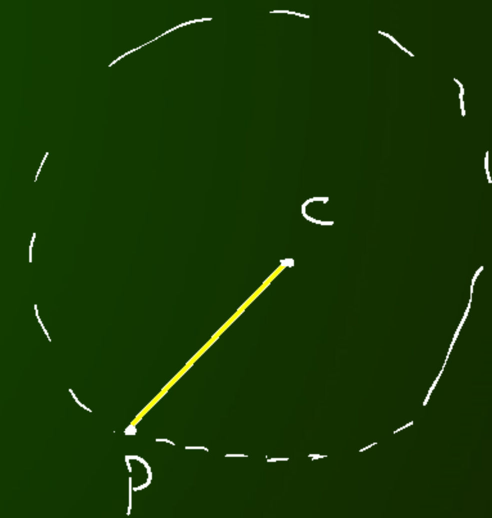
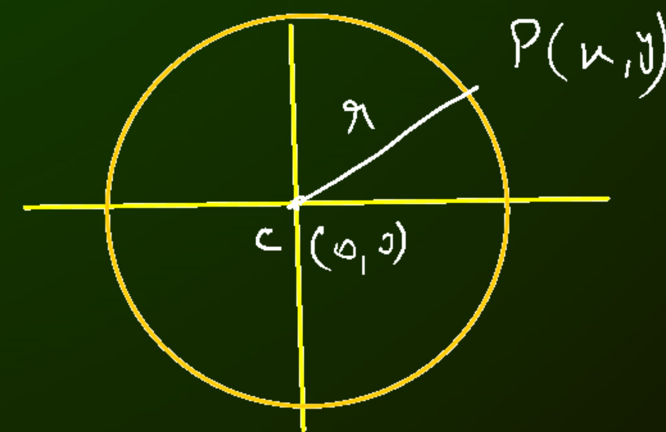
Circles

Def^n Locus of a pt. whose distance from a fixed pt. 'c' is said to be circle provided cP is constant.



Note: Fixed pt is said to be center & fixed distance is said to be radius.



$$cP = r$$

$$\sqrt{x^2+y^2} = r$$

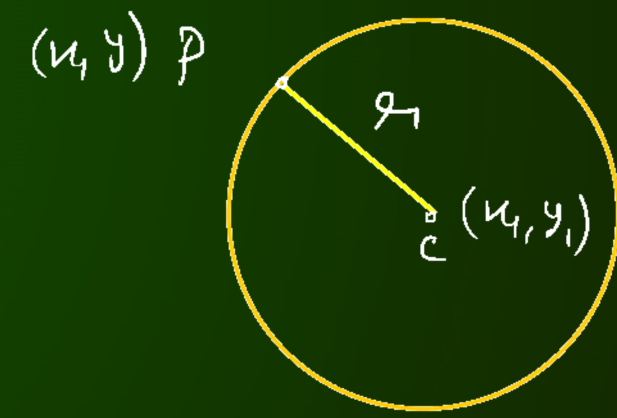
$$\Rightarrow \boxed{x^2+y^2 = r^2} \;;$$

$$CP = r$$

$$(x-x_1)^2 + (y-y_1)^2 = r^2$$

$$x^2 + x_1^2 - 2x_1x + y^2 + y_1^2 - 2y_1y - r^2 = 0$$

$$x^2 + y^2 - 2x_1x - 2y_1y + (x_1^2 + y_1^2 - r^2) = 0$$



$c = (1, 2)$, $r = 5$

$P = (x, y)$

$$CP = 5$$

$$(x-1)^2 + (y-2)^2 = 5^2$$

$$\Rightarrow x^2 + 1 - 2x + y^2 + 4 - 4y = 25$$

$$\Rightarrow \boxed{x^2 + y^2 - 2x - 4y - 20 = 0}$$

$$S \equiv ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$$

$$\Delta = 0 \; ; \; \Delta \neq 0$$

General second degree eq. of a circle:-

$S \equiv ax^2+by^2+2hxy+2gx+2fy+c=0$ is said to be a circle

if i) $\Delta \neq 0$

ii) $a=b$

iii) $h=0$

$\frac{1}{4}+\frac{1}{9}-1$

$=\frac{9+4-36}{36}$

$\left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{3}\right)^2=1$

$x^2+\frac{1}{4}-x+y^2+\frac{1}{9}-\frac{2}{3}y-1=0$

$x^2+y^2-x-\frac{2}{3}y-\frac{23}{36}=0$

or $\boxed{36x^2+36y^2-36x-24y-23=0}$

$$\begin{vmatrix} 36 & 0 & -18 \\ 0 & 36 & -12 \\ -18 & -12 & -23 \end{vmatrix} = 36(-36\times 23 - 144) - 0 - 18(0 + 18\times 36) = -ve \neq 0$$

Q.) verify $x^2+y^2-6x+2y+1=0$

Soln

$a=b$ ✓

$h=0$ ✓

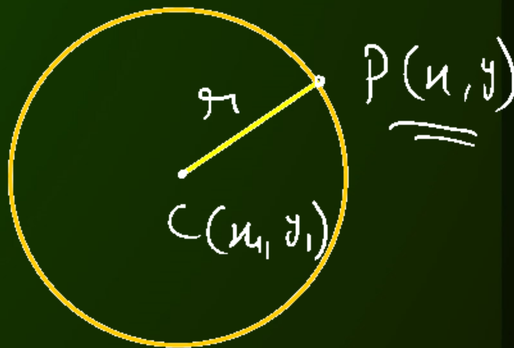
$$\Delta = \begin{vmatrix} 1 & 0 & -3 \\ 0 & 1 & 1 \\ -3 & 1 & 1 \end{vmatrix} = 1(\cancel{1}-\cancel{1}) - 0() - 3(0+3)$$

$$= -9 \neq 0$$

Various forms of a circle:

1:) center - radius form:

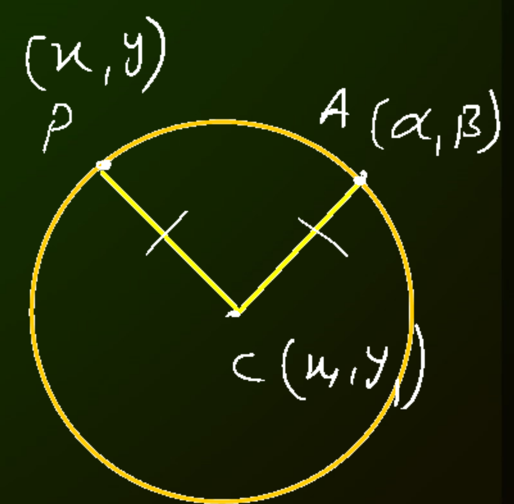
Eq. of the circle with center $C(x_1, y_1)$ and radius 'r' is



$CP = r$

$$\boxed{(x-x_1)^2 + (y-y_1)^2 = r^2}$$

Note: If $C = (0,0)$ then circle is $x^2 + y^2 = r^2$



2:) Center & a pt on the circumference is given

Eq. of a cle passing through $A(\alpha, \beta)$ and having center $C(x_1, y_1)$ is

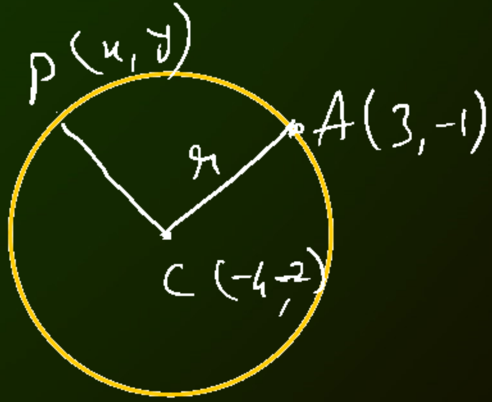
$$CP = CA$$

$$\Rightarrow \boxed{(x-x_1)^2+(y-y_1)^2 = (\alpha-x_1)^2+(\beta-y_1)^2}$$

Q.) Find the eq. of a circle whose $c=(1,2)$ & $r=3$

Sol^n $x^2+y^2-2x-4y-4=0$

Q.) Find the eq. of a circle which is passing through $A(3,-1)$ and having center $(-4,-2)$



Sol^n $x^2+y^2+8x+4y-30=0$

Q) Find the eq. of the circle whose center be the pt. of intersection of $3x-y-2=0$; $5x+y=6$ and having radius '2'

Sol'n $c=(1,1)$, $P=(x,y)$

$cP = r$

$(x-1)^2+(y-1)^2 = 2^2$

$x^2+y^2-2x-2y+2 = 4$

$\therefore\ x^2+y^2-2x-2y-2=0$

Q) Find the eq. of a circle whose center is $(12,-5)$ and having area of the circle 49π.

Sol'n $A=\pi r^2 \Rightarrow 49\pi = \pi r^2 \Rightarrow \boxed{r=7}$

$\therefore$ eq of cle is

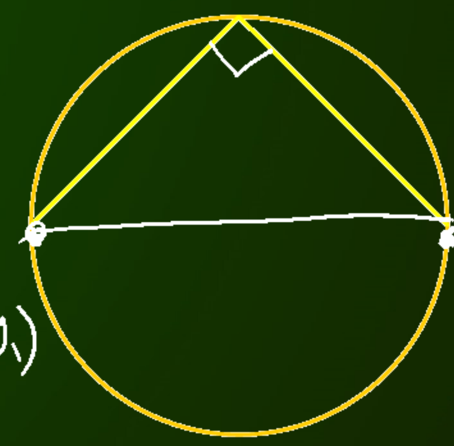
$(x-12)^2+(y+5)^2 = 7^2$

3) Diameter form:-

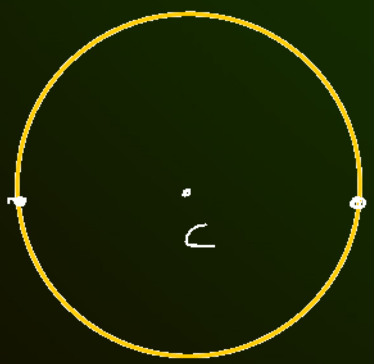
Eq. of the circle whose ends of the diameter

$A(x_1, y_1)$ & $B(x_2, y_2)$ is

$$\boxed{(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0}$$



Ex:- $A(3,6)$, $B(-1,4)$



$c=(1,5)$

$r = BC = \sqrt{2^2+1^2} = \sqrt{5}$

$(x-1)^2+(y-5)^2=(\sqrt{5})^2$

$\Rightarrow x^2+1-2x+y^2+25-10y=5$

$\Rightarrow x^2+y^2-2x-10y+21=0$

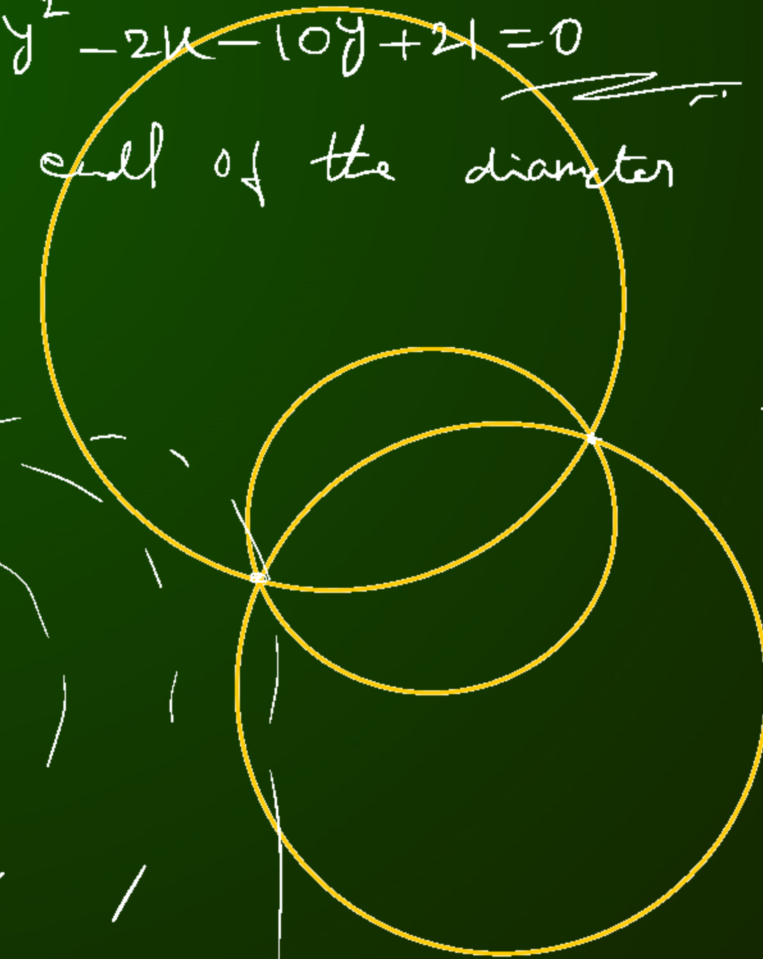
Alt:- $(x-3)(x+1)+(y-6)(y-4)=0 \Rightarrow x^2+x-3x-3+y^2-4y-6y+24=0$

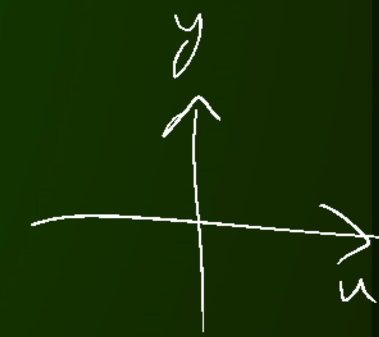
$x^2+y^2-2x-10y+21=0$

Q.) Find the eq. of the circle whose ends of the diameter

$A(a,0)$, $B(0,b)$

Soln $x(x-a)+y(y-b)=0$



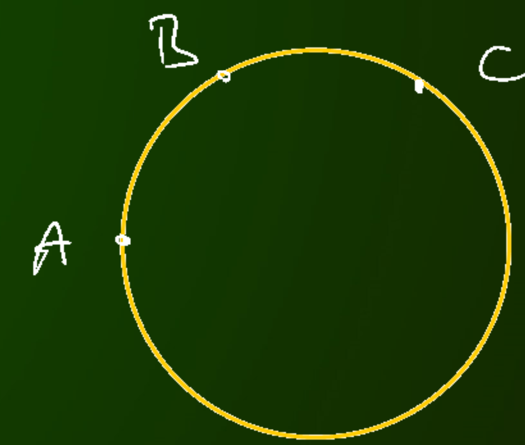


4.) 3 point form:-

$x^2+y^2+2gx+2fy+c=0$

Eq. of the ○le passing through $A(x_1, y_1)$, $B(x_2, y_2)$ & $C(x_3, y_3)$ is

$x^2+y^2+2gx+2fy+c=0$



Ex: Find the eq. of the ○le passing through

$A(1,1)$, $B(1,2)$, $C(2,1)$

Sol: Let $x^2+y^2+2gx+2fy+c=0$ — (1)

$A \in (1) \Rightarrow 1+1+2g+2f+c=0 \Rightarrow 2g+2f+c=-2$ — (2)

$B \in (1) \Rightarrow 1+4+2g+4f+c=0 \Rightarrow 2g+4f+c=-5$ — (3)

$C \in (1) \Rightarrow 4+1+4g+2f+c=0 \Rightarrow 4g+2f+c=-5$ — (4)

$(2)-(3) \Rightarrow -2f=3 \Rightarrow f=-\frac{3}{2}$

$(1) - (4) \Rightarrow -2g + 2f = 0 \Rightarrow f = g = -\frac{3}{2}$

$(2) \Rightarrow 2g + 2f + c = -2 \Rightarrow -3 - 3 + c = -2 \Rightarrow \boxed{c = 4}$

$\therefore x^2 + y^2 - 3x - 3y + 4 = 0$

Q.) $(2,-1), (3,2), (-1,0)$

$x^2 + y^2 - 3x + \cancel{y} + \cancel{2} = 0$ ✗

$4g - 2f + c = -5$

$6g + 4f + c = -13$

$-2g + c = -1$

$2g + 6f = -8$

$8g + 4f = -12$

$8g + 24f = -32$

$-20f = 20$

$f = -1$

$g = -1$

$c = -3$

$\boxed{x^2 + y^2 - 2x - 2y - 3 = 0}$ ✓

$\cancel{4} + 1 - \cancel{4} + 2 - 3 \quad 9 + 4 - 6 - 4 - 3 \quad 1 + 2 - 3$

5) General circle form:-

Eq. of a circle which is in the form of $x^2+y^2+2gx+2fy+c=0$ is said to be general form with center $(-g,-f)$

radius $=\sqrt{g^2+f^2-c}$

$a^2+2ab+b^2=(a+b)^2$

$a^2+2ab=(a+b)^2-b^2$

$x^2+2ax=(x+a)^2-a^2$

$x^2+ax=\left(x+\frac{a}{2}\right)^2-\left(\frac{a}{2}\right)^2$

$$\underbrace{x^2+2gx}+y^2+2fy+c=0$$

$$(x+g)^2-g^2+(y+f)^2-f^2+c=0$$

$$\Rightarrow \quad (x+g)^2+(y+f)^2=g^2+f^2-c$$

$x-(-g)$

$x-x_1$

$$(x-x_1)^2+(y-y_1)^2=r^2$$

Center $=(-g,-f)$

radius $=\sqrt{g^2+f^2-c}$

Q.) Find the center and radius of the circle $x^2+y^2-6x+8y-2=0$

Sol'n $C=(3,-4)$

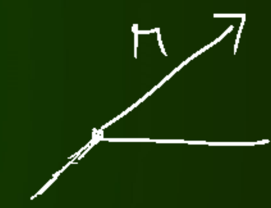
$$r=\sqrt{(3)^2+(-4)^2+2} = \sqrt{27}$$

Q.) $x^2+y^2+10x+2y-1=0$

$(-5,-1)$; $r=\sqrt{27}$

6.) Parametric form of a circle

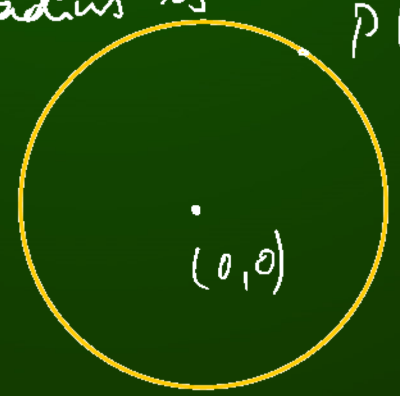
$$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r \quad , \quad \theta \in [0, 2\pi)$$



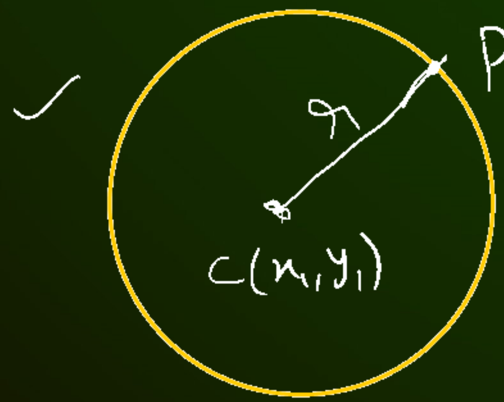
where (x_1, y_1) be the center and radius is 'r'

Any pt. on this circle is having

co-ordinates as P(



$p(x, y) = (r\cos\theta, r\sin\theta)$



$P\,(x_1 + r\cos\theta,\ y_1 + r\sin\theta)$

$x^2+y^2+2gx+2fy+c=0$

$c=(-g,-f)$

$r=\sqrt{g^2+f^2-c}$

$\therefore\ x^2+y^2+2gx+2fy+g^2=0$

✓

✓

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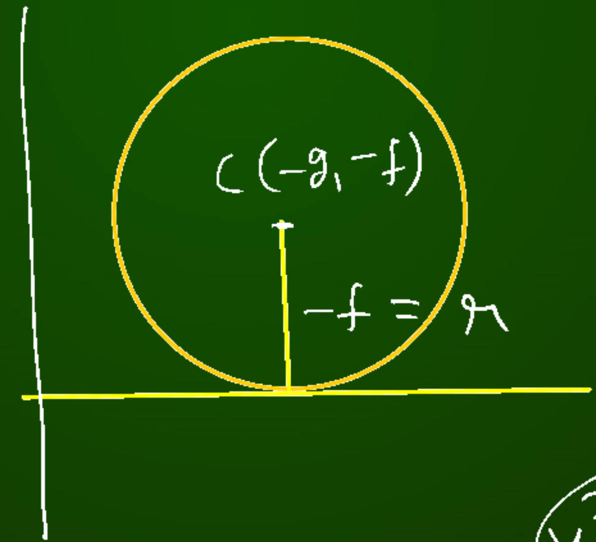


$-f=r$

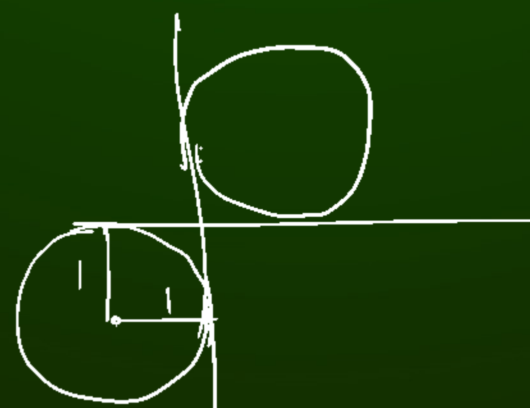
$\cancel{f^2}=g^2+\cancel{f^2}-c$

$\boxed{g^2=c}$

$x^2+y^2-4x+9y+4=0$

$x^2+y^2-4x-6y+9=0$ ✓

$x^2+y^2-6x-6y+9=0$



$(x+1)^2+(y+1)^2=1^2$

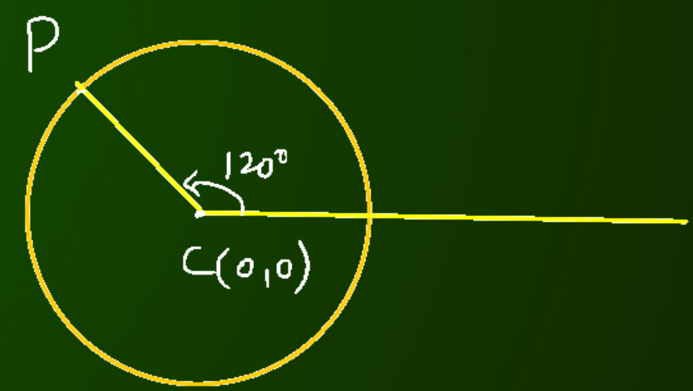
$x^2+1+2x+y^2+\cancel{1}+2y=\cancel{1}$

$x^2+y^2+2x+2y+1=0$

$c=(-1,-1)$ ✓

$r=1$ ✓

Q.) Find the point on the circle $x^2+y^2=16$ at an inclanation of angle $\underline{\underline{120^\circ}}$



Sol'n. $\boxed{P(x_1+r\cos\theta,\ y_1+r\sin\theta)}$

$P(0+4\cos 120^\circ,\ 0+4\sin 120^\circ)$

$= P\left(4\left(\frac{-1}{2}\right),\ 4\left(\frac{\sqrt{3}}{2}\right)\right)$

$= P(-2,\ 2\sqrt{3})$

Q.) $x^2+y^2-2x+6y+1=0$, $\theta = 30^\circ$

$c=(1,-3)$; $r=\sqrt{\cancel{1}+9-\cancel{1}}=3$

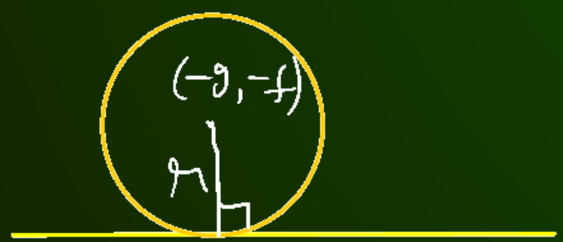
$P\left(1+3\left(\frac{\sqrt{3}}{2}\right),\ -3+3\left(\frac{1}{2}\right)\right) = \left(1+\frac{3\sqrt{3}}{2},\ -\frac{3}{2}\right)$

Condition for a circle which touches co-ordinate Axis:-

$$S \equiv x^2+y^2+2gx+2fy+c=0$$

$C=(-g,-f)$

i) **S=0 Touches x-axis** :- $r=\sqrt{g^2+f^2-c}$

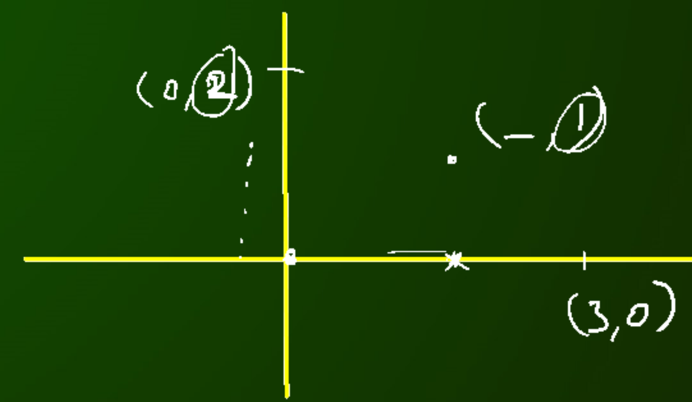


$$\boxed{-f=r}$$

$$\Rightarrow f^2=r^2$$

$$\cancel{f^2}=g^2+\cancel{f^2}-c$$

$$\boxed{c=g^2}$$



$$x^2+y^2+2gx+2fy+f^2=0$$

$$x^2+2gx+(y+f)^2=0$$

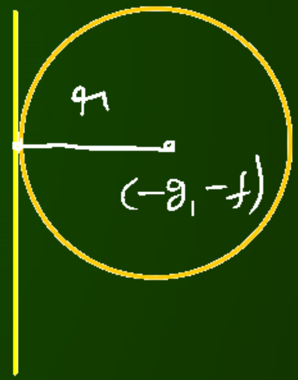
$\therefore \quad x^2+y^2+2gx+2fy+g^2=0$

Note: $(x+g)^2+y^2+2fy=0$, to x-axis

ii) $S=0$ touches Y-axis :-

$$\boxed{-g = r} \quad \& \quad \boxed{f^2 = c}$$

$$\cancel{g^2} = \cancel{g^2} + f^2 - c$$



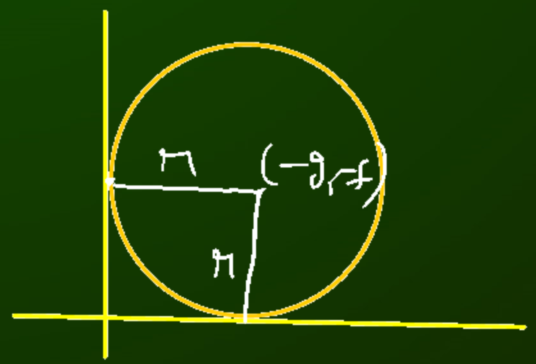
$x^2 + y^2 + 4x + 6y + 9 = 0$

$x^2 + 4x + (y+3)^2 = 0$

eq. of such one is $x^2 + y^2 + 2gx + 2fy + f^2 = 0$

iii) $S=0$ touches both the Axis :-

$$-g = -f = r \quad \& \quad c = \boxed{f^2 = g^2}$$



$x^2 + y^2 + 6x + 6y + 9 = 0$

$(x+3)^2 + y^2 + 6y = 0$

$x^2 + 6x + (y+3)^2 = 0$

$x^2 + y^2 + 2gx + 2gy + g^2 = 0$

&

$x^2 + y^2 + 2gx - 2gy + g^2 = 0$

Ex:

i) $x^2+y^2-16x+8y+64=0$ touches x-axis

ii) $x^2+y^2-16x-6y+9=0$ " y-axis

iii) $x^2+y^2-16x-16y+64=0$ " both the axis in I-Quadrant

iv) $x^2+y^2+4x+4y+4=0$ " " " III - "

v) $x^2+y^2-4x+4y+4=0$ " " " IV - "

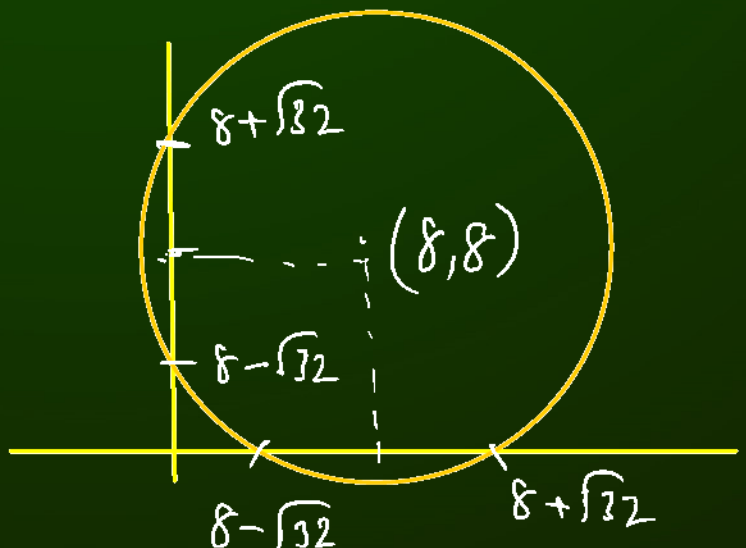
$c=(2,-2)$

vi) $x^2+y^2-16x-16y+32=0$

$(x-8)^2+y^2-16y+32$

$x^2-16x+32=0$

$(x-8)^2-32=0 \Rightarrow \boxed{x=8\pm\sqrt{32}}$

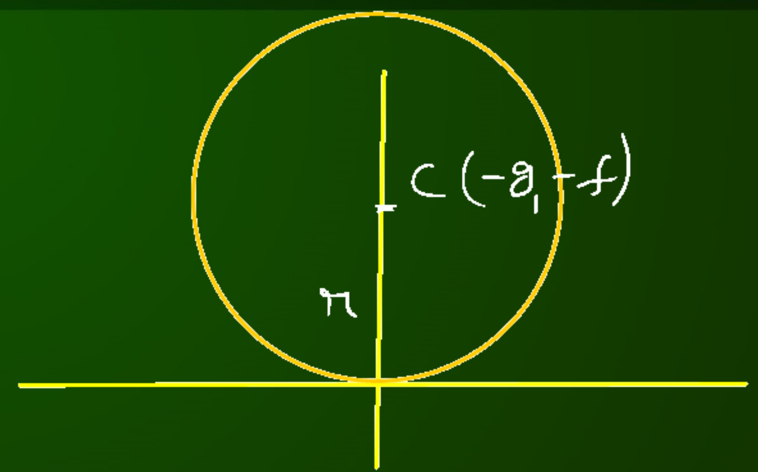


iv) Touching x-axis at O(0,0)

$$-g = 0$$

$$c = g^2 = 0$$

$$\boxed{x^2 + y^2 + 2fy = 0}$$

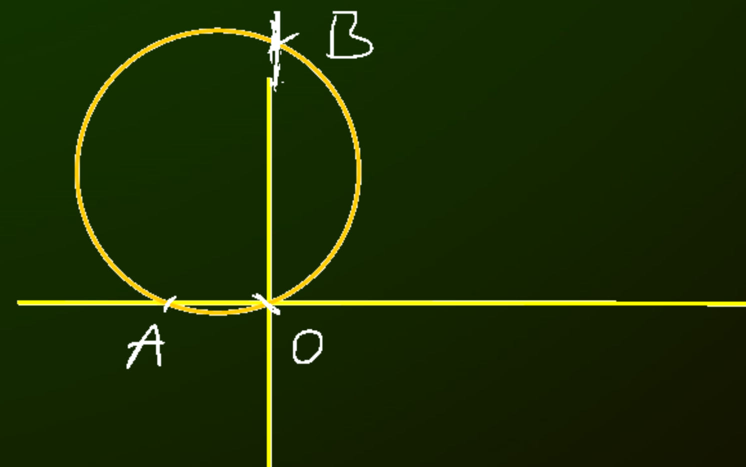


v) Touching y-axis at O(0,0)

$$x^2 + y^2 + 2gx = 0$$

vi) Passes through origin $\quad c = 0$

$$x^2 + y^2 + \underline{2g}x + 2fy = 0$$



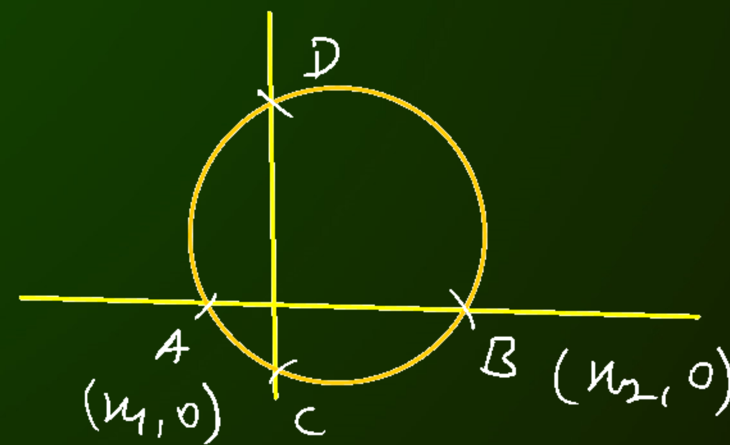
Note:- put $y=0$ to get x-intercept

$$x^2+2gx=0 \Rightarrow x_1=0 \quad \& \quad x_2=-2g$$

$|x_2-x_1|$

$x(x+2g)$

$$\rightarrow \text{x-intercept} = |x_2-x_1| = |-2g-0|$$
$$= |2g|$$



Similarly y-intercept $= |y_2-y_1| = |2f|$

Note: $x^2+y^2+2gx+2fy+c=0$

$y=0 \Rightarrow \quad x^2+2gx+c=0$

$$x_1+x_2=-2g, \quad x_1x_2=c$$

$(a-b)^2=(a+b)^2-4ab$

$$AB=|x_2-x_1| = \sqrt{(x_2+x_1)^2-4x_1x_2}$$

$$|x_2-x_1| = \sqrt{4g^2-4c} = 2\sqrt{g^2-c} \quad ; \quad CD = 2\sqrt{f^2-c}$$

Q.) Find the length of the X-intercept, Y-intercept of a circle

$x^2+y^2-6x+8y=0$

Sol:- $|x_2-x_1| = |2g| = 6$

$|y_2-y_1| = |2f| = 8$

Q.) Find the length of the X, Y-intercepts of a circle

$x^2+y^2+12x-8y-3=0$

Sol:- $|x_2-x_1| = 2\sqrt{g^2-c} = 2\sqrt{36+3} = 2\sqrt{39}$

$|y_2-y_1| = 2\sqrt{19}$

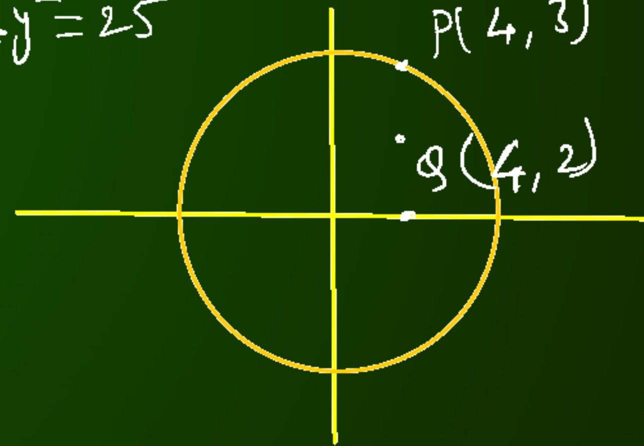
Position of a point w.r.t circle:

$x^2+y^2=25$

$\cdot R(4,6)$

$P(4,3)$

$Q(4,2)$



$S = x^2+y^2-25=0$

$S_P = 4^2+3^2-25$

$P(\alpha,\beta)$ lies in the same plane of a circle

$S_Q = 4^2+2^2-25 = -5<0$

$S \equiv x^2+y^2-r^2=0$

now

$S_R = 4^2+6^2-25 = 27>0$

If $S_P<0$ then P lies inside of the circle

If $S_P = 0$ " P " on the circle

If $S_P > 0$ " P " outside of " .

S_P

where $S_P = \alpha^2+\beta^2-r^2$

Note:- Above Th. is true for the circle $x^2+y^2+2gx+2fy+c=0$ also.

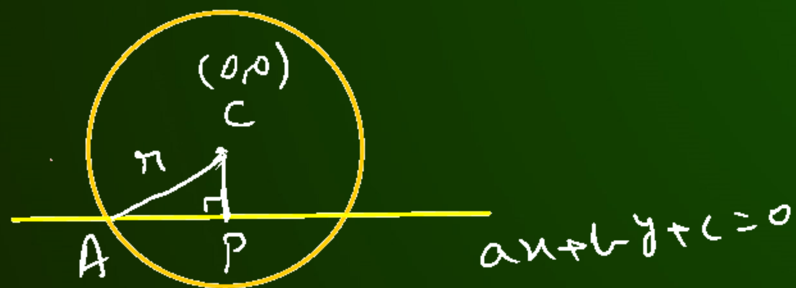
Q.) Find the position of the pt w.r.t the cle $x^2+y^2+10x-20y-3=0$

for the pt's i) $P(-4,2)$: $S_p = 16+4-40-40-3 < 0$

ii) $P(2,1)$: $S_p = 4+1+\cancel{20}-\cancel{20}-3 > 0$

Position of a line w.r.t the circle: $S \equiv x^2+y^2=r^2$; $C=(0,0)$; radius $= r$

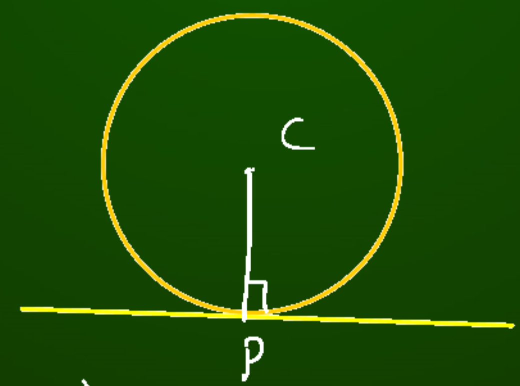
$$L \equiv ax+by+c=0$$



i) $CP < r$

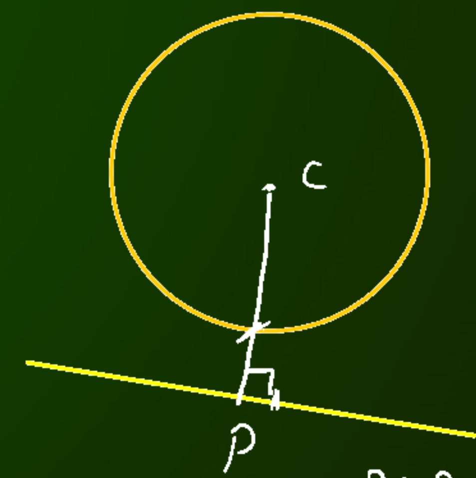
$$CP = \frac{|a(0)+b(0)+c|}{\sqrt{a^2+b^2}}$$

$$CP = \frac{c}{\sqrt{a^2+b^2}} < r$$



ii) $CP = r$

$$\frac{c}{\sqrt{a^2+b^2}} = r$$



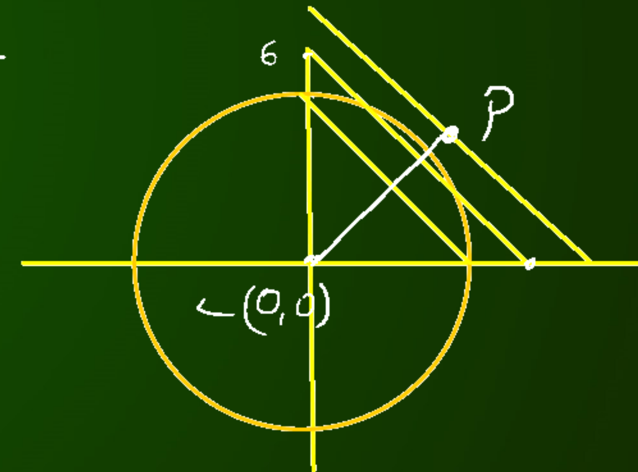
$CP > r$

$$\frac{c}{\sqrt{a^2+b^2}} > r$$

Q.) $x+y=n$, $n \in N$

$x^2+y^2=25$

Find the least 'n' such that $\boxed{x+y=n}$, $n \in N$ is not a tangent nor a secant to $x^2+y^2=25$



Sol: $CP > r \Rightarrow \dfrac{|0+0-n|}{\sqrt{2}} > 5$

$\Rightarrow n > 5\sqrt{2}$

$n > 7.1$

$\boxed{n = 8}$

1.414×5

7.1

Q.) Find the relation between $x^2+y^2=81$; $x-y=15$

$14+7.+$

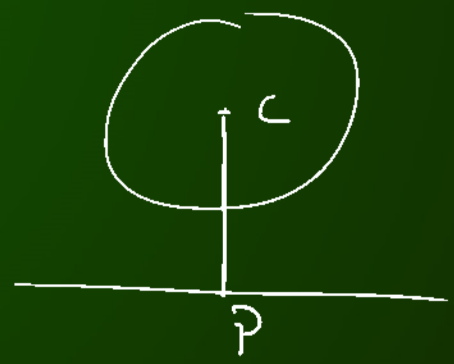
Sol'n $c=(0,0)$; $r=9$

$$CP = \frac{|0-0-15|}{\sqrt{2}}$$

$$CP = \frac{15}{\sqrt{2}} = \frac{15\sqrt{2}}{2} = 10.5\_\_ > 9$$

$CP > r$

$\frac{20\sqrt{13}}{13}$



1.414×15

$21.\_\_$

$35^2 = 1225$

3.6

$\frac{20\sqrt{13}}{13}$

$\sqrt{6} < \sqrt{13}$

$< \sqrt{13}\left(\frac{20}{13}\right)$

Q.) $x^2+y^2+2x-4y-1=0$; $c=(-1,2)$; $r=\sqrt{6}$

$2x-3y=12$

$$CP = \frac{|20|}{\sqrt{13}} = \frac{20}{3.6} \sim 5.23 > r$$

$x = \frac{20}{\sqrt{13}}$

$x^2 = \frac{400}{13} = \_\_\_$

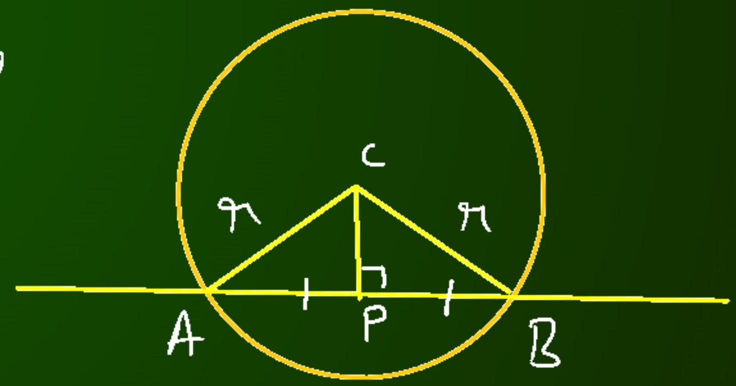
$CP > r$:

Finding length of a chord:

$S \equiv x^2+y^2+2gx+2fy+c=0$

$L \equiv lx+my+n=0$

$AB = 2\sqrt{r^2-d^2}$



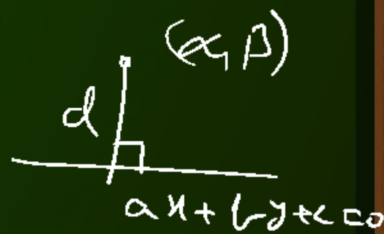
$\triangle APC$

$AP^2 + CP^2 = AC^2$

$AP^2 + d^2 = r^2$

$AP = \sqrt{r^2-d^2}$

$$d = \frac{|a\alpha+b\beta+c|}{\sqrt{a^2+b^2}}$$



$AB = 2(AP)$

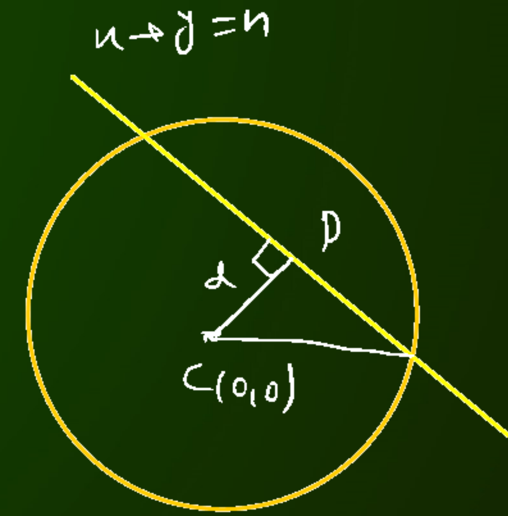
$AB = 2\sqrt{r^2-d^2}$

Q.) Find the length of a chord formed by $x^2+y^2=49$ to the line $3x+4y-8=0$

$d = \dfrac{|8|}{5}$

$$AB = 2\sqrt{r^2-d^2}$$
$$= 2\sqrt{49-\frac{64}{25}} = 2\sqrt{\frac{49\times25-64}{25}} = \frac{2\sqrt{1161}}{5}$$

Q.) For the circle $x^2+y^2=9$; $x+y=n,\ n\in N$ is a chord intersect at A_i, B_i then find $(A_1B_1)^2+(A_2B_2)^2+(A_3B_3)^2+\cdots$



Sol:

$C=(0,0)$; $r=3$

$L\equiv x+y=n$

$$CP=d=\frac{|-n|}{\sqrt{2}}$$

$$A_iB_i=2\sqrt{r^2-d^2}=2\sqrt{9-\frac{n^2}{2}}=2\sqrt{\frac{18-n^2}{2}}$$

$18-n^2>0$

$n^2<18$

$n<\sqrt{18}<5$

$n=1,2,3,4$

$$(A_iB_i)^2=4\left(\frac{18-n^2}{2}\right)=2(18-n^2)$$

$$(A_1B_1)^2+(A_2B_2)^2+(A_3B_3)^2+(A_4B_4)^2=2(18-1^2)+2(18-2^2)+2(18-3^2)+2(18-4^2)$$

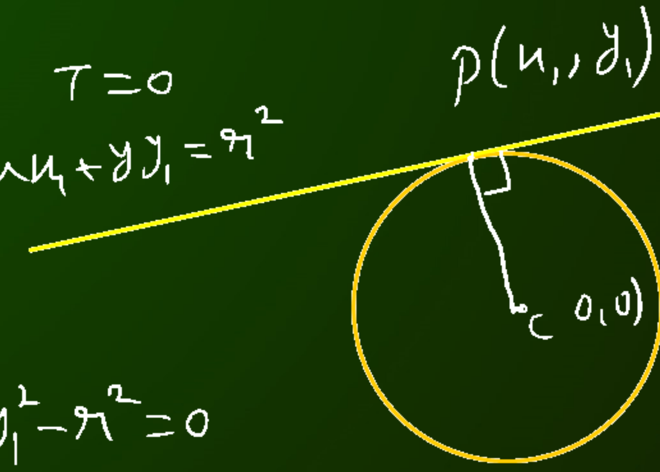
$$=4(2\times18)-2(1^2+2^2+3^2+4^2)=144-60=84$$

Finding Eq. of Tangent :-

i) **Point form :-** Eq. of Tangent to the circle $x^2+y^2=r^2$ from a pt $P(x_1, y_1)$ which lies on the circle is

$T=0$

$xx_1+yy_1=r^2$



$$\boxed{T=0}$$

$$\Rightarrow \boxed{xx_1+yy_1=r^2}$$

$S_p=0$

$\Rightarrow x_1^2+y_1^2-r^2=0$

Pf :-

$$y-y_1=-\frac{x_1}{y_1}(x-x_1)$$

$$yy_1-y_1^2=-xx_1+x_1^2$$

$$xx_1+yy_1=x_1^2+y_1^2 \Rightarrow xx_1+yy_1=r^2$$

Note: Eq. of Tangent from $P(x_1, y_1)$ lies on the circle $x^2+y^2+2gx+2fy+c=0$ to the same circle is

$$\boxed{T=0}$$

$$xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$$

$x^2 \rightarrow xx_1$

$y^2 \rightarrow yy_1$

$2x \rightarrow x+x_1$

$2y \rightarrow y+y_1$

$2xy \rightarrow x_1y+xy_1$

Q) Eq. of Tangent to the circle $x^2+y^2=25$ from $P(-3,4)$?

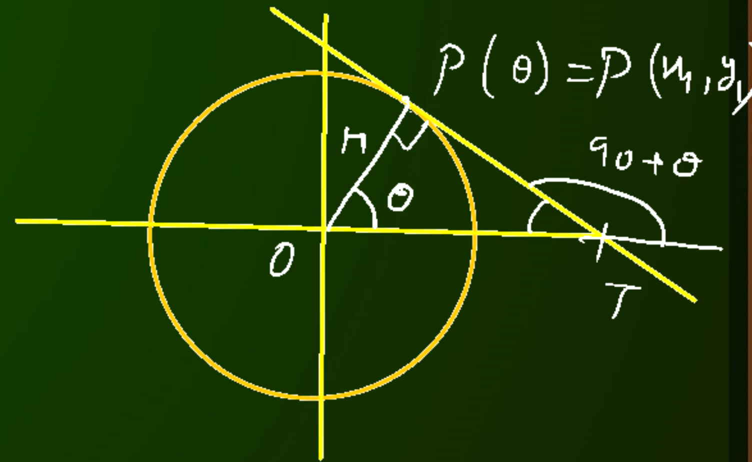
Sol: $T=0 \Rightarrow x(-3)+y(4)=25$

2) <u>Parametric form</u>:- $P(\theta) = (r\cos\theta, r\sin\theta)$ be a pt on the cle

$x^2+y^2=r^2$ then eq. of tangent at $P(\theta)$ is

$$\boxed{x\cos\theta + y\sin\theta = r}$$

$\angle POT = \theta$



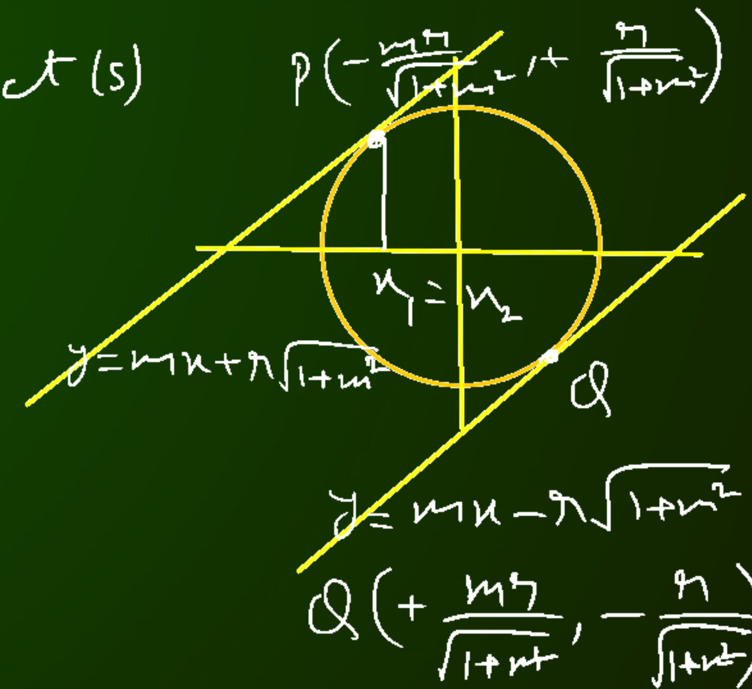
$$xx_1 + yy_1 = r^2 \Rightarrow x(r\cos\theta) + y(r\sin\theta) = r^2$$

$$\Rightarrow x\cos\theta + y\sin\theta = r$$

Q.) Find the eq. of tangent to the circle $x^2+y^2=25$ at $P(60^\circ)$

$$x + \sqrt{3}y = 10$$

3.) Slope form:- Eq. of Tangent with slope 'm' to the circle $x^2+y^2=r^2$ is $y=mx\pm r\sqrt{1+m^2}$ and the pt. of contact(s) are $\left(\mp\frac{mr}{\sqrt{1+m^2}}, \pm\frac{r}{\sqrt{1+m^2}}\right)$



$y=mx+c \Rightarrow x^2+(mx+c)^2=r^2$

$\Rightarrow x^2+m^2x^2+c^2+2mcx=r^2$

$\Rightarrow (1+m^2)x^2+2mcx+c^2-r^2=0$

$x_1=x_2 \Rightarrow D=0 \Rightarrow (2mc)^2-4(1+m^2)(c^2-r^2)=0$

$\cancel{4m^2c^2}-4(c^2-r^2+\cancel{m^2c^2}-m^2r^2)=0$

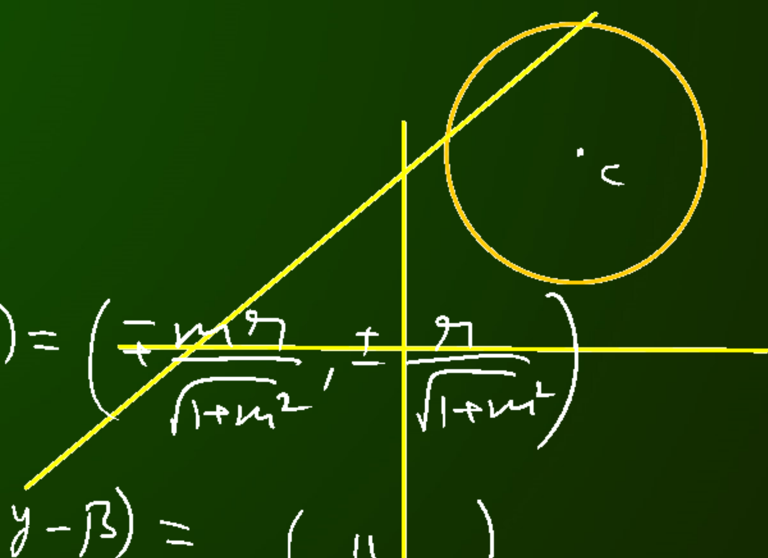
$\Rightarrow c^2-r^2-m^2r^2=0$

$\Rightarrow c^2=r^2(1+m^2) \Rightarrow c=\pm r\sqrt{1+m^2}$

Note: 1.) If the Tangent is $y = mx + r\sqrt{1+m^2}$ then the pt. of contact is $P\left(-\frac{mr}{\sqrt{1+m^2}}, +\frac{r}{\sqrt{1+m^2}}\right)$

2.) $(x-\alpha)^2+(y-\beta)^2=r^2$

$\boxed{X^2+Y^2=r^2} \Rightarrow Y = mX \pm r\sqrt{1+m^2}$ & $P(X,Y) = \left(\frac{\mp mr}{\sqrt{1+m^2}}, \frac{\pm r}{\sqrt{1+m^2}}\right)$

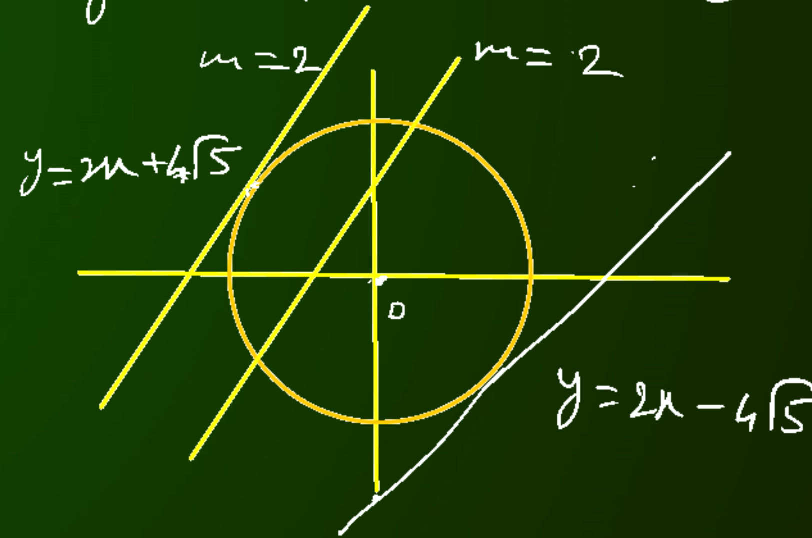


$\rightarrow$ $y-\beta = m(x-\alpha) \pm r\sqrt{1+m^2}$ & $P(x-\alpha, y-\beta) = (\;'' \;)$

$$P(x,y) = \left(\alpha \mp \frac{mr}{\sqrt{1+m^2}}, \beta \pm \frac{r}{\sqrt{1+m^2}}\right)$$

3.) To find eq. of Tangent from an external pt to the circle use slope form and put $P(\alpha,\beta)$ in this eq. then we will get a quadratic in 'm', find 'm' and put in the eq. of Tangent.

Q.) Find the eq. of Tangent ||el to the line $y = 2x + 1$ to the circle $x^2 + y^2 = 16$



Sol: $r = 4$ & $m = 2$

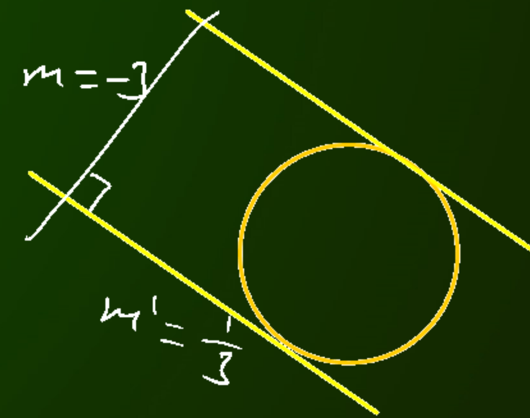
eq. of Tangent $y = mx \pm r\sqrt{1+m^2}$

$y = 2x \pm 4\sqrt{1+4}$

$y = 2x \pm 4\sqrt{5}$

$y = 2x + 4\sqrt{5}$ $\qquad$ $y = 2x - 4\sqrt{5}$

Q.) Find the eq. of Tangent $\perp$er to the line $3x+y+1=0$ to the circle $x^2+y^2=49$ and also find pt of contacts ?



Sol: slope of $3x+y+1=0$ is $m=-3$

Slope of the Tangent is $m'=\frac{1}{3}$ ✓

radius of the cle is $r=7$

$\therefore$ eq. of Tangent is $y=mx \pm r\sqrt{1+m^2}$

$$y=\frac{1}{3}x \pm 7\sqrt{1+\left(\frac{1}{3}\right)^2}$$

$$y=\frac{x}{3} \pm \frac{7\sqrt{10}}{3}$$

$$\therefore \boxed{3y = x \pm 7\sqrt{10}}$$

pt. of contact's are

$$\left(\mp\frac{mr}{\sqrt{1+m^2}}, \pm\frac{r}{\sqrt{1+m^2}}\right) = \left(\mp\frac{\frac{7}{\cancel{3}}}{\frac{\sqrt{10}}{\cancel{3}}}, \pm\frac{7}{\frac{\sqrt{10}}{3}}\right)$$

$$\left(\mp\frac{7}{\sqrt{10}}, \pm\frac{21}{\sqrt{10}}\right)$$

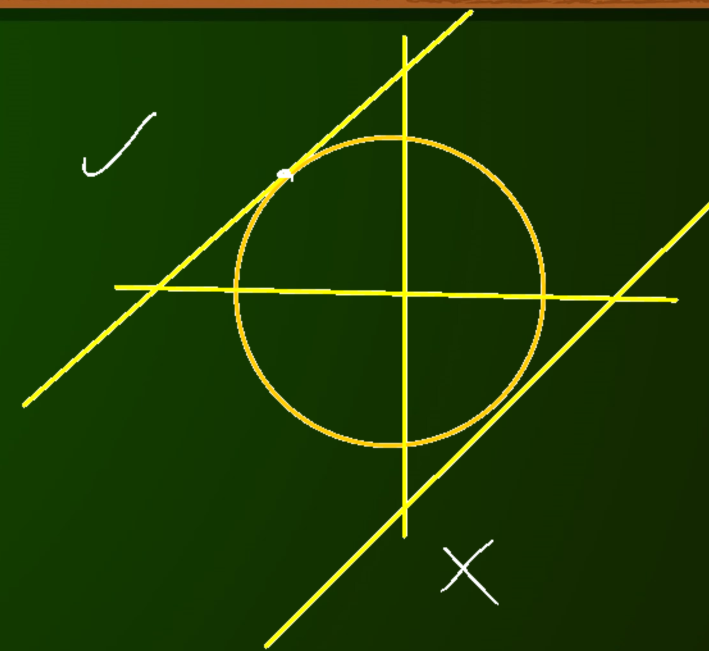
$$y = mx \pm r\sqrt{1+m^2}$$

$m = 1$

$r = 5$

$$y = x \pm 5\sqrt{2}$$

$$y = x + 5\sqrt{2} \quad , \quad P\left(-\frac{5}{\sqrt{2}}, +\frac{5}{\sqrt{2}}\right)$$



Q.) Find the eq. of tangent having slope $m=-1$ to the circle $(x-2)^2+(y+3)^2=25$ and also find the pt of contacts.

$$y = mx \pm r\sqrt{1+m^2}$$

Sol:-

$$y+3 = -1(x-2) \pm 5\sqrt{2}$$

$$y+3 = -x+2 \pm 5\sqrt{2}$$

$$\boxed{y = -x-1 \pm 5\sqrt{2}}$$

$+$ $(-, +)$

$$P\left(2 \mp \frac{(-1)(5)}{\sqrt{2}}, -3 \pm \frac{5}{\sqrt{2}}\right)$$

$l \equiv y = mx \pm \sqrt{1+m^2}$ —— (1)

$(-2,0) \in L \Rightarrow 0 = -2m \pm \sqrt{1+m^2}$

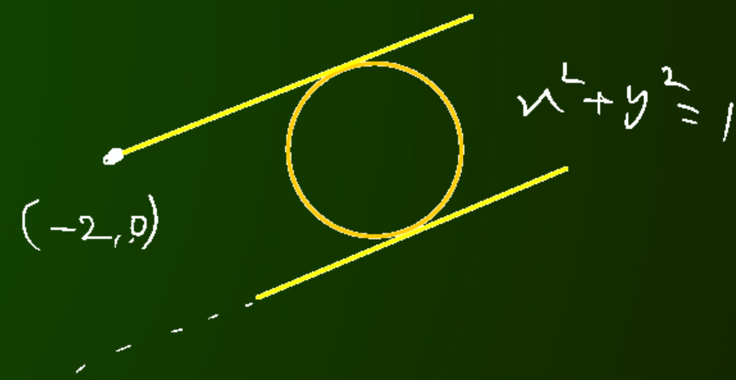
$\Rightarrow 4m^2 = 1+m^2$

$\Rightarrow 3m^2 = 1$

$\Rightarrow m = \pm \frac{1}{\sqrt{3}}$

$0 = \frac{2}{\sqrt{3}} + \frac{2}{\sqrt{3}}$

$\cancel{0 = -\frac{(-2)}{\sqrt{3}} - \frac{2}{\sqrt{3}}}$



$m = \frac{1}{\sqrt{3}}$: $y = \frac{1}{\sqrt{3}}x \pm \sqrt{1+\frac{1}{3}} \Rightarrow y = \frac{x}{\sqrt{3}} \pm \frac{2}{\sqrt{3}}$

$y = \frac{x}{\sqrt{3}} + \frac{2}{\sqrt{3}}$ ✓

$y = \frac{x}{\sqrt{3}} - \frac{2}{\sqrt{3}}$ ✗

$m = -\frac{1}{\sqrt{3}}$: $y = -\frac{x}{\sqrt{3}} \pm \frac{2}{\sqrt{3}}$

$y = -\frac{x}{\sqrt{3}} + \frac{2}{\sqrt{3}}$ ✗

$y = -\frac{x}{\sqrt{3}} - \frac{2}{\sqrt{3}}$ ✓

Q.) Find the eq. of Tangent from $P(3,0)$ to the circle $x^2+y^2=4$

Sol: $S_p = 9+0-4 = 5 > 0$ $\Rightarrow$ $P(3,0)$ lies outside of the circle, so we can get 2 tangents.

Let $y = mx \pm 2\sqrt{1+m^2}$ be the Tangent — (1)

$P(3,0)$ lies on (1) $\Rightarrow$ $0 = 3m \pm 2\sqrt{1+m^2}$

$\Rightarrow 9m^2 = 4+4m^2 \Rightarrow 5m^2 = 4$

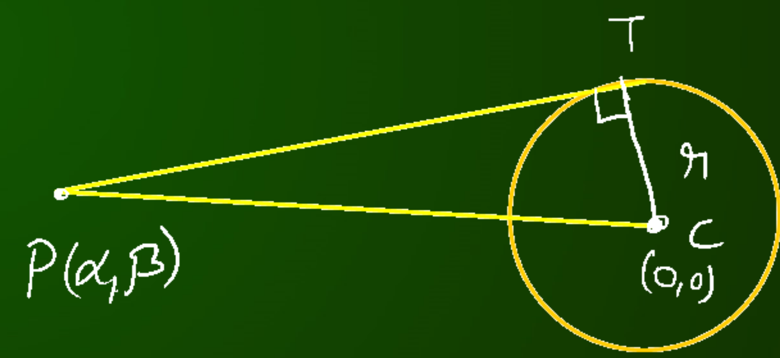
$\Rightarrow m = \pm \frac{2}{\sqrt{5}}$

$\sqrt{5}y = 2x - 6$ ✓

$\sqrt{5}y = -2x + 6$ ✓

Length of the Tangent:-

$P(\alpha,\beta)$ be an external pt of a c/e $x^2+y^2=r^2$ then the distance PT is said to be Length of the Tangent where 'T' be the pt. of contact.



$x^2+y^2=r^2$

$x^2 \to xx_1$
$y^2 \to yy_1$

$T = xx_1+yy_1-r^2$

$x \to \alpha$
$y \to \beta$

$S_1 = \alpha^2+\beta^2-r^2$

$PC = \sqrt{\alpha^2+\beta^2}$, $CT = r$

$PT^2 = PC^2 - CT^2$

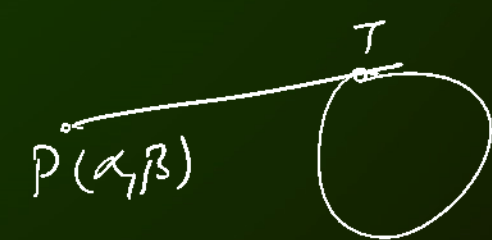
$PT^2 = \alpha^2+\beta^2-r^2$

$\boxed{PT = \sqrt{\alpha^2+\beta^2-r^2}} \Rightarrow \boxed{PT = \sqrt{S_1}}$

$PT = \sqrt{S_1}$

Note:-

$S \equiv x^2+y^2+2gx+2fy+c=0$



$PT = \sqrt{S_1}$

$= \sqrt{\alpha^2+\beta^2+2g\alpha+2f\beta+c}$

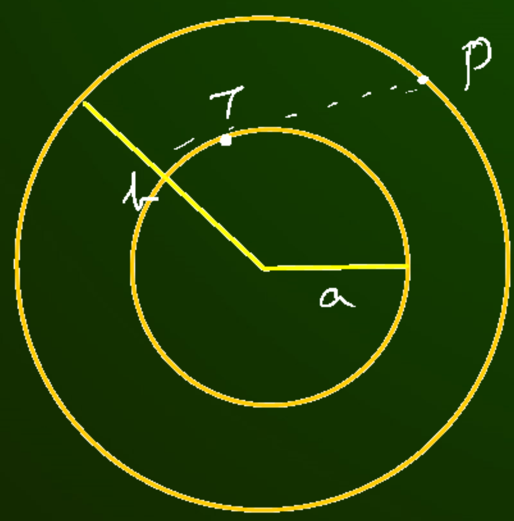
Q.) $x^2+y^2-4x-6y-2=0$, $P(5,6)$

$$PT=\sqrt{25+36-20+36-2}=\sqrt{75}=5\sqrt{3}$$

$x^2+y^2=b^2$ to $x^2+y^2=a^2$

Q.)

$P(b\cos\theta, b\sin\theta)$



$$PT=\sqrt{b^2c^2_\theta+b^2s^2_\theta-a^2}$$

$$\boxed{PT=\sqrt{b^2-a^2}}$$