INSTITUTO POLITÉCNICO NACIONAL

ESCUELA SUPERIOR DE CÓMPUTO

TEORÍA DE COMUNICACIONES Y SEÑALES

EVIDENCIA 1.5

PRESENTA

PALACIOS CABRERA ALBERTO

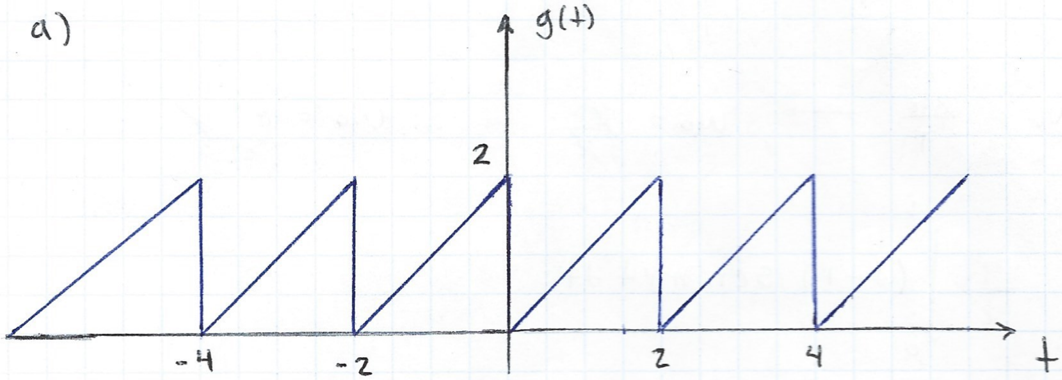
3CV16

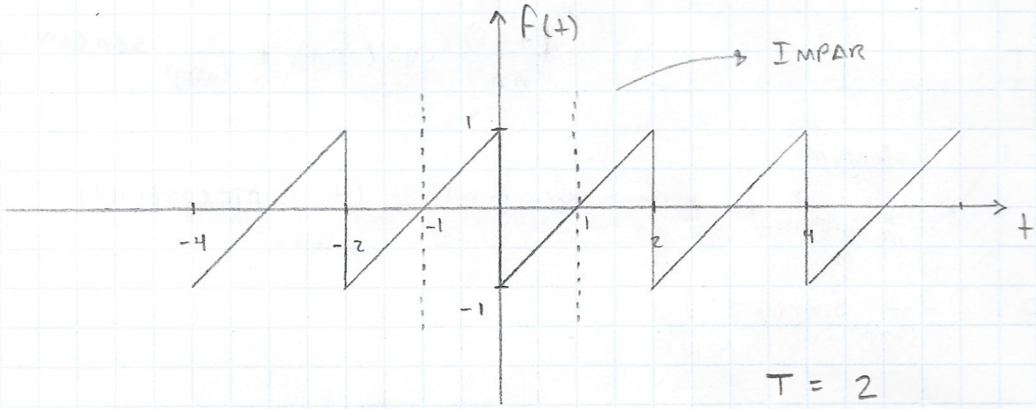
PROFESORA
ARZATE GORDILLO JACQUELINE

11 de marzo de 2021

Tarea 1.5 Encuentra la STF de g(t) a partir de la STF de otra función con paridad

a)





$T = 2$

$a_0 = a_n = 0$

$m = 1$

$y - y_1 = m(t - t_1)$

$y - (0) = 1(t + 1)$

$y = t + 1$

$-1 < t < 0$

$y - y_1 = m(t - t_1)$

$y - (-1) = 1(t - 0)$

$y + 1 = t \longrightarrow y = t - 1$

$y = t - 1$

$0 < t < 1$

$$b_n = \frac{4}{T}\int_0^{\frac{T}{2}} f(t)\ \text{sen}\, n\omega_o t\, dt$$

$$\omega_o = \frac{2\pi}{T} \longrightarrow \omega_o = \frac{\cancel{2}\pi}{\cancel{2}} \quad \therefore \quad \omega_o = \pi$$

$$b_n = \frac{\cancel{4}}{\cancel{2}}\int_0^1 (t-1)\ \text{sen}\, n\pi t\, dt$$

$$\frac{d}{dt} \qquad \int \qquad '' \ '' = -\frac{(t-1)}{n\pi}\cdot\cos(n\pi t) + \frac{1}{(n\pi)^2}\ \text{sen}(n\pi t)$$

$+ \quad t \longrightarrow t-1 \qquad \text{sen}\, n\pi t$

$- \quad \longrightarrow 1 \qquad -\frac{1}{n\pi}\cos n\pi t \qquad '' \ '' = \dfrac{\text{sen}(n\pi t) - (t-1)\, n\pi \cos(n\pi t)}{(n\pi)^2}$

$+ \quad 0 \qquad -\frac{1}{(n\pi)^2}\ \text{sen}\, n\pi t$

$$b_n = 2\left[\frac{\text{sen}(n\pi t) - (t-1)\, n\pi\cos(n\pi t)}{(n\pi)^2}\right]\Bigg|_0^1$$

$$b_n = 2\left[\frac{\overset{0}{\text{sen}(n\pi)} - \overset{0}{\cancel{(1-1)}}\, n\pi\cos(n\pi)}{(n\pi)^2} - \frac{\overset{0}{\cancel{\text{sen}(n\pi\cdot 0)}} - (0-1)\, n\pi \overset{1}{\cancel{\cos(n\pi\cdot 0)}}}{(n\pi)^2}\right]$$

$$b_n = 2\left[-\frac{\cancel{-}(\cancel{-}1)\, n\pi}{(n\pi)^2}\right] = 2\left[-\frac{n\pi}{(n\pi)^{\cancel{2}}}\right] \quad \therefore\ b_n = -\frac{2}{n\pi}$$

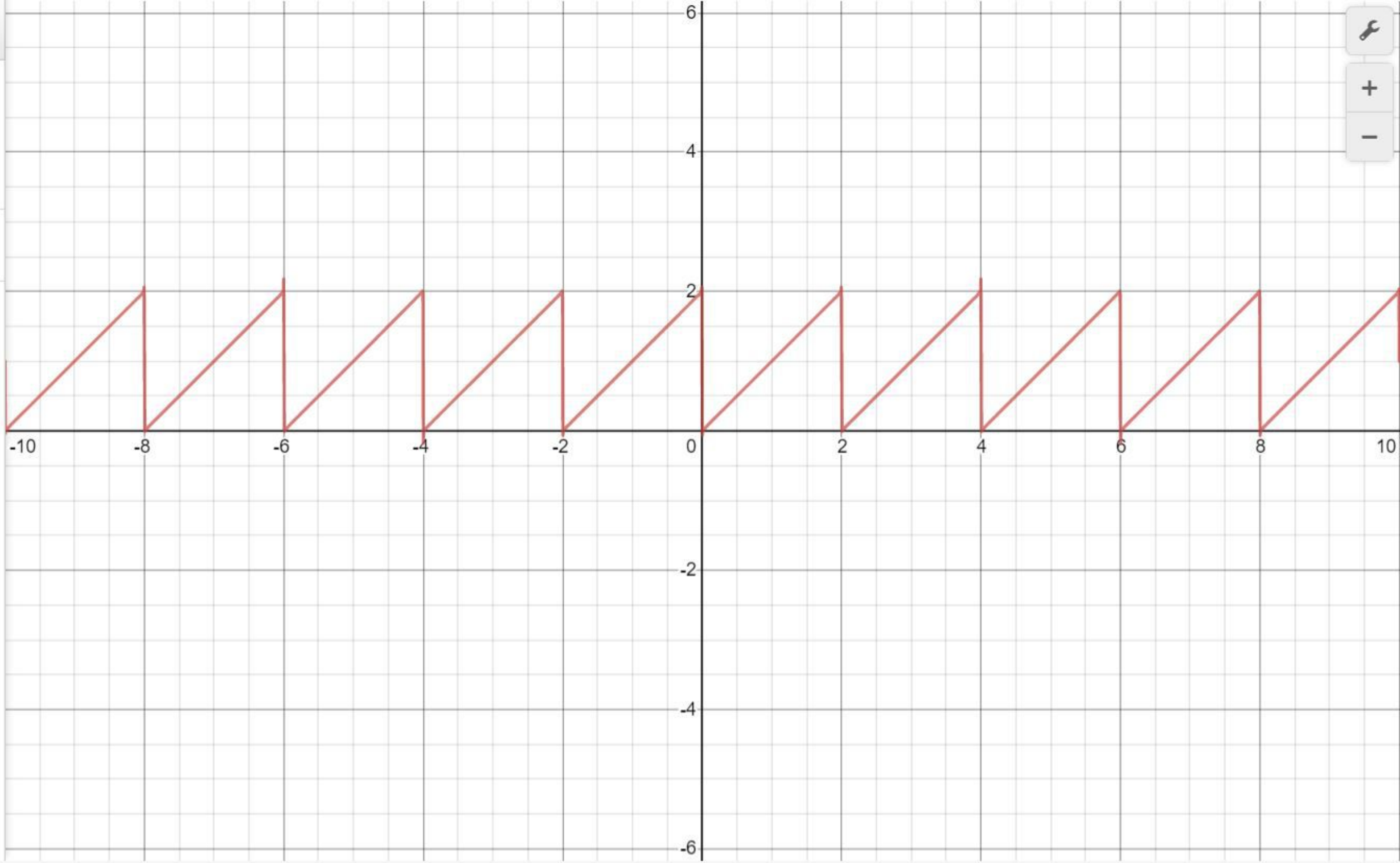
$$\therefore\ g(t) = 1 + \sum_{n=1}^{\infty} -\frac{2}{n\pi}\cdot \text{sen}\, n\pi t$$

1

$$f = 1 + \sum_{n=1}^{1000} -\frac{2}{n\pi}\sin(n\pi t)$$

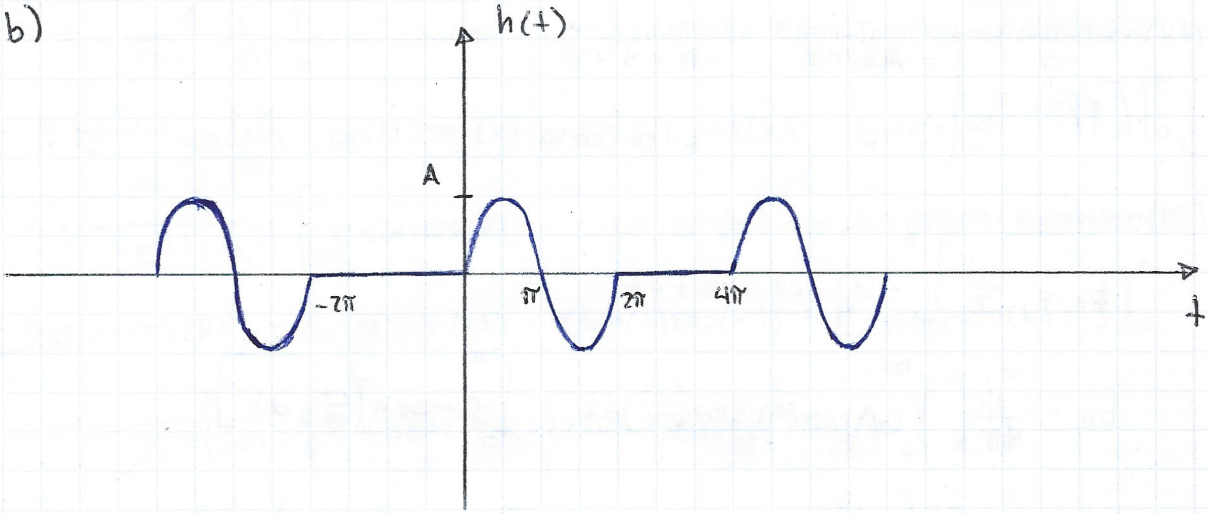
2

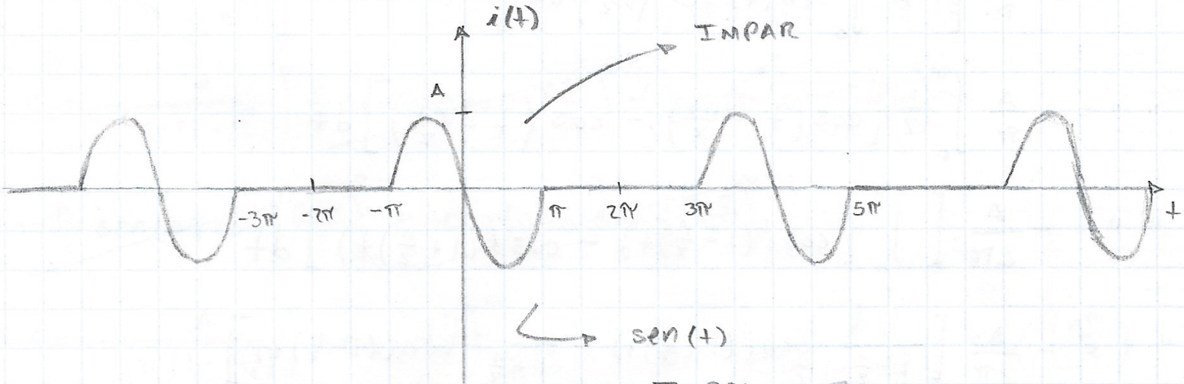
3



impulsado por desmos

b)





$T = 2\pi$

$$i(t)\Big|_{t=t-\pi} = i(t-\pi) = h(t)$$

$$a_0 = a_n = 0$$

$$w_0 = \frac{2\pi}{T} \longrightarrow w_0 = \frac{2\pi}{4\pi}$$

$$w_0 = \frac{1}{2}$$

$$t_0 < t < t_0 + T$$

$$-2\pi < t < 2\pi$$

$$2\pi = t_0 + T$$

$$2\pi = -2\pi + T$$

$$\therefore \; T = 4\pi$$

$$i(t) = \begin{cases} -A\,\text{sen}\,t & -\pi < t < \pi \\ 0 & \text{otro caso} \end{cases}$$

$$b_n = \frac{4}{T}\int_0^{\frac{T}{2}} i(t)\,\text{sen}\,n w_0 t\,dt$$

$$b_n = \frac{4}{4\pi}\left[\int_0^{\pi} -A\,\text{sen}(t)\,\text{sen}\left(\frac{nt}{2}\right)dt + \cancel{\int_0^{2\pi} 0\;\text{sen}\left(\frac{nt}{2}\right)dt}\right]$$

$$'' \; '' = \frac{1}{\pi}\left[-A\int_0^{\pi}\text{sen}(t)\,\text{sen}\left(\frac{nt}{2}\right)dt\right]$$

$$'' \; '' = -\frac{A}{\pi}\int_0^{\pi}\frac{1}{2}\left[\cos\left(t-\frac{nt}{2}\right) - \cos\left(t+\frac{nt}{2}\right)\right]dt$$

$$'' \; '' = -\frac{A}{2\pi}\left[\int_0^{\pi}\left[\cos\left(\left(1-\frac{n}{2}\right)t\right) - \cos\left(\left(1+\frac{n}{2}\right)t\right)\right]dt\right]$$

$$'' \; '' = -\frac{A}{2\pi}\left[\frac{1}{1-\frac{n}{2}}\cdot\text{sen}\left(\left(1-\frac{n}{2}\right)t\right) - \frac{1}{1+\frac{n}{2}}\cdot\text{sen}\left(\left(1+\frac{n}{2}\right)t\right)\right]$$

$$'' \; '' = -\frac{A}{2\pi}\left[\frac{\left(1+\frac{n}{2}\right)\text{sen}\left(\left(1-\frac{n}{2}\right)t\right) - \left(1-\frac{n}{2}\right)\text{sen}\left(\left(1+\frac{n}{2}\right)t\right)}{\left(1-\frac{n^2}{4}\right)}\right]\Bigg|_0^{\pi}$$

$$'' \; '' = -\frac{A}{2\pi\left(1-\frac{n^2}{4}\right)}\left[\left(1+\frac{n}{2}\right)\text{sen}\left(\left(1-\frac{n}{2}\right)t\right) - \left(1-\frac{n}{2}\right)\text{sen}\left(\left(1+\frac{n}{2}\right)t\right)\right]\Bigg|_0^{\pi} \qquad \boxed{K = \frac{n}{2}}$$

$$'' \; '' = -\frac{A}{2\pi(1-K^2)}\left[(1+K)\,\text{sen}(t-Kt) - (1-K)\,\text{sen}(t+Kt)\right]\Bigg|_0^{\pi}$$

$$'' \; '' = -\frac{A}{2\pi(1-K^2)}\Big[(1+K)\big(\text{sen}(t)\cos(Kt) - \cos(t)\,\text{sen}(Kt)\big)$$

$$- (1-K)\big(\text{sen}(t)\cos(Kt) + \cos(t)\,\text{sen}(Kt)\big)\Big]\Bigg|_0^{\pi}$$

$$bn = -\frac{A}{2\pi(1-K^2)}\Big[sen(t)\cos(Kt) - \cos(t)sen(Kt) + K sen(t)\cos(Kt) - K\cos(t)sen(Kt)$$

$$-\big(sen(t)\cos(Kt) + \cos(t)sen(Kt) - K sen(t)\cos(Kt) - K\cos(t)sen(Kt)\big)\Big]\Big|_0^{\pi}$$

$$'' \; '' = -\frac{A}{2\pi(1-K^2)}\Big[\cancel{sen(t)\cos(Kt)} - \cos(t)sen(Kt) + K sen(t)\cos(Kt) - \cancel{K\cos(t)sen(Kt)}$$

$$-\cancel{sen(t)\cos(Kt)} - \cos(t)sen(Kt) + K sen(t)\cos(Kt) + \cancel{K\cos(t)sen(Kt)}\Big]\Big|_0^{\pi}$$

$$'' \; '' = -\frac{A}{\cancel{2}\pi(1-K^2)}\Big[\cancel{2}K sen(t)\cos(Kt) - \cancel{2}\cos(t)sen(Kt)\Big]\Big|_0^{\pi}$$

$$'' \; '' = -\frac{A}{\pi(1-K^2)}\Big[K sen(t)\cos(Kt) - \cos(t)sen(Kt)\Big]\Big|_0^{\pi}$$

$$'' \; '' = -\frac{A}{\pi(1-\frac{n^2}{4})}\Big[\cancel{\frac{n}{2}sen(\pi)\cos\left(\frac{n\pi}{2}\right)}^{\to 0} - \cancel{\cos(\pi)}^{\to -1} sen\left(\frac{n\pi}{2}\right)$$

$$-\cancel{\frac{n}{2}sen(0)\cos\left(\frac{n\cdot 0}{2}\right)}^{\to 0} + \cancel{\cos(0)\cdot sen\left(\frac{n\cdot 0}{2}\right)}^{\to 0}\Big]$$

$$'' \; '' = -\frac{A}{\pi(1-\frac{n^2}{4})}\Big[sen\left(\frac{n\pi}{2}\right)\Big] \quad \therefore \; bn = -\frac{A}{\pi(1-\frac{n^2}{4})}\cdot sen\left(\frac{n\pi}{2}\right)$$

$n \neq 2$

Para b_1

$$\lim_{n\to 1} bn = -\frac{A}{\pi(1-\frac{1}{4})}\cdot \cancel{sen\left(\frac{\pi}{2}\right)}^{\to 1} = -\frac{A}{\pi\left(\frac{3}{4}\right)} \quad \therefore \; b_1 = \frac{4A}{3\pi}$$

Para b_2

$$\lim_{n\to 2} bn = -\frac{A}{\pi\cancel{(1-\frac{4}{4})}^{\to 0}}\cdot \cancel{sen\left(\frac{2\cdot\pi}{2}\right)}^{\to 0} = \frac{0}{0}$$ INDETERMINACIÓN

Aplicando L'Hopital

$$\lim_{n \to 2} b_n = -\frac{\frac{d}{dn}\left(A \operatorname{sen}\left(\frac{n\pi}{2}\right)\right)}{\frac{d}{dn}\left(\pi\left(1-\frac{n^2}{4}\right)\right)} = -\frac{A}{\pi}\frac{\frac{d}{dn}\left(\operatorname{sen}\left(\frac{n\pi}{2}\right)\right)}{\frac{d}{dn}\left(1-\frac{n^2}{4}\right)} = -\frac{A \cdot \frac{\pi}{2}\cos\left(\frac{n\pi}{2}\right)}{\pi\left(-\frac{1}{4}\cdot 2n\right)}$$

$$'' \;\; '' = \cancel{-}\frac{\frac{A\pi\cos\left(\frac{n\pi}{2}\right)}{2}}{\cancel{-}\frac{\pi 2n}{4}} = \frac{\cancel{4\pi} A\cos\left(\frac{n\pi}{2}\right)}{\cancel{4\pi} n} = \frac{A\cos\left(\frac{n\pi}{2}\right)}{n}$$

$$\lim_{n \to 2} b_n = \frac{A\overbrace{\cos\left(\frac{2\pi}{2}\right)}^{-1}}{2} = -\frac{A}{2} \quad \therefore \; b_2 = -\frac{A}{2}$$

$$\therefore \; i(t) = -\frac{4A}{3\pi}\cdot \operatorname{sen}\left(\frac{1}{2}t\right) - \frac{A}{2}\operatorname{sen}(t) + \sum_{n=3}^{\infty} -\frac{A}{\pi\left(1-\frac{n^2}{4}\right)}\cdot \operatorname{sen}\left(\frac{n\pi}{2}\right)\cdot \operatorname{sen}\left(\frac{nt}{2}\right)$$

$$\therefore \; i(t)\Big|_{t=t-\pi} = h(t) = -\frac{4A}{3\pi}\cdot \operatorname{sen}\left(\frac{1}{2}(t-\pi)\right) - \frac{A}{2}\cdot \operatorname{sen}(t-\pi)$$

$$-\frac{A}{\pi}\sum_{n=3}^{\infty}\frac{\operatorname{sen}\left(\frac{n\pi}{2}\right)}{1-\frac{n^2}{4}}\cdot \operatorname{sen}\left(\frac{n}{2}(t-\pi)\right)$$

$$f = -\frac{4A}{3\pi}\cdot\sin\left(\frac{1}{2}(t-\pi)\right) - \frac{A}{2}\cdot\sin(t-\pi) - \frac{A}{\pi}\cdot\sum_{n=3}^{1000}\frac{\sin\left(\frac{n\pi}{2}\right)}{1-\frac{n^2}{4}}\cdot\sin\left(\frac{n}{2}(t-\pi)\right)$$

$A = 1$

−10 10