AABW - Assignment 2a

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3rd of October, 2024

1 Introduction

1.1 Variables

- $p_f^{(i)} \in \mathbb{R}$ profit for floor f apartment i.
- $a_{fs}^{(i)} \in \mathbb{R}$ area of apartment i on floor f which has sector s (sector free can be ignored to reduce variables).
- $x_{fb} \in \mathbb{B}$ is 1 if floor f has design b.
- $y_{fs}^{(i)} \in \mathbb{B}$ is 1 if floor f apartment i has sector s assigned.
- $t_{fo} \in \mathbb{B}$ is 1 if floor f has owner o.
- $v_{fb} \in \mathbb{B}$ is 1 if floor f has design b and is assigned to owner investor.
- (optional) $z_f \in \mathbb{B}$ is 1 for floor f if at least one apartment is assigned to the social sector.

1.2 Sets

- $\mathcal{B}$ set of all building designs {aa, ab, . . . , ee}
- $\mathcal{F}$ set of all floors {1,. . . ,$f_{\max}$}
- $\mathcal{S}$ set of all sectors {social, middle, free}
- $\mathcal{O}$ set of all owners {corporation, investor, private}

1.3 Parameters

- α_{b_i} area of apartment i with design b
- $\alpha_b^{\min}$ area of apartment with minimum area of design b
- $\rho_{so}^{b_i}$ profit of apartment i in design b when assigned to sector s with owner o (w.l.o.g assuming profit is positive)
- I^{b_i} is 1 if design b has at least i number of apartments
- $I^{b_i}_{\text{corporation,social}}$ is 1 if design b apartment i has area $\geq 40m^2$ (the minimal apartment size for social sector and corporation owner assignments)

- $I^{b_i}_{\text{corporation,middle}}$ is 1 if design b apartment i has area $\geq 50m^2$
- $I^{b_i}_{s,\text{free}}$ is 1 if design b apartment i has area $\geq 60m^2$
- $f_{\max}$ the max number of floors
- $i_{\max}$ the maximum number of apartments in any design in $\mathcal{B}$
- M a sufficiently large constant
- $fraction\_investor$ the least fraction of apartments that should be owned by investors (in this case 0.7).
- avg_area_social the minimum average area size of an apartment of the "social" sector (in this case, $40m^2$)
- avg_area_middle the minimum average area size of an apartment of the "middle" sector (in this case, $50m^2$)
- $fraction\_social$ the minimum fraction of apartments that should be in the social sector (in this case, 0.4)
- $fraction\_middle$ the minimum fraction of apartments that should be in the middle sector (in this case, 0.4)

1.4 Model

Unless anything else is written the $\forall$ should be for all in the set with the associated capital letter. That is $\forall f \Leftrightarrow \forall f \in \mathcal{F}$ etc.

$$\max \sum_{\forall f} \sum_{i=1}^{i_{\max}} p_f^{(i)} \tag{1}$$

$$\sum_{\forall b} x_{fb} = 1 \qquad \forall f \tag{2}$$

$$\sum_{\forall s} y_{fs}^{(i)} = \sum_{\forall b} I^{b_i} x_{fb} \qquad \forall f, i = 1, \ldots, i_{\max} \tag{3}$$

$$\sum_{\forall o} t_{fo} = 1 \qquad \forall f \tag{4}$$

$$p_f^{(i)} \leq \sum_{\forall b} \rho_{bso}^{(i)} I^{b_i} x_{fb} + (1 - y_{fs}^{(i)})M + (1 - t_{fo})M \qquad \forall f, \forall s, \forall o, i = 1, \ldots, i_{\max} \tag{5}$$

$$p_f^{(i)} \leq \sum_{\forall b} I^{b_i} x_{fb} M \qquad \forall f, i = 1, \ldots, i_{\max} \tag{6}$$

$$a_{fs}^{(i)} \leq \alpha_{b_i} + (1 - x_{fb})M + (1 - y_{fs}^{(i)})M \qquad \forall f, \forall b, \forall s, i = 1, \ldots, i_{\max} \tag{7}$$

$$a_{fs}^{(i)} \leq y_{fs}^{(i)} M \qquad \forall f, \forall s, i = 1, \ldots, i_{\max} \tag{8}$$

$$v_{fb} \leq x_{fb} \qquad \forall f, \forall b \tag{9}$$

$$v_{fb} \leq t_{f,\text{investor}} \qquad \forall f, \forall b \tag{10}$$

$$v_{fb} \geq x_{fb} + y_{f,\text{investor}} - 1 \qquad \forall f, \forall b \tag{11}$$

$$\sum_{\forall f} \sum_{i=1}^{i_{\max}} y_{f,\text{social}}^{(i)} \geq \sum_{\forall f} \sum_{\forall s} \sum_{i=1}^{i_{\max}} y_{fs}^{(i)} fraction\_social \tag{12}$$

$$\sum_{\forall f} \sum_{i=1}^{i_{\max}} y_{f,\text{middle}}^{(i)} \geq \sum_{\forall f} \sum_{\forall s} \sum_{i=1}^{i_{\max}} y_{fs}^{(i)} fraction\_middle \tag{13}$$

$$\sum_{\forall f} \sum_{i=1}^{i_{\max}} a_{f,\text{social}}^{(i)} \alpha_{b_i} \geq \sum_{\forall f} \sum_{i=1}^{i_{\max}} y_{f,\text{social}}^{(i)} avg\_area\_social \tag{14}$$

$$\sum_{\forall f} \sum_{i=1}^{i_{\max}} a_{f,\text{middle}}^{(i)} \alpha_{b_i} \geq \sum_{\forall f} \sum_{i=1}^{i_{\max}} y_{f,\text{middle}}^{(i)} avg\_area\_middle \tag{15}$$

$$\tag{16}$$

$$y^{(i)}_{f,\text{social}} \leq \sum_{\forall b} I^{b_i}_{\text{corporation, social}} x_{fb} + (1 - t_{f,\text{corporation}})$$
$$\forall f, i = 1, \ldots, i_{\max} \tag{17}$$
$$y^{(i)}_{f,\text{middle}} \leq \sum_{\forall b} I^{b_i}_{\text{corporation, middle}} x_{fb} + (1 - t_{f,\text{corporation}})$$
$$\forall f, i = 1, \ldots, i_{\max} \tag{18}$$
$$y^{(i)}_{f,\text{free}} \leq \sum_{\forall b} I^{b_i}_{s,\text{free}} x_{fb}$$
$$\forall f, \forall s, i = 1, \ldots, i_{\max} \tag{19}$$
$$\sum_{i=1}^{i_{\max}} y^{(i)}_{f,\text{free}} \leq (1 - t_{f,\text{corporation}})M \qquad \forall f \tag{20}$$
$$\sum_{\forall f} \sum_{\forall b} \sum_{i=1}^{i_{\max}} v_{fb} I^{b_i} \geq \sum_{\forall f} \sum_{\forall s} \sum_{i=1}^{i_{\max}} y^{(i)}_{fs} fraction\_investor \tag{21}$$
$$\sum_{\forall b} \alpha^{\min}_b x_{fb} \leq \sum_{\forall b} \alpha^{\min}_b x_{f+1,b} \qquad \forall f \in \mathcal{F} \setminus \{f_{\max}\} \tag{22}$$
$$p^{(i)}_f, a^{(i)}_{fs} \geq 0 \qquad \forall f, \forall s, i = 1, \ldots, i_{\max} \tag{23}$$

1.4.1 Optional Restriction 11

$$z_f \geq \frac{1}{f_{\max}} \sum_{i=1}^{i_{\max}} y^{(i)}_{f,\text{social}} \qquad \forall f \tag{24}$$
$$\sum_{i=1}^{i_{\max}} y^{(i)}_{f,\text{free}} \leq (1 - z_f)M \qquad \forall f \tag{25}$$

1.5 Restriction Equations

The restriction all have one (or more) associated equations in the model, they are here presented.

1. Equation (12) ensures the number of apartments assigned to the social sector is at least 40% ($fraction\_social$) of all apartments.
2. Equation (13) ensures the number of apartments assigned to the middle sector is at least 40% ($fraction\_middle$) of all apartments.
3. Equation (14) ensures the average area of all apartments in the social sector is at least $40m^2$ (avg_area_social), the equation (7) and (8) are ensuring that we only get the apartment size that is actually assigned to sector s (the area of $a^{(i)}_{fs}$ maybe less than the actual area, however this constraint can always be satisfied, since it is upper bounded by the area).
4. Equation (15) ensures the average area of all apartments in the middle sector is at least $50m^2$ (avg_area_middle), the equation (7) and (8) are ensuring that we only get the apartment size that is actually assigned to sector s.

5. Equation (17) ensures that the minimal area if we assign an apartment to the social sector with owner corporation is at least $40m^2$ ($I^{b_i}_{\text{corporation,social}}$).

6. Equation (18) ensures that the minimal area if we assign an apartment to the middle sector with owner corporation is at least $50m^2$ ($I^{b_i}_{\text{corporation,middle}}$).

7. Equation (19) ensures that the minimal area if we assign an apartment to a free sector is at least $60m^2$ ($I^{b_i}_{s,\text{free}}$).

8. Equation (20) ensures that corporation cannot buy apartments that are assigned to the free sector

9. Equation (21) ensures that at least 70% ($fraction\_investor$) of the apartments are assigned to investors, with the help of equations (9), (10), and (11), which helps assign v_{fb} to be 1 only when an investor is chosen for that floor and design b is choosen.

10. This restriction is automatically ensured through the definition of t_{fo} and equation (4)

11. (optional) Equation (25) ensures that if an apartment is assigned to the social sector on that floor, then no apartments on that floor can be assigned to the free sector.

12. Equation (22) ensures that the floors has increasing (or the same) size of the area of the smalles apartment on that floor.