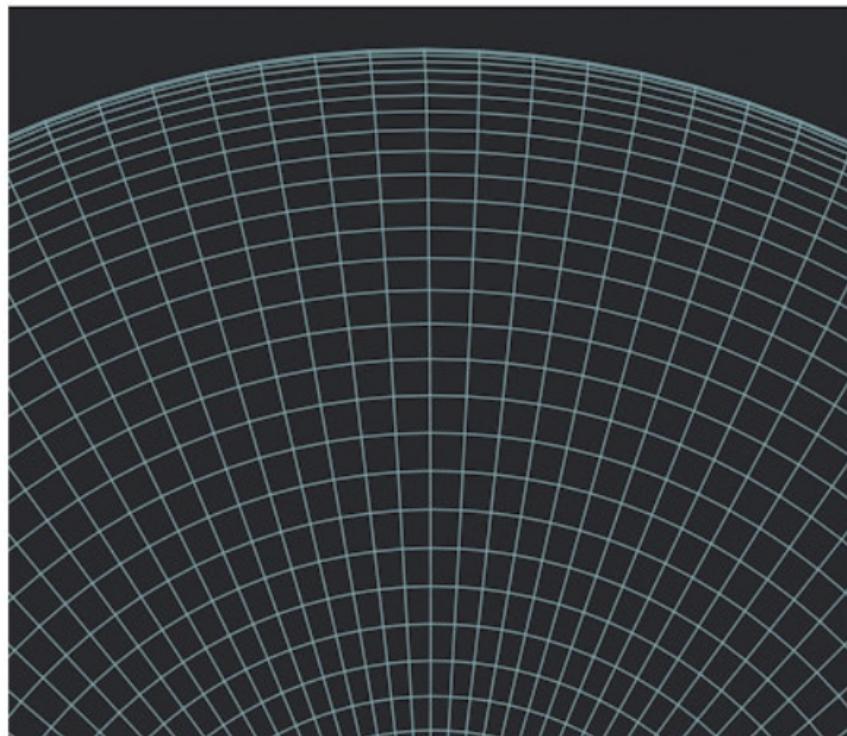

INTERNATIONAL MACROECONOMICS

A MODERN APPROACH

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slides

Chapter 5

Current Account Determination in a Production Economy

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Motivation

- So far, we abstracted from investment. Unrealistic but useful.
- Investment, which consists of spending in capital goods such as machines, new structures, equipment, and inventories, is an important component of aggregate demand amounting to around 20 percent of GDP in most countries.
- Investment is the most volatile component of aggregate demand, and, as such, it is important for understanding movements in the current account over the business cycle.

Preview

- In this chapter we add investment in physical capital, I_t , to our theoretical model of current account determination. To do so we incorporate a second economic agent, namely, **the firm**.
- We will derive the saving schedule, $S(r_1; A_1, A_2)$, the investment schedule, $I(r_1; A_2)$, and the current account schedule, $CA(r_1; A_1, A_2)$, which are the building blocks for the graphical current account analysis.
- We study the adjustment of the economy to: (a) temporary productivity shocks; (b) anticipated future productivity shocks; (c) world interest rate shocks; and (d) changes in the terms of trade.

Let's begin by modeling the investment choice of firms.

A production economy

- 2-period economy
- Output no longer an endowment, instead output is produced with physical capital.

The production function for output in period t , Q_t :

$$Q_t = A_t F(I_{t-1})$$

$A_t > 0$ for $t = 1, 2$ denotes the level of technology in period t ; I_{t-1} denotes investment in physical capital in period $t-1$, which becomes productive in period t . That is, the investment decision of period $t-1$ determines the capital stock in period t .

3 Properties of the function $F(\cdot)$:

1. Output is zero when capital is zero: $F(0) = 0$.
2. Output is increasing in capital: $F'(I_t) > 0$.
3. Output increases with capital at a decreasing rate: $F''(I_t) < 0$.

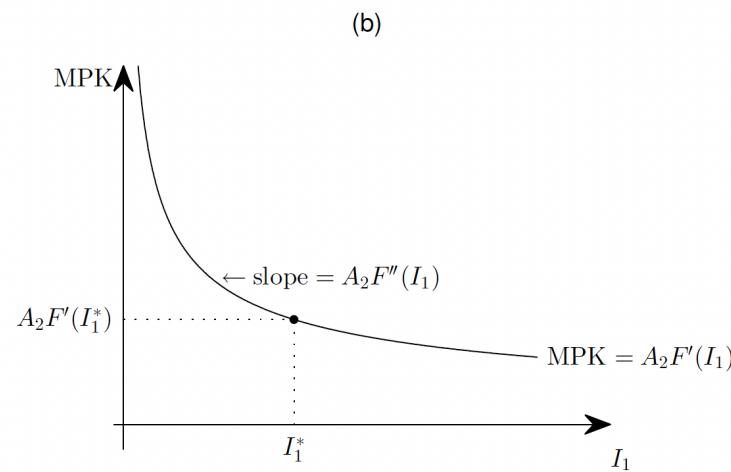
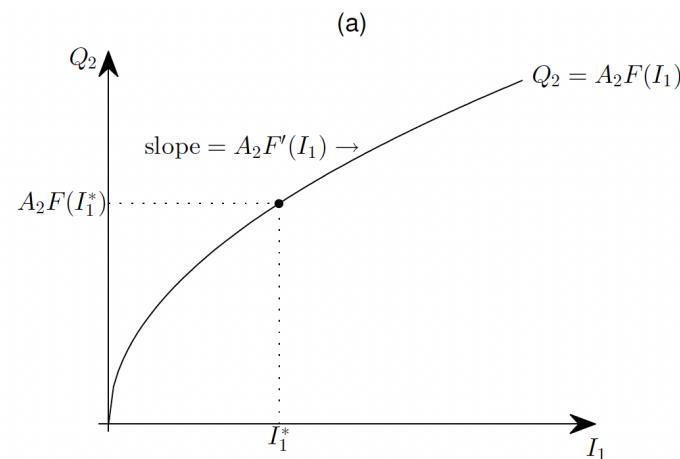
The marginal product of capital (MPK) is defined as

$$\text{MPK} = A_t F'(I_{t-1})$$

Property 2 says that the MPK is positive.

Property 3 says that the MPK is declining in capital. This property is known as *diminishing marginal product of capital*.

The production function and the marginal product of capital



Observations about the figure:

- Panel (a) displays the production function, $AF(I)$.
- Panel (b) displays the marginal product of capital as a function of the level of capital.

When $I_1 = 0$, production is nil. Output increases with investment, but at a decreasing rate.

Panels (a) and (b) are related by the fact that the slope of the production function at a given level of investment equals the level of the marginal product of capital at the same level of investment.

Example

Consider the production function

$$Q_2 = \sqrt{I_1}.$$

In this case, $A_2 = 1$ and $F(I_1)$ is the square root function.

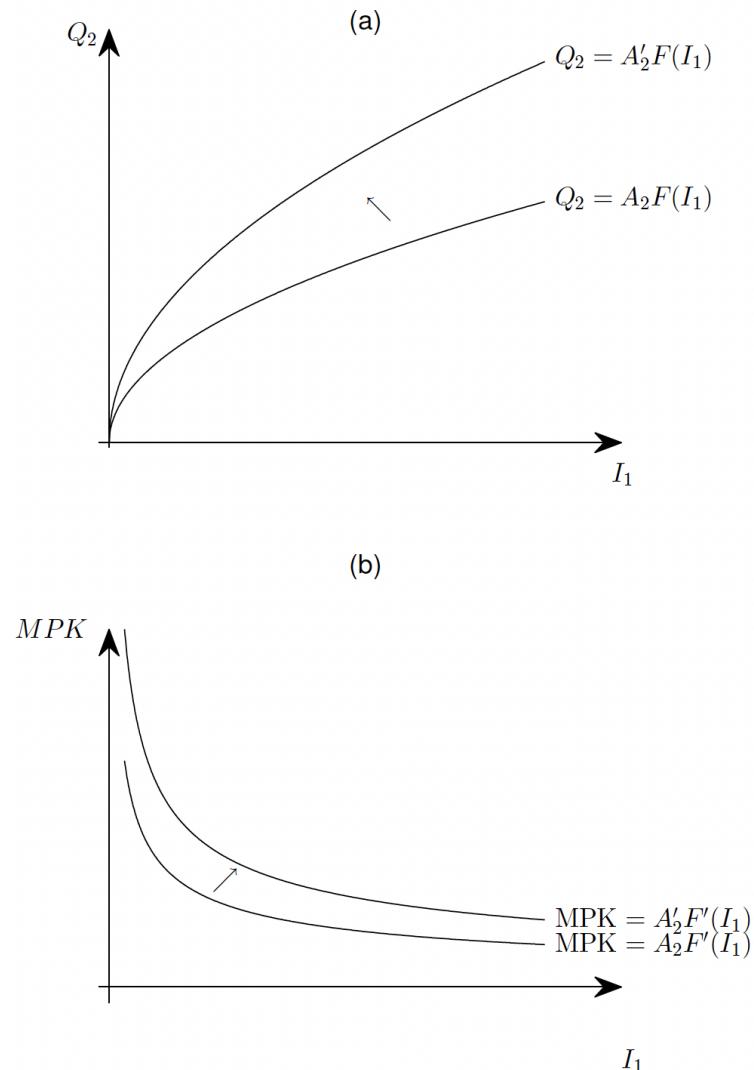
Are properties 1-3 satisfied?

$\sqrt{0} = 0$: property 1 holds.

$\text{MPK} = \frac{1}{2}I_1^{-1/2} > 0$: property 2 holds, MPK positive.

$d\text{MPK}/dI_1 = -\frac{1}{4}I_1^{-3/2} < 0$: property 3 holds, MPK diminishing.

Effect of an increase in productivity on the production function and the marginal product of capital schedule



An increase in the level of technology from A_2 to $A'_2 > A_2$ rotates the production function counter-clockwise around the origin and shifts the marginal product of capital schedule up and to the right.

How do firms decide about the level of investment?

The investment decision of the firm

Period 1: Firms borrow to finance purchases of investment goods.
 D_1^f = debt assumed by the firm in period 1. I_1 = investment goods purchased in period 1.

$$D_1^f = I_1. \quad (1)$$

Period 2: Firms produce and then sell the output: $A_2 F(I_1)$. Repay loan including interest: $(1 + r_1)D_1^f$.

Π_2 = firm profits in period 2:

$$\Pi_2 = A_2 F(I_1) - (1 + r_1)D_1^f. \quad (2)$$

Use $D_1^f = I_1$ to express profits as

$$\Pi_2 = A_2 F(I_1) - (1 + r_1)I_1. \quad (3)$$

The optimal level of investment

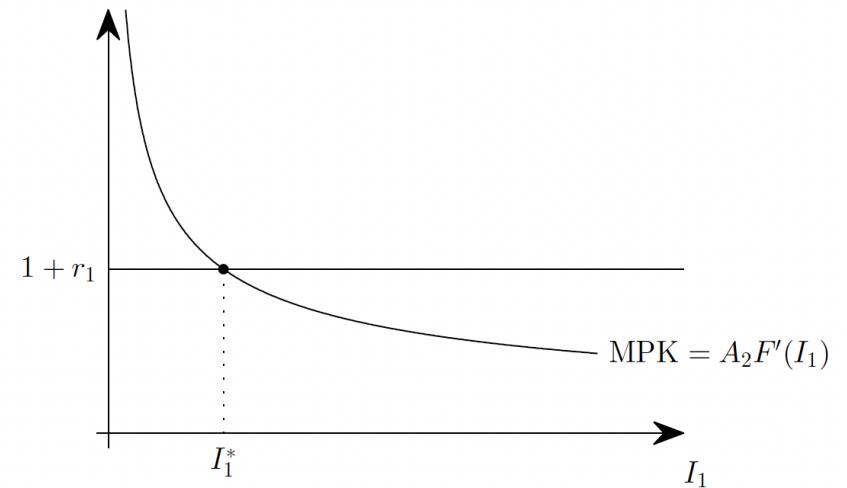
Profit Maximization Problem of the Firm

$$\max_{\{I_1\}} \Pi_2 = A_2 F(I_1) - (1 + r_1) I_1.$$

First-order optimality condition:

$$A_2 F'(I_1) = 1 + r_1. \quad (4)$$

The downward sloping line is the marginal product of capital schedule. The horizontal line depicts the marginal cost of capital schedule, which is equal to the gross interest rate, $1 + r_1$, for any level of investment. Firms invest up to the point where the marginal product of capital equals the marginal cost of capital, I_1^* . Profits are given by the area below the marginal product of capital schedule and above the marginal cost of capital.

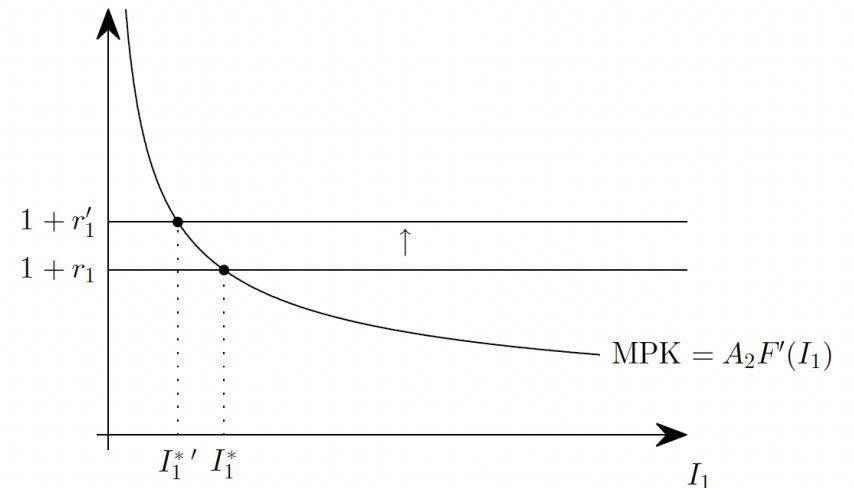


Effect of an increase in the interest rate on investment

An increase in the interest rate from r_1 to $r'_1 > r_1$ shifts the marginal cost schedule up. As a result, the optimal level of investment falls from I_1^* to $I_1^{* \prime}$.

Profits also fall as the interest rate increases. This can be seen graphically from the fact that the triangular area below the marginal product of capital schedule and above the marginal cost of capital schedule gets smaller as the latter shifts up.

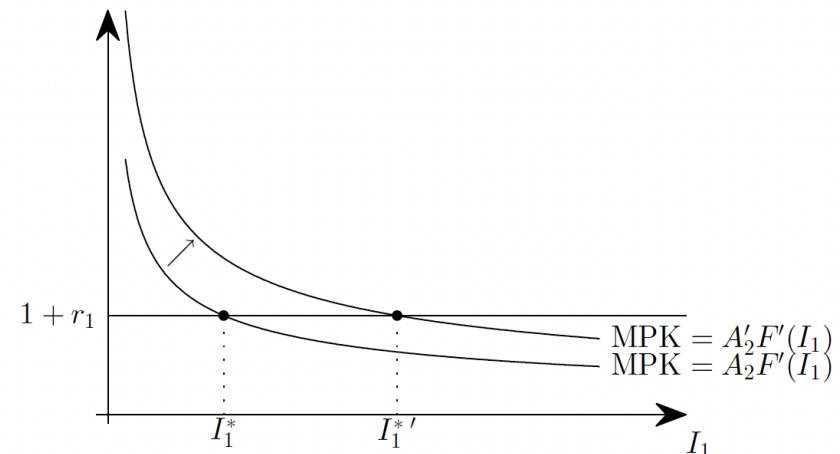
It follows that investment and profits both are a decreasing function of the interest rate.



The Effect of a Productivity Increase on the Firm's Investment Choice

Suppose A_2 increases to $A'_2 > A_2$. The positive productivity shock shifts the MPK schedule up and to the right. The firm expands investment until the new MPK schedule meets the marginal cost of capital schedule. Investment increases from I_1^* to $I_1^{* \prime}$. Profits increase, because the area below the MPK schedule and above the marginal cost schedule expands.

It follows that, all other things equal, both investment and profits are an increasing function of the technology factor A_2 .



The Profit Function in Period 2

Combining the last two results we have that profits are a decreasing function of the interest rate and an increasing function of the level of productivity. We can then write

$$\Pi_2 = \Pi_2(r_1, A_2),$$

- +

where the function $\Pi_2(\cdot, \cdot)$ is decreasing in its first argument, the interest rate, and increasing in its second argument, the productivity factor.

The Profit Function in Period 1

Profits in period 1, denoted Π_1 , are given by

$$\Pi_1 = A_1 F(I_0) - (1 + r_0) D_0^f, \quad (5)$$

with

$$D_0^f = I_0. \quad (6)$$

The variables I_0 , D_0^f , and r_0 are all predetermined in period 1 and are therefore taken as exogenous by the firm in period 1. Higher interest rates, r_0 , lower profits and a higher productivity factor in period 1, A_1 , raises profits.

$$\Pi_1 = \Pi_1(r_0, A_1). \\ \quad - \quad +$$

The Investment Schedule

- Aggregate investment is the sum of the investment decisions of individual firms.
- Assume that all firms are identical. Then aggregate investment behaves just like investment at the firm level, decreasing in the interest rate and increasing in the level of technology:

$$I_1 = I(r_1; A_2), \quad (7)$$

where now I_1 denotes aggregate investment in period 1.

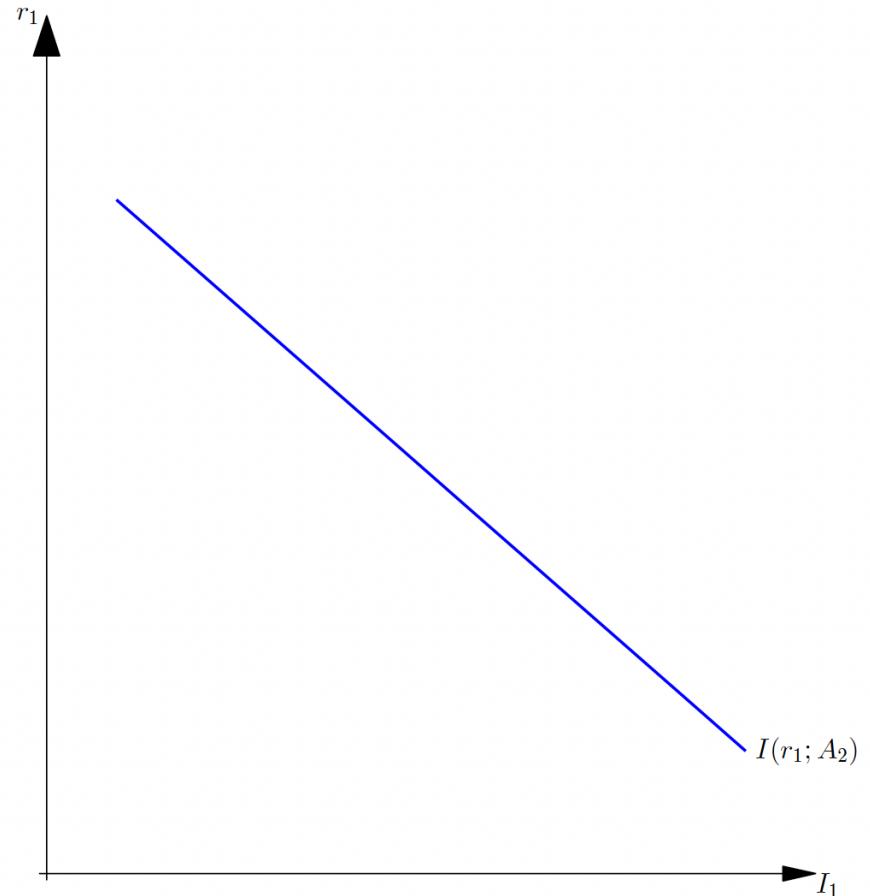
We will refer to this function as the investment schedule.

The Investment Schedule

The investment schedule

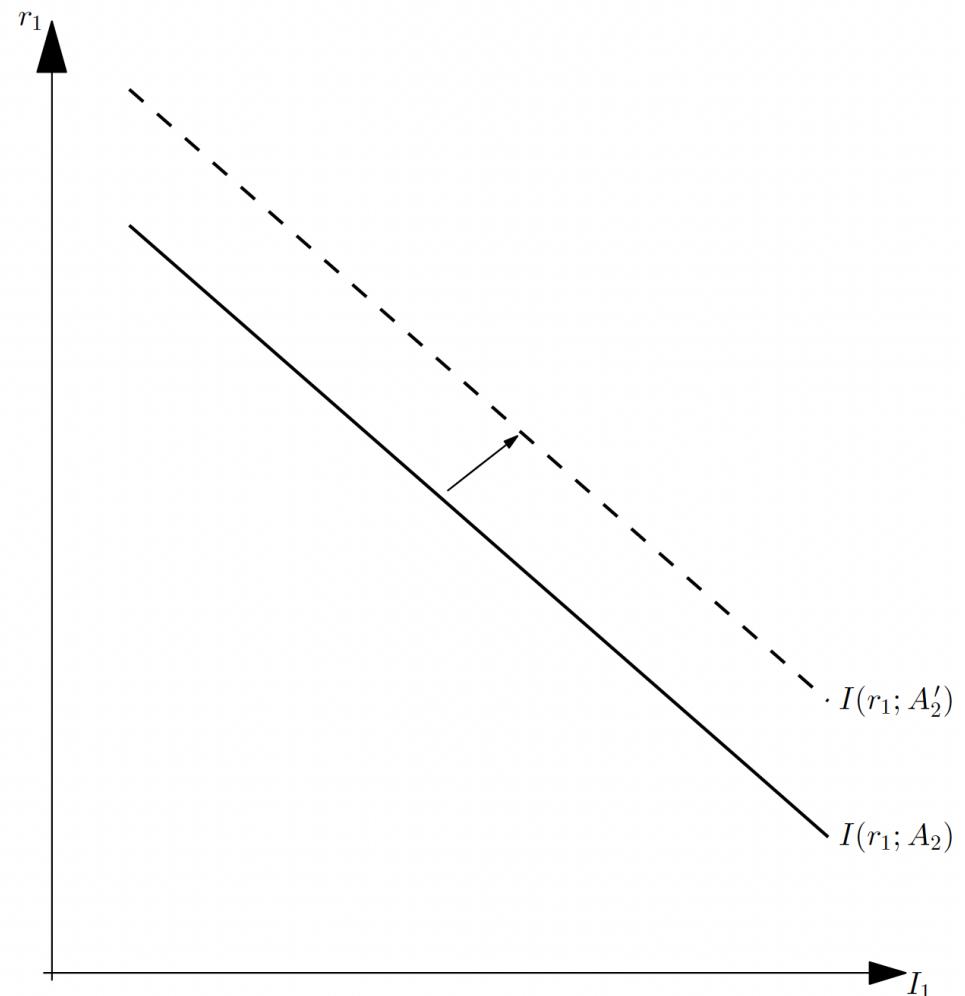
$$I_1 = I(r_1; A_2)$$

relates the aggregate level of investment and the interest rate, given the productivity factor A_2 . The investment schedule slopes downward because the profit maximizing level of investment is decreasing in the marginal cost of capital.



The effect of an increase in productivity on the investment schedule

The figure depicts the effect of an increase in the productivity parameter from A_2 to A'_2 . The positive productivity shock shifts the investment schedule up and to the right because for every level of the interest rate, the profit-maximizing level of investment is now higher.



How is the problem of households affected with the introduction of investment?

The Consumption-Saving Decision of Households

The budget constraints of households in the production economy are almost the same as in the endowment economy.

The only difference is that now households are the owners of firms.

Consequently, instead of receiving an endowment each period, households receive profit payments from firms, $\Pi_1(r_0, A_1)$ in period 1 and $\Pi_2(r_1, A_2)$ in period 2.

Period-1 budget constraint:

$$C_1 + B_1^h - B_0^h = \Pi_1(r_0, A_1) + r_0 B_0^h \quad (8)$$

Period-2 budget constraint:

$$C_2 = \Pi_2(r_1, A_2) + (1 + r_1)B_1^h \quad (9)$$

The intertemporal budget constraint:

$$C_1 + \frac{C_2}{1 + r_1} = (1 + r_0)B_0^h + \Pi_1(r_0, A_1) + \frac{\Pi_2(r_1, A_2)}{1 + r_1} \quad (10)$$

The household chooses C_1 and C_2 to maximize its utility function

$$U(C_1) + \beta U(C_2) \quad (11)$$

subject to the intertemporal budget constraint

$$C_2 = (1 + r_1)(\bar{Y} - C_1) \quad (12)$$

where

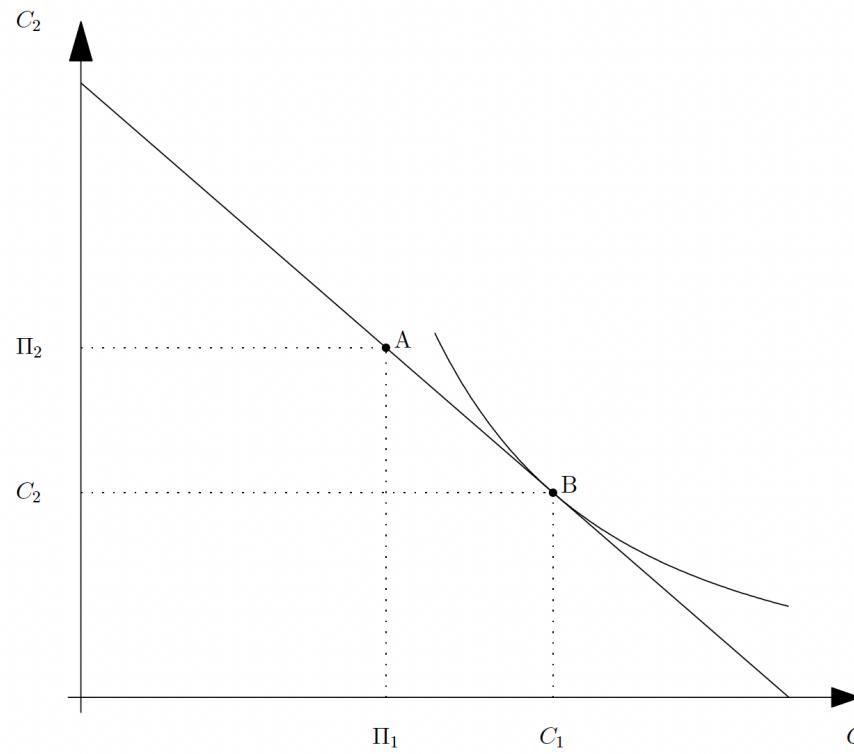
$$\bar{Y} = (1 + r_0)B_0^h + \Pi_1(r_0, A_1) + \frac{\Pi_2(r_1, A_2)}{1 + r_1}$$

The household takes \bar{Y} as exogenously given. The optimality conditions are the intertemporal budget constraint (12) and the Euler equation

$$\frac{U'(C_1)}{\beta U'(C_2)} = 1 + r_1. \quad (13)$$

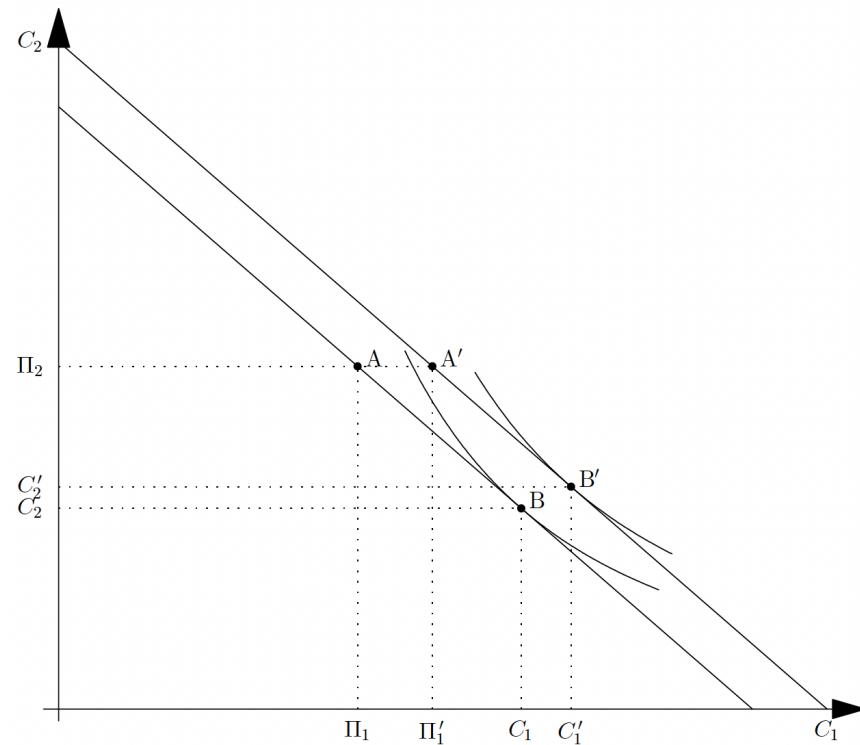
Note that these optimality conditions are identical to those pertaining to the endowment economy.

The optimal intertemporal consumption choice in the production economy



The figure depicts the optimal choice of consumption in periods 1 and 2. The intertemporal budget constraint is the straight downward sloping line. It crosses the profit path (Π_1, Π_2) at point A and has a slope equal to $-(1 + r_1)$. The optimal consumption path (C_1, C_2) is at point B, where an indifference curve is tangent to the intertemporal budget constraint. The figure is drawn under the assumption that the household starts period 1 with a zero net asset position, $B_0^h = 0$.

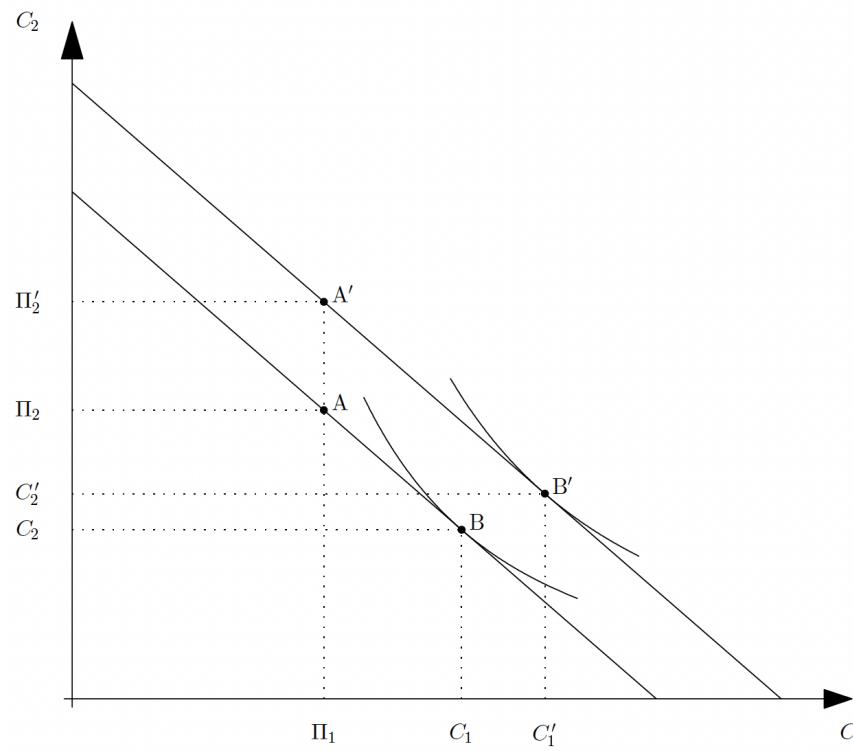
Effect of a Temporary Increase in Productivity on Consumption



The figure depicts the adjustment of consumption to an increase in the productivity of capital in period 1 from A_1 to $A'_1 > A_1$ holding constant the productivity of capital in period 2, A_2 , assuming that $B_0^h = 0$.

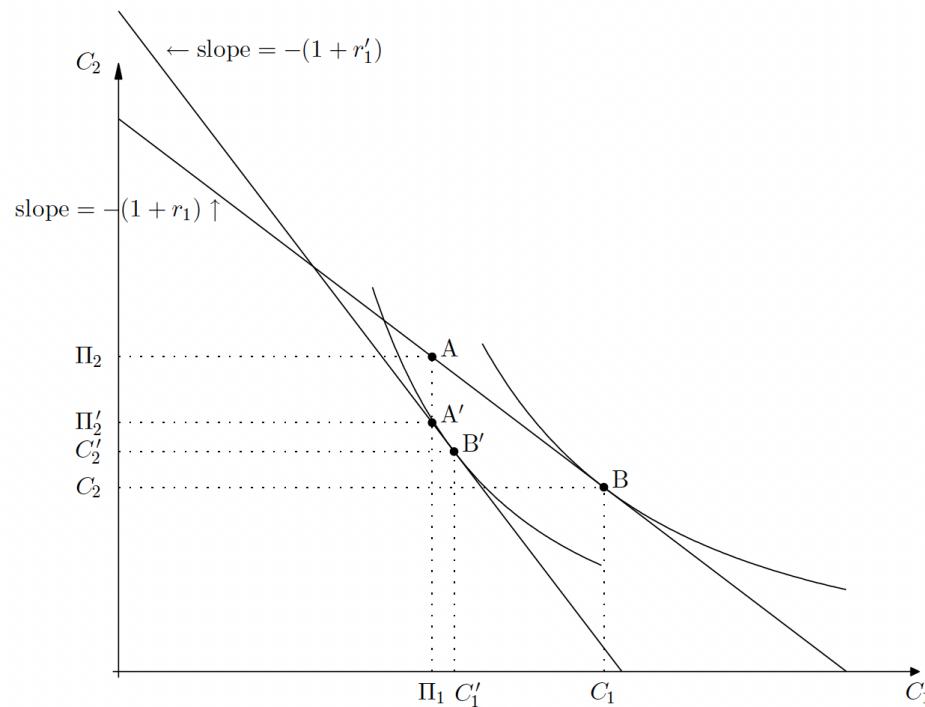
Prior to the productivity shock, the profit path is at point A and the optimal consumption path is at point B. When productivity increases, period-1 profits increase from Π_1 to Π'_1 , and period-2 profits remain unchanged. The new profit path is at point A'. The intertemporal budget constraint shifts in a parallel fashion out and to the right. By normality, consumption in both periods must increase. The new consumption path is at point B'. Period-1 consumption increases by less than period-1 profits to allow period-2 consumption to increase as well.

Effect of an Anticipated Future Productivity Increase on Consumption



The figure depicts the effect of an increase in the productivity of capital in period 2 from A_2 to A'_2 , holding the productivity of capital in period 1, A_1 , constant, for the case that $B_0^h = 0$. The initial path of profits is at point A, and the initial optimal consumption path is at point B. The anticipated positive productivity shock increases period-2 profits. The new path of profits is at point A'. The intertemporal budget constraint shifts out and to the right in a parallel fashion. By normality consumption in both periods must increase. The new consumption path is at point B'. The increase in period-1 consumption is financed by an increase in borrowing.

Effect of an Increase in the Interest Rate on Consumption



The figure depicts the effect of an increase in the interest rate from r_1 to $r'_1 > r_1$, for the case that $B_0^h = 0$. The initial path of profits is at point A, and the initial optimal consumption path is at point B. The increase in the interest rate gives rise to two negative income effects and one substitution effect all lowering period-1 consumption. First, it causes a reduction in profits in period 2 and thus represents a negative income effect. The new path of profits, point A', is directly below the original one, point A. Second, the increase in the interest rate makes the intertemporal budget constraint steeper. Because at point B, households were borrowing, the rise in r_1 represents a negative income effect. Finally, the increase in r_1 induces a substitution effect whereby households substitute future for current consumption. The new optimal consumption path, (C'_1, C'_2) , is point B'.

Taken together the above results imply that period-1 consumption is decreasing in the interest rate and increasing in current and expected future productivity. We can therefore write

$$C_1 = C(r_1, A_1, A_2) \quad (14)$$

- + +

The Saving Schedule

The Saving Schedule

National saving in period 1 is the difference between national income, Y_1 and consumption, C_1 :

$$S_1 = Y_1 - C_1$$

National income is the sum of net investment income and output,

$$Y_1 = r_0 B_0 + A_1 F(I_0),$$

where B_0 is the country's net foreign asset position at the beginning of period 1. In turn, B_0 is the sum of the household's net asset position, B_0^h , and the firm's net asset position, $-D_0^f$, that is,

$$B_0 = B_0^h - D_0^f. \tag{15}$$

As r_0 , B_0 , I_0 are predetermined, the only shifter of national income in period 1 is productivity:

$$Y_1 = Y(A_1). \quad (16)$$

+

Combining expressions (14) and (16) we have

$$S_1 = Y(A_1) - C(r_1, A_1, A_2).$$

+ - + +

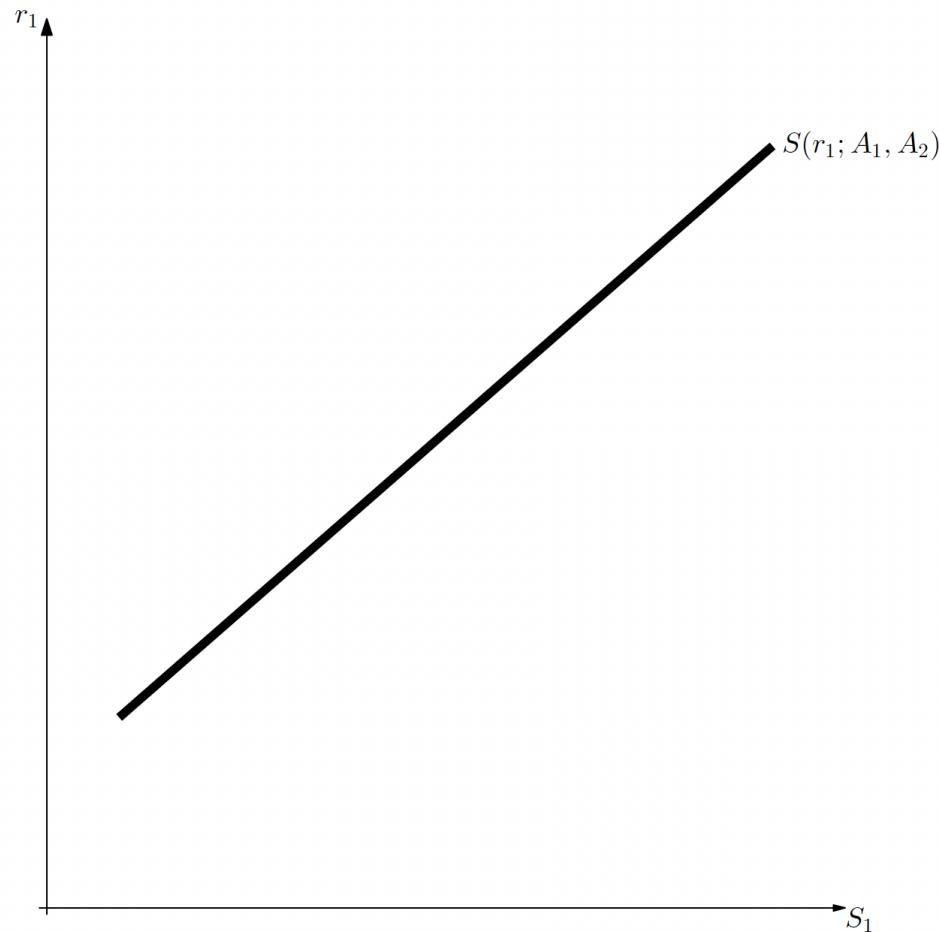
National saving is increasing in the interest rate, r_1 , and decreasing in the future expected level of productivity, A_2 . But what about A_1 ? Recall that an increase in A_1 increases C_1 by less than output. Thus, national saving increases unambiguously with an increase in A_1 . National saving can then be written as

$$S_1 = S(r_1; A_1, A_2). \quad (17)$$

+ + -

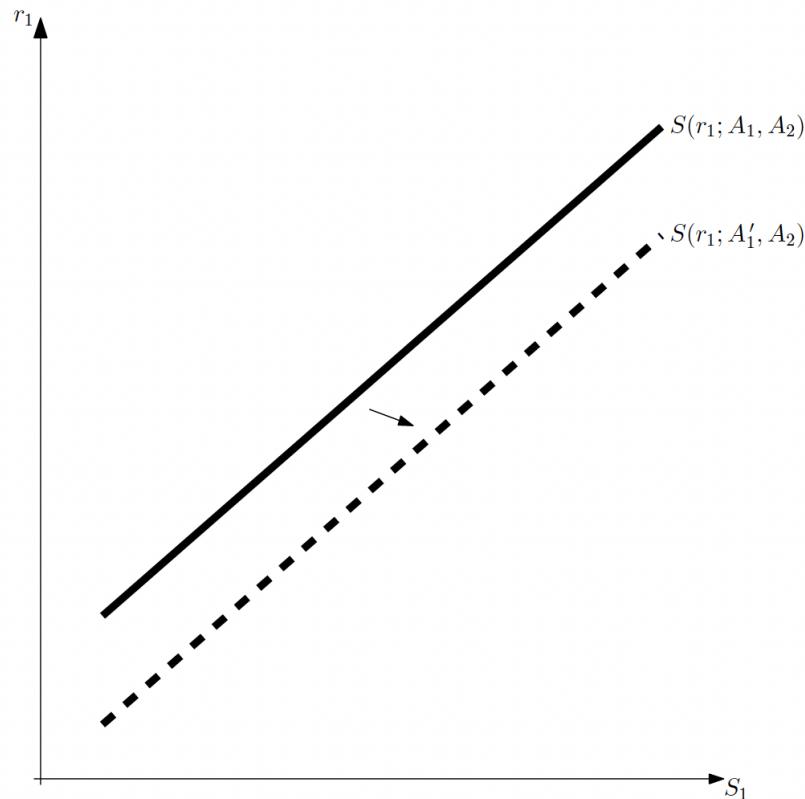
The Saving Schedule: $S_1 = S(r_1; A_1, A_2)$

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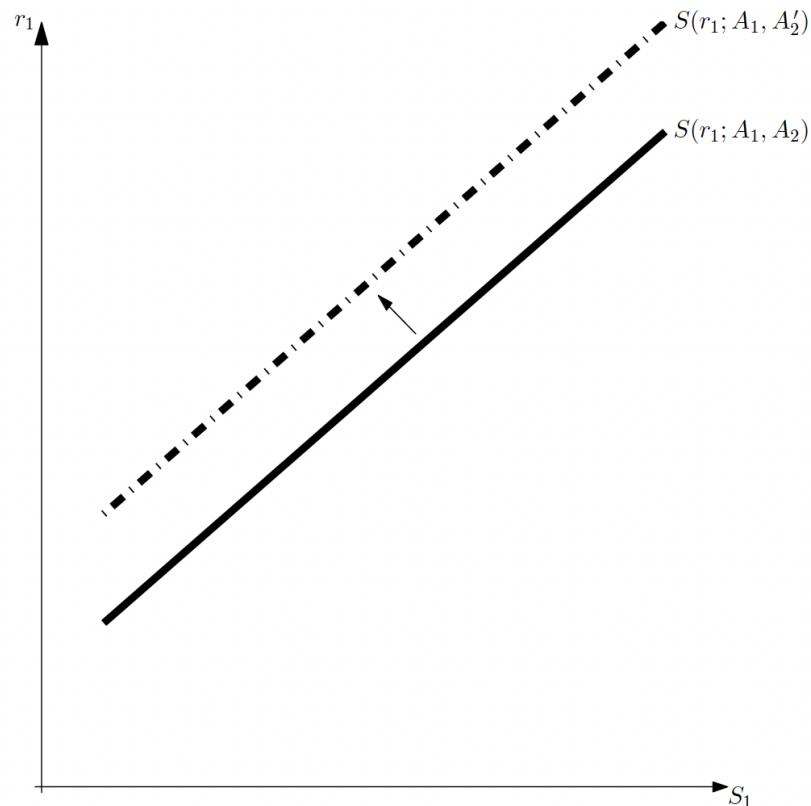
The saving schedule relates national saving to the interest rate. It slopes upward because an increase in the interest rate induces households to postpone current consumption and increase their holdings of interest bearing assets.

A temporary productivity increase shifts the saving schedule to the right



An increase in the productivity parameter in period 1 from A_1 to $A'_1 > A_1$ holding A_2 constant shifts the savings schedule down and to the right, because at every level of the interest rate households save part of the additional profit income generated by the increase in productivity for future consumption.

An anticipated future productivity increase shifts the saving schedule to the left



An anticipated increase in the productivity parameter from A_2 to $A'_2 > A_2$ leaves current income unchanged but increases future income. In period 1, households use some of the future increase in profits to increase consumption. Consequently, at every given level of the interest rate, saving declines.

The Current Account Schedule

The Current Account Schedule

The current account is defined as national saving minus investment

$$CA_1 = S_1 - I_1.$$

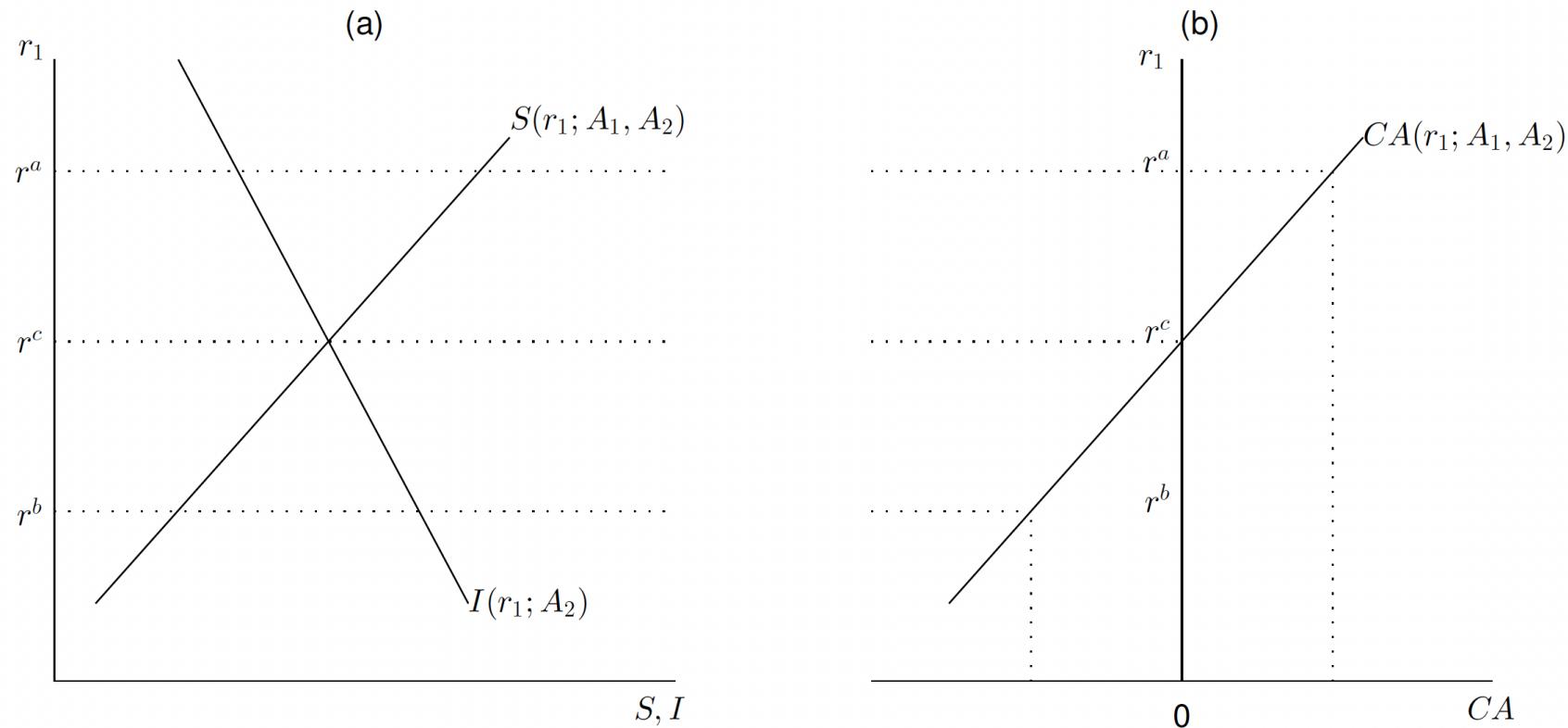
Use the saving schedule, (17), and the investment schedule, (7), to obtain

$$\begin{aligned} CA_1 &= S_1 - I_1 \\ &= S(r_1; A_1, A_2) - I(r_1; A_2) \\ &= CA(r_1; A_1, A_2) \end{aligned}$$

This equation represents the current account schedule, which expresses the current account as an increasing function of the interest rate with productivity as shifters of this function.

Saving, Investment and the Current Account

The Metzler Diagram



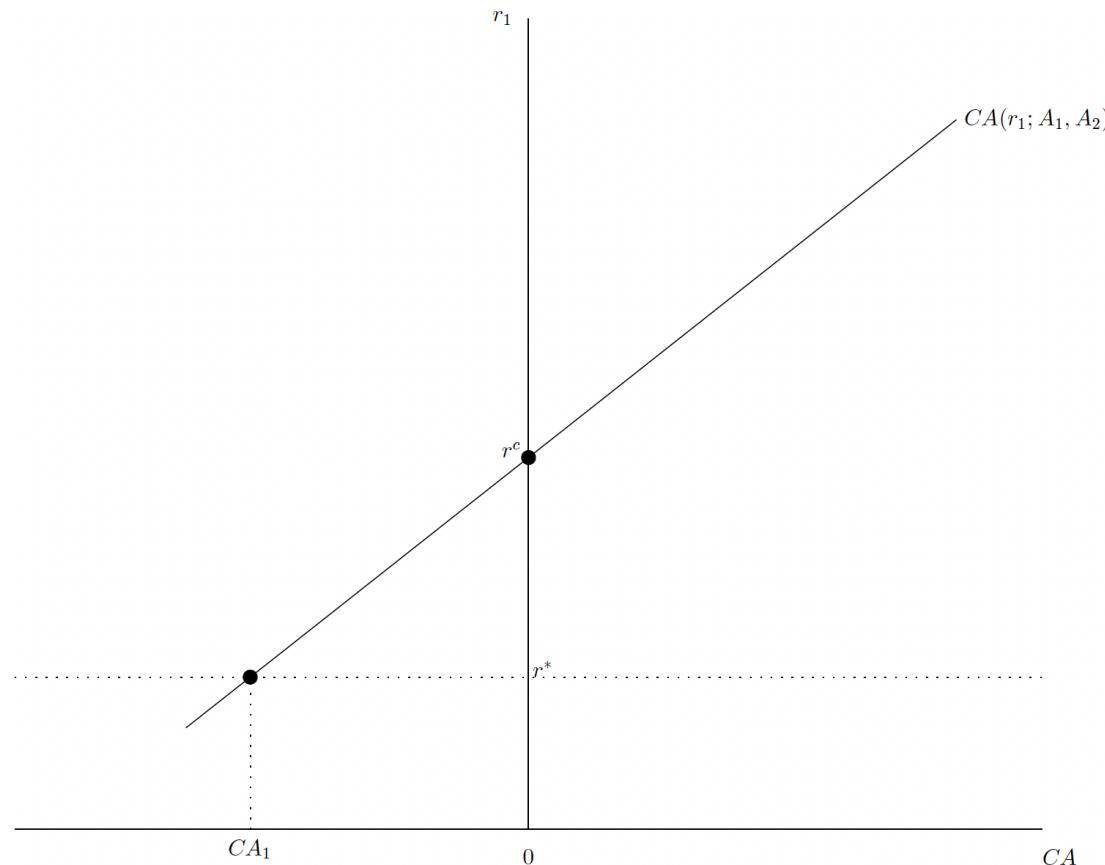
The figure presents a graphical derivation of the current account schedule. Panel (a) depicts the saving and investment schedule. Panel (b) depicts the current account schedule, which is the horizontal difference between the saving and investment schedule.

Equilibrium in the Production Economy

We assume that the country enjoys free capital mobility. Therefore, in equilibrium the domestic interest rate must equal the world interest rate, r^* :

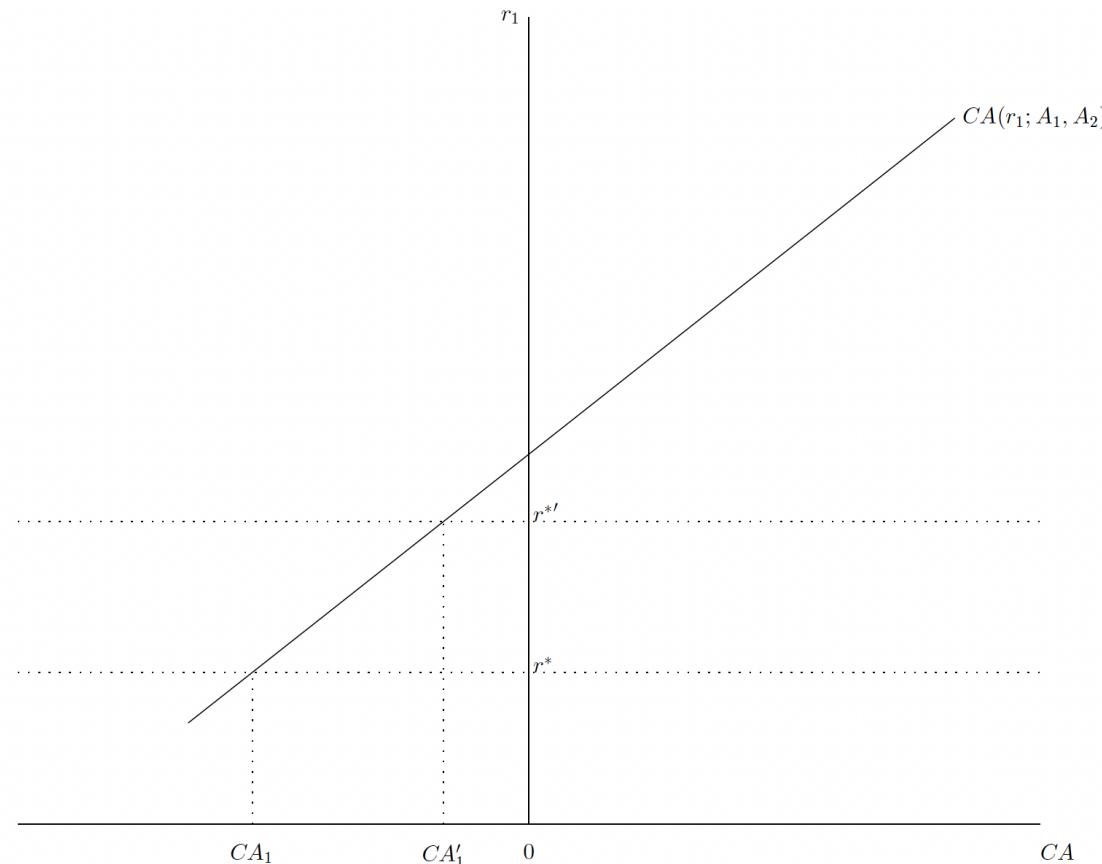
$$r_1 = r^*. \quad (18)$$

Current Account Determination in the Production Economy



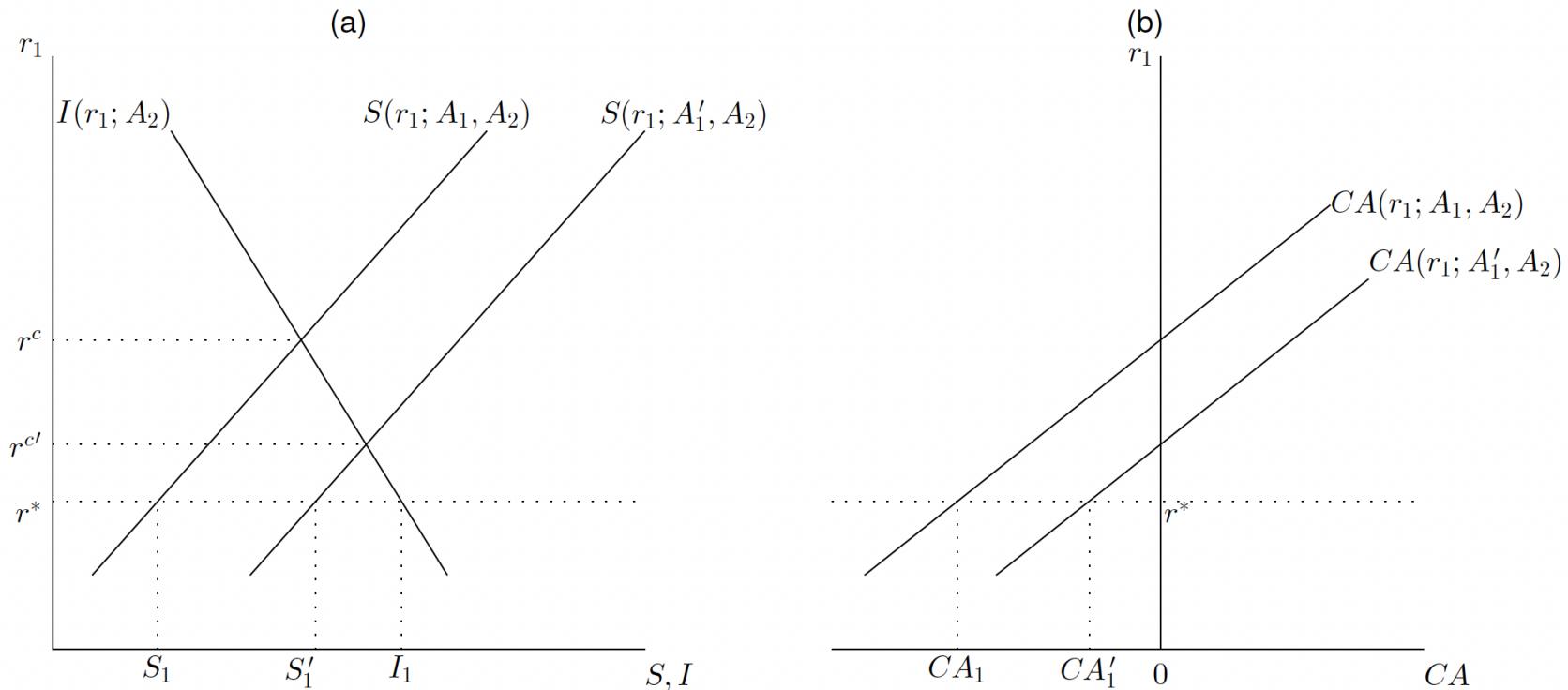
Under free capital mobility the current account is determined by the intersection of the current account schedule and the world interest rate, r^* . In a closed economy the current account is nil, and the domestic interest rate is r^c and is determined by the intersection of the current account schedule with the vertical axis.

Current Account Adjustment to an Increase in the World Interest Rate



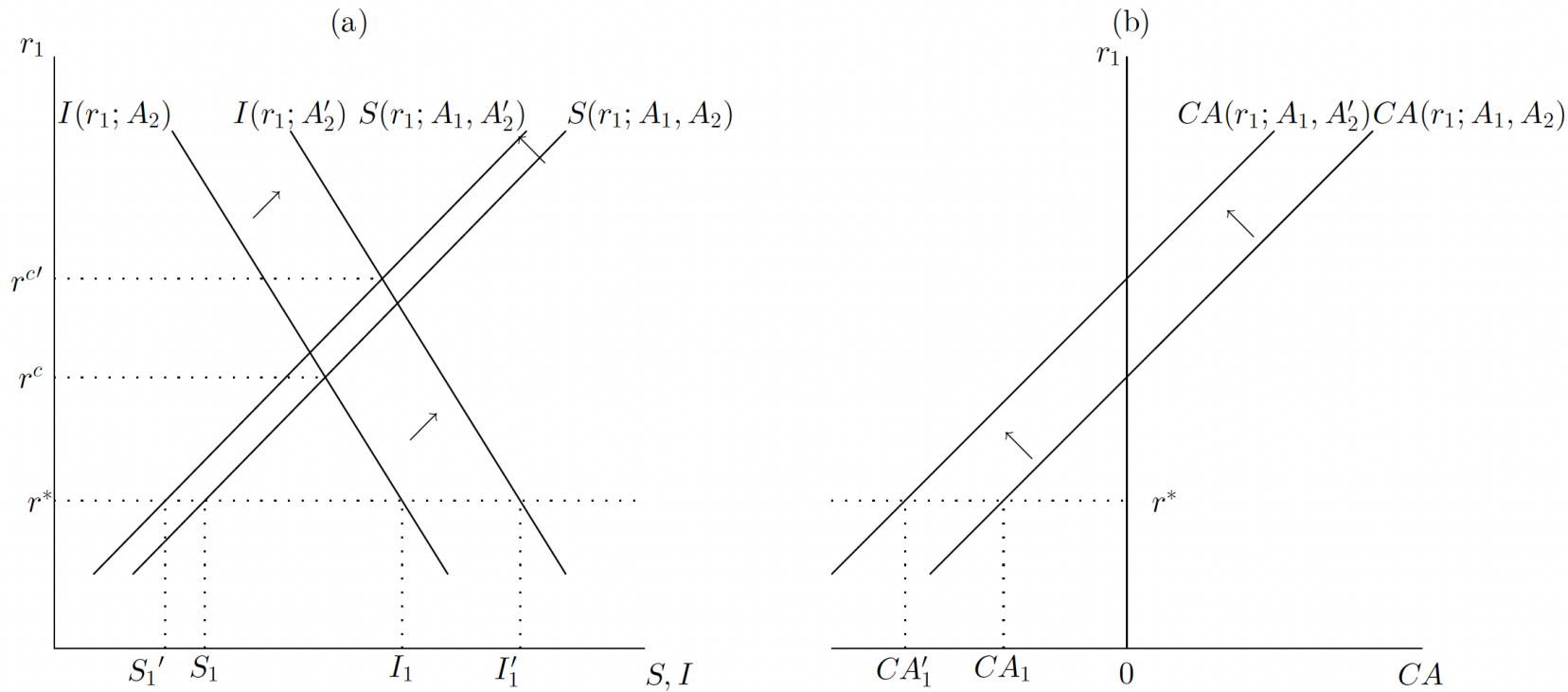
The initial world interest rate is r^* and the equilibrium current account is CA_1 . The world increases from r^* to $r'^* > r^*$. The higher interest rate leads to an improvement in the current account from CA_1 to CA'_1 .

Current account adjustment to a temporary increase in productivity



Productivity in period 1 increases from A_1 to $A'_1 > A_1$. The increase in A_1 shifts the saving schedule down and to the right. The investment schedule is unchanged. The current account schedule shifts down and to the right. In the new equilibrium the current account improves to CA'_1 , saving increases to S'_1 , and investment remains unchanged. In the closed economy, the increase in A_1 causes a fall in the domestic interest rate from r^c to r^{cl} and higher saving and investment.

Current account adjustment to an expected future increase in productivity



The productivity parameter is expected to increase from A_2 to A'_2 . The investment schedule shifts to the right and the saving schedule shifts to the left. The current account schedule shifts to the left. In the open economy, investment increases from I_1 to I'_1 , saving falls from S_1 to S'_1 , and the current account deteriorates from CA_1 to CA'_1 . In the closed economy, the interest rate increases from r^c to $r^{c'}$.

Adjustment of the Production Economy to Changes in the World Interest Rate and Productivity in Open and Closed Economies

	$r^* \uparrow$		$A_1 \uparrow$		$A_2 \uparrow$	
	Open	Closed	Open	Closed	Open	Closed
S_1	↑	—	↑	↑	↓	↑*
I_1	↓	—	—	↑	↑	↑*
CA_1	↑	—	↑	—	↓	—
r_1	↑	—	—	↓	—	↑

The table summarizes the effect of three different shocks on saving (S_1), investment (I_1), the current account (CA_1), and the domestic interest rate (r_1). The shocks considered are an increase in the world interest rate ($r^* \uparrow$), a temporary increase in productivity ($A_1 \uparrow$), and a future expected increase in productivity ($A_2 \uparrow$). Two different economic environments are considered: free capital mobility (Open) and a closed economy (Closed). * Note that the result that investment and saving increase in the closed economy in response to an increase in A_2 need not hold. It depends on the assumption that the horizontal shift in the saving schedule is smaller than that of the investment schedule. But this does not always have to be the case.

The Terms of Trade and The Current Account

The Terms of Trade and The Current Account

In chapter 4, we arrived at the conclusion that terms of trade shocks are just like endowment shocks. We will now show that this result extends to the production economy.

Assumptions:

- the country produces the export good (oil) and consumes the import good (food)
- physical capital is an import good
- the relative price of capital in terms of food is unity
- $TT_t = P_t^X/P_t^M$, terms of trade in period t , defined as the relative price of the export good (oil) in terms of the import good (food).

Profits in period 1: $\Pi_1 = TT_1 A_1 F(I_0) - (1 + r_0) I_0$

Profits in period 2: $\Pi_2 = TT_2 A_2 F(I_1) - (1 + r_1) I_1$

The profit maximization problem of firms

In period 1, the firm chooses I_1 to maximize Π_2 taking A_2 , TT_2 , and r_1 as given. The profit maximization condition of the firm is

$$TT_2 A_2 F'(I_1) = 1 + r_1$$

Same as equation (4), except that instead of A_2 it features $TT_2 A_2$.

→ a change in the terms of trade in period 2 has exactly the same effect on investment as a change in productivity in period 2.

Investment schedule in the two good economy:

$$I_1 = I(r_1; \underset{-}{TT}_2 \underset{+}{A}_2).$$

The Household's Problem in the Two Good Production Economy

The household's intertemporal budget constraint continues to be (10), with the only difference that now profits in periods 1 and 2 depend, respectively, on TT_1A_1 and TT_2A_2 instead of A_1 and A_2 . Therefore:

$$C_1 = C(r_1, \underset{-}{TT_1}A_1, \underset{+}{TT_2}A_2).$$

National income equals $Y_1 = r_0B_0 + Q_1 = r_0B_0 + TT_1A_1F(I_0)$, which also depends on TT_1A_1 instead of just A_1 .

Saving, $S_1 = Y_1 - C_1$, the difference between national income and consumption, behaves exactly as in the one-good economy except that, again, TT_1A_1 and TT_2A_2 take the place of A_1 and A_2 , respectively:

$$S_1 = S(r_1, \underset{+}{TT_1}A_1, \underset{+}{TT_2}A_2).$$

The current account schedule in the two good production economy

$$CA_1 = CA(r_1, \underset{+}{TT_1} A_1, \underset{+}{TT_2} \underset{-}{A_2}).$$

It follows that the current account adjustment to terms of trade shocks can be read off the table on slide 38 by replacing A_1 by $A_1 TT_1$ and A_2 by $A_2 TT_2$. In particular, a temporary terms of trade improvement (an increase in TT_1) produces an increase in saving, an improvement in the current account, and no change in investment. And an anticipated future improvement in the terms of trade (an increase in TT_2) causes a fall in saving, an expansion in investment, and a deterioration of the current account.

An Application: Giant Oil Discoveries

Are the predictions of the intertemporal production model of saving, investment, output, and current account determination studied in this chapter empirically compelling?

To answer this question, we examine the dynamics triggered by giant oil discoveries. In the context of our model, news of a giant oil discovery can be interpreted as an anticipated increase in the productivity of capital, that is, as an anticipated increase in A_2 .

The reason why a discovery is an anticipated productivity shock is that it takes time and extensive investment in oil production facilities to extract the oil and bring it to market. The average delay from discovery to production is estimated to be between 4 and 6 years.

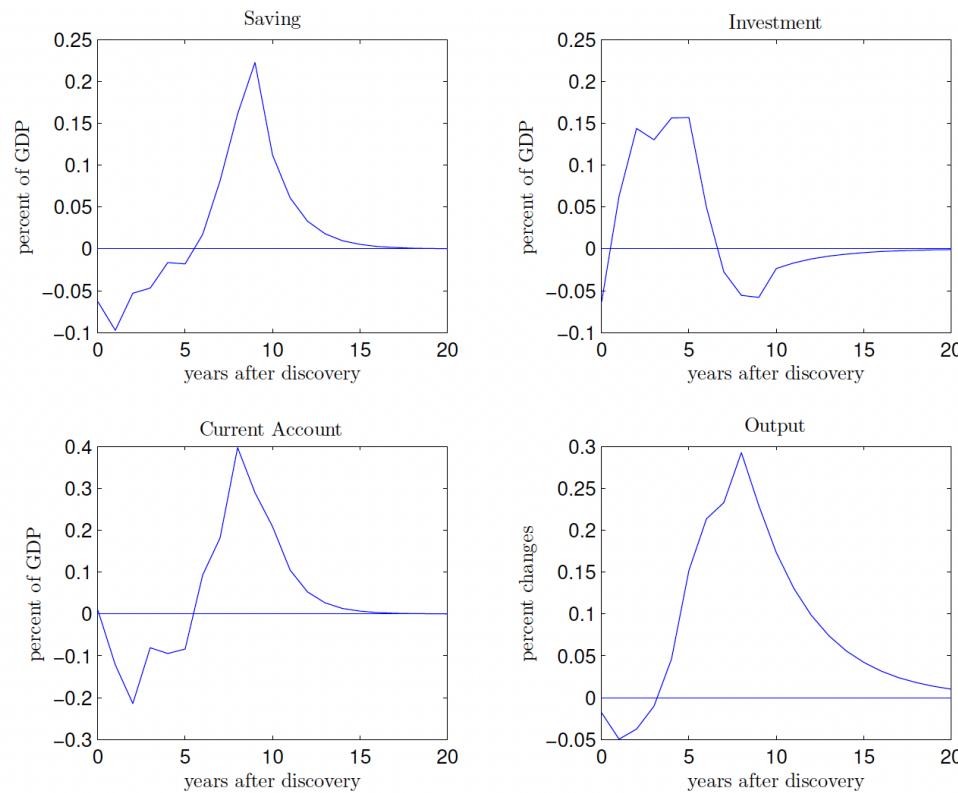
What Are Giant Oil Discoveries?

Arezki, Ramey, and Sheng analyze the effects of giant oil discoveries using data from 180 countries over the period 1970 to 2012.

They define a giant oil discovery as a discovery of an oil or gas field that contains at least 500 million barrels of ultimately recoverable oil equivalent. Giant oil discoveries are really big, with a median value of 9 percent of a year's GDP.

The sample contains in total 371 giant oil discoveries, which took place in 64 different countries (so 116 of the 180 countries in the sample experienced no giant oil discoveries over the sample period).

The dynamic responses are estimated using dynamic panel model estimation with distributed lags. Essentially, one runs a regression of the variable of interest, say the current account, onto its own lags, and current and lagged values of the giant oil discovery, and other control variables, such as a constant and time and country fixed effects.



The figure displays the estimated dynamic effect of an oil discovery on saving, investment, the current account, and output. The size of the oil discovery is 9 percent of GDP. Saving, investment, and the current account are expressed in percent of GDP. Output is expressed in percent deviation from trend. Data source. Arezki, Rabah, Valerie A. Ramey, and Liugang Sheng, "News Shocks in Open Economies: Evidence from Giant Oil Discoveries," *Quarterly Journal of Economics* 132, February 2017, 103-155, Online Appendix, Table D.I.

What Does the Figure Tell Us?

- Upon news of the giant oil discovery, investment experiences a boom that lasts for about 5 years.
- Saving declines and stays below normal for about 5 years, before rising sharply for several years.
- The current account deteriorates for 5 years and then experiences a reversal with a peak in year 8.
- Output is relatively stable until the fifth year, and then experiences a boom.
- The investment boom and the fall in saving and the current account last for roughly the delay from discovery to production typical in the oil industry.

- The estimated empirical responses are qualitatively consistent with the predictions of the theoretical model: The giant oil discovery induces oil companies to invest in the construction of drilling platforms. In addition, households anticipate higher profit income from the future exports of oil and as a result increase consumption and cut saving. Both the expansion in investment and the contraction in saving contribute to current account deficits in the initial years following the discovery.
- We conclude that the observed macroeconomic dynamics triggered by giant oil discoveries give credence to the intertemporal model of the current account studied in this chapter. This result is important because the model developed in this chapter is the backbone of many models in international macroeconomics.

Summing Up (1/2)

- Firms invest up to a point at which the marginal product of capital equals the gross interest rate.
- In the space (I_1, r_1) the investment schedule is downward sloping. An expected increase in the productivity of capital, A_2 , shifts the investment schedule up and to the right.
- In the space (S_1, r_1) , the saving schedule is upward sloping. An increase in the current productivity of capital, A_1 , shifts the saving schedule down and to the right. An expected increase in A_2 shifts the saving schedule up and to the left.
- The current account schedule is the horizontal difference between the saving and the investment schedules. In the space (CA_1, r_1) the current account schedule is upward sloping. An increase in A_1 shifts the current account schedule down and to the right. An expected increase in A_2 shifts the current account schedule up and to the left.

Summing Up (2/2)

- An increase in the world interest rate causes an improvement in the current account, an increase in saving, and a decrease in investment.
- An increase in A_1 produces an improvement in the current account. And an expected increase in A_2 causes a deterioration in the current account.
- Terms of trade shocks are just like productivity shocks. So their effects can be read off the last two bullets.
- The observed dynamics of saving, investment, and the current account triggered by giant oil discoveries around the world over the period 1970 to 2012 are consistent with the predictions of the intertemporal model of current account determination developed in this chapter.