

Support and Feedback Class 4 (week 6)
Current Account Determination in a Production Economy

Pre-Class Review Questions Solutions

A. Pre-class Review Questions These review questions are designed to help you reflect on the lecture material and check your understanding. You must complete them before the support and feedback class. At the start of the class, you will need to show your answers to your tutor, who will record both your attendance and preparation. If you have any doubts about the questions, you can ask your tutor at the beginning of the session. Please note that review questions will not be discussed in detail during the class; however, brief answers will be posted on Moodle before the test and exam.

1. **An Economy With Investment:** Consider a two-period model of a small open economy with a single good each period. Let preferences of the representative household be described by the utility function:

$$U(C_1, C_2) = \ln(C_1) + \ln(C_2),$$

where C_1 and C_2 denote, respectively, consumption in periods 1 and 2 and \ln denotes the natural logarithm. In period 1, the household receives an endowment of $Q_1 = 10$. In period 2, the household receives profits, denoted by Π_2 , from the firms it owns. Households and firms have access to financial markets where they can borrow or lend at the interest rate r_1 . (r_1 is the interest rate on assets held between periods 1 and 2.) Firms invest in period 1 to be able to produce goods in period 2. The production technology in period 2 is given by:

$$Q_2 = \sqrt{I_1},$$

where Q_2 and I_1 denote, respectively, output in period 2 and investment in period 1. Assume that there exists free international capital mobility and that the world interest rate, r^* , is 10% per period (i.e., $r^* = 0.1$). Finally, assume that the economy's initial net foreign asset position is zero ($B_0^* = 0$).

Solutions:

- (a) Compute the firm's optimal levels of period-1 investment and period-2 profits.

Given the production function $Q_2 = \sqrt{I_1}$, the marginal product of capital is $MPK = \frac{1}{2}(I_1)^{-\frac{1}{2}}$. Equating this with the cost of capital $1 + r_1 = 1 + r^* = 1.10$, we find the optimal level of investment in period 1:

$$I_1 = 0.21.$$

The implied level of output and profits in period 2 are

$$Q_2 = \sqrt{0.21},$$

$$\Pi_2 = \sqrt{0.21} - 1.10 \times 0.21 = 0.23.$$

- (b) State the maximization problem of the representative household and solve for the optimal levels of consumption in periods 1 and 2.

The household maximizes its lifetime utility

$$U(C_1, C_2) = \ln(C_1) + \ln(C_2)$$

subject to the intertemporal budget constraint

$$C_1 + \frac{C_2}{1+r_1} = (1+r_1)B_0^* + Q_1 + \frac{\Pi_2}{1+r_1}.$$

The marginal utilities of consumption are

$$U_1(C_1, C_2) = \frac{1}{C_1}, \quad U_2(C_1, C_2) = \frac{1}{C_2}.$$

With free capital mobility and $r^* = 0.10$, the intertemporal optimality condition (??) reads

$$-\frac{1/C_1}{1/C_2} = -(1+0.10),$$

which implies

$$C_2 = 1.10 \times C_1.$$

Since $B_0^* = 0$, $Q_1 = 10$ and $\Pi_2 = 0.23$, the budget constraint requires that

$$C_1 + \frac{C_2}{1+r_1} = 10.21.$$

Combining the last two conditions we obtain the optimal consumption plan:

$$C_1 = 5.11, \quad C_2 = 5.62.$$

- (c) Find the country's net foreign asset position at the end of period 1, the trade balance in periods 1 and 2, and the current account in periods 1 and 2.

Since $I_2 = 0$ in a two-period economy, the trade balances are

$$TB_1 = Q_1 - C_1 - I_1 = 10 - 5.11 - 0.21 = 4.68,$$

$$TB_2 = Q_2 - C_2 - I_2 = 0.46 - 5.62 - 0 = -5.16.$$

The implied net foreign assets position is

$$B_1^* = (1+r_0)B_0^* + TB_1 = 4.68.$$

The current account balances are then

$$CA_1 = r_0 B_0^* + TB_1 = 4.68,$$

$$CA_2 = r_1 B_1^* + TB_2 = 0.1 \times 4.68 - 5.16 = -4.69.$$

- (d) Now consider an investment surge. Specifically, assume that as a result of a technological improvement, the production technology becomes $Q_2 = 2\sqrt{I_1}$. Find the equilibrium levels of savings, investment, the trade balance, the current account, and the country's net foreign asset position in period 1. Compare your results with those obtained in items 1-3. providing interpretation and intuition.

The production function is now $Q_2 = 2\sqrt{I_1}$, so the marginal product of capital becomes $MPK = (I_1)^{-\frac{1}{2}}$. Equating this with the cost of capital again, we find a new investment level:

$$I_1 = 0.83.$$

Output and profits in period 2 are larger:

$$Q_2 = \sqrt{0.83} = 1.82,$$

$$\Pi_2 = 1.82 - 1.10 \times 0.83 = 0.91.$$

With this level of income, the intertemporal budget constraint requires that

$$C_1 + \frac{C_2}{1+r_1} = 10.83.$$

Combined with the intertemporal optimality condition $C_2 = 1.10 \times C_1$ again, this implies the new consumption plan

$$C_1 = 5.415, \quad C_2 = 5.957,$$

which implies the following trade balances:

$$TB_1 = 10 - 5.415 - 0.83 = 3.755,$$

$$TB_2 = 1.82 - 5.957 = -4.137.$$

The net foreign asset position and the current account balances are computed through the usual steps:

$$B_1^* = (1+r_0) B_0^* + TB_1 = 3.76,$$

$$CA_1 = r_0 B_0^* + TB_1 = 3.76,$$

$$CA_2 = r_1 B_1^* + TB_2 = 0.1 \times 3.755 - 4.14 = -3.76.$$

Higher productivity in period 2 raises both C_1 and C_2 . The household reduces its savings in period 1 to bring forward a portion of this productivity gain: the NFA is 3.76 instead of 4.68 in period 1. Because of this consumption smoothing, the country runs smaller trade and CA surpluses in period 1.