
                                                            13 for(int i=0;i<n;i++)</pre>
24 void atualiza(int no, int i, int j, int pos, int
                                                            14 {
      novo_valor)
                                                            15
                                                                   atual += delta[i];
25 {
                                                            16
                                                                   v[i] += atual;
26
       if(i==j)
                                                            17 }
27
           val[no]=novo_valor;
                                                              4.4 Seg-Tree-Farao
      }else
29
30
           int esq = 2*no;
                                                            1 typedef struct
           int dir = 2*no+1;
                                                            2 {
32
           int meio = (i+j)/2;
                                                                  pii prefix, sufix, total, maximo;
```

34

if (pos <= meio)</pre>

4 } no;

```
6 int noleft[MAX], noright[MAX]; //Guarda os valores
                                                                     dir)):
       dos nos para que nao sejam calculados novamente
      nas querys
7 int v[MAX];
                                                            74 int main()
8 no arvore[MAX];
                                                            75 {
                                                                    int T, N, Q, A, B;
9
                                                            76
10 pii somar(pii a, pii b) // une pairs
                                                            77
                                                                    no aux;
11 {
                                                            78
       return mp(a.f+b.f, a.s+b.s);
                                                                    scanf("%d", &T);
12
                                                            79
13 }
                                                            80
                                                                    while (T--)
14
                                                            81
15 no
     une(no 1, no r)
                                                            82
                                                                        scanf("%d", &N);
16 {
                                                            83
                                                                        for(int i=1;i<=N;i++)</pre>
       if(1.total.s==0)
17
                                                            84
18
           return r;
                                                            85
                                                                            scanf("%d", &v[i]); //Elementos da arvore
       if(r.total.s==0)
19
                                                            86
20
           return 1;
                                                            87
                                                                        noleft[1]=1; noright[1]=N;
                                                                        monta(1);
21
                                                            88
                                                                        cin >> 0:
23
                                                            90
      m.prefix = max(l.prefix, somar(l.total, r.prefix) 91
                                                                        while (Q--)
24
      ); //prefixo
                                                                        {
      m.sufix = max(r.sufix, somar(r.total, l.sufix)); 93
                                                                            scanf("%d%d", &A, &B); //Intervalo da
25
       //sufixo
      m.total = somar(1.total, r.total); //Soma de
                                                                            aux = busca(1, A, B);
26
                                                            94
       todos os elementos da subarvore
                                                                            printf("%d %d\n", aux.maximo.f, aux.
                                                             95
      m.maximo = max(max(1.maximo, r.maximo), somar(1.
                                                                   maximo.s);
       sufix, r.prefix)); //Resultado para cada
                                                                        }
                                                            96
       subarvore
                                                            97
                                                                   }
28
                                                            98
29
       return m;
                                                            99
30 }
                                                            100
                                                                    return 0;
                                                            101
31
32 no makenozero()
                                                               4.5
                                                                      BIT-2D
33 {
34
      m.prefix=m.sufix=m.total=m.maximo=mp(0,0);
35
                                                             1 // BIT 2D
       return m;
36
37 }
                                                             3 int bit[MAX][MAX];
38
                                                             4
39 no makeno(int k)
                                                             5 int sum(int x, int y)
40 {
                                                             6 {
41
                                                                    int resp=0;
42
      m.prefix=m.sufix=m.total=m.maximo=mp(k,1);
       return m;
43
                                                                    for(int i=x;i>0;i-=i&-i)
                                                             9
44 }
                                                                        for(int j=y;j>0;j-=j&-j)
                                                             10
45
                                                                            resp+=bit[i][j];
46 void monta(int n)
                                                             12
47 {
                                                            13
                                                                    return resp;
       if(noleft[n] == noright[n])
48
                                                            14 }
49
                                                            15
           arvore[n]=makeno(v[noleft[n]]);
50
                                                            16 void update(int x, int y, int delta)
                                                            17 {
       }
                                                                    for(int i=x;i<MAX;i+=i&-i)</pre>
                                                            18
                                                            19
                                                                        for (int j=y; j < MAX; j+=j&-j)</pre>
       int mid = (noleft[n]+noright[n])/2;
54
                                                                            bit[i][j]+=delta;
                                                            20
       noleft[2*n]=noleft[n]; noright[2*n]=mid;
                                                            21 }
       noleft[2*n+1]=mid+1; noright[2*n+1]=noright[n];
56
                                                            22
57
                                                            23 int query(int x1, y1, x2, y2)
       monta(2*n);
58
                                                            24 {
59
       monta(2*n+1):
                                                                    return sum(x2,y2) - sum(x2,y1) - sum(x1,y2) + sum
60
                                                                    (x1,y1);
       arvore[n]=une(arvore[2*n], arvore[2*n+1]);
61
                                                            26 }
62 }
                                                               4.6 BIT
64 no busca(int n, int esq, int dir)
65 {
66
       if(noleft[n]>=esq and noright[n]<=dir)</pre>
                                                             1 // (BIT) Fenwick Tree
           return arvore[n];
67
       if(noright[n] < esq or noleft[n] > dir)
68
                                                             3 int bit[MAX]:
           return makenozero();
69
70
                                                             5 int soma(int x)
       return une(busca(2*n, esq, dir),busca(2*n+1, esq, _6 {
71
```

```
int resp=0;
7
      // for(int i=x;i>0;i-=i&-i)
9
      // resp+=bit[i];
10
11
      while (x > 0)
12
13
          resp += bit[x];
14
           x = (x & -x);
15
16
17
18
       return resp;
19 }
20
21 int query(int L, R)
22 {
23
       return soma(R)-soma(L);
24 }
26 void update(int x, int v)
27 {
       // for(;x<=n;x+=x&-x)
              bit[x] += v;
29
      while(x <= N)</pre>
31
32
           bit[x] += v;
33
           x += (x \& -x);
34
35
36 }
```

4.7 Union-Find

```
1 // Union-Find Functions
3 int pai[MAX], peso[MAX];
5 int find(int aux)
       if(pai[aux] == aux)
          return aux;
9
          return pai[aux]=find(pai[aux], pai);
11 }
13 void join(int x, int y)
14 {
15
       x = find(x);
      y = find(y);
16
17
      if(pesos[x]<pesos[y])</pre>
18
          pai[x] = y;
19
       else if(pesos[x]>pesos[y])
20
         pai[y] = x;
21
       else if(pesos[x] == pesos[y])
23
24
           pai[x] = y;
25
           pesos[y]++;
26
27 }
29 int main()
30 €
       for(int i=1;i<=N;i++)</pre>
31
32
           pai[i]=i;
33 }
```

5 Math

5.1 Linear-Diophantine-Equation

```
1 // Linear Diophantine Equation
2 int gcd(int a, int b, int &x, int &y)
3 {
       if (a == 0)
4
           x = 0; y = 1;
6
           return b;
 8
      int x1, y1;
9
      int d = gcd(b%a, a, x1, y1);
10
       x = y1 - (b / a) * x1;
11
12
       y = x1;
       return d;
13
14 }
15
16 bool find_any_solution(int a, int b, int c, int &x0,
       int &y0, int &g)
17 €
18
       g = gcd(abs(a), abs(b), x0, y0);
       if (c % g)
19
           return false;
20
21
      x0 *= c / g;
22
      y0 *= c / g;
       if (a < 0) x0 = -x0;
24
       if (b < 0) y0 = -y0;
25
26
       return true;
27 }
_{29} // All solutions
_{30} // x = x0 + k*b/g

_{31} // y = y0 - k*a/g
        Factorization-sqrt
```

```
1 // Factorization of a number in sqrt(n)
3 int main()
4 {
       11 N:
5
       vector < int > div;
       cin >> N;
       for(11 i=2;i*i<=N;i++)</pre>
10
11
            if(N\%i==0)
12
                vet.pb(i);
14
                while (N\%i==0)
15
16
                    N/=i:
17
18
       if (N!=1)
19
           vet.pb(N);
20
21
       return 0;
22
23 }
```

5.3 Modular-Exponentiation

```
y = y >> 1;
                                                                        while(d == 1);
12
                                                             65
13
           x=(x*x)\mbox{\em mod};
                                                             66
      7
                                                                    while(d == n);
14
                                                             67
15
      return res;
                                                             68
16 }
                                                             69
                                                                    return d;
                                                            70 }
  5.4 Miller-Habin
                                                             72 ll fexp(ll a, ll b, ll c)
                                                             73 {
1 #include <bits/stdc++.h>
                                                             74
                                                                    11 \text{ ans} = 1;
2 #define mod 1000000007
                                                                    while(b)
3 #define Pi 3.14159265358979311599796346854
                                                             75
4 #define INF 0x3f3f3f3f
                                                             76
                                                                    {
                                                                        if(b & 1)
5 #define MAX 1000010
                                                             77
                                                                           ans = mul(ans, a, c);
                                                             78
6 #define f first
                                                                        a = mul(a, a, c);
7 #define s second
                                                             79
                                                                        b /= 2;
                                                             80
8 #define 11 long long
9 #define pb push_back
                                                             81
                                                                    }
                                                                    return ans;
10 #define mp make_pair
                                                             82
11 #define pii pair <int, int>
                                                            83 }
12 #define vi vector<int>
                                                             84
                                                             85 bool rabin(ll n)
13 #define vii vector< pii >
14 #define sws ios_base::sync_with_stdio(false);cin.tie(86 {
                                                                    if (n <= 1)
      NULL)
                                                            87
                                                                        return 1;
15 #define forn(i, n) for(int i=0; i<(int)(n); i++)
                                                                    if(n <= 3)</pre>
16 #define mdc(a, b) (__gcd((a), (b)))
                                                             89
17 #define mmc(a, b) (((a)/_{gcd}(a, b)) * b)
                                                                       return 1;
                                                             90
18 #define endl '\n'
                                                             91
                                                                    ll s=0, d=n-1;
19 #define teto(a, b) (a+b-1)/b
                                                             92
                                                             93
                                                                    while (d\%2==0)
                                                                    {
21 using namespace std;
                                                             94
                                                                        d/=2;
                                                             95
23 ll llrand()
                                                             96
                                                                        s++;
                                                            97
24 {
      11 tmp = rand();
                                                             98
25
                                                                    for(int k = 0; k < 64*4; k++)
                                                            99
      return (tmp << 31) | rand();</pre>
                                                            100
27 }
                                                                        11 a = (11rand()\%(n - 3)) + 2;
                                                                        ll x = fexp(a, d, n);
29 ll add(ll a, ll b, ll c)
                                                            102
                                                                        if(x != 1 and x != n-1)
                                                            103
30 €
                                                                        {
                                                            104
31
       return (a + b)%c;
32 }
                                                            105
                                                                             for(int r = 1; r < s; r++)</pre>
                                                            106
                                                                                 x = mul(x, x, n);
34 ll mul(ll a, ll b, ll c)
                                                            107
                                                            108
                                                                                 if(x == 1)
35 {
                                                                                     return 0;
                                                            109
36
       11 ans = 0:
       while(b)
                                                            110
                                                                                 if(x == n-1)
37
                                                                                     break:
38
           if(b & 1)
39
                                                                             if(x != n-1)
40
              ans = add(ans, a, c);
                                                            113
                                                            114
                                                                                 return 0;
           a = add(a, a, c);
41
                                                                        }
           b /= 2;
                                                            115
42
       }
                                                            116
43
       return ans;
                                                            117
44
                                                                    return 1;
45 }
                                                            118
                                                            119 }
46
                                                            120
47 ll rho(ll n)
                                                            121
48 {
                                                            122 int main()
       ll x, c, y, d, k;
49
       int i;
                                                            123 {
50
                                                            124
51
       do{
                                                                    //freopen("input.txt", "r", stdin);
           i = 1;
                                                            125
52
                                                            126
                                                                    //freopen("output.txt", "w", stdout);
           x = llrand()%n;
53
           c = llrand()%n;
                                                            127
54
                                                                    11 N, resp;
                                                            128
55
           y = x, k = 4;
                                                            129
                                                                    vector<1l> div;
           do{
56
               if(++i == k)
                                                            130
57
                                                            131
                                                                    cin >> N;
58
                {
                    y = x;
                                                            132
                                                                    resp = N;
59
                                                            133
                    k *= 2;
                                                            134
                                                                    while(N>1 and !rabin(N))
               }
61
               x = add(mul(x, x, n), c, n);
                                                            135
                                                                        11 d = rho(N);
                                                            136
               d = \_\_gcd(abs(x - y), n);
63
           }
                                                            137
                                                                        if(!rabin(d))
64
```

```
continue;
138
                                                              49
139
            div.pb(d);
                                                              50
                                                                          while(d == 1);
            while (N\%d==0)
140
                                                              51
               N/=d;
                                                                     while(d == n);
141
                                                              52
                                                              53
       if(N!=resp and N!=1)
                                                                     return d;
                                                              54
143
           div.pb(N);
                                                              55 }
144
145
                                                              56
                                                              57 int main()
146
        if(div.empty())
                                                              58 {
           cout << resp << endl;</pre>
                                                                      srand(time(0));
                                                              59
148
149
                                                              60
150
                                                              61
                                                                     11 N;
            for(int i=0;i<(int)div.size();i++)</pre>
                                                                     cin >> N;
                                                              62
151
               resp = __gcd(resp, div[i]);
152
                                                              63
                                                                     11 \text{ div} = \text{rho}(N);
153
                                                              64
                                                                     cout << div << " " << N/div << endl;
            cout << resp << endl;</pre>
                                                              65
                                                              66
                                                                     // Finding all divisors
       return 0;
157
                                                              68
158
                                                              69
159 }
                                                                     vector < 11 > div;
                                                              70
                                                              71
   5.5 Pollard-Rho
                                                                     while(N>1 and !rabin(N))
                                                              72
                                                              73
                                                              74
                                                                          11 d = rho(N);
 1 // Pollard Rho Algorithm
                                                              75
                                                                          if(!rabin(d))
                                                                              continue;
 3 #include <bits/stdc++.h>
                                                              76
                                                              77
                                                                          div.pb(d);
 4 #define ll long long
                                                                          while (N%d==0)
                                                              78
                                                                              N/=d;
                                                              79
 6 using namespace std;
                                                              80
                                                                     if (N!=resp and N!=1)
                                                              81
 8 11 11rand()
                                                              82
                                                                          div.pb(N);
        11 tmp = rand();
                                                              83
                                                                      return 0;
        return (tmp << 31) | rand();</pre>
                                                              84
11
                                                              85
                                                              86 }
14 ll add(ll a, ll b, ll c)
                                                                       Verif-primo
                                                                 5.6
       return (a + b)%c;
16
                                                               1 // prime verification sqrt(N)
 17 }
18
19 ll mul(ll a, ll b, ll c)
                                                               3 bool eh_primo(long long N)
20 {
       11 \text{ ans} = 0;
                                                                     if(N==2)
21
                                                               5
22
       while(b)
                                                                         return true;
                                                               6
        {
                                                                      else if (N==1 \text{ or } N\%2==0)
23
24
            if(b & 1)
                                                                        return false;
                ans = add(ans, a, c);
                                                                     for(long long i=3;i*i<=N;i+=2)</pre>
            a = add(a, a, c);
                                                                        if(N\%i==0)
26
                                                              1.0
            b /= 2;
                                                                             return false;
                                                              11
       }
28
                                                              12
                                                                     return true;
                                                              13 }
       return ans;
30 }
                                                                 5.7 Crivo
31
32 ll rho(ll n)
33 {
                                                               1 // Sieve of Eratosthenes
        ll x, c, y, d, k;
35
        int i;
                                                               3 int N:
        do{
36
                                                               4 vector <bool > primos(100010, true);
            i = 1;
37
                                                               5 cin >> N;
           x = llrand()%n;
38
39
           c = llrand()%n;
                                                               7 primos[0]=false;
            y = x, k = 4;
40
                                                               8 primos[1]=false;
            do{
41
                if(++i == k)
42
                                                              10 for(int i=2;i<=N;i++)</pre>
43
                {
                                                                    if(primos[i])
                                                              11
                     y = x;
                                                              12
                                                                          for(int j=i+i; j<=N; j+=i)</pre>
                     k *= 2;
45
                                                                              primos[j]=false;
                                                              13
                x = add(mul(x, x, n), c, n);
47
                                                               5.8 formulas
```

 $d = _gcd(abs(x - y), n);$

48

```
1 int sum_x2(11 N)
                                                                         cpx even=out[i];
                                                              67
2 {
                                                              68
                                                                         cpx odd=out[i+size/2];
       return (2*N*N*N + 3*N*N + N)/6;
                                                                         out[i] = even+EXP(i*step,dir)*odd;
                                                              69
4 }
                                                                         out[i+size/2] = even+EXP((i+size/2)*step,dir)*
                                                              70
                                                                     odd:
         FFT-golfbot
                                                                     }
                                                              71
                                                              72 }
#include <bits/stdc++.h>
                                                              73
                                                              74 int main()
3 using namespace std;
                                                              75 {
                                                                     for(int i=0;i<=N;++i)</pre>
                                                              76
5 const int N = (1<<19);</pre>
                                                              77
6 const double two_pi = 4 * acos(0);
                                                              78
                                                                         coss[i]=cos(two_pi*i/N);
                                                                         sins[i]=sin(two_pi*i/N);
                                                              79
8 struct cpx
                                                              80
                                                                     7
                                                                     while(cin >> n) // Numero de tacadas possiveis
                                                              81
9 {
                                                              82
       cpx(){}
10
                                                                         fill(x,x+N+100,0);
       cpx(double aa): a(aa){}
                                                              83
       cpx(double aa,double bb):a(aa),b(bb){}
                                                                         fill(a,a+N+100,0);
                                                                         for (int i=0; i < n; ++i)</pre>
       double a;
                                                              85
       double b;
                                                              86
14
                                                                              cin >> p; // Distancia das tacadas
       double modsq(void) const
                                                              87
                                                                              x[p]=1;
                                                              88
                                                                         }
                                                              89
17
           return a*a+b*b:
                                                                         for (int i=0;i<N+100;++i)</pre>
       }
                                                              90
18
                                                              91
19
       cpx bar(void) const
                                                                              b[i]=cpx(x[i],0);
                                                              92
20
           return cpx(a,-b);
                                                              93
21
                                                                         cin >> m; // Querys
       }
                                                              94
22
                                                                         for (int i=0; i < m; ++i)</pre>
23 };
                                                              95
                                                              96
24
                                                                              cin >> a[i]; // Distancia da query
25 cpx b[N+100];
                                                              97
                                                             98
26 cpx c[N+100];
27 cpx B[N+100];
                                                             99
                                                                         FFT(b,B,1,N,1);
                                                                         for (int i=0; i < N; ++i)</pre>
28 cpx C[N+100];
                                                             100
                                                                              C[i]=B[i]*B[i];
29 int a[N+100];
                                                                         FFT(C,c,1,N,-1);
30 int x[N+100];
31 double coss[N+100], sins[N+100];
                                                             103
                                                                         for (int i=0; i < N; ++i)</pre>
32 int n,m,p;
                                                             104
                                                                              c[i]=c[i]/N;
                                                                         int cnt=0;
                                                             105
                                                             106
                                                                          for (int i=0;i<m;++i)</pre>
34 cpx operator +(cpx a,cpx b)
                                                                             if(c[a[i]].a>0.5 || x[a[i]])
                                                             107
                                                             108
                                                                                  cnt++:
36
       return cpx(a.a+b.a,a.b+b.b);
                                                             109
                                                                         cout << cnt << endl;</pre>
37 }
                                                             110
38
                                                             111
                                                                     return 0;
39 cpx operator *(cpx a,cpx b)
                                                             112 }
40 {
       return cpx(a.a*b.a-a.b*b.b,a.a*b.b+a.b*b.a);
41
                                                                5.10 Modular-Factorial
42 }
43
44 cpx operator /(cpx a,cpx b)
                                                              1 // C++ program to comput n! % p using Wilson's
                                                                     Theorem
       cpx r = a*b.bar();
                                                              2 #include <bits/stdc++.h>
46
       return cpx(r.a/b.modsq(),r.b/b.modsq());
                                                              3 using namespace std;
48 }
                                                              5 int power(int x, unsigned int y, int p)
49
50 cpx EXP(int i, int dir)
                                                              6 {
                                                                     int res = 1;
51 {
       return cpx(coss[i],sins[i]*dir);
                                                                     x = x \% p;
52
53 }
                                                                     while (y > 0)
                                                              10
55 void FFT(cpx *in,cpx *out,int step,int size,int dir)
56 €
                                                                         if (y & 1)
                                                              12
       if(size<1) return;</pre>
                                                                             res = (res * x) % p;
       if(size==1)
58
                                                              14
                                                                         y = y >> 1;
59
                                                              15
60
           out [0] = in [0];
                                                              16
                                                                         x = (x * x) \% p;
           return;
                                                              17
61
                                                                     return res;
                                                              18
       FFT(in,out,step*2,size/2,dir);
                                                              19 }
63
       FFT(in+step,out+size/2,step*2,size/2,dir);
       for(int i=0;i<size/2;++i)</pre>
                                                              21 int modInverse(int a, int p)
65
                                                              22 {
66
```

```
return power(a, p-2, p);
23
24 }
25
26 int modFact(int n, int p)
27 {
       if (p <= n)
28
           return 0;
29
30
       int res = (p - 1);
31
32
       for(int i = n + 1; i < p; i++)
33
34
           res = (res * modInverse(i, p)) % p;
35
       return res;
36 }
37
38 int main()
39 {
       int n = 25, p = 29;
40
41
       cout << modFact(n, p);</pre>
       return 0;
42
43 }
```

5.11 Kamenetsky

```
1 // Number of digits in n! O(1)
3 #define Pi 3.14159265358979311599796346854
4 #define Eul 2.71828182845904509079559829842
6 long long findDigits(int n)
7 {
      double x;
9
      if (n < 0)
10
          return 0:
11
      if (n == 1)
12
          return 1;
13
14
      x = ((n * log10(n / euler) + log10(2 * Pi * n))
15
      /2.0)):
      return floor(x) + 1;
17
18 }
```

Misc

6.1 Bitwise

a^=b;

16

```
1 // Bitwise
      unsigned char a = 5, b = 9; // a = (00000101), b 69
3
      = (00001001)
      AND -
                      a&b
                            // The result is 00000001
5
      (1)
                            // The result is 00001101
      OR -
                      alb
      (13)
      XOR -
                      a^b
                            // The result is 00001100
      (12)
                      ~a
      NOT -
                            // The result is 11111010
      (250)
                     b<<1 // The result is 00010010
      Left shift -
      (18)
      Right shift - b>>1 // The result is 00000100
10
11
      // Exchange two int variables
12
13
           a^=b;
          b^=a;
```

```
// Even or Odd
    (x & 1)? printf("Odd"): printf("Even");
// Turn on the j-th bit
    int S = 34; //(100010)
    int j = 3;
    S = S | (1 << j);
// Turn off the j-th bit
    int S = 42; //(101010)
    int j = 1;
    S &= ~(1<<j)
    S == 40 //(101000)
// Check the j-th element
    int S = 42; //(101010)
    int j = 3;
    T = S & (1 << j); // T = 0
// Exchange o j-th element
    S = (1 << j)
// Position of the first bit on
    T = (S & (-S))
    T \rightarrow 4 \text{ bit ligado } //(1000)
// Most significant digit of N
    double K = log10(N);
    K = K - floor(K);
    int X = pow(10, K);
// Number of digits in N
    X =floor(log10(N)) + 1;
// Power of two
    bool isPowerOfTwo(int x)
    {
        return x && (!(x&(x-1)));
```

Strings

7.1 $_{ m KMP}$

```
1 //KMP Algorithm
3 #include <bits/stdc++.h>
_5 // Fills lps[] for given patttern pat[0..M-1]
6 void computeLPSArray(char* pat, int M, int* lps)
7 {
      // length of the previous longest prefix suffix
9
      int len = 0;
10
      lps[0] = 0; // lps[0] is always 0
12
      // the loop calculates lps[i] for i = 1 to M-1
13
```

17

18

19

20

21

22

23

24

25

27

29

30

31

32

33

34

36

37

38

39

40

41

42

43

44

45 46

47

48

49

50

51

53

54

56

57

58

59

60

61

62

63

65

67

68

```
int i = 1;
                                                                   // Preprocess the pattern (calculate lps[] array)
14
                                                            51
15
       while (i < M) {
                                                            52
                                                                   computeLPSArray(pat, M, lps);
          if (pat[i] == pat[len]) {
16
                                                            53
                                                                   int i = 0; // index for txt[]
17
               len++;
                                                            54
               lps[i] = len;
                                                                   int j = 0; // index for pat[]
                                                            55
               i++;
                                                                   while (i < N) {
                                                            56
19
           }
                                                                       if (pat[j] == txt[i]) {
20
           else // (pat[i] != pat[len])
                                                                           j++;
21
                                                            58
                                                                           i++;
22
                                                            59
               // This is tricky. Consider the example. 60
                                                                       }
               // AAACAAAA and i = 7. The idea is
24
                                                            61
       similar
                                                            62
                                                                       if (j == M) {
               // to search step.
                                                                           printf("Found pattern at index %d ", i -
                                                            63
               if (len != 0) {
                                                                   j);
26
                   len = lps[len - 1];
                                                                            j = lps[j - 1];
27
                                                            64
                                                                       }
                                                            65
28
                   // Also, note that we do not
                                                            66
       increment
                                                                       // \  \, \text{mismatch after j matches}
                                                            67
                    // i here
                                                                       else if (i < N and pat[j] != txt[i]) {</pre>
               }
                                                                          // Do not match lps[0..lps[j-1]]
31
                                                            69
                                                                   characters,
// they will match anyway
               else // if (len == 0)
32
33
                                                            70
                    lps[i] = 0;
                                                            71
                                                                           if (j != 0)
34
                    i++;
                                                            72
                                                                               j = lps[j - 1];
               }
                                                            73
                                                                            else
36
37
           }
                                                            74
                                                                                i = i + 1;
      }
                                                                       }
38
                                                            75
                                                                   }
39 }
                                                            76
                                                            77 }
41 // Prints occurrences of txt[] in pat[]
                                                            78
42 void KMPSearch(char* pat, char* txt)
                                                            79
                                                            _{80} // Driver program to test above function
43 {
       int M = strlen(pat);
                                                            81 int main()
44
45
      int N = strlen(txt);
                                                            82 {
                                                                   char txt[] = "ABABDABACDABABCABAB";
                                                            83
46
                                                                   char pat[] = "ABABCABAB";
       // create lps[] that will hold the longest prefix 84\,
                                                                   KMPSearch(pat, txt);
       suffix
                                                            85
       // values for pattern
                                                                   return 0;
48
                                                            86
      int lps[M];
                                                            87 }
49
50
```