Demonstration about the insolubility of the Solitaire Mancala

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Hypothesis:

3.1 Solitaire Mancala Rules

SR001: Initial Setup – Mancala board. The game shall be played using a standard Mancala board consisting of two rows, where each row has 6 holes and 1 store. The board is defined as:

store01	hole01	hole02	hole03	hole04	hole05	hole06	store02
	hole07	hole08	hole09	hole10	hole11	hole12	

SR002: Initial Setup – initial configuration. The game shall be initiated with 48 stones randomly split into the 12 holes (hole01, hole02, … hole12; see SR001), leaving each hole with at least 1 stone (all 48 stones shall all be placed in the board, leaving the stores empty).

SR003: Selection of hole before first round. The player shall select a hole (in any row, thus any of the 12 holes). SR004: A round. When a hole is selected, all stones in the selected hole are distributed either clockwise or anti-clockwise (the user can choose), as follows:

SR004(A): Dropping clockwise. One stone is placed in each hole starting with the hole next to the selected one, in clockwise direction:

- If the number of stones remaining to be distributed is more than 1 after dropping in hole06 or hole07, then store02 or store01 respectively is skipped, and the next stone is dropped in hole12 or hole01 respectively. The round is over when there are no more stones to distribute. If the game is not over (see SR005 and SR006), a new round starts as 4 described in SR004, where the selected hole will be the one where the last stone of this round was dropped.
- If the number of stones remaining to be distributed is 1 after dropping in hole06, this stone is dropped in store02. The round is over. If the game is not over (see SR005 and SR006), a new round starts as described in SR004, where the selected hole will be hole06.
- If the number of stones remaining to be distributed is 1 after dropping in hole07, this stone is dropped in store01. The round is over. If the game is not over (see SR005 and SR006), a new round starts as described in SR004, where the selected hole will be hole07.

SR004(B): Dropping anti-clockwise. One stone is placed in each hole starting with the hole next to the selected one, in anti-clockwise direction:

- If the number of stones remaining to be distributed is more than 1 after dropping in hole01 or hole12, the store01 or store02 respectively is skipped, and the next stone is dropped in hole07 or hole06 respectively. The round is over when there are no more stones to distribute. If the game is not over (see SR005 and SR006), a new round starts as described in SR004, where the selected hole will be the one where the last stone of this round was dropped.

- If the number of stones remaining to be distributed is 1 after dropping in hole01, this stone is dropped in store01. The round is over. If the game is not over (see SR005 and SR006), a new round starts as described in SR004, where the selected hole will be hole01.
- If the number of stones remaining to be distributed is 1 after dropping in hole12, this stone is dropped in store02. The round is over. If the game is not over (see SR005 and SR006), a new round starts as described in SR004, where the selected hole will be hole12.

SR005: Game over – player loses. When the last stone distributed in a round is placed in an empty hole, the player loses and the game is over.

SR006: Game over – player wins. The player wins the game if no stone remains in any of the 12 holes.

Thesis:

The player never wins.

The condition described in paragraph SR006 is unreachable using game rules.

store01	0	0	0	0	0	0	store02
	0	0	0	0	0	0	

Definitions:

$\mathbf{s_i}$: valid game configuration (see SR001, SR002 and SR003);

I: set of initial configurations (see SR002);

G: set of game over losers configurations (see SR005);

x: number of stones | $0 \leq x \leq 48$;

$S(s_i)$: set of valid successors of the game configuration s_i (a successor of s_i is a valid game configuration reachable from s_i using game rules (see SR004(A) and SR004(B)));

$P(s_i)$: set of valid predecessors of the game configuration s_i (a predecessor of s_i is a valid game configuration from which it is possible to reach s_i using game rules (see SR004(A) and SR004(B)));

s_i → SR004(A): Dropping clockwise. → s_j: s_j is reachable from s_i using game rules (see SR004(A));

s_i → SR004(B): Dropping anti-clockwise. → s_j: s_j is reachable from s_i using game rules (see SR004(B));

hole: is the selected hole.

e.g.

store01	x	**x**	x	x	x	x	store02
	x	x	x	x	x	x	

Demonstration:

D1:

- if selected hole is not equal at hole01, hole06, hole07, hole12 and the number of stones in the selected hole is equal to 1 then the configuration is a game over losers configuration.

s_1

store01	x	**1**	x	x	x	x	store02
	x	x	x	x	x	x	

s_2

store01	x	x	**1**	x	x	x	store02
	x	x	x	x	x	x	

s_3

store01	x	x	x	**1**	x	x	store02
	x	x	x	x	x	x	

s_4

store01	x	x	x	x	**1**	x	store02
	x	x	x	x	x	x	

s_5

store01	x	x	x	x	x	x	store02
	x	**1**	x	x	x	x	

s_6

store01	x	x	x	x	x	x	store02
	x	x	**1**	x	x	x	

s_7

store01	x	x	x	x	x	x	store02
	x	x	x	**1**	x	x	

s_8

store01	x	x	x	x	x	x	store02
	x	x	x	x	**1**	x	

$s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8 \in G$ (see SR004(A), SR004(B) and SR005)

D2:

- if s_i is a game over losers configuration then it has not successors.

$S(s_1), S(s_2), S(s_3), S(s_4), S(s_5), S(s_6), S(s_7), S(s_8) \in \emptyset$ (see SR005)

D3:

- if s_i is a game over losers configuration then it is not a valid predecessor (see predecessor definition and D2).

$s_i \in G \Rightarrow \nexists\ s_j \mid s_i \in P(s_j)$

D4:

- to win the game you must reach the game over goal configuration s_9 (see SR006).

s_9

store01	0	0	0	0	0	0	store02
	0	0	0	0	0	0	

D5:

- valid predecessors of the game over goal configuration s_9 (see predecessor definition).

$P(s_9)$:

s_{10}

store01	**1**	0	0	0	0	0	store02
	0	0	0	0	0	0	

$s_{10} \rightarrow$ SR004(B): Dropping anti-clockwise. $\rightarrow s_9$

s_{11}

store01	0	0	0	0	0	**1**	store02
	0	0	0	0	0	0	

$s_{11} \rightarrow$ SR004(A): Dropping clockwise. $\rightarrow s_9$

s_{12}

store01	0	0	0	0	0	0	store02
	1	0	0	0	0	0	

$s_{12} \rightarrow$ SR004(A): Dropping clockwise. $\rightarrow s_9$

s_{13}

store01	0	0	0	0	0	0	store02
	0	0	0	0	0	**1**	

$s_{13} \rightarrow$ SR004(B): Dropping anti-clockwise. $\rightarrow s_9$

P(s_{10}):

s_{14}

store01	0	**2**	0	0	0	0	store02
	0	0	0	0	0	0	

s_{14} → SR004(B): Dropping anti-clockwise. → s_{10}

s_{15}

store01	0	**1**	0	0	0	0	store02
	0	0	0	0	0	0	

s_{15} → SR004(B): Dropping anti-clockwise. → s_{10}

s_{15} is not a valid predecessor of s_{10} (see D1 and D3)

P(s_{14}):

s_{16}

store01	**1**	1	0	0	0	0	store02
	0	0	0	0	0	0	

s_{16} → SR004(A): Dropping clockwise. → s_{14}

s_{17}

store01	0	1	**1**	0	0	0	store02
	0	0	0	0	0	0	

s_{17} → SR004(B): Dropping anti-clockwise. → s_{14}

s_{17} is not a valid predecessor of s_{14} (see D1 and D3)

P(s_{16}):

s_{18}

store01	0	0	**2**	0	0	0	store02
	0	0	0	0	0	0	

s_{18} → SR004(B): Dropping anti-clockwise. → s_{16}

s_{19}

store01	0	0	**3**	0	0	0	store02
	0	0	0	0	0	0	

s_{19} → SR004(B): Dropping anti-clockwise. → s_{16}

P(s_{18}):

s_{20}

store01	0	**1**	1	0	0	0	store02
	0	0	0	0	0	0	

s_{20} → SR004(A): Dropping clockwise. → s_{18}

s_{20} is not a valid predecessor of s_{18} (see D1 and D3)

s_{21}

store01	0	0	1	**1**	0	0	store02
	0	0	0	0	0	0	

s_{21} → SR004(B): Dropping anti-clockwise. → s_{18}

s_{21} is not a valid predecessor of s_{18} (see D1 and D3)

P(s_{19}):

s_{22}

store01	0	**1**	2	0	0	0	store02
	0	0	0	0	0	0	

s_{22} → SR004(A): Dropping clockwise. → s_{19}

s_{22} is not a valid predecessor of s_{19} (see D1 and D3)

s_{23}

store01	0	0	2	**1**	0	0	store02
	0	0	0	0	0	0	

s_{23} → SR004(B): Dropping anti-clockwise. → s_{19}

s_{23} is not a valid predecessor of s_{19} (see D1 and D3)

D6:

- $s_9, s_{10}, s_{11}, s_{12}, s_{13}, s_{14}, s_{16}, s_{18}, s_{19} \notin I$

$\exists$ $hole_i$ | stones of $hole_i = 0$ (see SR002)

D7:

- a state s_i is unreachable using game rules if

$P(s_i) \in \emptyset$ *or* $(\forall s_j \in P(s_i) \mid s_j \notin I$ *and* s_j is unreachable using game rules)

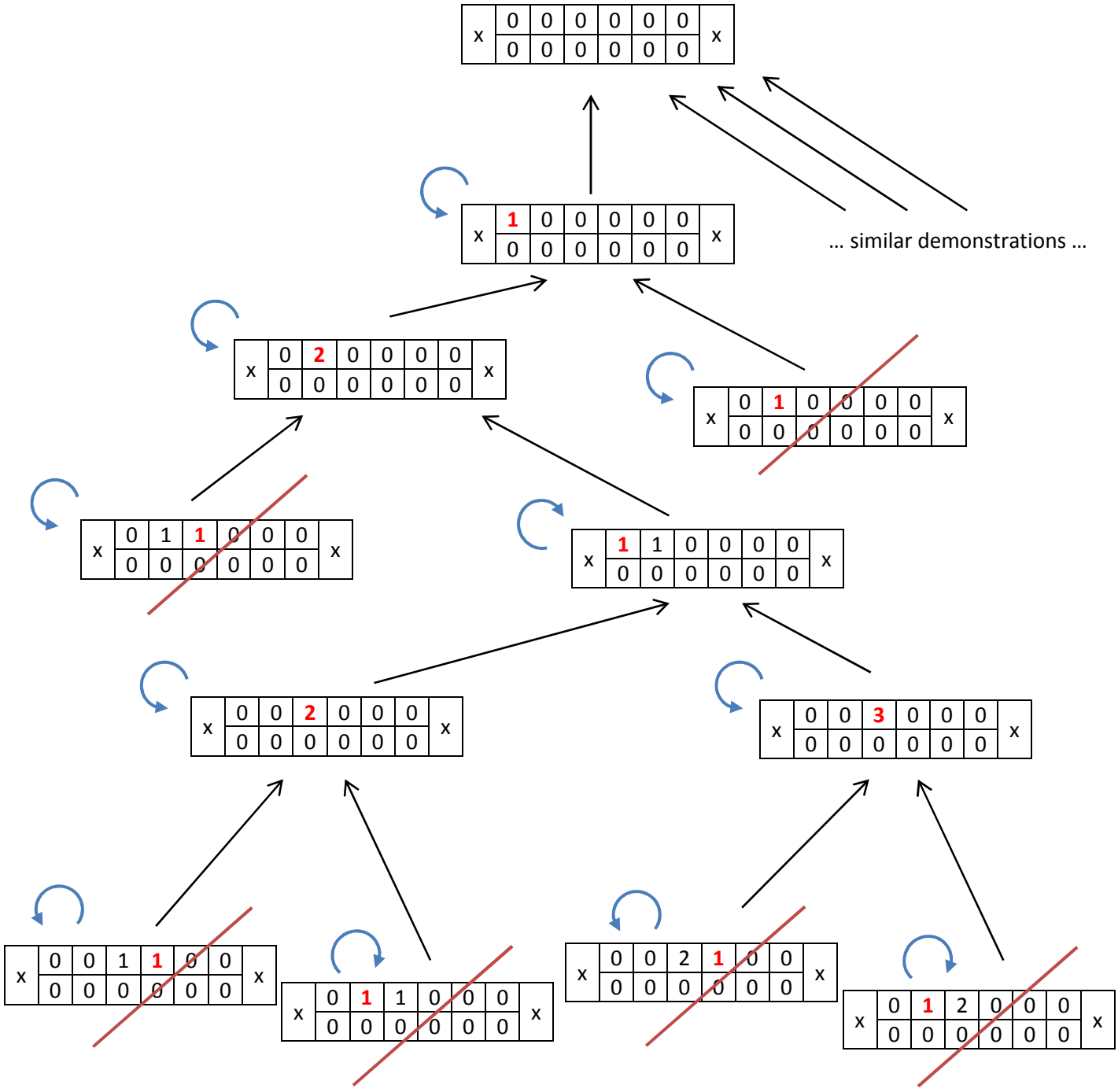
(see SR002, SR003, SR004(A) and SR004(B))

Thesis:

- s_9 is unreachable using game rules (see D5, D6 and D7)

(There are similar demonstrations for configurations s_{11}, s_{12}, s_{13}.)

Q.E.D.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