

**Support and Feedback Class 4 (week 6)**

**Current Account Determination in a Production Economy**

**A. Pre-Class Review Questions**

1. **An Economy With Investment:** Consider a two-period model of a small open economy with a single good each period. Let preferences of the representative household be described by the utility function:

$$U(C_1, C_2) = \ln(C_1) + \ln(C_2),$$

where  $C_1$  and  $C_2$  denote, respectively, consumption in periods 1 and 2 and  $\ln$  denotes the natural logarithm. In period 1, the household receives an endowment of  $Q_1 = 10$ . In period 2, the household receives profits, denoted by  $\Pi_2$ , from the firms it owns. Households and firms have access to financial markets where they can borrow or lend at the interest rate  $r_1$ . ( $r_1$  is the interest rate on assets held between periods 1 and 2.) Firms invest in period 1 to be able to produce goods in period 2. The production technology in period 2 is given by:

$$Q_2 = \sqrt{I_1},$$

where  $Q_2$  and  $I_1$  denote, respectively, output in period 2 and investment in period 1. Assume that there exists free international capital mobility and that the world interest rate,  $r^*$ , is 10% per period (i.e.,  $r^* = 0.1$ ). Finally, assume that the economy's initial net foreign asset position is zero ( $B_0^* = 0$ ).

- (a) Compute the firm's optimal levels of period-1 investment and period-2 profits.
- (b) State the maximization problem of the representative household and solve for the optimal levels of consumption in periods 1 and 2.
- (c) Find the country's net foreign asset position at the end of period 1, the trade balance in periods 1 and 2, and the current account in periods 1 and 2.
- (d) Now consider an investment surge. Specifically, assume that as a result of a technological improvement, the production technology becomes  $Q_2 = 2\sqrt{I_1}$ . Find the equilibrium levels of savings, investment, the trade balance, the current account, and the country's net foreign asset position in period 1. Compare your results with those obtained in items 1-3. providing interpretation and intuition.

**B. In-Class Questions**

1. Consider a two-period model of a small open economy with a single good each period. Let preferences of the representative household be described by the utility function:

$$U(C_1, C_2) = \sqrt{C_1 C_2},$$

where  $C_1$  and  $C_2$  denote, respectively, consumption in periods 1 and 2. In period 1, the household receives an endowment of  $Q_1 = 10$ . In period 2, the household receives profits,

denoted by  $\Pi = 2$ , from the firms it owns. Households and firms have access to financial markets where they can borrow or lend at the interest rate  $r_1$ . Firms borrow in period 1 to invest in physical capital. They are subject to a collateral constraint of the form

$$D_1^f \leq \kappa_1$$

where  $D_1^f$  denotes the amount of debt assumed by the firm in period 1 and  $\kappa_1$  denotes the value of the firm's collateral. Suppose that  $\kappa_1$  equals 4. In turn, firms use the physical capital purchased in period 1 to produce final goods in period 2. The production technology in period 2 is given by

$$Q_2 = 6I_1^{\frac{1}{3}},$$

where  $Q_2$  and  $I_1$  denote, respectively, output in period 2 and investment in period 1. Assume that there exists free international capital mobility and that the world interest rate,  $r^*$ , is 10% per period (i.e.,  $r^* = 0.1$ ). Finally, assume that the economy's initial net foreign asset position is zero ( $B_0^* = 0$ ).

- (a) Compute the firm's optimal levels of period-1 investment and period-2 profits. Is the collateral constraint binding in period 1? Explain.
- (b) State the maximization problem of the representative household and derive the associated optimality condition.
- (c) Solve for the equilibrium levels of period 1 consumption, the country's net foreign asset position ( $B_1^*$ ), the trade balance, and the current account.
- (d) Now suppose that a financial panic causes banks to lower their assessment of the value of firms' collateral. Specifically, suppose that  $\kappa_1$  falls from 4 to 1. Solve for the equilibrium levels of investment, consumption, the trade balance, the current account, and the country's net asset position in period 1, and output and profits in period 2. Provide intuition.
- (e) *A Bailout:* Suppose that as a way to mitigate the financial crisis, in period 1 the government levies a tax on households, denoted  $T_1$ , and lends the proceeds to firms at the world interest rate. Let  $T_1 = 0.5$ , and let  $D^{fG}$  denote the debt that firms owe to the government and  $D^{fB}$  the debt that firms owe to private banks. Continue to assume that lending of private banks to firms is limited by the collateral constraint  $D^{fB} \leq \kappa_1$  and that  $\kappa_1 = 1$ . In period 2, the government collects loan payments from firms and rebates the whole amount (including interest) to households in the form of a subsidy. State the household's and firm's optimization problems. Compute the equilibrium levels of investment, consumption, the trade balance, the current account, and the country's net foreign asset position in period 1 and output and profits in period 2.
- (f) Is the bailout welfare improving? Answer this question by computing the lifetime welfare of the representative household with and without bailout. Discuss your result.

### BC. Self-study Questions

1. **Getting familiar with different utilities:** Redo the in-class exercise above assuming the following utility functions:

(a)

$$U(C_1, C_2) = \ln(C_1) + \ln(C_2),$$

(b)

$$U(C_1, C_2) = C_1 - \frac{1}{2}C_2^2,$$

[P.S. The numbers could potentially get ugly. Do not worry about that and try to approximate them, in case.]