2) Determinar las constantes a, b y c de tal manera que $y''' + ay'' + by' + cy = 0$ tenga la solución:

$$y = c_1 e^{-x} + e^{2x}(c_2 \operatorname{sen} 4x - c_3 \cos 4x)$$

COMO LA EC. AUXILIAR DE $y''' + ay'' + by' + cy = 0$ ES

$$\lambda^3 + a\lambda^2 + b\lambda + c = 0$$

ESTO IMPLICA QUE LA EC. DIFERENCIAL TIENE 3 RAICES: $\lambda_1, \lambda_2, \lambda_3$

Y COMO LA SOLUCION ES:

$$y = \underbrace{c_1 e^{-x}}_{\text{REAL}} + \underbrace{e^{-2x}(c_2 \operatorname{sen} 4x - c_3 \cos 4x)}_{\text{COMPLEJO}}$$

POR LA FORMA: $y = e^{\alpha x}(A\cos\beta x + B \operatorname{sen}\beta x)$

$\lambda_1 = -1$

$\lambda_2 = \alpha + i\beta \longrightarrow \lambda_2 = -2 + 4i$

$\lambda_3 = \alpha - i\beta \longrightarrow \lambda_3 = -2 - 4i$

POR LO TANTO: $(\lambda+1)[\lambda+(2-4i)][\lambda+(2+4i)] = 0$

$$(\lambda+1)(\lambda+(2-4i))(\lambda+(2+4i)) = 0$$

$$[\lambda^2 + (2-4i)\lambda + \lambda + (2-4i)][\lambda + (2+4i)] = 0$$

$$\lambda^3 + (2-4i)\lambda^2 + \lambda^2 + (2-4i)\lambda + (2+4i)\lambda^2 + (2+4i)(2-4i)\lambda + (2-4i)\lambda + (2+4i)(2-4i) = 0$$

$$\lambda^3 + (2-4i)\lambda^2 + \lambda^2 + (2-4i)\lambda + (2+4i)\lambda^2 + (4-8i+8i-16i^2)\lambda + (2+4i)\lambda + (4-\cancel{8i}+\cancel{8i}-16i^2) = 0$$

$$\lambda^3 + (2-\cancel{4i}+2+\cancel{4i}+1)\lambda^2 + (2-\cancel{4i}+4-\cancel{8i}+\cancel{8i}-16i^2+2+\cancel{4i})\lambda + (20) = 0$$

$$\lambda^3 + (5)\lambda^2 + (2+4+16)\lambda + 20 = 0$$

$$\lambda^3 + (5)\lambda + (24)\lambda + 20 = 0$$

COMO ES DE LA FORMA:

$$\lambda^3 + a\lambda^2 + b\lambda + c = 0$$

$$\boxed{a = 5 \;,\; b = 24 \;,\; c = 20}$$

Tema:

3) Resolver la sig. ecuación diferencial

$$(x^2+2y')y''+2xy'=0$$

$y'=p$

$y''=p'$

$(x^2+2p)p'+2xp=0$

$(x^2+2p)p'=-2xp$

$p'=-\dfrac{2xp}{x^2+2p} \longrightarrow \dfrac{1}{p'}=-\dfrac{x^2+2p}{2px}$

$\dfrac{1}{p'}=-\dfrac{x^2}{2px}+\cancel{\dfrac{2p}{2px}} \longrightarrow \dfrac{1}{p'}=-\dfrac{x}{2p}+\dfrac{1}{x}$

$p'=-\dfrac{2p}{x}+x$

$p'+\overbrace{\dfrac{2}{x}}^{Q(x)}p=\overbrace{x}^{S(x)} \longrightarrow p=p_h+p_p$

$p_h=c_1e^{-\int Q(x)} \longrightarrow p_h=c_1e^{-\int \frac{2}{x}dx} \therefore p_h=c_1e^{-2\ln|x|}$

$p_h=c_1\cancel{e}^{\cancel{\ln}|x^{-2}|}$

$p_h=c_1x^{-2} \longrightarrow p_h=c_1x^{-2}$

$p_p=e^{-\int Q(x)}\int e^{\int Q(x)}\cdot S(x)\,dx \longrightarrow p_p=x^{-2}\int x^2\cdot x\,dx$

$p_p=x^{-2}\int x^3\,dx \longrightarrow p_p=x^{-2}\dfrac{x^4}{4}$

$p_p=\dfrac{x^2}{4}$

$p=c_1x^{-2}+\dfrac{x^2}{4}$

Sust y' por p

$y'=c_1x^{-2}+\dfrac{x^2}{4}$

Tema:

$$\frac{dy}{dx} = C_1 x^{-2} + \frac{x^2}{4}$$

$$\int dy = \int C_1 x^{-2} + \frac{x^2}{4}$$

$$y = \int C_1 x^{-2} + \int \frac{x^2}{4} \longrightarrow y = C_1\left(-\frac{1}{x}\right) + \frac{1}{4}\left(\frac{x^3}{3}\right) + C_2$$

$$y = \frac{1}{12}x^3 - \frac{1}{x}C_1 + \underline{C_2}$$

Nombre:
Tema:

Día	Mes	Año	Folio

4) Resolver la ecuación diferencial por el método de coeficientes indeter.

$$y'' + 2y' = 2x + 5 - e^{-2x}$$

$r^2 + 2r = 0$

$(r^2 + 2r) = 0$

$r(r+2) = 0$

$\therefore \boxed{r_1 = 0 \wedge r_2 = -2}$

$Y_c = C_1 + C_2 e^{-2x}$

Comparando LA EC. DIF CON LA FORMA GENERAL $y'' + 2y' = g(x)$

$$y'' + 2y' = 2x + 5 - e^{-2x}$$
$$y'' + 2y' = g(x)$$

$=$

$\therefore \quad g(x) = 2x + 5 - e^{-2x} \longrightarrow$ SOL. PARTICULAR

COMO ES UNA SOL PARTICULAR ASUMIMOS QUE EN FORMA CUADRÁTICA SE EXPRESARÁ:

$$Y_p = Ax^2 + Bx + Cxe^{-2x}$$

DERIVANDO Y_p POR PRIMERA VEZ

$Y_p' = 2Ax + B + C(e^{-2x} - 2xe^{-2x})$

DERIVANDO Y_p POR SEGUNDA VEZ

$Y_p'' = 2A + C(-2e^{-2x} - 2(-2xe^{-2x} + e^{-2x}))$

$Y_p'' = 2A + C(-2e^{-2x} + 4xe^{-2x} - 2e^{-2x})$

$Y_p'' = 2A + 4Cxe^{-2x} - 4Ce^{-2x}$

Nombre:

Tema:

SUST EN $y'' + 2y' = 2x + 5 - e^{-2x}$ LA EC y_p'' y y_p'

$$2A + 4Cxe^{-2x} - 4Ce^{-2x} + 2(2Ax + B + C(e^{-2x} - 2xe^{-2x}) = 2x + 5 - e^{-2x}$$

$$2A + \cancel{4Cxe^{-2x}} - \cancel{4Ce^{-2x}} + 4Ax + 2B + \cancel{2Ce^{-2x}} - \cancel{4Cxe^{-2x}} = 2x + 5 - e^{-2x}$$

$$2A + 4Ax - 2Ce^{-2x} + 2B = 2x + 5 - e^{-2x}$$

$$4Ax + (2A + 2B) - 2Ce^{-2x} = 2x + 5 - e^{-2x}$$

$4A = 2 \longrightarrow A = \frac{2}{4} \therefore A = \frac{1}{2}$

$2A + 2B = 5 \longrightarrow 2(\frac{1}{2}) + 2B = 5 \rightarrow 1 + 2B = 5 \rightarrow 2B = 4 \therefore B = 2$

$-2C = -1 \longrightarrow C = \frac{-1}{-2} \therefore C = \frac{1}{2}$

SUSTITUYENDO EN y_p

$$y_p = \frac{1}{2}x^2 + 2x + \frac{1}{2}xe^{-2x}$$

$$y = y_c + y_p$$

SUST y_c y y_p PARA OBTENER y

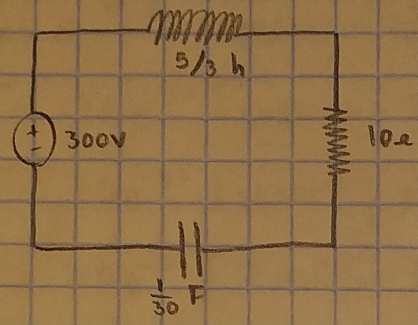
$$y = C_1 + C_2e^{-2x} + \frac{1}{2}x^2 + 2x + \frac{1}{2}xe^{-2x}$$

Nombre:

Tema:

Día | Mes | Año | Folio

5) Encuentre la carga en el capacitor y la corriente en el circuito LRC, con $L\ 5/3\ h$, $R = 10\Omega$, $C = 1/30\ F$, $E(t) = 300V$, $q(0) = 0C$, $I(0) = 0A$. Determine la carga máxima



LKV

$$\frac{5}{3}\frac{dI}{dt} + 10I + 30Q = 300$$

$$\boxed{I = \frac{dQ}{dt} \wedge \frac{dI}{dt} = \frac{d^2Q}{dt^2}}$$

$$\left(\frac{5}{3}\frac{d^2Q}{dt^2} + 10\frac{dQ}{dt} + 30Q = 300\right)\frac{3}{5}$$

$$\frac{d^2Q}{dt^2} + 6\frac{dQ}{dt} + 18Q = 180$$

$$Q'' + 6Q' + 18Q = 180 \ldots (1)$$

$$Q = Qh + Qp$$

PARA Qh

$$\lambda^2 + 6\lambda + 18 = 0$$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

AL SER RAICES COMPLEJAS LA SOLUCIÓN ES DE LA FORMA

$$\lambda = \frac{-6 \pm \sqrt{6^2 - 4(1)(18)}}{2(1)} = \frac{-6 \pm \sqrt{36 - 72}}{2}$$

$$y_h = e^{\alpha x}\left[A\cos\beta x + B\operatorname{sen}\beta x\right]$$

$$\lambda = \frac{-6 \pm \sqrt{-36}}{2} \quad \therefore \quad \lambda = \frac{-6 \pm 6i}{2}$$

$$Qh = e^{-3t}\left[C_1\cos(3t) + C_2\operatorname{sen}(3t)\right]$$

PARA LA ECUACION NO HOMOGENEA (PARTICULAR) SE TIENE QUE $Qp = A$

$$\lambda_1 = -3 + 3i \qquad \lambda = -3 - 3i$$

DONDE $\lambda = \alpha + \beta i$

$$Qp' = 0$$

$$\alpha = -3 \quad \wedge \quad \beta = 3$$

$$Qp'' = 0$$

SUST Qp, Qp' y Qp'' EN (1)

$$\cancel{0} + \cancel{6(0)} + 18A = 180$$

$$A = \frac{180}{18} \quad \therefore \quad A = 10 \longrightarrow Qp = 10$$

$Q = Q_h + Q_p$

$Q = e^{-3t}\left[C_1 \cos(3t) + C_2 \operatorname{sen}(3t)\right] + 10$

Si $Q(0) = 0$

$0 = e^{-3(0)}\left[C_1 \cos(3(0)) + C_2 \operatorname{sen}(3(0))\right] + 10$

$0 = C_1 + 10 \quad \therefore \quad C_1 = -10$

SUSTITUYENDO C_1

$Q = e^{-3t}\left[-10 \cos(3t) + C_2 \operatorname{sen}(3t)\right] + 10$

COMO $Q' = I$, DERIVAMOS Q

$Q' = -3e^{-3t}\left[-10\cos(3t) + C_2 \operatorname{sen}(3t)\right] + e^{-3t}\left[30 \operatorname{sen}(3t) + 3C_2 \cos(3t)\right]$

$Q' = e^{-3t}\left[30 \operatorname{sen}(3t) + 3C_2 \cos(3t)\right] - 3e^{-3t}\left[-10\cos(3t) + C_2 \operatorname{sen}(3t)\right]$

Si $I(0) = 0 \rightarrow Q'(0) = 0$

$0 = \cancel{e^{-3(0)}}^{1}\left[\cancel{30 \operatorname{sen}(3(0))}^{0} + 3C_2 \cancel{\cos(3(0))}^{1}\right] - 3\cancel{e^{-3(0)}}^{1}\left[-10\cancel{\cos(3(0))}^{1} + \cancel{C_2 \operatorname{sen}(3(0))}^{0}\right]$

$0 = 3C_2 + 30 \rightarrow 3C_2 = -30$

$C_2 = -\frac{30}{3} \quad \therefore \quad C_2 = -10$

SUST EN Q

$Q = e^{-3t}\left[-10\cos(3t) - 10 \operatorname{sen}(3t)\right] + 10$

$\therefore \quad Q = 10 - 10e^{-3t}\left[\cos(3t) - \operatorname{sen}(3t)\right]$

SUST EN $Q' \rightarrow I$

$$I = e^{-3t}\left[30\,sen(3t) + 3(-10)\cos(3t)\right] - 3e^{-3t}\left[-10\cos(3t) + (-10)\,sen(3t)\right]$$

$$I = e^{-3t}\left[30\,sen(3t) - 30\cos(3t)\right] - 3e^{-3t}\left[-10\cos(3t) - 10\,sen(3t)\right]$$

$$I = 30e^{-3t}\left[\cos(3t) + sen(3t)\right] + 30e^{-3t}\left[sen(3t) - \cos(3t)\right]$$

$$I = 30e^{-3t}\left[\cancel{\cos(3t)} + sen(3t) + sen(3t) - \cancel{\cos(3t)}\right]$$

$$I = 30e^{-3t}\left[2\,sen(3t)\right] \quad \therefore \quad I = 60e^{-3t}\left[sen(3t)\right]$$

PARA CARGA EN CAPACITOR POR LKV

$$Q_C = 30\,Q$$

$$Q_C = 30\left(10 - 10e^{-3t}\left[\cos(3t) - sen(3t)\right]\right)$$

$$Q_C = 30 - 30e^{-3t}\left[\cos(3t) - sen(3t)\right]$$

$$Q_C = 30\left(1 - e^{-3t}\left[\cos(3t) - sen(3t)\right]\right.$$

PARA CARGA MAX

$$\lim_{t \to \infty} Q(t) = Q_{MAX} \longrightarrow Q_{MAX} = 10 - \cancel{10e^{-3(\infty)}}^{\,0}\left[\cos(3(\infty)) - sen(3(\infty))\right]$$

$$Q_{MAX} = 10$$

Tema:

6) Resuelva la ecuación diferencial dada

$$x^3 \frac{d^3y}{dx^3} - 2x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 8y = 0$$

$x^3y''' - 2x^2y'' - 2xy' + 8y = 0 \longrightarrow$ $\boxed{\text{CAUCHY EULER}}$

$y = x^r$

$y' = rx^{r-1}$

$y'' = r(r-1)x^{r-2}$

$y''' = r(r-1)(r-2)x^{r-3}$

SUST EN LA EC DIFERENCIAL

$x^3[(r)(r-1)(r-2)x^{r-3}] - 2x^2[(r)(r-1)x^{r-2}] - 2x[rx^{r-1}] + 8[x^r] = 0$

$[(r)(r-1)(r-2)\cancel{(x^3)(x^{r-3})}^{x^r}] - 2[(r)(r-1)\cancel{(x^2)(x^{r-2})}^{x^r}] - 2[r\cancel{(x)(x^{r-1})}^{x^r}] + 8x^r = 0$

$[(r)(r-1)(r-2)x^r] - 2[(r)(r-1)x^r] - 2[rx^r] + 8x^r = 0$

$x^r[(r^2-r)(r-2)] - x^r[2(r^2-r)] - x^r[2r] + x^r(8) = 0$

$x^r[r^3 - 2r^2 - r^2 + 2r] - x^r[2r^2 - 2r] - x^r[2r] + x^r(8) = 0$

$x^r[r^3 - 2r^2 - r^2 + 2r - 2r^2 + \cancel{2r} - \cancel{2r} + 8] = 0$

$x^r[r^3 - 5r^2 + 2r + 8] = 0$

COMO $y = x^r$, SERA SOLUCIÓN DE LA EC. DIFERENCIAL SIEMPRE QUE "r" SEA RAIZ DE LA ECUACIÓN CUBICA:

$$r^3 - 5r^2 + 2r + 8 = 0$$

Nombre:	Día	Mes	Año	Folio
Tema:				

POR DIVISIÓN SINTETICA

1	-5	2	8	
	-1	6	-8	-1
(1)	-6	8	0	

$\therefore \; \underline{r_1 = -1}$

POR LO QUE

$(r+1)(r^2 - 6r + 8) = 0$

FACTORIZANDO: $r^2 - 6r + 8$

$(r-4)(r-2) = 0$

$r - 4 = 0 \qquad r - 2 = 0$

$r = 4 \qquad r = 2$

$\underline{r_2 = 4} \qquad \underline{r_3 = 2}$

ENTONCES: $r^3 - 5r^2 + 2r + 8 = 0$

$(r+1)(r-4)(r-2) = 0 \qquad \boxed{\text{CON } r_1 = -1,\ r_2 = 4,\ r_3 = 2}$

COMO LAS RAICES SON REALES Y DIFERENTES ENTRE SI, LA SOLUCIÓN DE LA EC. DIFERENCIAL GENERAL ES DE LA FORMA: $y = c_1 x^{r_1} + c_2 x^{r_2} + c_3 x^{r_3}$

$$\therefore \quad \underline{y = C_1 x^{-1} + C_2 x^{4} + C_3 x^{2}}$$