EDAN20
Language Technology
http://cs.lth.se/edan20/
Chapter 5: Counting Words

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Text Segmentation

Figure: Latin inscriptions on the *lapis niger*. *Corpus inscriptionum latinarum*, CIL I, 1. Picture from Wikipedia

Getting the Words from a Text: Tokenization

Arrange a list of characters:

`[l, i, s, t, ’ ’, o, f, ’ ’, c, h, a, r, a, c, t, e, r, s]`

into words:

`[list, of, characters]`

Sometimes tricky:

- Dates: 28/02/96
- Numbers: 9,812.345 (English), 9 812,345 (French and German) 9.812,345 (Old fashioned French)
- Abbreviations: km/h, m.p.h.,
- Acronyms: S.N.C.F.

Tokenizers use rules (or regexes) or statistical methods.

Tokenizing in Python: Using the Boundaries

Simple program

```
import re

one_token_per_line = re.sub('\s+', '\n', text)
```

Punctuation

```
import regex as re

spaced_tokens = re.sub('([\p{S}\p{P}])', r' \1 ', text)
one_token_per_line = re.sub('\s+', '\n', spaced_tokens)
```

Tokenizing in Python: Using the Content

Simple program

```
import regex as re

re.findall('\p{L}+', text)
```

Punctuation

```
spaced_tokens = re.sub('([\p{S}\p{P}])', r' \1 ', text)
re.findall('[\p{S}\p{P}\p{L}]+', spaced_tokens)
```

Improving Tokenization

The tokenization algorithm is word-based and defines a content

It does not work on nomenclatures such as Item #N23-SW32A, dates, or numbers

Instead it is possible to improve it using a boundary-based strategy with spaces (using for instance `\s`) and punctuation

But punctuation signs like commas, dots, or dashes can also be parts of tokens

Possible improvements using microgrammars

At some point, need of a dictionary:

Can't → can n't, *we'll* → we 'll

J'aime → j' aime but *aujourd'hui*

Sentence Segmentation

As for tokenization, segmenters use either rules (or regexes) or statistical methods.

Grefenstette and Tapanainen (1994) used the Brown corpus and experimented increasingly complex rules

Most simple rule: a period corresponds to a sentence boundary: 93.20% correctly segmented

Recognizing numbers:

`[0-9]+(\/[0-9]+)+` Fractions, dates

`([+\-])?[0-9]+(\.)?[0-9]*%` Percent

`([0-9]+,?)+(\.[0-9]+|[0-9]+)*` Decimal numbers

93.78% correctly segmented

Abbreviations

Common patterns (Grefenstette and Tapanainen 1994):

- single capitals: *A.*, *B.*, *C.*,
- letters and periods: *U.S. i.e. m.p.h.*,
- capital letter followed by a sequence of consonants: *Mr. St. Assn.*

Regex	**Correct**	**Errors**	**Full stop**
`[A-Za-z]\.`	1,327	52	14
`[A-Za-z]\.([A-Za-z0-9]\.)+`	570	0	66
`[A-Z][bcdfghj-np-tvxz]+\.`	1,938	44	26
Totals	3,835	96	106

Correct segmentation increases to 97.66%
With an abbreviation dictionary to 99.07%

Counting Words With Unix Tools

1. `tr -cs 'A-Za-z' '\n' <input_file |`
   Tokenize the text in `input_file`, where `tr` behaves like Perl `tr`: We have one word per line and the output is passed to the next command.
2. `tr 'A-Z' 'a-z' |`
   Translate the uppercase characters into lowercase letters and pass the output to the next command.
3. `sort |`
   Sort the words. The identical words will be grouped in adjacent lines.
4. `uniq -c |`
   Remove repeated lines. The identical adjacent lines will be replaced with one single line. Each unique line in the output will be preceded by the count of its duplicates in the input file (`-c`).
5. `sort -rn |`
   Sort in the reverse (`-r`) numeric (`-n`) order: Most frequent words first.
6. `head -5`
   Print the five first lines of the file (the five most frequent words).

Counting Words in Python

```
def tokenize(text):
    words = re.findall('\p{L}+', text)
    return words

def count_unigrams(words):
    frequency = {}
    for word in words:
        if word in frequency:
            frequency[word] += 1
        else:
            frequency[word] = 1
    return frequency
```

Counting Words in Python (Cont'd)

```
if __name__ == '__main__':
    text = sys.stdin.read().lower()
    words = tokenize(text)
    frequency = count_unigrams(words)
    for word in sorted(frequency.keys()):
        print(word, '\t', frequency[word])
```

Posting Lists

Many websites, such as Wikipedia, index their texts using an inverted index. Each word in the dictionary is linked to a posting list that gives all the documents where this word occurs and its positions in a document.

Collection

D1: Chrysler plans new investments in Latin America.

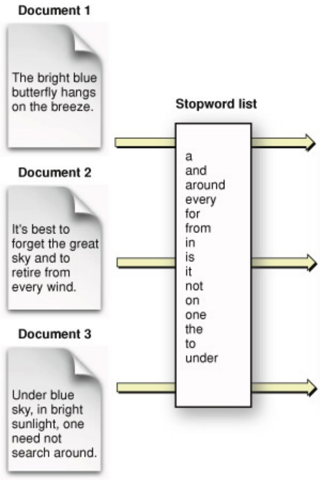
D2: Chrysler plans major investments in Mexico.

Index

Words	Posting lists
America	(D1, 7)
Chrysler	(D1, 1) → (D2, 1)
in	(D1, 5) → (D2, 5)
investments	(D1, 4) → (D2, 4)
Latin	(D1, 6)
major	(D2, 3)
Mexico	(D2, 6)
new	(D1, 3)
plans	(D1, 2) → (D2, 2)

Lucene is a high quality open-source indexer.
(`http://lucene.apache.org/`)

Inverted Index (Source Apple)



Inverted index

ID	Term	Document
1	best	2
2	blue	1, 3
3	bright	1, 3
4	butterfly	1
5	breeze	1
6	forget	2
7	great	2
8	hangs	1
9	need	3
10	retire	2
11	search	3
12	sky	2, 3
13	wind	2

http://developer.apple.com/library/mac/documentation/UserExperience/Conceptual/SearchKitConcepts/index.html

Information Retrieval: The Vector Space Model

The vector space model represents a document in a space of words.

Documents \Words	w_1	w_2	w_3	...	w_m
D_1	$C(w_1, D_1)$	$C(w_2, D_1)$	$C(w_3, D_1)$	...	$C(w_m, D_1)$
D_2	$C(w_1, D_2)$	$C(w_2, D_2)$	$C(w_3, D_2)$	...	$C(w_m, D_2)$
...					
D_n	$C(w_1, D_n)$	$C(w_2, D_n)$	$C(w_3, D_n)$	...	$C(w_m, D_n)$

It was created for information retrieval to compute the similarity of two documents or to match a document and a query.
We compute the similarity of two documents through their dot product.

The Vector Space Model: Example

A collection of two documents D1 and D2:

D1: Chrysler plans new investments in Latin America.

D2: Chrysler plans major investments in Mexico.

The vectors representing the two documents:

D.	america	chrysler	in	investments	latin	major	mexico	new	plans
1	1	1	1	1	1	0	0	1	1
2	0	1	1	1	0	1	1	0	1

The vector space model represents documents as bags of words (BOW) that do not take the word order into account.

The dot product is $\vec{D}1 \cdot \vec{D}2 = 0+1+1+1+0+0+0+0+1 = 4$

Their cosine is $\frac{\vec{D}1 \cdot \vec{D}2}{||\vec{D}1||.||\vec{D}2||} = \frac{4}{\sqrt{7}.\sqrt{6}} = 0.62$

Giving a Weight

Word clouds give visual weights to words

Image: Courtesy of Jonas Wisbrant

$TF \times IDF$

The frequency alone might be misleading

Document coordinates are in fact $tf \times idf$: Term frequency by inverted document frequency.

Term frequency $tf_{i,j}$: frequency of term j in document i

Inverted document frequency: $idf_j = \log(\frac{N}{n_j})$

Document Similarity

Documents are vectors where coordinates could be the count of each word:
$\vec{d} = (C(w_1), C(w_2), C(w_3), ..., C(w_n))$
The similarity between two documents or a query and a document is given by their cosine:

$$\cos(\vec{q}, \vec{d}) = \frac{\sum_{i=1}^{n} q_i d_i}{\sqrt{\sum_{i=1}^{n} q_i^2} \sqrt{\sum_{i=1}^{n} d_i^2}}.$$

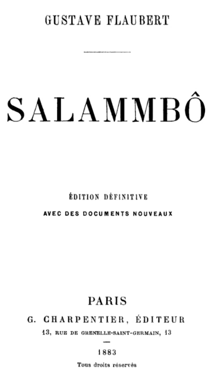
Dimension Reduction

One-hot encoding of TFIDF encoding can produce very long vectors:
Imagine a vocabulary one one million words per language with 100 languages.
A solution is to produce dense vectors also called word embeddings using a dimension reduction
This reduction is very close to principal component analysis or singular value decomposition
It can be automatically obtained through training or initialized with pretrained vectors

A Data Set: *Salammbô*

A corpus is a collection – a body – of texts.

French original	English translation


GUSTAVE FLAUBERT

SALAMMBÔ

ÉDITION DÉFINITIVE

AVEC DES DOCUMENTS NOUVEAUX

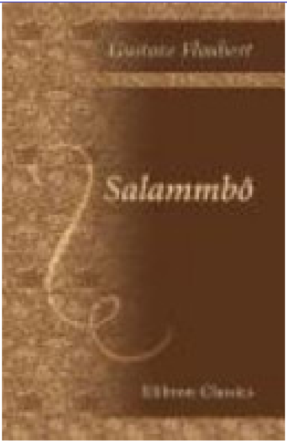
PARIS

G. CHARPENTIER, ÉDITEUR

13, RUE DE GRENELLE-SAINT-GERMAIN, 13

1883

Tous droits réservés



Supervised Learning

Letter counts from *Salammbô*

Chapter	French # characters	French # *A*	English # characters	English # *A*
Chapter 1	36,961	2,503	35,680	2,217
Chapter 2	43,621	2,992	42,514	2,761
Chapter 3	15,694	1,042	15,162	990
Chapter 4	36,231	2,487	35,298	2,274
Chapter 5	29,945	2,014	29,800	1,865
Chapter 6	40,588	2,805	40,255	2,606
Chapter 7	75,255	5,062	74,532	4,805
Chapter 8	37,709	2,643	37,464	2,396
Chapter 9	30,899	2,126	31,030	1,993
Chapter 10	25,486	1,784	24,843	1,627
Chapter 11	37,497	2,641	36,172	2,375
Chapter 12	40,398	2,766	39,552	2,560
Chapter 13	74,105	5,047	72,545	4,597
Chapter 14	76,725	5,312	75,352	4,871
Chapter 15	18,317	1,215	18,031	1,119

Data set: `https://github.com/pnugues/ilppp/tree/master/programs/ch04/salammbo`

Principal Component Analysis

We will use a small dataset to explain principal component analysis: The characters in *Salammbô*

Ch.	'a'	'b'	'c'	'd'	'e'	'f'	'g'	'h'	'i'	'j'	'k'	'l'	'm'	'n'	'o'	'p'	'q'	'r'	's'	't'	'u'	'v'	'w'	'x'	'y'	'z'	'à'
01_fr	2503	365	857	1151	4312	264	349	295	1945	65	4	1946	726	1896	1372	789	248	1948	2996	1938	1792	414	0	129	94	20	128
02_fr	2992	391	1006	1388	4993	319	360	350	2345	81	6	2128	823	2308	1560	977	281	2376	3454	2411	2069	499	0	175	89	23	136
03_fr	1042	152	326	489	1785	136	122	126	784	41	7	816	397	778	612	315	102	792	1174	856	707	147	0	42	31	7	39
04_fr	2487	303	864	1137	4158	314	331	287	2028	57	3	1796	722	1958	1318	773	274	2000	2792	2031	1734	422	0	138	81	27	110
05_fr	2014	268	645	949	3394	223	215	242	1617	67	3	1513	651	1547	1053	672	166	1601	2192	1736	1396	315	1	83	67	18	90
06_fr	2805	368	910	1266	4535	332	384	378	2219	97	3	1900	841	2179	1569	868	285	2205	3065	2293	1895	453	0	151	80	39	131
07_fr	5062	706	1770	2398	8512	623	622	620	4018	126	19	3726	1596	3851	2823	1532	468	4015	5634	4116	3518	844	0	272	148	71	246
08_fr	2643	325	869	1085	4229	307	317	359	2102	85	4	1857	811	2041	1367	833	239	2132	2814	2134	1788	437	0	135	64	30	130
09_fr	2126	289	771	920	3599	278	289	279	1805	52	6	1499	619	1711	1130	651	187	1719	2404	1763	1448	348	0	119	58	20	90
10_fr	1784	249	546	805	3002	179	202	215	1319	60	5	1462	598	1246	922	557	172	1242	1769	1423	1191	270	0	65	61	11	73
11_fr	2641	381	817	1078	4306	263	277	330	1985	114	0	1886	900	1966	1356	763	230	1912	2564	2218	1737	425	0	114	61	25	101
12_fr	2766	373	935	1237	4618	329	350	349	2273	65	2	1955	812	2285	1419	865	272	2276	3131	2274	1923	455	0	149	98	37	129
13_fr	5047	725	1730	2273	8678	648	566	642	3940	140	22	3746	1597	3984	2736	1550	425	4081	5599	4387	3480	767	0	288	119	41	209
14_fr	5312	689	1754	2149	8870	628	630	673	4278	143	2	3780	1610	4255	2713	1599	512	4271	5770	4467	3697	914	0	283	145	41	224
15_fr	1215	173	402	582	2195	150	134	148	969	27	6	950	387	906	697	417	103	985	1395	1037	893	206	0	63	36	3	48
01_en	2217	451	729	1316	3967	596	662	2060	1823	22	200	1204	656	1851	1897	525	19	1764	1942	2547	704	258	653	29	401	18	0
02_en	2761	551	777	1548	4543	685	769	2530	2163	13	284	1319	829	2218	2237	606	21	2019	2411	3083	861	295	769	37	475	31	0
03_en	990	183	271	557	1570	279	253	875	783	4	82	520	333	816	828	194	13	711	864	1048	298	94	254	8	145	15	0
04_en	2274	454	736	1315	3814	595	559	1978	1835	22	198	1073	690	1771	1865	514	33	1726	1918	2704	745	245	663	60	467	19	0
05_en	1865	400	553	1135	3210	515	525	1693	1482	7	153	949	571	1468	1586	517	17	1357	1646	2178	663	194	568	26	330	33	0
06_en	2606	518	797	1509	4237	687	669	2254	2097	26	216	1239	763	2174	2231	613	25	1931	2192	2955	899	277	733	49	464	37	0
07_en	4805	913	1521	2681	7834	1366	1163	4379	3838	42	416	2434	1461	3816	4091	1040	39	3674	4060	5369	1552	465	1332	74	843	52	0
08_en	2396	431	702	1416	4014	621	624	2171	2011	24	216	1152	748	2085	1947	527	33	1915	1966	2765	789	266	695	65	379	24	0
09_en	1993	408	653	1096	3373	575	517	1766	1648	16	146	861	629	1728	1698	442	20	1561	1626	2442	683	208	560	25	328	18	0
10_en	1627	359	451	933	2690	477	409	1475	1196	7	131	789	506	1266	1369	325	23	1211	1344	1759	502	181	410	31	255	20	0
11_en	2375	437	643	1364	3790	610	644	2217	1830	16	217	1122	799	1833	1948	486	23	1720	1945	2424	767	246	632	20	457	39	0
12_en	2560	489	757	1566	4331	677	650	2348	2033	28	234	1102	746	2125	2105	581	32	1939	2152	3046	750	278	721	35	418	40	0
13_en	4597	987	1462	2689	7963	1254	1201	4278	3634	39	432	2281	1493	3774	3911	1099	49	3577	3894	5540	1379	437	1374	77	673	49	0
14_en	4871	948	1439	2799	8179	1335	1140	4534	3829	36	427	2218	1534	4053	3989	1019	36	3689	3946	5858	1490	539	1377	90	856	49	0
15_en	1119	229	335	683	1994	323	281	1108	912	9	112	579	351	924	1004	305	9	863	997	1330	310	108	330	14	150	9	0

Table: Character counts per chapter, where the fr and en suffixes designate the language, either French or English

Each chapter is modeled by a vector of characters.

Character Counts

	French															English										
	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	01	02	03	04	05	06	07	08	09	10	11
a	2503	2992	1042	2487	2014	2805	5062	2643	2126	1784	2641	2766	5047	5312	1215	2217	2761	990	2274	1865	2606	4805	2396	1993	1627	2375
b	365	391	152	303	268	368	706	325	289	249	381	373	725	689	173	451	551	183	454	400	518	913	431	408	359	437
c	857	1006	326	864	645	910	1770	869	771	546	817	935	1730	1754	402	729	777	271	736	553	797	1521	702	653	451	643
d	1151	1388	489	1137	949	1266	2398	1085	920	805	1078	1237	2273	2149	582	1316	1548	557	1315	1135	1509	2681	1416	1096	933	1364
e	4312	4993	1785	4158	3394	4535	8512	4229	3599	3002	4306	4618	8678	8870	2195	3967	4543	1570	3814	3210	4237	7834	4014	3373	2690	3790
f	264	319	136	314	223	332	623	307	278	179	263	329	648	628	150	596	685	279	595	515	687	1366	621	575	477	610
g	349	360	122	331	215	384	622	317	289	202	277	350	566	630	134	662	769	253	559	525	669	1163	624	517	409	644
h	295	350	126	287	242	378	620	359	279	215	330	349	642	673	148	2060	2530	875	1978	1693	2254	4379	2171	1766	1475	2217
i	1945	2345	784	2028	1617	2219	4018	2102	1805	1319	1985	2273	3940	4278	969	1823	2163	783	1835	1482	2097	3838	2011	1648	1196	1830
j	65	81	41	57	67	97	126	85	52	60	114	65	140	143	27	22	13	4	22	7	26	42	24	16	7	16
k	4	6	7	3	3	3	19	4	6	5	0	2	22	2	6	200	284	82	198	153	216	416	216	146	131	217
l	1946	2128	816	1796	1513	1900	3726	1857	1499	1462	1886	1955	3746	3780	950	1204	1319	520	1073	949	1239	2434	1152	861	789	1122
m	726	823	397	722	651	841	1596	811	619	598	900	812	1597	1610	387	656	829	333	690	571	763	1461	748	629	506	799
n	1896	2308	778	1958	1547	2179	3851	2041	1711	1246	1966	2285	3984	4255	906	1851	2218	816	1771	1468	2174	3816	2085	1728	1266	1833
o	1372	1560	612	1318	1053	1569	2823	1367	1130	922	1356	1419	2736	2713	697	1897	2237	828	1865	1586	2231	4091	1947	1698	1369	1948
p	789	977	315	773	672	868	1532	833	651	557	763	865	1550	1599	417	525	606	194	514	517	613	1040	527	442	325	486
q	248	281	102	274	166	285	468	239	187	172	230	272	425	512	103	19	21	13	33	17	25	39	33	20	23	23
r	1948	2376	792	2000	1601	2205	4015	2132	1719	1242	1912	2276	4081	4271	985	1764	2019	711	1726	1357	1931	3674	1915	1561	1211	1720
s	2996	3454	1174	2792	2192	3065	5634	2814	2404	1769	2564	3131	5599	5770	1395	1942	2411	864	1918	1646	2192	4060	1966	1626	1344	1945
t	1938	2411	856	2031	1736	2293	4116	2134	1763	1423	2218	2274	4387	4467	1037	2547	3083	1048	2704	2178	2955	5369	2765	2442	1759	2424
u	1792	2069	707	1734	1396	1895	3518	1788	1448	1191	1737	1923	3480	3697	893	704	861	298	745	663	899	1552	789	683	502	767
v	414	499	147	422	315	453	844	437	348	270	425	455	767	914	206	258	295	94	245	194	277	465	266	208	181	246
w	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	653	769	254	663	568	733	1332	695	560	410	632
x	129	175	42	138	83	151	272	135	119	65	114	149	288	283	63	29	37	8	60	26	49	74	65	25	31	20
y	94	89	31	81	67	80	148	64	58	61	61	98	119	145	36	401	475	145	467	330	464	843	379	328	255	457
z	20	23	7	27	18	39	71	30	20	11	25	37	41	41	3	18	31	15	19	33	37	52	24	18	20	39
à	128	136	39	110	90	131	246	130	90	73	101	129	209	224	48	0	0	0	0	0	0	0	0	0	0	0
â	36	50	9	43	67	42	50	43	24	18	40	33	55	75	20	0	0	0	0	0	0	0	0	0	0	0
æ	0	1	0	0	0	0	1	0	2	0	0	0	3	0	2	0	0	0	0	0	0	0	0	0	0	0
ç	35	28	10	22	24	30	46	34	16	16	34	23	61	56	17	0	0	0	0	0	0	0	0	0	0	0
è	102	147	49	138	112	122	232	119	99	68	108	151	237	260	58	0	0	0	0	0	0	0	0	0	0	0
é	423	513	194	424	367	548	966	502	370	304	438	480	940	1019	221	0	0	0	0	0	0	0	0	0	0	0
ê	43	68	24	36	44	57	96	54	43	53	68	60	126	94	32	0	0	0	0	0	0	0	0	0	0	0
ë	1	0	0	0	1	0	2	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
î	17	20	12	15	11	15	42	11	8	15	26	13	32	28	12	0	0	0	0	0	0	0	0	0	0	0
ï	2	0	0	2	8	12	9	1	2	5	15	3	5	2	0	0	0	0	0	0	0	0	0	0	0	0
ô	20	20	27	15	23	15	41	14	13	38	50	15	37	45	24	0	0	0	0	0	0	0	0	0	0	0
ù	14	9	4	6	18	14	30	6	5	3	7	11	24	21	7	0	0	0	0	0	0	0	0	0	0	0
û	7	9	7	4	15	15	38	8	15	10	9	14	30	21	11	0	0	0	0	0	0	0	0	0	0	0
œ	5	5	2	8	7	9	9	5	3	5	7	0	13	12	6	0	0	0	0	0	0	0	0	0	0	0

Table: Character counts per chapter in French, left part, and English, right part

Each characters is modeled by a vector of chapters.

Singular Value Decomposition

There are as many as 40 characters: the 26 unaccented letters from *a* to *z* and the 14 French accented letters

Singular value decomposition (SVD) reduces these dimensions, while keeping the resulting vectors semantically close

$\mathbf{X}$ is the $m \times n$ matrix of the letter counts per chapter, in our case, $m = 30$ and $n = 40$.

We can rewrite $\mathbf{X}$ as:

$$\mathbf{X} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^\intercal,$$

where $\mathbf{U}$ is a matrix of dimensions $m \times m$, $\mathbf{\Sigma}$, a diagonal matrix of dimensions $m \times n$, and $\mathbf{V}$, a matrix of dimensions $n \times n$.

The diagonal terms of $\mathbf{\Sigma}$ are called the **singular values** and are traditionally arranged by decreasing value.

We keep the highest values and set the rest to zero.

Code Example

Jupyter Notebook 3.1-SVD

Word Sequences

Words have specific contexts of use.

Pairs of words like *strong* and *tea* or *powerful* and *computer* are not random associations.

Psychological linguistics tells us that it is difficult to make a difference between *writer* and *rider* without context

A listener will discard the improbable *rider of books* and prefer *writer of books*

A language model is the statistical estimate of a word sequence.

Originally developed for speech recognition

The language model component enables to predict the next word given a sequence of previous words: *the writer of books, novels, poetry*, etc. and not *the writer of hooks, nobles, poultry*, . . .

N-Grams

The types are the distinct words of a text while the tokens are all the words or symbols.

The phrases from *Nineteen Eighty-Four*

> *War is peace*
> *Freedom is slavery*
> *Ignorance is strength*

have 9 tokens and 7 types.

Unigrams are single words

Bigrams are sequences of two words

Trigrams are sequences of three words

Trigrams

Word	Rank	More likely alternatives
We	9	*The This One Two A Three Please In*
need	7	*are will the would also do*
to	1	
resolve	85	*have know do...*
all	9	*the this these problems...*
of	2	*the*
the	1	
important	657	*document question first...*
issues	14	*thing point to...*
within	74	*to of and in that...*
the	1	
next	2	*company*
two	5	*page exhibit meeting day*
days	5	*weeks years pages months*

Counting Bigrams With Unix Tools

1. `tr -cs ’A-Za-z’ ’\n’ < input_file > token_file`
   Tokenize the input and create a file with the unigrams.
2. `tail +2 < token_file > next_token_file`
   Create a second unigram file starting at the second word of the first tokenized file (+2).
3. `paste token_file next_token_file > bigrams`
   Merge the lines (the tokens) pairwise. Each line of `bigrams` contains the words at index i and $i+1$ separated with a tabulation.
4. And we count the bigrams as in the previous script.

Counting Bigrams in Python

```
bigrams = [tuple(words[inx:inx + 2])
           for inx in range(len(words) - 1)]
```

The rest of the count_bigrams function is nearly identical to count_unigrams. As input, it uses the same list of words:

```
def count_bigrams(words):
    bigrams = [tuple(words[inx:inx + 2])
               for inx in range(len(words) - 1)]
    frequencies = {}
    for bigram in bigrams:
        if bigram in frequencies:
            frequencies[bigram] += 1
        else:
            frequencies[bigram] = 1
    return frequencies
```

Probabilistic Models of a Word Sequence

$$\begin{aligned} P(S) &= P(w_1, ..., w_n), \\ &= P(w_1)P(w_2|w_1)P(w_3|w_1, w_2)...P(w_n|w_1, ..., w_{n-1}), \\ &= \prod_{i=1}^{n} P(w_i|w_1, ..., w_{i-1}). \end{aligned}$$

The probability $P(\textit{It was a bright cold day in April})$ from *Nineteen Eighty-Four* corresponds to
It to begin the sentence, then *was* knowing that we have *It* before, then *a* knowing that we have *It was* before, and so on until the end of the sentence.

$$\begin{aligned} P(S) = \; & P(It) \times P(was|It) \times P(a|It, was) \times P(bright|It, was, a) \times ... \\ & \times P(April|It, was, a, bright, ..., in). \end{aligned}$$

Approximations

Bigrams:

$$P(w_i|w_1, w_2, ..., w_{i-1}) \approx P(w_i|w_{i-1}),$$

Trigrams:

$$P(w_i|w_1, w_2, ..., w_{i-1}) \approx P(w_i|w_{i-2}, w_{i-1}).$$

Using a trigram language model, $P(S)$ is approximated as:

$$\begin{aligned} P(S) \approx\ & P(It) \times P(was|It) \times P(a|It, was) \times P(bright|was, a) \times ... \\ & \times P(April|day, in). \end{aligned}$$

Maximum Likelihood Estimate

Bigrams:

$$P_{MLE}(w_i|w_{i-1}) = \frac{C(w_{i-1}, w_i)}{\sum_{w} C(w_{i-1}, w)} = \frac{C(w_{i-1}, w_i)}{C(w_{i-1})}.$$

Trigrams:

$$P_{MLE}(w_i|w_{i-2}, w_{i-1}) = \frac{C(w_{i-2}, w_{i-1}, w_i)}{C(w_{i-2}, w_{i-1})}.$$

Conditional Probabilities

A common mistake in computing the conditional probability $P(w_i|w_{i-1})$ is to use

$$\frac{C(w_{i-1}, w_i)}{\#bigrams}.$$

This is not correct. This formula corresponds to $P(w_{i-1}, w_i)$.
The correct estimation is

$$P_{MLE}(w_i|w_{i-1}) = \frac{C(w_{i-1}, w_i)}{\sum_w C(w_{i-1}, w)} = \frac{C(w_{i-1}, w_i)}{C(w_{i-1})}.$$

Proof:

$$P(w_1, w_2) = P(w_1)P(w_2|w_1) = \frac{C(w_1)}{\#words} \times \frac{C(w_1, w_2)}{C(w_1)} = \frac{C(w_1, w_2)}{\#words}$$

Training the Model

The model is trained on a part of the corpus: the **training set**

It is tested on a different part: the **test set**

The vocabulary can be derived from the corpus, for instance the 20,000 most frequent words, or from a lexicon

It can be closed or open

A closed vocabulary does not accept any new word

An open vocabulary maps the new words, either in the training or test sets, to a specific symbol, `<UNK>`

Probability of a Sentence: Unigrams

<s> *A good deal of the literature of the past was, indeed, already being transformed in this way* </s>

w_i	$C(w_i)$	#words	$P_{MLE}(w_i)$
<s>	7072	–	
a	2482	108140	0.023
good	53	108140	0.00049
deal	5	108140	$4.62\ 10^{-5}$
of	3310	108140	0.031
the	6248	108140	0.058
literature	7	108140	$6.47\ 10^{-5}$
of	3310	108140	0.031
the	6248	108140	0.058
past	99	108140	0.00092
was	2211	108140	0.020
indeed	17	108140	0.00016
already	64	108140	0.00059
being	80	108140	0.00074
transformed	1	108140	$9.25\ 10^{-6}$
in	1759	108140	0.016
this	264	108140	0.0024
way	122	108140	0.0011
</s>	7072	108140	0.065

Probability of a Sentence: Bigrams

<s> A good deal of the literature of the past was, indeed, already being transformed in this way </s>

w_{i-1}, w_i	$C(w_{i-1}, w_i)$	$C(w_{i-1})$	$P_{MLE}(w_i \mid w_{i-1})$
<s> *a*	133	7072	0.019
a good	14	2482	0.006
good deal	0	53	0.0
deal of	1	5	0.2
of the	742	3310	0.224
the literature	1	6248	0.0002
literature of	3	7	0.429
of the	742	3310	0.224
the past	70	6248	0.011
past was	4	99	0.040
was indeed	0	2211	0.0
indeed already	0	17	0.0
already being	0	64	0.0
being transformed	0	80	0.0
transformed in	0	1	0.0
in this	14	1759	0.008
this way	3	264	0.011
way </s>	18	122	0.148

Sparse Data

Given a vocabulary of 20,000 types, the potential number of bigrams is $20,000^2 = 400{,}000{,}000$
With trigrams $20,000^3 = 8{,}000{,}000{,}000{,}000$
Methods:

- Laplace: add one to all counts
- Linear interpolation:

$$P_{DelInterpolation}(w_n|w_{n-2}, w_{n-1}) = \lambda_1 P_{MLE}(w_n|w_{n-2}w_{n-1}) + \lambda_2 P_{MLE}(w_n|w_{n-1}) + \lambda_3 P_{MLE}(w_n),$$

- Good-Turing: The discount factor is variable and depends on the number of times a n-gram has occurred in the corpus.
- Back-off

Laplace's Rule

$$P_{Laplace}(w_{i+1}|w_i) = \frac{C(w_i, w_{i+1}) + 1}{\sum_w (C(w_i, w) + 1)} = \frac{C(w_i, w_{i+1}) + 1}{C(w_i) + Card(V)},$$

w_i, w_{i+1}	$C(w_i, w_{i+1}) + 1$	$C(w_i) + Card(V)$	$P_{Lap}(w_{i+1}\|w_i)$
<s> *a*	133 + 1	7072 + 8635	0.0085
a good	14 + 1	2482 + 8635	0.0013
good deal	0 + 1	53 + 8635	0.00012
deal of	1 + 1	5 + 8635	0.00023
of the	742 + 1	3310 + 8635	0.062
the literature	1 + 1	6248 + 8635	0.00013
literature of	3 + 1	7 + 8635	0.00046
of the	742 + 1	3310 + 8635	0.062
the past	70 + 1	6248 + 8635	0.0048
past was	4 + 1	99 + 8635	0.00057
was indeed	0 + 1	2211 + 8635	0.000092
indeed already	0 + 1	17 + 8635	0.00012
already being	0 + 1	64 + 8635	0.00011
being transformed	0 + 1	80 + 8635	0.00011
transformed in	0 + 1	1 + 8635	0.00012
in this	14 + 1	1759 + 8635	0.0014
this way	3 + 1	264 + 8635	0.00045
way </s>	18 + 1	122 + 8635	0.0022

Good–Turing

Laplace's rule shifts an enormous mass of probability to very unlikely bigrams. Good–Turing's estimation is more effective
Let's denote N_c the number of n-grams that occurred exactly c times in the corpus.
N_0 is the number of unseen n-grams, N_1 the number of n-grams seen once, N_2 the number of n-grams seen twice The frequency of n-grams occurring c times is re-estimated as:

$$c* = (c+1)\frac{E(N_{c+1})}{E(N_c)},$$

Unseen n-grams: $c* = \dfrac{N_1}{N_0}$ and N-grams seen once: $c* = \dfrac{2N_2}{N_1}$.

Good-Turing for *Nineteen eighty-four*

Nineteen eighty-four contains 37,365 unique bigrams and 5,820 bigram seen twice.
Its vocabulary of 8,635 words generates $86352^2 = 74{,}563{,}225$ bigrams whose 74,513,701 are unseen.
New counts:

- Unseen bigrams: $\dfrac{37,365}{74,513,701} = 0.0005.$
- Unique bigrams: $2 \times \dfrac{5820}{37,365} = 0.31.$
- Etc.

Freq. of occ.	N_c	$c*$	**Freq. of occ.**	N_c	$c*$
0	74,513,701	0.0005	5	719	3.91
1	37,365	0.31	6	468	4.94
2	5,820	1.09	7	330	6.06
3	2,111	2.02	8	250	6.44
4	1,067	3.37	9	179	8.93

Backoff

If there is no bigram, then use unigrams:

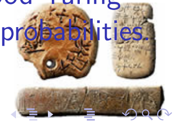
$$P_{\text{Backoff}}(w_i|w_{i-1}) = \begin{cases} \tilde{P}(w_i|w_{i-1}), & \text{if } C(w_{i-1}, w_i) \neq 0, \\ \alpha P(w_i), & \text{otherwise.} \end{cases}$$

$$P_{\text{Backoff}}(w_i|w_{i-1}) = \begin{cases} P_{\text{MLE}}(w_i|w_{i-1}) = \dfrac{C(w_{i-1}, w_i)}{C(w_{i-1})}, & \text{if } C(w_{i-1}, w_i) \neq 0, \\ P_{\text{MLE}}(w_i) = \dfrac{C(w_i)}{\#\text{words}}, & \text{otherwise.} \end{cases}$$

Backoff: Example

w_{i-1}, w_i	$C(w_{i-1}, w_i)$		$C(w_i)$	$P_{Backoff}(w_i \mid w_{i-1})$
<s>			7072	—
<s> *a*	133		2482	0.019
a good	14		53	0.006
good deal	0	backoff	5	$4.62\ 10^{-5}$
deal of	1		3310	0.2
of the	742		6248	0.224
the literature	1		7	0.00016
literature of	3		3310	0.429
of the	742		6248	0.224
the past	70		99	0.011
past was	4		2211	0.040
was indeed	0	backoff	17	0.00016
indeed already	0	backoff	64	0.00059
already being	0	backoff	80	0.00074
being transformed	0	backoff	1	$9.25\ 10^{-6}$
transformed in	0	backoff	1759	0.016
in this	14		264	0.008
this way	3		122	0.011
way </s>	18		7072	0.148

The figures we obtain are not probabilities. We can use the Good-Turing technique to discount the bigrams and then scale the unigram probabilities. This is the Katz backoff.



Quality of a Language Model (I)

The quality of a language model corresponds to its accuracy in predicting word sequences: $P(w_1, ..., w_n)$: The higher, the better.
We derive the model (the statistics) from a training set and evaluate this quality on a long unseen sequence sequence: The test set.
With the *n*-gram approximations, we have:

$$P(w_1, ..., w_n) = \prod_{i=1}^{n} P(w_i) \quad \text{Unigrams}$$

$$P(w_1, ..., w_n) = P(w_1) \prod_{i=2}^{n} P(w_i|w_{i-1}) \quad \text{Bigrams}$$

$$P(w_1, ..., w_n) = P(w_1)P(w_2|w_1) \prod_{i=3}^{n} P(w_i|w_{i-2}, w_{i-1}) \quad \text{Trigrams}$$

etc.

Quality of a Language Model (II)

The probability value will depend on the length of the sequence. We take the geometric mean instead to standardize across different lengths:

$$\sqrt[n]{\prod_{i=1}^{n} P(w_i)} \qquad \text{Unigrams}$$

$$\sqrt[n]{P(w_1)\prod_{i=2}^{n} P(w_i|w_{i-1})} \qquad \text{Bigrams}$$

...

In practice, we use the log to compute the per word probability of a word sequence, the entropy rate:

$$H(L) = -\frac{1}{n}\log_2 P(w_1, ..., w_n).$$

Here the lower, the better

The figures are usually presented with the perplexity metric:

$$PP(p,m) = 2^{H(L)}.$$

Mathematical Background

Entropy rate: $H_{rate} = -\frac{1}{n} \sum_{w_1,\ldots,w_n \in L} p(w_1,\ldots,w_n) \log_2 p(w_1,\ldots,w_n)$,

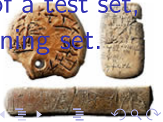
Cross entropy:

$$H(p,m) = -\frac{1}{n} \sum_{w_1,\ldots,w_n \in L} p(w_1,\ldots,w_n) \log_2 m(w_1,\ldots,w_n).$$

We have:

$$\begin{aligned} H(p,m) &= \lim_{n\to\infty} -\frac{1}{n} \sum_{w_1,\ldots,w_n \in L} p(w_1,\ldots,w_n) \log_2 m(w_1,\ldots,w_n), \\ &= \lim_{n\to\infty} -\frac{1}{n} \log_2 m(w_1,\ldots,w_n). \end{aligned}$$

We compute the cross entropy on the complete word sequence of a test set, governed by p, using a bigram or trigram model, m, from a training set.



Other Statistical Formulas

- Mutual information (The strength of an association):

$$I(w_i, w_j) = \log_2 \frac{P(w_i, w_j)}{P(w_i)P(w_j)} \approx \log_2 \frac{N \cdot C(w_i, w_j)}{C(w_i)C(w_j)}.$$

- T-score (The confidence of an association):

$$\begin{aligned} t(w_i, w_j) &= \frac{mean(P(w_i, w_j)) - mean(P(w_i))mean(P(w_j))}{\sqrt{\sigma^2(P(w_i, w_j)) + \sigma^2(P(w_i)P(w_j))}}, \\ &\approx \frac{C(w_i, w_j) - \frac{1}{N}C(w_i)C(w_j)}{\sqrt{C(w_i, w_j)}}. \end{aligned}$$

T-Scores with Word *set*

Word	Frequency	Bigram *set* + word	*t*-score
up	134,882	5512	67.980
a	1,228,514	7296	35.839
to	1,375,856	7688	33.592
off	52,036	888	23.780
out	12,3831	1252	23.320

Source: Bank of English

Mutual Information with Word *surgery*

Word	Frequency	Bigram word + *surgery*	Mutual info
arthroscopic	3	3	11.822
pioneeing	3	3	11.822
reconstructive	14	11	11.474
refractive	6	4	11.237
rhinoplasty	5	3	11.085

Source: Bank of English

Mutual Information in Python

```
def mutual_info(words, freq_unigrams, freq_bigrams):
    mi = {}
    factor = len(words) * len(words) / (len(words) - 1)
    for bigram in freq_bigrams:
        mi[bigram] = (
            math.log(factor * freq_bigrams[bigram] /
                     (freq_unigrams[bigram[0]] *
                      freq_unigrams[bigram[1]]), 2))
    return mi
```

T-Scores in Python

```
def t_scores(words, freq_unigrams, freq_bigrams):
    ts = {}
    for bigram in freq_bigrams:
        ts[bigram] = ((freq_bigrams[bigram] -
                       freq_unigrams[bigram[0]] *
                       freq_unigrams[bigram[1]] /
                       len(words)) /
                       math.sqrt(freq_bigrams[bigram]))
    return ts
```

Word Embeddings

We can extend singular value decomposition from characters to words.
The rows will represent the words in the corpus, and the columns, documents,
We can replace documents by a context of a few words to the left and to the right of the focus word: w_i.
A context C_j is then defined by a window of $2K$ words centered on the word:

$$w_{i-K}, w_{i-K+1}, \ldots, w_{i-1}, w_{i+1}, \ldots, w_{i+K-1}, w_{i+K},$$

where the context representation uses a bag of words.
We can even reduce the context to a single word to the left or to the right of w_i and use bigrams.

Word Embeddings

We store the word-context pairs (w_i, C_j) in a matrix.
Each matrix element measures the association strength between word w_i and context C_j, for instance mutual information.
Mutual information, often called pointwise mutual information (the strength of an association) is defined as:

$$I(w_i, w_j) = \log_2 \frac{P(w_i, w_j)}{P(w_i)P(w_j)} \approx \log_2 \frac{N \cdot C(w_i, w_j)}{C(w_i)C(w_j)}.$$

D#\Words	C_1	C_2	C_3	...	C_n
w_1	$MI(w_1, C_1)$	$MI(w_1, C_2)$	$MI(w_1, C_3)$	...	$MI(w_1, C_n)$
w_2	$MI(w_2, C_1)$	$MI(w_2, C_2)$	$MI(w_2, C_3)$	...	$MI(w_2, C_n)$
w_3	$MI(w_3, C_1)$	$MI(w_3, C_2)$	$MI(w_3, C_3)$	...	$MI(w_3, C_n)$
...	...	...	...	...	...
w_m	$MI(w_m, C_1)$	$MI(w_m, C_2)$	$MI(w_m, C_3)$	...	$MI(w_m, C_n)$

Word Embeddings

We compute the word embeddings with a singular value decomposition, where we truncate the matrix $\mathbf{U\Sigma}$ to 50, 100, 300, or 500 dimensions. The word embeddings are the rows of this matrix.

We usually measure the similarity between two embeddings $\vec{u}$ and $\vec{v}$ with the cosine similarity:

$$\cos(\vec{u},\vec{v}) = \frac{\vec{u}\cdot\vec{v}}{||\vec{u}||\cdot||\vec{v}||},$$

ranging from -1 (most dissimilar) to 1 (most similar) or with the cosine distance ranging from 0 (closest) to 2 (most distant):

$$1-\cos(\vec{u},\vec{v}) = 1-\frac{\vec{u}\cdot\vec{v}}{||\vec{u}||\cdot||\vec{v}||}.$$

Popular Word Embeddings

Embeddings from large corpora are obtained with iterative techniques

Some popular embedding algorithms with open source programs:

- word2vec: `https://github.com/tmikolov/word2vec`
- GloVe: Global Vectors for Word Representation `https://nlp.stanford.edu/projects/glove/`
- ELMo: `https://allennlp.org/elmo`
- fastText: `https://fasttext.cc/`

To derive word embeddings, you will have to apply these programs on a very large corpus

Embeddings for many languages are also publicly available. You just download them

gensim is a Python library to create word embeddings from a corpus.
`https://radimrehurek.com/gensim/index.html`