Kinematic considerations for a Toy Generative Model for Jets

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In this notes, we study the kinematics requirements and implications for a Toy Generative Model of Jets. There is a companion note describing the model. Let start by considering a a 2-body decay. In the parent rest frame, we have the parent momentum $p_p^\mu = p_{\rm L}^\mu + p_{\rm R}^\mu = (\sqrt{s}, 0, 0, 0)$. From requiring 4-momentum conservation, the children energies are given by

$$\begin{aligned} E_{\rm L} &= \frac{\sqrt{s}}{2}\left(1 + \frac{m_{\rm L}^2}{s} - \frac{m_{\rm R}^2}{s}\right) \\ E_{\rm R} &= \frac{\sqrt{s}}{2}\left(1 + \frac{m_{\rm R}^2}{s} - \frac{m_{\rm L}^2}{s}\right) \end{aligned} \tag{1}$$

and the magnitude of their 3-momentum by

$$|\vec{p}| = \frac{\sqrt{s}}{2}\bar{\beta} = \frac{\sqrt{s}}{2}\sqrt{1 - \frac{2(m_{\rm L}^2 + m_{\rm R}^2)}{s} + \frac{(m_{\rm L}^2 - m_{\rm R}^2)^2}{s^2}} \tag{2}$$

0.1 Simplest case: $m_{\mathbf{L}} = m_{\mathbf{R}} = m$

In this section we will specialize to the case where $m_{\rm L} = m_{\rm R} = m$ (we will comment on the general case in the next subsection).

$$\begin{aligned} E_{\rm L} = E_{\rm R} &= \frac{\sqrt{s}}{2} = E \\ |\vec{p}| &= E\sqrt{1 - \frac{m^2}{E^2}} \end{aligned} \tag{3}$$

If we apply a boost to the lab frame, with factor $\beta\hat{n}$, we obtain for the children

3-momentum

$$
\begin{aligned}
\vec{p}^{\,l}_{\mathrm{L}} &= -\gamma E\beta\hat{n} \,-\, E\sqrt{1-\frac{m^2}{E^2}}\;\left[\hat{r}+(\gamma-1)(\hat{r}\cdot\hat{n})\hat{n}\right] \\
\vec{p}^{\,l}_{\mathrm{R}} &= -\gamma E\beta\hat{n} \,+\, E\sqrt{1-\frac{m^2}{E^2}}\;\left[\hat{r}+(\gamma-1)(\hat{r}\cdot\hat{n})\hat{n}\right]
\end{aligned}
\tag{4}
$$

with $\hat{r}$ a unit vector that specifies the direction of the children momentum in the parent rest frame. For the parent we have

$$
\vec{p}^{\,l}_{p} = -2\gamma E\beta\hat{n} \tag{5}
$$

From (5), we can rewrite (4) as:

$$
\begin{aligned}
\vec{p}^{\,l}_{\mathrm{L}} &= \frac{1}{2}\vec{p}^{\,l}_{p} - \vec{\Delta} \\
\vec{p}^{\,l}_{\mathrm{R}} &= \frac{1}{2}\vec{p}^{\,l}_{p} + \vec{\Delta}
\end{aligned}
\tag{6}
$$

with

$$
\vec{\Delta} = E\sqrt{1-\frac{m^2}{E^2}}\;\left[\hat{r}+(\gamma-1)(\hat{r}\cdot\hat{n})\hat{n}\right] \tag{7}
$$

Let's study the dependence of $|\vec{\Delta}| = \Delta(E, m, \hat{r}, \gamma, \hat{n})$. We will work in the (y,z) plane.

- $\gamma = \frac{E_p}{m_p}$ or $\gamma\beta = |\vec{p}_p|/m_p$
- If we draw an angle $\phi \in \{-\pi, \pi\}$ from a uniform distribution, we get in the $(\hat{y}, \hat{z})$ plane, $\hat{r} = (\sin\phi, \cos\phi)$.
- At each step, we draw ϕ and m, and boost the children to the lab frame. Next, we promote each child to a parent and repeat the process. Thus, the unit vector $\hat{n}$ is fixed by the previous step and given by $\hat{n} = (\sin\theta_{\mathrm{p}}, \cos\theta_{\mathrm{p}})$ with $\theta_{\mathrm{p}} = \tan^{-1}\left(\frac{(p_{\mathrm{p}})_y}{(p_{\mathrm{p}})_z}\right)$ the parent angle with respect to the $\hat{z}$ axis in the lab frame.
- At each step, $E_{\mathrm{L}} = E_{\mathrm{R}} = E = \frac{\sqrt{s}}{2} = \frac{m_{\mathrm{p}}}{2}$ is fixed by the parent mass.

As a result, we obtain $|\Delta| = \Delta(m, \phi)$ given by

$$
\begin{aligned}
\Delta_y &= \frac{m_{\mathrm{p}}}{2}\sqrt{1-4\frac{m^2}{m_{\mathrm{p}}^2}}\left[\sin\phi + \left(\frac{m_{\mathrm{p}}}{2m}-1\right)\cos(\phi-\theta_{\mathrm{p}})\sin\theta_{\mathrm{p}}\right] \\
\Delta_z &= \frac{m_{\mathrm{p}}}{2}\sqrt{1-4\frac{m^2}{m_{\mathrm{p}}^2}}\left[\cos\phi + \left(\frac{m_{\mathrm{p}}}{2m}-1\right)\cos(\phi-\theta_{\mathrm{p}})\cos\theta_{\mathrm{p}}\right]
\end{aligned}
\tag{8}
$$

0.2 General case, with $m_{\mathbf{L}} \neq m_{\mathbf{R}}$

In this case, (6) becomes

$$\begin{aligned} \vec{p}^{\,l}_{\mathrm{L}} &= \frac{E_{\mathrm{L}}}{E_{\mathrm{p}}}\vec{p}^{\,l}_{\mathrm{p}} - |\vec{p}| \;\; [\hat{r} + (\gamma - 1)(\hat{r}\cdot\hat{n})\hat{n}] \\ \vec{p}^{\,l}_{\mathrm{R}} &= \frac{E_{\mathrm{R}}}{E_{\mathrm{p}}}\vec{p}^{\,l}_{\mathrm{p}} + |\vec{p}| \;\; [\hat{r} + (\gamma - 1)(\hat{r}\cdot\hat{n})\hat{n}] \end{aligned} \tag{9}$$

where $|\vec{p}|$ is as in (2):

$$|\vec{p}| = \frac{\sqrt{s}}{2}\bar{\beta} = \frac{\sqrt{s}}{2}\sqrt{1 - \frac{2(m_{\mathrm{L}}^2 + m_{\mathrm{R}}^2)}{s} + \frac{(m_{\mathrm{L}}^2 - m_{\mathrm{R}}^2)^2}{s^2}}$$