Correlation between shear wave velocity and elastic modulus at large strain level

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Correlation between elastic modulus and shear wave velocity at large strain level

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ABSTRACT
Geotechnical studies on large projects need *constitutive* relations for modeling deformation *behavior* of soils and construction *materials*. Modeling is particularly important for zoned dams to examine the compatibility of deformations between pervious and impervious zones, generally made of different materials. The determination of the deformation behavior of granular materials, especially coarse granular soils, requires special equipment's and remains a complex issue. The small strain shear modulus (G_{max}) is an attractive parameter as it can be measured in the field under the true in-situ conditions using non-destructive tests (e.g., MMASW). However, the exploitation of these measures requires the establishment of correlations between this modulus (G_{max}) and other soil parameters at large deformations (e.g., E, E_{oed}, ν). This can be achieved by measuring simultaneously the oedometric module and the shear wave velocity (V_s) in the laboratory on materials having a similar gradation to that used for the geotechnical structure construction. This paper presents the results of experimental work carried out at the soil mechanics laboratory of the University of Sherbrooke on different sands. The purpose of this study is to establish correlations between the large deformation modulus measured in oedometer (E_{oed}) and the shear wave velocity (V_s) or shear modulus measured at low strains (G_{max}) on granular materials. These relationships can be utilized to predict, from in-situ V_s measurement (e.g., MMASW), the modules at large deformations required for numerical modeling of various soil materials.

RÉSUMÉ
Les études géotechniques sur les grands projets doivent avoir des relations constitutives pour la modélisation du comportement de déformation des matériaux de construction ou de fondation. La modélisation est particulièrement importante pour les barrages zonés pour examiner la compatibilité des déformations entre les zones perméables et imperméables, généralement faites de matériaux différents. La détermination du comportement en déformation des matériaux granulaires, en particulier les sols granulaires grossiers, nécessite un équipement spécial et complexe. Pour cette raison, le module de cisaillement à petites déformations (G_{max}) est un paramètre intéressant, car il peut être mesuré sur le terrain sous les véritables conditions in situ sans intrusion (MMASW). Cependant, l'exploitation de ces mesures nécessite la mise en place de la relation entre ce module (G_{max}) et les paramètres du sol à de grandes déformations (E, Eoed, ν). Ceci peut être réalisé en mesurant simultanément le module œdométrique et la vitesse des ondes de cisaillement (Vs) en laboratoire sur des matériaux ayant une gradation similaire à celle utilisée pour la construction de la structure géotechnique. Cet article présente les résultats de travaux expérimentaux menés au laboratoire de mécanique des sols de l'Université de Sherbrooke pour différents matériaux granulaires. Le but de cette étude est d'établir des relations entre le module à grande déformation mesurée à l'oedomètre (Eoed, grande déformation) et la vitesse des ondes de cisaillement ou le module de cisaillement calculé à de faibles déformations (G_{max}) sur les matériaux granulaires. Ces relations peuvent être utilisées pour prédire, à partir de la vitesse in-situ de l'onde de cisaillement (MMASW), les modules à des déformations importantes nécessaires pour la modélisation numérique de divers matériaux du sol.

1 INTRODUCTION

A rigorous design of foundations should account for both the short- and long-term behavior of the supporting ground. The long-term behavior (i.e., large deformation) of soil has received a great deal of attention in geotechnical researches, and it is commonly evaluated using traditional tests such as triaxial and direct shear tests. In contrast, the short-term behavior, where the soil properties are assumed to be almost linear, is rather disregarded in the geotechnical community; it is, therefore, determined by geophysical methods. In fact, the characterization of the mechanical properties of soils in this small-strain range (ε $<10^{-3}$ to 10^{-6}) is fundamental in many civil engineering applications such as embankment and for very important construction project such as nuclear plant. Moreover, the small strain shear modulus (G_{max}) is an attractive parameter as it can be measured in the field under the true in-situ conditions using non-destructive tests (e.g., MMASW), and it can be exploited more if it is correlated with other soil parameters at large deformations (e.g., E, E_{oed}, ν).

The purpose of this paper is to establish correlations between the deformation modulus measured in oedometric condition (E_{oed}, large deformation condition) and the propagation velocity of shear waves (V_s) or initial shear modulus measured at low strains (G_{max}) of granular materials similar to those commonly used in the construction of dams and dikes or that can be found in several natural soil deposits supporting other types of structures. More specifically, the current paper illustrates through measuring V_s of three different granular materials with different particle size distribution curves using the piezoelectric ring-actuator (P-RAT) (Karray et al. 2015) that the initial shear modulus (G_{max}) of a soil specimen can be correlated with the oedometric modulus (E_{oed}). The obtained relationships could be used to evaluate the behavior of foundations and structures constructed on granular materials from the direct (in situ) measurement of the ground shear wave velocity.

2 IMPORTANCE OF G_{MAX}

The shear stiffness of granular soils at small strain (less than 0.001%) which could be considered as the region of the true linear elastic behavior of the soil, usually denoted G_0 or G_{max}, is a key parameter in major geotechnical applications involving deep excavations beside existing buildings, tunneling, integral bridge abutments, bridge piers, pile foundations, liquefaction evaluation or earthquake ground response analysis (e,g, Bui 2009). Several studies demonstrated the important part of this parameter related to the seismic hazard (e.g. Riepl et al., 2000; Louie, 2001; Wang and Hao, 2002; Thompson et al., 2010; Theilen-Willige, 2010). In seismic active areas where crucial concerns in the design of geotechnical systems, the maximum shear modulus is involved in earthquake ground response analyses (e.g., Kramer 1996), liquefaction potential (e.g., Youd et al. 2001), and soils characterization (e.g., Robertson et al. 1995). On the other hand, G_{max} is also used in static geotechnical applications especially in foundation engineering as it has been an intensive focus by many several researches (e,g, Imai and Yoshimura (1976), Tatham (1982), Willkens et al. (1984), Eberhart-Phillips et al. (1989), Keceli (1990), Jongmans (1992), Sully and Campanella (1995), and Pyrak-Nolte et al. (1996)).

Some other studies proposed empirical correlations of G_{max} and other geotechnical parameter such as the ultimate bearing capacity of soils as reported by Abd El-Rahman et al. (1992) who used, in their ultimate bearing capacity equation, the logarithm of shear wave velocity which is related directly to G_{max}:

$$G_{max} = \rho(v_s)^2 \qquad [1]$$

where ρ is the dry density of the soil.

An explicit expression for the allowable bearing pressure using V_s has been developed by Turker (2004). On the other hand the importance of such parameter (G_{max}) is illustrated and mentioned in several design criteria in which the maximum deformation does not exceed (10^{-3}).

Some more examples showing the interest of the geotechnical engineering in understanding the behavior of soils at these low levels of deformation are given in Sauzeat (2003). In one of these examples related to large-scale projects such as the construction of nuclear accelerators, large-scale load tests were carried out on the site for every detail with high precision. The results provide a deformation of order not higher than 10^{-4}. Another example could be cited as a second practical case for which the deformations remain low during the digging of a tunnel in London, in clay soil (Burland (1989)). In this project, the deformations slightly exceed 10^{-3} in the limit diameter of the tube above the tunnel. In the rest of massive, deformations are smaller than 10^{-4}. Finally, a third practical example which is the study done by Jardine and Potts (1988) on the behavior of pile foundation in traction.

Other studies have been done in Japan on several cases of geotechnical engineering structures for which laboratory tests were performed, since the nature of the soil and geotechnical conditions that characterizes this country, high accuracy is required. These tests, mostly on precision triaxial devices aimed determining the stiffness of soil to deformations of intervals less than 10^{-4}. These results have been indexed by Koseki et al (2001) then used in the calculation codes to predict the deformations of the soil.

In fact, the small-strain shear modulus started to be involved in modeling for static and dynamic soil analysis in the late 1970's (Seed and Idris 1970). Since that time, the majority of small-strain stiffness models can be classified in three subclasses:

- Nonlinear (para-elastic) stress – strain law (e.g. models from soil dynamic, Jardine model)
- Kinematic hardening elastoplastic models formulated in stress space (e.g. multi surface models)
- Other models that include strain space based formulations (e.g. Simpson Brick model, Intergranular strain).

3 OEDOMETRIC MODULUS

Oedometric modulus represents the elasticity modulus in oedometric conditions. In fact, elasticity modulus could be defined as the ability of material to resist the excessive deformation during loading. In a large strain range, the deformations are irreversible and feature by very predominate viscous effects. The damping coefficient stabilizes and tends towards a maximum value. Rigidities are very low compared to those in the range of very small deformations. This behavior is clearly represented through the well-known degradation curves. The oedometer modulus at large strain range will be focused on in this study.

The deformation in the case of oedometric test is a purely axial (vertical) compressive strain type as the rigid walls of the oedometric cell do not allow the potential of radial deformation. However, although only an axial stress is applied to the sample, there is indeed a radial stress because lateral constrain applied to the granular medium from the sidewalls. The one-dimensional consolidation oedometer test is in fact in a general use in foundation design and the laboratory modeling of soil foundation behavior is acceptable in most cases of foundations as they approximate a real condition a fortiori; the foundation area is larger, and the relative thickness of the compressible subsoil layer is smaller (Lenk 2009).

Many relationships have been established between the oedometric modulus (E_{oed}) and other material modulus/parameter. A summary of most of these relationships is included in the following equations:

$$E_{oed}=\frac{1-\upsilon}{(1+\upsilon)\cdot(1-2\cdot\upsilon)}\cdot E \qquad [2]$$

$$E_{oed}=K+\frac{4}{3}\cdot G \qquad [3]$$

$$E_{oed}=\frac{2(\nu-1)}{2\nu-1}G \qquad [4]$$

$$E_{oed}=\frac{\sigma'}{Cc/2.3}\cdot(1+e_0)=\frac{\sigma'}{\lambda}\cdot(1+e_0) \qquad [5]$$

4 TESTING EQUIPMENT

The current experimental investigation is conducted using the piezoelectric ring-actuator (P-RAT) technique (Karray et al. 2015). The piezoelectric ring-actuator technique P-RAT was developed at the University of Sherbrooke (Gamal El Dean, 2007; Ethier, 2009, Ben Romdhan et al 2014, Karray et al 2015). The device consist of two piezoelectric rings incorporated in the traditional oedometer cell, first ring transmits the shear wave after converting the electrical voltage input and a receiver which diffuses data to acquisition card. A bishop oedmeter type with a small cell (60 mm diameter) was used through this study.

5 THE USED SOIL SAMPLES

A series of experimental test using the P-RAT is conducted on three different sand types: Péribonka sand (portion < 5 mm), EM1 sand, and Champagne sand. The physical properties of these types of sands are listed in Table 1.

Soil sample	Peribonka	Em1	Champagne
G_s	2.7	2.69	2.7
D_{50}	0.411	0.6	0.37
C_u	4	5	7
e_{max}	0.85	0.82	0.74
e_{min}	**0.35**	**0.41**	**0.37**

Table 1. Physical properties of the used sand samples.

6 RESULTS AND DISCUTION

The following discussion will be focused on the measured stress-deformation curves as they directly related to the oedometer modulus under investigation. A large number of tests in the laboratory on the different sand samples were used to construct the stress-strain curves in oedometric conditions. Figures 1-3 show the variation of the measured vertical strain (ε_v) of Péribonka, EM1, and Champagne sands, respectively with the applied vertical stress in Kpa. Each sand sample was teste at four different initial densities.

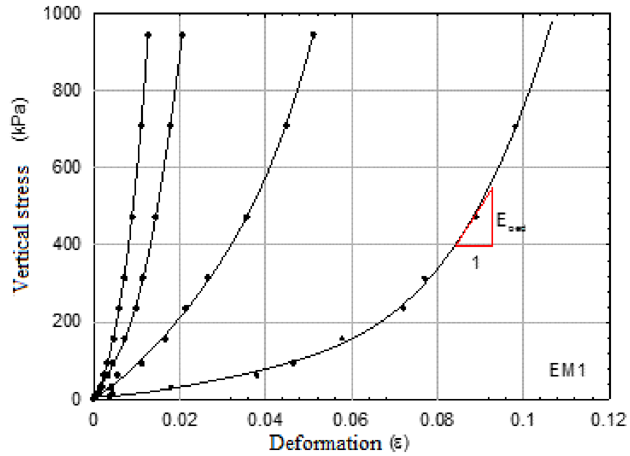

Figure 1. Vertical stress-vertical strain curve for Em1

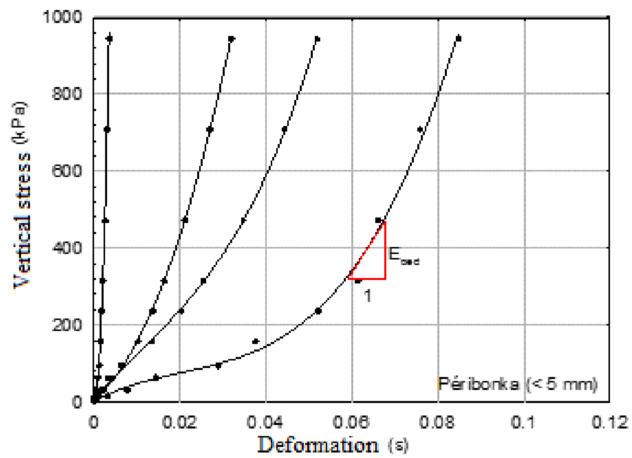

Figure 2. Vertical stress-vertical strain curve for Peribonka.

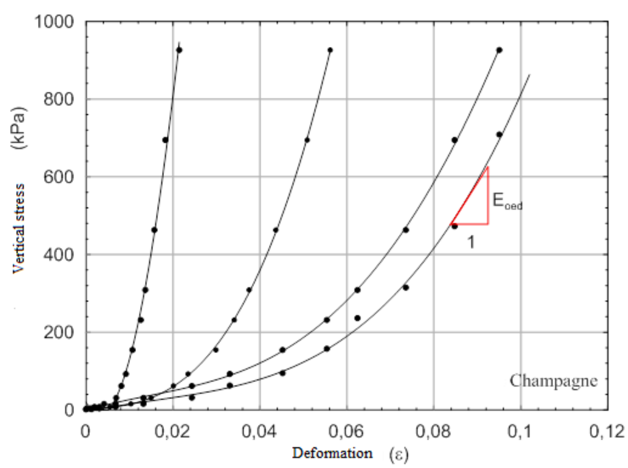

Figure 3. Vertical stress-vertical strain curve for Champagne.

The variation of the applied vertical stress (σ'_v) with the measured vertical strain (ε_v) presented in Figs. 1-3 can be approximated in the form of a polynomial function as:

$$\sigma'_v = a \cdot \varepsilon_v^{\,2} \qquad [6]$$

where a is a correlative parameter.

Theoretically, the oedometric modulus can be determined at each stress state (i.e., stage of loading) obtained by differentiating the polynomial function in Eq. 6 as the oedometric modulus at any stress state corresponds to the tangent of the stress-strain curve as indicated in Figs. 1-3.

$$E_{oed} = \frac{\partial \sigma'_v}{\partial \varepsilon_v} \qquad [7]$$

Combining Eq. 6 and 7 yields:

$$E_{oed} = 2.\sqrt{a}.\sqrt{\sigma'_v} \qquad [8]$$

Eq. 8 shows that the experimentally determined oedometric modulus obtained from the current tests is a function of the square root of the vertical stress. With this equation (Eq. 8), it's possible to have a value of oedometric modulus at each state of stress.

It is well-known that the shear wave velocity, V_s is a function of $\sigma'^{.25}_v$ according to (e.g., Hardin and Richart, 1963; Hardin and Black, 1967; Hardin Drnevich 1972) among other. The later correlation has been also confirmed by tests conducted on the three tested soils as shown in Fig. 4 on tests performed on Péribonka sand.

The obtained oedometric modulus at different stress state (void ratio) of the three tested materials are normalized with respect to the applied vertical stress following the work by Ohde 1939 and plotted against the void ratio in Fig. 5. The observed trend and range of values are consistent with those suggested by Ohde 1951 (between 10 to 75 MPa for granular soil).

Similarly, the measured G_{max} at different stress state (void ratio) of the three tested materials are normalized with respect to the applied vertical stress and plotted against the void ratio in Fig. 6.

With a rigorous examination of Figs. 5 and 6, an impressive ascertainment can be revealed in that the correlations between both E_{oed} and G_{max} with the void ratio shows similar trend. In other words, the effect of the change of void ratio on the measured E_{oed} and G_{max} appears to be similar. Based on the experimental results obtained from this study, it is possible to write for a given soil:

$$\frac{E_{Oed}}{G_{max}} = A.\exp(B.e) \qquad [9]$$

Figure 7 shows the variation of the ratio of modules (E_{oed} / G_{max}) for the three tested samples. It seems that for loose materials the ratio between the modules is around 0.1 while for dense soil this ratio significantly increases to be around 0.4. The variation of the ratio (E_{oed}/G_{max}) in Fig. 7 seems to be logic and agrees well with the literature.

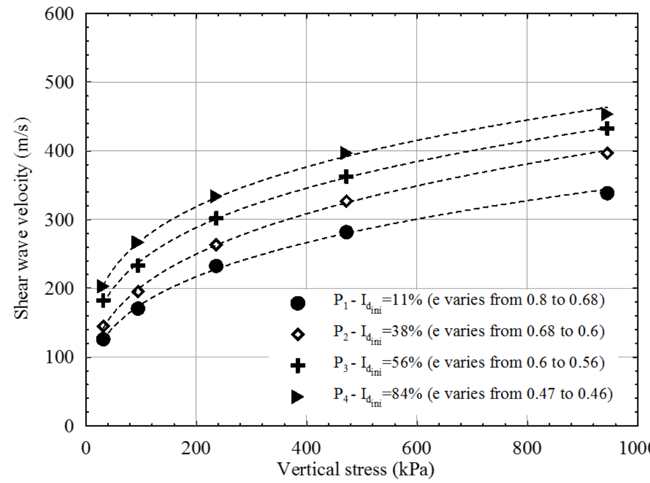

Figure. 4 Shear wave velocity of Péribonka sand as a function of the vertical applied stress

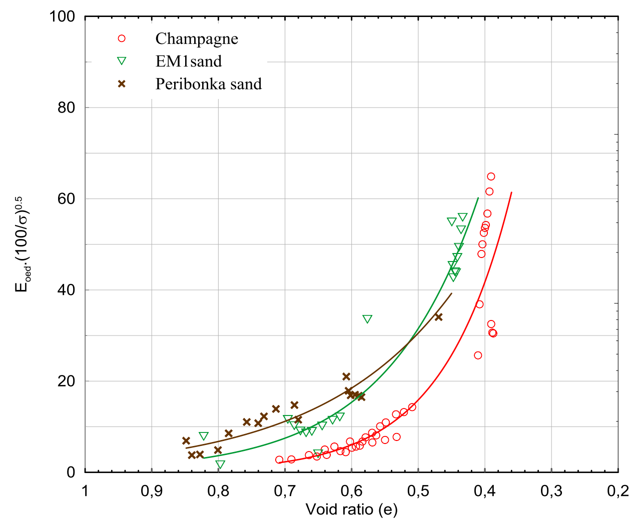

Figure 5 Normalized oedometric modulus as a function of void ratio.

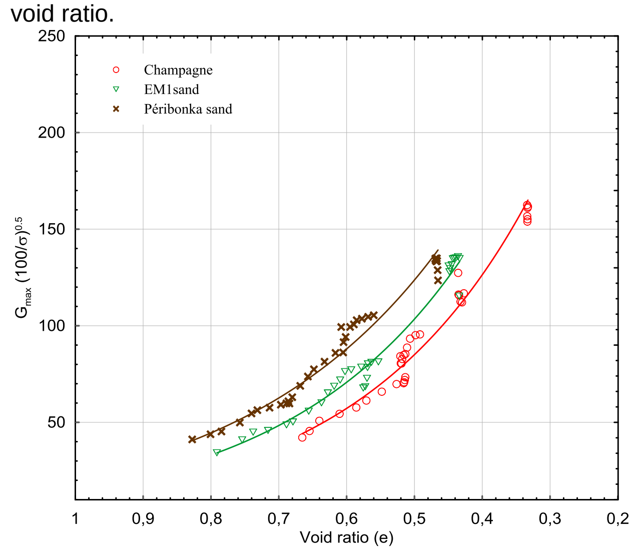

Figure 6: Normalized shear modulus as a function of void ratio.

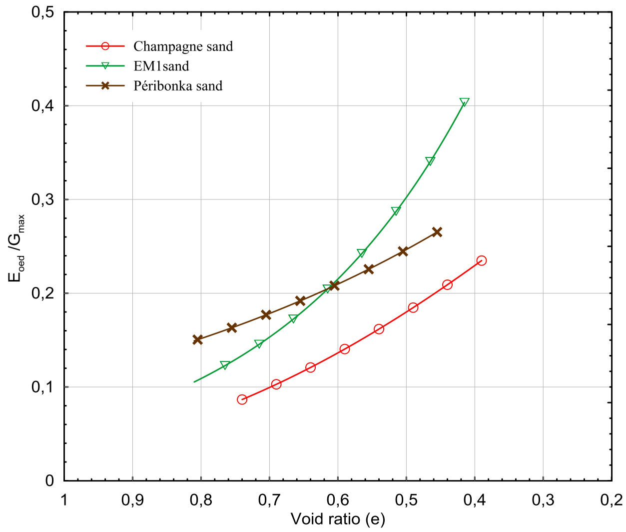

Figure 7. Correlation between oedometric modulus and initial shear modulus.

From practical point of view, this correlation can be used in settlement prediction and design of shallow foundation. The performance of the proposed oedometric, maximum shear modulus ratio is examined against experimental data in term of pressure – settlement curves provided by Bouassida 2015. Bouassida 2015 conducted comprehensive set tests on model shallow foundation in order to develop a design chart for shallow foundation based on small-strain soil characteristics. The proposed E_{oed}/G_{max} correlation was involved in the numerical model using the computer code, FLAC. Although, Fig. 8 shows a slight difference between experimental data conducted by the P-RAT, the comparison can be considered as good for all practical design purposes.

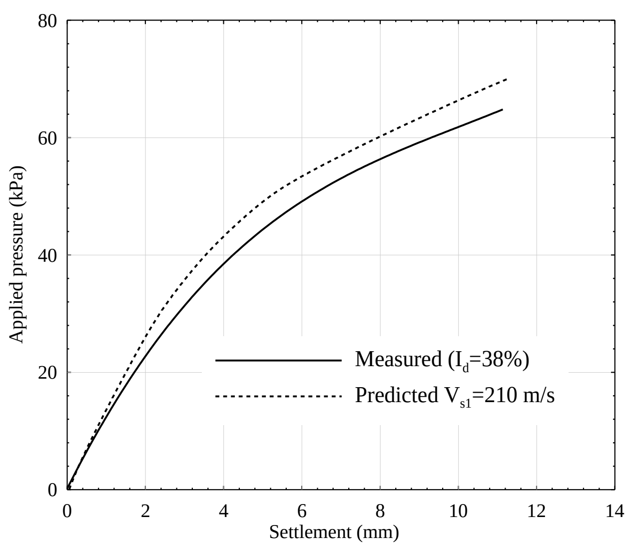

Figure 8 Experimental and predicted (adopting the proposed E_{oed}/G_{max} correlation) pressure-settlement curve: Champagne sand with Id = 38%.

7 CONCLUSION

This paper presents the results of experimental work using the piezoelectric ring-actuator (P-RAT) carried out at the soil mechanics laboratory of the University of Sherbrooke on different sands in order to establish correlations between the large deformation modulus measured in odometer (E_{oed}) and the shear wave velocity (V_s) or shear modulus measured at low strains (G_{max}). These relationships can be utilized to predict, from in-situ V_s measurement (e.g., MMASW), the modules at large deformations required for numerical modeling of various soil materials. The proposed G_{max}-E_{ode} is verified against data from the literature and used successfully to predicted the stress-strain foundation characteristics measured in the laboratory using model tests and those from numerical modelling using the computer code, FLAC

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