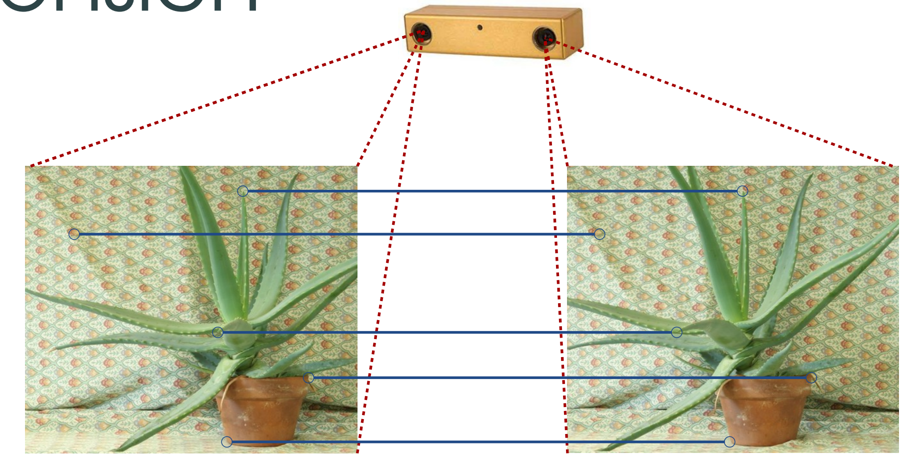
Università degli Studi di Padova

3D Data Processing

Lab 1

Problem Formulation

- Search in a pair of stereo images for corresponding pixels that are projections of the same 3D point
- Epipolar constraints reduces the search to one dimension

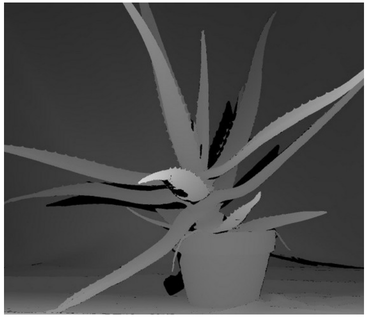


Dense Matching

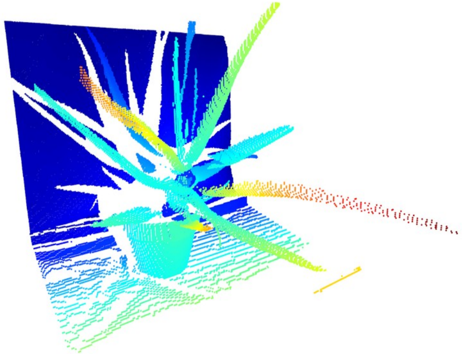
Find correspondences for all points that are projected in both images

Left image

Right image



Disparity map

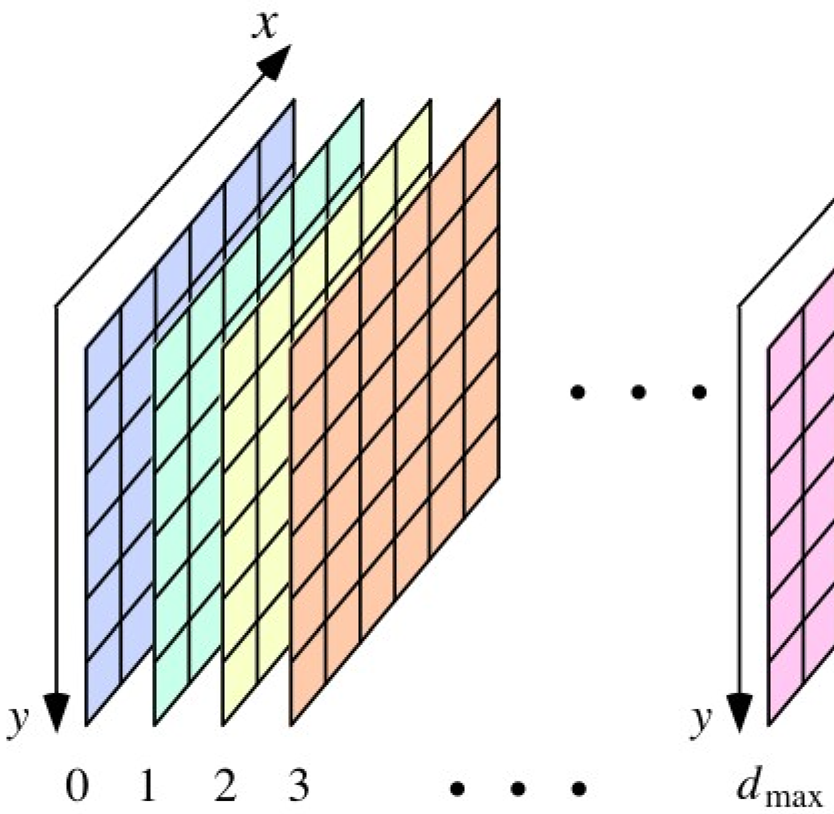


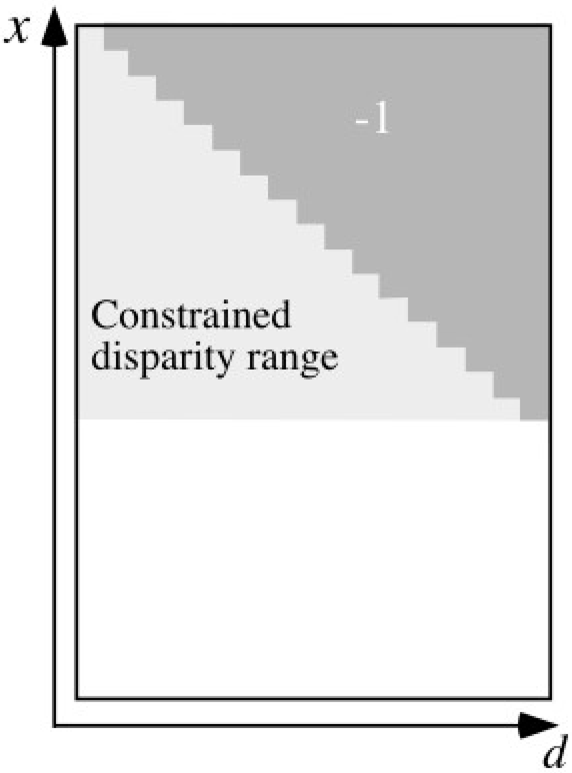
Point cloud

Assumptions

- Most scene points are visible from both viewpoints
- Corresponding image regions are similar
- We consider rectified rectilinear stereo rigs: the epipolar lines are the rows of the images (**scanlines**)

Cost Volume





Cost Volume

```
for(int r = window_height_/2 + 1; r < height_ - window_height_/2 - 1; r++)
{
  for(int c = window_width_/2 + 1; c < width_ - window_width_/2 - 1; c++)
  {
    for(int d=0; d<disparity_range_; d++)
    {
      long cost = 0;
      // do stuff
      cost_[r][c][d]=cost;
    }
  }
}
```

// do stuff:
compute E_data, using one of the methods seen in class (SSD, SAD, ZNCC, CENSUS)

Global Methods

- Enforce smoothness constraints, i.e. disparity is **piecewise smooth**
- Enforce ordering constraints
- Greater computational complexity

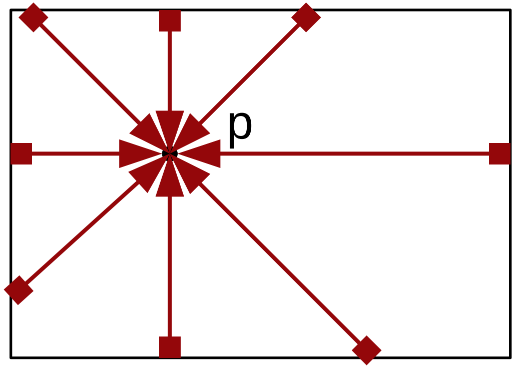
Semi-Global Matching

- Approximate global methods by aggregating costs for a number of directions (from 2D to multiple 1D areas of interest)
- Minimization along individual image rows can be performed efficiently in polynomial time using Dynamic Programming

H Hirschmuller "Stereo processing by semiglobal matching and mutual information",
Pattern Analysis and Machine Intelligence, IEEE Transactions on 30 (2), 328-341

Semi-Global Matching

- For each direction, start from one end point and go toward **p**
- For each pixel along the direction, update the following dynamic programming equation (the result at step i depends on the result at step i-1):



$$E(p_i, d) = E_{data}(p_i, d) + E_{smooth}(p_i, p_{i-1}) - \min_{0 \leq \Delta \leq d_{\max}} E(p_{i-1}, \Delta)$$

Restrict the range of resulting values, without affecting the minimization procedure

where:

$$E_{smooth}(p, q) = \min \begin{cases} E(q, f_q) & \text{if } f_p = f_q \\ E(q, f_q) + c_1 & \text{if } |f_p - f_q| = 1 \\ \min_{0 \leq \Delta \leq d_{\max}} E(q, \Delta) + c_2(p, q) & \text{if } |f_p - f_q| > 1 \end{cases}$$

Exercise: Semi-Global Matching

Consider the stereo matching problem for the following 7 x 1 left and right images:

2	3	1	2	3	3	1

1	2	3	1	4	0	2

1) Compute the right to left **cost volume**, considering positive disparities, $d >= 0$ with $d_{max} = 3$ and data cost defined by the sum of absolute differences (SAD) computed in 1 x 1 windows. Use value -1 to set "no disparity assigned"
2) Given the matching cost computed in 1), consider the Semi-Global Matching method with the following simplified dynamic programming equations:

$$E(p_i, d) = E_{data}(p_i, d) + E_{smooth}(p_i, p_{i-1})$$

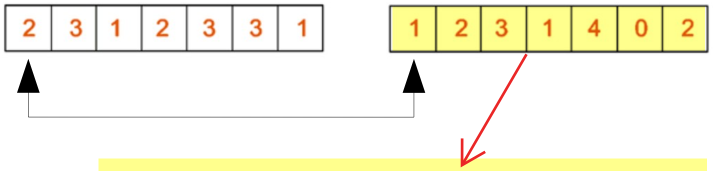
$$E_{smooth}(p, q) = \min \begin{cases} E(q, f_q) & \text{if } f_p = f_q \\ E(q, f_q) + c_1 & \text{if } |f_p - f_q| = 1 \\ \min_{0 \leq \Delta \leq d_{\max}} E(q, \Delta) + c_2 & \text{if } |f_p - f_q| > 1 \end{cases}$$

where $c_1 = 1$ and $c_2 = 2$. Compute the integration matrix for the highlighted pixel and scanline:

2	3	1	2	3	3	1

1	2	3	$\boxed{1}$	4	0	2

Exercise Solution



COST VOLUME

disparity

0	1	1	2	1	1	3	1
1	2	1	1	2	1	1	-1
2	0	0	0	2	3	-1	-1
3	1	1	0	0	-1	-1	-1

using SAD right to left with a 1x1 window, means taking the module of the difference between the pixel on the right and the disparity-adjusted pixel on the left

notice that the disparity-adjusted pixel may go outside the left image --> -1

Exercise Solution

2	3	1	2	3	3	1

1	2	3	1	4	0	2

$$E(p_i, d) = E_{data}(p_i, d) + E_{smooth}(p_i, p_{i-1})$$

$$E_{smooth}(p, q) = \min \begin{cases} \boxed{E(q, f_q)} & \text{if } f_p = f_q \\ \boxed{E(q, f_q) + c_1} & \text{if } |f_p - f_q| = 1 \\ \boxed{\min_{0 \leq \Delta \leq d_{\max}} E(q, \Delta) + c_2} & \text{if } |f_p - f_q| > 1 \end{cases}$$

0				
1				
2				
3				

Exercise Solution

2	3	1	2	3	3	1

1	2	3	1	4	0	2

$$E(p_i, d) = E_{data}(p_i, d) + E_{smooth}(p_i, p_{i-1})$$

$$E_{smooth}(p, q) = \min \begin{cases} \boxed{E(q, f_q)} & \text{if } f_p = f_q \\ \boxed{E(q, f_q) + c_1} & \text{if } |f_p - f_q| = 1 \\ \boxed{\min_{0 \leq \Delta \leq d_{\max}} E(q, \Delta) + c_2} & \text{if } |f_p - f_q| > 1 \end{cases}$$

0	1			
1	2			
2	0			
3	1			

Exercise Solution

2	3	1	2	3	3	1

q

1	2	3	$\boxed{1}$	4	0	2

p

$$E(p_i, d) = E_{data}(p_i, d) + E_{smooth}(p_i, p_{i-1})$$

$$E_{smooth}(p, q) = \min \begin{cases} \boxed{E(q, f_q)} & \text{if } f_p = f_q \\ \boxed{E(q, f_q) + c_1} & \text{if } |f_p - f_q| = 1 \\ \boxed{\min_{0 \leq \Delta \leq d_{\max}} E(q, \Delta) + c_2} & \text{if } |f_p - f_q| > 1 \end{cases}$$

0	1			
1	2			
2	0			
3	1			

1 + min(1, 2+1, 0+2)=2

1 + min(2, 1+1, 0+1, 0+2)=2

0 + min(0, 2+1, 1+1, 0+2)=0

1 + min(1, 0+1, 0+2)=2

Exercise Solution

2	3	1	2	3	3	1

1	2	3	1	4	0	2

$$E(p_i, d) = E_{data}(p_i, d) + E_{smooth}(p_i, p_{i-1})$$

$$E_{smooth}(p,q) = \min \begin{cases} \boxed{E(q, f_q)} & \text{if } f_p = f_q \\ \boxed{E(q, f_q) + c_1} & \text{if } |f_p - f_q| = 1 \\ \boxed{\min_{0 \le \Delta \le d_{\max}} E(q, \Delta) + c_2} & \text{if } |f_p - f_q| > 1 \end{cases}$$

0	1			
1	2			
2	0			
3	1			

1 + min(1, 2+1, 0+2)=2

1 + min(2, 1+1, 0+1, 0+2)=2

0 + min(0, 2+1, 1+1, 0+2)=0

1 + min(1, 0+1, 0+2)=2

Exercise Solution

2	3	1	2	3	3	1

1	2	3	$\boxed{1}$	4	0	2

$$E(p_i, d) = E_{data}(p_i, d) + E_{smooth}(p_i, p_{i-1})$$

$$E_{smooth}(p,q) = \min \begin{cases} \boxed{E(q, f_q)} & \text{if } f_p = f_q \\ \boxed{E(q, f_q) + c_1} & \text{if } |f_p - f_q| = 1 \\ \boxed{\min_{0 \le \Delta \le d_{\max}} E(q, \Delta) + c_2} & \text{if } |f_p - f_q| > 1 \end{cases}$$

0	1	2		
1	2	2		
2	0	0		
3	1	2		

2 + min(2, 2+1, 0+2)=4

1 + min(2, 2+1, 0+1, 0+2)=2

0 + min(0, 2+1, 2+1, 0+2)=0

0 + min(2, 0+1, 0+2)=1

Exercise Solution

2	3	1	2	3	3	1

1	2	3	1	4	0	2

$$E(p_i, d) = E_{data}(p_i, d) + E_{smooth}(p_i, p_{i-1})$$

$$E_{smooth}(p,q) = \min \begin{cases} \boxed{E(q, f_q)} & \text{if } f_p = f_q \\ \boxed{E(q, f_q) + c_1} & \text{if } |f_p - f_q| = 1 \\ \boxed{\min_{0 \le \Delta \le d_{\max}} E(q, \Delta) + c_2} & \text{if } |f_p - f_q| > 1 \end{cases}$$

0	1	2	4	
1	2	2	2	
2	0	0	0	
3	1	2	1	

1 + min(4, 2+1, 0+2)=3

2 + min(2, 4+1, 0+1, 0+2)=3

2 + min(0, 2+1, 1+1, 0+2)=2

0 + min(1, 0+1, 0+2)=1

Exercise Solution

2	3	1	2	3	3	1

1	2	3	$\boxed{1}$	4	0	2

$$E(p_i, d) = E_{data}(p_i, d) + E_{smooth}(p_i, p_{i-1})$$

$$E_{smooth}(p, q) = \min \begin{cases} \boxed{E(q, f_q)} & \text{if } f_p = f_q \\ \boxed{E(q, f_q) + c_1} & \text{if } |f_p - f_q| = 1 \\ \boxed{\min_{0 \le \Delta \le d_{\max}} E(q, \Delta) + c_2} & \text{if } |f_p - f_q| > 1 \end{cases}$$

0	1	2	4	3
1	2	2	2	3
2	0	0	0	2
3	1	2	1	1