

# Notebook - Maratona de Programação

# Tiago de Souza Fernandes

Sumário			6	Misc		11
-	A1 *4	0		6.1	Bitwise	11
1	Algoritmos	2	7	Str	ings	11
	1.1 Iterative-BS	2	'		KMP	11
<b>2</b>	Grafos	2				- 11
	2.1 BFS	2				
	2.2 Find-bridges	2				
	2.3 Dijkstra	2				
	2.4 Floyd-Warshall	3				
	2.5 Kruskal	3				
	2.6 DFS	3				
	2.7 Represent	3				
	2.8 Prim	3				
•						
3	Geometria	4				
	3.1 Inter-Retas	4				
4	ED	4				
	4.1 Iterative-SegTree	4				
	4.2 Recursive-SegTree	5				
	4.3 Delta-Encoding	5				
	4.4 Seg-Tree-Farao	5				
	4.5 BIT-2D	6				
	4.6 BIT	6				
	4.7 Union-Find	7				
۲	Math	7				
5	Math 5.1 Linear-Diophantine-Equation	<b>7</b> 7				
	5.2 Factorization-sqrt	7				
	5.3 Modular-Exponentiation	7				
	5.4 Miller-Habin	8				
	5.5 Pollard-Rho	9				
	5.6 Verif-primo	9				
	5.7 Crivo	9				
	5.8 FFT-golfbot	9				
	5.9 Modular-Factorial	_				
	5.10 Kamanatalar	11				

## 1 Algoritmos

#### 1.1 Iterative-BS

```
1 int main()
2 {
       int l=1, r=N;
       int res=-1;
       while(1 <= r)</pre>
6
            int m = (1 + r)/2;
            if(!ver(m))
            {
10
                 1 = m+1;
11
            }
12
            else
13
            {
                 res = m:
15
16
                 r = m-1:
            }
17
18
       cout << res << endl;</pre>
19
20
       return 0;
22 }
```

#### 2 Grafos

#### 2.1 BFS

```
1 //BFS (Breadth First Search) O(V+A)
3 void BFS(int x)
       int atual, v, u;
       queue < int > fila;
       fila.push(x);
       componente[x] = valor;
9
       atual = 0;
10
       while(!fila.empty())
11
12
13
           v = fila.front();
           fila.pop();
14
           for(int i = 0;i < (int)vizinhos[v].size();i</pre>
16
       ++)
           {
               u = vizinhos[v][i];
18
                if (componente[u] == -1)
20
                    componente[u] = componente[v];
21
22
                    fila.push(u);
23
           }
24
       }
25
```

### 2.2 Find-bridges

```
#define vi vector<int>

vector< vector<int> > grafo;
vector<bool> visited;
vi t, low;
int timer=0;

void find_bridges(int v, int p=-1)
{
```

```
visited[v] = true;
10
11
       t[v] = low[v] = timer++;
       for(int i=0;i<(int)grafo[v].size();i++)</pre>
12
14
            int vert = grafo[v][i];
           if (vert == p)
15
                continue;
16
           if (visited[vert])
17
                low[v] = min(low[v], t[vert]);
18
19
           else
           {
20
21
                find_bridges(vert, v);
                low[v] = min(low[v], low[vert]);
22
                if(low[to] > t[v])
23
                    IS_BRIDGE(v, vert);
24
25
26
       }
27 }
29 int main()
30 {
       timer = 0;
31
       visited.assign(N+1, false);
32
       t.assign(N+1, 0);
       low.assign(N+1, 0);
34
35
       for(int i=0;i<N;i++)</pre>
36
           if(!visited[i])
37
38
                find_bridges(1);
39
40
       return 0;
41 }
```

#### 2.3 Dijkstra

```
1 // Dijkstra - Shortest Path
3 #define pii pair<int, int>
4 #define vi vector<int>
5 #define vii vector< pair<int,int> >
6 #define INF 0x3f3f3f3f
8 vector < vii > grafo;
9 vi distancia;
10 priority_queue < pii, vii, greater <pii>> fila;
12 void dijkstra(int k)
13 {
14
       int dist, vert, aux;
       distancia[k]=0;
16
       fila.push(mp(k, 0));
18
       while(!fila.empty())
19
       {
20
21
           aux=fila.top().f;
22
           fila.pop();
23
           for(int i=0; i<grafo[aux].size(); i++)</pre>
25
                vert=grafo[aux][i].f;
26
27
               dist=grafo[aux][i].s;
28
                if (distancia[vert]>distancia[aux]+dist)
                    distancia[vert] = distancia[aux] + dist;
30
                    fila.push(mp(vert, distancia[vert]));
32
33
           }
       }
34
35 }
37 int main()
```

```
38 €
39
       dist.assign(N+1, INF);
      grafo.assign(N+1, vii());
40
41
      for(int i=0; i<M; i++)</pre>
43
           cin >> a >> b >> p;
           grafo[a].pb(mp(b, p));
45
           grafo[b].pb(mp(a, p));
46
47
48 }
       Floyd-Warshall
1 // Floyd Warshall
```

#### 2.5 Kruskal

```
1 // Kruskal - Minimum Spanning Tree
3 typedef struct
       int A, B;
       int dist;
7 } vertice;
9 vertice grafo[MAX];
10 int pai[MAX];
12 int find(int X) // Union-Find
13 €
       if(pai[X] == X)
14
15
           return X;
16
17
           return pai[X]=find(pai[X]);
18 }
19
20 void join(int X, int Y)
21 {
       int paix = find(X);
22
       int paiy = find(Y);
23
       pai[paix]=paiy;
25 }
26
27 bool comp(vertice A, vertice B)
28 {
       return A.dist < B.dist;</pre>
29
30 }
31
32 void kruskal()
33 {
       for(int i=1;i<=N;i++)</pre>
           pai[i]=i;
35
37
       for(int i=1;i<=M;i++)</pre>
          cin >> grafo[i].A >> grafo[i].B >> grafo[i].
38
39
       sort(grafo+1, grafo+M+1, comp);
41
       for(int i=1;i<M;i++)</pre>
42
```

```
43
44
            if (find(grafo[i].A)!=find(grafo[i].B))
            {
45
                 join(grafo[i].A, grafo[i].B);
46
47
                 soma+=grafo[i].dist;
            }
48
       }
49
50
       cout << soma << endl;</pre>
51
52 }
```

#### 2.6 DFS

#### 2.7 Represent

```
_1 // Grafos
3 // List of edges
      vector < pair <int, int> > arestas;
      arestas.push_back(make_pair(1, 2));
      arestas.push_back(make_pair(1, 3));
9 // Adjacency Matrix
10
      int grafo[10][10];
11
12
       grafo[1][2] = grafo[2][1] = 1;
13
      grafo[1][3] = grafo[3][1] = 2;
14
15
16 // Adjacency List
17
18
      vector < int > vizinhos[10];
19
       vizinhos[1].push_back(2);
       vizinhos[1].push_back(2);
```

#### 2.8 Prim

```
int x, y;
       while (1)
                                                            6 } pnt;
18
19
           int davez = -1;
                                                            8 bool collinear(pnt p, pnt q, pnt r)
20
                                                            9 {
           while(!fila.empty())
                                                                   if(q.x<=max(p.x,r.x) && q.x>=min(p.x,r.x) && q.y
22
                                                            10
                                                                   <=max(p.y,r.y) && q.y>=min(p.y,r.y))
23
               int atual = fila.top().second;
                                                                       return true;
24
               fila.pop();
25
                                                            12
                                                                   return false;
                                                            13
               if (!processado[atual])
                                                            14 }
27
                    davez = atual;
29
                                                            int orientation(pnt p, pnt q, pnt r)
                   break;
                                                            17 {
30
               }
31
                                                            18
                                                                   int val=(q.y-p.y)*(r.x-q.x)-(q.x-p.x)*(r.y-q.y);
           }
32
                                                            19
                                                            20
                                                                   if(val==0)
           if(davez == -1)
                                                                       return 0;
34
                                                            21
               break:
                                                                   else if(val>0)
36
                                                            23
                                                                       return 1;
           processado[davez] = true;
                                                                   else
                                                            24
37
                                                                       return 2;
           for(int i = 0;i < (int)vizinhos[davez].size() 26 }</pre>
39
       ;i++)
           {
                                                            28 bool intersect(pnt p1, pnt q1, pnt p2, pnt q2)
40
                                                            29 {
41
               int dist = vizinhos[davez][i].first;
                                                                   int o1 = orientation(p1, q1, p2);
42
                                                            30
                                                                   int o2 = orientation(p1, q1, q2);
               int atual = vizinhos[davez][i].second;
43
                                                            31
                                                                   int o3 = orientation(p2, q2, p1);
                                                            32
               if( distancia[atual] > dist && !
                                                                   int o4 = orientation(p2, q2, q1);
45
                                                            33
      processado[atual])
                                                            34
                                                                   if(o1!=o2 \text{ and } o3!=o4)
46
               {
                                                            35
                    distancia[atual] = dist;
                                                                       return true;
47
                                                            36
                   fila.push( pii(distancia[atual],
      atual));
                                                                   if(o1==0 && collinear(p1, p2, q1))
                                                            38
                                                                       return true;
           }
50
                                                            40
                                                                   if (o2==0 && collinear(p1, q2, q1))
51
                                                            41
                                                            42
                                                                       return true;
      int custo_arvore = 0;
53
                                                            43
54
       for(int i = 1; i <= n; i++)</pre>
                                                            44
                                                                   if(o3==0 && collinear(p2, p1, q2))
           custo_arvore += distancia[i];
                                                                       return true;
55
                                                            45
56
                                                            46
57
      return custo_arvore;
                                                            47
                                                                   if(o4==0 && collinear(p2, q1, q2))
58 }
                                                                       return true;
                                                            48
59
                                                            49
60 int main(){
                                                                   return false;
                                                            50
                                                            51
                                                            52 }
      cin >> n >> m;
62
63
      for(int i = 1;i <= m;i++){</pre>
                                                                    ED
64
                                                              4
65
           int x, y, tempo;
                                                                     Iterative-SegTree
           cin >> x >> y >> tempo;
                                                              4.1
67
68
           vizinhos[x].pb( pii(tempo, y) );
                                                            1 // Segment Tree Iterativa - Range maximum query
69
           vizinhos[y].pb( pii(tempo, x) );
70
71
                                                             3 #define N 100010
72
      cout << Prim() << endl;</pre>
                                                             5 struct Segtree
74
                                                             6 {
75
      return 0;
                                                                   int t[2*N]={0};
76 }
                                                             9
                                                                   void build()
  3
       Geometria
                                                                   {
                                                            10
                                                                       for(int i=N-1; i>0; i--)
                                                            11
                                                                           t[i]=max(t[i<<1], t[1<<1|1]);
                                                            12
  3.1 Inter-Retas
                                                            13
                                                            14
1 // Intersection between lines
                                                                   int query(int 1, int r)
                                                            15
                                                            16
```

17

18

int ans=0:

for (i+=N, r+=N; l< r; l>>=1, r>>=1)

3 typedef struct

```
{
                                                                           atualiza(esq, i, meio, pos, novo_valor);
19
                                                            36
20
               if (1&1)
                                                            37
                                                                       else
                   ans=max(ans, t[1++]);
                                                                           atualiza(dir, meio+1, j, pos, novo_valor)
21
                                                           38
               if (r&1)
                   ans=max(ans, t[--r]);
           }
                                                                       if (val[esq]>val[dir])
24
                                                           40
                                                                           val[no]=val[esq];
                                                            41
           return ans:
26
                                                           42
                                                                           val[no]=val[dir];
27
                                                           43
                                                           44
                                                                   }
       void update(int p, int value)
                                                           45 }
29
30
                                                           47 int consulta(int no, int i, int j, int A, int B)
           for(t[p+=n]=value; p>1; p>>=1)
31
               t[p>>1] = max(t[p], t[p^1]);
                                                           48 {
32
       7
33
                                                           49
                                                                   if(i>B || j<A)</pre>
                                                                       return -1;
34
                                                           50
35 };
                                                            51
                                                                   if(i>=A and j<=B)
                                                                       return val[no];
36
                                                           52
37 int main()
                                                            53
                                                                   int esq = 2*no;
38 {
                                                           54
       Segtree st;
                                                                   int dir = 2*no+1;
39
                                                           55
40
                                                           56
                                                                   int meio = (i+j)/2;
       for(int i=0;i<n;i++)</pre>
                                                           57
41
                                                                   int resp_esq = consulta(esq, i, meio, A, B);
                                                                   int resp_dir = consulta(dir, meio+1, j, A, B);
           cin >> aux:
43
                                                           59
          st.t[N+i] = aux; //Leaves are stored in
44
                                                           60
       continuous nodes with indices starting with N
                                                           61
                                                                   if (resp_dir==-1)
                                                                       return resp_esq;
45
                                                           62
                                                                   if(resp_esq==-1)
46
                                                            63
      st.build();
                                                                      return resp_dir;
47
                                                           64
      x = st.query(inicio, fim);
                                                           65
48
       st.update(ind, value);
                                                                   if (resp_esq>resp_dir)
49
                                                           66
                                                           67
                                                                      return resp_esq;
50
51 }
                                                           68
                                                                       return resp_dir;
                                                           69
  4.2 Recursive-SegTree
                                                           70 }
                                                           71
1 // Segment Tree Recursiva - Range maximum query
                                                           72 int main()
                                                           73 {
                                                                   monta(1, N, 1);
                                                           74
3 vector<int> val(MAX, 0);
4 vector < int > vet(N);
                                                           75
                                                                   atualiza(1, 1, N, pos, valor);
                                                                   x = consulta(1, 1, N, inicio, fim);
                                                           76
                                                           77
6 void monta(int i, int j, int no)
                                                           78 }
       if(i==j)
                                                                     Delta-Encoding
9
       {
10
           val[no]=vet[i];
           return;
                                                            1 // Delta encoding
12
13
                                                            3 for(int i=0;i<q;i++)</pre>
      int esq = 2*no;
14
                                                            4 {
       int dir = 2*no+1;
                                                                   int 1, r, x;
      int meio = (i+j)/2;
16
                                                                   cin >> 1 >> r >> x;
17
                                                                   delta[1] += x;
                                                            7
      monta(i, meio, esq);
18
                                                                   delta[r+1] -= x;
                                                            8
      monta(meio+1, j, dir);
19
                                                            9 }
20
                                                           10
       val[no]=max(val[esq], val[dir]);
21
                                                           11 int atual = 0;
22 }
23
                                                            13 for(int i=0;i<n;i++)</pre>
24 void atualiza(int no, int i, int j, int pos, int
                                                           14 {
      novo_valor)
                                                            15
                                                                   atual += delta[i];
25 {
                                                            16
                                                                   v[i] += atual;
26
       if(i==j)
                                                            17 }
27
           val[no]=novo_valor;
                                                              4.4 Seg-Tree-Farao
      }else
29
30
           int esq = 2*no;
                                                            1 typedef struct
           int dir = 2*no+1;
                                                            2 {
32
           int meio = (i+j)/2;
                                                                  pii prefix, sufix, total, maximo;
```

34

if (pos <= meio)</pre>

4 } no;

```
6 int noleft[MAX], noright[MAX]; //Guarda os valores
                                                                     dir)):
       dos nos para que nao sejam calculados novamente
      nas querys
7 int v[MAX];
                                                            74 int main()
8 no arvore[MAX];
                                                            75 {
                                                                    int T, N, Q, A, B;
9
                                                            76
10 pii somar(pii a, pii b) // une pairs
                                                            77
                                                                    no aux;
11 {
                                                            78
       return mp(a.f+b.f, a.s+b.s);
                                                                    scanf("%d", &T);
12
                                                            79
13 }
                                                            80
                                                                    while (T--)
14
                                                            81
15 no
     une(no 1, no r)
                                                            82
                                                                        scanf("%d", &N);
16 {
                                                            83
                                                                        for(int i=1;i<=N;i++)</pre>
       if(1.total.s==0)
17
                                                            84
18
           return r;
                                                            85
                                                                            scanf("%d", &v[i]); //Elementos da arvore
       if(r.total.s==0)
19
                                                            86
20
           return 1;
                                                            87
                                                                        noleft[1]=1; noright[1]=N;
                                                                        monta(1);
21
                                                            88
                                                                        cin >> 0:
23
                                                            90
      m.prefix = max(l.prefix, somar(l.total, r.prefix) 91
                                                                        while (Q--)
24
      ); //prefixo
                                                                        {
      m.sufix = max(r.sufix, somar(r.total, l.sufix)); 93
                                                                            scanf("%d%d", &A, &B); //Intervalo da
25
       //sufixo
      m.total = somar(1.total, r.total); //Soma de
                                                                            aux = busca(1, A, B);
26
                                                            94
       todos os elementos da subarvore
                                                                            printf("%d %d\n", aux.maximo.f, aux.
                                                             95
      m.maximo = max(max(1.maximo, r.maximo), somar(1.
                                                                   maximo.s);
       sufix, r.prefix)); //Resultado para cada
                                                                        }
                                                            96
       subarvore
                                                            97
                                                                   }
28
                                                            98
29
       return m;
                                                            99
30 }
                                                            100
                                                                    return 0;
                                                            101
31
32 no makenozero()
                                                               4.5
                                                                      BIT-2D
33 {
34
      m.prefix=m.sufix=m.total=m.maximo=mp(0,0);
35
                                                             1 // BIT 2D
       return m;
36
37 }
                                                             3 int bit[MAX][MAX];
38
                                                             4
39 no makeno(int k)
                                                             5 int sum(int x, int y)
40 {
                                                             6 {
41
                                                                    int resp=0;
42
      m.prefix=m.sufix=m.total=m.maximo=mp(k,1);
       return m;
43
                                                                    for(int i=x;i>0;i-=i&-i)
                                                             9
44 }
                                                                        for(int j=y;j>0;j-=j&-j)
                                                             10
45
                                                                            resp+=bit[i][j];
46 void monta(int n)
                                                             12
47 {
                                                            13
                                                                    return resp;
       if(noleft[n] == noright[n])
48
                                                            14 }
49
                                                            15
           arvore[n]=makeno(v[noleft[n]]);
50
                                                            16 void update(int x, int y, int delta)
                                                            17 {
       }
                                                                    for(int i=x;i<MAX;i+=i&-i)</pre>
                                                            18
                                                            19
                                                                        for (int j=y; j < MAX; j+=j&-j)</pre>
       int mid = (noleft[n]+noright[n])/2;
54
                                                                            bit[i][j]+=delta;
                                                            20
       noleft[2*n]=noleft[n]; noright[2*n]=mid;
                                                            21 }
       noleft[2*n+1]=mid+1; noright[2*n+1]=noright[n];
56
                                                            22
57
                                                            23 int query(int x1, y1, x2, y2)
       monta(2*n);
58
                                                            24 {
59
       monta(2*n+1):
                                                                    return sum(x2,y2) - sum(x2,y1) - sum(x1,y2) + sum
60
                                                                    (x1,y1);
       arvore[n]=une(arvore[2*n], arvore[2*n+1]);
61
                                                            26 }
62 }
                                                               4.6 BIT
64 no busca(int n, int esq, int dir)
65 {
66
       if(noleft[n]>=esq and noright[n]<=dir)</pre>
                                                             1 // (BIT) Fenwick Tree
           return arvore[n];
67
       if(noright[n] < esq or noleft[n] > dir)
68
                                                             3 int bit[MAX]:
           return makenozero();
69
70
                                                             5 int soma(int x)
       return une(busca(2*n, esq, dir),busca(2*n+1, esq, _6 {
71
```

```
int resp=0;
7
      // for(int i=x;i>0;i-=i&-i)
9
      // resp+=bit[i];
10
11
      while (x > 0)
12
13
          resp += bit[x];
14
           x = (x & -x);
15
16
17
18
       return resp;
19 }
20
21 int query(int L, R)
22 {
23
       return soma(R)-soma(L);
24 }
26 void update(int x, int v)
27 {
       // for(;x<=n;x+=x&-x)
              bit[x] += v;
29
      while(x <= N)</pre>
31
32
           bit[x] += v;
33
           x += (x \& -x);
34
35
36 }
```

#### 4.7 Union-Find

```
1 // Union-Find Functions
3 int pai[MAX], peso[MAX];
5 int find(int aux)
       if(pai[aux] == aux)
          return aux;
9
          return pai[aux]=find(pai[aux], pai);
11 }
13 void join(int x, int y)
14 {
15
       x = find(x);
      y = find(y);
16
17
      if(pesos[x]<pesos[y])</pre>
18
          pai[x] = y;
19
       else if(pesos[x]>pesos[y])
20
         pai[y] = x;
21
       else if(pesos[x] == pesos[y])
23
24
           pai[x] = y;
25
           pesos[y]++;
26
27 }
29 int main()
30 €
       for(int i=1;i<=N;i++)</pre>
31
32
           pai[i]=i;
33 }
```

#### 5 Math

#### 5.1 Linear-Diophantine-Equation

```
1 // Linear Diophantine Equation
2 int gcd(int a, int b, int &x, int &y)
3 {
       if (a == 0)
4
           x = 0; y = 1;
6
           return b;
 8
      int x1, y1;
9
      int d = gcd(b%a, a, x1, y1);
10
       x = y1 - (b / a) * x1;
11
12
       y = x1;
       return d;
13
14 }
15
16 bool find_any_solution(int a, int b, int c, int &x0,
       int &y0, int &g)
17 €
18
       g = gcd(abs(a), abs(b), x0, y0);
       if (c % g)
19
           return false;
20
21
      x0 *= c / g;
22
      y0 *= c / g;
       if (a < 0) x0 = -x0;
24
       if (b < 0) y0 = -y0;
25
26
       return true;
27 }
_{29} // All solutions
_{30} // x = x0 + k*b/g

_{31} // y = y0 - k*a/g
        Factorization-sqrt
```

```
1 // Factorization of a number in sqrt(n)
3 int main()
4 {
       11 N:
5
       vector < int > div;
       cin >> N;
       for(11 i=2;i*i<=N;i++)</pre>
10
11
            if(N\%i==0)
12
                vet.pb(i);
14
                while (N\%i==0)
15
16
                    N/=i:
17
18
       if (N!=1)
19
           vet.pb(N);
20
21
       return 0;
22
23 }
```

#### 5.3 Modular-Exponentiation

```
y = y >> 1;
                                                                        while(d == 1);
12
                                                             65
13
           x=(x*x)\mbox{\em mod};
                                                             66
      7
                                                                    while(d == n);
14
                                                             67
15
      return res;
                                                             68
16 }
                                                             69
                                                                    return d;
                                                            70 }
  5.4 Miller-Habin
                                                             72 ll fexp(ll a, ll b, ll c)
                                                             73 {
1 #include <bits/stdc++.h>
                                                             74
                                                                    11 \text{ ans} = 1;
2 #define mod 1000000007
                                                                    while(b)
3 #define Pi 3.14159265358979311599796346854
                                                             75
4 #define INF 0x3f3f3f3f
                                                             76
                                                                    {
                                                                        if(b & 1)
5 #define MAX 1000010
                                                             77
                                                                           ans = mul(ans, a, c);
                                                             78
6 #define f first
                                                                        a = mul(a, a, c);
7 #define s second
                                                             79
                                                                        b /= 2;
                                                             80
8 #define 11 long long
9 #define pb push_back
                                                             81
                                                                    }
                                                                    return ans;
10 #define mp make_pair
                                                             82
11 #define pii pair <int, int>
                                                            83 }
12 #define vi vector<int>
                                                             84
                                                             85 bool rabin(ll n)
13 #define vii vector< pii >
14 #define sws ios_base::sync_with_stdio(false);cin.tie(86 {
                                                                    if (n <= 1)
      NULL)
                                                            87
                                                                        return 1;
15 #define forn(i, n) for(int i=0; i<(int)(n); i++)
                                                                    if(n <= 3)</pre>
16 #define mdc(a, b) (__gcd((a), (b)))
                                                             89
17 #define mmc(a, b) (((a)/_{gcd}(a, b)) * b)
                                                                       return 1;
                                                             90
18 #define endl '\n'
                                                             91
                                                                    ll s=0, d=n-1;
19 #define teto(a, b) (a+b-1)/b
                                                             92
                                                             93
                                                                    while (d\%2==0)
                                                                    {
21 using namespace std;
                                                             94
                                                                        d/=2;
                                                             95
23 ll llrand()
                                                             96
                                                                        s++;
                                                            97
24 {
      11 tmp = rand();
                                                             98
25
                                                                    for(int k = 0; k < 64*4; k++)
                                                            99
      return (tmp << 31) | rand();</pre>
                                                            100
27 }
                                                                        11 a = (11rand()\%(n - 3)) + 2;
                                                                        ll x = fexp(a, d, n);
29 ll add(ll a, ll b, ll c)
                                                            102
                                                                        if(x != 1 and x != n-1)
                                                            103
30 €
                                                                        {
                                                            104
31
       return (a + b)%c;
32 }
                                                            105
                                                                             for(int r = 1; r < s; r++)</pre>
                                                            106
                                                                                 x = mul(x, x, n);
34 ll mul(ll a, ll b, ll c)
                                                            107
                                                            108
                                                                                 if(x == 1)
35 {
                                                                                     return 0;
                                                            109
36
       11 ans = 0:
       while(b)
                                                            110
                                                                                 if(x == n-1)
37
                                                                                     break:
38
           if(b & 1)
39
                                                                             if(x != n-1)
40
              ans = add(ans, a, c);
                                                            113
                                                            114
                                                                                 return 0;
           a = add(a, a, c);
41
                                                                        }
           b /= 2;
                                                            115
42
       }
                                                            116
43
       return ans;
                                                            117
44
                                                                    return 1;
45 }
                                                            118
                                                            119 }
46
                                                            120
47 ll rho(ll n)
                                                            121
48 {
                                                            122 int main()
       ll x, c, y, d, k;
49
       int i;
                                                            123 {
50
                                                            124
51
       do{
                                                                    //freopen("input.txt", "r", stdin);
           i = 1;
                                                            125
52
                                                            126
                                                                    //freopen("output.txt", "w", stdout);
           x = llrand()%n;
53
           c = llrand()%n;
                                                            127
54
                                                                    11 N, resp;
                                                            128
55
           y = x, k = 4;
                                                            129
                                                                    vector<1l> div;
           do{
56
               if(++i == k)
                                                            130
57
                                                            131
                                                                    cin >> N;
58
                {
                    y = x;
                                                            132
                                                                    resp = N;
59
                                                            133
                    k *= 2;
                                                            134
                                                                    while(N>1 and !rabin(N))
               }
61
               x = add(mul(x, x, n), c, n);
                                                            135
                                                                        11 d = rho(N);
                                                            136
               d = \_\_gcd(abs(x - y), n);
63
           }
                                                            137
                                                                        if(!rabin(d))
64
```

```
continue;
138
                                                              49
139
            div.pb(d);
                                                              50
                                                                          while(d == 1);
            while (N\%d==0)
140
                                                              51
               N/=d;
                                                                     while(d == n);
141
                                                              52
       if(N!=resp and N!=1)
                                                                     return d;
                                                              54
143
           div.pb(N);
                                                              55 }
145
                                                              56
                                                              57 int main()
146
        if(div.empty())
                                                              58 {
           cout << resp << endl;</pre>
                                                                      srand(time(0));
                                                              59
148
149
                                                              60
                                                                     11 N;
150
                                                              61
            for(int i=0;i<(int)div.size();i++)</pre>
                                                                     cin >> N;
                                                              62
151
               resp = __gcd(resp, div[i]);
152
                                                              63
                                                                     11 \text{ div} = \text{rho}(N);
153
                                                              64
                                                                     cout << div << " " << N/div << endl;
            cout << resp << endl;</pre>
                                                              65
                                                              66
                                                                     // Finding all divisors
       return 0;
157
                                                              68
158
                                                              69
                                                                     vector<ll> div;
159 }
                                                              70
                                                              71
   5.5 Pollard-Rho
                                                                     while(N>1 and !rabin(N))
                                                              73
                                                              74
                                                                          11 d = rho(N);
 1 // Pollard Rho Algorithm
                                                              75
                                                                          if(!rabin(d))
                                                                              continue;
 3 #include <bits/stdc++.h>
                                                              76
                                                                          div.pb(d);
 4 #define ll long long
                                                                          while (N\%d==0)
                                                              78
                                                                              N/=d;
                                                              79
 6 using namespace std;
                                                              80
                                                                     if (N!=resp and N!=1)
                                                              81
 8 11 11rand()
                                                              82
                                                                          div.pb(N);
       11 tmp = rand();
                                                              83
                                                                      return 0;
        return (tmp << 31) | rand();</pre>
                                                              84
11
                                                              85
                                                              86 }
14 ll add(ll a, ll b, ll c)
                                                                       Verif-primo
                                                                 5.6
       return (a + b)%c;
16
 17 }
                                                               1 // prime verification sqrt(N)
19 ll mul(ll a, ll b, ll c)
                                                               3 bool eh_primo(long long N)
20 {
                                                               4 {
       11 \text{ ans} = 0;
                                                                     if(N==2)
21
                                                               5
                                                                         return true;
22
       while(b)
                                                                     else if (N==1 \text{ or } N\%2==0)
23
            if(b & 1)
                                                                         return false;
                ans = add(ans, a, c);
                                                                     for(long long i=3;i*i<=N;i+=2)</pre>
                                                               9
                                                                       if(N\%i==0)
            a = add(a, a, c);
                                                              10
26
            b /= 2;
                                                              11
                                                                              return false;
       }
                                                                     return true;
28
                                                              12
       return ans;
30 }
                                                                 5.7 Crivo
31
32 ll rho(ll n)
33 {
                                                               1 // Sieve of Eratosthenes
        ll x, c, y, d, k;
35
        int i;
                                                               3 int N;
        do{
36
                                                               4 vector <bool > primos (100010, true);
            i = 1;
37
                                                               5 cin >> N;
           x = llrand()%n;
38
39
           c = llrand()%n;
                                                               7 primos [0] = false;
            y = x, k = 4;
40
                                                               8 primos[1]=false;
            do{
41
                if(++i == k)
42
                                                              10 for(int i=2;i<=N;i++)</pre>
43
                                                                     if(primos[i])
                                                              11
                     y = x;
                                                                          for(int j=i+i; j<=N; j+=i)</pre>
                                                              12
                     k *= 2;
45
                                                                              primos[j]=false;
                                                                      FFT-golfbot
                                                                 5.8
                x = add(mul(x, x, n), c, n);
47
```

 $d = \_gcd(abs(x - y), n);$ 

```
#include <bits/stdc++.h>
                                                              73
                                                              74 int main()
                                                              75 {
3 using namespace std;
                                                              76
                                                                      for(int i=0;i<=N;++i)</pre>
5 \text{ const int } N = (1 << 19);
                                                              77
6 const double two_pi = 4 * acos(0);
                                                                          coss[i]=cos(two_pi*i/N);
                                                              78
                                                                          sins[i]=sin(two_pi*i/N);
                                                              79
8 struct cpx
                                                              80
                                                                     while(cin >> n) // Numero de tacadas possiveis
9 {
                                                              81
       cpx(){}
                                                                     {
10
                                                              82
       cpx(double aa): a(aa){}
                                                                          fill(x,x+N+100,0);
11
                                                              83
       cpx(double aa, double bb):a(aa),b(bb){}
                                                                          fill(a,a+N+100,0);
                                                                          for(int i=0;i<n;++i)</pre>
       double a;
                                                              85
       double b:
                                                              86
14
       double modsq(void) const
                                                                              cin >> p; // Distancia das tacadas
15
                                                              87
                                                                              x[p]=1;
16
                                                              88
17
           return a*a+b*b;
                                                              89
                                                                          }
       }
                                                                          for (int i=0;i<N+100;++i)</pre>
18
                                                              90
       cpx bar(void) const
19
                                                                              b[i]=cpx(x[i],0);
20
       {
                                                              92
           return cpx(a,-b);
21
                                                              93
                                                                          cin >> m; // Querys
       }
                                                              94
22
23 };
                                                                          for (int i=0; i < m; ++i)</pre>
                                                              95
                                                                              cin >> a[i]; // Distancia da query
25 cpx b[N+100];
                                                              97
26 cpx c[N+100];
                                                              98
                                                                          FFT(b,B,1,N,1);
27 cpx B[N+100];
                                                              99
28 cpx C[N+100];
                                                                          for (int i=0; i < N; ++i)</pre>
                                                             100
29 int a[N+100];
                                                                              C[i]=B[i]*B[i];
                                                             101
                                                                          FFT(C,c,1,N,-1);
30 int x[N+100];
31 double coss[N+100], sins[N+100];
                                                                          for (int i=0; i < N; ++i)</pre>
                                                                              c[i]=c[i]/N;
32 int n,m,p;
                                                             104
                                                                          int cnt=0;
33
34 cpx operator +(cpx a,cpx b)
                                                             106
                                                                          for(int i=0;i<m;++i)</pre>
                                                                              if(c[a[i]].a>0.5 || x[a[i]])
35 {
                                                             107
       return cpx(a.a+b.a,a.b+b.b);
36
                                                             108
                                                                                   cnt++;
                                                                          cout << cnt << endl;</pre>
37 }
                                                             109
                                                             110
38
39 cpx operator *(cpx a,cpx b)
                                                                     return 0;
                                                             112 }
40 €
41
       return cpx(a.a*b.a-a.b*b.b,a.a*b.b+a.b*b.a);
                                                                       Modular-Factorial
                                                                 5.9
42 }
43
44 cpx operator /(cpx a,cpx b)
                                                               1 // C++ program to comput n! % p using Wilson's
45 {
                                                                     Theorem
       cpx r = a*b.bar();
46
                                                               2 #include <bits/stdc++.h>
       return cpx(r.a/b.modsq(),r.b/b.modsq());
47
                                                               3 using namespace std;
48 }
49
                                                               5 int power(int x, unsigned int y, int p)
50 cpx EXP(int i,int dir)
                                                               6 {
51 {
                                                                     int res = 1;
       return cpx(coss[i],sins[i]*dir);
52
                                                                     x = x \% p;
53 }
                                                                     while (y > 0)
54
55 void FFT(cpx *in,cpx *out,int step,int size,int dir)
56 €
                                                                          if (y & 1)
       if(size<1) return;</pre>
57
                                                                              res = (res * x) % p;
                                                              13
       if(size==1)
58
                                                              14
59
       {
                                                                          y = y >> 1;
           out[0]=in[0];
60
                                                                          x = (x * x) \% p;
                                                              16
61
           return;
                                                                     }
                                                              17
62
                                                              18
                                                                      return res;
       FFT(in,out,step*2,size/2,dir);
63
                                                              19 }
       FFT(in+step,out+size/2,step*2,size/2,dir);
64
                                                              20
65
       for(int i=0;i<size/2;++i)</pre>
                                                              21 int modInverse(int a, int p)
66
       {
                                                              22 {
           cpx even=out[i];
67
                                                              23
                                                                      return power(a, p-2, p);
           cpx odd=out[i+size/2];
68
                                                              24 }
           out[i] = even+EXP(i*step,dir)*odd;
69
           out[i+size/2] = even+EXP((i+size/2)*step,dir)* 26 int modFact(int n, int p)
70
       odd:
                                                              27 {
71
       }
                                                                      if (p \ll n)
                                                              28
72 }
                                                                          return 0;
                                                              29
```

```
30
31
       int res = (p - 1);
32
       for(int i = n + 1; i < p; i++)</pre>
33
34
           res = (res * modInverse(i, p)) % p;
       return res:
35
36 }
37
38 int main()
39 {
       int n = 25, p = 29;
40
       cout << modFact(n, p);</pre>
41
42
       return 0;
43 }
```

#### 5.10 Kamenetsky

```
1 // Number of digits in n! O(1)
3 #define Pi 3.14159265358979311599796346854
4 #define Eul 2.71828182845904509079559829842
6 long long findDigits(int n)
7 {
      double x;
8
9
      if (n < 0)
10
          return 0;
11
      if (n == 1)
          return 1;
13
14
      x = ((n * log10(n / euler) + log10(2 * Pi * n))
15
      /2.0));
16
      return floor(x) + 1;
17
18 }
```

## 6 Misc

#### 6.1 Bitwise

```
1 // Bitwise
      unsigned char a = 5, b = 9; // a = (00000101), b
3
      = (00001001)
      AND -
                      a&b
                            // The result is 00000001
      (1)
                            // The result is 00001101
      OR -
                      alb
6
      (13)
      XOR -
                      a^b
                            // The result is 00001100
      (12)
                            // The result is 11111010
      NOT -
                      ~ a
      (250)
                      b<<1 // The result is 00010010
      Left shift -
      (18)
      Right shift - b>>1 // The result is 00000100
      // Exchange two int variables
13
14
          a^=b;
          b^=a;
15
          a^=b;
16
17
      // Even or Odd
18
19
           (x & 1)? printf("Odd"): printf("Even");
20
      // Turn on the j-th bit
22
23
```

```
int S = 34; //(100010)
    int j = 3;
    S = S | (1 << j);
// Turn off the j-th bit
    int S = 42; //(101010)
    int j = 1;
    S \&= (1 << j)
    S == 40 //(101000)
// Check the j-th element
    int S = 42; //(101010)
    int j = 3;
    T = S & (1 << j); // T = 0
// Exchange o j-th element
    S ^= (1<<j)
// Position of the first bit on
    T = (S & (-S))
    T \rightarrow 4 \text{ bit ligado } //(1000)
// Most significant digit of N
    double K = log10(N);
    K = K - floor(K);
    int X = pow(10, K);
// Number of digits in N
    X = floor(log10(N)) + 1;
// Power of two
    bool isPowerOfTwo(int x)
    {
        return x && (!(x&(x-1)));
```

## 7 Strings

24

25

26

27

29

31

32

33

34

35

36

37

38

39 40

41

43

44

45 46

47

48

49

50

51

53

54

56

57

58

59

60

61

62

63 64

65

68

#### 7.1 KMP

```
1 //KMP Algorithm
3 #include <bits/stdc++.h>
5 // Fills lps[] for given patttern pat[0..M-1]
6 void computeLPSArray(char* pat, int M, int* lps)
7 {
      // length of the previous longest prefix suffix
8
9
      int len = 0;
10
      lps[0] = 0; // lps[0] is always 0
      // the loop calculates lps[i] for i = 1 to M-1
13
14
      int i = 1;
      while (i < M) {
15
          if (pat[i] == pat[len]) {
               len++:
17
               lps[i] = len;
19
               i++;
           }
20
```

```
int i = 0; // index for txt[]
int j = 0; // index for pat[]
           else // (pat[i] != pat[len])
                                                              54
21
22
                                                              55
                // This is tricky. Consider the example. {\it 56}
                                                                     while (i < N) {
23
               // AAACAAAA and i = 7. The idea is
                                                                         if (pat[j] == txt[i]) {
                                                             57
24
       similar
                                                                             j++;
                // to search step.
                                                                              i++;
                                                              59
25
                                                                         }
                if (len != 0) {
                                                              60
26
                    len = lps[len - 1];
27
                                                              61
                                                                         if (j == M) {
                                                              62
28
                    // Also, note that we do not
                                                                              printf("Found pattern at index %d ", i -
                                                              63
       increment
                                                                     j);
                    // i here
30
                                                              64
                                                                              j = lps[j - 1];
                                                                         }
                }
31
                                                              65
                else // if (len == 0)
                                                              66
32
                                                                          // \  \, {\tt mismatch \  \, after \  \, j \  \, matches}
33
                                                              67
                    lps[i] = 0;
                                                                         else if (i < N and pat[j] != txt[i]) {
                                                              68
34
                                                                             // Do not match lps[0..lps[j-1]]
35
                    i++;
                                                              69
                }
                                                                     characters,
36
           }
                                                                             // they will match anyway
       }
                                                                              if (j != 0)
38
                                                              71
39 }
                                                              72
                                                                                  j = lps[j - 1];
40
                                                              73
                                                                                  i = i + 1;
41 // Prints occurrences of txt[] in pat[]
                                                              74
42 void KMPSearch(char* pat, char* txt)
                                                              75
                                                                         }
                                                                     }
43 {
                                                              76
                                                              77 }
44
       int M = strlen(pat);
       int N = strlen(txt);
45
46
       // create lps[] that will hold the longest prefix 80 // Driver program to test above function
                                                             81 int main()
       suffix
       // values for pattern
48
                                                                     char txt[] = "ABABDABACDABABCABAB";
       int lps[M];
49
                                                              83
50
                                                              84
                                                                     char pat[] = "ABABCABAB";
       // Preprocess the pattern (calculate lps[] array) 85
                                                                     KMPSearch(pat, txt);
       computeLPSArray(pat, M, lps);
                                                                     return 0;
52
                                                              86
                                                              87 }
```