UNIDAD CENTRAL DEL VALLE DEL CAUCA
FACULTAD DE INGENIERÍA
PROGRAMAS: INGENIERÍA BIOMÉDICA, INGENIERÍA ELECTRÓNICA, INGENIERÍA DE SISTEMAS
ASIGNATURA: ANÁLISIS NUMÉRICO, Sem 2020 02
TALLER No 4: Interpolación y Aproximación polinomial

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1. Lagrange polynomials

1. For the given functions $f(x)$, let $x_0 = 0$, $x_1 = 0.6$ $x_2 = 0.9$. Construct interpolation polynomials of degree at most one and at most two to approximate $f(0.45)$, and find the absolute error.

   a. $f(x) = \cos x$ b. $f(x) = \ln(x+1)$ c. $f(x) = \tan x$

2. Construct the Lagrange interpolating polynomials for the following functions, and find a bound for the absolute error on the interval $[x_0, x_n]$.

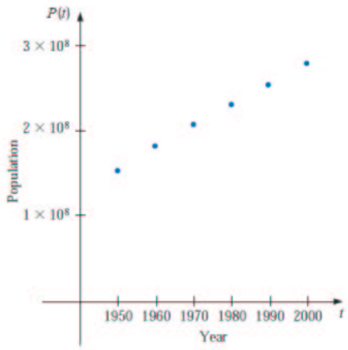
   a. $f(x) = e^{2x}\cos 3x$, $x_0 = 0$, $x_1 = 0.3$, $x_2 = 0.6$, $n = 2$
   b. $f(x) = \sin(\ln x)$, $x_0 = 2.0$, $x_1 = 2.4$, $x_2 = 2.6$, $n = 2$
   c. $f(x) = \cos x + \sin x$, $x_0 = 0$, $x_1 = 0.25$, $x_2 = 0.5$, $x_3 = 1.0$, $n = 2$

3. Sea $f(x) = e^x$, para $0 \leq x \leq 2$.

   a. Approximate $f(0.25)$ using linear interpolation with $x_0 = 0$, y $x_1 = 0.5$.
   b. Approximate $f(0.75)$ using linear interpolation with $x_0 = 0.5$, y $x_1 = 1$.
   c. Approximate $f(0.25)$ y $f(0.75)$ by using the second interpolating polynomial with $x_0 = 0$, $x_1 = 1$ y $x_2 = 2$.
   d. Which approximations are better and why?

4. A census of the population of the United States is taken every 10 years. The following table lists the population, in thousands of people, from 1950 to 2000, and the data are also represented in the figure.

Year	1950	1960	1970	1980	1990	2000
Population(in thousands)	151.326	179.323	203.302	226.542	249.633	281.422



a. Use Lagrange interpolation to approximate the population in the years, 1940, 1975 y 2020.

b. The population in 1940 was approximately 132.165.000. How accurate do you think your 1975 and 2020 figures are?

5. The error function defined by

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

gives the probability that any one of a series of trials will lie within x units of the mean, assuming that the trials have a normal distribution with mean 0 and standard deviation $\frac{\sqrt{2}}{2}$. This integral cannot be evaluated in terms of elementary functions, so an approximating technique must be used.

a. Integrate the Maclaurin series for e^{-t^2} to show that

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)k!}$$

b. Use the Maclaurin series to construct a table for $\text{erf}(x)$ that is accurate to within 10^{-4} for $\text{erf}(x_i)$, where $x_i = 0.2i$, for $i = 0, 1, \cdots, 5$

c. Use both linear interpolation and quadratic interpolation to obtain an approximation to $\text{erf}(\frac{1}{3})$. Which approach seems most feasible?

2. Divided Differences

6. Given

$$\begin{aligned} P_n(x) = & f[x_0] + f[x_0, x_1](x - x_0) + a_2(x - x_0)(x - x_1) \\ & + a_3(x - x_0)(x - x_1)(x - x_2) + \cdots \\ & + a_n(x - x_0). \dots .(x - x_{n-1}) \end{aligned}$$

Use $P_n(x_2)$ to show that $a_2 = f[x_0, x_1, x_2]$

7. Use Newton's Divided-Difference formula, to construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specified value using each of the polynomials.

a. $f(8.4)$ Si

x	8.1	8.3	8.6	8.7
$f(x)$	16.94410	17.56492	18.50515	18.82091

b. $f(0.9)$ Si

x	0.6	0.7	0.8	1.0
$f(x)$	-0.17694460	0.01375227	0.22363362	0.65809197

8. Use Newton the forward-difference formula to construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specified value using each of the polynomials..

a. $f(-\frac{1}{3})$ Si

b. $f(0.25)$ Si

x	0.1	0.2	0.3	0.4
$f(x)$	-0.62049958	0.28398668	0.00660095	0.24822440

9. Use the Newton backward-difference formula to construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specified value using each of the polynomials.

   a. $f(-\frac{1}{3})$ Si

x	-0.75	-0.5	-0.25	0
$f(x)$	-0.07181250	0.02475000	0.33493750	1.10100000

   b. $f(0.25)$ Si

x	0.1	0.2	0.3	0.4
$f(x)$	-0.62049958	0.28398668	0.00660095	0.24822440

10. a. Approximate $f(0.05)$ using the following data and the Newton forward-difference formula:

x	0.0	0.2	0.4	0.6	0.8
$f(x)$	1.00000	1.22140	1.49182	1.82212	2.22554

   b. Use the Newton backward-difference formula to approximate $f(0.65)$.

   c. Use Stirling's formula to approximate $f(0.43)$.

11. For a function f, the forward-divided differences are given by

$x_0 = 0.0$	$f[x_0]$		
		$f[x_0, x_1]$	
$x_1 = 0.4$	$f[x_1]$		$f[x_0, x_1, x_2] = \frac{50}{7}$
		$f[x_1, x_2] = 10$	
$x_2 = 0.7$	$f[x_2] = 6$		

Determine the missing entries in the table.

3. Cubic Spline Interpolation

12. Determine the natural cubic spline S that interpolates the data $f(0) = 0$, $f(1) = 1$, y $f(2) = 2$.

13. Determine the clamped cubic spline S that interpolates the data $f(0) = 0$, $f(1) = 1$, y $f(2) = 2$ and satisfies $S'(0) = S'(2) = 1$.

14. Construct the natural cubic spline for the following data

a.

x	$f(x)$
8.3	17.56492
8.6	18.50515

b.

x	$f(x)$
-0.5	-0.0247500
-0.25	0.3347935
0.00	1.1010000

c.

x	$f(x)$
0.1	-0.62049958
0.2	-0.28398668
0.3	-0.00660095
0.4	-0.24842440

15. The data in Exercise 14 were generated using the following functions. Use the cubic splines constructed in Exercise 14 for the given value of x to approximate $f(x)$ and $f'(x)$, and calculate the actual error.

   a. $f(x) = x\ln(x)$; aproximadamente $f(8.4)$ y $f'(8.4)$.

   b. $f(x) = x^3 + 4.001x^2 + 4.002x + 1.01$; aproximadamente $f(-\frac{1}{3})$ y $f'(-\frac{1}{3})$.

c. $f(x) = x \cos x - 2x^2 + 3x - 1$; aproximadamente $f(0.25)$ y $f'(0.25)$.

16. Construct the clamped cubic spline using the data of Exercise 14 and the fact that

    a. $f'(8.3) = 3.116256$ y $f'(8.6) = 3.151762$.
    b. $f'(-0.5) = 0.7510000$ y $f'(0.0) = 4.0020000$.
    c. $f'(0.1) = 3.58502082$ y $f'(0.4) = 2.16529366$.

17. Construct a natural cubic spline to approximate $f(x) = \cos(\pi x)$ by using the values given by $x_0 = 0$, $x_1 = 0.25$, $x_2 = 0.5$, $x_3 = 0.75$, $x_4 = 1.0$. Integrate the spline over $[0, 1]$, and compare the result to $\int_0^1 \cos(\pi x)\, dx = 0$. Use the derivatives of the spline to approximate $f'(0.5)$ and $f''(0.5)$. Compare these approximations to the actual values.

18. Repeat Exercise 17, constructing instead the clamped cubic spline with $f'(0) = f'(1) = 0$.

19. A natural cubic spline S on $[0, 2]$ is defined by

$$S(x) = \begin{cases} S_0(x) = 1 + 2x - x^3, & \text{si } 0 \leq x < 1, \\ S_1(x) = 2 + b(x-1) + c(x-1)^2 + d(x-1)^3, & \text{si } 1 \leq x \leq 2 \end{cases}$$

    Find b, c and d.

20. A natural cubic spline S is defined by

$$S(x) = \begin{cases} S_0(x) = 1 + B(x-1) - D(x-1)^3, & \text{si } 1 \leq x < 2, \\ S_1(x) = 1 + b(x-2) - \frac{3}{4}(x-2)^2 + d(x-2)^3, & \text{si } 2 \leq x \leq 3 \end{cases}$$

    If S interpolates the data $(1, 1)$, $(2, 1)$ y $(3, 0)$, obtenga B, D, b y d.

21. A car traveling along a straight road is clocked at a number of points. The data from the observations are given in the following table, where the time is in seconds, the distance is in feet, and the speed is in feet per second.

Tiempo	0	3	5	8	13
Distancia	0	225	383	623	993
Velocidad	75	77	80	74	72

    a. Use a clamped cubic spline to predict the position of the car and its speed when $t = 10s$.
    b. Use the derivative of the spline to determine whether the car ever exceeds a $55\frac{mi}{h}$ speed limit on the road; if so, what is the first time the car exceeds this speed?
    c. What is the predicted maximum speed for the car?