

# EDAN20

## Language Technology

<http://cs.lth.se/edan20/>

### Chapter 5: Counting Words

Pierre Nugues

Lund University  
[Pierre.Nugues@cs.lth.se](mailto:Pierre.Nugues@cs.lth.se)  
[http://cs.lth.se/pierre\\_nugues/](http://cs.lth.se/pierre_nugues/)

September 7, 2015



# Counting Words and Word Sequences

Words have specific contexts of use.

Pairs of words like *strong* and *tea* or *powerful* and *computer* are not random associations.

Psychological linguistics tells us that it is difficult to make a difference between *writer* and *rider* without context

A listener will discard the improbable *rider of books* and prefer *writer of books*

A language model is the statistical estimate of a word sequence.

Originally developed for speech recognition

The language model component enables to predict the next word given a sequence of previous words: *the writer of books, novels, poetry*, etc. and not *the writer of hooks, nobles, poultry*, ...



# Getting the Words from a Text: Tokenization

Arrange a list of characters:

```
[l, i, s, t, ' ', o, f, ' ', c, h, a, r, a, c, t, e, r, s]
```

into words:

```
[list, of, characters]
```

Sometimes tricky:

- Dates: 28/02/96
- Numbers: 9,812.345 (English), 9 812,345 (French and German)  
9.812,345 (Old fashioned French)
- Abbreviations: km/h, m.p.h.,
- Acronyms: S.N.C.F.

Tokenizers use rules (or regexes) or statistical methods.



# Tokenizing in Perl

```
$text = <>;  
while ($line = <>) {  
    $text .= $line;  
}  
$text =~ tr/a-zâäãäæçéèêëîïôöœßùûüÿA-ZÀÂÃÄÅÆÇÈÉÊËÎÏÔÖŒÙÚÛÜÝ  
    '\-,.?!:; /\n/cs;  
$text =~ s/([, .?!:;]) /\n$1\n/g;  
$text =~ s/\n+ /\n/g;  
print $text;
```



## Elementary tokenizers:

```
String[] tokenize(String text) {  
    //String [] words = text.split("[\\s\\-,:;!?.'\"«»()-...&'\"'*-]+");  
    //String [] words =  
    // text.split("^a-zâãäåæçèéêëîïôöøßûüÿA-ZÀÁÂÃÄÅÇÈÉÊËÏÎÖÜŸ'\\-]+");  
    //String [] words = text.split("\\W+"); // Not unicode friendly  
    String[] words = text.split("\\P{L}+");  
    return words;  
}  
  
String tokenize2(String text) {  
    String words = text.replaceAll("\\P{L}+", " ");  
    return words;  
}
```



# Improving Tokenization

The tokenization algorithm is word-based and defines a content  
It does not work on nomenclatures such as Item #N23-SW32A, dates, or numbers

Instead it is possible to improve it using a boundary-based strategy with spaces (using for instance \s) and punctuation

But punctuation signs like commas, dots, or dashes can also be parts of tokens

Possible improvements using microgrammars

At some point, need of a dictionary:

*Can't* → can n't, *we'll* → we 'll

*J'aime* → j' aime but *aujourd'hui*



# Sentence Segmentation

As for tokenization, segmenters use either rules (or regexes) or statistical methods.

Grefenstette and Tapanainen (1994) used the Brown corpus and experimented increasingly complex rules

Most simple rule: a period corresponds to a sentence boundary: 93.20% correctly segmented

Recognizing numbers:

$[0-9]+(\backslash/[0-9]+)+$

Fractions, dates

$([+\backslash-])?[0-9]+(\backslash.)?[0-9]*\%$

Percent

$([0-9]+,(?)+(\backslash.[0-9]+|[0-9]+)*$

Decimal numbers

93.78% correctly segmented



# Abbreviations

Common patterns (Grefenstette and Tapanainen 1994):

- single capitals: *A.*, *B.*, *C.*,
- letters and periods: *U.S. i.e. m.p.h.*,
- capital letter followed by a sequence of consonants: *Mr. St. Assn.*

Regex	Correct	Errors	Full stop
[A-Za-z]\.	1,327	52	14
[A-Za-z]\. ([A-Za-z0-9]\. )+	570	0	66
[A-Z] [bcdfghj-np-tvxz]+\.	1,938	44	26
<b>Totals</b>	<b>3,835</b>	<b>96</b>	<b>106</b>

Correct segmentation increases to 97.66%  
 With an abbreviation dictionary to 99.07%





# N-Grams

The types are the distinct words of a text while the tokens are all the words or symbols.

The phrases from *Nineteen Eighty-Four*

*War is peace*

*Freedom is slavery*

*Ignorance is strength*

have 9 tokens and 7 types.

Unigrams are single words

Bigrams are sequences of two words

Trigrams are sequences of three words



# Trigrams

Word	Rank	More likely alternatives
<i>We</i>	9	<i>The This One Two A Three Please In</i>
<i>need</i>	7	<i>are will the would also do</i>
<i>to</i>	1	
<i>resolve</i>	85	<i>have know do. . .</i>
<i>all</i>	9	<i>the this these problems. . .</i>
<i>of</i>	2	<i>the</i>
<i>the</i>	1	
<i>important</i>	657	<i>document question first. . .</i>
<i>issues</i>	14	<i>thing point to. . .</i>
<i>within</i>	74	<i>to of and in that. . .</i>
<i>the</i>	1	
<i>next</i>	2	<i>company</i>
<i>two</i>	5	<i>page exhibit meeting day</i>
<i>days</i>	5	<i>weeks years pages months</i>



# Counting Words in Perl: Useful Features

Useful instructions and features: `split`, `sort`, and associative arrays (hash tables, dictionaries):

```
@words = split(/\n/, $text);
```

```
$wordcount{"a"} = 21;
```

```
$wordcount{"And"} = 10;
```

```
$wordcount{"the"} = 18;
```

```
keys %wordcount
```

```
sort array
```



# Counting Words in Perl

```
$text = <>;  
while ($line = <>) {  
    $text .= $line;  
}  
$text =~ tr/a-zâäâäæçéèêëîïôöœßùûüÿA-ZÀÂÃÄÅÆÇÈÉÊËÎÏÔÖŒÛÜŸ  
    '\-,.?!:; /\n /cs;  
$text =~ s/([, .?!:;]) /\n$1\n/g;  
$text =~ s/\n+ /\n/g;  
@words = split(/\n/, $text);
```



# Counting Words in Perl (Cont'd)

```
for ($i = 0; $i <= $#words; $i++) {  
    if (!exists($frequency{$words[$i]})) {  
        $frequency{$words[$i]} = 1;  
    } else {  
        $frequency{$words[$i]}++;  
    }  
}  
  
foreach $word (sort keys %frequency){  
    print "$frequency{$word} $word\n";  
}
```



# Counting Bigrams in Perl

```
@words = split(/\n/, $text);
for ($i = 0; $i < $#words; $i++) {
    $bigrams[$i] = $words[$i] . " " . $words[$i + 1];
}
for ($i = 0; $i < $#words; $i++) {
    if (!exists($frequency_bigrams{$bigrams[$i]})) {
        $frequency_bigrams{$bigrams[$i]} = 1;
    } else {
        $frequency_bigrams{$bigrams[$i]}++;
    }
}
foreach $bigram (sort keys %frequency_bigrams){
    print "$frequency_bigrams{$bigram} $bigram \n";
}
```



# Counting Words in Java: Useful Features

Useful instructions and data structures:

```
Map<String, Integer> count(String[] words) {  
    Map<String, Integer> counts = new TreeMap<>();  
    for (String word : words) {  
        if (counts.get(word) == null) {  
            counts.put(word, 1);  
        } else {  
            counts.put(word, counts.get(word) + 1);  
        }  
    }  
    return counts;  
}
```



# Counting Bigrams in Java

```
Map<String, Integer> count(String[] words) {  
    Map<String, Integer> counts = new TreeMap<>();  
    for (int i = 0; i < words.length - 1; i++) {  
        String bigram = words[i] + "\t" + words[i + 1];  
        if (counts.get(bigram) == null) {  
            counts.put(bigram, 1);  
        } else {  
            counts.put(bigram, counts.get(bigram) + 1);  
        }  
    }  
    return counts;  
}
```





# Counting Words With Unix Tools

❶ `tr -cs 'A-Za-z' '\n' <input_file |`

Tokenize the text in `input_file`, where `tr` behaves like Perl `tr`: We have one word per line and the output is passed to the next command.

❷ `tr 'A-Z' 'a-z' |`

Translate the uppercase characters into lowercase letters and pass the output to the next command.

❸ `sort |`

Sort the words. The identical words will be grouped in adjacent lines.

❹ `uniq -c |`

Remove repeated lines. The identical adjacent lines will be replaced with one single line. Each unique line in the output will be preceded by the count of its duplicates in the input file (`-c`).

❺ `sort -rn |`

Sort in the reverse (`-r`) numeric (`-n`) order: Most frequent words first

❻ `head -5`

Print the five first lines of the file (the five most frequent words)



# Counting Bigrams With Unix Tools

- ❶ `tr -cs 'A-Za-z' '\n' < input_file > token_file`  
Tokenize the input and create a file with the unigrams.
- ❷ `tail +2 < token_file > next_token_file`  
Create a second unigram file starting at the second word of the first tokenized file (+2).
- ❸ `paste token_file next_token_file > bigrams`  
Merge the lines (the tokens) pairwise. Each line of bigrams contains the words at index  $i$  and  $i + 1$  separated with a tabulation.
- ❹ And we count the bigrams as in the previous script.



# Probabilistic Models of a Word Sequence

$$\begin{aligned}P(S) &= P(w_1, \dots, w_n), \\&= P(w_1)P(w_2|w_1)P(w_3|w_1, w_2)\dots P(w_n|w_1, \dots, w_{n-1}), \\&= \prod_{i=1}^n P(w_i|w_1, \dots, w_{i-1}).\end{aligned}$$

The probability  $P(\textit{It was a bright cold day in April})$  from *Nineteen Eighty-Four* corresponds to

$\textit{It}$  to begin the sentence, then  $\textit{was}$  knowing that we have  $\textit{It}$  before, then  $\textit{a}$  knowing that we have  $\textit{It was}$  before, and so on until the end of the sentence.

$$\begin{aligned}P(S) &= P(\textit{It}) \times P(\textit{was}|\textit{It}) \times P(\textit{a}|\textit{It}, \textit{was}) \times P(\textit{bright}|\textit{It}, \textit{was}, \textit{a}) \times \dots \\&\quad \times P(\textit{April}|\textit{It}, \textit{was}, \textit{a}, \textit{bright}, \dots, \textit{in}).\end{aligned}$$



# Approximations

Bigrams:

$$P(w_i | w_1, w_2, \dots, w_{i-1}) \approx P(w_i | w_{i-1}),$$

Trigrams:

$$P(w_i | w_1, w_2, \dots, w_{i-1}) \approx P(w_i | w_{i-2}, w_{i-1}).$$

Using a trigram language model,  $P(S)$  is approximated as:

$$P(S) \approx P(It) \times P(was|It) \times P(a|It, was) \times P(bright|was, a) \times \dots \\ \times P(April|day, in).$$



# Maximum Likelihood Estimate

Bigrams:

$$P_{MLE}(w_i|w_{i-1}) = \frac{C(w_{i-1}, w_i)}{\sum_w C(w_{i-1}, w)} = \frac{C(w_{i-1}, w_i)}{C(w_{i-1})}.$$

Trigrams:

$$P_{MLE}(w_i|w_{i-2}, w_{i-1}) = \frac{C(w_{i-2}, w_{i-1}, w_i)}{C(w_{i-2}, w_{i-1})}.$$



# Conditional Probabilities

A common mistake in computing the conditional probability  $P(w_i|w_{i-1})$  is to use

$$\frac{C(w_{i-1}, w_i)}{\# \text{bigrams}}.$$

This is not correct. This formula corresponds to  $P(w_{i-1}, w_i)$ .  
The correct estimation is

$$P_{MLE}(w_i|w_{i-1}) = \frac{C(w_{i-1}, w_i)}{\sum_w C(w_{i-1}, w)} = \frac{C(w_{i-1}, w_i)}{C(w_{i-1})}.$$

Proof:

$$P(w_1, w_2) = P(w_1)P(w_2|w_1) = \frac{C(w_1)}{\# \text{words}} \times \frac{C(w_1, w_2)}{C(w_1)} = \frac{C(w_1, w_2)}{\# \text{words}}$$



# Training the Model

The model is trained on a part of the corpus: the **training set**

It is tested on a different part: the **test set**

The vocabulary can be derived from the corpus, for instance the 20,000 most frequent words, or from a lexicon

It can be closed or open

A closed vocabulary does not accept any new word

An open vocabulary maps the new words, either in the training or test sets, to a specific symbol, <UNK>



# Probability of a Sentence: Unigrams

*<s> A good deal of the literature of the past was, indeed, already being transformed in this way </s>*

$w_i$	$C(w_i)$	#words	$P_{MLE}(w_i)$
<s>	7072	–	
<i>a</i>	2482	115212	0.023
<i>good</i>	53	115212	0.00049
<i>deal</i>	5	115212	$4.62 \cdot 10^{-5}$
<i>of</i>	3310	115212	0.031
<i>the</i>	6248	115212	0.058
<i>literature</i>	7	115212	$6.47 \cdot 10^{-5}$
<i>of</i>	3310	115212	0.031
<i>the</i>	6248	115212	0.058
<i>past</i>	99	115212	0.00092
<i>was</i>	2211	115212	0.020
<i>indeed</i>	17	115212	0.00016
<i>already</i>	64	115212	0.00059
<i>being</i>	80	115212	0.00074
<i>transformed</i>	1	115212	$9.25 \cdot 10^{-6}$
<i>in</i>	1759	115212	0.016
<i>this</i>	264	115212	0.0024
<i>way</i>	122	115212	0.0011
</s>	7072	115212	0.065





# Probability of a Sentence: Bigrams

*<s> A good deal of the literature of the past was, indeed, already being transformed in this way </s>*

$w_{i-1}, w_i$	$C(w_{i-1}, w_i)$	$C(w_{i-1})$	$P_{MLE}(w_i   w_{i-1})$
<i>&lt;s&gt; a</i>	133	7072	0.019
<i>a good</i>	14	2482	0.006
<i>good deal</i>	0	53	0.0
<i>deal of</i>	1	5	0.2
<i>of the</i>	742	3310	0.224
<i>the literature</i>	1	6248	0.0002
<i>literature of</i>	3	7	0.429
<i>of the</i>	742	3310	0.224
<i>the past</i>	70	6248	0.011
<i>past was</i>	4	99	0.040
<i>was indeed</i>	0	2211	0.0
<i>indeed already</i>	0	17	0.0
<i>already being</i>	0	64	0.0
<i>being transformed</i>	0	80	0.0
<i>transformed in</i>	0	1	0.0
<i>in this</i>	14	1759	0.008
<i>this way</i>	3	264	0.011
<i>way &lt;/s&gt;</i>	18	122	0.148



# Sparse Data

Given a vocabulary of 20,000 types, the potential number of bigrams is  $20,000^2 = 400,000,000$

With trigrams  $20,000^3 = 8,000,000,000,000$

Methods:

- Laplace: add one to all counts
- Linear interpolation:

$$P_{\text{DelInterpolation}}(w_n | w_{n-2}, w_{n-1}) = \lambda_1 P_{MLE}(w_n | w_{n-2} w_{n-1}) + \lambda_2 P_{MLE}(w_n | w_{n-1}) + \lambda_3 P_{MLE}(w_n),$$

- Good-Turing: The discount factor is variable and depends on the number of times a n-gram has occurred in the corpus.
- Back-off



# Laplace's Rule

$$P_{Laplace}(w_{i+1}|w_i) = \frac{C(w_i, w_{i+1}) + 1}{\sum_w (C(w_i, w) + 1)} = \frac{C(w_i, w_{i+1}) + 1}{C(w_i) + \text{Card}(V)},$$

$w_i, w_{i+1}$	$C(w_i, w_{i+1}) + 1$	$C(w_i) + \text{Card}(V)$	$P_{Lap}(w_{i+1} w_i)$
<s> a	133 + 1	7072 + 8635	0.0085
a good	14 + 1	2482 + 8635	0.0013
good deal	0 + 1	53 + 8635	0.00012
deal of	1 + 1	5 + 8635	0.00023
of the	742 + 1	3310 + 8635	0.062
the literature	1 + 1	6248 + 8635	0.00013
literature of	3 + 1	7 + 8635	0.00046
of the	742 + 1	3310 + 8635	0.062
the past	70 + 1	6248 + 8635	0.0048
past was	4 + 1	99 + 8635	0.00057
was indeed	0 + 1	2211 + 8635	0.000092
indeed already	0 + 1	17 + 8635	0.00012
already being	0 + 1	64 + 8635	0.00011
being transformed	0 + 1	80 + 8635	0.00011
transformed in	0 + 1	1 + 8635	0.00012
in this	14 + 1	1759 + 8635	0.0014
this way	3 + 1	264 + 8635	0.00045
way </s>	18 + 1	122 + 8635	0.0022



# Good–Turing

Laplace's rule shifts an enormous mass of probability to very unlikely bigrams. Good–Turing's estimation is more effective

Let's denote  $N_c$  the number of n-grams that occurred exactly  $c$  times in the corpus.

$N_0$  is the number of unseen n-grams,  $N_1$  the number of n-grams seen once,  $N_2$  the number of n-grams seen twice. The frequency of n-grams occurring  $c$  times is re-estimated as:

$$c^* = (c + 1) \frac{E(N_{c+1})}{E(N_c)},$$

Unseen n-grams:  $c^* = \frac{N_1}{N_0}$  and N-grams seen once:  $c^* = \frac{2N_2}{N_1}$ .



# Good-Turing for *Nineteen eighty-four*

*Nineteen eighty-four* contains 37,365 unique bigrams and 5,820 bigram seen twice.

Its vocabulary of 8,635 words generates  $8635^2 = 74,563,225$  bigrams whose 74,513,701 are unseen.

New counts:

- Unseen bigrams:  $\frac{37,365}{74,513,701} = 0.0005$ .
- Unique bigrams:  $2 \times \frac{5820}{37,365} = 0.31$ .
- Etc.

Freq. of occ.	$N_c$	$c^*$	Freq. of occ.	$N_c$	$c^*$
0	74,513,701	0.0005	5	719	3.91
1	37,365	0.31	6	468	4.94
2	5,820	1.09	7	330	6.06
3	2,111	2.02	8	250	6.44
4	1,067	3.37	9	179	8.93



# Backoff

If there is no bigram, then use unigrams:

$$P_{\text{Backoff}}(w_i | w_{i-1}) = \begin{cases} \tilde{P}(w_i | w_{i-1}), & \text{if } C(w_{i-1}, w_i) \neq 0, \\ \alpha P(w_i), & \text{otherwise.} \end{cases}$$

$$P_{\text{Backoff}}(w_i | w_{i-1}) = \begin{cases} P_{\text{MLE}}(w_i | w_{i-1}) = \frac{C(w_{i-1}, w_i)}{C(w_{i-1})}, & \text{if } C(w_{i-1}, w_i) \neq 0, \\ P_{\text{MLE}}(w_i) = \frac{C(w_i)}{\# \text{words}}, & \text{otherwise.} \end{cases}$$



# Backoff: Example

$w_{i-1}, w_i$	$C(w_{i-1}, w_i)$	$C(w_i)$	$P_{\text{Backoff}}(w_i   w_{i-1})$
<s>		7072	—
<s> a	133	2482	0.019
a good	14	53	0.006
good deal	0	5	$4.62 \cdot 10^{-5}$
deal of	1	3310	0.2
of the	742	6248	0.224
the literature	1	7	0.00016
literature of	3	3310	0.429
of the	742	6248	0.224
the past	70	99	0.011
past was	4	2211	0.040
was indeed	0	17	0.00016
indeed already	0	64	0.00059
already being	0	80	0.00074
being transformed	0	1	$9.25 \cdot 10^{-6}$
transformed in	0	1759	0.016
in this	14	264	0.008
this way	3	122	0.011
way </s>	18	7072	0.148

The figures we obtain are not probabilities. We can use the Good-Turing technique to discount the bigrams and then scale the unigram probabilities. This is the Katz backoff.



# Quality of a Language Model

Per word probability of a word sequence:  $H(L) = -\frac{1}{n} \log_2 P(w_1, \dots, w_n)$ .

Entropy rate:  $H_{rate} = -\frac{1}{n} \sum_{w_1, \dots, w_n \in L} p(w_1, \dots, w_n) \log_2 p(w_1, \dots, w_n)$ ,

Cross entropy:

$$H(p, m) = -\frac{1}{n} \sum_{w_1, \dots, w_n \in L} p(w_1, \dots, w_n) \log_2 m(w_1, \dots, w_n).$$

We have:

$$\begin{aligned} H(p, m) &= \lim_{n \rightarrow \infty} -\frac{1}{n} \sum_{w_1, \dots, w_n \in L} p(w_1, \dots, w_n) \log_2 m(w_1, \dots, w_n), \\ &= \lim_{n \rightarrow \infty} -\frac{1}{n} \log_2 m(w_1, \dots, w_n). \end{aligned}$$

We compute the cross entropy on the complete word sequence of a test set, governed by  $p$ , using a bigram or trigram model,  $m$ , from a training set.

Perplexity:

$$PP(p, m) = 2^{H(p, m)}.$$





# Other Statistical Formulas

- Mutual information (The strength of an association):

$$I(w_i, w_j) = \log_2 \frac{P(w_i, w_j)}{P(w_i)P(w_j)} \approx \log_2 \frac{N \cdot C(w_i, w_j)}{C(w_i)C(w_j)}.$$

- T-score (The confidence of an association):

$$\begin{aligned} t(w_i, w_j) &= \frac{\text{mean}(P(w_i, w_j)) - \text{mean}(P(w_i))\text{mean}(P(w_j))}{\sqrt{\sigma^2(P(w_i, w_j)) + \sigma^2(P(w_i)P(w_j))}}, \\ &\approx \frac{C(w_i, w_j) - \frac{1}{N}C(w_i)C(w_j)}{\sqrt{C(w_i, w_j)}}. \end{aligned}$$



# T-Scores with Word *set*

Word	Frequency	Bigram <i>set</i> + word	<i>t</i> -score
<i>up</i>	134,882	5512	67.980
<i>a</i>	1,228,514	7296	35.839
<i>to</i>	1,375,856	7688	33.592
<i>off</i>	52,036	888	23.780
<i>out</i>	12,3831	1252	23.320

Source: Bank of English



# Mutual Information with Word *surgery*

Word	Frequency	Bigram word + <i>surgery</i>	Mutual info
<i>arthroscopic</i>	3	3	11.822
<i>pioneering</i>	3	3	11.822
<i>reconstructive</i>	14	11	11.474
<i>refractive</i>	6	4	11.237
<i>rhinoplasty</i>	5	3	11.085

Source: Bank of English



# Mutual Information and T-Scores in Perl

...

```
@words = split(/\n/, $text);
for ($i = 0; $i < $#words; $i++) {
    $bigrams[$i] = $words[$i] . " " . $words[$i + 1];
}
for ($i = 0; $i <= $#words; $i++) {
    $frequency{$words[$i]}++;
}
for ($i = 0; $i < $#words; $i++) {
    $frequency_bigrams{$bigrams[$i]}++;
}
```



# Mutual Information in Perl

```
for ($i = 0; $i < $#words; $i++) {  
    $mutual_info{$bigrams[$i]} = log(($#words + 1) *  
        $frequency_bigrams{$bigrams[$i]}/  
        ($frequency{$words[$i]} * $frequency{$words[$i + 1]}))/  
        log(2);  
}  
  
foreach $bigram (keys %mutual_info){  
    @bigram_array = split(/ /, $bigram);  
    print $mutual_info{$bigram}, " ", $bigram, "\t",  
        $frequency_bigrams{$bigram}, "\t",  
        $frequency{$bigram_array[0]}, "\t",  
        $frequency{$bigram_array[1]}, "\n";  
}
```



# T-Scores in Perl

```
for ($i = 0; $i < $#words; $i++) {  
    $t_scores{$bigrams[$i]} = ($frequency_bigrams{$bigrams[$i]}  
        - $frequency{$words[$i]} *  
        $frequency{$words[$i + 1]} / ($#words + 1)) /  
        sqrt($frequency_bigrams{$bigrams[$i]});  
}  
  
foreach $bigram (keys %t_scores) {  
    @bigram_array = split(/ /, $bigram);  
    print $t_scores{$bigram}, " ", $bigram, "\t",  
        $frequency_bigrams{$bigram}, "\t",  
        $frequency{$bigram_array[0]}, "\t",  
        $frequency{$bigram_array[1]}, "\n";  
}
```



# Information Retrieval: The Vector Space Model

The vector space model represents a document in a space of words.

Documents \ Words	$w_1$	$w_2$	$w_3$	...	$w_m$
$D_1$	$C(w_1, D_1)$	$C(w_2, D_1)$	$C(w_3, D_1)$	...	$C(w_m, D_1)$
$D_2$	$C(w_1, D_2)$	$C(w_2, D_2)$	$C(w_3, D_2)$	...	$C(w_m, D_2)$
...					
$D_n$	$C(w_1, D_n)$	$C(w_2, D_n)$	$C(w_3, D_n)$	...	$C(w_m, D_n)$

It was created for information retrieval to compute the similarity of two documents or to match a document and a query.

We compute the similarity of two documents through their dot product.



# The Vector Space Model: Example

A collection of two documents D1 and D2:

D1: Chrysler plans new investments in Latin America.

D2: Chrysler plans major investments in Mexico.

The vectors representing the two documents:

D.	america	chrysler	in	investments	latin	major	mexico	new	plans
1	1	1	1	1	1	0	0	1	1
2	0	1	1	1	0	1	1	0	1

The vector space model represents documents as bags of words (BOW) that do not take the word order into account.

The dot product is  $\vec{D1} \cdot \vec{D2} = 0 + 1 + 1 + 1 + 0 + 0 + 0 + 0 + 1 = 4$

Their cosine is  $\frac{\vec{D1} \cdot \vec{D2}}{\|\vec{D1}\| \cdot \|\vec{D2}\|} = \frac{4}{\sqrt{7} \cdot \sqrt{6}} = 0.62$





Word clouds give visual weights to words



Image: Courtesy of Jonas Wisbrant

# $TF \times IDF$

The frequency alone might be misleading

Document coordinates are in fact  $tf \times idf$ : Term frequency by inverted document frequency.

Term frequency  $tf_{i,j}$ : frequency of term  $j$  in document  $i$

Inverted document frequency:  $idf_j = \log\left(\frac{N}{n_j}\right)$



# Document Similarity

Documents are vectors where coordinates could be the count of each word:  
 $\vec{d} = (C(w_1), C(w_2), C(w_3), \dots, C(w_n))$

The similarity between two documents or a query and a document is given by their cosine:

$$\cos(\vec{q}, \vec{d}) = \frac{\sum_{i=1}^n q_i d_i}{\sqrt{\sum_{i=1}^n q_i^2} \sqrt{\sum_{i=1}^n d_i^2}}.$$



# Posting Lists

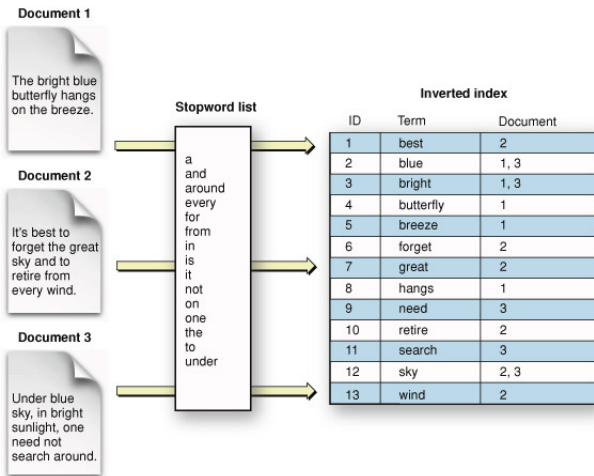
Many websites, such as Wikipedia, index their texts using an inverted index. Each word in the dictionary is linked to a posting list that gives all the documents where this word occurs and its positions in a document.

Words	Posting lists
<i>America</i>	(D1, 7)
<i>Chrysler</i>	(D1, 1) → (D2, 1)
<i>in</i>	(D1, 5) → (D2, 5)
<i>investments</i>	(D1, 4) → (D2, 4)
<i>Latin</i>	(D1, 6)
<i>major</i>	(D2, 3)
<i>Mexico</i>	(D2, 6)
<i>new</i>	(D1, 3)
<i>plans</i>	(D1, 2) → (D2, 2)

Lucene is a high quality open-source indexer.  
(<http://lucene.apache.org/>)



# Inverted Index (Source Apple)



<http://developer.apple.com/library/mac/documentation/UserExperience/Conceptual/SearchKitConcepts/index.html>

