EDAN20
Language Technology
http://cs.lth.se/edan20/
Chapter 5: Counting Words

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Text Segmentation

Figure: Latin inscriptions on the *lapis niger*. *Corpus inscriptionum latinarum*, CIL I, 1. Picture from Wikipedia

Getting the Words from a Text: Tokenization

Arrange a list of characters:

`[l, i, s, t, ' ', o, f, ' ', c, h, a, r, a, c, t, e, r, s]`

into words:

`[list, of, characters]`

Sometimes tricky:

- Dates: 28/02/96
- Numbers: 9,812.345 (English), 9 812,345 (French and German) 9.812,345 (Old fashioned French)
- Abbreviations: km/h, m.p.h.,
- Acronyms: S.N.C.F.

Tokenizers use rules (or regexes) or statistical methods.

Tokenizing in Python: Using the Boundaries

Simple program

```
import re

one_token_per_line = re.sub('\s+', '\n', text)
```

Punctuation

```
import regex as re

spaced_tokens = re.sub('([\p{S}\p{P}])', r' \1 ', text)
one_token_per_line = re.sub('\s+', '\n', spaced_tokens)
```

Tokenizing in Python: Using the Content

Simple program

```
import regex as re

re.findall(’\p{L}+’, text)
```

Punctuation

```
spaced_tokens = re.sub(’([\p{S}\p{P}])’, r’ \1 ’, text)
re.findall(’[\p{S}\p{P}\p{L}]+’, spaced_tokens)
```

Improving Tokenization

The tokenization algorithm is word-based and defines a content

It does not work on nomenclatures such as Item #N23-SW32A, dates, or numbers

Instead it is possible to improve it using a boundary-based strategy with spaces (using for instance `\s`) and punctuation

But punctuation signs like commas, dots, or dashes can also be parts of tokens

Possible improvements using microgrammars

At some point, need of a dictionary:

Can't → can n't, *we'll* → we 'll

J'aime → j' aime but *aujourd'hui*

Sentence Segmentation

As for tokenization, segmenters use either rules (or regexes) or statistical methods.

Grefenstette and Tapanainen (1994) used the Brown corpus and experimented increasingly complex rules

Most simple rule: a period corresponds to a sentence boundary: 93.20% correctly segmented

Recognizing numbers:

<table>
<tr><td><code>[0-9]+(\/[0-9]+)+</code></td><td>Fractions, dates</td></tr>
<tr><td><code>([+\-])?[0-9]+(\.)?[0-9]*%</code></td><td>Percent</td></tr>
<tr><td><code>([0-9]+,?)+(\.[0-9]+|[0-9]+)*</code></td><td>Decimal numbers</td></tr>
</table>

93.78% correctly segmented

Abbreviations

Common patterns (Grefenstette and Tapanainen 1994):

- single capitals: *A.*, *B.*, *C.*,
- letters and periods: *U.S. i.e. m.p.h.*,
- capital letter followed by a sequence of consonants: *Mr. St. Assn.*

Regex	**Correct**	**Errors**	**Full stop**
`[A-Za-z]\.`	1,327	52	14
`[A-Za-z]\.([A-Za-z0-9]\.)+`	570	0	66
`[A-Z][bcdfghj-np-tvxz]+\.`	1,938	44	26
Totals	3,835	96	106

Correct segmentation increases to 97.66%
With an abbreviation dictionary to 99.07%

Counting Words With Unix Tools

1. `tr -cs 'A-Za-z' '\n' <input_file |`
   Tokenize the text in `input_file`, where `tr` behaves like Perl `tr`: We have one word per line and the output is passed to the next command.
2. `tr 'A-Z' 'a-z' |`
   Translate the uppercase characters into lowercase letters and pass the output to the next command.
3. `sort |`
   Sort the words. The identical words will be grouped in adjacent lines.
4. `uniq -c |`
   Remove repeated lines. The identical adjacent lines will be replaced with one single line. Each unique line in the output will be preceded by the count of its duplicates in the input file (`-c`).
5. `sort -rn |`
   Sort in the reverse (`-r`) numeric (`-n`) order: Most frequent words first.
6. `head -5`
   Print the five first lines of the file (the five most frequent words).

Counting Words in Python

```
def tokenize(text):
    words = re.findall('\p{L}+', text)
    return words

def count_unigrams(words):
    frequency = {}
    for word in words:
        if word in frequency:
            frequency[word] += 1
        else:
            frequency[word] = 1
    return frequency
```

Counting Words in Python (Cont'd)

```
if __name__ == ’__main__’:
    text = sys.stdin.read().lower()
    words = tokenize(text)
    frequency = count_unigrams(words)
    for word in sorted(frequency.keys()):
        print(word, ’\t’, frequency[word])
```

Posting Lists

Many websites, such as Wikipedia, index their texts using an inverted index. Each word in the dictionary is linked to a posting list that gives all the documents where this word occurs and its positions in a document.

Collection

D1: Chrysler plans new investments in Latin America.

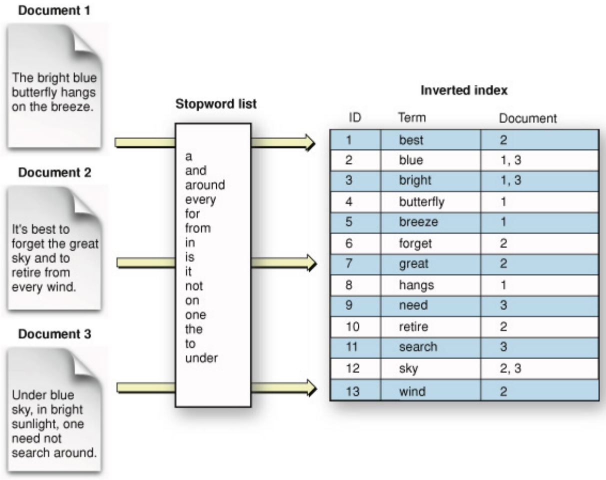
D2: Chrysler plans major investments in Mexico.

Index

Words	Posting lists
America	(D1, 7)
Chrysler	(D1, 1) → (D2, 1)
in	(D1, 5) → (D2, 5)
investments	(D1, 4) → (D2, 4)
Latin	(D1, 6)
major	(D2, 3)
Mexico	(D2, 6)
new	(D1, 3)
plans	(D1, 2) → (D2, 2)

Lucene is a high quality open-source indexer. (`http://lucene.apache.org/`)

Inverted Index (Source Apple)



ID	Term	Document
1	best	2
2	blue	1, 3
3	bright	1, 3
4	butterfly	1
5	breeze	1
6	forget	2
7	great	2
8	hangs	1
9	need	3
10	retire	2
11	search	3
12	sky	2, 3
13	wind	2

http://developer.apple.com/library/mac/documentation/UserExperience/Conceptual/SearchKitConcepts/index.html

Information Retrieval: The Vector Space Model

The vector space model represents a document in a space of words.

Documents \Words	w_1	w_2	w_3	...	w_m
D_1	$C(w_1, D_1)$	$C(w_2, D_1)$	$C(w_3, D_1)$	...	$C(w_m, D_1)$
D_2	$C(w_1, D_2)$	$C(w_2, D_2)$	$C(w_3, D_2)$	...	$C(w_m, D_2)$
...					
D_n	$C(w_1, D_n)$	$C(w_2, D_n)$	$C(w_3, D_n)$	...	$C(w_m, D_n)$

It was created for information retrieval to compute the similarity of two documents or to match a document and a query.
We compute the similarity of two documents through their dot product.

The Vector Space Model: Example

A collection of two documents D1 and D2:

D1: Chrysler plans new investments in Latin America.

D2: Chrysler plans major investments in Mexico.

The vectors representing the two documents:

D.	america	chrysler	in	investments	latin	major	mexico	new	plans
1	1	1	1	1	1	0	0	1	1
2	0	1	1	1	0	1	1	0	1

The vector space model represents documents as bags of words (BOW) that do not take the word order into account.

The dot product is $\vec{D}1 \cdot \vec{D}2 = 0+1+1+1+0+0+0+0+1 = 4$

Their cosine is $\frac{\vec{D}1 \cdot \vec{D}2}{||\vec{D}1||.||\vec{D}2||} = \frac{4}{\sqrt{7}.\sqrt{6}} = 0.62$

Giving a Weight

Word clouds give visual weights to words

Image: Courtesy of Jonas Wisbrant

$TF \times IDF$

The frequency alone might be misleading

Document coordinates are in fact $tf \times idf$: Term frequency by inverted document frequency.

Term frequency $tf_{i,j}$: frequency of term j in document i

Inverted document frequency: $idf_j = \log(\frac{N}{n_j})$

Document Similarity

Documents are vectors where coordinates could be the count of each word:
$\vec{d} = (C(w_1), C(w_2), C(w_3), ..., C(w_n))$
The similarity between two documents or a query and a document is given by their cosine:

$$\cos(\vec{q}, \vec{d}) = \frac{\sum\limits_{i=1}^{n} q_i d_i}{\sqrt{\sum\limits_{i=1}^{n} q_i^2}\sqrt{\sum\limits_{i=1}^{n} d_i^2}}.$$

Word Sequences

Words have specific contexts of use.

Pairs of words like *strong* and *tea* or *powerful* and *computer* are not random associations.

Psychological linguistics tells us that it is difficult to make a difference between *writer* and *rider* without context

A listener will discard the improbable *rider of books* and prefer *writer of books*

A language model is the statistical estimate of a word sequence.

Originally developed for speech recognition

The language model component enables to predict the next word given a sequence of previous words: *the writer of books, novels, poetry*, etc. and not *the writer of hooks, nobles, poultry*, ...

N-Grams

The types are the distinct words of a text while the tokens are all the words or symbols.

The phrases from *Nineteen Eighty-Four*

> *War is peace*
> *Freedom is slavery*
> *Ignorance is strength*

have 9 tokens and 7 types.

Unigrams are single words

Bigrams are sequences of two words

Trigrams are sequences of three words

Trigrams

Word	Rank	More likely alternatives
We	9	*The This One Two A Three Please In*
need	7	*are will the would also do*
to	1	
resolve	85	*have know do...*
all	9	*the this these problems...*
of	2	*the*
the	1	
important	657	*document question first...*
issues	14	*thing point to...*
within	74	*to of and in that...*
the	1	
next	2	*company*
two	5	*page exhibit meeting day*
days	5	*weeks years pages months*

Counting Bigrams With Unix Tools

1. `tr -cs 'A-Za-z' '\n' < input_file > token_file`
   Tokenize the input and create a file with the unigrams.
2. `tail +2 < token_file > next_token_file`
   Create a second unigram file starting at the second word of the first tokenized file (+2).
3. `paste token_file next_token_file > bigrams`
   Merge the lines (the tokens) pairwise. Each line of `bigrams` contains the words at index i and $i+1$ separated with a tabulation.
4. And we count the bigrams as in the previous script.

Counting Bigrams in Python

```
bigrams = [tuple(words[inx:inx + 2])
           for inx in range(len(words) - 1)]
```

The rest of the count_bigrams function is nearly identical to count_unigrams. As input, it uses the same list of words:

```
def count_bigrams(words):
    bigrams = [tuple(words[inx:inx + 2])
               for inx in range(len(words) - 1)]
    frequencies = {}
    for bigram in bigrams:
        if bigram in frequencies:
            frequencies[bigram] += 1
        else:
            frequencies[bigram] = 1
    return frequencies
```

Probabilistic Models of a Word Sequence

$$
\begin{aligned}
P(S) &= P(w_1, ..., w_n),\\
&= P(w_1)P(w_2|w_1)P(w_3|w_1,w_2)...P(w_n|w_1,...,w_{n-1}),\\
&= \prod_{i=1}^{n} P(w_i|w_1,...,w_{i-1}).
\end{aligned}
$$

The probability $P(\mathit{It\ was\ a\ bright\ cold\ day\ in\ April})$ from *Nineteen Eighty-Four* corresponds to
It to begin the sentence, then *was* knowing that we have *It* before, then *a* knowing that we have *It was* before, and so on until the end of the sentence.

$$
\begin{aligned}
P(S) = {} & P(It) \times P(was|It) \times P(a|It, was) \times P(bright|It, was, a) \times ...\\
& \times P(April|It, was, a, bright, ..., in).
\end{aligned}
$$

Approximations

Bigrams:

$$P(w_i|w_1, w_2, ..., w_{i-1}) \approx P(w_i|w_{i-1}),$$

Trigrams:

$$P(w_i|w_1, w_2, ..., w_{i-1}) \approx P(w_i|w_{i-2}, w_{i-1}).$$

Using a trigram language model, $P(S)$ is approximated as:

$$\begin{aligned} P(S) \quad \approx \quad & P(It) \times P(was|It) \times P(a|It, was) \times P(bright|was, a) \times \ldots \\ & \times P(April|day, in). \end{aligned}$$

Maximum Likelihood Estimate

Bigrams:

$$P_{MLE}(w_i|w_{i-1}) = \frac{C(w_{i-1}, w_i)}{\sum\limits_{w} C(w_{i-1}, w)} = \frac{C(w_{i-1}, w_i)}{C(w_{i-1})}.$$

Trigrams:

$$P_{MLE}(w_i|w_{i-2}, w_{i-1}) = \frac{C(w_{i-2}, w_{i-1}, w_i)}{C(w_{i-2}, w_{i-1})}.$$

Conditional Probabilities

A common mistake in computing the conditional probability $P(w_i|w_{i-1})$ is to use

$$\frac{C(w_{i-1}, w_i)}{\#bigrams}.$$

This is not correct. This formula corresponds to $P(w_{i-1}, w_i)$.
The correct estimation is

$$P_{MLE}(w_i|w_{i-1}) = \frac{C(w_{i-1}, w_i)}{\sum_w C(w_{i-1}, w)} = \frac{C(w_{i-1}, w_i)}{C(w_{i-1})}.$$

Proof:

$$P(w_1, w_2) = P(w_1)P(w_2|w_1) = \frac{C(w_1)}{\#words} \times \frac{C(w_1, w_2)}{C(w_1)} = \frac{C(w_1, w_2)}{\#words}$$

Training the Model

The model is trained on a part of the corpus: the **training set**

It is tested on a different part: the **test set**

The vocabulary can be derived from the corpus, for instance the 20,000 most frequent words, or from a lexicon

It can be closed or open

A closed vocabulary does not accept any new word

An open vocabulary maps the new words, either in the training or test sets, to a specific symbol, `<UNK>`

Probability of a Sentence: Unigrams

<s> *A good deal of the literature of the past was, indeed, already being transformed in this way* </s>

w_i	$C(w_i)$	#words	$P_{MLE}(w_i)$
<s>	7072	–	
a	2482	115212	0.023
good	53	115212	0.00049
deal	5	115212	$4.62\ 10^{-5}$
of	3310	115212	0.031
the	6248	115212	0.058
literature	7	115212	$6.47\ 10^{-5}$
of	3310	115212	0.031
the	6248	115212	0.058
past	99	115212	0.00092
was	2211	115212	0.020
indeed	17	115212	0.00016
already	64	115212	0.00059
being	80	115212	0.00074
transformed	1	115212	$9.25\ 10^{-6}$
in	1759	115212	0.016
this	264	115212	0.0024
way	122	115212	0.0011
</s>	7072	115212	0.065

Probability of a Sentence: Bigrams

<s> *A good deal of the literature of the past was, indeed, already being transformed in this way* </s>

w_{i-1}, w_i	$C(w_{i-1}, w_i)$	$C(w_{i-1})$	$P_{MLE}(w_i \| w_{i-1})$
<s> *a*	133	7072	0.019
a good	14	2482	0.006
good deal	0	53	0.0
deal of	1	5	0.2
of the	742	3310	0.224
the literature	1	6248	0.0002
literature of	3	7	0.429
of the	742	3310	0.224
the past	70	6248	0.011
past was	4	99	0.040
was indeed	0	2211	0.0
indeed already	0	17	0.0
already being	0	64	0.0
being transformed	0	80	0.0
transformed in	0	1	0.0
in this	14	1759	0.008
this way	3	264	0.011
way </s>	18	122	0.148

Sparse Data

Given a vocabulary of 20,000 types, the potential number of bigrams is $20,000^2 = 400{,}000{,}000$

With trigrams $20,000^3 = 8{,}000{,}000{,}000{,}000$

Methods:

- Laplace: add one to all counts
- Linear interpolation:

$$\begin{aligned} P_{DelInterpolation}(w_n|w_{n-2}, w_{n-1}) &= \lambda_1 P_{MLE}(w_n|w_{n-2}w_{n-1}) + \\ &\quad \lambda_2 P_{MLE}(w_n|w_{n-1}) + \lambda_3 P_{MLE}(w_n), \end{aligned}$$

- Good-Turing: The discount factor is variable and depends on the number of times a n-gram has occurred in the corpus.
- Back-off

Laplace's Rule

$$P_{Laplace}(w_{i+1}|w_i) = \frac{C(w_i, w_{i+1}) + 1}{\sum_w (C(w_i, w) + 1)} = \frac{C(w_i, w_{i+1}) + 1}{C(w_i) + Card(V)},$$

w_i, w_{i+1}	$C(w_i, w_{i+1}) + 1$	$C(w_i) + Card(V)$	$P_{Lap}(w_{i+1}\|w_i)$
<s> *a*	133 + 1	7072 + 8635	0.0085
a good	14 + 1	2482 + 8635	0.0013
good deal	0 + 1	53 + 8635	0.00012
deal of	1 + 1	5 + 8635	0.00023
of the	742 + 1	3310 + 8635	0.062
the literature	1 + 1	6248 + 8635	0.00013
literature of	3 + 1	7 + 8635	0.00046
of the	742 + 1	3310 + 8635	0.062
the past	70 + 1	6248 + 8635	0.0048
past was	4 + 1	99 + 8635	0.00057
was indeed	0 + 1	2211 + 8635	0.000092
indeed already	0 + 1	17 + 8635	0.00012
already being	0 + 1	64 + 8635	0.00011
being transformed	0 + 1	80 + 8635	0.00011
transformed in	0 + 1	1 + 8635	0.00012
in this	14 + 1	1759 + 8635	0.0014
this way	3 + 1	264 + 8635	0.00045
way </s>	18 + 1	122 + 8635	0.0022

Good–Turing

Laplace's rule shifts an enormous mass of probability to very unlikely bigrams. Good–Turing's estimation is more effective

Let's denote N_c the number of n-grams that occurred exactly c times in the corpus.

N_0 is the number of unseen n-grams, N_1 the number of n-grams seen once,N_2 the number of n-grams seen twice The frequency of n-grams occurring c times is re-estimated as:

$$c* = (c+1)\frac{E(N_{c+1})}{E(N_c)},$$

Unseen n-grams: $c* = \frac{N_1}{N_0}$ and N-grams seen once: $c* = \frac{2N_2}{N_1}$.

Good-Turing for *Nineteen eighty-four*

Nineteen eighty-four contains 37,365 unique bigrams and 5,820 bigram seen twice.

Its vocabulary of 8,635 words generates $86352^2 = 74{,}563{,}225$ bigrams whose 74,513,701 are unseen.

New counts:

- Unseen bigrams: $\dfrac{37,365}{74,513,701} = 0.0005.$
- Unique bigrams: $2 \times \dfrac{5820}{37,365} = 0.31.$
- Etc.

Freq. of occ.	N_c	$c*$	Freq. of occ.	N_c	$c*$
0	74,513,701	0.0005	5	719	3.91
1	37,365	0.31	6	468	4.94
2	5,820	1.09	7	330	6.06
3	2,111	2.02	8	250	6.44
4	1,067	3.37	9	179	8.93

Backoff

If there is no bigram, then use unigrams:

$$P_{\text{Backoff}}(w_i|w_{i-1}) = \begin{cases} \tilde{P}(w_i|w_{i-1}), & \text{if } C(w_{i-1}, w_i) \neq 0, \\ \alpha P(w_i), & \text{otherwise.} \end{cases}$$

$$P_{\text{Backoff}}(w_i|w_{i-1}) = \begin{cases} P_{\text{MLE}}(w_i|w_{i-1}) = \dfrac{C(w_{i-1}, w_i)}{C(w_{i-1})}, & \text{if } C(w_{i-1}, w_i) \neq 0, \\ P_{\text{MLE}}(w_i) = \dfrac{C(w_i)}{\#\text{words}}, & \text{otherwise.} \end{cases}$$

Backoff: Example

w_{i-1}, w_i	$C(w_{i-1}, w_i)$		$C(w_i)$	$P_{Backoff}(w_i \vert w_{i-1})$
<s>			7072	—
<s> *a*	133		2482	0.019
a good	14		53	0.006
good deal	0	backoff	5	$4.62\ 10^{-5}$
deal of	1		3310	0.2
of the	742		6248	0.224
the literature	1		7	0.00016
literature of	3		3310	0.429
of the	742		6248	0.224
the past	70		99	0.011
past was	4		2211	0.040
was indeed	0	backoff	17	0.00016
indeed already	0	backoff	64	0.00059
already being	0	backoff	80	0.00074
being transformed	0	backoff	1	$9.25\ 10^{-6}$
transformed in	0	backoff	1759	0.016
in this	14		264	0.008
this way	3		122	0.011
way </s>	18		7072	0.148

The figures we obtain are not probabilities. We can use the Good-Turing technique to discount the bigrams and then scale the unigram probabilities. This is the Katz backoff.

Quality of a Language Model

Per word probability of a word sequence: $H(L) = -\frac{1}{n}\log_2 P(w_1,...,w_n)$.
Entropy rate: $H_{rate} = -\frac{1}{n}\sum_{w_1,...,w_n \in L} p(w_1,...,w_n)\log_2 p(w_1,...,w_n)$,

Cross entropy:

$$H(p,m) = -\frac{1}{n}\sum_{w_1,...,w_n \in L} p(w_1,...,w_n)\log_2 m(w_1,...,w_n).$$

We have:

$$\begin{aligned} H(p,m) &= \lim_{n\to\infty} -\frac{1}{n}\sum_{w_1,...,w_n \in L} p(w_1,...,w_n)\log_2 m(w_1,...,w_n), \\ &= \lim_{n\to\infty} -\frac{1}{n}\log_2 m(w_1,...,w_n). \end{aligned}$$

We compute the cross entropy on the complete word sequence of a test set, governed by p, using a bigram or trigram model, m, from a training set.
Perplexity:

$$PP(p,m) = 2^{H(p,m)}.$$

Other Statistical Formulas

- Mutual information (The strength of an association):

$$I(w_i, w_j) = \log_2 \frac{P(w_i, w_j)}{P(w_i)P(w_j)} \approx \log_2 \frac{N \cdot C(w_i, w_j)}{C(w_i)C(w_j)}.$$

- T-score (The confidence of an association):

$$\begin{aligned} t(w_i, w_j) &= \frac{mean(P(w_i, w_j)) - mean(P(w_i))mean(P(w_j))}{\sqrt{\sigma^2(P(w_i, w_j)) + \sigma^2(P(w_i)P(w_j))}}, \\ &\approx \frac{C(w_i, w_j) - \frac{1}{N}C(w_i)C(w_j)}{\sqrt{C(w_i, w_j)}}. \end{aligned}$$

T-Scores with Word *set*

Word	Frequency	Bigram *set* + word	*t*-score
up	134,882	5512	67.980
a	1,228,514	7296	35.839
to	1,375,856	7688	33.592
off	52,036	888	23.780
out	12,3831	1252	23.320

Source: Bank of English

Mutual Information with Word *surgery*

Word	Frequency	Bigram word + *surgery*	Mutual info
arthroscopic	3	3	11.822
pioneeing	3	3	11.822
reconstructive	14	11	11.474
refractive	6	4	11.237
rhinoplasty	5	3	11.085

Source: Bank of English

Mutual Information in Python

```
def mutual_info(words, freq_unigrams, freq_bigrams):
    mi = {}
    factor = len(words) * len(words) / (len(words) - 1)
    for bigram in freq_bigrams:
        mi[bigram] = (
            math.log(factor * freq_bigrams[bigram] /
                     (freq_unigrams[bigram[0]] *
                      freq_unigrams[bigram[1]]), 2))
    return mi
```

T-Scores in Python

```
def t_scores(words, freq_unigrams, freq_bigrams):
    ts = {}
    for bigram in freq_bigrams:
        ts[bigram] = ((freq_bigrams[bigram] -
                      freq_unigrams[bigram[0]] *
                      freq_unigrams[bigram[1]] /
                      len(words)) /
                      math.sqrt(freq_bigrams[bigram]))
    return ts
```