

# Demonstration about the insolubility of the Solitaire Mancala

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## Hypothesis:

### 3.1 Solitaire Mancala Rules

**SR001: Initial Setup – Mancala board.** The game shall be played using a standard Mancala board consisting of two rows, where each row has 6 holes and 1 store. The board is defined as:

store01	hole01	hole02	hole03	hole04	hole05	hole06	store02
	hole07	hole08	hole09	hole10	hole11	hole12	

**SR002: Initial Setup – initial configuration.** The game shall be initiated with 48 stones randomly split into the 12 holes (hole01, hole02, ... hole12; see SR001), leaving each hole with at least 1 stone (all 48 stones shall all be placed in the board, leaving the stores empty).

**SR003: Selection of hole before first round.** The player shall select a hole (in any row, thus any of the 12 holes). **SR004:** A round. When a hole is selected, all stones in the selected hole are distributed either clockwise or anti-clockwise (the user can choose), as follows:

**SR004(A): Dropping clockwise.** One stone is placed in each hole starting with the hole next to the selected one, in clockwise direction:

- If the number of stones remaining to be distributed is more than 1 after dropping in hole06 or hole07, then store02 or store01 respectively is skipped, and the next stone is dropped in hole12 or hole01 respectively. The round is over when there are no more stones to distribute. If the game is not over (see SR005 and SR006), a new round starts as 4 described in SR004, where the selected hole will be the one where the last stone of this round was dropped.
- If the number of stones remaining to be distributed is 1 after dropping in hole06, this stone is dropped in store02. The round is over. If the game is not over (see SR005 and SR006), a new round starts as described in SR004, where the selected hole will be hole06.
- If the number of stones remaining to be distributed is 1 after dropping in hole07, this stone is dropped in store01. The round is over. If the game is not over (see SR005 and SR006), a new round starts as described in SR004, where the selected hole will be hole07.

**SR004(B): Dropping anti-clockwise.** One stone is placed in each hole starting with the hole next to the selected one, in anti-clockwise direction:

- If the number of stones remaining to be distributed is more than 1 after dropping in hole01 or hole12, the store01 or store02 respectively is skipped, and the next stone is dropped in hole07 or hole06 respectively. The round is over when there are no more stones to distribute. If the game is not over (see SR005 and SR006), a new round starts as described in SR004, where the selected hole will be the one where the last stone of this round was dropped.

- SR006: Game over – player wins.** The player wins the game if no stone remains in any of the 12 holes.

The condition described in paragraph SR006 is unreachable using game rules.

store01	x	x	x	x	x	x	store02
	x	x	x	x	x	x	

## Demonstration:

### D1:

- if selected hole is not equal at hole01, hole06, hole07, hole12 and the number of stones in the selected hole is equal to 1 then the configuration is a game over losers configurations.

$$S_1$$

store01	x	<b>1</b>	x	x	x	x	store02
	x	x	x	x	x	x	

$$S_2$$

store01	x	x	<b>1</b>	x	x	x	store02
	x	x	x	x	x	x	

$$S_3$$

store01	x	x	x	<b>1</b>	x	x	store02
	x	x	x	x	x	x	

$$S_4$$

store01	x	x	x	x	<b>1</b>	x	store02
	x	x	x	x	x	x	

$$S_5$$

store01	x	x	x	x	x	x	store02
	x	<b>1</b>	x	x	x	x	

$$S_6$$

store01	x	x	x	x	x	x	store02
	x	x	<b>1</b>	x	x	x	

$$S_7$$

store01	x	x	x	x	x	x	store02
	x	x	x	<b>1</b>	x	x	

$$S_8$$

store01	x	x	x	x	x	x	store02
	x	x	x	x	<b>1</b>	x	

$S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8 \in G$  (see SR004(A), SR004(B) and SR005)

### D2:

- if  $s_i$  is a game over losers configurations then it has not successors.

$S(s_1), S(s_2), S(s_3), S(s_4), S(s_5), S(s_6), S(s_7), S(s_8) \in \emptyset$  (see SR005)

**D3:**

- if  $s_i$  is a game over losers configurations then it is not a valid predecessor (see predecessor definition and D2).

$$s_i \in G \Rightarrow \nexists s_j \mid s_i \in P(s_j)$$

**D4:**

- to win the game you must reach the game over goal configurations  $s_9$  (see SR006).

$s_9$


store01	0	0	0	0	0	0	store02
	0	0	0	0	0	0	

**D5:**

- valid predecessors of the game over goal configurations  $s_9$  (see predecessor definition).

**P( $s_9$ ):**

$s_{10}$




store01	<b>1</b>	0	0	0	0	0	store02
	0	0	0	0	0	0	

$s_{10} \rightarrow \text{SR004(B): Dropping anti-clockwise.} \rightarrow s_9$


$s_{11}$

store01	0	0	0	0	0	<b>1</b>	store02
	0	0	0	0	0	0	



$s_{11} \rightarrow \text{SR004(A): Dropping clockwise.} \rightarrow s_9$

$s_{12}$




store01	0	0	0	0	0	0	store02
	<b>1</b>	0	0	0	0	0	

$s_{12} \rightarrow \text{SR004(A): Dropping clockwise.} \rightarrow s_9$


$s_{13}$

store01	0	0	0	0	0	0	store02
	0	0	0	0	0	<b>1</b>	




$s_{13} \rightarrow \text{SR004(B): Dropping anti-clockwise.} \rightarrow s_9$

**P(s<sub>10</sub>):**



s <sub>14</sub>							
store01	0	<b>2</b>	0	0	0	0	store02
	0	0	0	0	0	0	

s<sub>14</sub> → SR004(B): Dropping anti-clockwise. → s<sub>10</sub>




s <sub>15</sub>							
store01	0	<b>1</b>	0	0	0	0	store02
	0	0	0	0	0	0	

s<sub>15</sub> → SR004(B): Dropping anti-clockwise. → s<sub>10</sub>


s<sub>15</sub> is not a valid predecessor of s<sub>10</sub> (see D1 and D3)

**P(s<sub>14</sub>):**



s <sub>16</sub>							
store01	<b>1</b>	1	0	0	0	0	store02
	0	0	0	0	0	0	

s<sub>16</sub> → SR004(A): Dropping clockwise. → s<sub>14</sub>




s <sub>17</sub>							
store01	0	1	<b>1</b>	0	0	0	store02
	0	0	0	0	0	0	

s<sub>17</sub> → SR004(B): Dropping anti-clockwise. → s<sub>14</sub>


s<sub>17</sub> is not a valid predecessor of s<sub>14</sub> (see D1 and D3)

**P(s<sub>16</sub>):**



s <sub>18</sub>							
store01	0	0	<b>2</b>	0	0	0	store02
	0	0	0	0	0	0	


s<sub>18</sub> → SR004(B): Dropping anti-clockwise. → s<sub>16</sub>



s <sub>19</sub>							
store01	0	0	<b>3</b>	0	0	0	store02
	0	0	0	0	0	0	

s<sub>19</sub> → SR004(B): Dropping anti-clockwise. → s<sub>16</sub>


**P(s<sub>18</sub>):**



s <sub>20</sub>							
store01	0	<b>1</b>	1	0	0	0	store02
	0	0	0	0	0	0	

s<sub>20</sub> → SR004(A): Dropping clockwise. → s<sub>18</sub>

s<sub>20</sub> is not a valid predecessor of s<sub>18</sub> (see D1 and D3)




s <sub>21</sub>							
store01	0	0	1	<b>1</b>	0	0	store02
	0	0	0	0	0	0	

s<sub>21</sub> → SR004(B): Dropping anti-clockwise. → s<sub>18</sub>

s<sub>21</sub> is not a valid predecessor of s<sub>18</sub> (see D1 and D3)


**P(s<sub>19</sub>):**



$s_{22}$							
store01	0	1	2	0	0	0	store02
	0	0	0	0	0	0	

$s_{22} \rightarrow \text{SR004(A): Dropping clockwise.} \rightarrow s_{19}$

$s_{22}$  is not a valid predecessor of  $s_{19}$  (see D1 and D3)



$s_{23}$							
store01	0	0	2	1	0	0	store02
	0	0	0	0	0	0	

$s_{23} \rightarrow \text{SR004(B): Dropping anti-clockwise.} \rightarrow s_{19}$

$s_{23}$  is not a valid predecessor of  $s_{19}$  (see D1 and D3)

**D6:**

- $s_9, s_{10}, s_{11}, s_{12}, s_{13}, s_{14}, s_{16}, s_{18}, s_{19} \notin I$   
 $\exists \text{ hole}_i \mid \text{stones of hole}_i = 0$  (see SR002)

**D7:**

- a state  $s_i$  is unreachable using game rules if  
 $P(s_i) \in \emptyset$  or  $(\forall s_j \in P(s_i) \mid s_j \notin I \text{ and } s_j \text{ is unreachable using game rules})$   
 (see SR002, SR003, SR004(A) and SR004(B))

**Thesis:**

- $s_9$  is unreachable using game rules (see D5, D6 and D7)

(There are demonstrations similar to configurations  $s_{11}, s_{12}, s_{13}$ .)

**Q.E.D.**

