Gravitational waves from the early universe

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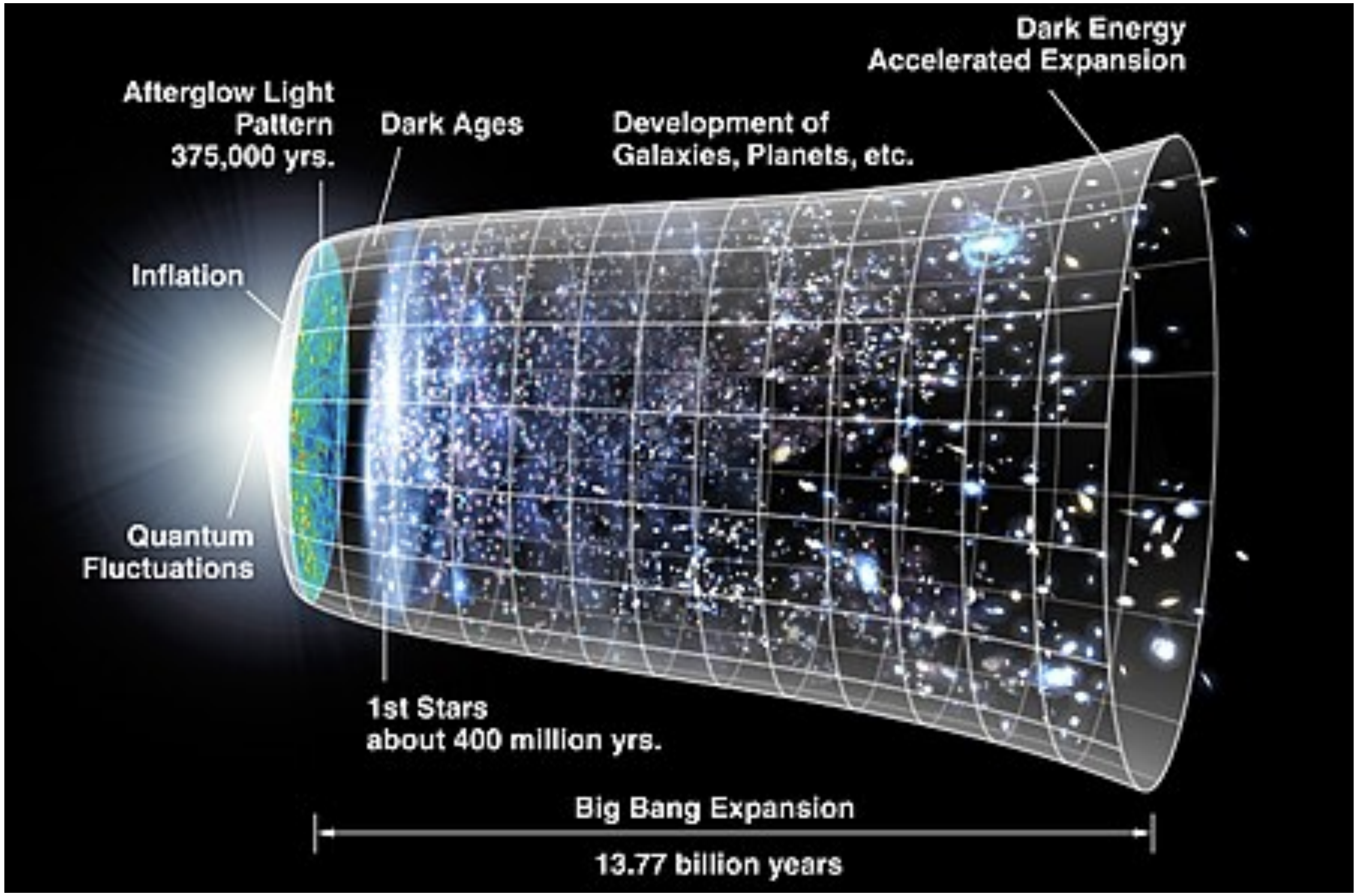
How can GW help to probe the universe?

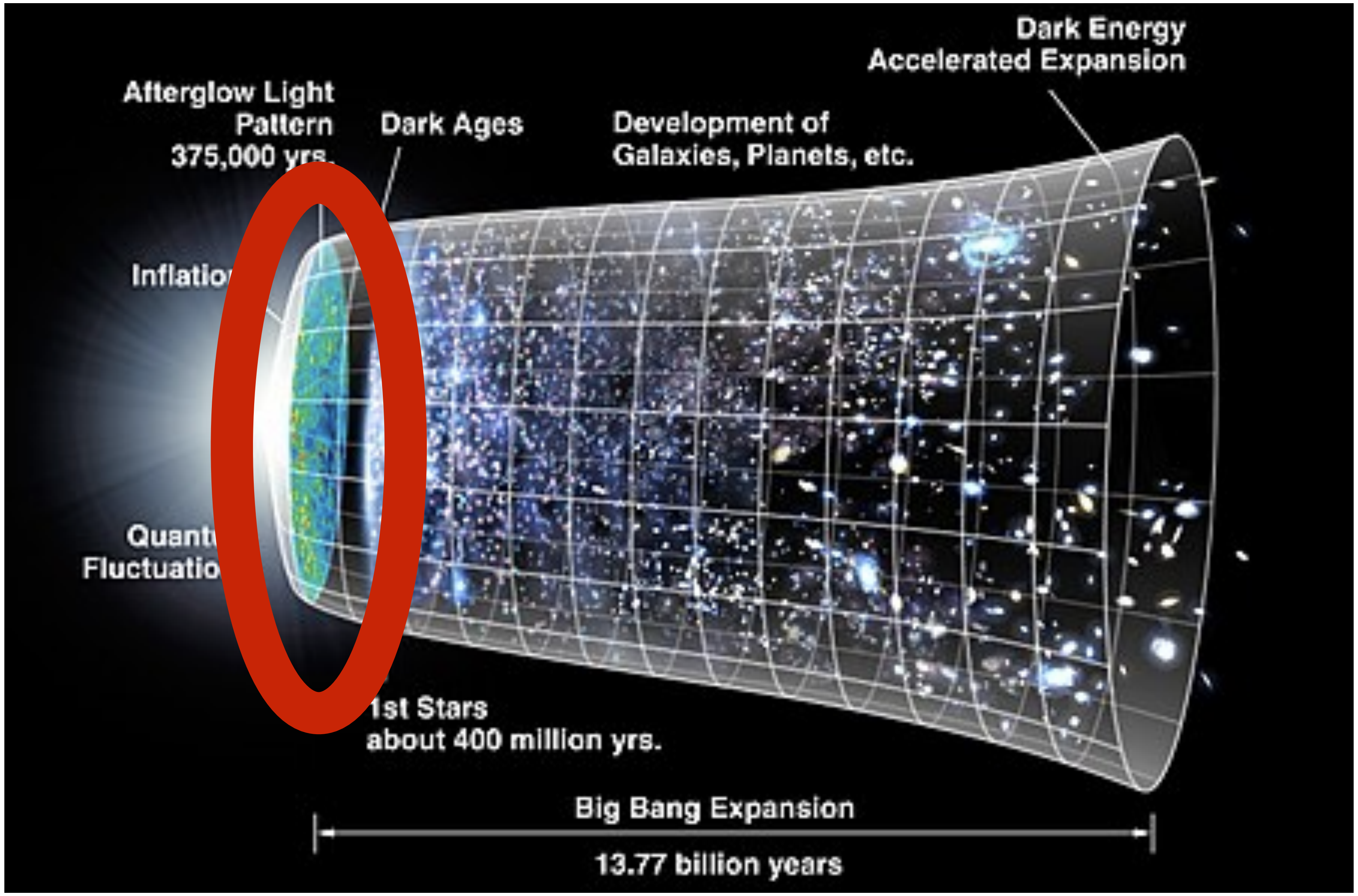


because of the weakness of the gravitational interaction the universe is "transparent" to GWs

$$\frac{\text{rate of interaction}}{\text{Hubble rate}} \qquad \frac{\Gamma(T)}{H(T)} \sim \frac{G^2\, T^5}{T^2/M_{Pl}} \sim \left(\frac{T}{M_{Pl}}\right)^3 < 1$$

How can GW help to probe the universe?

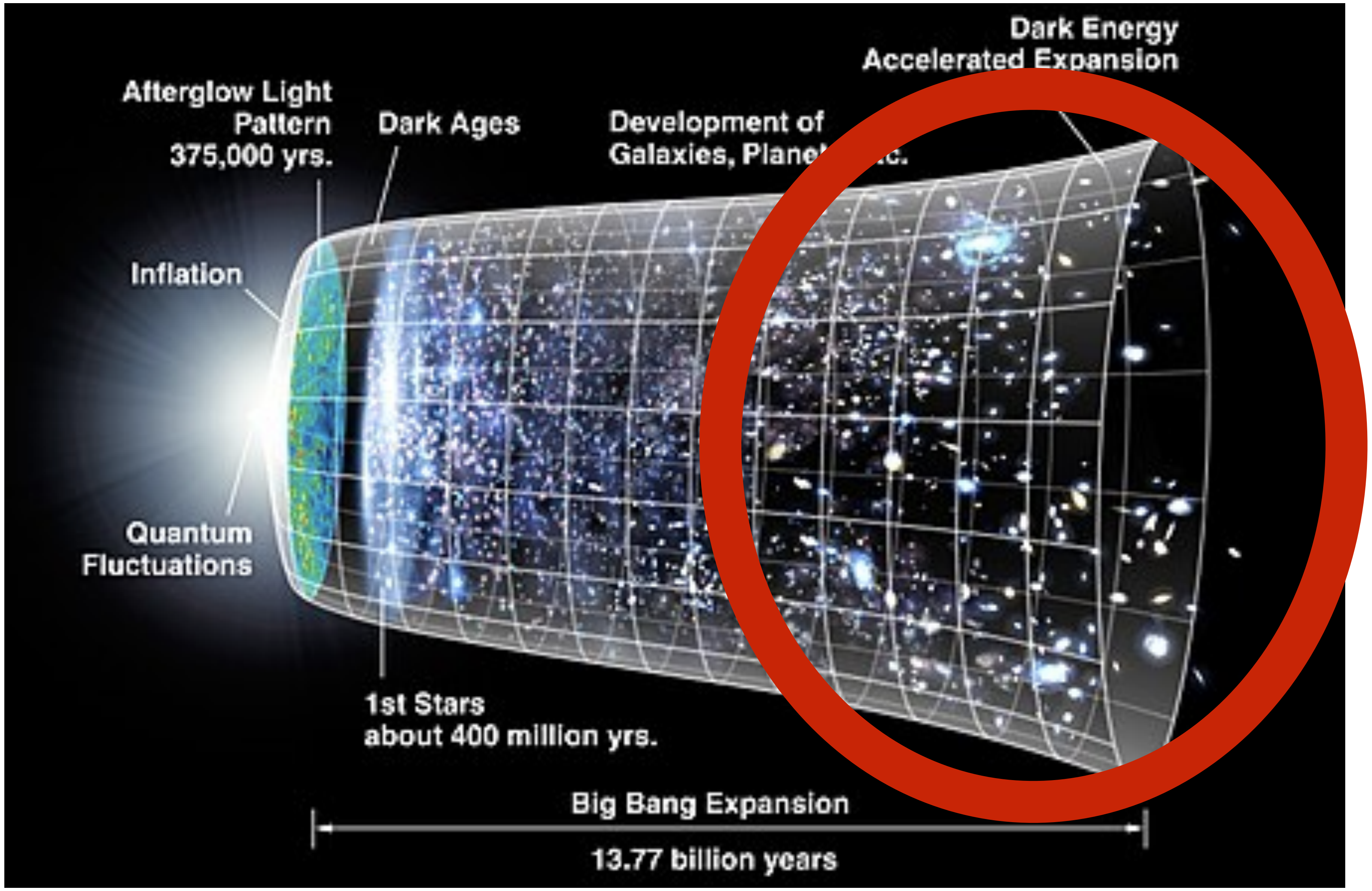
early universe



GW can bring direct information from the early universe: phenomena occurring in the early universe can produce stochastic GW backgrounds (SGWB) a fossil radiation like the CMB

tests of high energy phenomena

How can GW help to probe the universe?



Binaries of compact objects (black holes, neutron stars…) orbiting around each other and possibly merging emit GWs

Provide information on binary formation and evolution, cosmological structure formation, black hole growth and environment, tests of General Relativity in strong and weak regime, tests of the cosmic expansion…

Summary of the lecture

- GW equation of motion in FLRW and its relevant solutions
- Characterisation of a stochastic GW background from the early universe
- What is and will be known on the SGWB
- A few examples of SGWB sources with characteristic solutions

C.C. and D.G. Figueroa, “Cosmological backgrounds of GWs”, arXiv:1801.04268
M. Maggiore, “Gravitational waves”, volume 1 and 2, Oxford University Press

GW propagation equation in FLRW

- GWs emerge naturally in General Relativity as a causal theory of gravitation, in which there must be some form of radiation propagating information causally: GWs!

- "waves" are propagating perturbations over a background:

1. take a background space-time metric (the gravitational field)
2. define a small perturbation over this background metric
3. insert it into the equations that describe the space-time dynamics (Einstein equations)
4. (if everything goes well) one finds a dynamical solution for the perturbation which is propagating as a wave -> GWs!

1. $$ds^2 = -dt^2 + a^2(t)\left(\delta_{ij} + h_{ij}\right) dx^i dx^j$$

Neglecting scalar and vector perturbations

2. $$|h_{ij}| \ll 1 \qquad h_i^i = \partial_j h_i^j = 0$$

superimposed on the homogeneous and isotropic background

3. $$\bar{G}_{\mu\nu} + \delta G_{\mu\nu} = 8\pi G\left(\bar{T}_{\mu\nu} + \delta T_{\mu\nu}\right)$$

"The Cosmic Microwave Background", R. Durrer, Cambridge University Press, 2008

GW propagation equation in FLRW

4. $$\ddot{h}_{ij}(\mathbf{x},t) + 3\,H\,\dot{h}_{ij}(\mathbf{x},t) - \frac{\nabla^2}{a^2}\,h_{ij}(\mathbf{x},t) = 16\pi G\,\boxed{\Pi_{ij}(\mathbf{x},t)}$$

Source: tensor anisotropic stress

Perfect fluid

$$T_{\mu\nu} = \bar{T}_{\mu\nu} + \delta T_{\mu\nu}$$

In the cosmological context:
energy momentum tensor of the matter content of the universe (background + perturbations)

$$\delta T_{ij} = \bar{p}\,\delta g_{ij} + a^2[\delta p\,\delta_{ij} + (\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2)\sigma + 2\partial_{(i}v_{j)}\boxed{+\,\Pi_{ij}]}$$

$$(\partial_i v_i = 0,\ \partial_i\Pi_{ij} = 0,\ \Pi_{ii} = 0)$$

NO GWs FROM THE HOMOGENEOUS MATTER COMPONENT

COMMENT: In cosmology, the FLRW space-time is homogeneous and isotropic, so tensor modes can be defined also when $\lambda \sim L_B$ (exemple: horizon re-entry after inflation), but one cannot say these are GWs, unless modes are well within the horizon ($\lambda \ll L_B$)

GW propagation equation in FLRW

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In the cosmological context:
energy momentum tensor of the matter content of the universe (background + perturbations)

One exploits the translational invariance and performs a F.T. in space

$$h_{ij}(\mathbf{x},t) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^3}\, h_r(\mathbf{k},t)\, e^{-i\mathbf{k}\cdot\mathbf{x}}\, e^r_{ij}(\hat{\mathbf{k}})$$

$$\Pi_{ij}(\mathbf{x},t) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^3}\, \Pi_r(\mathbf{k},t)\, e^{-i\mathbf{k}\cdot\mathbf{x}}\, e^r_{ij}(\hat{\mathbf{k}})$$

GW propagation equation in FLRW

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The evolution equation decouples for each polarisation mode

$$h_r''(\mathbf{k},\eta) + 2\,\mathcal{H}\,h_r'(\mathbf{k},\eta) + k^2\,h_r(\mathbf{k},\eta) = 16\pi G\,a^2\,\Pi_r(\mathbf{k},\eta)$$

conformal time, Hubble factor and comoving wavenumber

GW propagation equation in FLRW

$$h_r''(\mathbf{k},\eta) + 2\,\mathcal{H}\,h_r'(\mathbf{k},\eta) + k^2\,h_r(\mathbf{k},\eta) = 16\pi G\,a^2\,\Pi_r(\mathbf{k},\eta)$$

Solution of the homogeneous equation

Power-law scale factor $a(\eta) = a_n \eta^n$

Covering matter (n=2) and radiation domination (n=1), and De Sitter inflation n=-1)

$$h_r(\mathbf{k},\eta) = \frac{A_r(\mathbf{k})}{a_n \eta^{n-1}}\,j_{n-1}(k\eta) + \frac{B_r(\mathbf{k})}{a_n \eta^{n-1}}\,y_{n-1}(k\eta)$$

Two notable limiting cases: sub-Hubble and super-Hubble modes

$$H_r(\mathbf{k},\eta) = a\,h_r(\mathbf{k},\eta) \qquad H_r''(\mathbf{k},\eta) + \left(k^2 - \frac{a''}{a}\right) H_r(\mathbf{k},\eta) = 0$$

$$a''/a \propto \mathcal{H}^2$$

GW propagation equation in FLRW

CASE 1: Sub-Hubble modes, relevant for **propagation after the source stops**

$$k^2 \gg \mathcal{H}^2 \qquad h_r(\mathbf{k},\eta) = \frac{A_r(\mathbf{k})}{a(\eta)}\, e^{ik\eta} + \frac{B_r(\mathbf{k})}{a(\eta)}\, e^{-ik\eta}$$

In this limit, GWs are plane waves with redshifting amplitude

What are the coefficients $A_r(\mathbf{k})$ and $B_r(\mathbf{k})$ from the initial condition?

Suppose the source operates in a time interval $\eta_{\rm fin}$ - $\eta_{\rm in}$ in the radiation dominated era

$$H_r^{\rm rad}(\mathbf{k},\eta<\eta_{\rm fin}) = \frac{16\pi G}{k}\int_{\eta_{\rm in}}^{\eta} d\tau\, a(\tau)^3\, \sin[k(\eta-\tau)]\,\Pi_r(\mathbf{k},\tau)$$

Matching at $\eta_{\rm fin}$ with the homogeneous solution to find the GW signal today

$$H_r^{\rm rad}(\mathbf{k},\eta>\eta_{\rm fin}) = A_r^{\rm rad}(\mathbf{k})\cos(k\eta) + B_r^{\rm rad}(\mathbf{k})\sin(k\eta)$$

$$A_r^{\rm rad}(\mathbf{k}) = \frac{16\pi G}{k}\int_{\eta_{\rm in}}^{\eta_{\rm fin}} d\tau\, a(\tau)^3\, \sin(-k\tau)\,\Pi_r(\mathbf{k},\tau)$$

$$B_r^{\rm rad}(\mathbf{k}) = \frac{16\pi G}{k}\int_{\eta_{\rm in}}^{\eta_{\rm fin}} d\tau\, a(\tau)^3\, \cos(k\tau)\,\Pi_r(\mathbf{k},\tau)$$

GW propagation equation in FLRW

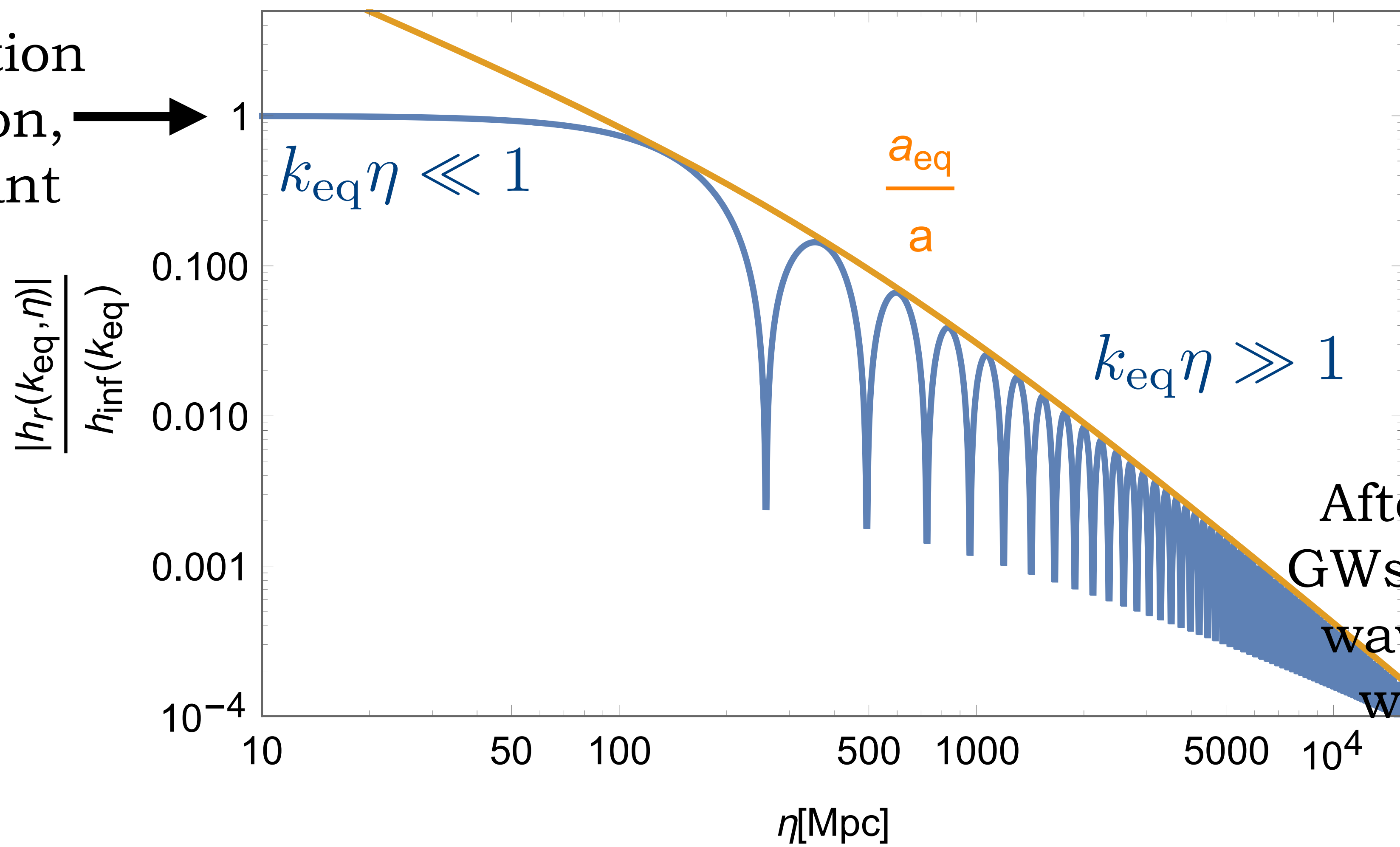
CASE 2: Super-Hubble modes, relevant for **inflationary tensor perturbations**

$$k^2 \ll \mathcal{H}^2 \qquad h_r(\mathbf{k},\eta) = A_r(\mathbf{k}) + \frac{B_r(\mathbf{k})}{a^3(\eta)}$$

Decaying mode, negligible

Full solution with inflationary initial conditions
Hubble re-entry at the radiation-matter transition

Initial condition from inflation, $A_r(\mathbf{k})$ constant

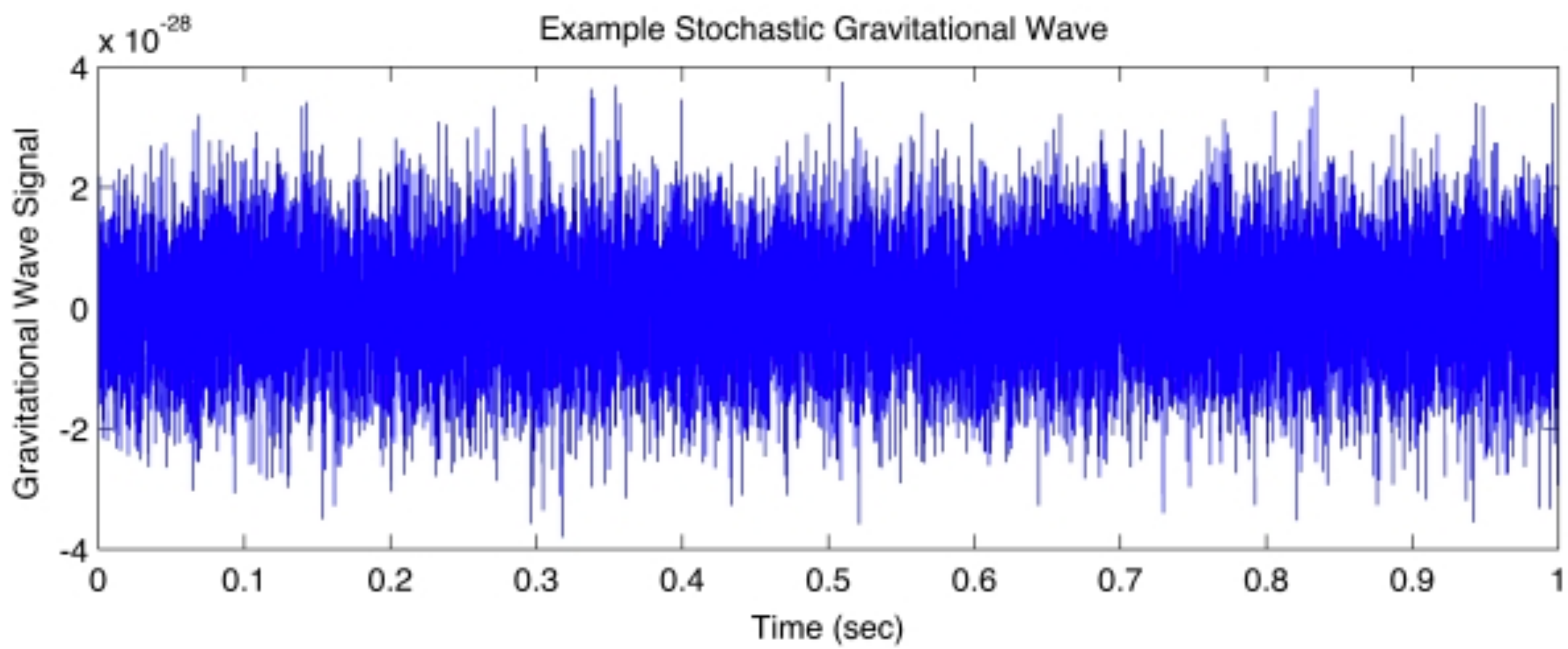


After Hubble re-entry, GWs oscillating as plane waves and redshifting with the expansion

Why sources in the early universe produce SGWBs?

A stochastic GW background is a signal for which *only the statistical properties can be accessed* because it is given by the incoherent superposition of sources that cannot be individually resolved

LIGO website



- For example, the superposition of deterministic GW signals from astrophysical binary sources with too low signal-to-noise ratio, or too much overlap in time and frequency -> confusion noise (Examples: LVK, LISA, PTAs…)

- Early universe GW sources produce SGWBs because they are homogeneously and isotropically distributed over the entire universe, and correlated on scales much smaller than the detector resolution

Why sources in the early universe produce SGWBs?

A GW source acting at time t_* in the early universe cannot produce a signal correlated on length/time scales larger than the causal horizon at that time

$$\boxed{\ell_* \leq H_*^{-1}}$$

ℓ_* characteristic length-scale of the source
(typical size of variation of the tensor anisotropic stresses)

Why sources in the early universe produce SGWBs?

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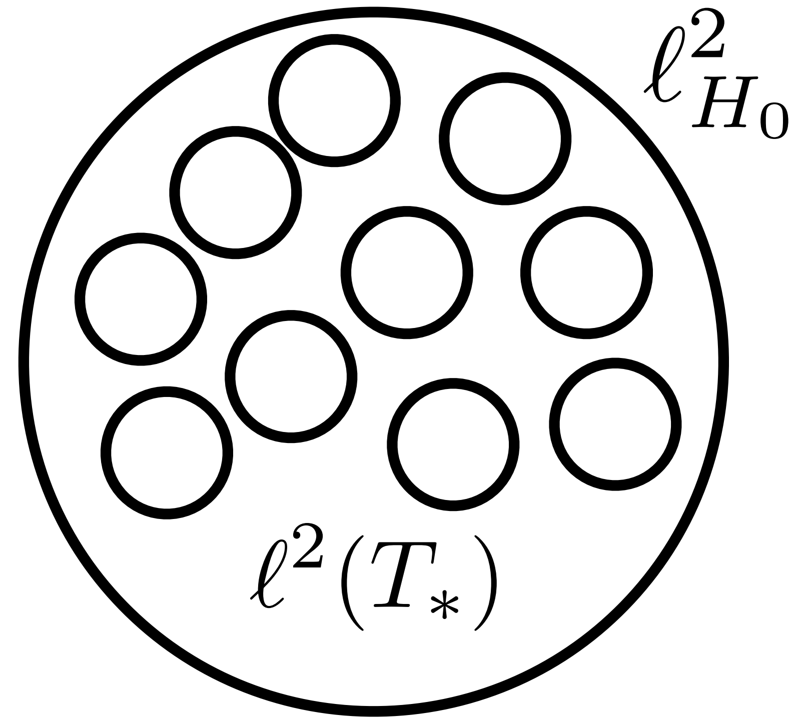


Angular size on the sky today of a region in which the SGWB signal is correlated

$$\Theta_* = \frac{\ell_*}{d_A(z_*)}$$

Angular diameter distance

Number of uncorrelated regions accessible today $\sim \Theta_*^{-2}$

Suppose a GW detector angular resolution of 10 deg $\longrightarrow$ $z_* \lesssim 17$

$$\Theta(z_* = 1090) \simeq 0.9\,\mathrm{deg} \qquad \Theta(T_* = 100\,\mathrm{GeV}) \simeq 10^{-12}\mathrm{deg}$$

Only the statistical properties of the signal can be accessed

Why sources in the early universe produce SGWBs?

- We access today the GW signal from many independent horizon volumes: $h_{ij}(\mathbf{x},t)$ must be treated as a random variable, only its statistical properties can be accessed, e.g. its correlator $\langle h_r(\mathbf{x},\eta_1)\, h_s(\mathbf{y},\eta_2)\rangle$

  where <…> is an ensemble average

- The universe is homogeneous and isotropic, so the GW source is operating everywhere at the same time with the same average properties ("a-causal" initial conditions from Inflation)

- Under the ergodic hypothesis, the ensemble average can be substituted with volume / time averages: we identify this average with the volume / time one necessary to define the GW energy momentum tensor

- Notable exception: *SGWB from Inflation* (intrinsic quantum fluctuations that become classical (stochastic) outside the horizon)

Characterisation of a primordial SGWB

The SGWB is in general homogenous and isotropic, unpolarised and Gaussian

As the FLRW space-time

$$\langle h_{ij}(\mathbf{x},\eta_1)\, h_{lm}(\mathbf{y},\eta_2)\rangle = F_{ijlm}(|\mathbf{x}-\mathbf{y}|,\eta_1,\eta_2)$$

Certainly some *induced anisotropy*, e.g. the dipole with respect to the cosmological frame

More challenging to detect than the "monopole"

If the sourcing process preserves parity

$$\langle h_{+2}(\mathbf{k},\eta)h_{+2}(\mathbf{k},\eta) - h_{-2}(\mathbf{k},\eta)h_{-2}(\mathbf{k},\eta)\rangle = \langle h_{+}(\mathbf{k},\eta)h_{\times}(\mathbf{k},\eta)\rangle = 0$$

Helicity basis $e^{\pm 2}_{ij} = \dfrac{e^{+}_{ij} \pm i\, e^{\times}_{ij}}{2}$

There are exceptions!

Central limit theorem: the signal comes from the superposition of many independent regions

Characterisation of a primordial SGWB

Power spectrum of the GW amplitude $h_c(k,t)$

$$\langle h_r(\mathbf{k},\eta)\, h_p^*(\mathbf{q},\eta)\rangle = \frac{8\pi^5}{k^3}\,\underbrace{\delta^{(3)}(\mathbf{k}-\mathbf{q})}_{\text{Statistical homogeneity and isotropy}}\,\delta_{rp}\, h_c^2(k,\eta)$$

Unpolarised

Gaussianity: the two-point correlation function is enough to fully describe the SGWB

$$\langle h_{ij}(\mathbf{x},\eta)\, h_{ij}(\mathbf{x},\eta)\rangle = 2\int_0^{+\infty}\frac{dk}{k}\, h_c^2(k,\eta)$$

Related to the variance of the GW amplitude in real space

For *freely propagating sub-Hubble modes*, and taking the time-average:

$$h_r(\mathbf{k},\eta) = \frac{A_r(\mathbf{k})}{a(\eta)}\, e^{ik\eta} + \frac{B_r(\mathbf{k})}{a(\eta)}\, e^{-ik\eta}$$

$$\langle h_r(\mathbf{k},\eta)\, h_p^*(\mathbf{q},\eta)\rangle = \frac{1}{a^2(\eta)}[\langle A_r(\mathbf{k})\, A_p^*(\mathbf{q})\rangle + \langle B_r(\mathbf{k})\, B_p^*(\mathbf{q})\rangle] \qquad h_c(k,\eta) \propto \frac{1}{a^2(\eta)}$$

Characterisation of a primordial SGWB

Power spectrum of the GW energy density $\frac{d\rho_{\rm GW}}{d\log k}$

$$\rho_{\rm GW} = \frac{\langle \dot{h}_{ij}(\mathbf{x},t)\,\dot{h}_{ij}(\mathbf{x},t)\rangle}{32\pi G} = \frac{\langle h'_{ij}(\mathbf{x},\eta)\,h'_{ij}(\mathbf{x},\eta)\rangle}{32\pi G\,a^2(\eta)} = \int_0^{+\infty}\frac{dk}{k}\,\frac{d\rho_{\rm GW}}{d\log k}$$

$$\langle h'_r(\mathbf{k},\eta)\,{h'_p}^*(\mathbf{q},\eta)\rangle = \frac{8\pi^5}{k^3}\,\delta^{(3)}(\mathbf{k}-\mathbf{q})\,\delta_{rp}\,{h'_c}^2(k,\eta)$$

For *freely propagating sub-Hubble modes*, and taking the time-average:

$${h'_c}^2(k,\eta) \simeq k^2\,h_c^2(k,\eta) \qquad\qquad \frac{d\rho_{\rm GW}}{d\log k} = \frac{k^2\,h_c^2(k,\eta)}{16\pi G\,a^2(\eta)}$$

$$\rho_{\rm GW} \propto \frac{1}{a(\eta)^4}$$

GW energy density scales like radiation for freely propagating sub-Hubble modes (free massless particles)

Characterisation of a primordial SGWB

GW energy density parameter

Evaluated today, for a source that operated at time η_*

$$h^2\,\Omega_{\rm GW}(k,\eta_0) = \frac{h^2\rho_*}{\rho_c}\,\left(\frac{a_*}{a_0}\right)^4\left(\frac{1}{\rho_*}\,\frac{d\rho_{\rm GW}}{d\log k}(k,\eta_*)\right)$$

To make connection with the <u>detection process</u> one assumes that

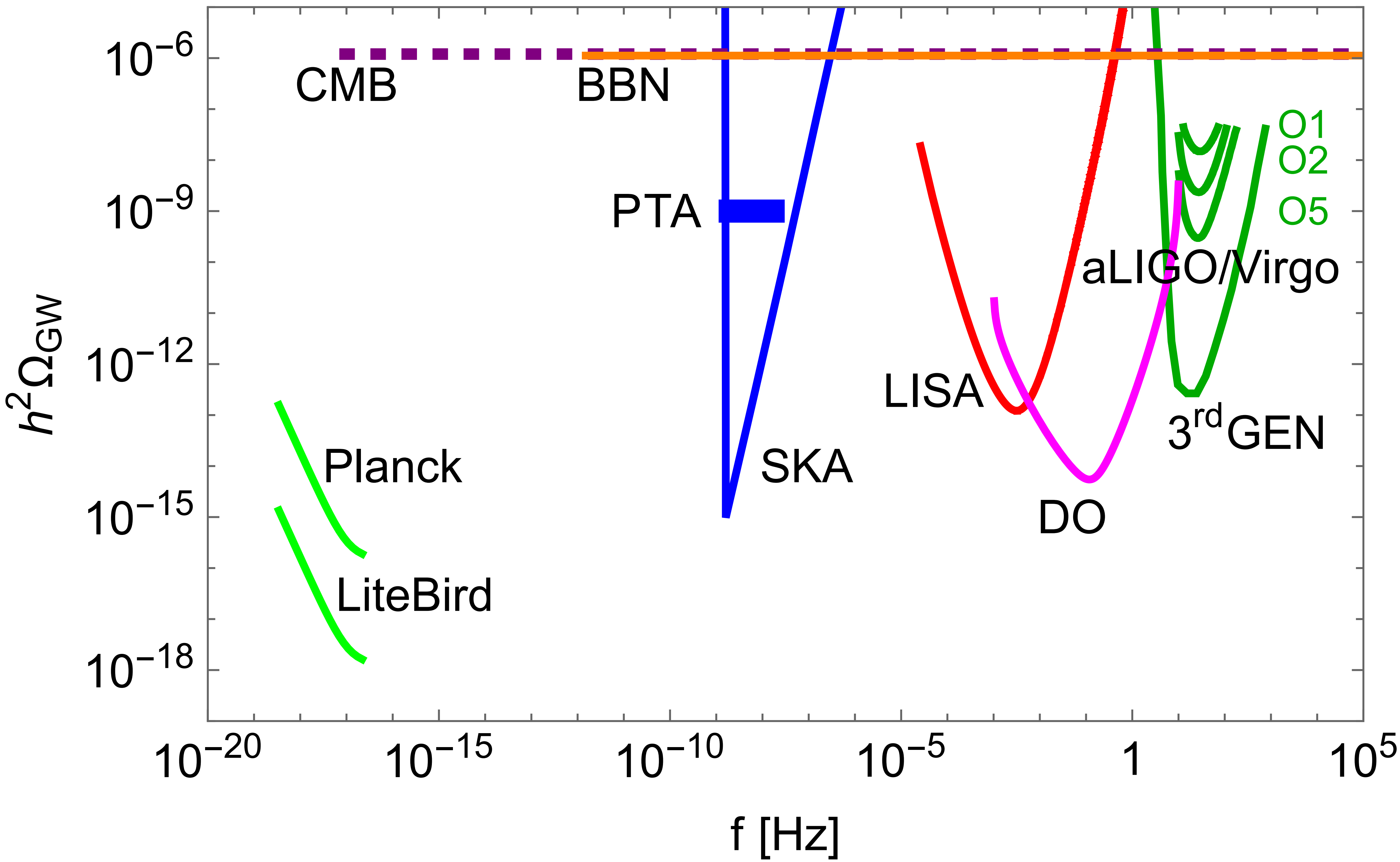
- The source has stopped operating so the waves are freely propagating
- The expansion of the universe is negligible over the time of the measurement so that the SGWB appears stationary
- One can F.T. in time as well $\quad f = \dfrac{1}{2\pi}\dfrac{k}{a_0}$

Power spectral density

$$\langle \bar{h}_r(f,\hat{\mathbf{k}})\bar{h}_p^*(g,\hat{\mathbf{q}})\rangle = a_0^4\,f^2\,g^2\langle A_r(\mathbf{k})\,A_p^*(\mathbf{q})\rangle = \\ = \frac{1}{8\pi}\,\delta(f-g)\,\delta^{(2)}(\hat{\mathbf{k}}-\hat{\mathbf{q}})\,\delta_{rp}\,S_h(f)$$

$$\Omega_{\rm GW}(f) = \frac{4\pi^2}{3H_0^2}\,f^3\,S_h(f)$$

What is/will be known about the SGWB



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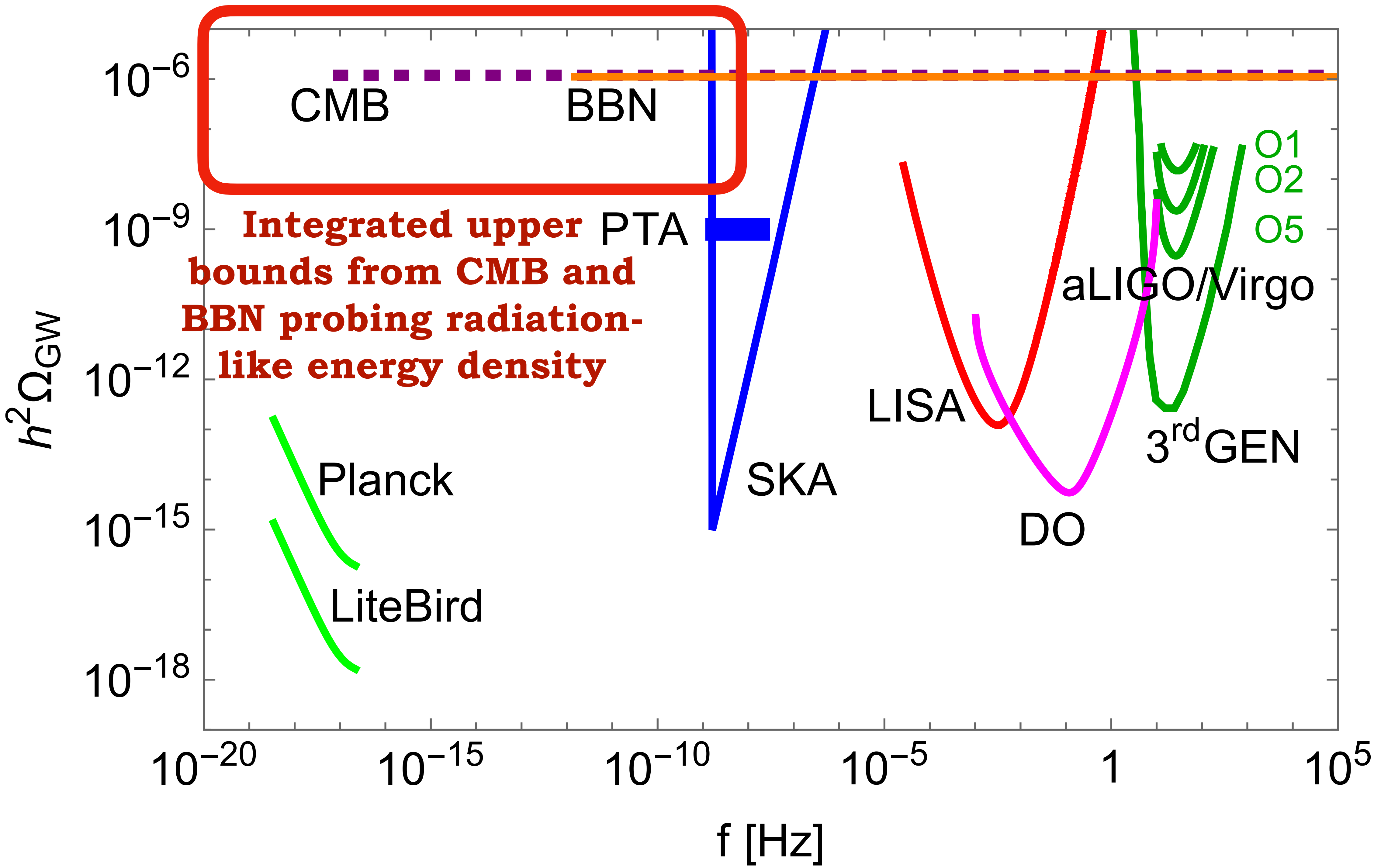


What is/will be known about the SGWB

- GW contribute to the energy density in the universe and change its background evolution

$$H^2(T) = \frac{8\pi G}{3}\Sigma_i \rho_i(T)$$

- The abundances of elements produced at Big Bang Nucleosynthesis (BBN) depend on the relative abundance of neutrons and protons, which depends on the Hubble scale at T ~ MeV

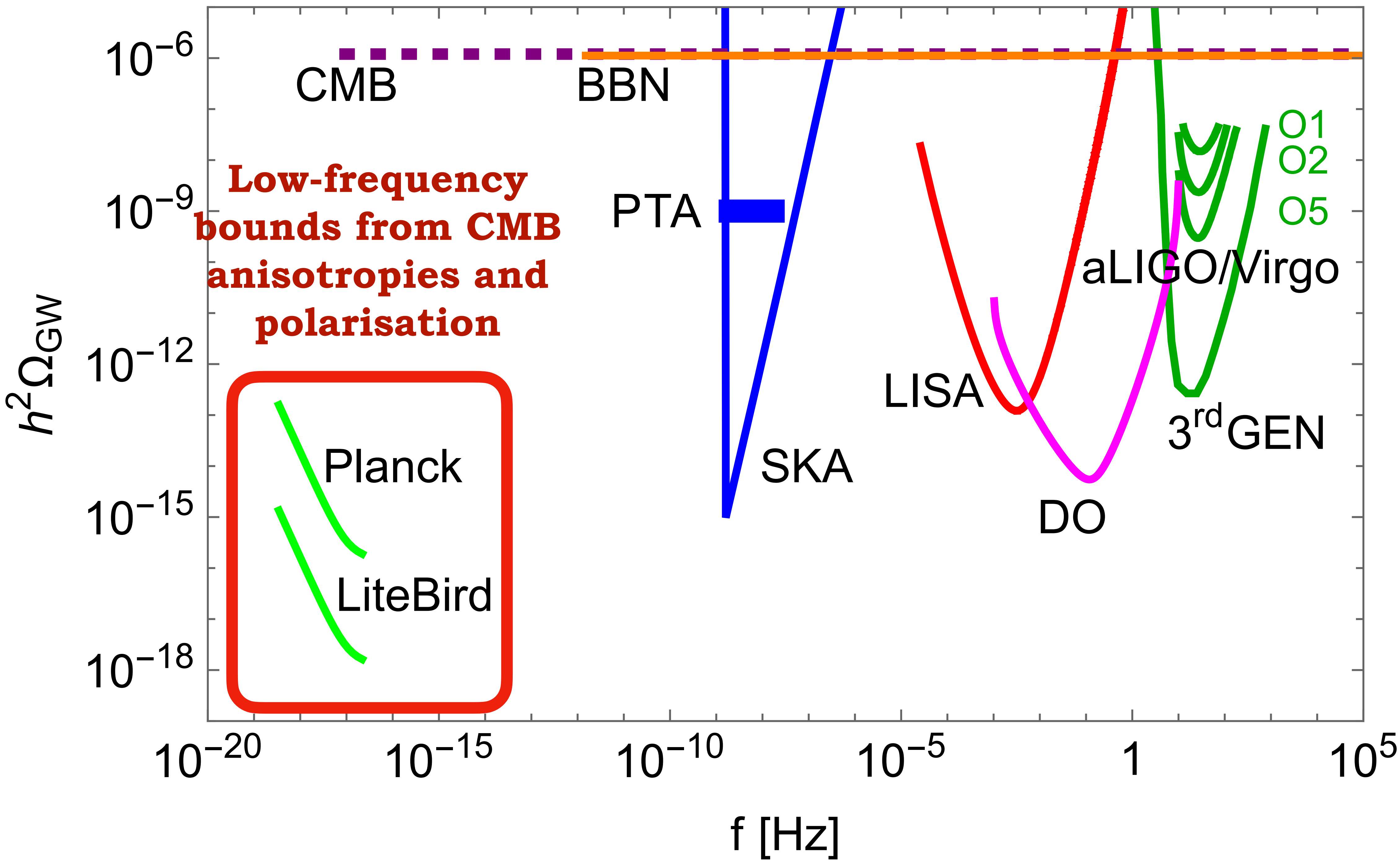
- The Cosmic Microwave Background (CMB) monopole and anisotropy spectrum depend on the Hubble scale at decoupling T ~ 0.3 eV, on the matter-radiation equality…

- Bounds on the *integrated GW energy density* at/previous to the BBN and CMB epochs

$$\left(\frac{\rho_{\rm GW}}{\rho_c}\right)_0 = \int \frac{df}{f}\,\Omega_{\rm GW}(f) = \Omega_\gamma^0 \left(\frac{g_S(T_0)}{g_S(T)}\right)^{4/3} \boxed{\left(\frac{\rho_{\rm GW}}{\rho_\gamma}\right)_T}$$

CAREFUL! Plot “wrong”…

What is/will be known about the SGWB



Cosmic microwave background

frequency range of detection: 10^{-18} Hz < f < 10^{-16} HZ

- temperature anisotropy: limit by Planck

$$\frac{\delta T}{T} = -\int_{t_{\rm dec}}^{t_0} \dot{h}_{ij}\, n^i n^j \, dt$$

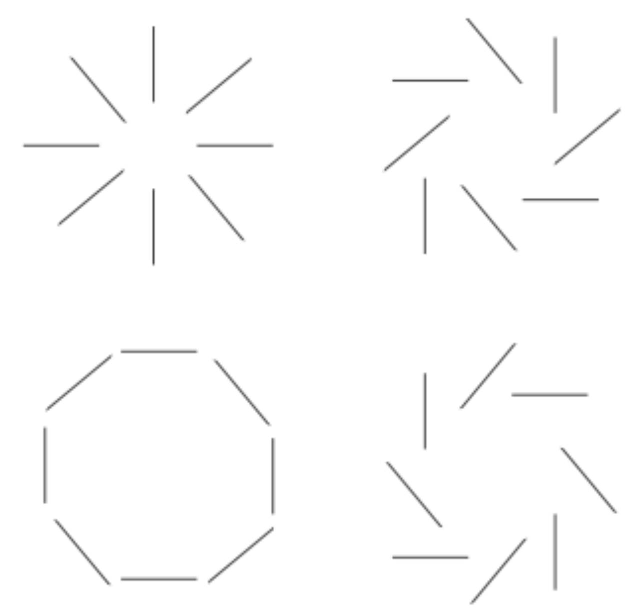
$$P_h(k) = A_t(k_0)\,(k\,\eta_0)^{n_t} \qquad C_{\ell,T}^{\Theta\Theta} \simeq \frac{\sqrt{\pi}}{3} A_t(k_0)\, \frac{\ell(\ell+2)!}{(\ell-2)!}\, \frac{\Gamma\left[\frac{7-n_t}{2}\right]\Gamma\left[\ell+\frac{n_t}{2}\right]}{\Gamma\left[4-\frac{n_t}{2}\right]\Gamma\left[\ell+7-\frac{n_t}{2}\right]}$$

$$\propto \ \ell^{n_t-2} \qquad \text{for} \quad 1 \ll \ell \lesssim 60$$

- polarisation: BB spectrum measured by BICEP and Planck generated at photon decoupling time, from Thomson scattering of electrons by a quadrupole temperature anisotropy in the photons

polarisation patterns

generated by primordial scalar and tensor perturbations

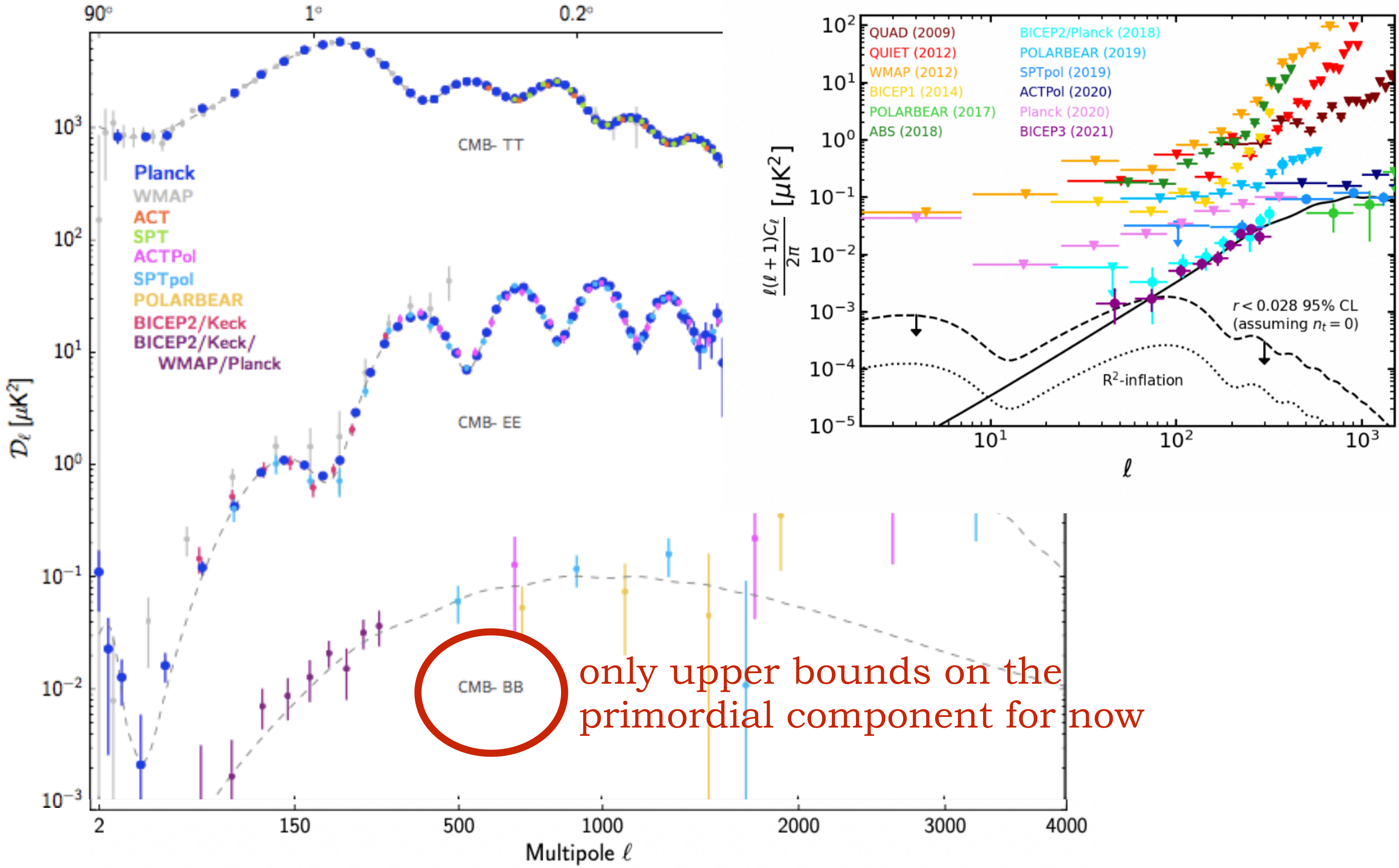


E mode B mode

generated only by primordial tensor perturbations or by foregrounds

Cosmic microwave background

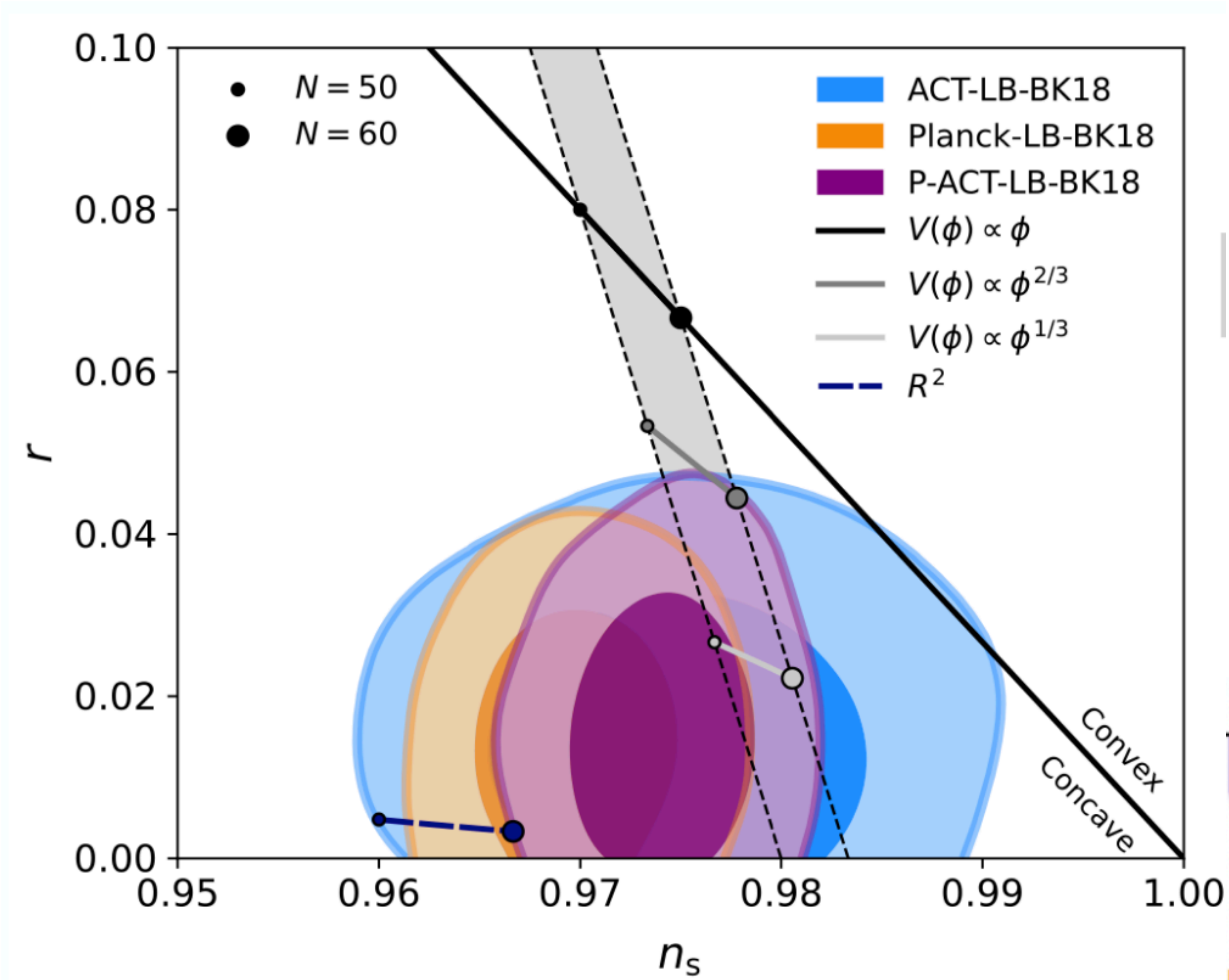
Galloni et al: arXiv:2208.00188



only upper bounds on the
primordial component for now

Planck collaboration: arXiv:1807.06205

Cosmic microwave background

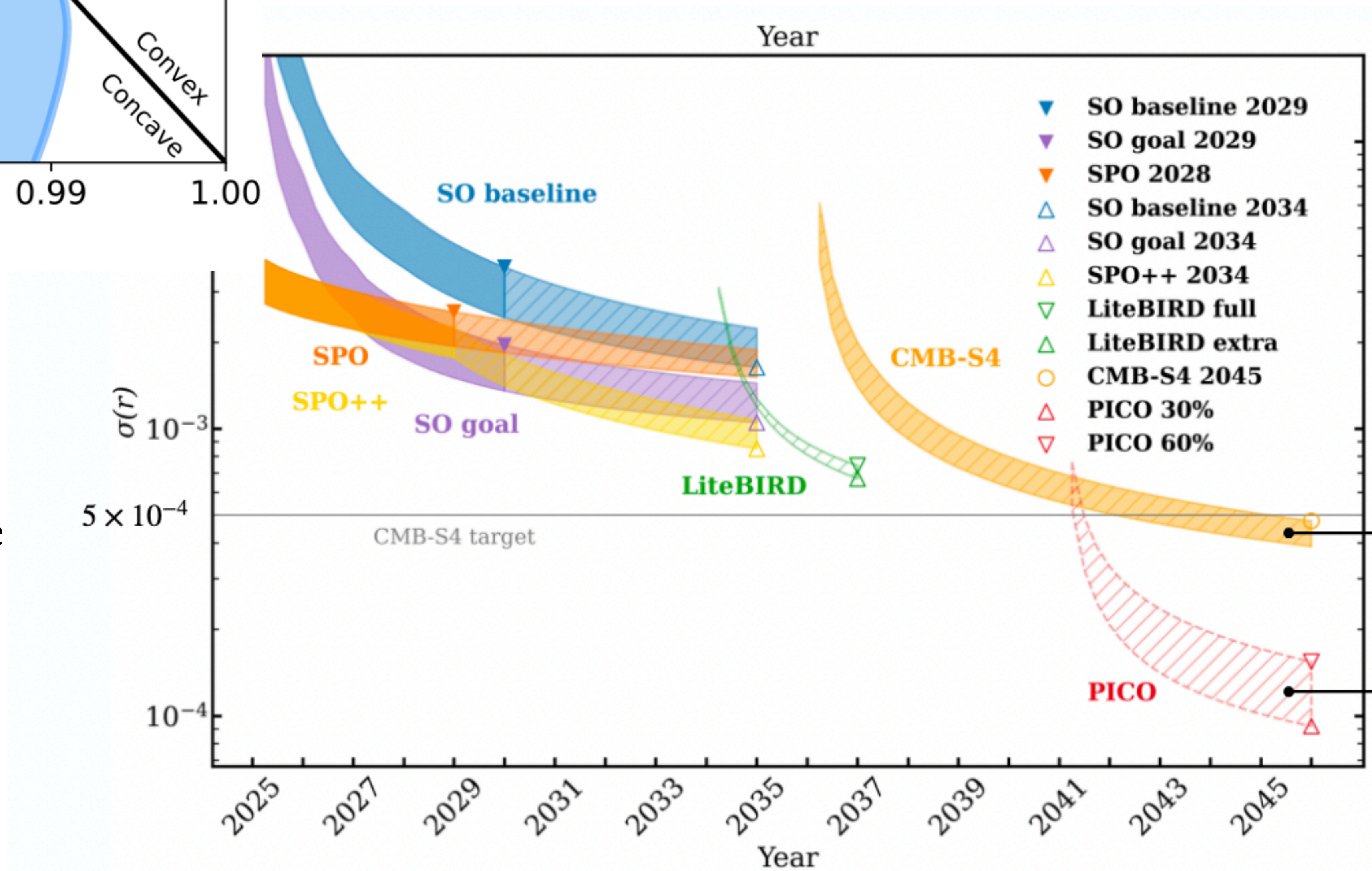


Calabrese et al, arXiv:2503.14454

This constraint is usually represented in the context of Inflation as a bound on the tensor-to-scalar ratio

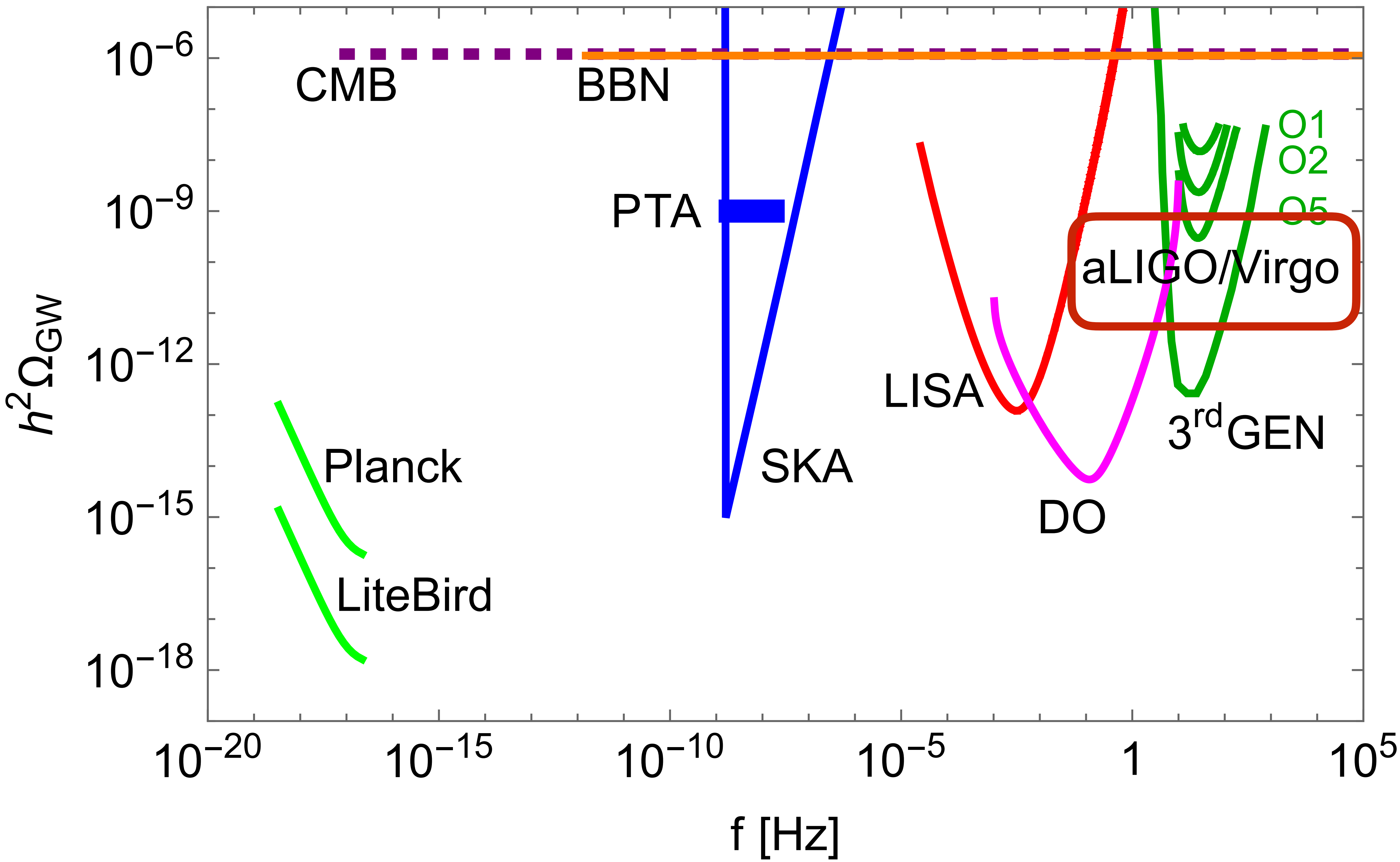
A. Challinor, https://indico-dpt.unige.ch/event/1/contributions/31/attachments/16/19/Challinor_Geneva2025.pdf

Projected sensitivity on the tensor-to-scalar ratio



What is/will be known about the SGWB

Present and future GW obsevatories:
LIGO VIRGO KAGRA



Earth-based interferometers

LIGO/Virgo (operating)

arm length L = 4 km

frequency range of detection:
10 Hz < f < 5kHZ

3rd generation ET, CE (future)

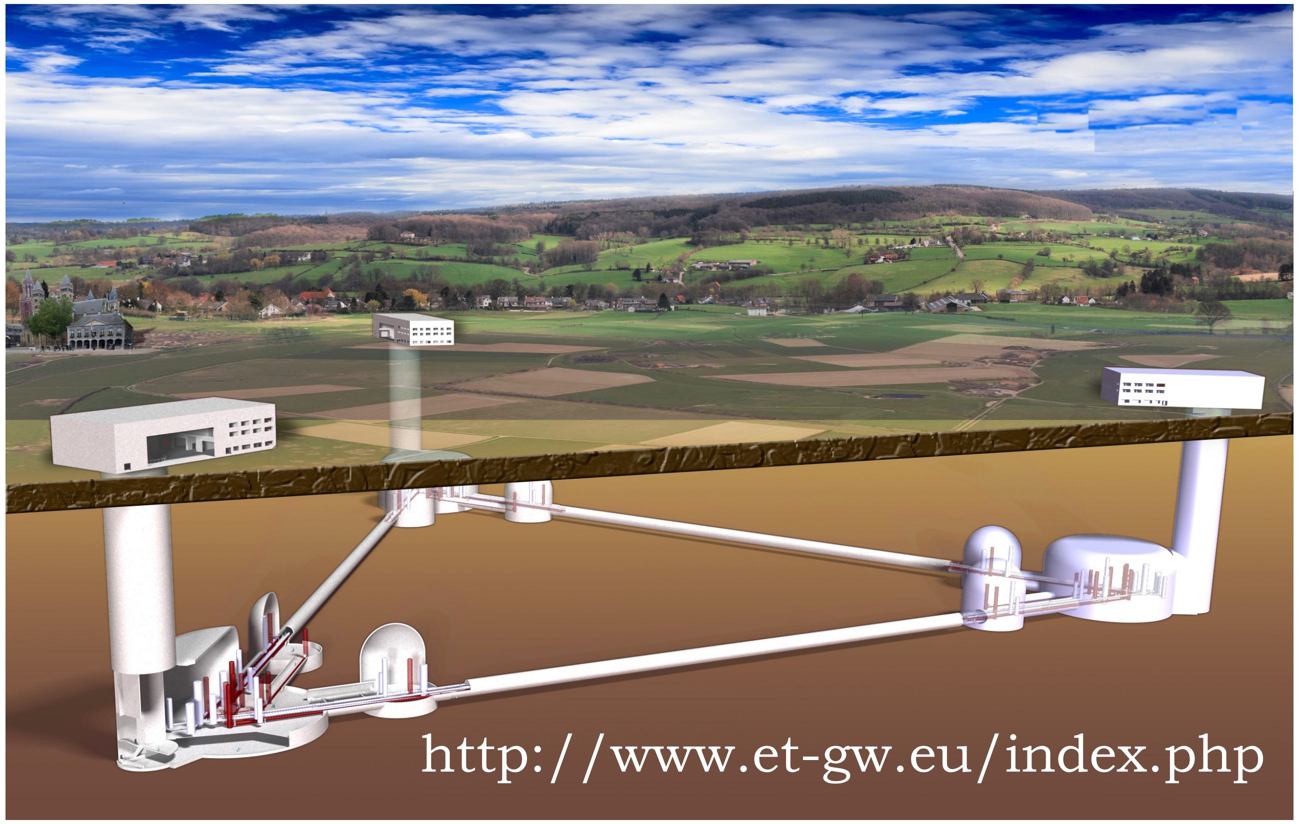
arm length L ~15-20 km

frequency range of detection:
1 Hz < f < 10^4 HZ

factor 20 improvement in sensitivity

DETECTION TARGETS:

- Black hole coalescing binaries of masses few to hundred solar masses (BHBs)
- Neutron Star and NS-BH binaries / SN explosions
- Stochastic GW background



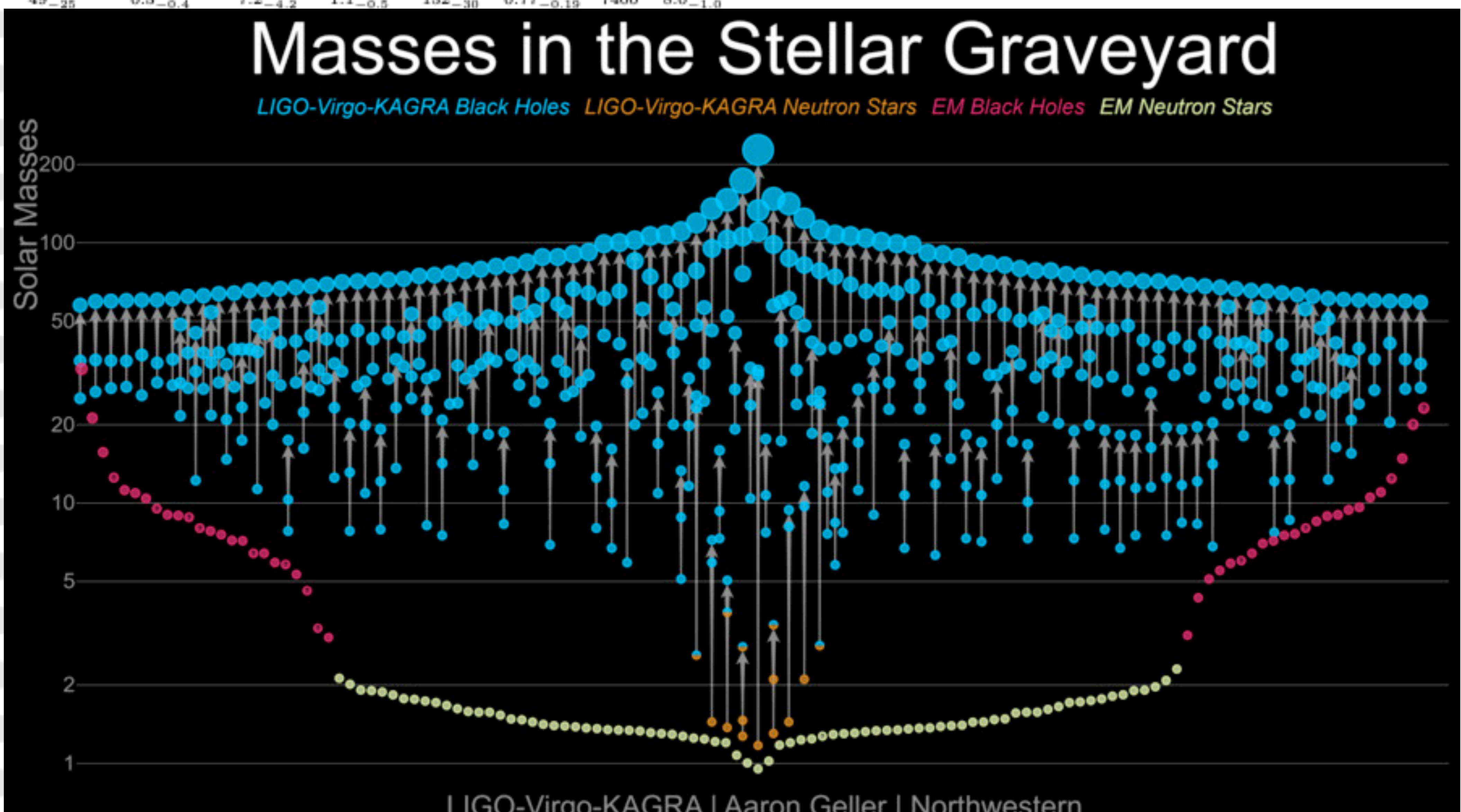
Earth-based interferometers

Candidate	M $[M_\odot]$	$\mathcal{M}$ $[M_\odot]$	m_1 $[M_\odot]$	m_2 $[M_\odot]$	$\chi_{\rm eff}$	$D_{\rm L}$ [Gpc]	z	$M_{\rm f}$ $[M_\odot]$	$\chi_{\rm f}$	$\Delta\Omega$ $[\rm deg^2]$	SNR
GW230518_125908	$9.61^{+0.76}_{-0.79}$	$2.80^{+0.06}_{-0.06}$	$8.17^{+0.84}_{-0.92}$	$1.45^{+0.13}_{-0.10}$	$-0.01^{+0.09}_{-0.11}$	$0.24^{+0.11}_{-0.10}$	$0.05^{+0.02}_{-0.02}$	$9.46^{+0.76}_{-0.80}$	$0.38^{+0.03}_{-0.03}$	490	$14.2^{+0.2}_{-0.4}$
GW230529_181500	$5.08^{+0.61}_{-0.60}$	$1.94^{+0.04}_{-0.04}$	$3.66^{+0.82}_{-1.21}$	$1.42^{+0.60}_{-0.22}$	$-0.10^{+0.12}_{-0.18}$	$0.2^{+0.1}_{-0.1}$	$0.04^{+0.02}_{-0.02}$	$4.92^{+0.62}_{-0.63}$	$0.58^{+0.08}_{-0.06}$	24000	$11.6^{+0.3}_{-0.4}$
GW230601_224134	107^{+22}_{-15}	$45.0^{+10.0}_{-7.3}$	64^{+17}_{-13}	44^{+14}_{-15}	$-0.03^{+0.27}_{-0.32}$	$3.7^{+2.1}_{-1.8}$	$0.60^{+0.28}_{-0.26}$	102^{+21}_{-14}	$0.67^{+0.12}_{-0.13}$	3300	$12.3^{+0.2}_{-0.3}$
GW230605_065343	$28.6^{+4.0}_{-2.8}$	$11.9^{+1.0}_{-0.9}$	$17.2^{+6.5}_{-3.5}$	$11.1^{+2.5}_{-2.7}$	$0.06^{+0.16}_{-0.10}$	$1.1^{+0.6}_{-0.5}$	$0.21^{+0.10}_{-0.09}$	$27.3^{+4.1}_{-2.7}$	$0.69^{+0.05}_{-0.05}$	1000	$10.5^{+0.3}_{-0.4}$
GW230606_004305	$63.4^{+13.4}_{-8.3}$	$26.5^{+5.7}_{-3.6}$	$37.6^{+13.4}_{-7.6}$	$25.8^{+8.1}_{-8.3}$	$-0.1^{+0.3}_{-0.3}$	$2.7^{+1.5}_{-1.4}$	$0.48^{+0.20}_{-0.22}$	$60.7^{+12.8}_{-7.9}$	$0.64^{+0.11}_{-0.14}$	1400	$10.3^{+0.3}_{-0.4}$
GW230608_205047	79^{+16}_{-11}	$32.8^{+7.3}_{-5.4}$	48^{+13}_{-11}	$31.0^{+11.0}_{-10.0}$	$0.04^{+0.25}_{-0.26}$	$3.5^{+2.2}_{-1.7}$	$0.58^{+0.29}_{-0.26}$	75^{+15}_{-11}	$0.69^{+0.11}_{-0.14}$	2200	$9.8^{+0.3}_{-0.5}$
GW230609_064958	$60.3^{+12.9}_{-8.0}$	$25.5^{+5.8}_{-3.6}$	$35.3^{+10.7}_{-6.9}$	$25.2^{+7.5}_{-7.5}$	$-0.1^{+0.2}_{-0.3}$	$3.4^{+1.9}_{-1.7}$	$0.56^{+0.25}_{-0.25}$	$57.8^{+12.2}_{-7.7}$	$0.64^{+0.09}_{-0.13}$	1700	$9.8^{+0.3}_{-0.5}$
GW230624_113103	$43.8^{+11.1}_{-6.6}$	$18.0^{+3.2}_{-2.4}$	$27.2^{+13.4}_{-6.9}$	$16.1^{+4.8}_{-4.5}$	$0.2^{+0.3}_{-0.3}$	$1.9^{+1.3}_{-1.0}$	$0.35^{+0.19}_{-0.16}$	$41.8^{+10.7}_{-6.3}$	$0.72^{+0.12}_{-0.11}$	1300	$9.7^{+0.4}_{-0.5}$
GW230627_015337	$14.2^{+0.8}_{-0.4}$	$6.02^{+0.16}_{-0.07}$	$8.37^{+1.67}_{-1.26}$	$5.79^{+0.95}_{-0.92}$	$0.02^{+0.08}_{-0.03}$	$0.31^{+0.06}_{-0.13}$	$0.07^{+0.01}_{-0.03}$	$13.5^{+0.8}_{-0.5}$	$0.68^{+0.02}_{-0.03}$	110	$28.5^{+0.1}_{-0.1}$
GW230628_231200	$59.3^{+8.6}_{-4.9}$	$25.5^{+3.8}_{-2.2}$	$32.5^{+5.9}_{-3.9}$	$27.1^{+4.9}_{-5.2}$	$-0.01^{+0.16}_{-0.16}$	$2.3^{+0.8}_{-1.1}$	$0.40^{+0.12}_{-0.17}$	$56.5^{+8.1}_{-4.6}$	$0.69^{+0.08}_{-0.06}$	660	$15.5^{+0.2}_{-0.3}$
GW230630_125806	84^{+26}_{-19}	$35.0^{+11.4}_{-8.5}$	51^{+20}_{-14}	33^{+15}_{-13}	$0.2^{+0.3}_{-0.3}$	$5.4^{+5.1}_{-3.0}$	$0.83^{+0.59}_{-0.41}$	80^{+25}_{-18}	$0.75^{+0.11}_{-0.16}$	4300	$8.1^{+0.4}_{-0.5}$
GW230630_234532	$16.8^{+2.1}_{-1.4}$	$7.06^{+0.57}_{-0.48}$	$10.0^{+3.5}_{-1.8}$	$6.64^{+1.46}_{-1.59}$	$-0.04^{+0.16}_{-0.08}$	$1.1^{+0.5}_{-0.5}$	$0.21^{+0.09}_{-0.09}$	$16.1^{+2.1}_{-1.4}$	$0.66^{+0.04}_{-0.05}$	1500	$9.4^{+0.3}_{-0.5}$
GW230702_185453	60^{+15}_{-11}	$22.9^{+4.0}_{-3.5}$	40^{+20}_{-13}	$18.0^{+8.5}_{-6.5}$	$0.05^{+0.29}_{-0.26}$	$2.4^{+1.9}_{-1.1}$	$0.43^{+0.27}_{-0.17}$	57^{+15}_{-11}	$0.64^{+0.13}_{-0.15}$	2500	$9.5^{+0.3}_{-0.5}$
GW230704_021211	$52.7^{+10.8}_{-8.2}$	$21.8^{+4.4}_{-3.2}$	$32.4^{+11.9}_{-7.9}$	$19.9^{+6.5}_{-6.0}$	$-0.01^{+0.21}_{-0.24}$	$2.7^{+1.8}_{-1.5}$	$0.48^{+0.25}_{-0.23}$	$50.5^{+10.6}_{-7.9}$	$0.66^{+0.10}_{-0.13}$	1500	$9.0^{+0.3}_{-0.5}$
GW230704_212616	139^{+49}_{-31}	55^{+23}_{-14}	89^{+42}_{-27}	49^{+30}_{-25}	$0.3^{+0.3}_{-0.4}$	$7.2^{+6.1}_{-4.2}$	$1.1^{+0.7}_{-0.5}$	132^{+47}_{-30}	$0.77^{+0.12}_{-0.19}$	7400	$8.0^{+1.0}_{-1.0}$
GW230706_104333	$27.9^{+4.4}_{-3.0}$	$11.8^{+1.7}_{-1.1}$	$16.3^{+5.4}_{-3.0}$								
GW230707_124047	82^{+19}_{-12}	$35.1^{+8.6}_{-5.3}$	$46.1^{+12.2}_{-8.2}$								
GW230708_053705	$51.9^{+10.2}_{-7.6}$	$22.2^{+4.4}_{-3.3}$	$29.1^{+8.2}_{-5.4}$								
GW230708_230935	103^{+21}_{-16}	$42.5^{+9.7}_{-7.8}$	64^{+20}_{-15}								
GW230709_122727	74^{+20}_{-14}	$31.0^{+9.4}_{-6.6}$	45^{+16}_{-11}								
GW230712_090405	46^{+22}_{-11}	$16.5^{+11.1}_{-3.2}$	$32.0^{+17.0}_{-10.0}$								
GW230723_101834	$27.5^{+4.3}_{-2.6}$	$11.4^{+1.6}_{-0.9}$	$16.7^{+5.9}_{-3.4}$								
GW230726_002940	$63.6^{+10.3}_{-7.9}$	$27.3^{+4.4}_{-3.4}$	$35.6^{+9.0}_{-5.9}$								
GW230729_082317	$20.3^{+5.0}_{-2.4}$	$8.33^{+0.94}_{-0.75}$	$12.3^{+7.6}_{-2.7}$								
GW230731_215307	$18.3^{+1.6}_{-1.0}$	$7.77^{+0.58}_{-0.36}$	$10.4^{+2.9}_{-1.4}$								
GW230803_033412	74^{+22}_{-16}	$30.5^{+9.3}_{-6.7}$	45^{+19}_{-12}								
GW230805_034249	$54.9^{+13.8}_{-9.7}$	$23.1^{+5.9}_{-4.2}$	$32.1^{+12.5}_{-7.3}$								
GW230806_204041	85^{+24}_{-15}	$35.8^{+10.7}_{-6.9}$	51^{+18}_{-12}								
GW230811_032116	$57.9^{+8.7}_{-6.6}$	$24.0^{+4.1}_{-2.4}$	$35.6^{+8.7}_{-7.5}$								
GW230814_061920	110^{+26}_{-20}	$45.5^{+11.9}_{-9.9}$	69^{+19}_{-17}								
GW230814_230901	$61.8^{+2.0}_{-2.1}$	$26.7^{+0.9}_{-1.0}$	$33.6^{+2.8}_{-2.2}$								
GW230819_171910	106^{+41}_{-22}	42^{+14}_{-11}	70^{+47}_{-21}								
GW230820_212515	96^{+22}_{-16}	$38.6^{+11.2}_{-9.8}$	62^{+23}_{-15}								
GW230824_033047	88^{+20}_{-14}	$37.0^{+9.1}_{-6.3}$	52^{+16}_{-12}								
GW230825_041334	71^{+22}_{-15}	$29.4^{+9.4}_{-6.3}$	43^{+17}_{-12}								
GW230831_015414	73^{+26}_{-15}	$30.7^{+10.6}_{-6.5}$	$42.0^{+18.0}_{-10.0}$								
GW230904_051013	$17.9^{+2.4}_{-1.6}$	$7.54^{+0.53}_{-0.60}$	$10.6^{+4.1}_{-1.9}$								
GW230911_195324	$55.3^{+7.8}_{-7.0}$	$23.1^{+3.4}_{-3.0}$	$33.6^{+8.1}_{-7.3}$								
GW230914_111401	$95.0^{+15.0}_{-10.0}$	$39.6^{+7.5}_{-6.2}$	59^{+12}_{-11}								
GW230919_215712	$49.0^{+4.2}_{-4.5}$	$21.0^{+1.8}_{-1.9}$	$27.3^{+5.5}_{-3.7}$								
GW230920_071124	$56.4^{+10.2}_{-8.0}$	$23.9^{+4.6}_{-3.4}$	$32.4^{+9.1}_{-6.2}$								
GW230922_020344	$68.6^{+8.9}_{-6.5}$	$29.2^{+3.6}_{-2.6}$	$39.3^{+10.0}_{-6.3}$								
GW230922_040658	125^{+36}_{-21}	52^{+17}_{-12}	76^{+28}_{-18}								
GW230924_124453	$51.8^{+7.0}_{-4.9}$	$22.3^{+3.1}_{-2.1}$	$28.8^{+5.9}_{-4.0}$								
GW230927_043729	$61.8^{+12.6}$	$26.5^{+5.5}$	$34.9^{+8.7}$								

Individually resolved BHBs, NSBs, NS-BH

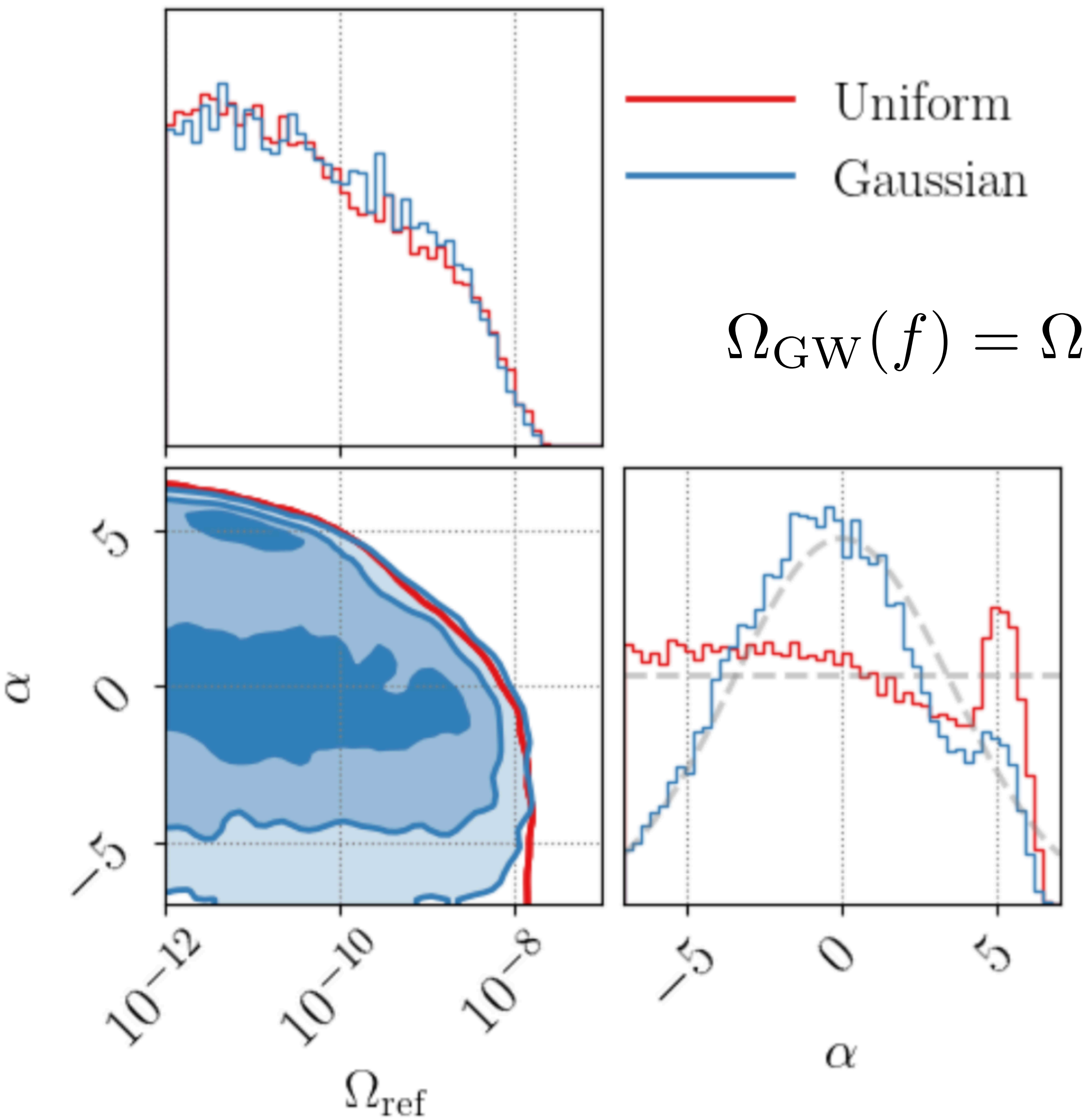
Last catalogue GWTC-4: 128 binary mergers detected, including NS-BH and NS-NS mergers

LVK Collaboration: arXiv::2508.18082



Earth-based interferometers

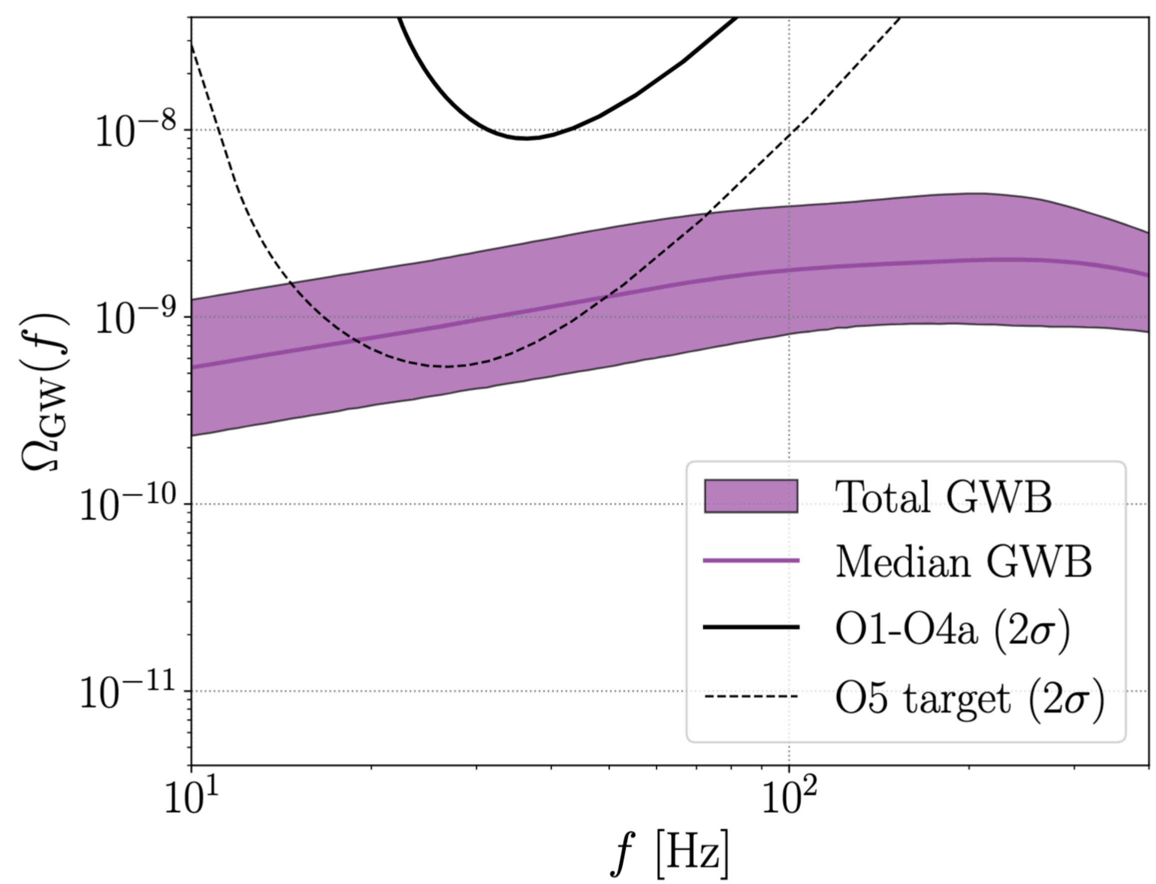
Stochastic GW background: for now, only upper bounds



$$\Omega_{\rm GW}(f) = \Omega_{\rm ref}\left(\frac{f}{25\,\mathrm{Hz}}\right)^{\alpha}$$

Most probably no cosmological SGWB detection by LVK, masked by astrophysical foreground detection expected for ~2030

SGWB from BHBs and NSBs

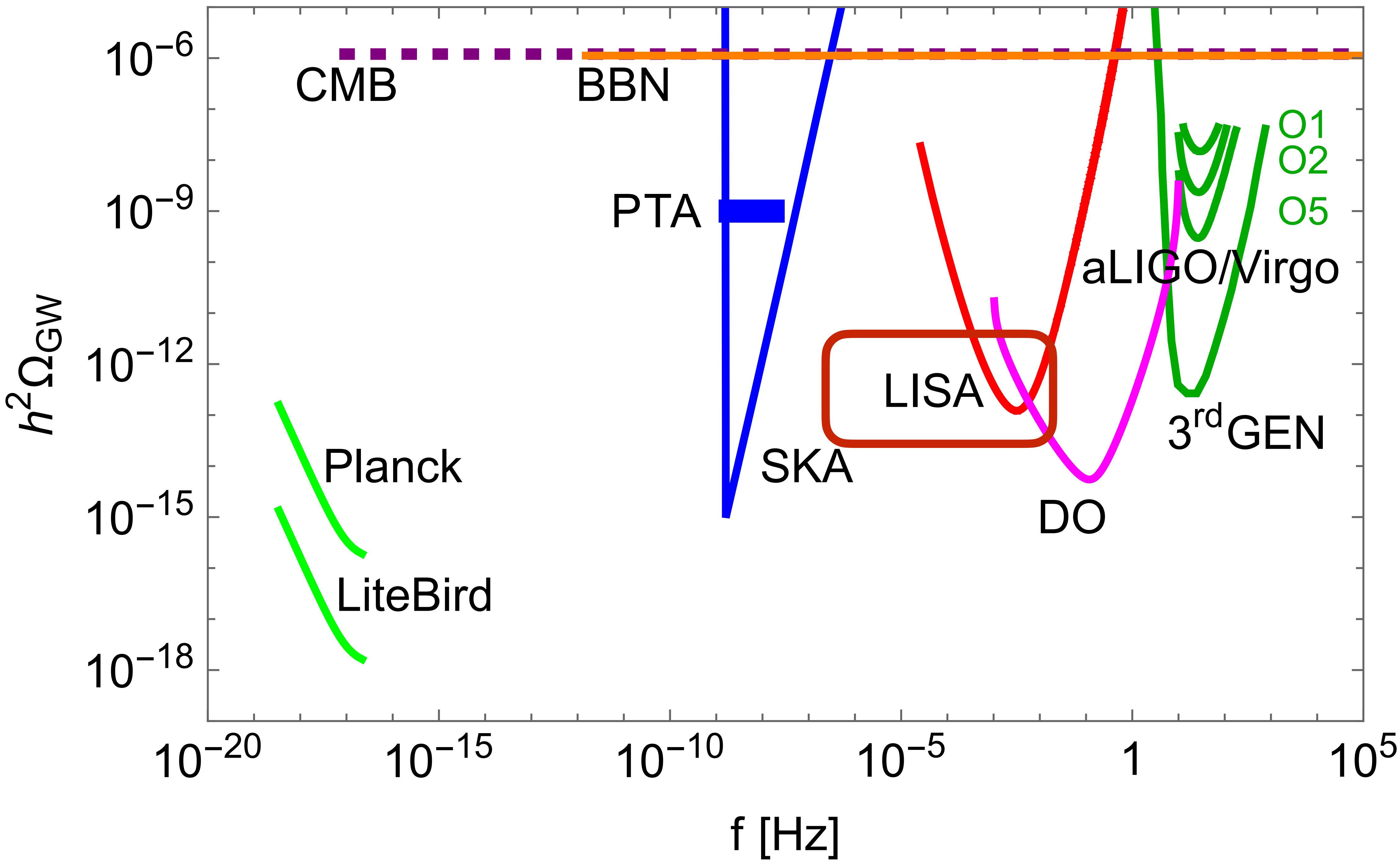


Detection via cross-correlation of signals from different detectors

LVK Collaborations, arXiv:2508.20721

What is/will be known about the SGWB

Present and future GW observatories: LISA



Space-based interferometers: LISA

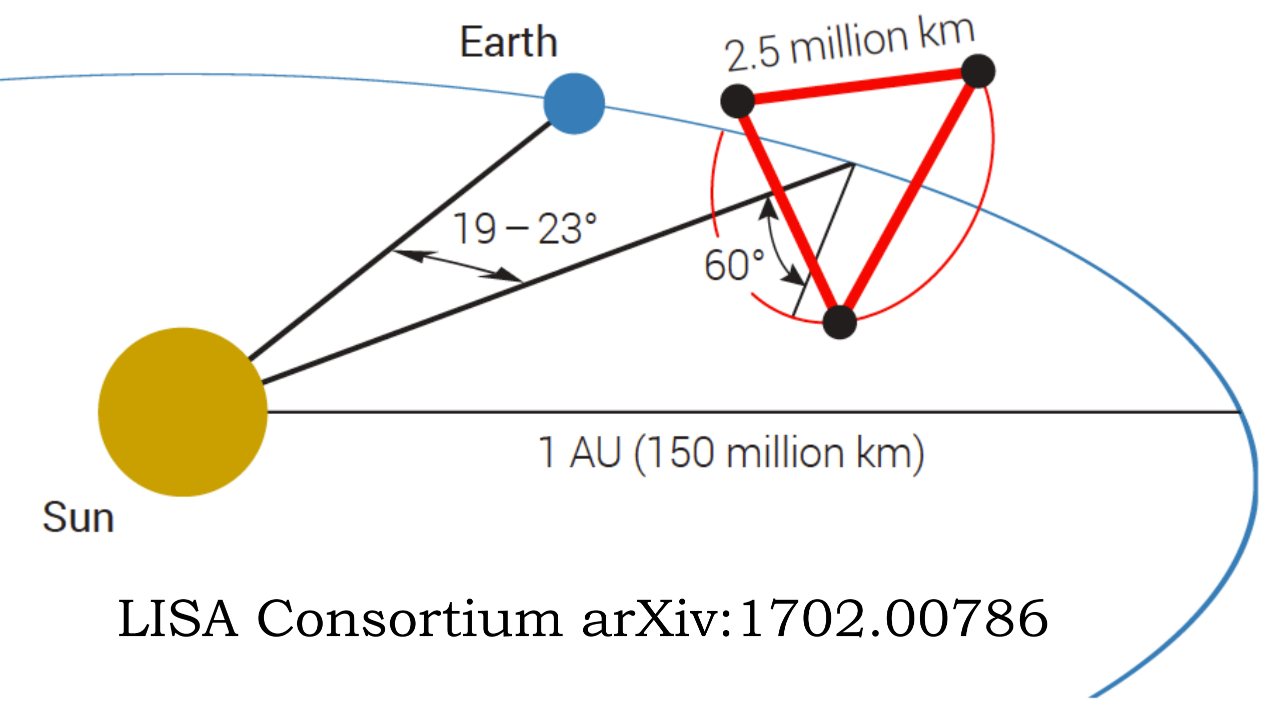
LISA: Laser Interferometer Space Antenna

- no seismic noise
- much longer arms than on Earth

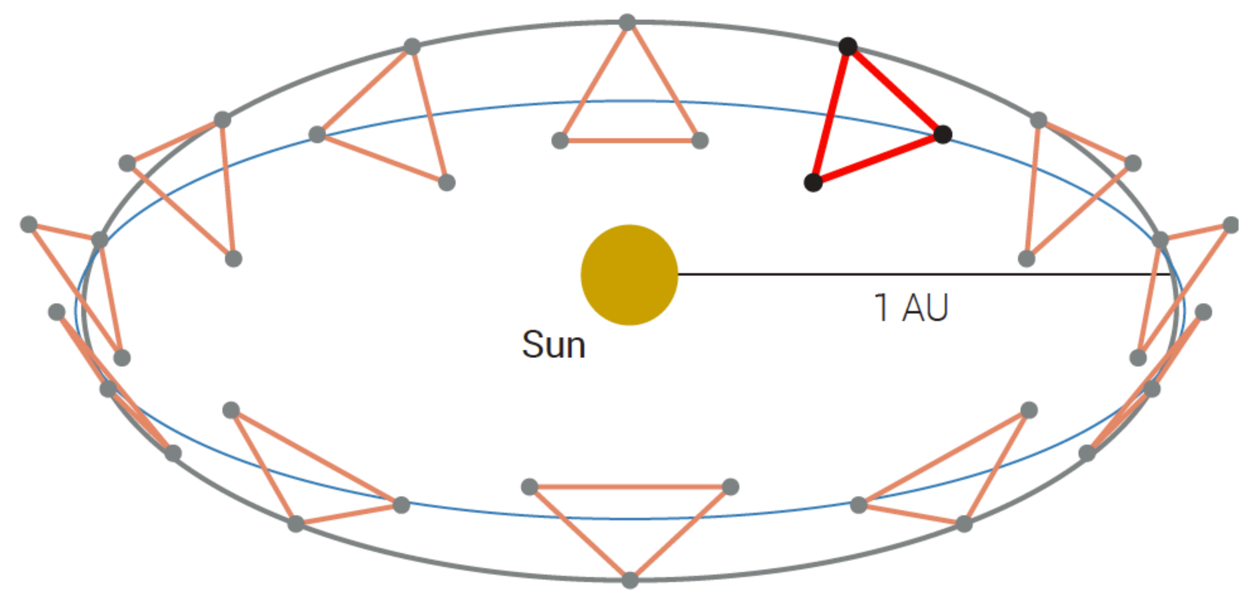
frequency range of detection:

$$10^{-4}\,\mathrm{Hz} < f < 1\,\mathrm{Hz}$$

- Launch in ~2035
- two masses in free fall per spacecraft
- 2.5 million km arms
- picometer displacement of masses

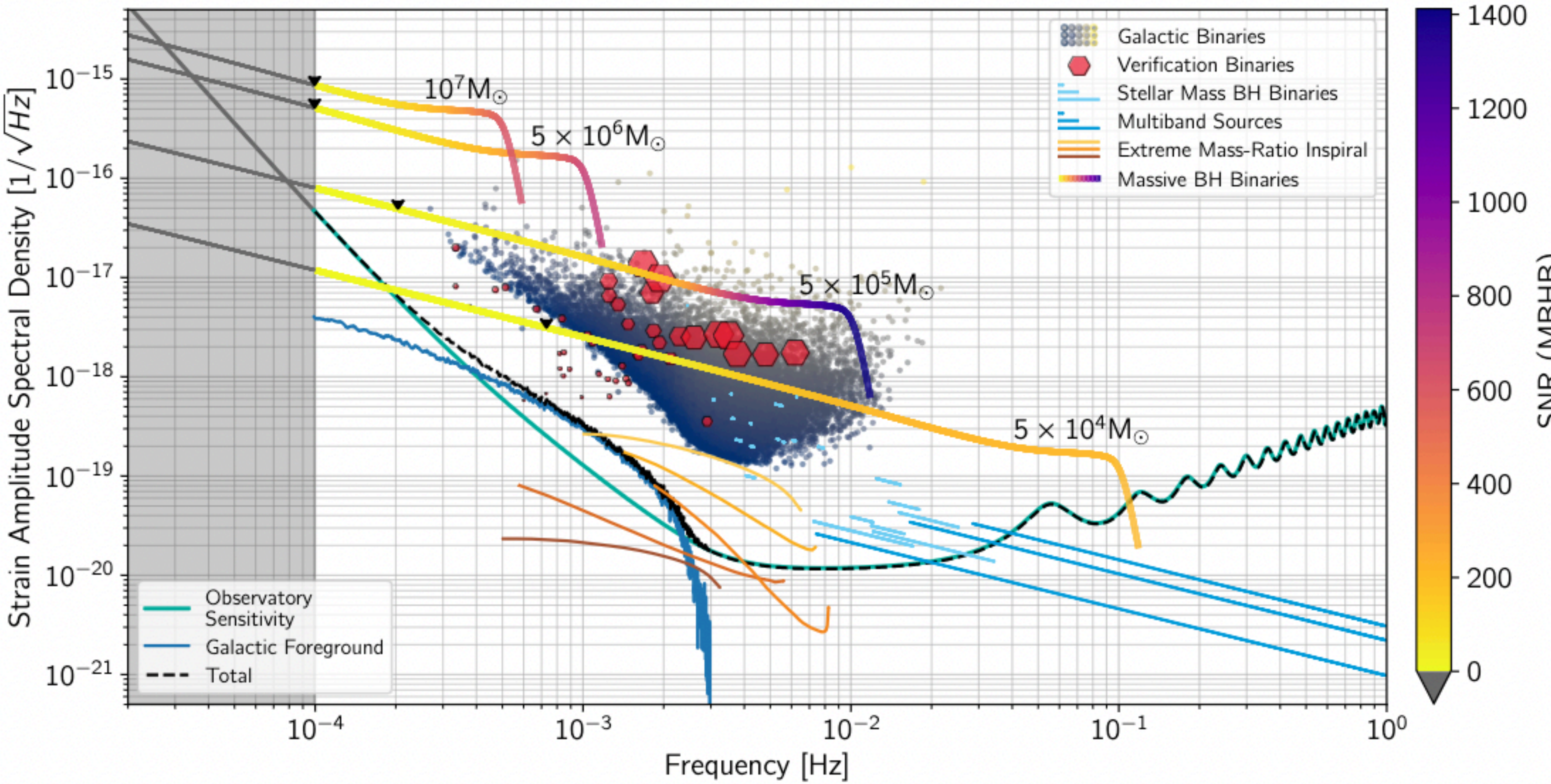


LISA Consortium arXiv:1702.00786



Space-based interferometers: LISA

DETECTION TARGETS:



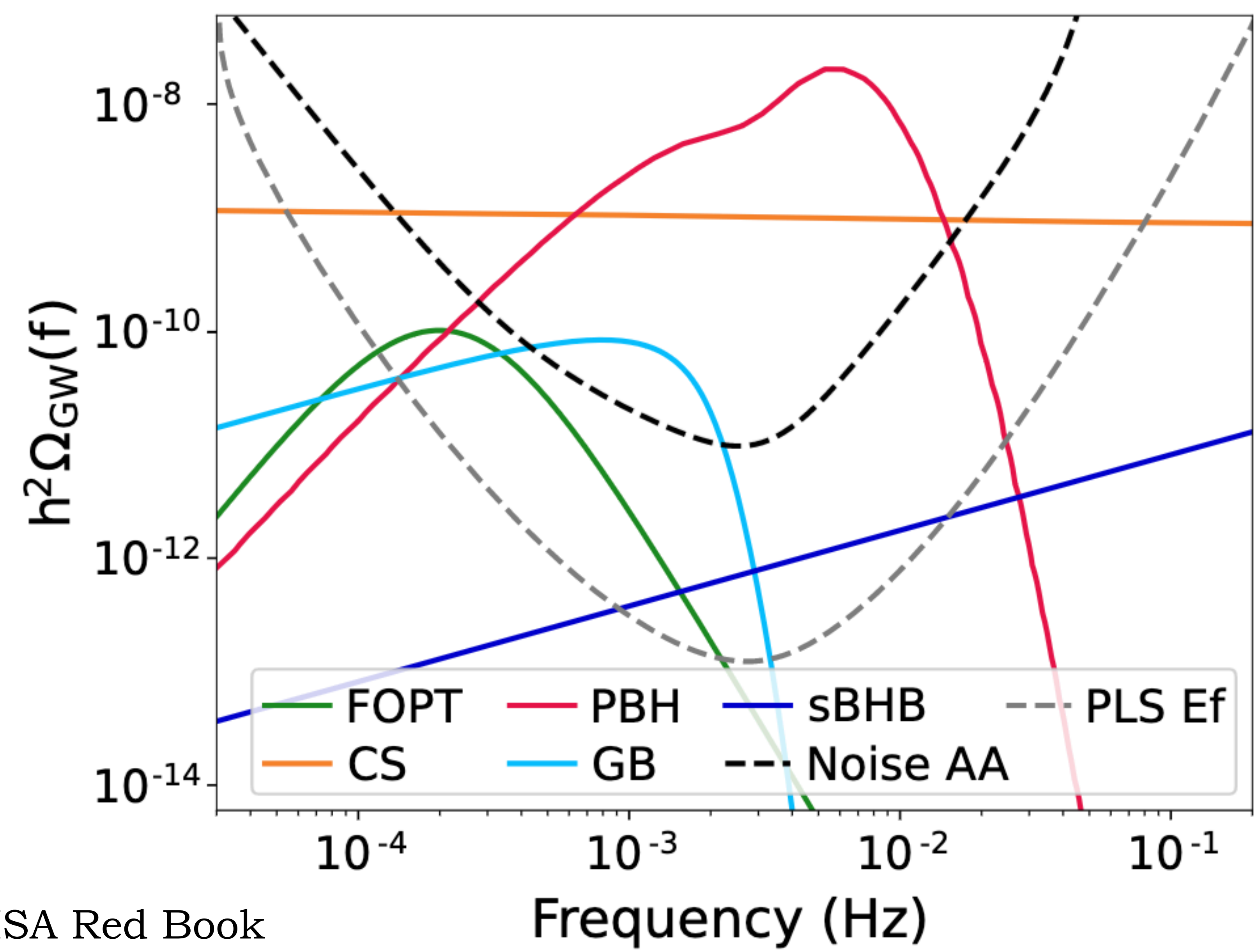
M. Colpi et al, arXiv:2402.07571 - LISA Red Book

Space-based interferometers: LISA

Stochastic GW background

- Confusion noise from the binaries in the Galaxy (mainly WD binaries)
- Confusion noise from extra-galactic binaries (WD binaries and BHBs)
- Candidates from the early universe, in particular at the EW scale

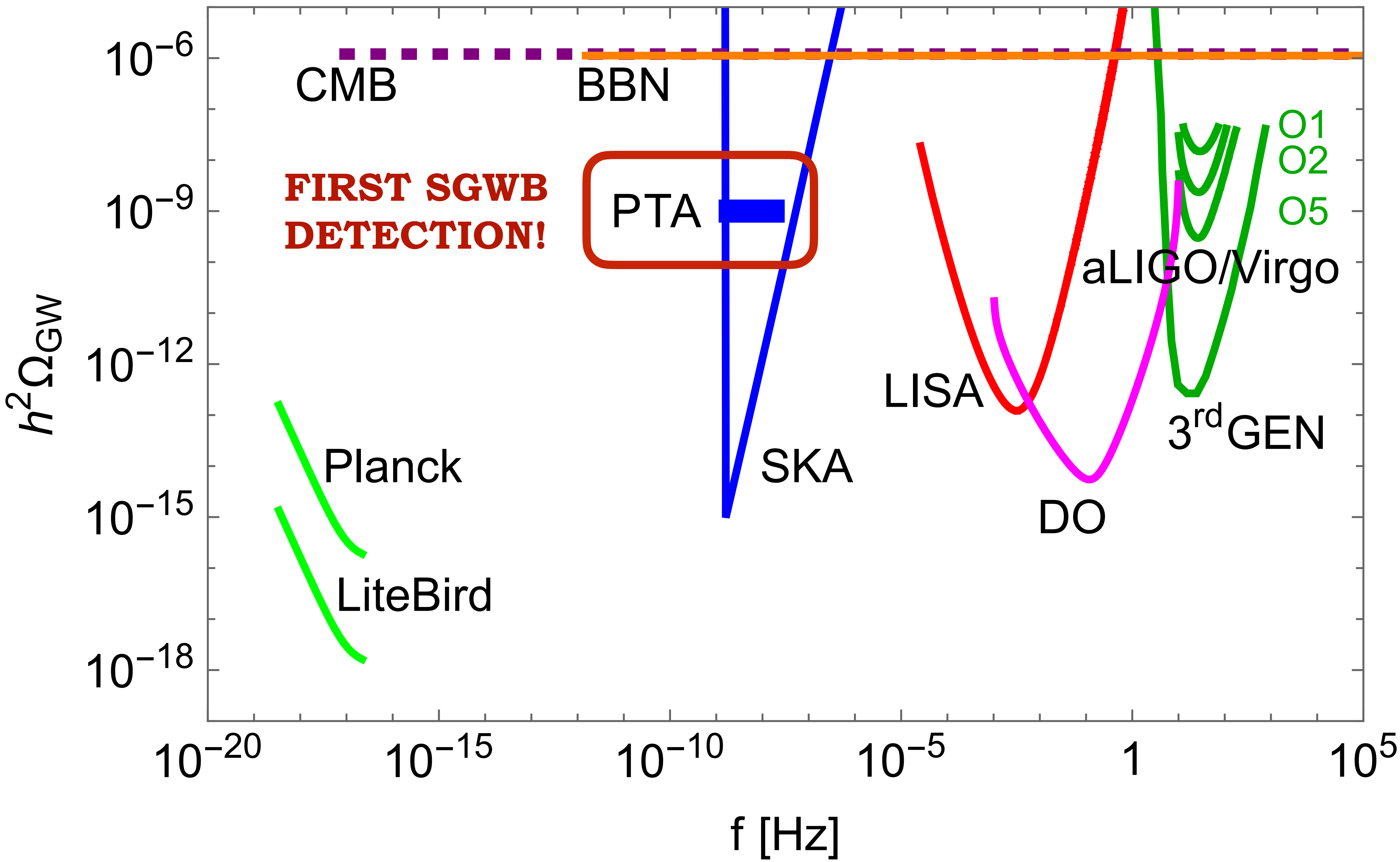
Detecting a SGWB with LISA is challenging: no cross-correlation, need to assume knowledge of the noise (possibility of null channels)



M. Colpi et al, arXiv:2402.07571 - LISA Red Book

What is/will be known about the SGWB

Present and future GW observatories:
Pulsar Timing Arrays



Pulsar timing arrays

CPTA, EPTA, NANOGrav, PPTA -> IPTA

frequency range of detection: 10^{-9} Hz < f < 10^{-7} HZ

- rotating, magnetised neutron stars emitting periodic radio-frequency EM pulses -> can be used as clocks in the sky
- the radio pulses are emitted at very regular time intervals, but their arrival times can be altered by a GW passing between the pulsar and the Earth
- First a timing model of the pulsar is constructed, which is then compared to observations to infer the *timing residuals* where the GW effect is looked for

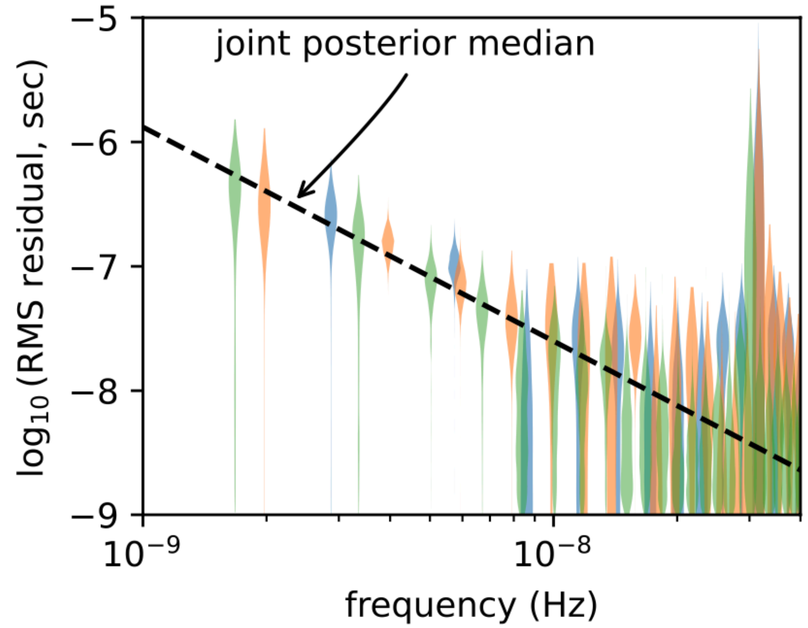
DETECTION TARGETS:

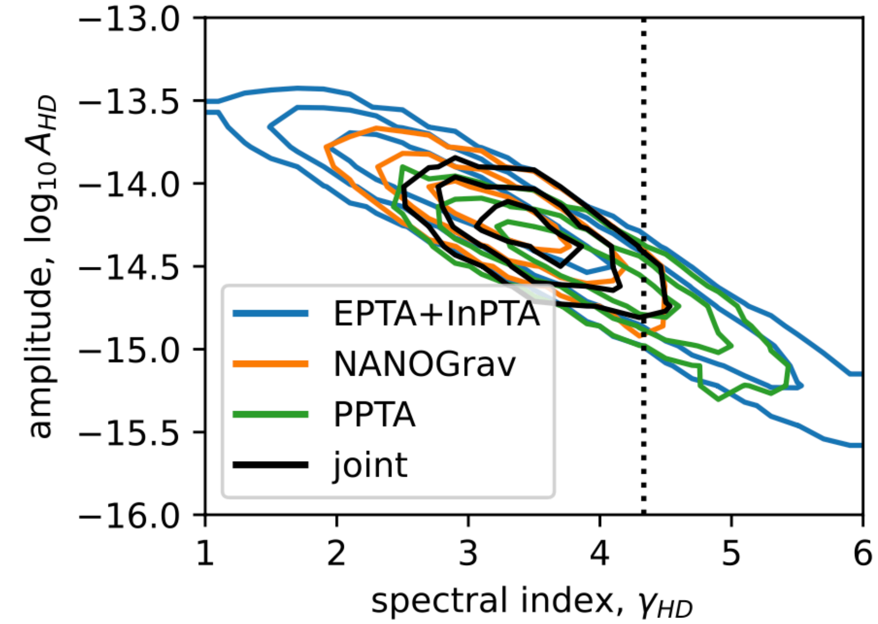
Individual emission and stochastic background from inspiralling Super Massive Black Hole Binaries (SMBHBs) with masses ~10^9 $M_\odot$ at the centre of galaxies

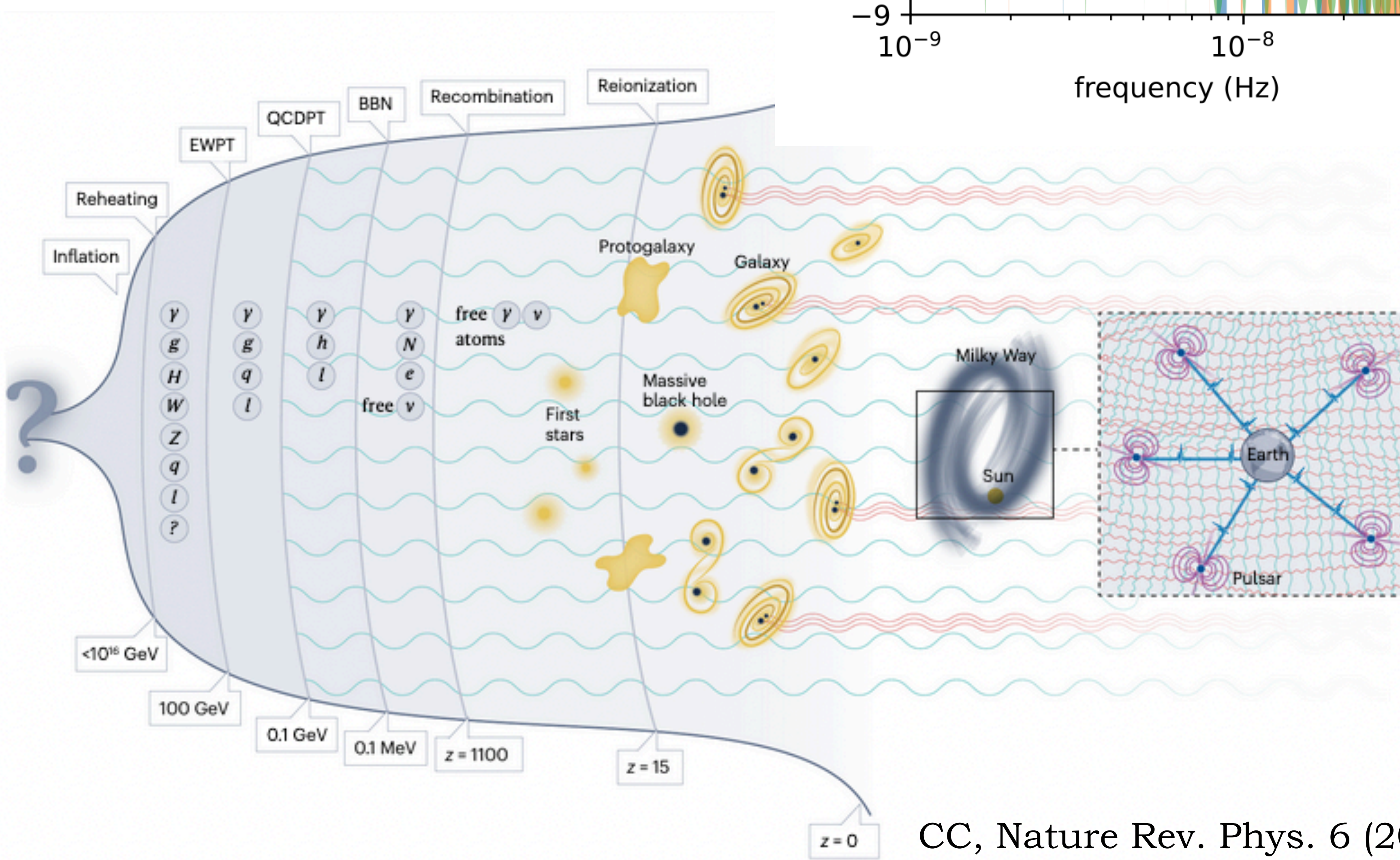
Pulsar timing arrays

- The slopes are shallower than 13/3 (but maybe the model isn't fully adapted…)
- The amplitude is consistent with the one from a SMBHBs SGWB
- All datasets are consistent within 1σ as shown by IPTA

IPTA Collaboration,
arXiv:2309.00693



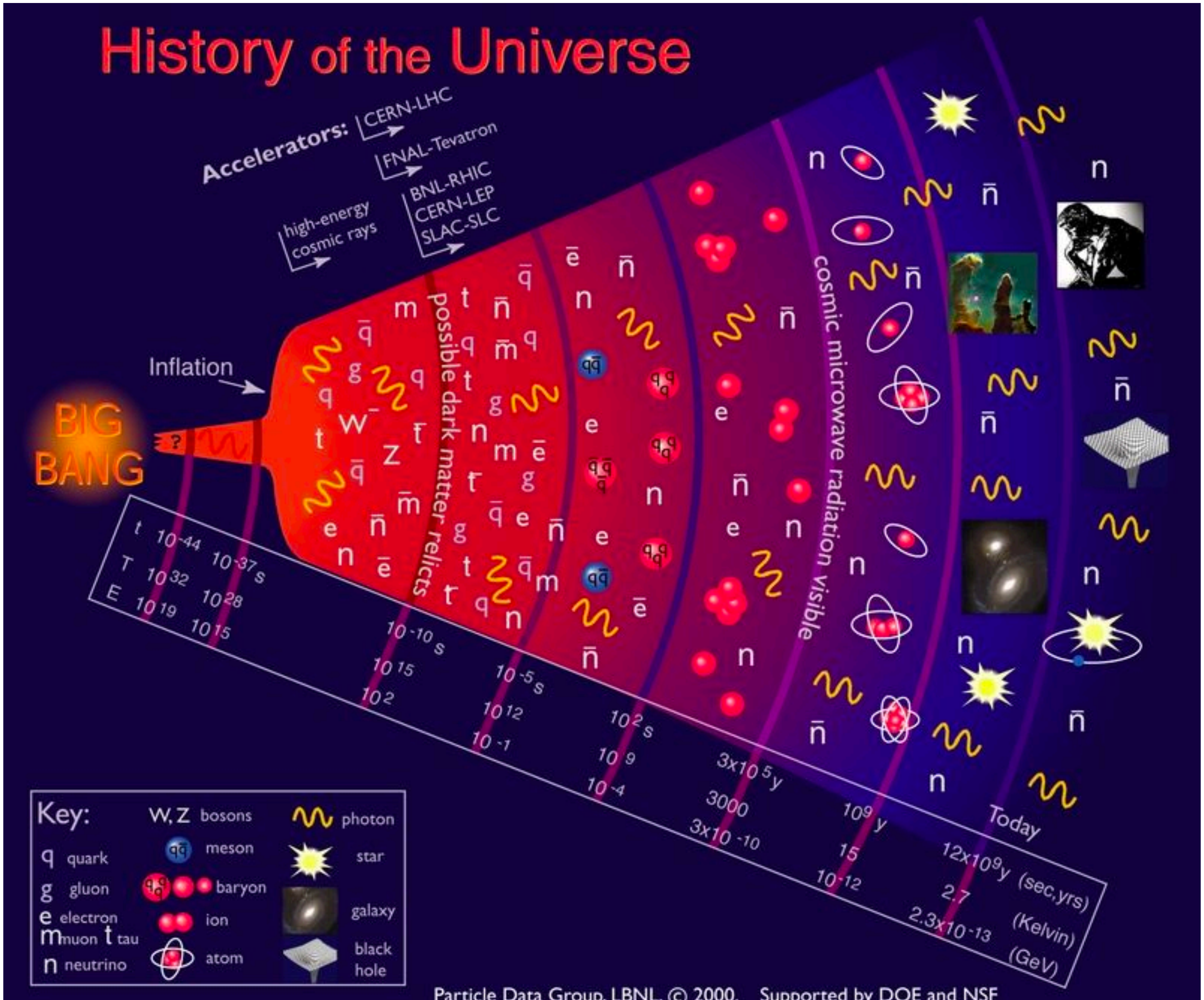




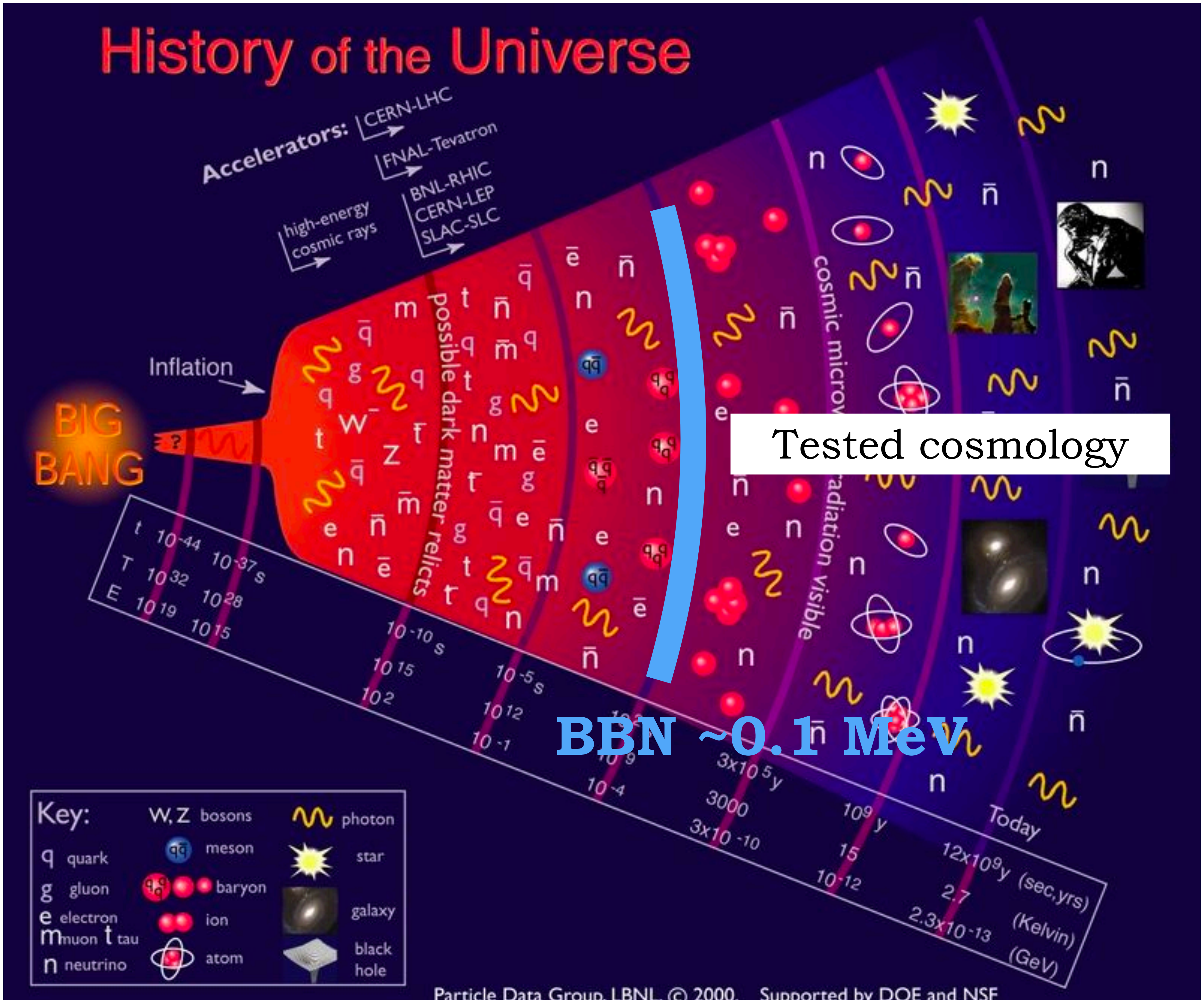
But the signal could also be of primordial origin…

CC, Nature Rev. Phys. 6 (2024), 5291-293

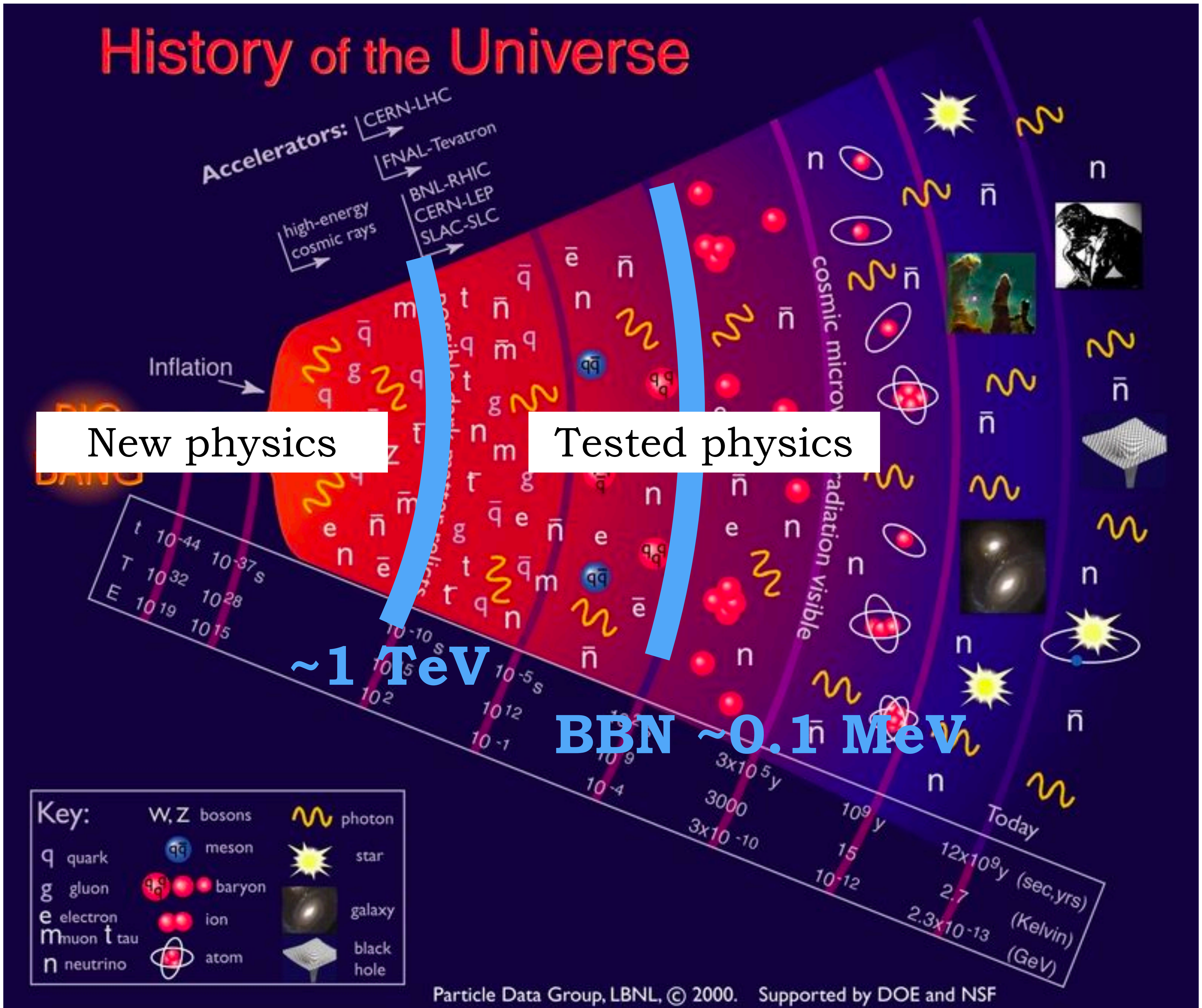
Examples of SGWB sources in the early universe



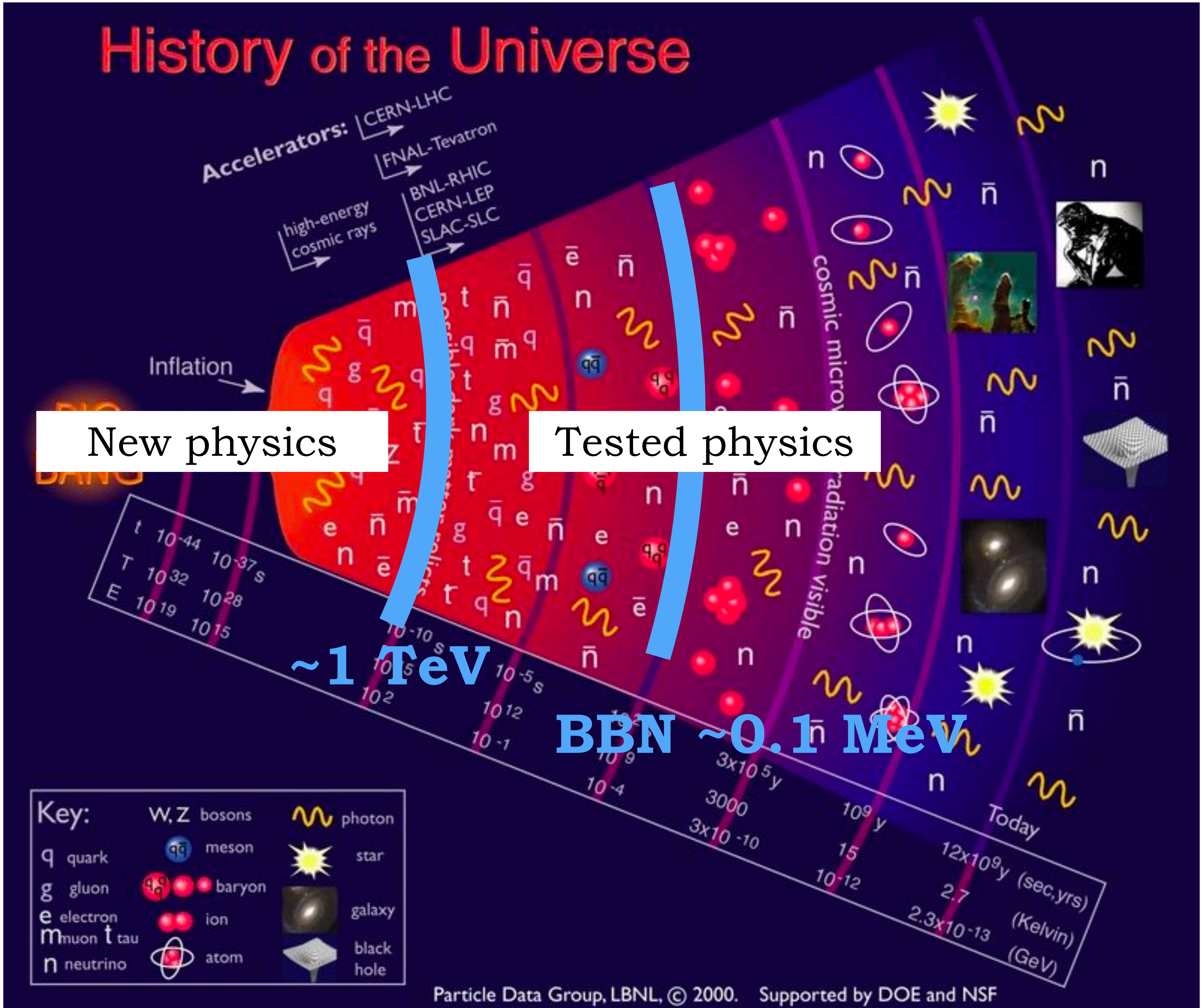
GWs can bring direct information from very early stages of the universe evolution, to which we have no direct access through em radiation —> amazing discovery potential



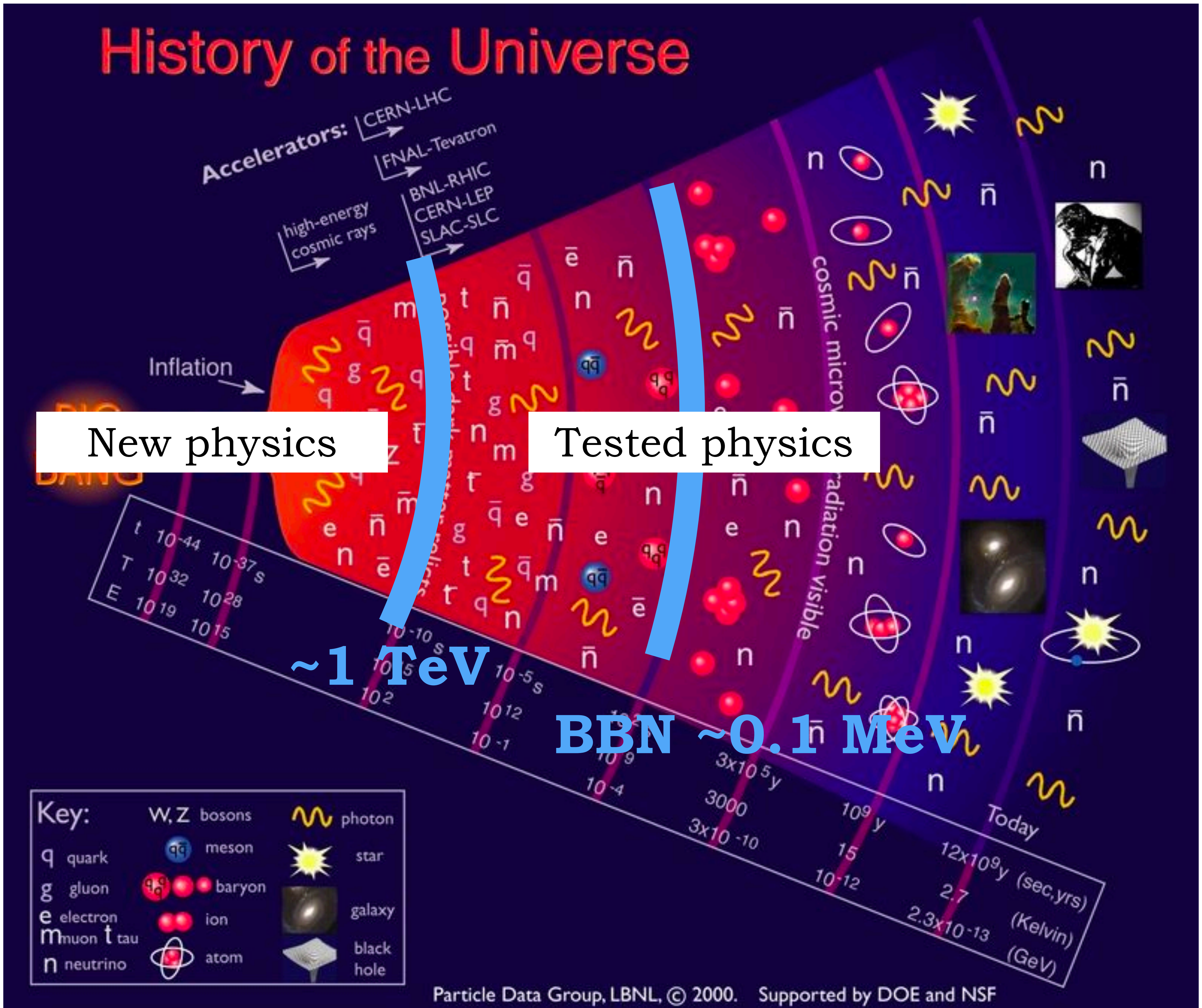
No guaranteed GW signal: predictions rely on untested phenomena, and are often difficult to estimate (non-linear dynamics, strongly coupled theories…)



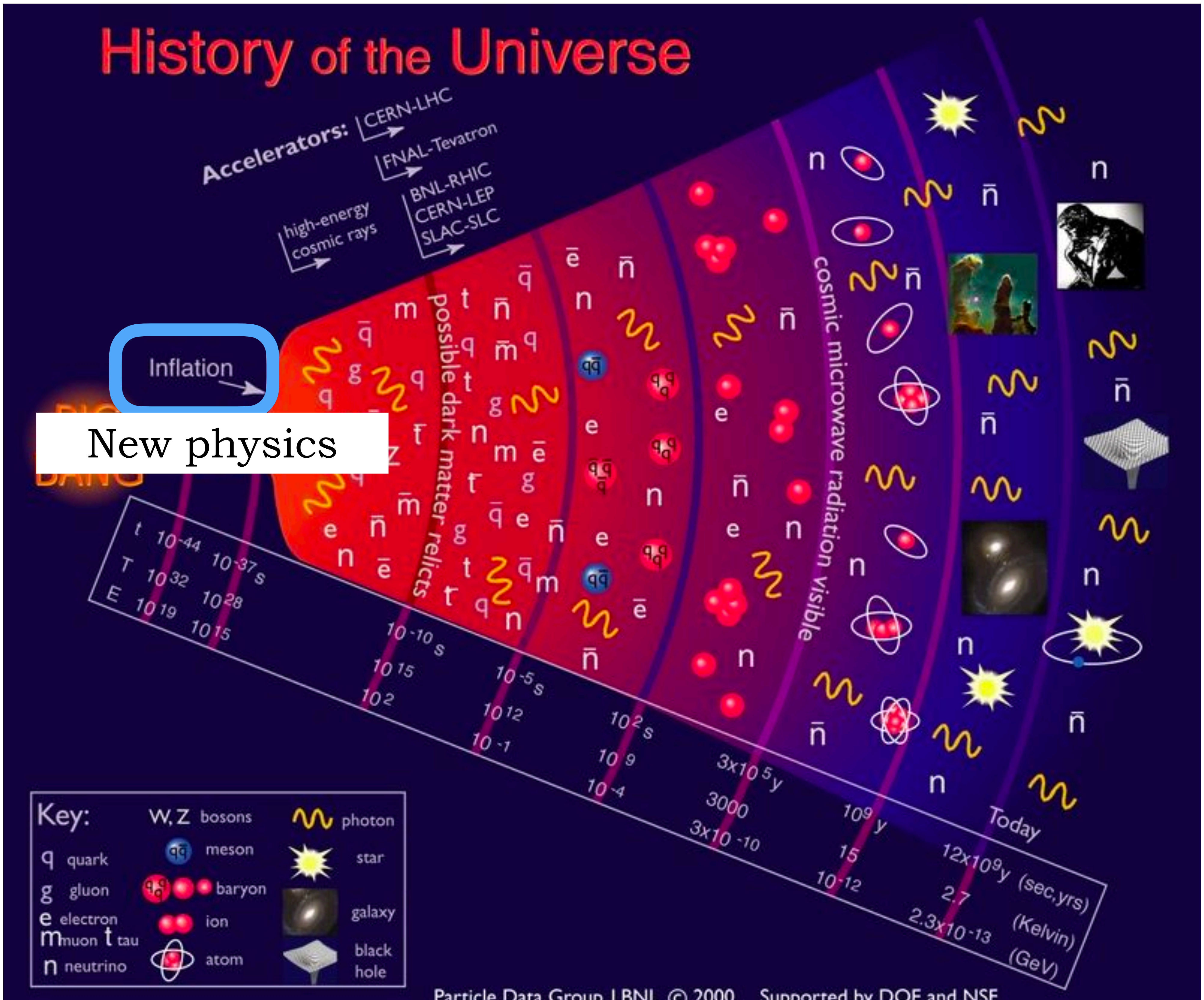
Many GW generation processes are related to PHASE TRANSITIONS



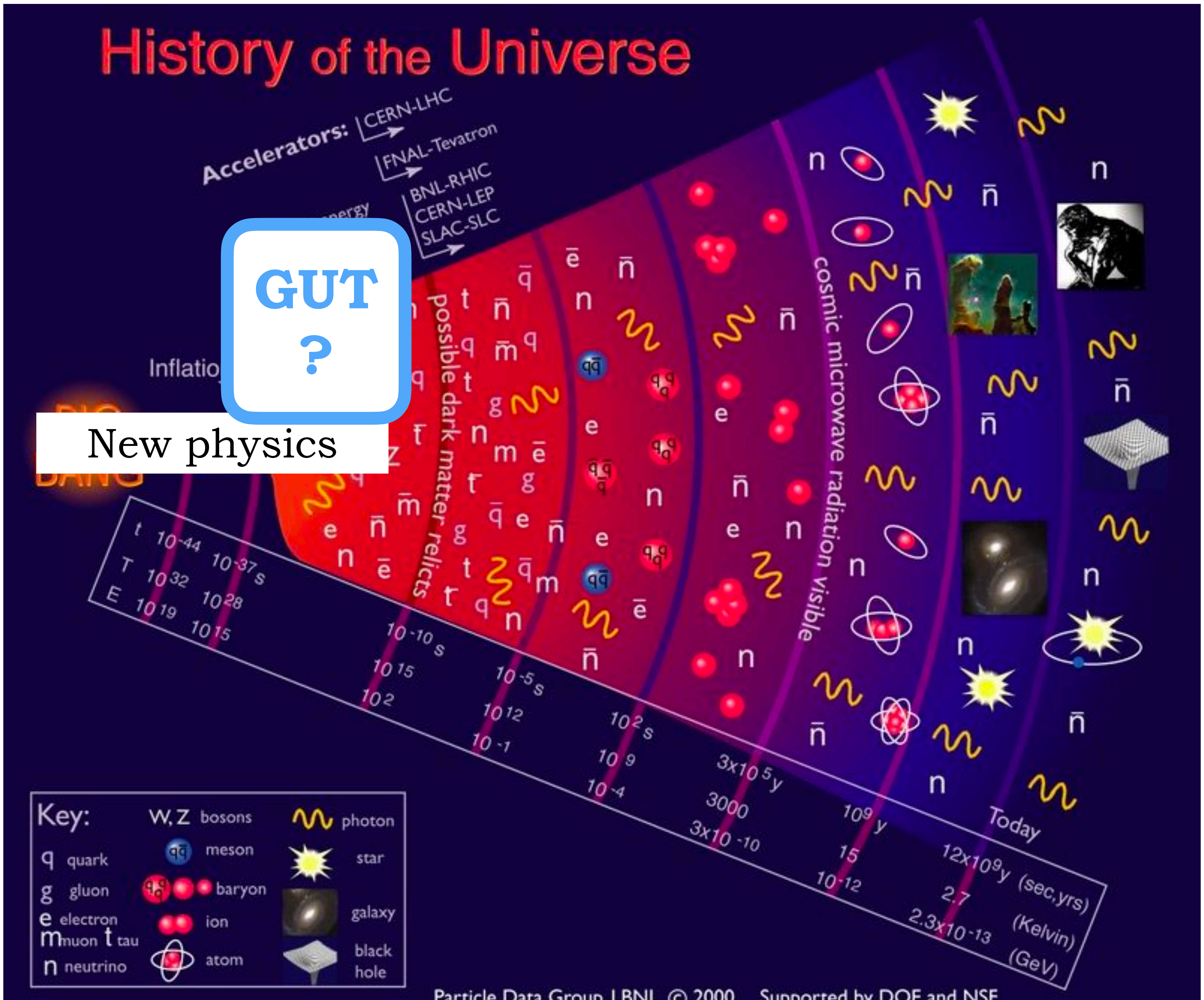
Phase transition: some field in the universe changes from one state to another, which has become more energetically favourable due to a change in external conditions (e.g. a change in temperature)



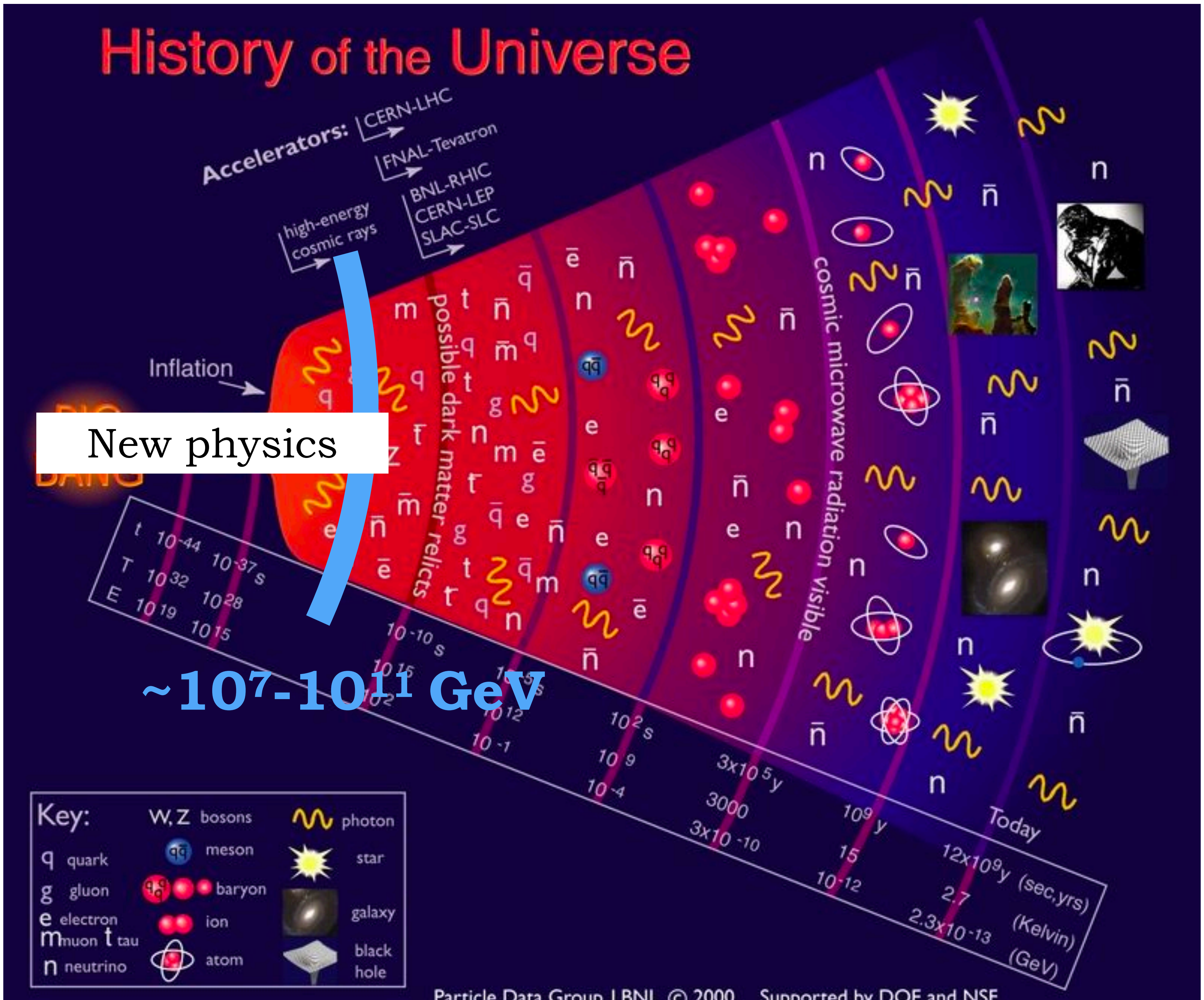
Inflation: phase transition of the Inflaton field



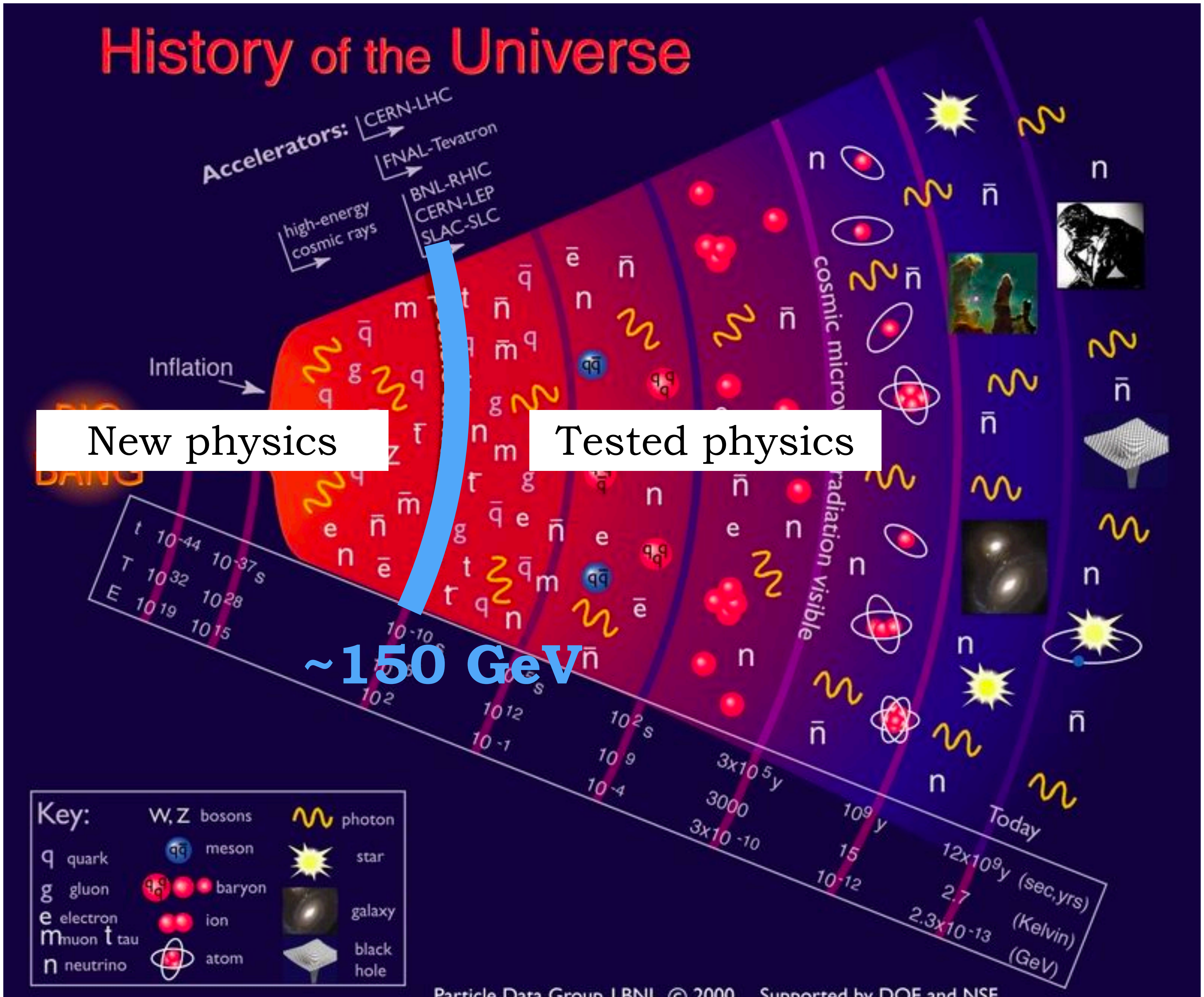
GUT phase transition or similar: related to the breaking of the symmetries of the high-energy theory describing the universe



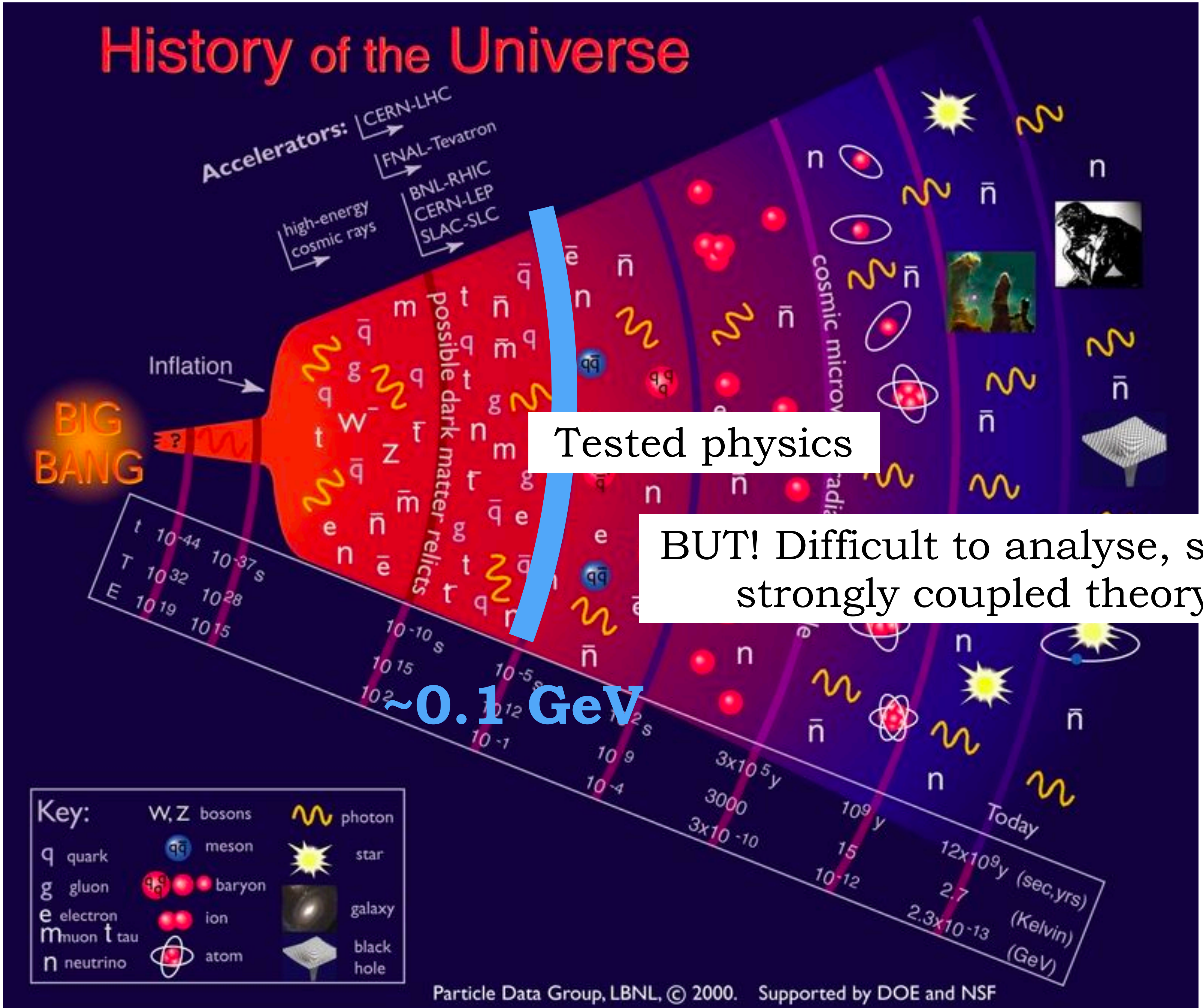
Peccei-Quinn phase transition: invoked to solve the strong CP problem



Electroweak phase transition: phase transition of the Higgs field, driven by the temperature decrease as the universe expands



QCD phase transition: phase transition related to the strong interaction, confinement of quarks into hadrons



SGWB from a stochastic source in the radiation era

$$h_r''(\mathbf{k},\eta) + 2\,\mathcal{H}\,h_r'(\mathbf{k},\eta) + k^2\,h_r(\mathbf{k},\eta) = 16\pi G\,a^2\,\Pi_r(\mathbf{k},\eta)$$

Possible sources of tensor anisotropic stress in the early universe:

- Scalar field gradients $\Pi_{ij} \sim [\partial_i\phi\partial_j\phi]^{TT}$
- Bulk fluid motion $\Pi_{ij} \sim [\gamma^2(\rho+p)v_i v_j]^{TT}$
- Gauge fields $\Pi_{ij} \sim [-E_i E_j - B_i B_j]^{TT}$
- Second order scalar perturbations, Π_{ij} from a combination of $\partial_i\Psi, \partial_i\Phi$
- …

The components of the anisotropic stress must be treated as
random variables
because we cannot access the detailed properties of the generation processes at the moment they operated

SGWB from a stochastic source in the radiation era

Anisotropic stress power spectral density at unequal time

unequal time correlator of the anisotropic stress

$$\langle \Pi_r(\mathbf{k},\tau)\,\Pi_p^*(\mathbf{q},\zeta)\rangle = \frac{(2\pi)^3}{4}\,\delta^{(3)}(\mathbf{k}-\mathbf{q})\,\delta_{rp}\,\Pi(k,\tau,\zeta)$$

SGWB from a stochastic source in the radiation era

unequal time correlator of the anisotropic stress

Anisotropic stress power spectral density at unequal time

$$\langle \Pi_r(\mathbf{k},\tau)\,\Pi_p^*(\mathbf{q},\zeta)\rangle = \frac{(2\pi)^3}{4}\,\delta^{(3)}(\mathbf{k}-\mathbf{q})\,\delta_{rp}\,\Pi(k,\tau,\zeta)$$

We now proceed with two approximate analytical solutions of the GW propagation equation:

- **Fast source** operating for less than one Hubble time -> peaked SGWB power spectrum

- **Continuous source** operating for several Hubble times -> extended SGWB power spectrum

SGWB from a stochastic source in the radiation era

unequal time correlator of the anisotropic stress

Anisotropic stress power spectral density at unequal time

$$\langle \Pi_r(\mathbf{k},\tau)\,\Pi_p^*(\mathbf{q},\zeta)\rangle = \frac{(2\pi)^3}{4}\,\delta^{(3)}(\mathbf{k}-\mathbf{q})\,\delta_{rp}\,\Pi(k,\tau,\zeta)$$

We now proceed with two approximate analytical solutions of the GW propagation equation:

- **Fast source** operating for less than one Hubble time -> peaked SGWB power spectrum

operating in a time interval $\eta_{\rm fin}$ - $\eta_{\rm in}$ in the radiation dominated era

Typical example: first order phase transition

$$H_r^{\rm rad}(\mathbf{k},\eta>\eta_{\rm fin}) = A_r^{\rm rad}(\mathbf{k})\cos(k\eta) + B_r^{\rm rad}(\mathbf{k})\sin(k\eta)$$

$$A_r^{\rm rad}(\mathbf{k}) = \frac{16\pi G}{k}\int_{\eta_{\rm in}}^{\eta_{\rm fin}} d\tau\, a(\tau)^3\,\sin(-k\tau)\,\Pi_r(\mathbf{k},\tau),$$

$$B_r^{\rm rad}(\mathbf{k}) = \frac{16\pi G}{k}\int_{x_{\rm in}}^{x_{\rm fin}} d\tau\, a(\tau)^3\,\cos(k\tau)\,\Pi_r(\mathbf{k},\tau)$$

SGWB from a **FAST** stochastic source in the radiation era

GW amplitude power spectrum today for modes $k\eta_0 \gg 1$

$$\langle h_r(\mathbf{k},\eta_0)\, h_p^*(\mathbf{q},\eta_0)\rangle = \frac{1}{a_0^2}\left[\langle A_r(\mathbf{k})\, A_p^*(\mathbf{q})\rangle + \langle B_r(\mathbf{k})\, B_p^*(\mathbf{q})\rangle\right]$$
$$= 8\pi^5\, \delta^{(3)}(\mathbf{k}-\mathbf{q})\, \delta_{rp}\, \frac{h_c^2(k,\eta_0)}{k^3}$$

GW energy density power spectrum today for modes $k\eta_0 \gg 1$

$$\frac{d\rho_{\rm GW}}{d\log k} = \frac{k^2\, h_c^2(k,\eta_0)}{16\pi G\, a_0^2}$$

(freely propagating sub-Hubble modes)

$$\frac{d\rho_{\rm GW}}{d\log k}(k,\eta_0) = \frac{4}{\pi}\,\frac{G}{a_0^4}k^3 \int_{\eta_{\rm in}}^{\eta_{\rm fin}} d\tau\, a^3(\tau) \int_{\eta_{\rm in}}^{\eta_{\rm fin}} d\zeta\, a^3(\zeta)\, \cos[k(\tau-\zeta)]\, \Pi(k,\tau,\zeta)$$

SGWB from a **FAST** stochastic source in the radiation era

GW amplitude power spectrum today for modes $k\eta_0 \gg 1$

$$\langle h_r(\mathbf{k},\eta_0)\, h_p^*(\mathbf{q},\eta_0)\rangle = \frac{1}{a_0^2}[\langle A_r(\mathbf{k})\, A_p^*(\mathbf{q})\rangle + \langle B_r(\mathbf{k})\, B_p^*(\mathbf{q})\rangle]$$
$$= 8\pi^5\, \delta^{(3)}(\mathbf{k}-\mathbf{q})\, \delta_{rp}\, \frac{h_c^2(k,\eta_0)}{k^3}$$

GW energy density power spectrum today for modes $k\eta_0 \gg 1$

$$\frac{d\rho_{\rm GW}}{d\log k} = \frac{k^2\, h_c^2(k,\eta_0)}{16\pi G\, a_0^2}$$

(freely propagating sub-Hubble modes)

$$\frac{d\rho_{\rm GW}}{d\log k}(k,\eta_0) = \frac{4}{\pi}\,\frac{G}{a_0^4}k^3 \int_{\eta_{\rm in}}^{\eta_{\rm fin}} d\tau\, \underbrace{\cancel{a^3(\tau)}}_{a_*^3} \int_{\eta_{\rm in}}^{\eta_{\rm fin}} d\zeta\, \underbrace{\cancel{a^3(\zeta)}}_{a_*^3}\, \underbrace{\cancel{\cos[k(\tau-\zeta)]}}_{\simeq 1}\, \underbrace{\cancel{\Pi(k,\tau,\zeta)}}_{\Pi(k)}$$

SUPPOSE:

$\Delta\eta = \eta_{\rm fin} - \eta_{\rm in} \ll \mathcal{H}_*^{-1}$ $\quad$ $k\eta_{\rm in} \ll 1$ $\quad$ $\Pi(k,\tau,\eta)$ constant over $\Delta\eta$

SGWB from a **FAST** stochastic source in the radiation era

GW energy density parameter today for modes $1/\eta_0 \ll k \ll 1/\eta_{in}$

$$h^2\Omega_{\rm GW}(k,\eta_0) = \frac{3}{2\pi^2} h^2\Omega^0_{\rm rad} \left(\frac{g_0}{g_*}\right)^{\frac{1}{3}} (\Delta\eta\mathcal{H}_*)^2 \left(\frac{\rho_\Pi}{\rho_{\rm rad}}\right)^2 \underbrace{(k\ell_*)^3 \tilde{P}_{\rm GW}(k)}$$

$$\Pi(k) = \ell_*^3\,\rho_\Pi^2\,\tilde{P}_{\rm GW}(k)$$

From the time integrals

SGWB from a **FAST** stochastic source in the radiation era

GW energy density parameter today for modes $1/\eta_0 \ll k \ll 1/\eta_{in}$

$$h^2\Omega_{\rm GW}(k,\eta_0) = \frac{3}{2\pi^2} h^2\Omega^0_{\rm rad}\left(\frac{g_0}{g_*}\right)^{\frac{1}{3}} (\Delta\eta\mathcal{H}_*)^2 \left(\frac{\rho_\Pi}{\rho_{\rm rad}}\right)^2 (k\ell_*)^3 \tilde{P}_{\rm GW}(k)$$

$$\mathcal{O}(10^{-9}) \qquad \mathcal{O}(10^{-6}) \qquad \mathcal{O}(10^{-3})$$

Value detected at PTA

Factor depending slightly on the generation epoch through the number of relativistic d.o.f.

Only slow, very anisotropic processes have the chance to generate detectable SGWB signals for sub-Hubble sources

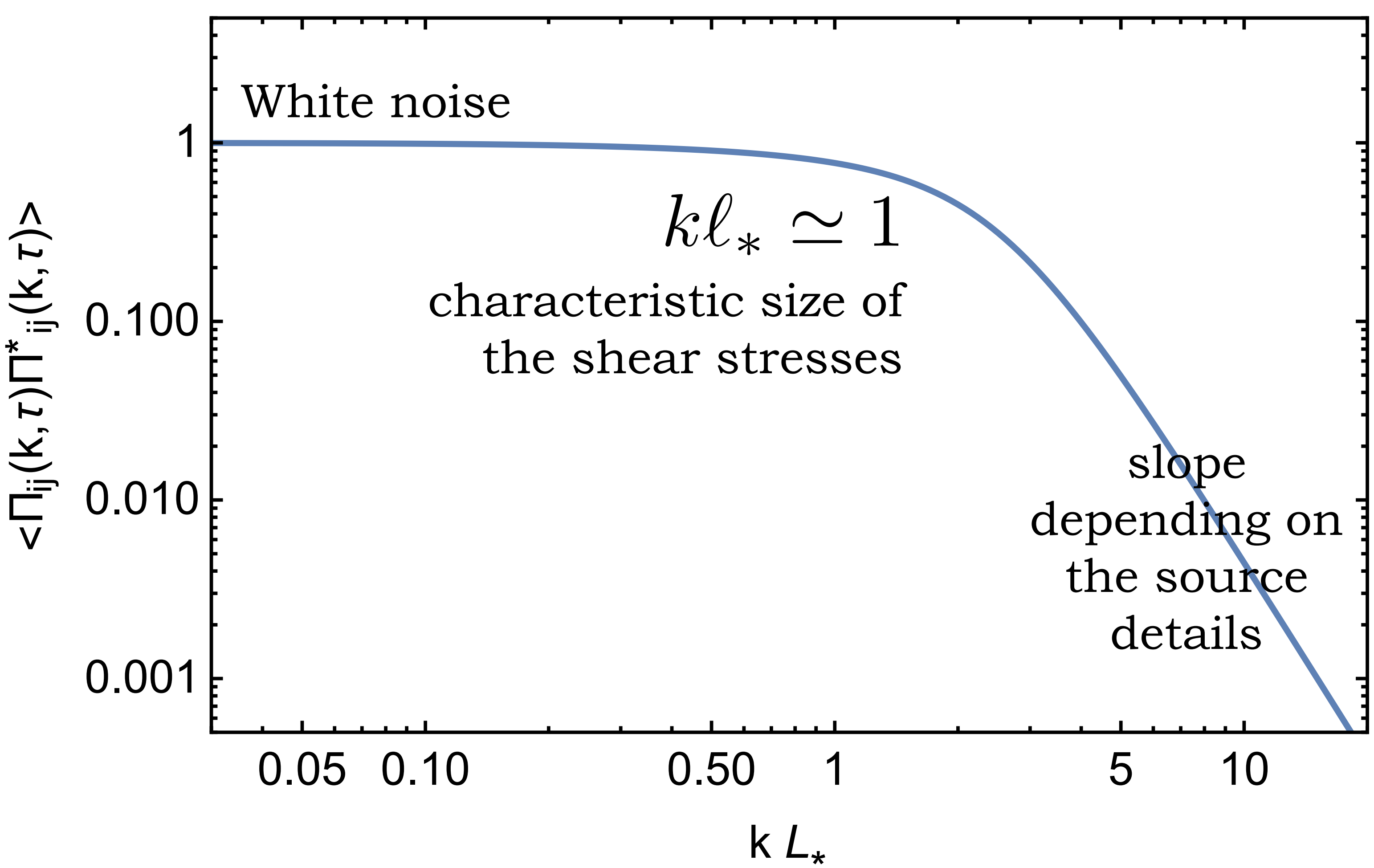
Value for detection at LISA

$$\mathcal{O}(10^{-11}) \qquad \mathcal{O}(10^{-6}) \qquad \mathcal{O}(10^{-5})$$

SGWB from a **FAST** stochastic source in the radiation era

GW energy density parameter today for modes $1/\eta_0 \ll k \ll 1/\eta_{in}$

$$h^2\Omega_{\rm GW}(k,\eta_0) = \frac{3}{2\pi^2} h^2\Omega^0_{\rm rad}\left(\frac{g_0}{g_*}\right)^{\frac{1}{3}} (\Delta\eta\mathcal{H}_*)^2 \left(\frac{\rho_\Pi}{\rho_{\rm rad}}\right)^2 (k\ell_*)^3 \tilde{P}_{\rm GW}(k)$$



Fast source:
independent on k for
large enough scales
(uncorrelated)

$$\boxed{\ell_* \leq H_*^{-1}}$$

SGWB from a **FAST** stochastic source in the radiation era

GW energy density parameter today for modes $1/\eta_0 \ll k \ll 1/\eta_{in}$

$$h^2\Omega_{\rm GW}(k,\eta_0) = \frac{3}{2\pi^2} h^2\Omega^0_{\rm rad}\left(\frac{g_0}{g_*}\right)^{\frac{1}{3}}(\Delta\eta\mathcal{H}_*)^2\left(\frac{\rho_\Pi}{\rho_{\rm rad}}\right)^2(k\ell_*)^3\tilde{P}_{\rm GW}(k)$$

$$1/\eta_0 \ll \underbrace{k \ll \mathcal{H}_*}_{\text{Range of validity of the solution}} \ll \underbrace{1/(a_*\ell_*)}_{\text{Causality of the sourcing process}}$$

$$\boxed{\Omega_{\rm GW}(k) \propto (k\ell_*)^3}$$

SGWB from a **FAST** stochastic source in the radiation era

- Characteristic time of the source evolution $$\delta t_c = \frac{\ell_*}{v_{\rm rms}}$$
- Characteristic time of the GW production from the Green's function: $$\delta t_{\rm gw} \sim \frac{1}{k}$$
- GW production goes faster than source evolution for all relevant wave-numbers including the spectrum peak $$k > \frac{v_{\rm rms}}{\ell_*}$$
- One assumes that the source is constant in time for a finite time interval (which can be larger than the Hubble time) $$\delta t_{\rm fin} \sim \mathcal{N} \delta t_c$$
- One can then easily integrate to find the GW spectrum

$$\boxed{h^2 \Omega_{\rm GW}(k, \eta_0) \propto h^2 \Omega^0_{\rm rad} \left(\frac{g_0}{g_*}\right)^{\frac{1}{3}} \left(\frac{\rho_\Pi}{\rho_{\rm rad}}\right)^2 (k\ell_*)^3 \tilde{P}_{\rm GW}(k) \begin{cases} \ln^2[1 + \mathcal{H}_* \delta t_{\rm fin}] & \text{if } k\,\delta t_{\rm fin} < 1 \\ \ln^2[1 + (k/\mathcal{H}_*)^{-1}] & \text{if } k\,\delta t_{\rm fin} \geq 1 \end{cases}}$$

A. Roper Pol et al, arXiv:2201.05630

SGWB from a **FAST** stochastic source in the radiation era

$$k_{\rm peak} \simeq 4\pi/\ell_*$$

$$\Omega_{\rm gw,\,peak} \propto \left(\frac{\rho_\Pi}{\rho_{\rm rad}}\right)^2 (\mathcal{H}_*\ell_*)^2$$

Transition from k^3 to k^1 at $k \simeq 1/\delta t_{\rm fin}$

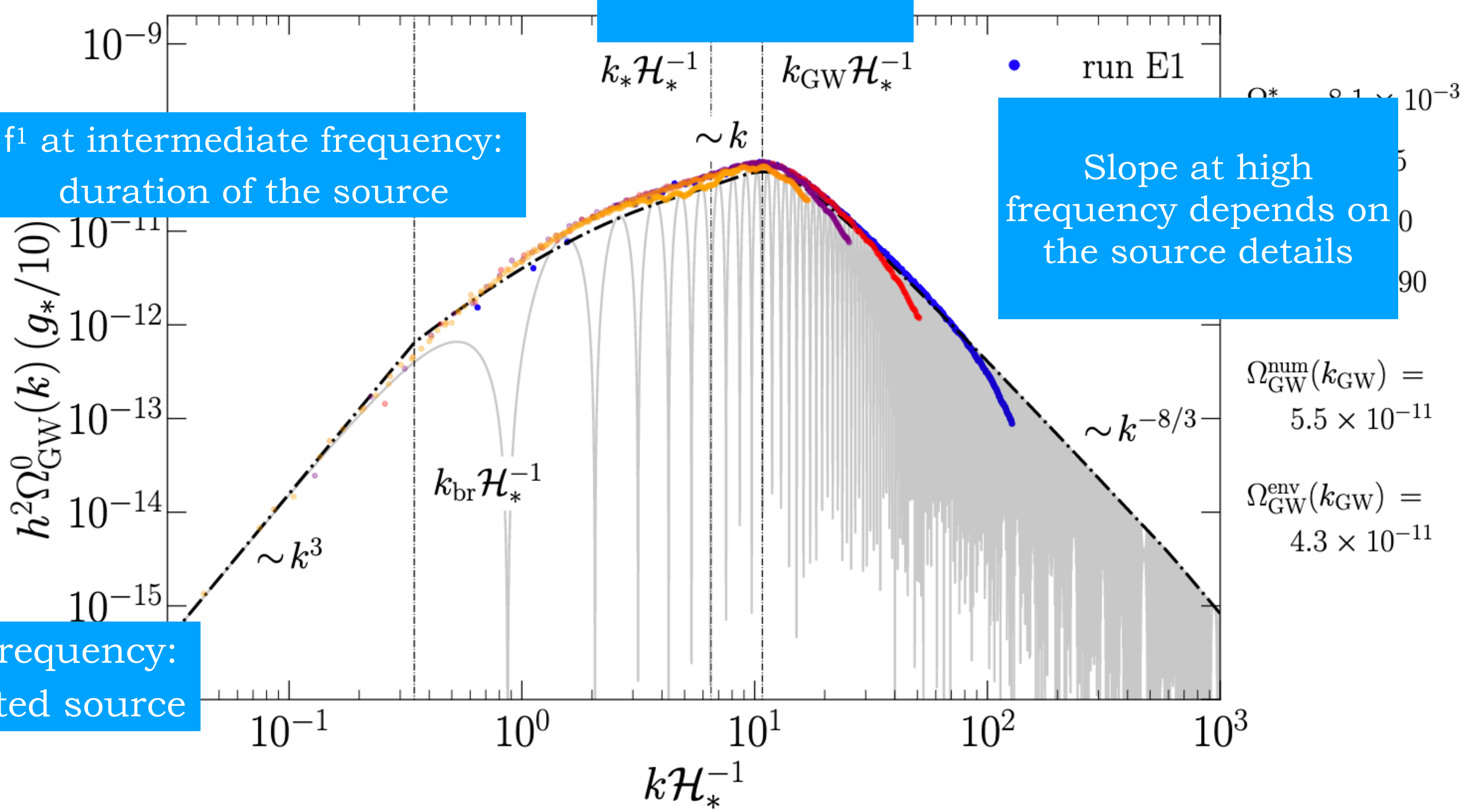
Can be smoother if $\delta t_{\rm fin} > 1/\mathcal{H}_*$

Peak at f ~ $1/\ell_*$

f[1] at intermediate frequency: duration of the source

Slope at high frequency depends on the source details

f[3] at low frequency: uncorrelated source



A. Roper Pol et al, arXiv:2201.05630

SGWB from a **CONTINUOUS** stochastic source in the radiation era

Typical example: topological defects

Suppose the source is operating continuously in the radiation dominated era

- No matching at the end time of the source
- No *free* sub-Hubble modes

$$H_r(\mathbf{k},\eta) = a\, h_r(\mathbf{k},\eta)$$

$$H_r^{\rm rad}(\mathbf{k},\eta) = \frac{16\pi G}{k}\int_{\eta_{\rm in}}^{\eta} d\tau\, a(\tau)^3\,\sin[k(\eta-\tau)]\,\Pi_r(\mathbf{k},\tau)$$

$$h_r{}'(\mathbf{k},\eta) = \frac{16\pi G}{a(\eta)}\int_{\eta_{\rm in}}^{\eta} d\tau\, a(\tau)^3\,\cos[k(\eta-\tau)]\,\Pi_r(\mathbf{k},\tau)$$

$$\langle h_r'(\mathbf{k},\eta)\, h_p'^{\,*}(\mathbf{q},\eta)\rangle = \frac{8\pi^5}{k^3}\,\delta^{(3)}(\mathbf{k}-\mathbf{q})\,\delta_{rp}\, h_c'^{\,2}(k,\eta) \qquad \frac{d\rho_{\rm GW}}{d\log k} = \frac{h_c'^{\,2}(k,\eta)}{16\pi G\, a^2(\eta)}$$

$$\frac{d\rho_{\rm GW}}{d\log k}(k,\eta) = \frac{4}{\pi}\frac{G}{a^4}\,k^3\int_{\eta_{\rm in}}^{\eta} d\tau\, a(\tau)^3\int_{\eta_{\rm in}}^{\eta} d\zeta\, a(\zeta)^3\,\mathcal{G}(k,\eta,\tau,\zeta)\,\Pi(k,\tau,\zeta)$$

SGWB from a **CONTINUOUS** stochastic source in the radiation era

Typical example: topological defects

Suppose the source is operating continuously in the radiation dominated era

- Scaling (property of the topological defects network)
- Decays very fast in off-diagonal $k\tau \neq k\zeta$
- Decays as a power law on the diagonal $k\tau = k\zeta$

D. Figueroa et al, arXiv:1212.5458

$$\Pi(k,\tau,\zeta) = \frac{v^4}{\sqrt{\tau\zeta}}\,\frac{\mathcal{U}(k\tau,k\zeta)}{a^2(\tau)a^2(\zeta)}$$

$$\frac{d\rho_{\rm GW}}{d\log k}(k,\eta) = \frac{4}{\pi}\frac{G}{a^4}\,k^3\int_{\eta_{\rm in}}^{\eta} d\tau\, a(\tau)^3 \int_{\eta_{\rm in}}^{\eta} d\zeta\, a(\zeta)^3\, \mathcal{G}(k,\eta,\tau,\zeta)\,\Pi(k,\tau,\zeta)$$

SGWB from a **CONTINUOUS** stochastic source in the radiation era

Typical example: topological defects

Suppose the source is operating continuously in the radiation dominated era

$$h^2\Omega_{\rm GW}(f) = \frac{32}{3}h^2\Omega_{\rm rad}\left(\frac{v}{M_{\rm Pl}}\right)^4 F_{\rm RD}^{[\mathcal{U}]}(\infty)$$

TODAY FLAT SPECTRUM
AT SUB-HORIZON MODES
IN THE RADIATION ERA

D. Figueroa et al, arXiv:1212.5458

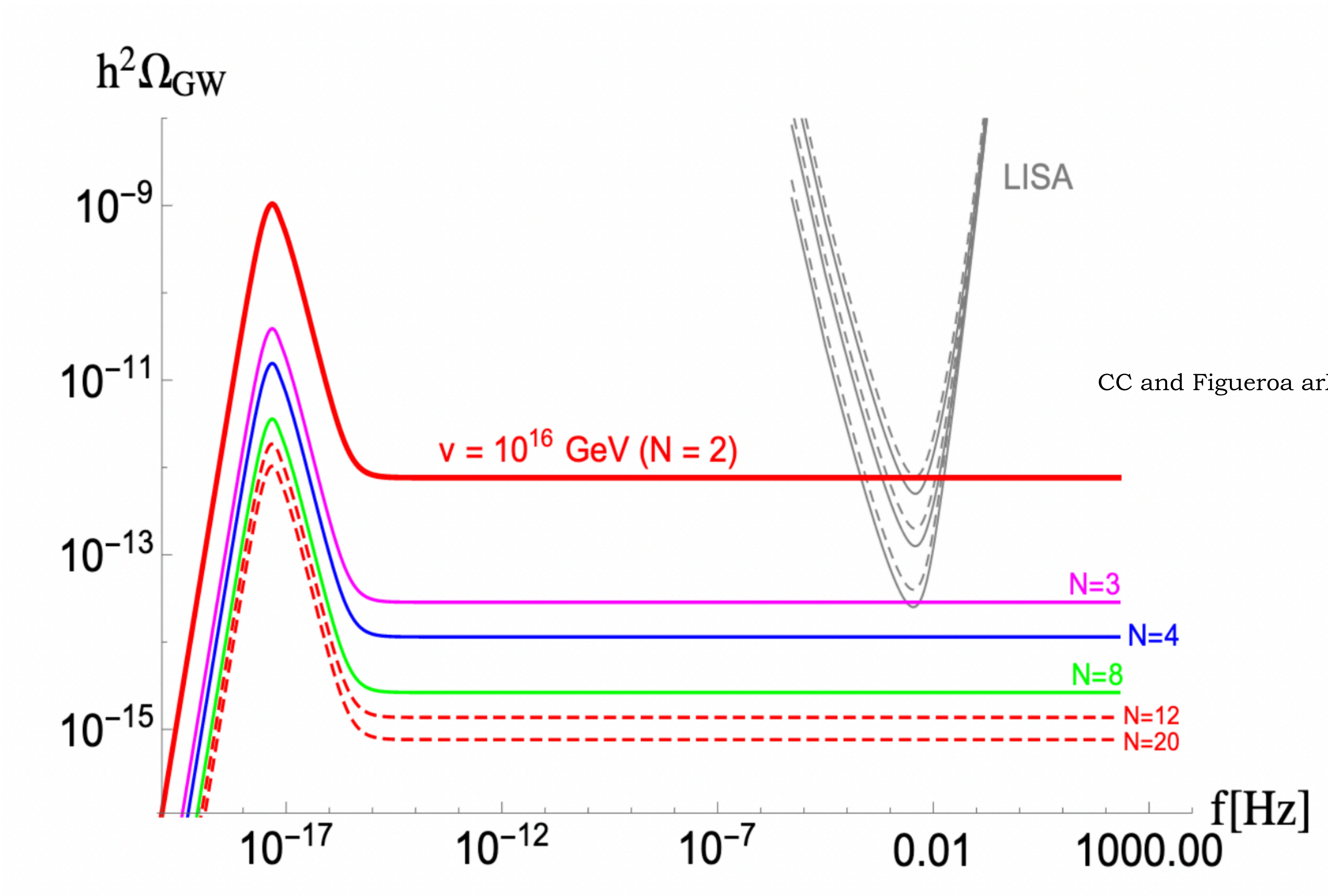
Progressively
independent on the
upper bound

$$\frac{d\rho_{\rm GW}}{d\log k}(k,\eta) = \frac{32}{3}\Omega_{\rm rad}\frac{\rho_c}{a^4}\left(\frac{v}{M_{\rm Pl}}\right)^4\int_{x_{\rm in}}^{x}dx_1\int_{x_{\rm in}}^{x}dx_2\,\sqrt{x_1x_2}\,\mathcal{G}(x,x_1,x_2)\,\mathcal{U}(x_1,x_2)$$

D. Figueroa et al, arXiv:1212.5458

SGWB from a **CONTINUOUS** stochastic source in the radiation era

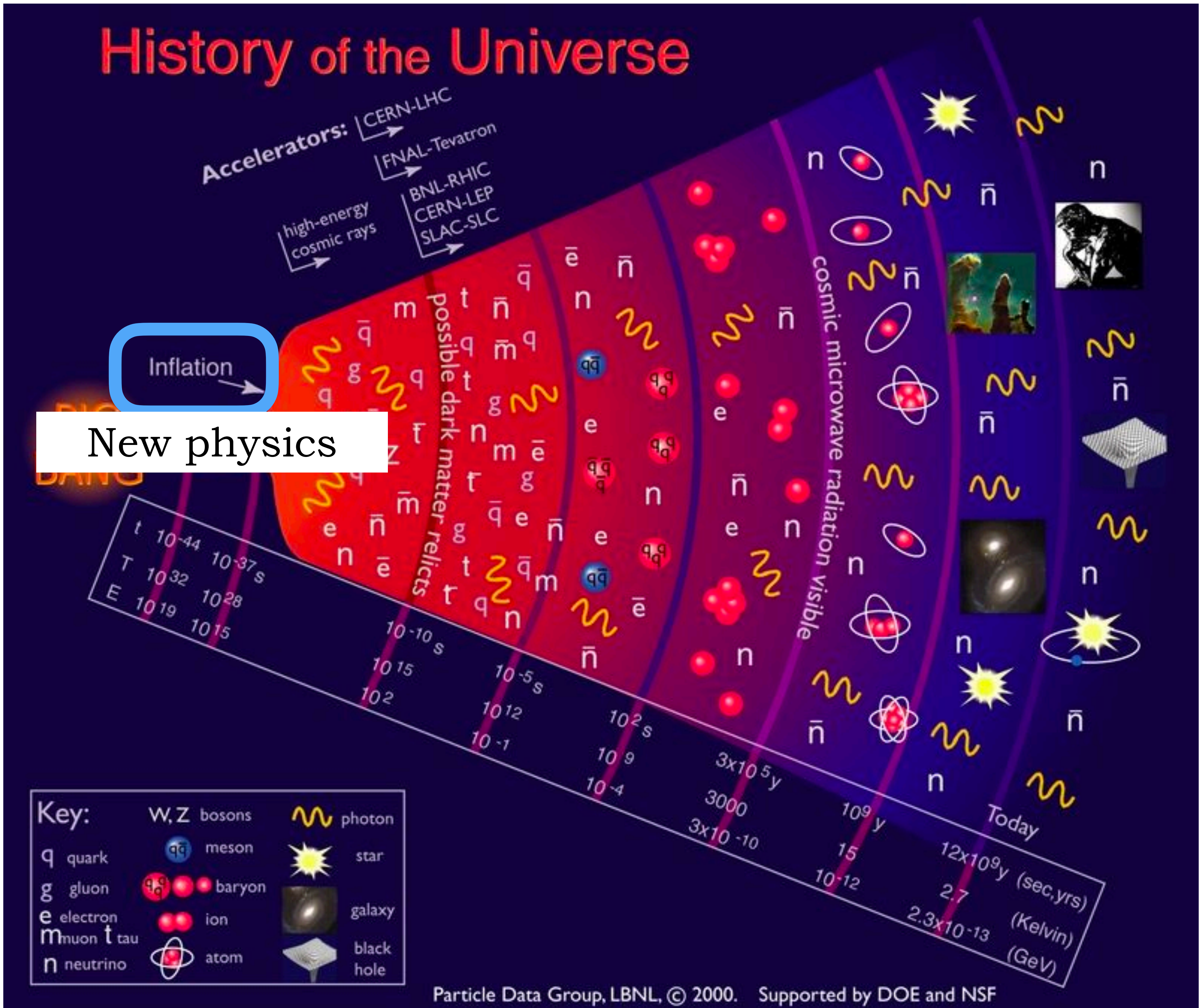
Typical example: topological defects



CC and Figueroa arXiv:1801.04268

Examples of signals

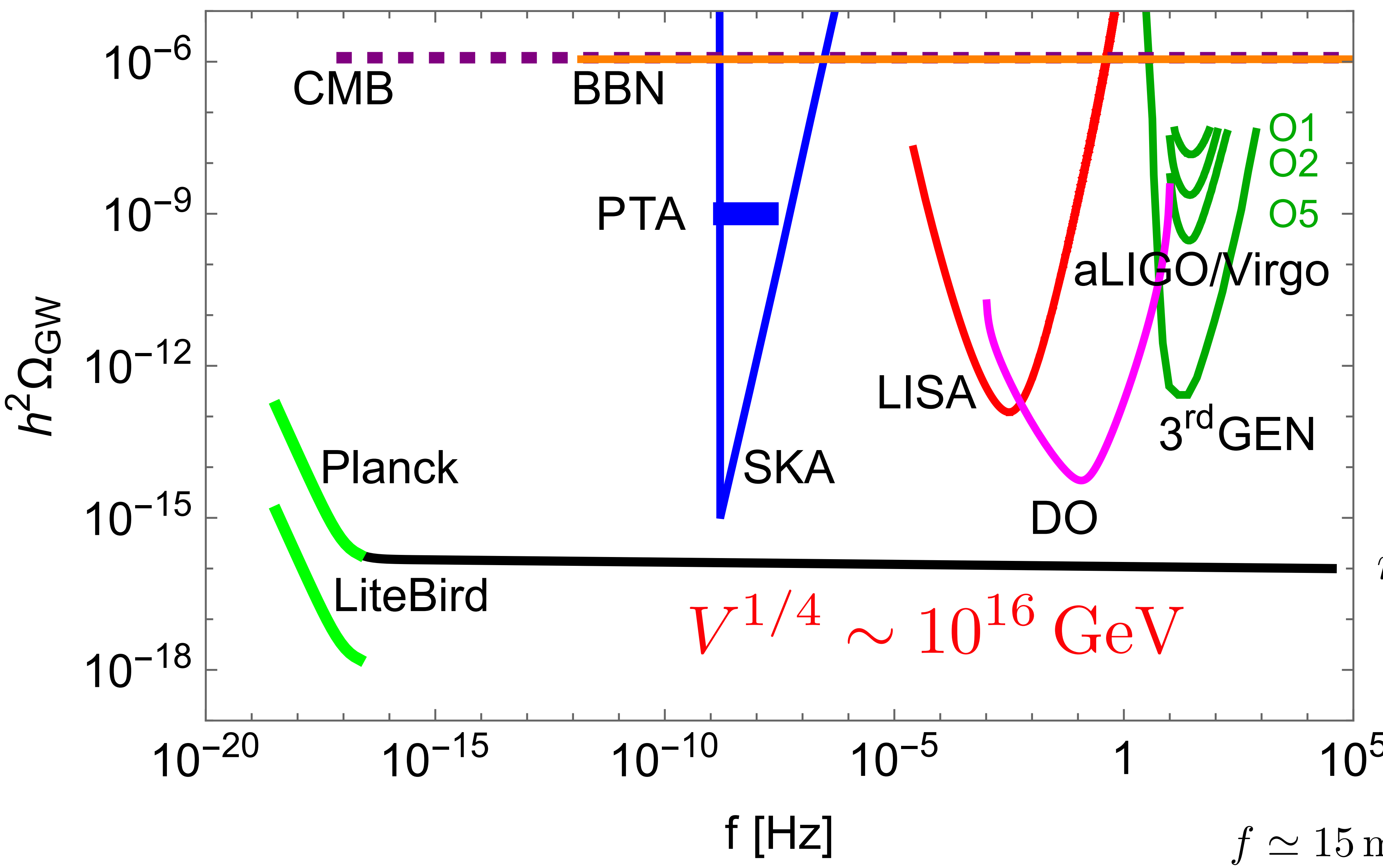
Inflation: phase transition of the Inflaton field



GW signal from (slow roll) inflation

Gw detectors offer the amazing opportunity to probe the inflationary power spectrum (and the model of inflation) down to the tiniest scales

BUT! The signal in the standard slow roll scenario is too low because of CMB observational bound



$n_T(\phi) \sim -2\epsilon(\phi)$

$$f \simeq 15\,\mathrm{mHz}\left(\frac{k}{10^{13}\,\mathrm{Mpc}^{-1}}\right)$$

GW signal from (slow roll) inflation

- tensor spectrum $\mathcal{P}_h = \frac{2}{\pi}\frac{H^2}{m_{Pl}^2}\left(\frac{k}{aH}\right)^{-2\epsilon}$ $\epsilon \equiv \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1$

- transfer function as modes re-enter the Hubble scale

$$\Omega_{\rm GW}(f) = \frac{3}{128}\,\Omega_{\rm rad}\, r\, \mathcal{P}_{\mathcal{R}}^{*}\left(\frac{f}{f_*}\right)^{n_T}\left[\frac{1}{2}\left(\frac{f_{\rm eq}}{f}\right)^2 + \frac{16}{9}\right]$$

- tensor to scalar ratio $r = \mathcal{P}_h/\mathcal{P}_{\mathcal{R}}$ $r_* < 0.038$ Planck+BICEP+ACT+BAO limit

- scalar amplitude at CMB pivot scale $\mathcal{P}_{\mathcal{R}}^{*} \simeq 2\cdot 10^{-9}$ $k_* = \frac{0.05}{\rm Mpc}$

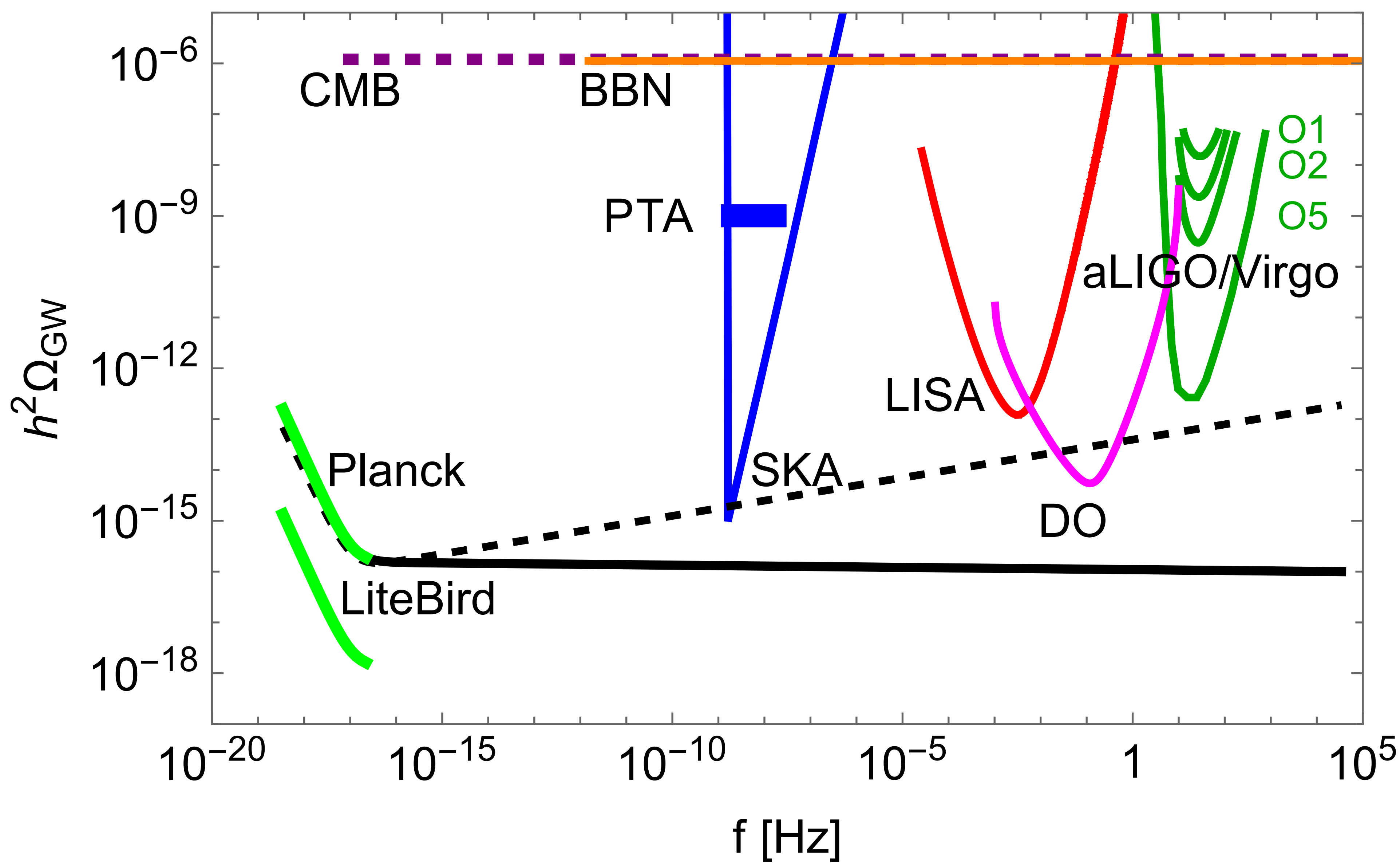
- GW signal extended in frequency: $H_0 \leq f \leq H_{\rm inf}$

continuous sourcing of GW as modes re-enter the Hubble horizon

GW signal from (non-standard) inflation

There is the possibility to enhance the signal going beyond the standard inflationary scenario: adding extra fields, modifying the inflaton potential, modifying the gravitational interaction, adding a phase with stiff equation of state...

$$H_r''(\mathbf{k},\eta) + \left(k^2 - \frac{a''}{a}\right) H_r(\mathbf{k},\eta) = \boxed{16\pi G\, a^3\, \Pi_r(\mathbf{k},\eta)}$$



Example: inflaton-gauge field coupling

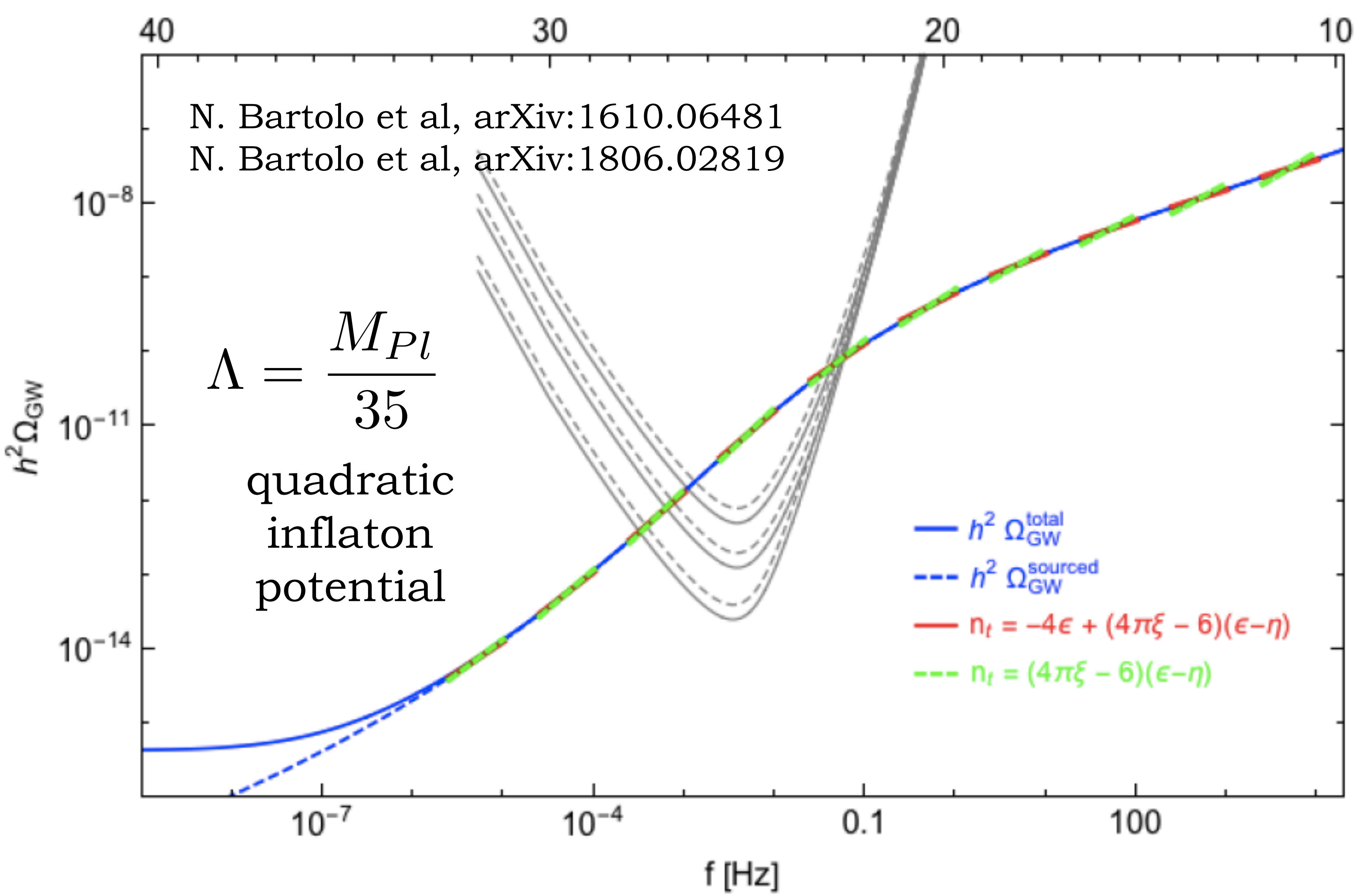
Add a term in the Lagrangian coupling pseudo-scalar inflaton to gauge fields

$$V(\phi) + \frac{\phi}{4\Lambda} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

Production of gauge fields and consequently of GWs through the source

$$\Pi_{ij} \sim [-E_i E_j - B_i B_j]^{TT}$$

EXAMPLE IN THE LISA BAND:



OTHER SIGNATURES/ CONSTRAINTS: non-gaussianity, chirality, primordial black holes

Predictions of the signal must be refined accounting for non-linearity of the system

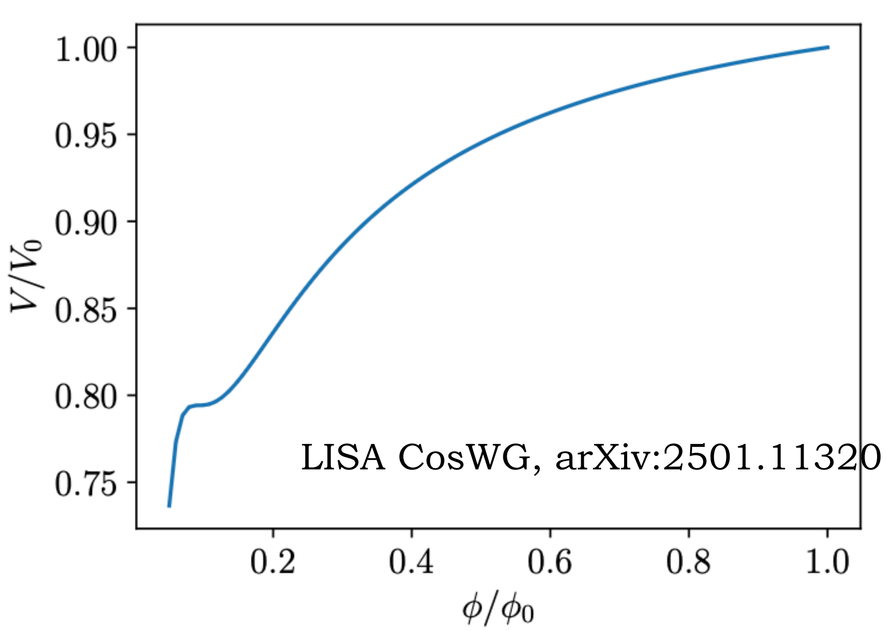
Example: GW signal from second order scalar perturbations and associated primordial black holes

- At linear order in cosmological perturbation theory, scalar and tensor perturbations are decoupled and evolve separately, but *at second order they mix*
- Gradients in the scalar component *can source the tensor component at second order*: since the scalar fluctuations are order of 10^{-5}, the tensors are small $\partial_i \Psi, \partial_i \Phi$
- However, *if the scalar component is enhanced*, the induced tensor component can be important (e.g. from a phase of ultra slow-roll close to reheating)
- The enhanced scalar density fluctuations can collapse upon horizon reentry and produce *primordial black holes* whose properties are linked to those of the tensor spectrum

$$\Omega_{\rm GW}(f) = \Omega_{\rm rad} \int_0^\infty \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u\, \mathcal{K}(u,v)\mathcal{P}_\mathcal{R}(uk)\mathcal{P}_\mathcal{R}(vk)$$

Second order in curvature perturbation

$\phi_0 = 3M_\mathrm{P}$ and $V_0 = 2.3\cdot 10^{-10} M_\mathrm{P}^4$



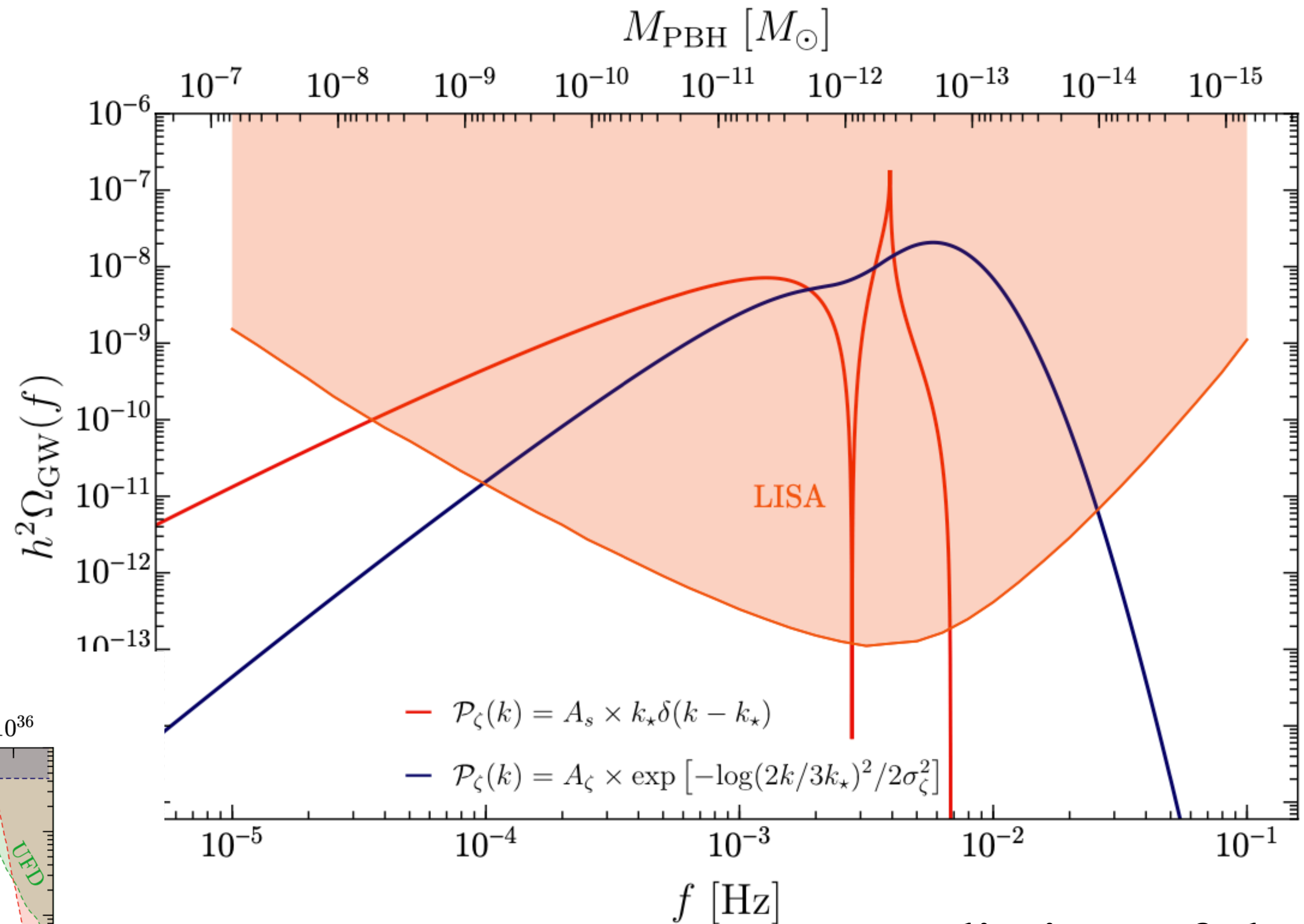
The signal depends on the shape of the curvature power spectrum, several phenomenological models are proposed

Example: GW signal from second order scalar perturbations and associated primordial black holes

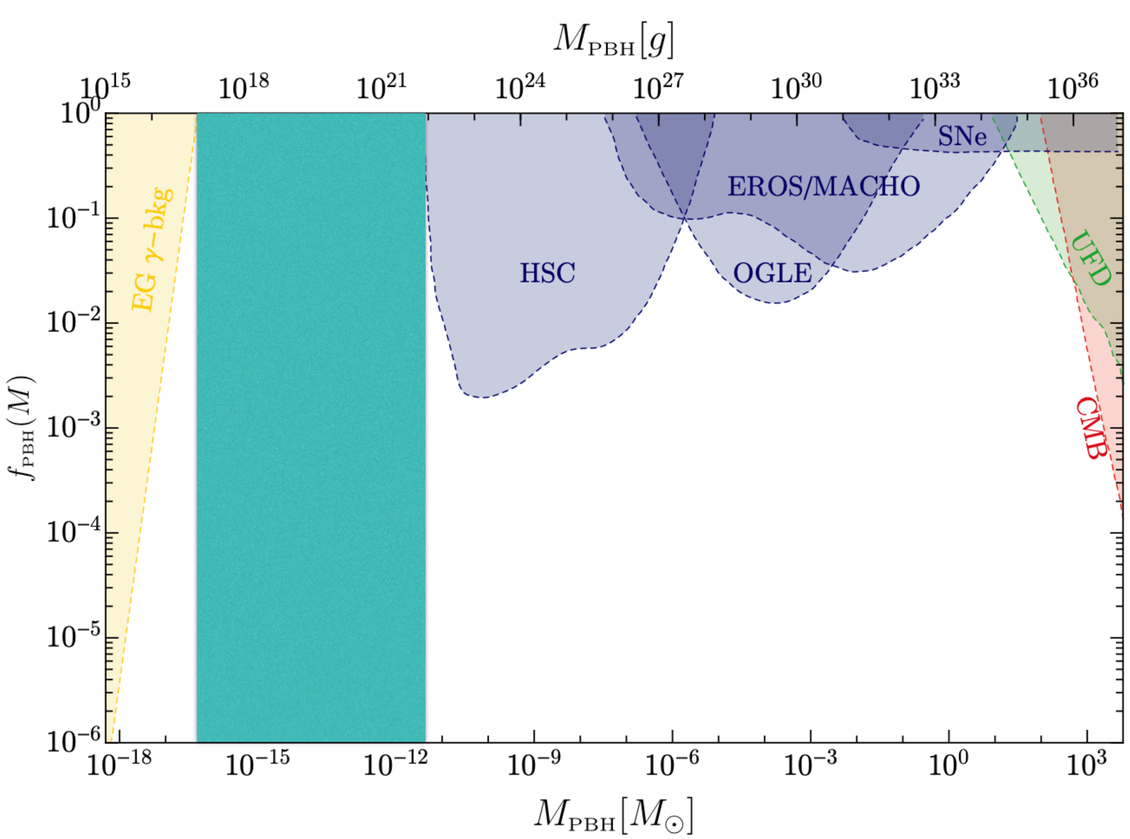
Source, Π_{ij} from a combination of

$$\partial_i \Psi, \partial_i \Phi$$

EXAMPLE OF SIGNAL IN THE LISA BAND:



Predictions of the signal are fairly model dependent



Interesting for LISA: PBH in the mass window in which they can be the totality of the Dark Matter

$$\frac{M_H}{M_\odot} \simeq 7 \times 10^{-11} \frac{k}{10^{12}\,{\rm Mpc}^{-1}}$$

A. Riotto, https://indico.math.cnrs.fr/event/5766/contributions/5153/attachments/2801/3587/Paris2021.pdf

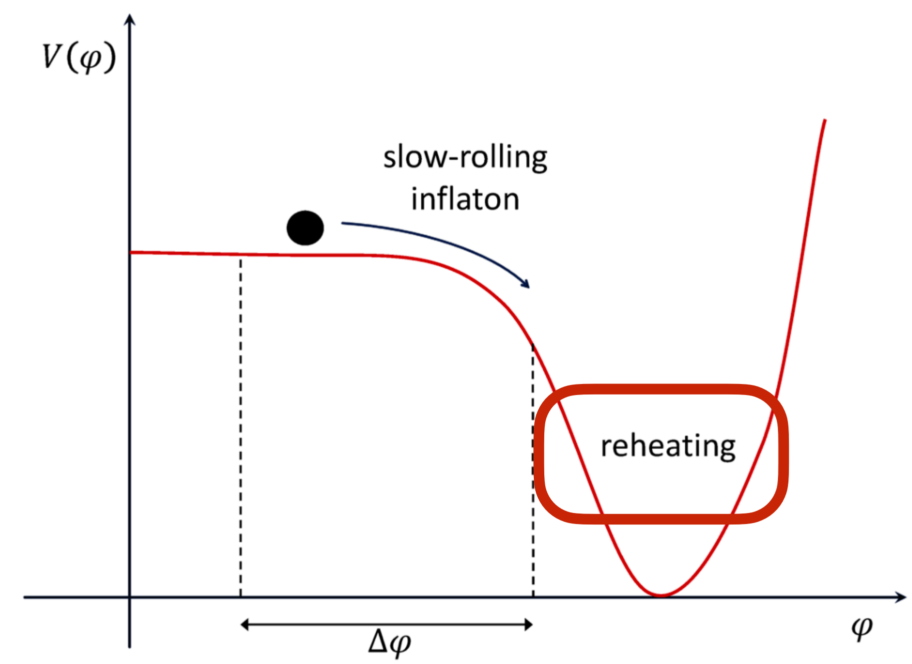
Example: resonant particle production at preheating

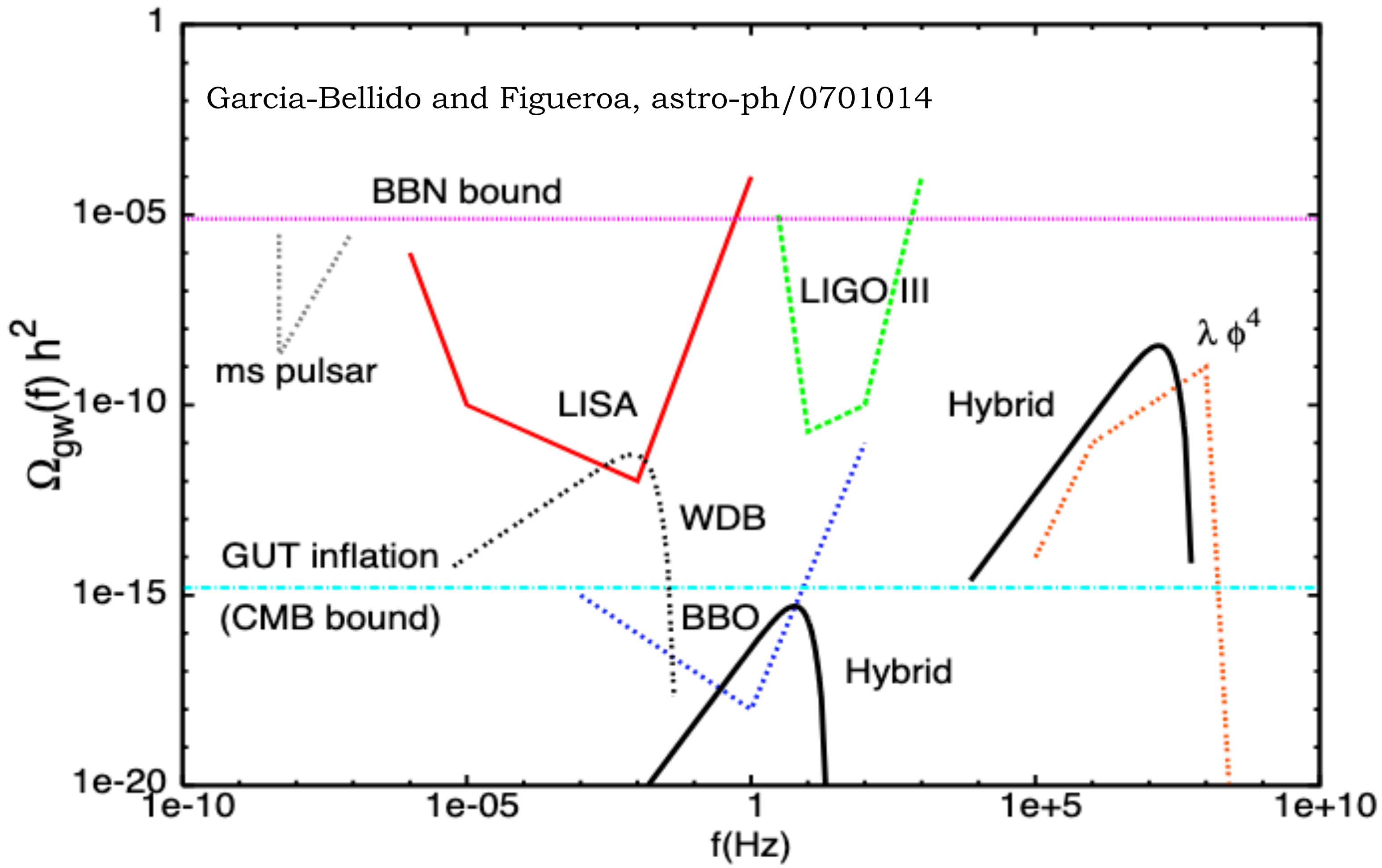
$$V(\phi) + \frac{1}{2} g^2 \phi^2 \chi^2$$

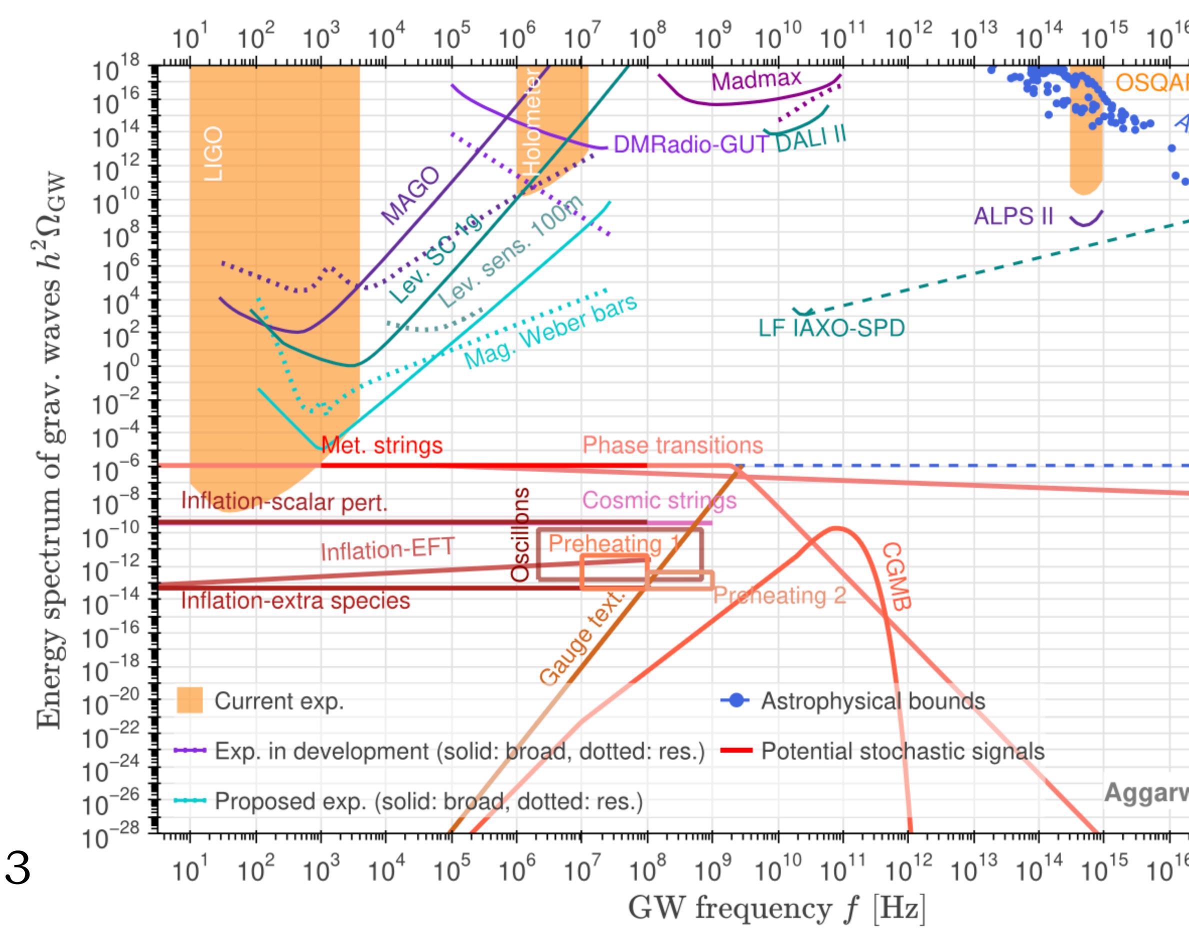
Kofman et al. arXiv:hep-ph/9704452

GW sourced from inhomogeneous field

$$\Pi_{ij} \sim \left[\partial_i \chi \partial_j \chi\right]^{TT}$$







high frequency detectors up to 10^{18} GeV, but sensitivity still above BBN and CMB bounds

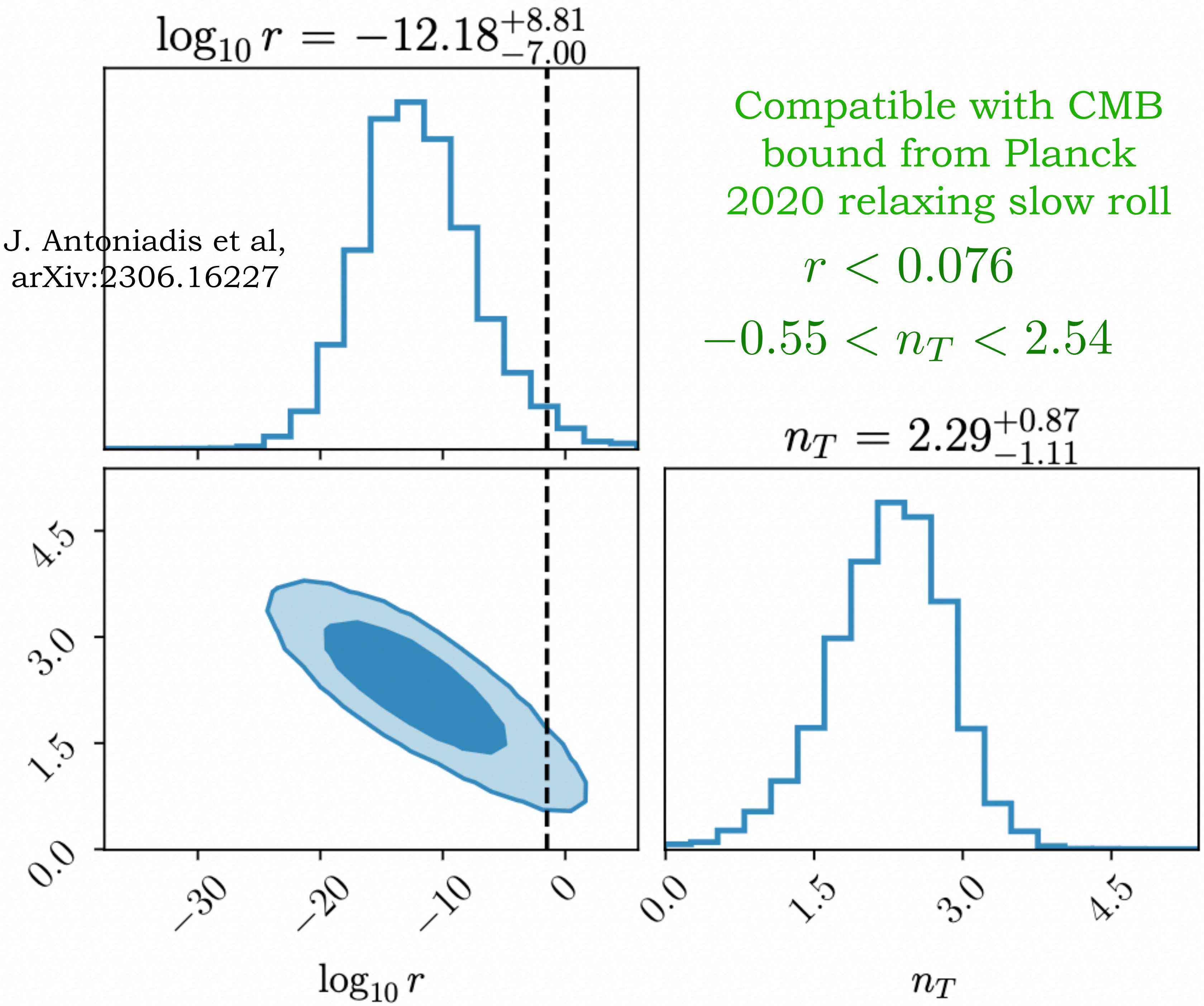
Aggarwal et al, arXiv:2501.11723

An example of possible detection at PTA?

Very small value of r at CMB scale

$$\Omega_{\rm GW}(f) = \frac{3}{128}\,\Omega_{\rm rad}\,r\,\mathcal{P}^*_{\mathcal{R}}\left(\frac{f}{f_*}\right)^{n_T}\left[\frac{1}{2}\left(\frac{f_{\rm eq}}{f}\right)^2 + \frac{16}{9}\right]$$

$$\times\left(\frac{f}{f_{\rm RD}}\right)^{\frac{2(3w-1)}{3w+1}}$$



J. Antoniadis et al, arXiv:2306.16227

Compatible with CMB bound from Planck 2020 relaxing slow roll

$$r < 0.076$$

$$-0.55 < n_T < 2.54$$

Would this be compatible with slow roll and a stiff equation of state? Marginally

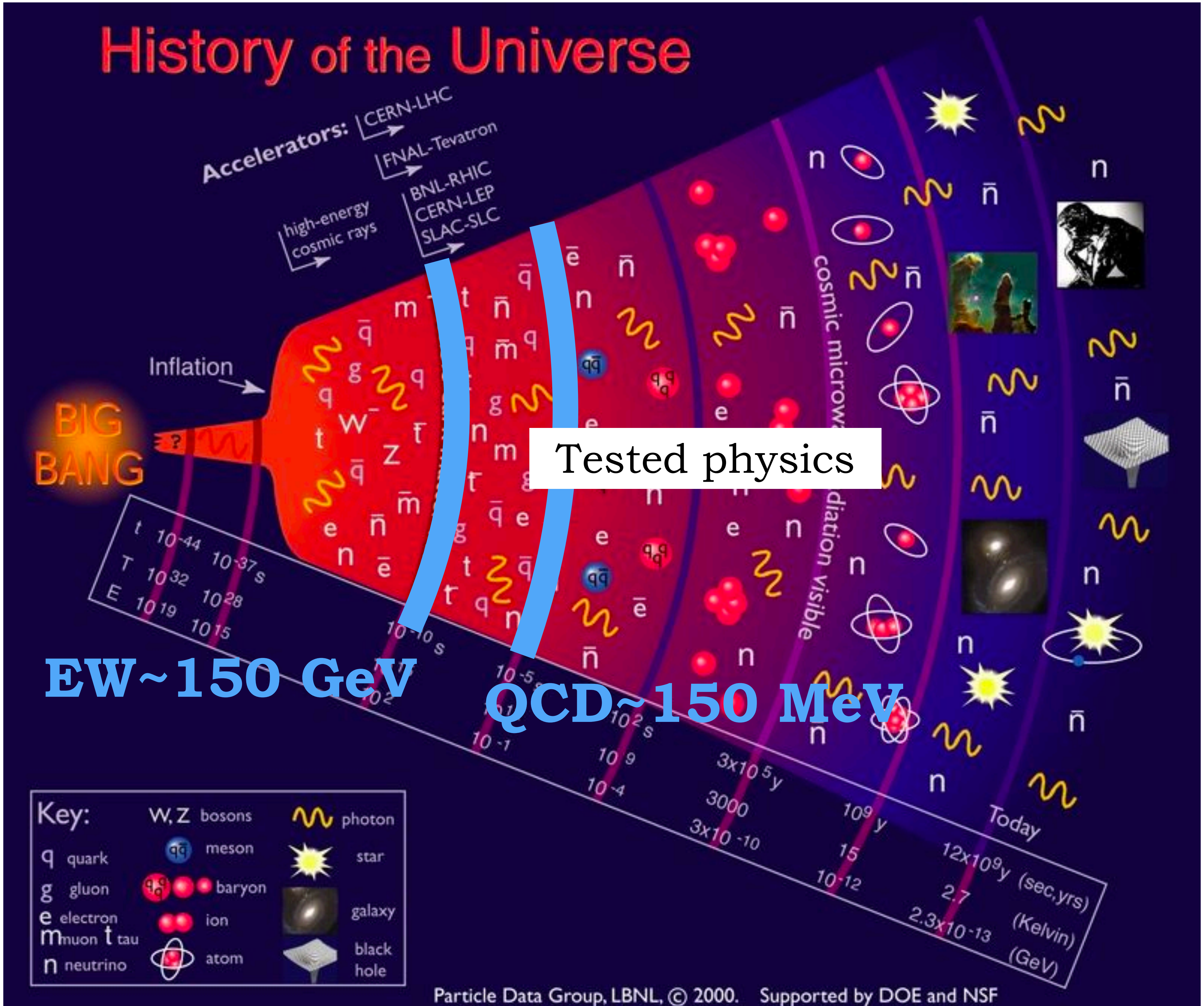
$$\gamma = 5 - n_T + \frac{2(1-3w)}{3w+1}$$

Bound between 0 and -2

$$\gamma_{\rm best\ fit} \simeq 2.7 \;\longrightarrow\; n_T \gtrsim 0.3$$

Strong degeneracy between the two parameters $n_T = -0.16\log_{10} r + 0.46$

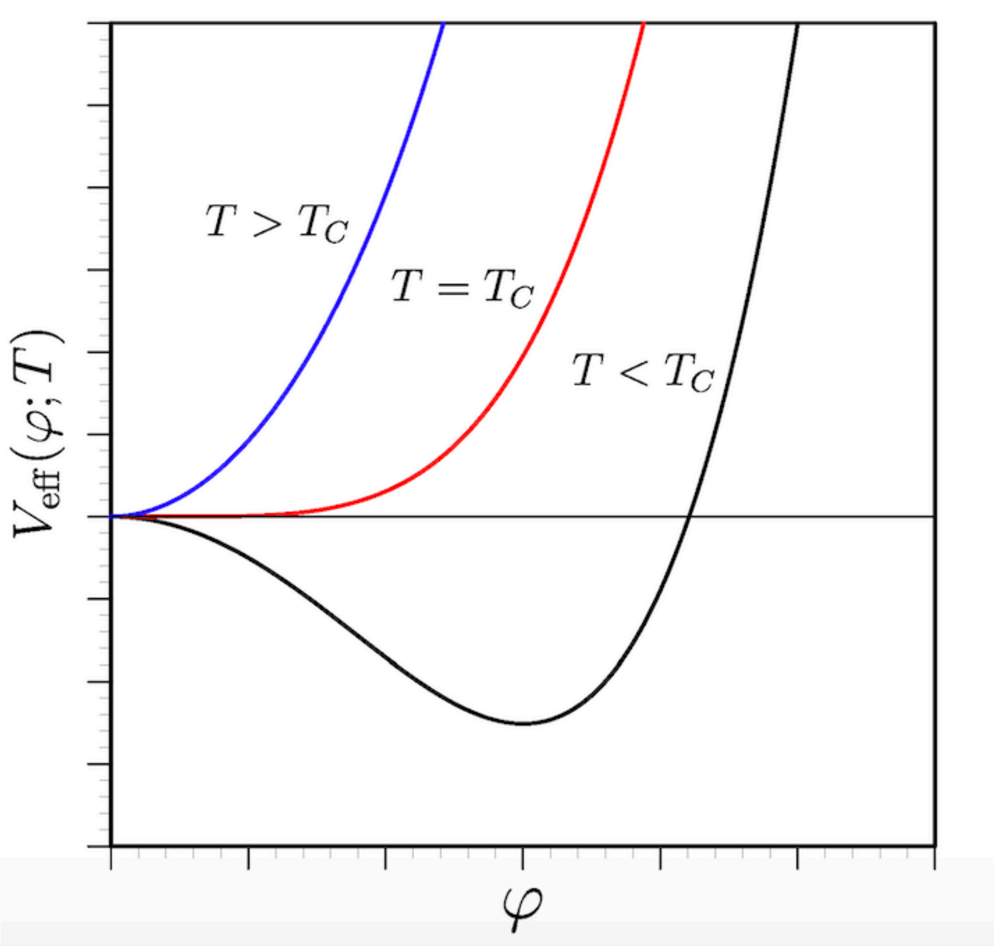
phase transitions predicted by the standard model of particle physics: electroweak and QCD



First order phase transitions

- We know that at least two PTs occurred in the universe, the EW one and the QCD one: according to the standard model, they are both **crossovers**
- However, *sizeable (detectable) GW generation requires a first order PT*, proceeding through the *nucleation of true vacuum bubbles*

Second order phase transition



First order phase transition

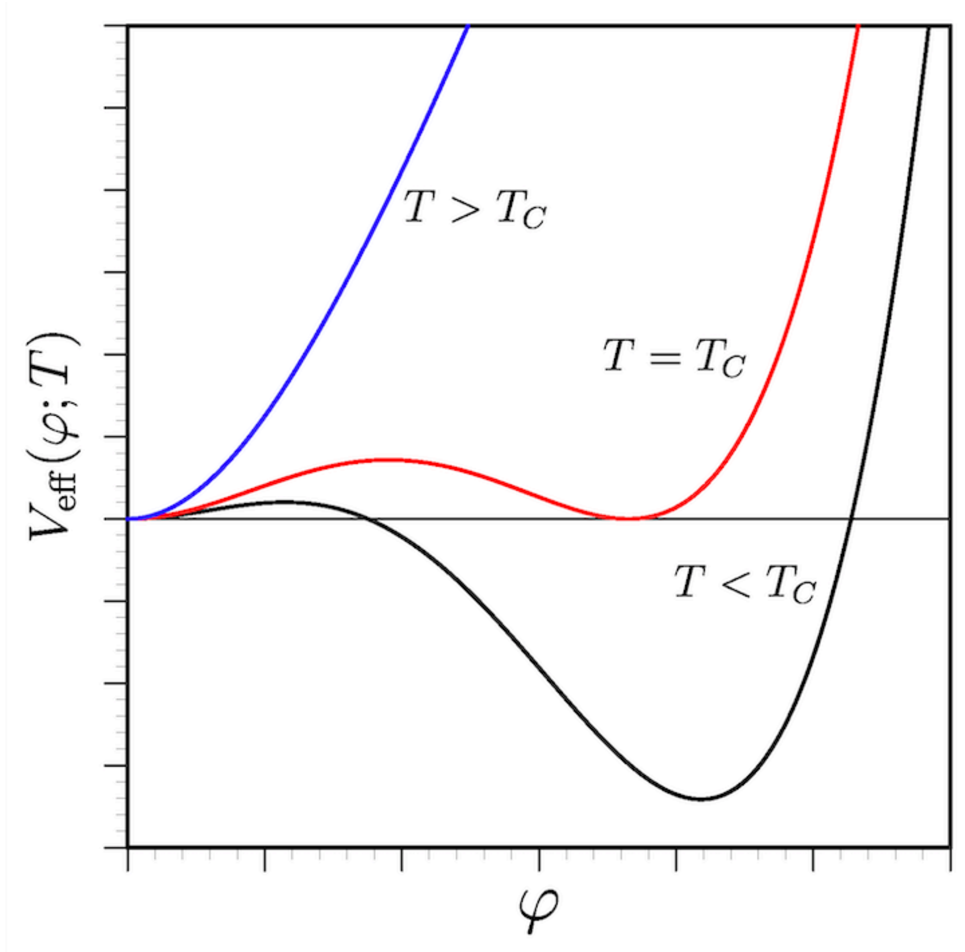


Image credit: E. Senaha, Symmetry 2020

First order phase transitions

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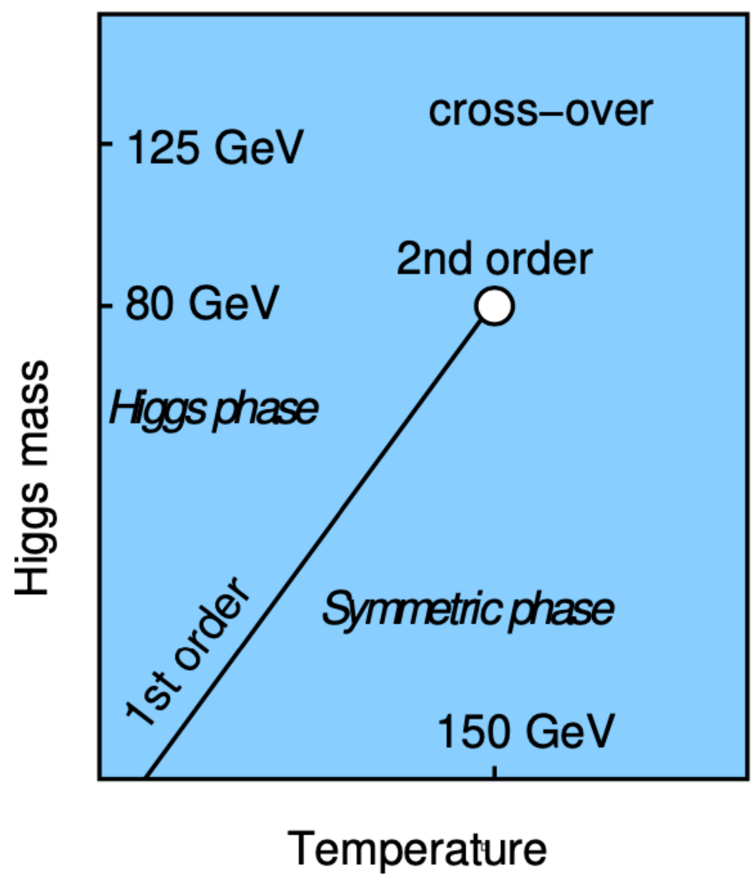
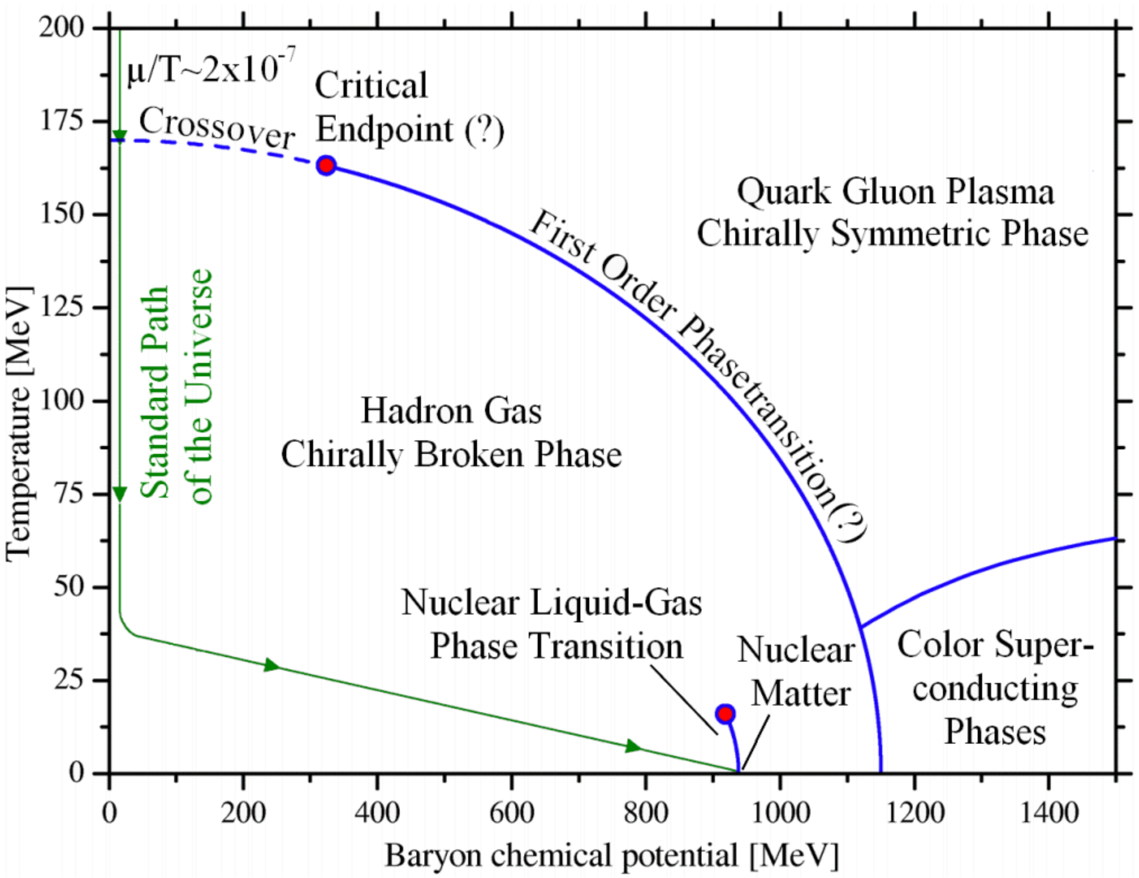
EWPT



M. Hindmarsh et al,
arXiv:2008.09136

QCDPT



T. Boekel and J. Schaffner-Bielich,
arXiv:1105.0832

First order phase transitions

- We know that at least two PTs occurred in the universe, the EW one and the QCD one: according to the standard model, they are both **crossovers**
- However, *sizeable (detectable) GW generation requires a first order PT*, proceeding through the *nucleation of true vacuum bubbles*

EWPT

might become first order in BSM EW sector extensions: SM + light scalars (SM+singlet, SUSY, 2HDM, composite Higgs…)

QCDPT

Depends on the conditions in the early universe: **might become first order if the lepton asymmetry in the universe is large**

D. Schwarz and Stuke, arXiv:0906.3434
M. Middeldorf-Wygas et al, arXiv:2009.00036

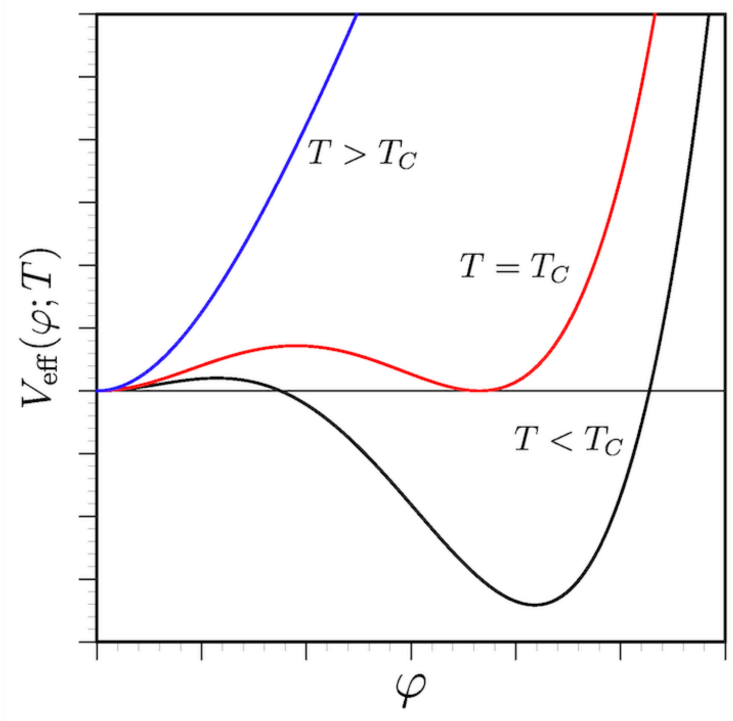
OTHER EXAMPLES OF POSSIBLE FOPTs:

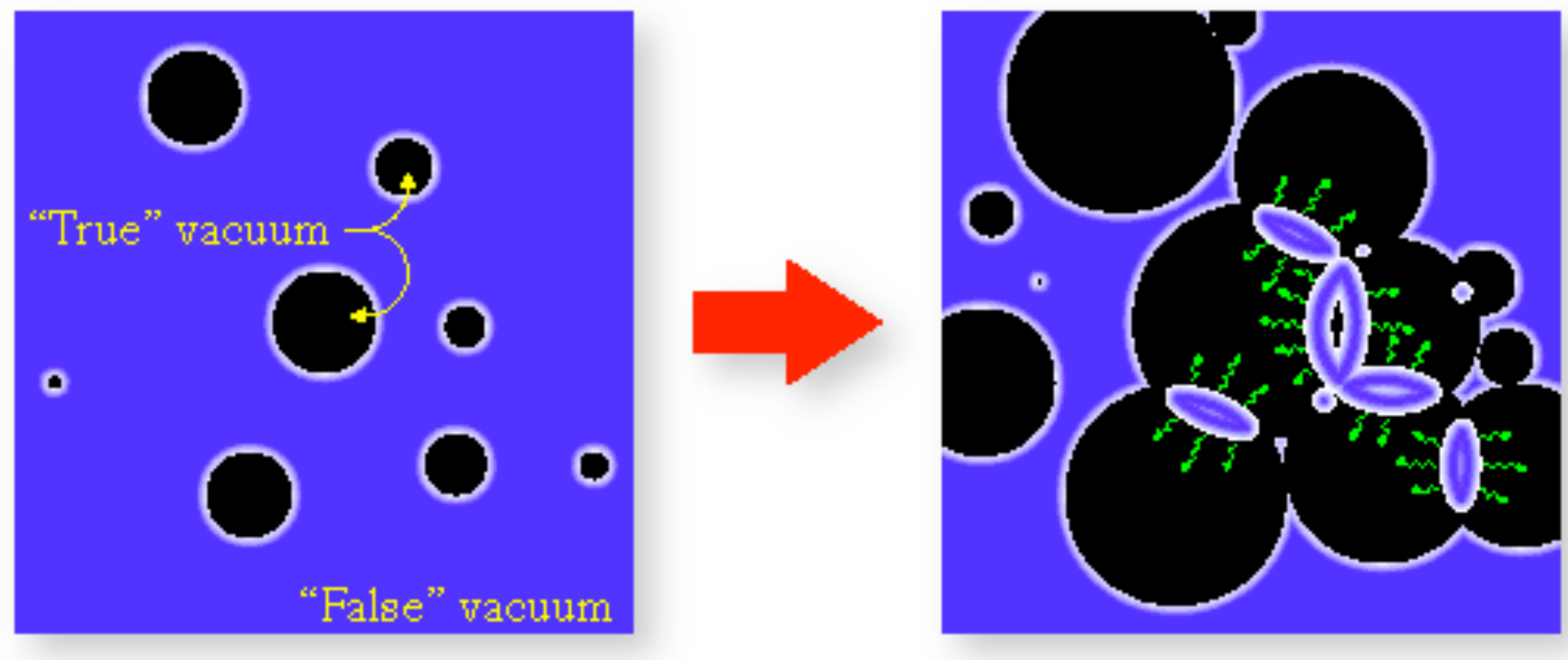
- **Effective approaches:** heavy new physic represented by higher dimensional operators
- **Conformal models:** e.g. conformal symmetry breaking with dilaton
- **New symmetries:** extend the SM with e.g. $U(1)_{B-L}$
- **Hidden sectors:** provide also dark matter candidates, PT can be as strong as one wants
- **Peccei Quinn** can be first order depending on the realisation

Opportunity to probe high energy physics scenarios beyond the standard model

First order phase transitions

Sources of tensor anisotropic stress (and thereby GWs)
at a first order phase transition:





Several processes, rich phenomenology!

- Bubble collision (scalar field gradients) $\Pi_{ij}^{TT} \sim [\partial_i\phi\partial_j\phi]^{TT}$
- Bulk fluid motion $\Pi_{ij}^{TT} \sim [\gamma^2(\rho+p)v_iv_j]^{TT}$ sound waves and/or turbulence
- Electromagnetic fields $\Pi_{ij}^{TT} \sim [-E_iE_j - B_iB_j]^{TT}$

First order phase transitions

The signal depends on the following parameters

- The temperature of the FOPT T_*
- The amount of energy available in the source K, connected to the PT strength
- The size of the anisotropic stresses, connected to the bubble size $R_* = v_w/\beta$
- The bubble wall velocity v_w

$T_*,\ \alpha,\ \dfrac{\beta}{H_*}$ → Determined by the effective potential

v_w, K → Determined by the bubble expansion dynamics and interaction, and by the fluid dynamics (sound speed fixed)

If the PT is strong and non-linearities in the bulk fluid develop: fraction of kinetic energy in turbulent motions $\varepsilon = \dfrac{K_{\rm turb}}{K}$

Most of these parameters are known (at least in principle) given a PT model + numerical simulations of the fluid dynamics

numerical simulations are necessary to infer the GW signal because of non-linear dynamics and/or complicated fluid shells profiles and/or intrinsic randomness of the process

First order phase transitions

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- The temperature of the FOPT T_*
- The amount of energy available in the source K, connected to the PT strength
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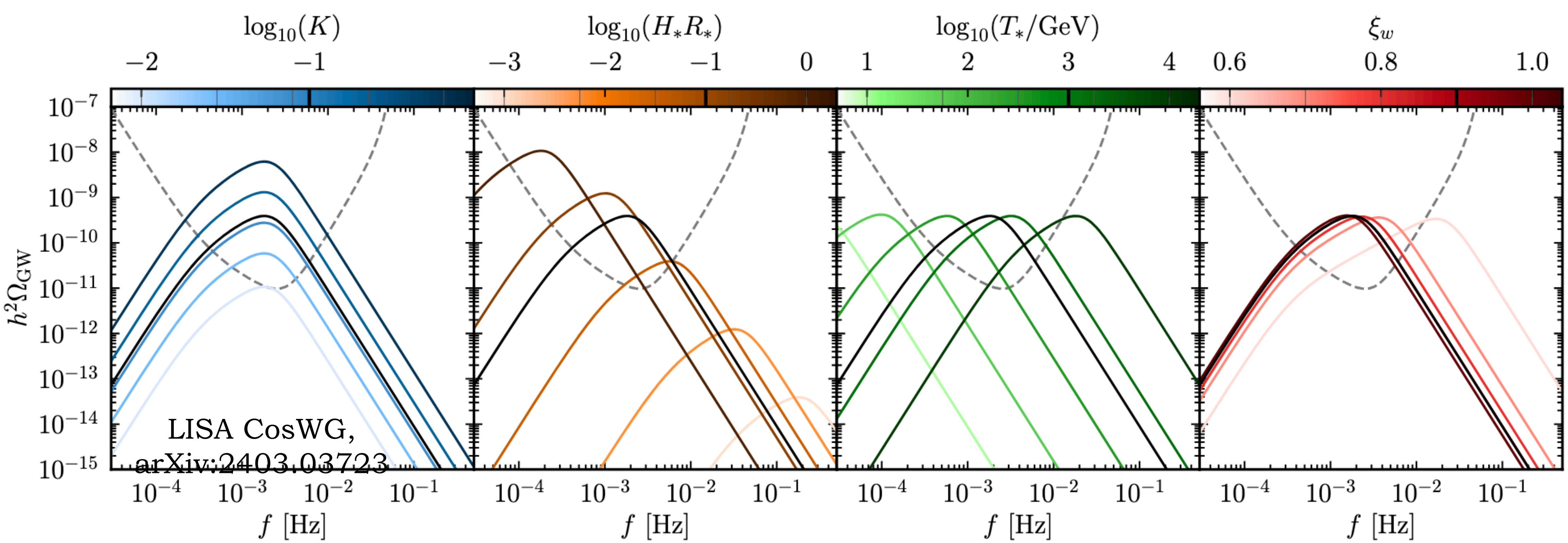


(b) sound waves (black: $K = 0.1$, $H_* R_* = 0.1$, $\xi_w = 0.9$, $T_* = 1\,\mathrm{TeV}$)

First order phase transitions

LIGO Virgo Kagra

$$1\,\mathrm{Hz} < f < 1000\,\mathrm{Hz} \quad \longrightarrow \quad 10^{6}\,\mathrm{GeV} \lesssim T_* \lesssim 10^{10}\,\mathrm{GeV}$$

LVK constraints from non-detection

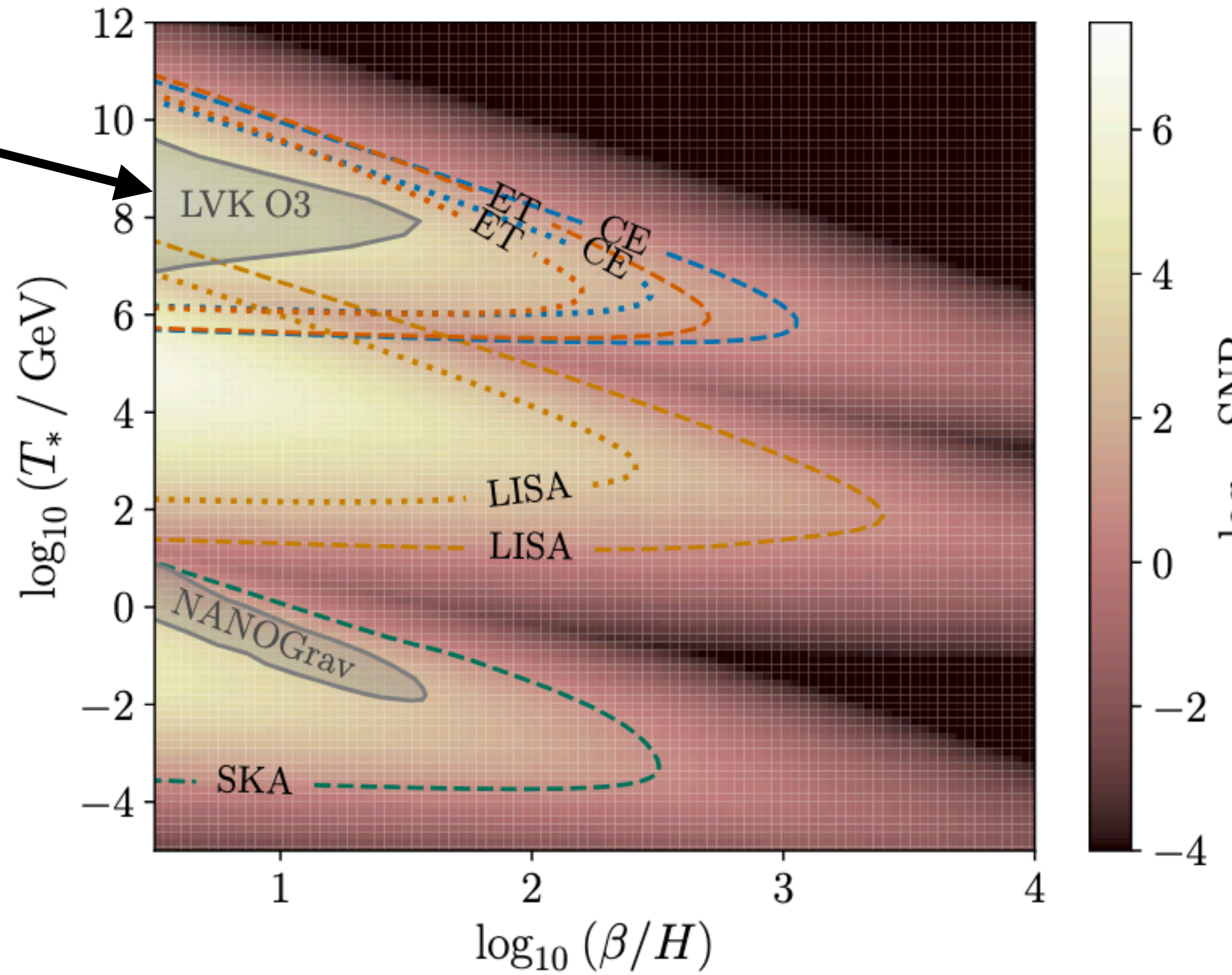
Badger et al, arXiv:2209.14707

Peccei-Quinn phase transition

$$T_{\mathrm{PQ}} \sim F_a$$

$$10^{7-8}\,\mathrm{GeV} \lesssim F_a \lesssim 10^{10-11}\,\mathrm{GeV}$$

CC et al, ArXiv:2406.02359



Parameter to which the signal amplitude is *inversely* proportional

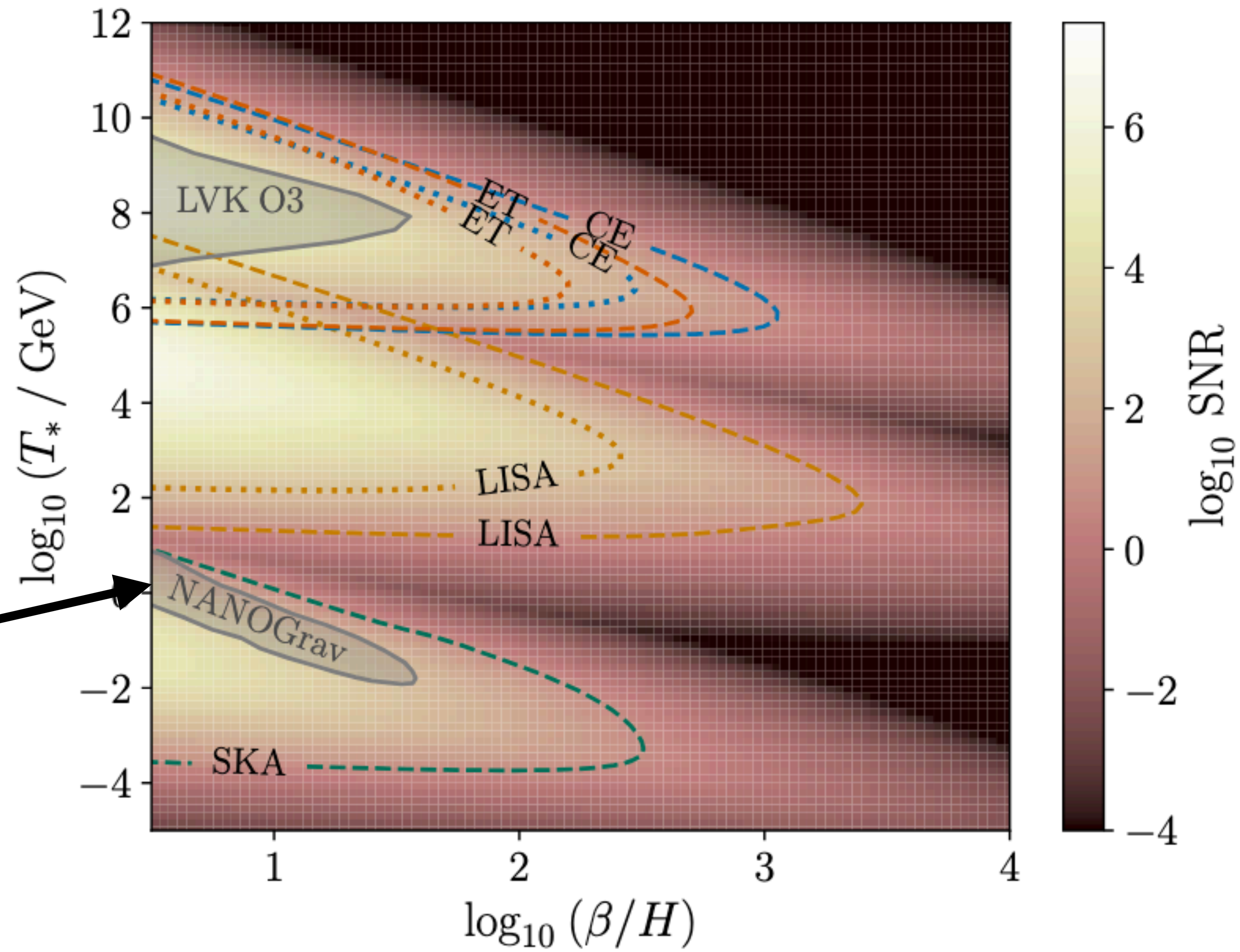
First order phase transitions

Pulsar Timing Arrays

$$10^{-9}\,\mathrm{Hz} < f < 10^{-7}\,\mathrm{Hz} \quad \longrightarrow \quad 1\,\mathrm{MeV} \lesssim T_* \lesssim 1\,\mathrm{GeV}$$

PTAs offer the possibility to probe the **QCD energy scale**

CC et al, ArXiv:2406.02359



Parameter space region that could explain the measurement

Afzal et al arXiv:2306.16219

Parameter to which the signal amplitude is *inversely* proportional

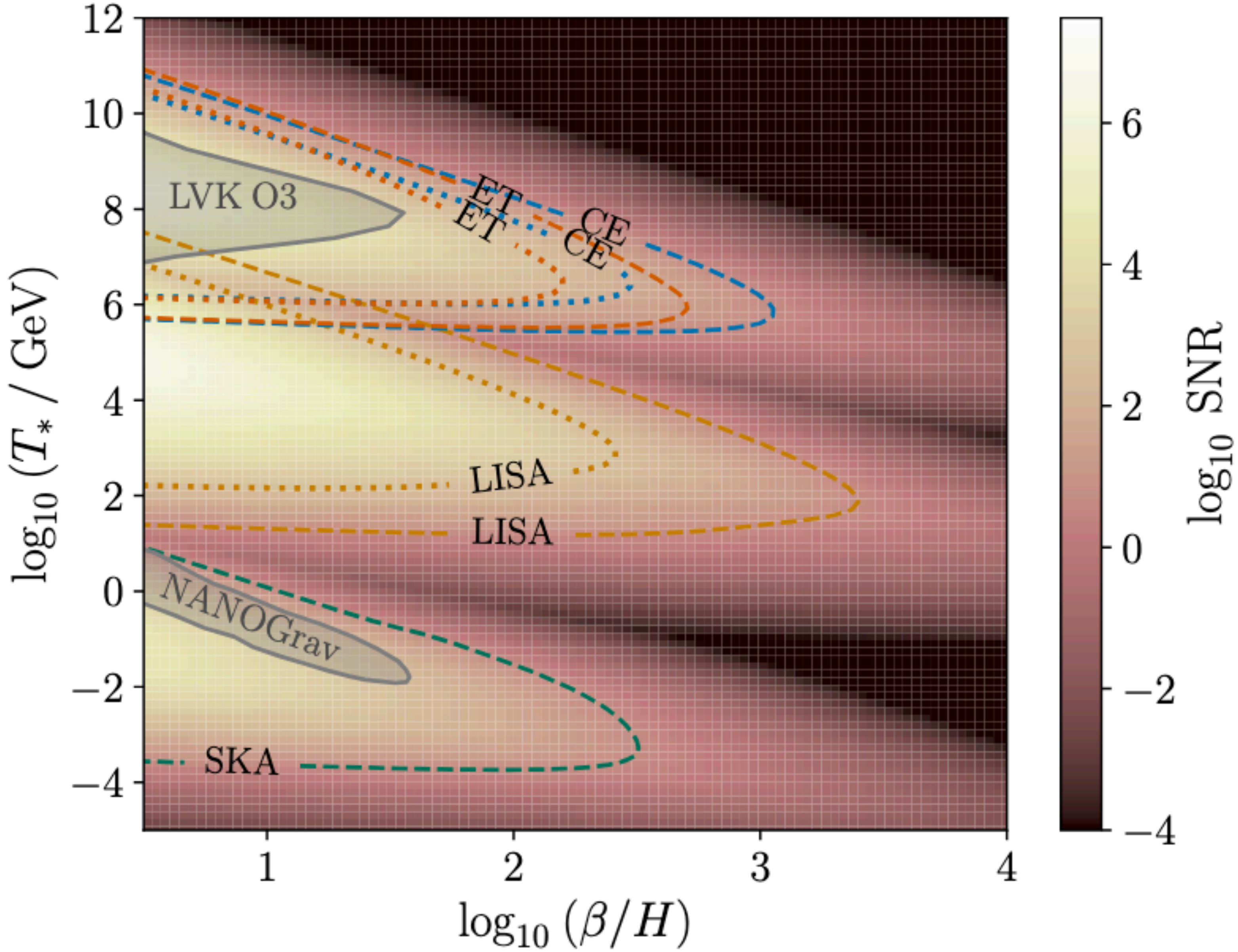
First order phase transitions

Laser interferometer space antenna

$$10^{-5}\,\mathrm{Hz} < f < 0.1\,\mathrm{Hz} \quad \longrightarrow \quad 10\,\mathrm{GeV} \lesssim T_* \lesssim 10^5\,\mathrm{GeV}$$

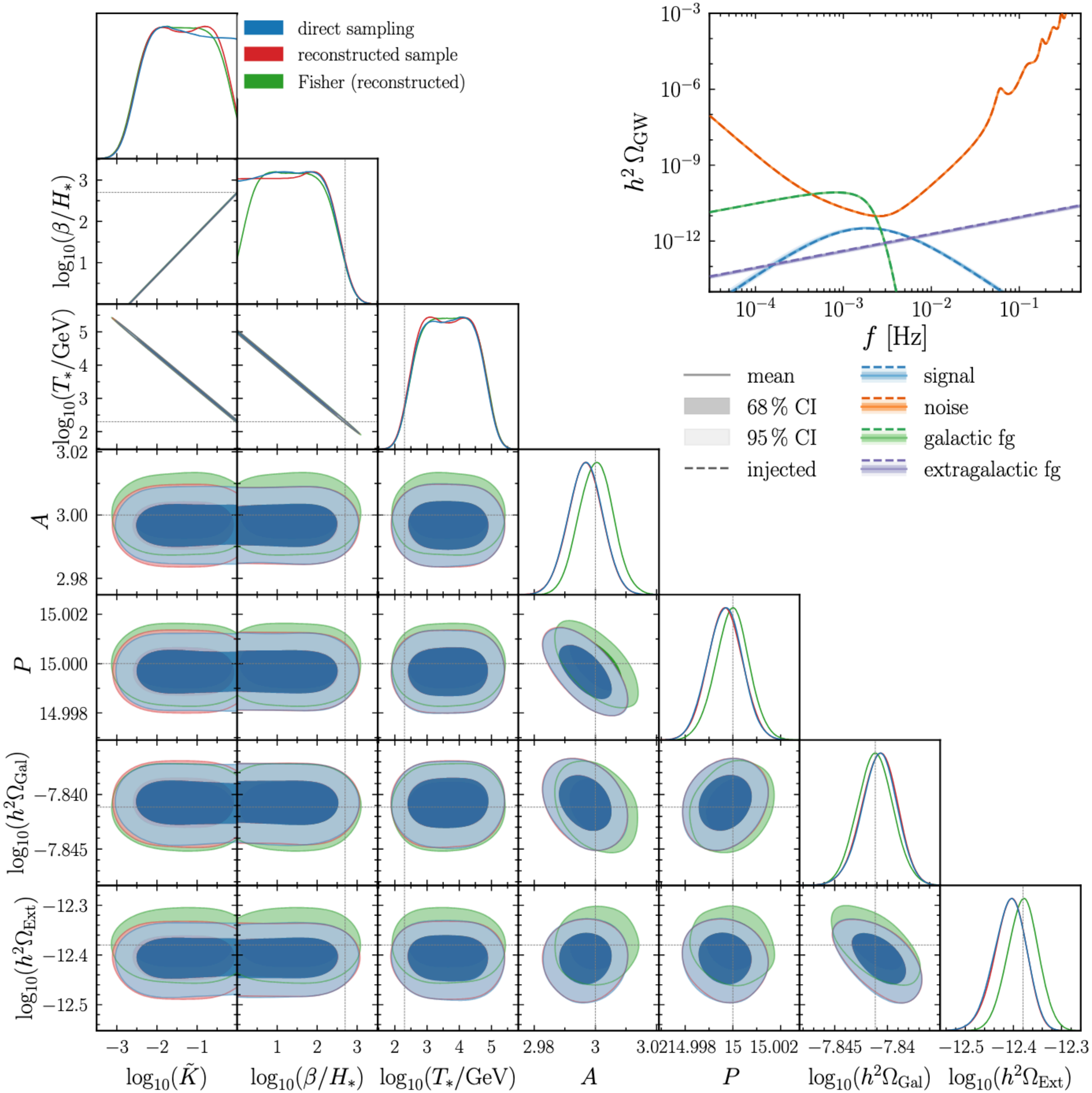
LISA offers the possibility to probe the **EW energy scale and beyond**

CC et al, ArXiv:2406.02359



Parameter to which the signal amplitude is *inversely* proportional

Examples of detectable signal from the EWPT

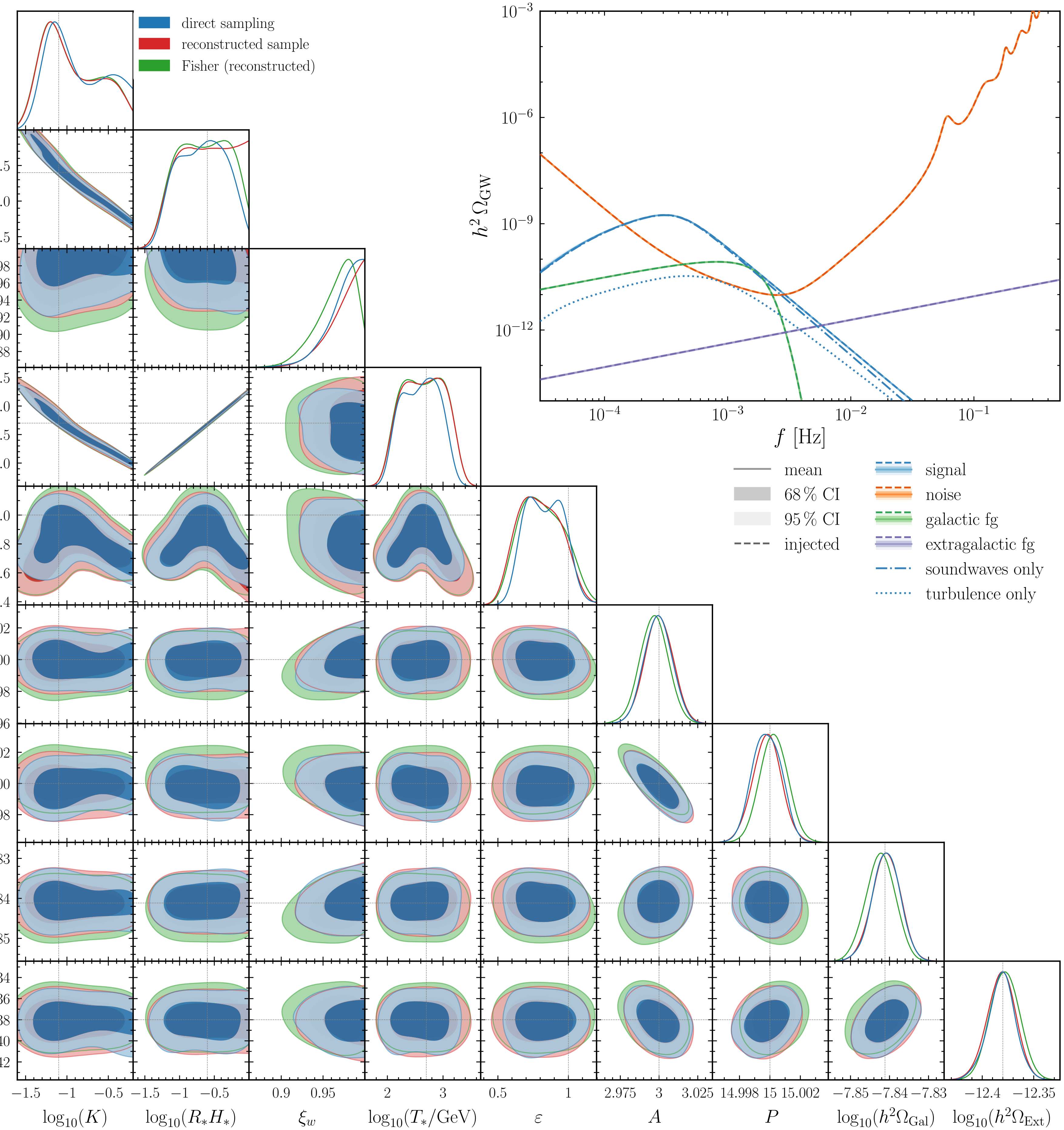


Template-based reconstruction of the *thermodynamic parameters* of the first order PT for **bubble collisions**

accounting for **foregrounds** and **assuming a two-parameters noise model**

LISA CosWG, arXiv:2403.03723

Examples of detectable signal from the EWPT



Template-based reconstruction of the *thermodynamic parameters* of the first order PT for **sound waves + turbulence**

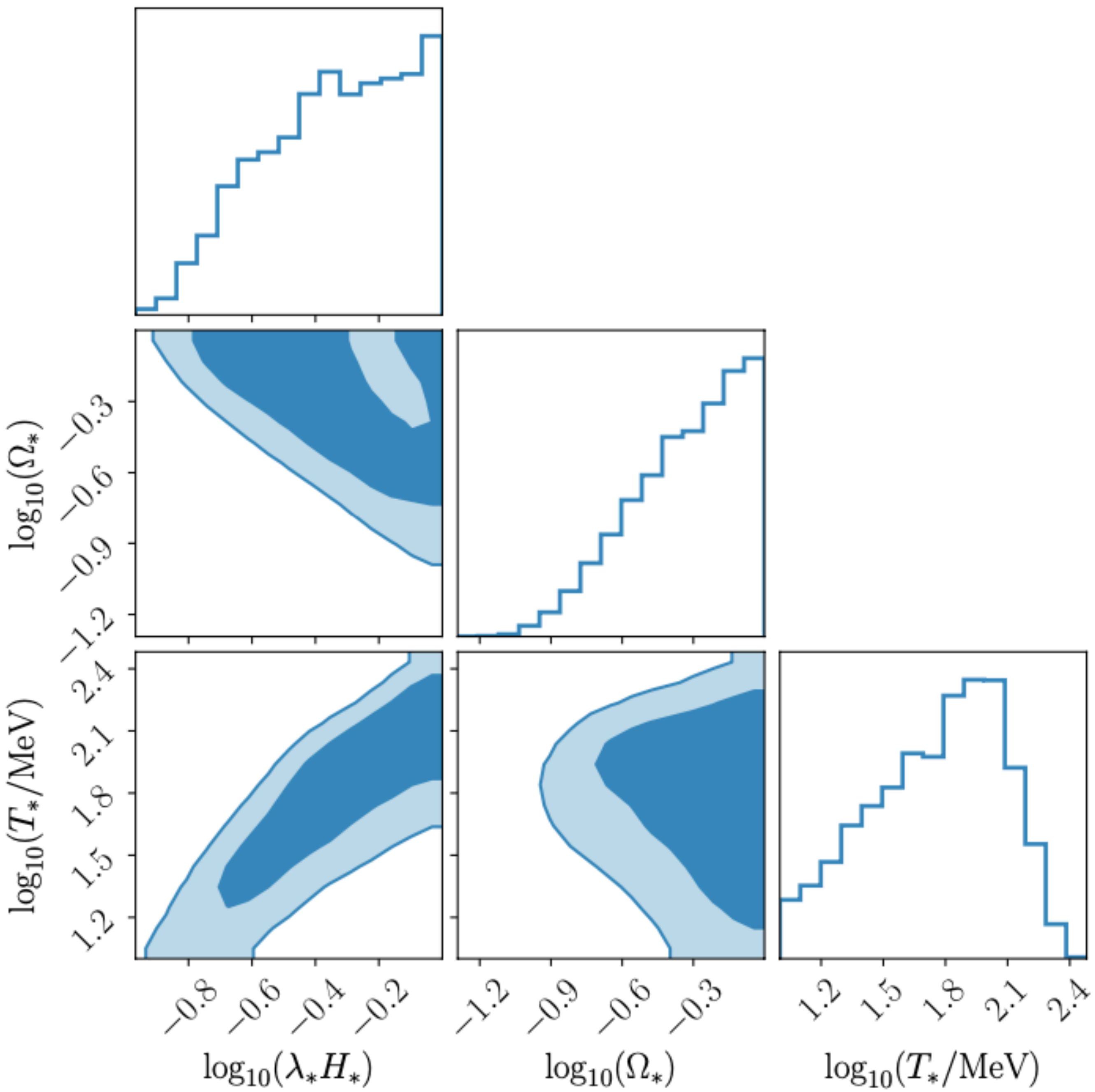
accounting for **foregrounds** and **assuming a two-parameters noise model**

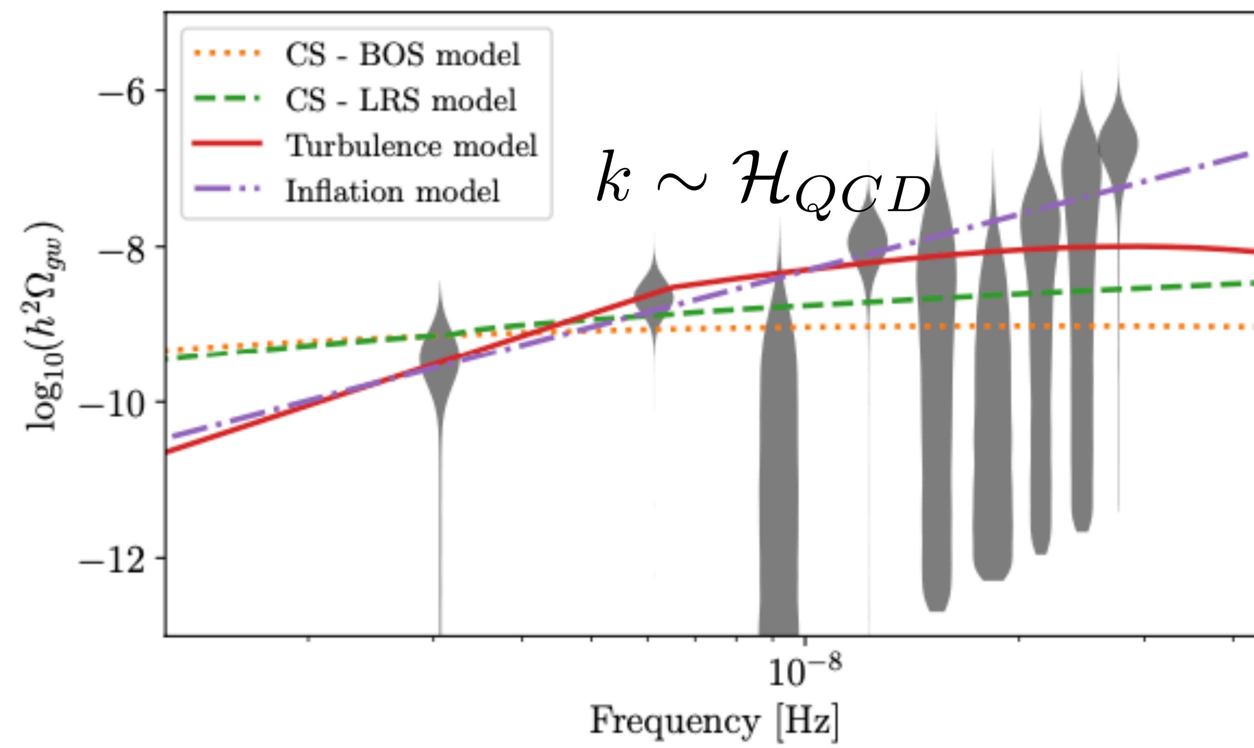
LISA CosWG, arXiv:2403.03723

An example of possible detection at PTA?

The PTA signal is compatible with GWs generated by MHD turbulence at the QCD scale

- T_* must be close to the QCD scale, the amount of energy available in anisotropic stress K must be high (at least 10% of the total energy density of the universe, and size of the anisotropic stresses $R_* = v_w/\beta$ must be close to the horizon





The the signal is fit with the low frequency tail, and the spectrum has a break at a scale comparable to the horizon at the QCD PT

To summarise:

- SGWB might reveal a powerful tool to probe the early universe and high energy physics

- The spectral shape must be predicted with good accuracy in order to disentangle the different sources (and also for foregrounds)

- General considerations about the characteristics of the spectral shape are possible in some cases, to pin down at least the class of SGWB sources

- Inflation: new physics but observationally compelling, extended GW signal in frequency, only accessible by CMB unless one goes beyond the standard slow roll scenario (there are well motivated scenarios!)

- Topological defects: amazing potential to probe high energy theory, but need to account for GW signal model dependent

- Electroweak PT: at the limit of tested physics, GW signal can be accessed/ constrained by LISA only for models beyond the standard model of particle physics

- QCD PT: tested physics but difficult to predict, GW signal can be accessed/ constrained by PTA only for models beyond the standard model of particle physics

- **SGWBs from the primordial universe might seem speculative but their potential to probe fundamental physics is great and amazing discoveries can be around the corner, especially after the PTA results!**