## 1 Variables

## 2 root

|   | var       | symbol  | documentation               | type     | units | eqs |
|---|-----------|---------|-----------------------------|----------|-------|-----|
| 8 | $F_{N,A}$ | F_N_A   | fudamental incidence matrix | network  |       |     |
| 5 | t         | t       | time                        | frame    | s     |     |
| 6 | $t^o$     | to      | starting time               | frame    | s     | 4   |
| 7 | $t^e$     | te      | end time                    | frame    | s     | 5   |
| 1 | #         | value   | numerical value             | constant |       |     |
| 2 | 1         | one     | numerical value one         | constant |       | 1   |
| 3 | 0         | zero    | numerical value zero        | constant |       | 2   |
| 4 | 0.5       | onehalf | numerical value one half    | constant |       | 3   |

# 3 physical

|     | var              | symbol   | documentation                                    | type       | units                     | eqs  |
|-----|------------------|----------|--|------------|---------------------------|------|
| 9   | $P_{N,A}$        | P_N_A    | projection from node to arc for arc properties   | projection |                           |      |
| 32  | $P_{NS,AS}$      | P_NS_AS  | projection node species to arc species           | projection |                           |      |
| 33  | $P_{K,NK}$       | P_K_NK   | projection of conversion to node conversion      | projection |                           |      |
| 34  | $P_{S,NS}$       | P_S_NS   | projection species to node species               | projection |                           |      |
| 35  | $P_{N,NK}$       | P_N_NK   | projection node to node conversion               | projection |                           |      |
| 36  | $P_{NS,KS}$      | P_NS_KS  | projection node species to conversion species    | projection |                           |      |
| 37  | $P_{A,NS}$       | P_A_NS   | projection arc to node species for conductivity  | projection |                           |      |
| 65  | $P_{NK,KS}$      | P_NK_KS  | projection node conversion to conversion species | projection |                           |      |
| 162 | $P_{NN}S_{N,NS}$ | P_N_NS   | projection of nodes onto the node species        | projection |                           |      |
| 10  | $r_{xN}$         | r_x      | x-coordinate                                     | frame      | m                         |      |
| 11  | $r_{yN}$         | r_y      | y-coordinate                                     | frame      | $\mid m \mid$             |      |
| 12  | $r_{zN}$         | r_z      | z coordinate                                     | frame      | m                         |      |
| 13  | $U_N$            | U        | fundamental state – internal energy              | state      | $kgm^2s^{-2}$             |      |
| 14  | $S_N$            | S        | fundamental state – entropy                      | state      | $kg  m^2  K^{-1}  s^{-2}$ |      |
| 15  | $V_N$            | V        | fundamental state – volume                       | state      | $m^3$                     |      |
| 16  | $n_{NS}$         | n        | fundamental state – molar mass                   | state      | mol                       | 86   |
| 20  | $H_N$            | Н        | enthalpy   | state      | $kg m^2 s^{-2}$           | 9 87 |
| 21  | $A_N$            | A        | Helmholtz energy                                 | state      | $kg m^2 s^{-2}$           | 10   |
| 22  | $G_N$            | G        | Gibbs free energy                                | state      | $kgm^2s^{-2}$             | 11   |
| 23  | $C_N$            | charge   | fundamental state – charge                       | state      | As                        |      |
| 24  | $A^v$            | Avogadro | Avogadro number                                  | constant   | $mol^{-1}$                |      |
| 17  | $p_N$            | p        | thermodynamic pressure                           | effort     | $kg  m^{-1}  s^{-2}$      | 6    |
| 18  | $T_N$            | Т        | temperature                                      | effort     | K                         | 7    |

|    | var        | symbol  | documentation                  | type           | units                       | eqs   |
|----|------------|---------|--------------------------------|----------------|-----------------------------|-------|
| 19 | $\mu_{NS}$ | chemPot | chemical potential             | effort         | $kg  m^2  mol^{-1}  s^{-2}$ | 8     |
| 27 | $Ue_N$     | Ue      | electrical potential – voltage | effort         | $kg m^2 A^{-1} s^{-3}$      | 14 95 |
| 28 | $v_{xN}$   | v_x     | velocitiy in x-direction       | secondaryState | $ms^{-1}$                   | 15    |
| 29 | $v_{yN}$   | v_y     | velocity in y-direction        | secondaryState | $ms^{-1}$                   | 16    |
| 30 | $v_{zN}$   | V_Z     | velocity in z-direction        | secondaryState | $ms^{-1}$                   | 17    |

### 4 control

|     | var               | symbol      | documentation                                   | type      | units                  | eqs |
|-----|-------------------|-------------|---|-----------|------------------------|-----|
| 136 | $x_N$             | х           | state   | state     |                        | 111 |
| 137 | $xo_N$            | xo          | initial state                                   | state     |                        | 109 |
| 129 | $A_{N,D}$         | A           | dynamic matrix                                  | constant  | $s^{-1}$               |     |
| 130 | $B_{A,D}$         | В           | input matrix                                    | constant  | $s^{-1}$               |     |
| 131 | $C_{N,A}$         | C           | measurement matrix                              | constant  |                        |     |
| 132 | $D_A$             | D           | diagonal event matrix (no dimensional problems) | constant  |                        |     |
| 133 | $y^o{}_A$         | setPoint    | set point                                       | constant  |                        | 119 |
| 134 | $m_A$             | meas        | measurements                                    | constant  |                        |     |
| 135 | $e_A$             | е           | control error                                   | constant  |                        | 108 |
| 139 | $1_{N,D}$         | I_N_D       | space transformation D to N                     | constant  |                        |     |
| 138 | $\dot{x}_D$       | dxdt        | differential state (ABCD) model                 | diffState | $s^{-1}$               | 110 |
| 141 | $\check{I}_N$     | Imeasured   | measured current                                | algebraic | A                      | 113 |
| 143 | $\check{U}^e{}_N$ | UeMeasured  | measured electrical potential                   | algebraic | $kg m^2 A^{-1} s^{-3}$ | 115 |
| 144 | $\check{\xi}$     | addMeasured | measured additive fraction                      | algebraic |                        | 116 |
| 145 | $RComputed_N$     | RComputed   | measured resistance                             | algebraic | $kg m^2 A^{-2} s^{-3}$ | 117 |
| 146 | store             | store       | quantities to be stored                         | algebraic |                        | 118 |
| 154 | $y_A$             | у           | output equation                                 | algebraic |                        | 126 |

### 5 reactions

|     | var             | symbol      | documentation   | type                    | units                           | eqs |
|-----|-----------------|-------------|---|-------------------------|---------------------------------|-----|
| 147 | $P_{NK}$        | P_NK        | reactions per node  | projection              |                                 |     |
| 38  | $K^o{}_K$       | Ко          | Arrhenius frequency factor                                      | constant                | $m^{-3}  mol  s^{-1}$           |     |
| 62  | $E^a{}_{NK}$    | Ea          | Arrhenius activation energy                                     | constant                | $kg  m^2  mol^{-1}  s^{-2}$     | 41  |
| 63  | $K_{NK}$        | K_NK        | Arrhenius reaction 'constant'                                   | constant                | $m^{-3}  mol  s^{-1}$           | 42  |
| 155 | B               | Boltzmann   | Boltzmann constant  | constant                | $kg  m^2  K^{-1}  s^{-2}$       |     |
| 157 | R               | GasConstant | gas constant  | constant                | $kg m^2 mol^{-1} K^{-1} s^{-2}$ | 127 |
| 158 | $N_{K,KS}$      | N_K_KS      | stoichiometry   | constant                |                                 |     |
| 159 | $N_{NK,KS}$     | N_NK_KS     | extended stoichiometrix matrix                                  | constant                |                                 | 128 |
| 60  | $T_{NK}$        | T_NK        | temperature of the reactive system                              | effort                  | K                               | 39  |
| 151 | $c_{NK,KS}$     | С           | concentration matrix reaction per node and species per reaction | secondaryState          | $m^{-3}  mol$                   | 123 |
| 152 | $c^o{}_{NK,KS}$ | со          | norming concentration   | ${\bf secondary State}$ | $m^{-3}  mol$                   | 124 |
| 153 | $x_{NK,KS}$     | x           | matrix of normed, dimensionless mole fractions                  | ${\bf secondary State}$ |                                 | 125 |
| 160 | $\phi_{NK}$     | phi         | probability function for reactions                              | ${\bf secondary State}$ |                                 | 129 |
| 163 | $	ilde{n}_{NS}$ | nProd       | the species production term                                     | ${\bf secondary State}$ | $mol  s^{-1}$                   | 130 |

### 6 material

|     | var             | symbol     | documentation  | type     | units                       | eqs   |
|-----|-----------------|------------|--|----------|-----------------------------|-------|
| 40  | $\lambda_S$     | Mm         | species molecular mass   | constant | $kg  mol^{-1}$              |       |
| 112 | ξ               | additive   | fraction of additives  | constant |                             | 88    |
| 42  | $C_{pN}$        | Ср         | total heat capacity at constant pressure                                   | property | $kg  m^2  K^{-1}  s^{-2}$   | 21    |
| 43  | $C_{VN}$        | Cv         | total heat capacity at constant volume                                     | property | $kg  m^2  K^{-1}  s^{-2}$   | 22    |
| 44  | $k_{xN}^q$      | kq_x       | thermal conductivity in x-direction  | property | $kg K^{-1} s^{-3}$          | 23    |
| 45  | $k_{yN}^q$      | kq_y       | thermal conductivity in y-direction  | property | $kg K^{-1} s^{-3}$          | 24    |
| 46  | $k_{zN}^q$      | kq_z       | thermal conductivity in z-direction'                                       | property | $kg K^{-1} s^{-3}$          | 25    |
| 47  | $k^q{}_N$       | kq         | thermal conductivity   | property | $kg K^{-1} s^{-3}$          | 26    |
| 48  | $k_{xN}^c$      | kc_x       | convecitve mass conductivity in x-direction                                | property | $m^{-1} s$                  | 27    |
| 49  | $k_{yN}^c$      | kc_y       | convecitve mass conductivity in y-direction                                | property | $m^{-1} s$                  | 28    |
| 50  | $k_{zN}^c$      | kc_z       | convecitve mass conductivity in z-direction                                | property | $m^{-1} s$                  | 29    |
| 51  | $k^c{}_N$       | kc         | convective mass conductivity   | property | $m^{-1} s$                  | 30    |
| 52  | $k_{xNS}^d$     | kd_x       | diffusional mass conductivity in x-direction                               | property | $kg^{-1} m^{-4} mol^2 s$    | 31    |
| 53  | $k_{yNS}^d$     | kd_y       | diffusional mass conductivity in y-direction                               | property | $kg^{-1} m^{-4} mol^2 s$    | 32    |
| 54  | $k_{zNS}^d$     | kd_z       | diffusional mass conductivity in z-direction                               | property | $kg^{-1} m^{-4} mol^2 s$    | 33    |
| 55  | $k^d_{NS}$      | kd         | diffusional mass condctivity   | property | $kg^{-1}  m^{-4}  mol^2  s$ | 34    |
| 56  | $h_{NS}$        | h          | partial molar enthalpies   | property | $kg  m^2  mol^{-1}  s^{-2}$ | 35    |
| 59  | $ ho_N$         | density    | density  | property | $kg m^{-3}$                 | 38    |
| 115 | $R^e{}_N$       | elResist   | electrical resistant   | property | $kg m^2 A^{-2} s^{-3}$      | 91 92 |
| 116 | $k^{e,\xi}{}_N$ | elConductC | simple model for the electrical conductivity as a function of the additive | property | $kg^{-1} m^{-2} A^2 s^3$    | 93    |

### 7 macroscopic

|     | var                 | symbol  | documentation   | type               | units           | eqs |
|-----|---------------------|---------|---|--------------------|-----------------|-----|
| 92  | $\hat{V}_A$         | fV      | volumetric flow   | transport          | $m^3  s^{-1}$   | 67  |
| 93  | $\hat{n}^d{}_{AS}$  | fnd_AS  | diffusional mass flow in a given stream                       | transport          | $mol  s^{-1}$   | 68  |
| 94  | $\hat{n}^d{}_{NS}$  | fnd     | net diffusional mass flow                                     | transport          | $mol  s^{-1}$   | 69  |
| 95  | $\hat{H}^d{}_A$     | fHd_A   | enthalpy flow per diffusional mass stream                     | transport          | $kg m^2 s^{-3}$ | 70  |
| 96  | $\hat{H}^d{}_N$     | fHd     | net enthaply stream due to diffusion                          | transport          | $kg m^2 s^{-3}$ | 71  |
| 97  | $d_A$               | d       | flow direction of convectional flow                           | transport          |                 | 72  |
| 102 | $\hat{H}^{c}{}_{A}$ | fHc_A   | convective enthalpy flow for given stream                     | transport          | $kg m^2 s^{-3}$ | 77  |
| 103 | $\hat{H}^c{}_N$     | fHc     | net convectional enthalpy stream                              | transport          | $kg m^2 s^{-3}$ | 78  |
| 104 | $\hat{w}_A$         | fw_A    | sample work stream  | transport          | $kg m^2 s^{-3}$ | 79  |
| 105 | $\hat{w}_N$         | fw      | net work stream   | transport          | $kg m^2 s^{-3}$ | 80  |
| 106 | $\hat{q}_{xA}$      | fq_A_x  | heat flow in x-direction for given stream                     | transport          | $kg m^2 s^{-3}$ | 81  |
| 107 | $\hat{q}_N$         | fq      | net heat flow   | transport          | $kg m^2 s^{-3}$ | 82  |
| 71  | $A_{yzN}$           | Ayz     | cross sectional area yz                                       | geometry           | $m^2$           | 48  |
| 72  | $A_{xzN}$           | Axz     | cross sectional area xz                                       | geometry           | $m^2$           | 49  |
| 73  | $A_{xyN}$           | Axy     | cross sectional area xy                                       | geometry           | $m^2$           | 50  |
| 70  | $F_{NS,AS}$         | F_NS_AS | species related incidence matrix                              | network            |                 |     |
| 90  | $D_{N,A}$           | D       | difference operator   | differenceOperator |                 |     |
| 91  | $D_{NS,AS}$         | D_NS_AS | difference operator for species topology                      | differenceOperator |                 |     |
| 109 | $H^o{}_N$           | Но      | initial enthalpy  | state              | $kg m^2 s^{-2}$ | 84  |
| 110 | $n^o{}_{NS}$        | no      | initial species   | state              | mol             | 85  |
| 127 | $1_S$               | one_S   | a vector of ones with the length of the ordinal number of $S$ | constant           |                 |     |
| 57  | $m_N$               | m       | total mass  | secondaryState     | kg              | 36  |

|     | var                | symbol      | documentation                                   | type              | units                  | eqs   |
|-----|--------------------|-------------|---|-------------------|------------------------|-------|
| 66  | $c_{NS}$           | С           | molar composition                               | secondaryState    | $m^{-3}  mol$          | 44    |
| 98  | $c_{AS}$           | c_AS        | concentration in convectional flow              | secondaryState    | $m^{-3}  mol$          | 73    |
| 99  | $\hat{n}^c{}_{AS}$ | fnc_AS      | molar convetional mass flow in the given stream | secondaryState    | $mol  s^{-1}$          | 74    |
| 100 | $\hat{n}^c{}_{NS}$ | fnc         | net molar convectional mass flow                | secondaryState    | $mol  s^{-1}$          | 75    |
| 126 | $\phi$             | intensities | collected intensities                           | secondaryState    |                        | 106   |
| 128 | $n^t{}_N$          | nTotal      | total number of moles                           | secondaryState    | mol                    | 107   |
| 101 | $\dot{n}_{NS}$     | dndt        | differential species balance                    | diffState         | $mol  s^{-1}$          | 76    |
| 108 | $\dot{H}_N$        | dHdt        | differential enthalpy balance                   | diffState         | $kgm^2s^{-3}$          | 83    |
| 118 | $\dot{U}^e{}_N$    | dUedt       | Kirkhoff first law                              | diffState         | $kg m^2 A^{-1} s^{-3}$ | 96 97 |
|     |                    |             |   |                   |                        | 98    |
| 113 | $I_N$              | i           | electrical current definition                   | internalTransport | A                      | 89    |

## 8 solid

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|--|-----|--------|---------------|------|-------|-----|

## 9 fluid

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|--|-----|--------|---------------|------|-------|-----|

# 10 liquid

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|--|-----|--------|---------------|------|-------|-----|

# 11 gas

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|--|-----|--------|---------------|------|-------|-----|

### 12 control-control

| var symbol documentation type units eqs |  |  | symbol | documentation | type | units | eqs |
|---|--|--|--------|---------------|------|-------|-----|
|---|--|--|--------|---------------|------|-------|-----|

# 13 gas-liquid

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|--|-----|--------|---------------|------|-------|-----|

## 14 gas-gas

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|--|-----|--------|---------------|------|-------|-----|

# 15 liquid-liquid

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|--|-----|--------|---------------|------|-------|-----|

# 16 gas-solid

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|--|-----|--------|---------------|------|-------|-----|

# 17 solid-solid

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|--|-----|--------|---------------|------|-------|-----|

# 18 liquid-solid

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|--|-----|--------|---------------|------|-------|-----|

### 19 material-material

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|--|-----|--------|---------------|------|-------|-----|

#### 20 reactions—reactions

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|--|-----|--------|---------------|------|-------|-----|

### 21 control-reactions

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|--|-----|--------|---------------|------|-------|-----|

#### 22 reactions-control

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|--|-----|--------|---------------|------|-------|-----|

#### 23 control-material

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|--|-----|--------|---------------|------|-------|-----|

## 24 material-control

|     | var | symbol    | documentation   | type | units | eqs |
|-----|-----|-----------|---|------|-------|-----|
| 124 | ξ   | _additive | link variable additive to interface material $\gg > { m control}$ | get  |       | 104 |

### ${\bf 25} \quad {\bf control-macroscopic}$

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|--|-----|--------|---------------|------|-------|-----|

### 26 macroscopic-control

|     | var    | symbol | documentation  | type | units                  | eqs |
|-----|--------|--------|--|------|------------------------|-----|
| 119 | $I_N$  | _i     | link variable i to interface macroscopic »> control  | get  | A                      | 99  |
| 125 | $T_N$  | _T     | link variable T to interface macroscopic »> control  | get  | K                      | 105 |
| 142 | $Ue_N$ | _Ue    | link variable Ue to interface macroscopic »> control | get  | $kg m^2 A^{-1} s^{-3}$ | 114 |

#### 27 reactions-material

|  | var | symbol | documentation | type | units | eqs |  |
|--|-----|--------|---------------|------|-------|-----|--|
|--|-----|--------|---------------|------|-------|-----|--|

### 28 material-reactions

|  | var | symbol | documentation | type | units | eqs |
|--|-----|--------|---------------|------|-------|-----|
|--|-----|--------|---------------|------|-------|-----|

### 29 reactions-macroscopic

|     | var             | symbol | documentation  | type | units         | eqs |
|-----|-----------------|--------|--|------|---------------|-----|
| 164 | $	ilde{n}_{NS}$ | _nProd | link variable nProd to interface reactions $\gg$ macroscopic | get  | $mol  s^{-1}$ | 131 |

## macroscopic-reactions

|    | var      | symbol | documentation   | type | units         | eqs |
|----|----------|--------|---|------|---------------|-----|
| 67 | $c_{NS}$ | _c     | link variable c to interface macroscopic $\gg>$ reactions | get  | $m^{-3}  mol$ | 45  |

### material-macroscopic

|    | var         | symbol   | documentation  | type | units                       | eqs |
|----|-------------|----------|--|------|-----------------------------|-----|
| 41 | $\lambda_S$ | _Mm      | link variable Mm to interface material »> macroscopic  | get  | $kgmol^{-1}$                | 20  |
| 74 | $ ho_N$     | _density | link variable density to interface material $\gg >$ macroscopic  | get  | $kg  m^{-3}$                | 51  |
| 75 | $h_{NS}$    | _h       | link variable h to interface material »> macroscopic   | get  | $kg  m^2  mol^{-1}  s^{-2}$ | 52  |
| 76 | $k_{xN}^q$  | _kq_x    | $\begin{array}{ c c c c c c } link \ variable \ kq \ x \ to \ interface \ material \ >> \ macroscopic \\ \hline \end{array}$ | get  | $kg K^{-1} s^{-3}$          | 53  |
| 77 | $Cv_N$      | _Cv      | $\begin{tabular}{ll} link variable Cv to interface material $> $ macroscopic \\ \end{tabular}$                               | get  | $kg  m^2  K^{-1}  s^{-2}$   | 54  |
| 78 | $k_{yN}^q$  | _kq_y    | link variable kq y to interface material »> macroscopic  | get  | $kg K^{-1} s^{-3}$          | 55  |
| 79 | $k_{zN}^q$  | _kq_z    | link variable kq z to interface material »> macroscopic  | get  | $kg K^{-1} s^{-3}$          | 56  |
| 80 | $k^q{}_N$   | _kq      | $   \   link \ variable \ kq \ to \ interface \ material \ >> \ macroscopic $  | get  | $kg K^{-1} s^{-3}$          | 57  |
| 81 | $k_{xN}^c$  | _kc_x    | link variable kc x to interface material »> macroscopic  | get  | $m^{-1} s$                  | 58  |
| 82 | $Cp_N$      | _Cp      | $\begin{array}{ c c c c c c c c c c c c c c c c c c c$   | get  | $kg  m^2  K^{-1}  s^{-2}$   | 59  |
| 83 | $k_{yN}^c$  | _kc_y    | link variable kc y to interface material »> macroscopic  | get  | $m^{-1} s$                  | 60  |
| 84 | $k_{zN}^c$  | _kc_z    | link variable kc z to interface material »> macroscopic  | get  | $m^{-1} s$                  | 61  |
| 85 | $k^c{}_N$   | _kc      | link variable kc to interface material »> macroscopic  | get  | $m^{-1} s$                  | 62  |
| 86 | $k_{xNS}^d$ | _kd_x    | link variable kd x to interface material $\gg$ macroscopic   | get  | $kg^{-1} m^{-4} mol^2 s$    | 63  |
| 87 | $k_{yNS}^d$ | _kd_y    | link variable kd y to interface material »> macroscopic  | get  | $kg^{-1} m^{-4} mol^2 s$    | 64  |
| 88 | $k_{zNS}^d$ | _kd_z    | link variable kd z to interface material $\gg >$ macroscopic   | get  | $kg^{-1} m^{-4} mol^2 s$    | 65  |

|     | var        | symbol      | documentation   | type | units                       | eqs |
|-----|------------|-------------|---|------|-----------------------------|-----|
| 89  | $k^d_{NS}$ | _kd         | link variable kd to interface material $\gg$ macroscopic  | get  | $kg^{-1}  m^{-4}  mol^2  s$ | 66  |
| 117 | $R^e{}_N$  | _elConductC | link variable elConductC to interface material »> macroscopic   | get  | $kg^{-1} m^{-2} A^2 s^3$    | 94  |
| 140 | additive   | _additive   | $\begin{array}{c} \mbox{link variable additive to interface material } > \mbox{macroscopic} \\ \mbox{scopic} \end{array}$ | get  |                             | 112 |

### 32 macroscopic-material

|     | var   | symbol | documentation   | type | units | eqs |
|-----|-------|--------|---|------|-------|-----|
| 58  | $m_N$ | _m     | link variable m to interface macroscopic $\gg >$ material | get  | kg    | 37  |
| 114 | $i_N$ | _i     | link variable i to interface macroscopic »> material      | get  | A     | 90  |

# 33 Equations

### 34 Generic

| no | equation   | documentation                            | layer    |
|----|--|--|----------|
| 1  | 1 := Instantiate(#, #)                                   | numerical value 1                        | root     |
| 2  | 0 := Instantiate(#, #)                                   | numerical value zero                     | root     |
| 3  | 0.5 := Instantiate(#, #)                                 | numerical value one half                 | root     |
| 4  | $t^o := \text{Instantiate}(t, \#)$                       | starting time                            | root     |
| 5  | $t^e := \text{Instantiate}(t, \#)$                       | end time                                 | root     |
| 6  | $p_N := \left(-\frac{\partial U_N}{\partial V_N}\right)$ | thermodynamic pressure                   | physical |
| 7  | $T_N := \frac{\partial U_N}{\partial S_N}$               | temperature                              | physical |
| 8  | $\mu_{NS} := rac{\partial  U_N}{\partial  n_{NS}}$      | chemical potential                       | physical |
| 9  | $H_N := U_N - p_N \cdot V_N$                             | enthalpy                                 | physical |
| 10 | $A_N := U_N - T_N \cdot S_N$                             | Helmholtz energy                         | physical |
| 11 | $G_N := U_N + p_N \cdot V_N - T_N \cdot S_N$             | Gibbs free energy                        | physical |
| 14 | $Ue_N := (C_N)^{-1} \cdot U_N$                           | electrical potential – voltage           | physical |
| 15 | $v_{xN} := \frac{\partial r_{xN}}{\partial t}$           | velocitiy in x-direction                 | physical |
| 16 | $v_{yN} := \frac{\partial  r_{yN}}{\partial  t}$         | velocity in y direction                  | physical |
| 17 | $v_{zN} := \frac{\partial r_{zN}}{\partial t}$           | velocity in z-direction                  | physical |
| 21 | $C_{pN} := \frac{\partial H_N}{\partial T_N}$            | total heat capacity at constant pressure | material |

| no | equation   | documentation                                    | layer       |
|----|--|--|-------------|
| 22 | $C_{VN} := \frac{\partial U_N}{\partial T_N}$  | total heat capacity at constant volume           | material    |
| 23 | $k_{xN}^q := (V_N)^{-1} \cdot \frac{\partial U_N}{\partial T_N} \cdot v_{xN}$  | thermal conductivity in x-direction              | material    |
| 24 | $k_{yN}^q := (V_N)^{-1} \cdot \frac{\partial U_N}{\partial T_N} \cdot v_{yN}$  | thermal conductivity in y-direction              | material    |
| 25 | $k_{zN}^q := (V_N)^{-1} \cdot \frac{\partial U_N}{\partial T_N} \cdot v_{zN}$  | thermal conductivity in z-direction'             | material    |
| 26 | $k^q{}_N := \operatorname{Stack}\left(k^q_{xN}, k^q_{yN}, k^q_{zN}\right)$   | thermal conductivity                             | material    |
| 27 | $k_{xN}^c := \left(\lambda_S \overset{S \in NS}{\star} (\mu_{NS})^{-1}\right) \cdot (V_N)^{-1} \cdot \frac{\partial U_N}{\partial p_N} \cdot v_{xN}$ | convecitve mass conductivity in x-direction      | material    |
| 28 | $k_{yN}^c := \left(\lambda_S \overset{S \in NS}{\star} (\mu_{NS})^{-1}\right) \cdot (V_N)^{-1} \cdot \frac{\partial U_N}{\partial p_N} \cdot v_{yN}$ | convecitve mass conductivity in y-direction      | material    |
| 29 | $k_{zN}^c := \left(\lambda_S \overset{S \in NS}{\star} (\mu_{NS})^{-1}\right) \cdot (V_N)^{-1} \cdot \frac{\partial U_N}{\partial p_N} \cdot v_{zN}$ | convecitve mass conductivity in z-direction      | material    |
| 30 | $k^c{}_N := \operatorname{Stack}\left(k^c_{xN}, k^c_{yN}, k^c_{zN}\right)$   | convective mass conductivity                     | material    |
| 31 | $k_{xNS}^d := (\mu_{NS})^{-1} \cdot \left( v_{xN} \odot \left( (V_N)^{-1} \odot \frac{\partial U_N}{\partial \mu_{NS}} \right) \right)$              | diffusional mass conductivity in x-direction     | material    |
| 32 | $k_{yNS}^d := (\mu_{NS})^{-1} \cdot \left( v_{yN} \odot \left( (V_N)^{-1} \odot \frac{\partial U_N}{\partial \mu_{NS}} \right) \right)$              | diffusional mass conductivity in y-<br>direction | material    |
| 33 | $k_{zNS}^d := (\mu_{NS})^{-1} \cdot \left( v_{zN} \odot \left( (V_N)^{-1} \odot \frac{\partial U_N}{\partial \mu_{NS}} \right) \right)$              | diffusional mass conductivity in z-<br>direction | material    |
| 34 | $k^{d}_{NS} := \operatorname{Stack}\left(k^{d}_{xNS}, k^{d}_{yNS}, k^{d}_{zNS}\right)$   | diffusional mass condctivity                     | material    |
| 35 | $h_{NS} := H_N \odot \left( n_{NS} \right)^{-1}$   | partial molar enthalpies                         | material    |
| 36 | $m_N := \lambda_S \overset{S \in NS}{\star} n_{NS}$  | total mass                                       | macroscopic |

| no | equation  | documentation                                   | layer       |
|----|---|---|-------------|
| 38 | $\rho_N := m_N \cdot (V_N)^{-1}$  | density   | material    |
| 39 | $T_{NK} := P_{N,NK} \stackrel{N}{\star} T_N$  | temperature of the reactive system              | reactions   |
| 41 | $E^a{}_{NK} := \text{Instantiate}(R . T_{NK}, \#)$  | Arrhenius activation energy                     | reactions   |
| 42 | $K_{NK} := K^o{}_K \odot exp((-E^a{}_{NK}) \cdot (R \cdot T_{NK})^{-1})$  | Arrhenius reaction 'constant'                   | reactions   |
| 44 | $c_{NS} := (V_N)^{-1} \odot n_{NS}$   | molar composition                               | macroscopic |
| 48 | $A_{yzN} := r_{yN} . r_{zN}$  | cross sectional area yz                         | macroscopic |
| 49 | $A_{xzN} := r_{xN} \cdot r_{zN}$  | cross sectional area xz                         | macroscopic |
| 50 | $A_{xyN} := r_{xN} . r_{yN}$  | cross sectional area xy                         | macroscopic |
| 67 | $\hat{V}_A := \left(\rho_N\right)^{-1} \cdot k_{xN}^c \cdot A_{yzN} \cdot D_{N,A} \stackrel{N}{\star} p_N$      | volumetric flow                                 | macroscopic |
| 68 | $\hat{n}^d{}_{AS} := A_{yzN} \odot \left( -k^d_{xNS} \right) . D_{NS,AS} \overset{NS}{\star} \mu_{NS}$          | diffusional mass flow in a given stream         | macroscopic |
| 69 | $\hat{n}^d{}_{NS} := F_{NS,AS} \stackrel{AS}{\star} \hat{n}^d{}_{AS}$   | net diffusional mass flow                       | macroscopic |
| 70 | $\hat{H}^d{}_A := \left(F_{NS,AS} \overset{NS}{\star} h_{NS}\right) \overset{S \in AS}{\star} \hat{n}^d{}_{AS}$ | enthalpy flow per diffusional mass<br>stream    | macroscopic |
| 71 | $\hat{H}^d{}_N := F_{N,A} \stackrel{A}{\star} \hat{H}^d{}_A$  | net enthaply stream due to diffusion            | macroscopic |
| 72 | $d_A := \operatorname{sign}\left(F_{N,A} \stackrel{N}{\star} p_N\right)$  | flow direction of convectional flow             | macroscopic |
| 73 | $c_{AS} := (0.5 \cdot (F_{NS,AS} - d_A \odot  F_{NS,AS} )) *^{NS} c_{NS}$                                       | concentration in convectional flow              | macroscopic |
| 74 | $\hat{n}^c{}_{AS} := \hat{V}_A \odot c_{AS}$  | molar convetional mass flow in the given stream | macroscopic |
| 75 | $\hat{n}^c{}_{NS} := F_{NS,AS} \stackrel{AS}{\star} \hat{n}^c{}_{AS}$   | net molar convectional mass flow                | macroscopic |

| no | equation  | documentation  | layer       |
|----|---|--|-------------|
| 76 | $\dot{n}_{NS} := \hat{n}^c{}_{NS} + \hat{n}^d{}_{NS}$   | differential species balance   | macroscopic |
| 77 | $\hat{H}^c{}_A := \left(F_{NS,AS} \overset{NS}{\star} h_{NS}\right) \overset{S \in AS}{\star} \hat{n}^c{}_{AS}$ | convective enthalpy flow for given stream                                  | macroscopic |
| 78 | $\hat{H}^c{}_N := F_{N,A} \stackrel{A}{\star} \hat{H}^c{}_A$  | net convectional enthalpy stream   | macroscopic |
| 79 | $\hat{w}_A := \text{Instantiate}(\hat{H}^c{}_A, \#)$  | sample work stream   | macroscopic |
| 80 | $\hat{w}_N := F_{N,A} \overset{A}{\star} \hat{w}_A$   | net work stream  | macroscopic |
| 81 | $\hat{q}_{xA} := A_{yzN} \cdot k_{xN}^q \cdot D_{N,A} \stackrel{N}{\star} T_N$                                  | heat flow in x-direction for given stream                                  | macroscopic |
| 82 | $\hat{q}_N := F_{N,A} \stackrel{A}{\star} \hat{q}_{xA}$   | net heat flow  | macroscopic |
| 83 | $\dot{H}_N := \hat{H}^c{}_N + \hat{H}^d{}_N + \hat{q}_N + \hat{w}_N$  | differential enthalpy balance  | macroscopic |
| 84 | $H^o{}_N := \text{Instantiate}(H_N, \#)$  | initial enthalpy   | macroscopic |
| 85 | $n^o{}_{NS} := \text{Instantiate}(n_{NS}, \#)$  | initial species  | macroscopic |
| 86 | $n_{NS} := \int_{t^o}^{t^e} \dot{n}_{NS} \ dt$  | fundamental state – molar mass   | macroscopic |
| 87 | $H_N := \int_{t^o}^{t^e} \dot{H}_N \ dt$  | enthalpy   | macroscopic |
| 88 | $\xi := \operatorname{Instantiate}(\xi, \#)$  | fraction of additives  | material    |
| 89 | $I_N := rac{d  C_N}{d  t}$   | electrical current definition  | macroscopic |
| 91 | $R^e_N := (i_N)^{-1} \cdot Ue_N$  | electrical resistant   | material    |
| 92 | $R^e{}_N := \operatorname{Instantiate}(R^e{}_N, \#)$  | electrical resistant   | material    |
| 93 | $k^{e,\xi}{}_N := (R^e{}_N)^{-1} \cdot \xi$   | simple model for the electrical conductivity as a function of the additive | material    |

| no  | equation   | documentation                   | layer       |
|-----|--|---------------------------------|-------------|
| 95  | $Ue_N := (R^e{}_N)^{-1} . I_N$   | electrical potential – voltage  | macroscopic |
| 96  | $\dot{U}^e{}_N := 1 . Ue_N$  | Kirkhoff first law              | macroscopic |
| 97  | $\dot{U}^e{}_N := Root\left(Ue_N\right)$                                       | Kirkhoff first law              | macroscopic |
| 98  | $\dot{U}^e{}_N := \operatorname{Instantiate}(\dot{U}^e{}_N, 0)$                | Kirkhoff first law              | macroscopic |
| 106 | $\phi := \operatorname{MixedStack}(p_N, T_N, \mu_{NS}, c_{NS}, Ue_N)$          | collected intensities           | macroscopic |
| 107 | $n^t{}_N := 1_S \overset{S \in NS}{\star}  n_{NS}$                             | total number of moles           | macroscopic |
| 108 | $e_A := m_A - y^o{}_A$   | control error                   | control     |
| 109 | $xo_N := \text{Instantiate}(x_N, \#)$  | initial state                   | control     |
|     | $\dot{x}_D := A_{N,D} \overset{N}{\star} x_N + B_{A,D} \overset{A}{\star} e_A$ | differential state (ABCD) model | control     |
| 111 | $x_N := \int_{t^o}^{t^e} 1_{N,D} \stackrel{D}{\star} \dot{x}_D \ dt$           | state                           | control     |
| 113 | $\check{I}_N := I_N$   | measured current                | control     |
| 115 | $\check{U}^e{}_N := Ue_N$  | measured electrical potential   | control     |
| 116 | $\check{\xi} := \xi$   | measured additive fraction      | control     |

| no  | equation   | documentation                                  | layer     |
|-----|--|--|-----------|
| 117 | $RComputed_N := (\check{I}_N)^{-1} . \check{U}^e{}_N$  | measured resistance                            | control   |
| 118 | $store := \operatorname{MixedStack}\left(\check{I}_N, \check{U}^e{}_N, RComputed_N, \check{\xi}\right)$  | quantities to be stored                        | control   |
| 119 | $y^o{}_A := \operatorname{Instantiate}(y^o{}_A, \#)$   | set point                                      | control   |
| 123 | $c_{NK,KS} := P_{NK} \cdot \left( P_{NS,KS} \overset{NS}{\star} c_{NS} \right)$  | var doc :                                      | reactions |
| 124 | $c^o_{NK,KS} := \text{Instantiate}(c_{NK,KS}, \#)$   | norming concentration                          | reactions |
| 125 | $x_{NK,KS} := (c^o_{NK,KS})^{-1} \cdot c_{NK,KS}$  | matrix of normed, dimensionless mole fractions | reactions |
| 126 | $y_A := C_{N,A} \stackrel{N}{\star} x_N + D_A \cdot e_A$   | output equation                                | control   |
| 127 | $R:=A^v \cdot B$   | gas constant                                   | reactions |
| 128 | $N_{NK,KS} := P_{K,NK} \stackrel{K}{\star} N_{K,KS}$   | extended stoichiometrix matrix                 | reactions |
| 129 | $\phi_{NK} := \prod_{KS} x_{NK,KS}^{N_{NK,KS}}$  | probability function for reactions             | reactions |
| 130 | $\tilde{n}_{NS} := V_N \overset{N}{\star} \left( P_{N,NK} \overset{NK}{\star} \left( \left( K_{NK} . \phi_{NK} \right) . \left( P_{NS,KS} \overset{KS}{\star} N_{NK,KS} \right) \right) \right)$ | the species production term                    | reactions |

## 35 Interface Link Equation

| no | equation   | documentation      | layer                        |
|----|--|--------------------|------------------------------|
| 20 | $\lambda_S := \lambda_S$   | interface equation | material -> macro-<br>scopic |
| 37 | $m_N := m_N$   | interface equation | macroscopic -><br>material   |
| 45 | $c_{NS} := c_{NS}$   | interface equation | macroscopic -> reactions     |
| 51 | $ ho_N :=  ho_N$   | interface equation | material -> macro-<br>scopic |
| 52 | $h_{NS} := h_{NS}$   | interface equation | material -> macro-<br>scopic |
| 53 | $k_{xN}^q := k_{xN}^q$   | interface equation | material -> macro-<br>scopic |
| 54 | $Cv_N := C_{VN}$   | interface equation | material -> macro-<br>scopic |
| 55 | $egin{aligned} k_{yN}^q := k_{yN}^q \end{aligned}$               | interface equation | material -> macro-<br>scopic |
| 56 | $k_{zN}^q := k_{zN}^q$   | interface equation | material -> macro-<br>scopic |
| 57 | $\left  \begin{array}{c} k^q{}_N := k^q{}_N \end{array} \right $ | interface equation | material -> macro-<br>scopic |
| 58 | $k_{xN}^c := k_{xN}^c$   | interface equation | material -> macro-<br>scopic |

| no  | equation                   | documentation      | layer                        |
|-----|----------------------------|--------------------|------------------------------|
| 59  | $Cp_N := C_{pN}$           | interface equation | material -> macro-<br>scopic |
| 60  | $k_{yN}^c := k_{yN}^c$     | interface equation | material -> macro-<br>scopic |
| 61  | $k_{zN}^c := k_{zN}^c$     | interface equation | material -> macro-<br>scopic |
| 62  | $k^c{}_N := k^c{}_N$       | interface equation | material -> macro-<br>scopic |
|     | $k_{xNS}^d := k_{xNS}^d$   | interface equation | material -> macro-<br>scopic |
| 64  | $k_{yNS}^d := k_{yNS}^d$   | interface equation | material -> macro-<br>scopic |
| 65  | $k_{zNS}^d := k_{zNS}^d$   | interface equation | material -> macro-<br>scopic |
| 66  | $k^d{}_{NS} := k^d{}_{NS}$ | interface equation | material -> macro-<br>scopic |
| 90  | $i_N:=I_N$                 | interface equation | macroscopic -><br>material   |
| 94  | $R^e{}_N := k^{e,\xi}{}_N$ | interface equation | material -> macro-<br>scopic |
| 99  | $I_N:=I_N$                 | interface equation | macroscopic -> control       |
| 104 | $\xi := \xi$               | interface equation | material -> control          |

| no  | equation                         | documentation      | layer                        |
|-----|----------------------------------|--------------------|------------------------------|
| 105 | $T_N := T_N$                     | interface equation | macroscopic -> control       |
| 112 | $additive := \xi$                | interface equation | material -> macro-<br>scopic |
| 114 | $Ue_N:=Ue_N$                     | interface equation | macroscopic -> control       |
| 131 | $	ilde{n}_{NS} := 	ilde{n}_{NS}$ | interface equation | reactions -> macroscopic     |