

Appendix to Search Frictions and the Business Cycle in a Small Open Economy DSGE Model

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January 2020

Note: This appendix is not a self-contained document.

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A Equilibrium Conditions of the Search Model

The variables in uppercase that are not prices contain a unit root in equilibrium due to the presence of the non-stationary productivity shock. To have a stationary version of the model, we divided those variables in t by A_{t-1} . For the case of the Lagrange multiplier Λ_t , we multiply it by A_{t-1}^σ because it decreases along the balanced growth path.

The rational expectations equilibrium of the stationary version of the model model is the set of sequences (60 variables)

$$\left\{ \begin{array}{l} \lambda_t, q_t, c_t, i_t, k_t, \Theta_t, \tilde{\chi}, u_t, m_t, n_t, \rho_t, s_t, e_t, \bar{\mathcal{E}}_t, \mathcal{E}_t^*, \mathcal{U}_t, \Delta_t^1, \Delta_t^2, w_t^*, \epsilon_t, \mu_t, \mathcal{F}_t^*, \mathcal{W}_t^* \\ \bar{\mathcal{W}}_t, w_t, \bar{w}_t, \tilde{w}_t, h_t, y_t^C, x_t^H, x_t^F, y_t^H, y_t^F, p_t^w, \tilde{p}_t^H, \tilde{p}_t^F, f_t^H, f_t^F, mc_t^H, mc_t^F, r_t^K \\ v_t, \rho_t^n, \bar{f}_t, \psi_t, y_t^m, \pi_t, R_t, \xi_t, rer_t, \pi_t^s, x_t^{H*}, p_t^H, p_t^F, \Delta_t^H, \Delta_t^F, b_t^*, tb_t, y_t, p_t^Y \end{array} \right\}_{t=0}^\infty$$

such that for given initial values and exogenous sequences

$$\{\kappa_t, \varrho_t, \varpi_t, \rho_t^x, z_t, a_t, y_t^{Co}, g_t, R_t^*, \pi_t^*, \zeta_t, y_t^*, p_t^{Co*}\}_{t=0}^\infty$$

and assuming $F_t \sim \log N(0, \sigma_w^2)$, the following conditions are satisfied:

Household (5 equations)

$$\lambda_t = \left(c_t - \zeta \frac{c_{t-1}}{a_{t-1}} \right)^{-\sigma} \quad (1)$$

$$\begin{aligned} \frac{1}{q_t} &= \left[1 - \frac{\gamma}{2} \left(\frac{i_t}{i_{t-1}} a_{t-1} - \bar{a} \right)^2 - \gamma \left(\frac{i_t}{i_{t-1}} a_{t-1} - \bar{a} \right) \frac{i_t}{i_{t-1}} a_{t-1} \right] \varpi_t \\ &+ \frac{\beta}{a_t^\sigma} \gamma E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}}{\lambda_t} \frac{q_{t+1}}{q_t} \left(\frac{i_{t+1}}{i_t} a_t - \bar{a} \right) \left(\frac{i_{t+1}}{i_t} a_t \right)^2 \varpi_{t+1} \right\} \end{aligned} \quad (2)$$

$$q_t = \frac{\beta}{a_t^\sigma} E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}}{\lambda_t} [r_{t+1}^K + q_{t+1}(1 - \delta)] \right\} \quad (3)$$

$$\lambda_t = \frac{\beta}{a_t^\sigma} R_t E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}}{\pi_{t+1}} \right\} \quad (4)$$

$$k_t = (1 - \delta) \frac{k_{t-1}}{a_{t-1}} + \left(1 - \frac{\gamma}{2} \left(\frac{i_t}{i_{t-1}} a_{t-1} - \bar{a} \right)^2 \right) \varpi_t i_t \quad (5)$$

Preference shifter (2 equations)

$$\Theta_t = \tilde{\chi}_t \left(c_t - \zeta \frac{c_{t-1}}{a_{t-1}} \right)^{-\sigma} \quad (6)$$

$$\tilde{\chi}_t = \tilde{\chi}_{t-1}^{1-v} \left(c_t - \zeta \frac{c_{t-1}}{a_{t-1}} \right)^{\sigma v} \quad (7)$$

Labor market variables (6 equations)

$$m_t = \bar{m} v_t^{1-\mu} u_t^\mu \quad (8)$$

$$n_t = (1 - \rho_t) (n_{t-1} + m_{t-1}) \quad (9)$$

$$u_t = 1 - n_t \quad (10)$$

$$s_t = m_t / u_t \quad (11)$$

$$e_t = m_t / v_t \quad (12)$$

$$\rho_t = \rho_t^n + (1 - \rho_t^n) \rho_t^x \quad (13)$$

Average value of employment (5 equations): recall that $\Xi_{t,t+1} = \frac{\beta}{a_t^\sigma} \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}}{\lambda_t}$ and $\Gamma_t^W = a_t^{\alpha_w} \pi_t^{\vartheta_W} \bar{\pi}^{1-\vartheta_W}$

$$\bar{\mathcal{E}}_t = w_t h_t - \frac{\Theta_t \kappa_t h_t^{1+\phi}}{\lambda_t (1 + \phi)} + a_t E_t \Xi_{t,t+1} [(1 - \rho_{t+1}) (\mathcal{E}_{t+1}^* + \theta^w \Delta_{t+1}^1) + \rho_{t+1} \mathcal{U}_{t+1}] \quad (14)$$

$$\mathcal{E}_t^* = w_t^* h_t - \frac{\Theta_t \kappa_t h_t^{1+\phi}}{\lambda_t (1 + \phi)} + a_t E_t \Xi_{t,t+1} [(1 - \rho_{t+1}) (\mathcal{E}_{t+1}^* + \theta^w \Delta_{t+1}^2) + \rho_{t+1} \mathcal{U}_{t+1}] \quad (15)$$

$$\mathcal{U}_t = \bar{b} + a_t E_t \Xi_{t,t+1} [s_t (1 - \rho_{t+1}) \bar{\mathcal{E}}_{t+1} + (1 - s_t (1 - \rho_{t+1})) \mathcal{U}_{t+1}] \quad (16)$$

$$\Delta_t^1 = \left(\frac{\Gamma_{t-1}^W}{a_{t-1}} \frac{w_{t-1}}{w_t^*} - 1 \right) w_t^* h_t + a_t \theta^w E_t \Xi_{t,t+1} \Delta_{t+1}^1 \quad (17)$$

$$\Delta_t^2 = \left(\frac{\Gamma_{t-1}^W}{a_{t-1}} \frac{w_{t-1}^*}{w_t^*} - 1 \right) w_t^* h_t + a_t \theta^w E_t \Xi_{t,t+1} \Delta_{t+1}^2 \quad (18)$$

Negotiated wage (6 equations):

$$\epsilon_t = h_t + \theta^w \Gamma_t^W E_t \Xi_{t,t+1} (1 - \rho_{t+1}) \frac{\epsilon_{t+1}}{\pi_{t+1}} \quad (19)$$

$$\mu_t = h_t + \theta^w \Gamma_t^W (1 - e_t) E_t \Xi_{t,t+1} (1 - \rho_{t+1}) \frac{\mu_{t+1}}{\pi_{t+1}} \quad (20)$$

$$\mathcal{F}_t^* = p_t^w (1 - \alpha) \frac{y_t^H}{n_t} - w_t^* h_t - \psi_t + \frac{\bar{\omega}}{e_t} \quad (21)$$

$$\mathcal{W}_t^* = \frac{\varphi}{1 - \varphi} \frac{\epsilon_t}{\mu_t} \mathcal{F}_t^* \quad (22)$$

$$\bar{\mathcal{W}}_t = \bar{\mathcal{E}}_t - \bar{b} - a_t E_t \Xi_{t,t+1} [s_t (1 - \rho_{t+1}) \bar{\mathcal{W}}_{t+1} + \mathcal{U}_{t+1}] \quad (23)$$

$$\mathcal{W}_t^* = w_t^* h_t - \frac{\Theta_t \kappa_t h_t^{1+\phi}}{\lambda_t (1 + \phi)} - \bar{b} + a_t E_t \Xi_{t,t+1} (1 - \rho_{t+1}) \left(\theta^w \frac{\varphi}{1 - \varphi} \frac{\epsilon_{t+1}}{\mu_{t+1}} (1 - \Gamma_{t+1}^W) w_{t+1}^* h_{t+1} - s_t \bar{\mathcal{W}}_{t+1} + \mathcal{W}_{t+1}^* \right) \quad (24)$$

Average real wage (3 equations):

$$w_t = (1 - \theta^w) w_t^* + \theta^w \bar{w}_t \quad (25)$$

$$\bar{w}_t \pi_t = \frac{\Gamma_{t-1}^W}{a_{t-1} n_t} (1 - \rho_t) ((1 - s_{t-1}) w_{t-1} n_{t-1} + s_{t-1} \tilde{w}_{t-1}) \quad (26)$$

$$\tilde{w}_t \pi_t = (1 - \theta^w) w_t^* \pi_t + \frac{\theta^w \Gamma_{t-1}^W}{a_{t-1}} \tilde{w}_{t-1} \quad (27)$$

Individual hours (1 equation):

$$h_t = \left(p_t^w (1 - \alpha)^2 \frac{\lambda_t}{\Theta_t \kappa_t} \frac{y_t^H}{n_t} \right)^{\frac{1}{1+\phi}} \quad (28)$$

Final and composite goods (5 equations):

$$y_t^C = \left[(1 - o)^{\frac{1}{\eta}} (x_t^H)^{\frac{\eta-1}{\eta}} + o^{\frac{1}{\eta}} (x_t^F)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (29)$$

$$x_t^H = (1 - o) (p_t^H)^{-\eta} y_t^C \quad (30)$$

$$x_t^F = o (p_t^F)^{-\eta} y_t^C \quad (31)$$

$$y_t^H \Delta_t^H = z_t \left(\frac{k_{t-1}}{a_{t-1}} \right)^\alpha (a_t n_t h_t)^{1-\alpha} \quad (32)$$

$$y_t^F \Delta_t^F = y_t^m \quad (33)$$

Calvo-pricing of retail goods (9 equations): recall that $\Gamma_t^H = \pi_{t-1}^{\vartheta_H} \bar{\pi}^{1-\vartheta_H}$ and $\Gamma_t^F = \pi_{t-1}^{\vartheta_F} \bar{\pi}^{1-\vartheta_F}$

$$mc_t^H = p_t^w / p_t^H \quad (34)$$

$$mc_t^F = rer_t / p_t^F \quad (35)$$

$$f_t^H = (\tilde{p}_t^H)^{-\epsilon_H} mc_t^H y_t^H + a_t \theta_H E_t \Xi_{t,t+1} \left\{ \left(\frac{\Gamma_t^H}{\pi_{t+1}} \right)^{-\epsilon_H} \left(\frac{\tilde{p}_t^H}{\tilde{p}_{t+1}^H} \right)^{-\epsilon_H} \left(\frac{p_t^H}{p_{t+1}^H} \right)^{-1-\epsilon_H} f_{t+1}^H \right\} \quad (36)$$

$$f_t^H = (\tilde{p}_t^H)^{1-\epsilon_H} \left(\frac{\epsilon_H - 1}{\epsilon_H} \right) y_t^H + a_t \theta_H E_t \Xi_{t,t+1} \left\{ \left(\frac{\Gamma_t^H}{\pi_{t+1}} \right)^{1-\epsilon_H} \left(\frac{\tilde{p}_t^H}{\tilde{p}_{t+1}^H} \right)^{1-\epsilon_H} \left(\frac{p_t^H}{p_{t+1}^H} \right)^{-\epsilon_H} f_{t+1}^H \right\} \quad (37)$$

$$f_t^F = (\tilde{p}_t^F)^{-\epsilon_F} mc_t^F y_t^F + a_t \theta_F E_t \Xi_{t,t+1} \left\{ \left(\frac{\Gamma_t^F}{\pi_{t+1}} \right)^{-\epsilon_F} \left(\frac{\tilde{p}_t^F}{\tilde{p}_{t+1}^F} \right)^{-\epsilon_F} \left(\frac{p_t^F}{p_{t+1}^F} \right)^{-1-\epsilon_F} f_{t+1}^F \right\} \quad (38)$$

$$f_t^F = (\tilde{p}_t^F)^{1-\epsilon_F} \left(\frac{\epsilon_F - 1}{\epsilon_F} \right) y_t^F + a_t \theta_F E_t \Xi_{t,t+1} \left\{ \left(\frac{\Gamma_t^F}{\pi_{t+1}} \right)^{1-\epsilon_F} \left(\frac{\tilde{p}_t^F}{\pi_{t+1}} \right)^{1-\epsilon_F} \left(\frac{p_t^F}{p_{t+1}^F} \right)^{-\epsilon_F} f_{t+1}^F \right\} \quad (39)$$

$$1 = (1 - \theta_H) (\tilde{p}_t^H)^{1-\epsilon_H} + \theta_H \left(\frac{p_{t-1}^H}{p_t^H} \frac{\Gamma_t^H}{\pi_t} \right)^{1-\epsilon_H} \quad (40)$$

$$1 = (1 - \theta_F) (\tilde{p}_t^F)^{1-\epsilon_F} + \theta_F \left(\frac{p_{t-1}^F}{p_t^F} \frac{\Gamma_t^F}{\pi_t} \right)^{1-\epsilon_F} \quad (41)$$

$$\frac{\bar{\omega}}{e_t} = a_t E_t \Xi_{t,t+1} (1 - \rho_{t+1}) \left(p_{t+1}^w (1 - \alpha) \frac{y_{t+1}^H}{n_{t+1}} - w_{t+1} h_{t+1} - \psi_{t+1} + \frac{\bar{\omega}}{e_{t+1}} \right) \quad (42)$$

Intermediate goods firms (6 equations):

$$r_t^K = p_t^w \alpha \frac{y_t^H}{k_{t-1}} a_{t-1} \quad (43)$$

$$v_t \bar{\omega} = y_t^C - c_t - i_t - g_t - n_t \psi_t \quad (44)$$

$$\rho_t^n = 1 - \Phi \left(\frac{\ln \bar{f}_t}{\sigma_w} \right) \quad (45)$$

$$\bar{f}_t = p_t^w (1 - \alpha) \frac{y_t^H}{n_t} - w_t h_t + \frac{\bar{\omega}}{e_t} \quad (46)$$

$$\psi_t = \frac{\exp \left(\frac{\sigma_w^2}{2} \right) \Phi \left(\frac{\ln \bar{f}_t - \sigma_w^2}{\sigma_w} \right)}{1 - \rho_t^n} \quad (47)$$

$$y_t^m = c_t + i_t + g_t + x_t^{H*} + y_t^{Co} - y_t \quad (48)$$

Taylor rule (1 equation):

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\rho_R} \left[\left(\frac{\pi_t}{\bar{\pi}} \right)^{\alpha_\pi} \left(\frac{y_t}{y_{t-1}} \right)^{\alpha_y} \right]^{1-\rho_R} \exp(\varepsilon_t^R) \quad (49)$$

Rest of the world (4 equations):

$$\xi_t = \bar{\xi} \exp \left[-\psi \frac{rer_t b_t^* - rer \times b^*}{rer \times b^*} + \frac{\zeta_t - \zeta}{\zeta} \right] \quad (50)$$

$$\frac{rer_t}{rer_{t-1}} = \frac{\pi_t^s \pi_t^*}{\pi_t} \quad (51)$$

$$\lambda_t = \frac{\beta}{a_t^\sigma} R_t^* \xi_t E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\pi_{t+1}^S \lambda_{t+1}}{\pi_{t+1}} \right\} \quad (52)$$

$$x_t^{H*} = o^* \left(\frac{p_t^H}{rer_t} \right)^{-\eta^*} y_t^* \quad (53)$$

Aggregation and market clearing (7 equations):

$$y_t^H = x_t^H + x_t^{H*} \quad (54)$$

$$y_t^F = x_t^F \quad (55)$$

$$\Delta_t^H = (1 - \theta_H) (\tilde{p}_t^H)^{-\epsilon_H} + \theta_H \left(\frac{p_{t-1}^H}{p_t^H} \frac{\Gamma_t^H}{\pi_t} \right)^{-\epsilon_H} \Delta_{t-1}^H \quad (56)$$

$$\Delta_t^F = (1 - \theta_F) (\tilde{p}_t^F)^{-\epsilon_F} + \theta_F \left(\frac{p_{t-1}^F}{p_t^F} \frac{\Gamma_t^F}{\pi_t} \right)^{-\epsilon_F} \Delta_{t-1}^F \quad (57)$$

$$b_t^* + (1 - \chi) p_t^{Co*} y_t^{Co} = \frac{b_{t-1}^* R_{t-1}^* \xi_t}{a_{t-1}} + \frac{tb_t}{rer_t} \quad (58)$$

$$tb_t = p_t^H x_t^{H*} + rer_t p_t^{Co*} y_t^{Co} - rer_t y_t^m \quad (59)$$

$$p_t^Y y_t = c_t + i_t + g_t + tb_t \quad (60)$$

B Steady State of the Search Model

We show how to compute the steady state of the model for given values $h, u, \rho = p^{EU} / (1 - p^{UE})$, $s_{\rho x} = \rho/\rho_x$, e , p^H , $s^{tb} = tb/(p^Y y)$, $s^g = g/(p^Y y)$ and $s^{Co} = rer \times p^{Co*} y^{Co} / (p^Y y)$. The parameters \bar{m} , $\bar{\rho}_x$, $\bar{\omega}$, \bar{g} , \bar{y}^{Co} , $\bar{\kappa}$, $\bar{\pi}^*$, o^* and \bar{b} are determined endogenously while the values of the remaining parameters are taken as given.

From the exogenous processes $x_t = \{\varrho_t, \varpi_t, z_t, a_t, R_t^*, \zeta_t, y_t^*, p_t^{Co*}\}$,

$$x = \bar{x}.$$

From (6) and (7),

$$\Theta = 1.$$

From (50),

$$\xi = \bar{\xi}.$$

From (49),

$$\pi = \bar{\pi}.$$

From (4),

$$R = \frac{\pi a^\sigma}{\beta}.$$

From (2),

$$q = \varpi^{-1}.$$

From (3),

$$r^K = q \left(\frac{a^\sigma}{\beta} - 1 + \delta \right).$$

From (52),

$$\pi^S = \frac{a^\sigma \pi}{\beta R^* \xi}.$$

From (51),

$$\pi^* = \bar{\pi}^* = \frac{\pi}{\pi^s}.$$

From (40) and (41),

$$\tilde{p}_t^H = 1, \quad \tilde{p}_t^F = 1.$$

From (56) and (57),

$$\Delta^H = (\tilde{p}^H)^{-\epsilon_H}, \quad \Delta^F = (\tilde{p}^F)^{-\epsilon_F}.$$

From (36)-(37) and (38)-(39),

$$mc^H = \left(\frac{\epsilon_H - 1}{\epsilon_H} \right) \tilde{p}^H, \quad mc^F = \left(\frac{\epsilon_F - 1}{\epsilon_F} \right) \tilde{p}^F.$$

From (34),

$$p^w = p^H mc^H.$$

From (10),

$$n = 1 - u.$$

From $s_{\rho x} = \rho^x / \rho$,

$$\rho^x = \bar{\rho}^x = s_{\rho x} \rho.$$

From (13),

$$\rho^n = \frac{\rho - \rho^x}{1 - \rho^x}.$$

From (45),

$$\bar{f} = \exp(\sigma_w \Phi^{-1}(1 - \rho^n)).$$

From (47),

$$\psi = \frac{\exp\left(\frac{\sigma_w^2}{2}\right) \Phi\left(\frac{\ln \bar{f} - \sigma_w^2}{\sigma_w}\right)}{1 - \rho^n}.$$

From (42) and (46),

$$\bar{\omega} = ea^{1-\sigma}\beta(1-\rho)(\bar{f}-\psi).$$

From (9) and (12),

$$v = \frac{\rho n}{e(1-\rho)}.$$

From (12),

$$m = ev.$$

From (8),

$$\bar{m} = mv^{\mu-1}u^{-\mu}.$$

From (11),

$$s = m/u.$$

From (32) and (43),

$$k = a^2hn \left(\frac{\alpha p^w z}{\Delta^H r^K} \right)^{\frac{1}{1-\alpha}}.$$

From (32),

$$y^H = \frac{z \left(\frac{k}{a} \right)^\alpha (anh)^{1-\alpha}}{\Delta^H}.$$

From (46),

$$w = \frac{1}{h} \left(p^w (1-\alpha) \frac{y^H}{n} + \frac{\bar{\omega}}{e} - \bar{f} \right).$$

From (25)-(27),

$$w^* = w \left(\frac{1 - (1-\rho)(1-s)a^{\alpha_w-1}\theta^w}{1-\theta^w} \right) \left(1 + \frac{(1-\rho)s}{n} \frac{a^{\alpha_w-1}\theta^w}{1-a^{\alpha_w-1}\theta^w} \right)^{-1}.$$

From (27),

$$\tilde{w} = \frac{1-\theta^w}{1-a^{\alpha_w-1}\theta^w} w^*.$$

From (25),

$$\bar{w} = \frac{w}{\theta^w} - \frac{1-\theta^w}{\theta^w} w^*.$$

From (19),

$$\epsilon = \frac{h}{1-\theta^w a^{\alpha_w-\sigma} \beta (1-\rho)}.$$

From (20),

$$\mu = \frac{h}{1-\theta^w a^{\alpha_w-\sigma} \beta (1-\rho) (1-e)}.$$

From (21),

$$\mathcal{F}^* = p^w (1 - \alpha) \frac{y^H}{n} - w^* h - \psi + \frac{\bar{\omega}}{e}.$$

From (22),

$$\mathcal{W}^* = \frac{\varphi\epsilon}{(1 - \varphi)\mu} \mathcal{F}^*.$$

From (17),

$$\Delta^1 = \left(\frac{a^{\alpha_w-1} \pi w / w^* - 1}{1 - a^{1-\sigma} \theta^w \beta} \right) w^* h.$$

From (18),

$$\Delta^2 = \left(\frac{a^{\alpha_w-1} \pi - 1}{1 - a^{1-\sigma} \theta^w \beta} \right) w^* h.$$

From (14)-(16), (23) and (24),

$$\mathcal{U} = \frac{\alpha_7 - \alpha_3 + \alpha_5 + a^{1-\sigma} \beta s (1 - \rho) \alpha_7}{\alpha_8 + \alpha_4 - \alpha_6 + a^{1-\sigma} \beta s (1 - \rho) \alpha_8 - a^{1-\sigma} \beta}$$

where

$$\alpha_1 = \frac{w^* h - \frac{\Theta \kappa h^{1+\phi}}{\lambda(1+\phi)} + a^{1-\sigma} \beta (1 - \rho) \theta^w \Delta^2}{1 - a^{1-\sigma} \beta (1 - \rho)}, \quad \alpha_2 = \frac{a^{1-\sigma} \beta \rho}{1 - a^{1-\sigma} \beta (1 - \rho)},$$

$$\alpha_3 = wh - \frac{\Theta \kappa h^{1+\phi}}{\lambda(1+\phi)} + a^{1-\sigma} \beta (1 - \rho) (\theta^w \Delta^1 + \alpha_1),$$

$$\alpha_4 = ((1 - \rho) \alpha_2 + \rho) a^{1-\sigma} \beta, \quad \alpha_5 = -a^{1-\sigma} \beta s (1 - \rho) \alpha_3, \quad \alpha_6 = 1 - a^{1-\sigma} \beta (1 + s (1 - \rho) (\alpha_4 - 1)),$$

$$\alpha_7 = \frac{\left(1 + a^{1-\sigma} \beta (1 - \rho) \theta^w \frac{\varphi}{1-\varphi} \frac{\epsilon}{\mu} (1 - \Gamma^W) \right) w^* h - \frac{\Theta \kappa h^{1+\phi}}{\lambda(1+\phi)} - (1 - a^{1-\sigma} \beta (1 - \rho)) \mathcal{W}^* - \alpha_5}{a^{1-\sigma} \beta s (1 - \rho)}, \quad \alpha_8 = \frac{\alpha_6}{a^{1-\sigma} \beta s (1 - \rho)}$$

From (15),

$$\mathcal{E}^* = \alpha_1 + \alpha_2 \mathcal{U}$$

From (14),

$$\bar{\mathcal{E}} = \alpha_3 + \alpha_4 \mathcal{U}$$

From (16),

$$\bar{b} = \alpha_5 + \alpha_6 \mathcal{U}$$

From (24),

$$\bar{\mathcal{W}} = \alpha_7 - \alpha_8 \mathcal{U}$$

From (36) and $\Xi = \beta/a^\sigma$,

$$f^H = \frac{(\tilde{p}^H)^{-\epsilon_H} m c^H y^H}{1 - \beta a^{1-\sigma} \theta_H}.$$

From (5),

$$i = \frac{1 - (1 - \delta)/a}{\varpi} k.$$

From (29)-(31),

$$p^F = \left(\frac{1}{o} - \frac{1-o}{o} (p^H)^{1-\eta} \right)^{1/(1-\eta)}.$$

From (35),

$$rer = p^F mc^F.$$

From nominal GDP equal to $p^Y y = p^H y^H + p^F y^F + rer \times p^{Co*} y^{Co} - rer \times y^m - \psi n - \bar{\omega} v$,

$$p^Y y = \frac{p^H y^H + (\psi n + \bar{\omega} v) \left(p^F (1 - mc^F \Delta^F) o (p^F)^{-\eta} - 1 \right)}{1 - s^{Co} - p^F (1 - mc^F \Delta^F) o (p^F)^{-\eta} (1 - s^{tb})}.$$

From $s^{tb} = tb/(p^Y y)$, $s^g = g/(p^Y y)$ and $s^{Co} = rer p^{Co*} y^{Co}/(p^Y y)$,

$$tb = s^{tb} p^Y y, \quad g = \bar{g} = s^g p^Y y, \quad y^{Co} = \bar{y}^{Co} = \frac{s^{Co} p^Y y}{rer \times p^{Co*}}.$$

From (30), (44), (54) and (60),

$$x^{H*} = y^H - (1 - o) (p_t^H)^{-\eta} (c + i + g + n\psi + v\bar{\omega})$$

From (59), (33) and (55),

$$x_t^F = \frac{p^H x^{H*} + rer p^{Co*} y^{Co} - tb}{rer \times \Delta_t^F}.$$

From (31),

$$y^C = \frac{x^F}{o} (p^F)^\eta.$$

From (44),

$$c = y^C - i - g - n\psi - v\bar{\omega}.$$

From (1),

$$\lambda = \left(c - \zeta \frac{c}{a} \right)^{-\sigma}.$$

From (28),

$$\kappa = \bar{\kappa} = \frac{p^w \lambda (1 - \alpha)^2 y^H}{\Theta h^{1+\phi}} \frac{y^H}{n}.$$

From (7),

$$\tilde{\chi} = \left(c - \zeta \frac{c}{a} \right)^\sigma.$$

From (53),

$$o^* = \frac{x^{H*}}{y^*} \left(\frac{p^H}{rer} \right)^{\eta^*}.$$

From (58),

$$b^* = \frac{tb - (1 - \chi) rer \times p^{Co*} y^{Co}}{rer (1 - R^* \xi / a\pi^*)}.$$

From (54),

$$x^H = y^H - x^{H*}.$$

From (55),

$$y^F = x^F.$$

From (38),

$$f^F = \frac{(\tilde{p}^F)^{-\epsilon_F} y^F m c^F}{1 - \beta a^{1-\sigma} \theta_F}.$$

From (33),

$$y^m = y^F \Delta^F.$$

From (48),

$$y = c + i + g + x^{H*} + y^{Co} - y^m.$$

From (60),

$$p^Y = \frac{c + i + g + tb}{y}.$$

C Equilibrium Conditions of the Calvo Wages Model

The rational expectations equilibrium of the stationary version of the model is the set of sequences

$$\{\lambda_t, c_t, h_t, h_t^d, w_t, \tilde{w}_t, m c_t^W, f_t^W, \Delta_t^W, \gamma_t^W, i_t, k_t, r_t^K, q_t, y_t, y_t^C, y_t^F, y_t^H, x_t^F, x_t^H, x_t^{H*}, R_t, \xi_t, \pi_t, \pi_t^S, rer_t, p_t^H, \tilde{p}_t^H, p_t^F, \tilde{p}_t^F, p_t^Y, p_t^m, m c_t^H, f_t^H, \Delta_t^H, m c_t^F, f_t^F, \Delta_t^F, b_t^*, imp_t, tb_t\}_{t=0}^\infty,$$

(41 variables) such that for given initial values and exogenous sequences

$$\{\kappa_t, \varrho_t, \varpi_t, z_t, a_t, \zeta_t^o, \zeta_t^u, R_t^*, \pi_t^*, p_t^{Co*}, y_t^{Co}, y_t^*, g_t\}_{t=0}^\infty,$$

conditions (1)-(5), (29)-(31), (33)-(41), (43), (48)-(60), and the following conditions are satisfied:

$$w_t m c_t^W = \kappa \frac{h_t^\phi}{\lambda_t}, \quad (61)$$

$$\begin{aligned} f_t^W &= mc_t^W \tilde{w}_t^{-\epsilon_W} h_t^d \\ &+ \frac{\beta}{a_t^{\sigma-1}} \theta_W E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\gamma_t^W}{a_t \pi_{t+1}} \right)^{-\epsilon_W} \left(\frac{\tilde{w}_t}{\tilde{w}_{t+1}} \right)^{-\epsilon_W} \left(\frac{w_t}{w_{t+1}} \right)^{-1-\epsilon_W} f_{t+1}^W \right\}, \quad (62) \end{aligned}$$

$$\begin{aligned} f_t^W &= \tilde{w}_t^{1-\epsilon_W} h_t^d \left(\frac{\epsilon_W - 1}{\epsilon_W} \right) \\ &+ \frac{\beta}{a_t^{\sigma-1}} \theta_W E_t \left\{ \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{\gamma_t^W}{a_t \pi_{t+1}} \right)^{1-\epsilon_W} \left(\frac{\tilde{w}_t}{\tilde{w}_{t+1}} \right)^{1-\epsilon_W} \left(\frac{w_t}{w_{t+1}} \right)^{-\epsilon_W} f_{t+1}^W \right\}, \quad (63) \end{aligned}$$

$$1 = (1 - \theta_W) \tilde{w}_t^{1-\epsilon_W} + \theta_W \left(\frac{w_{t-1}}{w_t} \frac{\gamma_t^W}{a_{t-1} \pi_t} \right)^{1-\epsilon_W}, \quad (64)$$

$$\Delta_t^W = (1 - \theta_W) \tilde{w}_t^{-\epsilon_W} + \theta_w \left(\frac{w_{t-1}}{w_t} \frac{\gamma_t^W}{a_{t-1} \pi_t} \right)^{-\epsilon_W} \Delta_{t-1}^W, \quad (65)$$

$$y_t^H \Delta_t^H = z_t \left(\frac{k_{t-1}}{a_{t-1}} \right)^\alpha (a_t h_t^d)^{1-\alpha}, \quad (66)$$

$$w_t = p_t^m \alpha \frac{y_t^H}{h_t^d} a_{t-1}, \quad (67)$$

$$h_t = h_t^d \Delta_t^W, \quad (68)$$

$$y_t^C = c_t + i_t + g_t, \quad (69)$$

$$\gamma_t^W = a^{\alpha_W} \pi_t^{\vartheta_W} \bar{\pi}^{1-\vartheta_W}. \quad (70)$$

D Steady State of the Calvo Wages Model

We solve for the steady state for given values of h , p^H , $s^{tb} = tb/(p^Y y)$, $s^g = g/(p^Y y)$ and $s^{Co} = rer \times p^{Co*} y^{Co} / (p^Y y)$. The parameters $\bar{\pi}^*$, κ , o^* , \bar{g} and \bar{y}^{Co} are determined endogenously while the values of the remaining parameters are taken as given. The following equations are added to the steady state of the model with search frictions.

From (64),

$$\tilde{w} = \left(\frac{1 - \theta_W (\gamma^W / a \pi)^{1-\epsilon_W}}{1 - \theta_W} \right)^{\frac{1}{1-\epsilon_W}}.$$

From (65),

$$\Delta^W = \frac{1 - \theta_W}{1 - \theta_w (\gamma^W / a \pi)^{-\epsilon_W}} \tilde{w}^{-\epsilon_W}.$$

From (62)-(63),

$$mc^W = \left(\frac{\epsilon_W - 1}{\epsilon_W} \frac{1 - \beta a^{1-\sigma} (\gamma^W / a \pi)^{-\epsilon_W} \theta_W}{1 - \beta a^{1-\sigma} (\gamma^W / a \pi)^{1-\epsilon_W} \theta_W} \right) \tilde{w}.$$

From (68),

$$h^d = h/\Delta^W.$$

From (62),

$$f^W = \tilde{w}^{-\epsilon_W} h^d m c^W / \left(1 - \beta a^{1-\sigma} (\gamma^W/a\pi)^{-\epsilon_W} \theta_W \right).$$

From (56), (2) and (67),

$$w = \left[\frac{\alpha^\alpha (1-\alpha)^{1-\alpha} p^H m c^H z a^{1-\alpha}}{(r^K)^\alpha} \right]^{\frac{1}{1-\alpha}}.$$

From (56) and (67),

$$k = \frac{\alpha a w h^d}{(1-\alpha) r^K}.$$

From (66),

$$y^H = z (k/a)^\alpha (ah^d)^{1-\alpha} / \Delta^H.$$

From (69),

$$c = y^C - g - i.$$

From GDP equal to value added,

$$p^Y y = p^H y^H + p^Y y s^{Co} + p^F (1 - m c^F \Delta^F) o (p^F)^{-\eta} (1 - s^{tb}) p^Y y.$$

From (61),

$$\kappa = m c^W \lambda w / h^\phi.$$

The remaining equations are the same as in the model with search frictions, except for the equations corresponding to the labor market variables from the search model which are eliminated.

Table 1: Calibrated Parameters and Targeted Steady State Values.

Parameter	Description	Value	Source
<i>Search model</i>			
u	Unemployment rate in st. st.	0.08	Average (1987-2014)
e	Firm matching rate	0.7	Den Haan et al. (2000)
ρ	Total separation rate	0.0755	Jones and Naudon (2009)
ρ^x	Exog. separation rate	$\frac{2}{3}\rho$	Den Haan et al. (2000)
$\mu_{\tilde{c}}$	Log-normal mean of \tilde{c}	0	Normalization
<i>Standard model</i>			
ϵ_W	E. o. s. wages	11	Medina and Soto (2007)
<i>Common parameters</i>			
σ	Inverse intertemporal e. o. s.	1	Medina and Soto (2007)
α	Capital share in production	1-0.66	Medina and Soto (2007)
δ	Capital depreciation	0.06/4	Medina and Soto (2007)
ϵ_H	E. o. s. domestic aggregate	11	Medina and Soto (2007)
ϵ_F	E. o. s. imported aggregate	11	Medina and Soto (2007)
α_W	Indexation parameter	1	Medina and Soto (2007)
o	Share of F in Y^C	0.32	Average (1987-2014)
χ	Gov. share in commodity sector	0.61	Average (1987-2014)
s^{tb}	Trade balance to GDP in st. st.	0.04	Average (1987-2014)
s^g	Gov. cons. to GDP in st. st.	0.11	Average (1987-2014)
s^{Co}	Commod. prod. to GDP in st. st.	0.10	Average (1987-2014)
$\bar{\pi}$	Inflation in st. st.	3% p.a.	Inflation target in Chile
p^H	Relative price of H in st. st.	1	Normalization
h	Hours per worker in st. st.	0.3	Normalization
\bar{a}	Long-run growth	2% p.a.	Albagli et al. (2015)
β	Subjective discount factor	0.9995	MPR approx. 5%
R^*	Foreign rate in st. st.	4.5% p.a.	Fuentes and Gredig (2008)
ξ	Country premium in st. st.	1.5% p.a.	Average (1987-2014)

Note: All rates are annualized figures.

E Detailed Results and Robustness Checks

Table 2: Marginal Data Densities.

	Search				Standard
	With ρ_t^x	W/out ρ_t^x	Low ρ_t^n	W/out ρ_t^x , low ρ_t^n	
$\log p(X^T \text{ without } u^T \theta)$	-1453.17	-1456.04	-1443.33	-1443.18	-1473.38
$\log p(X^T \theta)$	-1505.99	-1525.49	-1488.75	-	-1473.38

Note: X^T denotes the full data set, X^T without u^T the set excluding the unemployment rate. For the model with search frictions, we also compute the marginal likelihoods shutting down the exogenous separation shock (ρ_t^x), and lowering endogenous separations in equilibrium. The marginal data densities are Laplace approximations at the mean of the posterior distribution.

Table 3: Second Moments.

Variable	Description	s.d. (%)			AC order 1			Correl. Non-mining GDP		
		Data	Search	Standard	Data	Search	Standard	Data	Search	Standard
$\Delta \log Y^{NM}$	Non-mining GDP	1.14	1.66	1.92	0.20	0.00	0.61	1.00	1.00	1.00
$\Delta \log C$	Consumption	1.21	1.28	1.72	0.36	0.47	0.66	0.64	0.21	0.56
$\Delta \log I$	Investment	3.75	4.69	6.02	0.20	0.47	0.74	0.22	0.60	0.75
π	Inflation	0.69	0.79	1.23	0.60	0.49	0.63	-0.33	-0.31	-0.20
R	MPR	0.42	0.47	0.85	0.88	0.88	0.94	-0.35	-0.07	-0.22
rer	Real exch. rate	5.17	6.69	10.29	0.75	0.76	0.89	-0.17	-0.38	-0.18
ξ	EMBIG Chile	0.15	0.26	0.27	0.83	0.94	0.95	-0.24	-0.10	-0.13
$\Delta \log W$	Real wage	0.58	0.69	1.25	0.36	0.44	0.48	0.24	0.48	0.59
$h \times n$	Total hours worked	1.87	1.97	7.61	0.73	0.90	0.89	0.06	0.13	0.1
u	Unemployment rate	1.43	1.37	-	0.96	0.87	-	0.11	-0.14	-

Note: The model moments are the theoretical moments at the posterior mean.

Table 4: Estimated Parameters.

Param.	Description	Posterior						
		Prior			Search		Standard	
		Dist.	Mean	s.d.	Mean	90% HPDI	Mean	90% HPDI
v	Wealth effect size	beta	0.5	0.25	0.10	[0.00, 0.21]	0.34	[0.00, 0.70]
ϕ	Inv. elast. of h	norm	2	2	5.09	[2.90, 7.30]	1.00	[0.00, 2.16]
ς	Habit formation	beta	0.7	0.1	0.75	[0.66, 0.83]	0.82	[0.74, 0.89]
ψ	Country prem. elast.	invg	0.01	Inf	0.005	[0.003, 0.007]	0.005	[0.003, 0.006]
η	E. o. s. H and F	invg	1.5	0.25	3.53	[2.87, 4.18]	1.74	[1.36, 2.09]
η^*	RER elast. of X^{H^*}	invg	0.25	0.1	0.23	[0.13, 0.32]	0.18	[0.12, 0.24]
γ	Inv. adj. cost	norm	4	1.5	0.81	[0.11, 1.77]	5.48	[3.59, 7.44]
φ	Bargaining power	beta	0.5	0.1	0.27	[0.09, 0.45]	—	—
σ_w	s.d. of F	norm	0.1	0.05	0.30	[0.25, 0.36]	—	—
μ	Match elast.	beta	0.5	0.1	0.75	[0.65, 0.84]	—	—
θ_W	Nom. W stickiness	beta	0.5	0.1	0.78	[0.67, 0.89]	0.90	[0.87, 0.95]
ϑ_W	Index. past infl. W	beta	0.5	0.15	0.24	[0.10, 0.36]	0.40	[0.21, 0.60]
θ_H	Calvo prob. H	beta	0.5	0.1	0.33	[0.24, 0.42]	0.56	[0.50, 0.61]
ϑ_H	Index. past infl. H	beta	0.5	0.15	0.51	[0.27, 0.76]	0.21	[0.08, 0.35]
θ_F	Calvo prob. F	beta	0.5	0.1	0.82	[0.77, 0.86]	0.57	[0.50, 0.63]
ϑ_F	Index. past infl. F	beta	0.5	0.15	0.64	[0.48, 0.80]	0.54	[0.29, 0.77]
ρ_R	MPR rule R_{t-1}	beta	0.75	0.1	0.83	[0.79, 0.87]	0.85	[0.82, 0.89]
α_π	MPR rule π_t	norm	1.5	0.1	1.55	[1.40, 1.70]	1.50	[1.36, 1.64]
α_y	MPR rule Δy_t	norm	0.125	0.05	0.15	[0.08, 0.23]	0.13	[0.05, 0.21]
ρ_ϱ	AC cons. pref. sh.	beta	0.75	0.1	0.64	[0.50, 0.79]	0.74	[0.60, 0.88]
ρ_κ	AC labor pref. sh.	beta	0.75	0.1	0.77	[0.64, 0.91]	0.74	[0.58, 0.90]
ρ_{ρ^x}	AC exo. sep. sh.	beta	0.75	0.1	0.88	[0.81, 0.95]	—	—
ρ_ϖ	AC inv. sh.	beta	0.75	0.1	0.65	[0.51, 0.79]	0.85	[0.78, 0.92]
ρ_z	AC temp. TFP sh.	beta	0.75	0.1	0.72	[0.59, 0.85]	0.50	[0.39, 0.61]
ρ_a	AC perm. TFP sh.	beta	0.375	0.1	0.35	[0.21, 0.49]	0.32	[0.20, 0.43]
ρ_{ζ^o}	AC obs. risk sh.	beta	0.75	0.1	0.87	[0.81, 0.94]	0.85	[0.77, 0.93]
ρ_{ζ^u}	AC unobs. risk sh.	beta	0.75	0.1	0.77	[0.69, 0.86]	0.74	[0.64, 0.85]
$\rho_{y^{Co}}$	AC commodity sh.	beta	0.75	0.1	0.95	[0.91, 0.98]	0.94	[0.90, 0.98]
σ_ϱ	s.d. cons. pref. sh.	invg	0.01	Inf	0.04	[0.03, 0.05]	0.06	[0.04, 0.08]
σ_κ	s.d. labor pref. sh.	invg	0.01	Inf	0.03	[0.02, 0.04]	0.13	[0.03, 0.23]
σ_{ρ^x}	s.d. separation sh.	invg	0.01	Inf	0.15	[0.10, 0.20]	—	—
σ_ϖ	s.d. inv. shock	invg	0.01	Inf	0.03	[0.01, 0.05]	0.08	[0.05, 0.10]
σ_z	s.d. temp. TFP sh.	invg	0.01	Inf	0.007	[0.005, 0.009]	0.019	[0.015, 0.022]
σ_a	s.d. perm. TFP sh.	invg	0.01	Inf	0.003	[0.002, 0.004]	0.019	[0.015, 0.023]
σ_{ζ^o}	s.d. obs. risk sh.	invg	0.003	Inf	0.0008	[0.0006, 0.0009]	0.001	[0.001, 0.001]
σ_{ζ^u}	s.d. unobs. risk sh.	invg	0.003	Inf	0.007	[0.004, 0.01]	0.009	[0.005, 0.013]
σ_R	s.d. MPR shock	invg	0.003	Inf	0.0017	[0.0014, 0.0019]	0.0016	[0.0013, 0.0019]
$\sigma_{y^{Co}}$	s.d. commodity sh.	invg	0.01	Inf	0.07	[0.04, 0.09]	0.07	[0.05, 0.09]

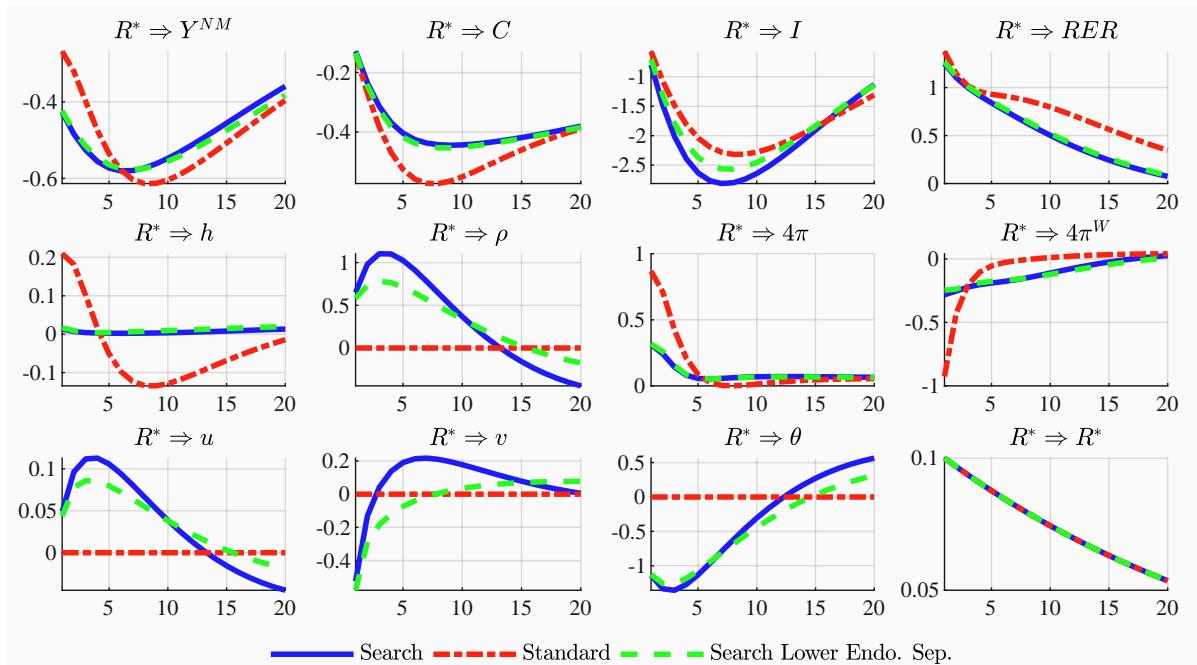
Note: The results are based on 500,000 draws from the posterior distribution using the Metropolis-Hastings (MH) algorithm, dropping the first 250,000 draws to achieve convergence. The average acceptance rate of the MH algorithm was approximately 25% for each model. HPDI are the highest posterior density intervals. The priors for the parameters ϕ and α_π were truncated at 0 and 1, respectively. The computations were conducted using Dynare 4.5.5.

Table 5: Variance Decomposition.

Variable	z	a	ϖ	y^{NM}	Shock										Co. prod. y_{Co}^*	Exo. sep. ρ^x			
					TFP trans.	TFP perm.	Inv. tech.	Total tech. sh.	Cons. pref.	Lab. sup.	Total pref.	MP e^R	Gov. rate	Risk obs.	Total risk	For. infl.	For. dem.		
					κ	κ	g	ζ^o	ζ^u	π^*	R^*	y^*	p_{Co*}	y^*					
y^{NM}	19	5	10	34	5	5	10	0	0	7	7	20	6	11	0	37	2	9	
c	1	3	2	5	16	0	16	0	0	8	8	9	13	41	0	64	6	0	
i	2	0	19	22	5	1	6	1	0	18	19	31	8	10	0	49	2	1	
π	24	2	33	60	3	4	7	12	0	0	7	7	3	4	1	0	9	0	
R	6	1	40	48	4	1	5	7	0	0	10	11	12	6	9	0	27	1	1
rer	1	0	3	5	0	0	1	1	0	2	45	46	19	20	6	0	46	1	0
ξ	0	0	2	2	0	0	0	0	0	38	3	40	27	16	11	0	54	2	0
w	5	2	2	9	2	0	2	0	0	1	14	15	26	13	29	0	68	5	2
$h \times n$	3	1	16	19	5	22	27	2	0	0	4	5	7	4	10	0	22	2	24
u	3	1	17	21	1	1	2	3	0	0	3	3	9	2	5	0	16	1	53
y^{NM}	2	2	79	83	3	7	9	1	0	0	1	1	3	1	2	0	6	0	—
c	0	12	10	23	22	1	23	0	0	0	5	5	8	8	29	0	44	4	—
i	0	14	66	80	3	1	4	0	0	0	2	2	7	2	4	0	13	1	—
π	18	17	20	55	4	1	5	5	0	0	20	20	7	7	2	0	15	0	—
R	4	12	43	60	6	1	8	3	0	0	12	13	9	4	4	0	17	0	—
rer	1	7	21	30	1	2	3	1	0	0	19	19	14	12	19	0	45	3	—
ξ	0	4	11	15	1	0	1	0	0	28	5	33	11	16	21	0	47	4	—
w	1	23	7	32	2	4	6	0	0	0	7	7	11	9	31	0	51	4	—
$h \times n$	9	24	48	81	2	10	12	1	0	0	2	2	1	1	2	0	4	0	—

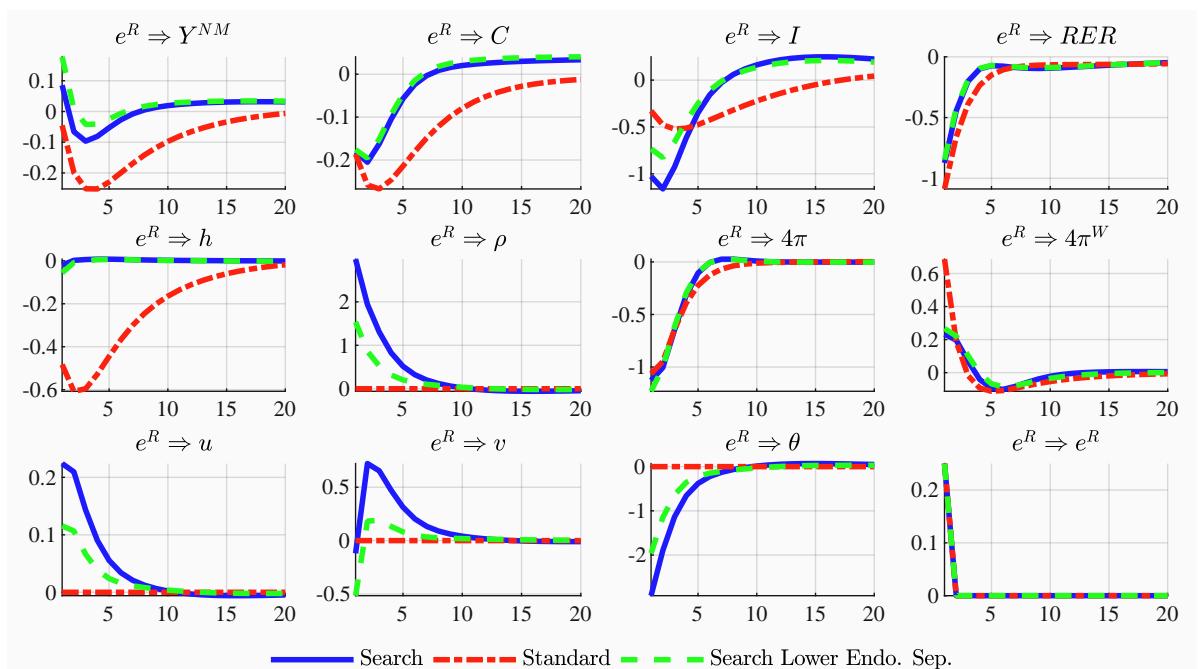
Note: The table entries are the fraction of the unconditional theoretical variances at the posterior mean (in %) explained by the shocks.

Figure 1: Impulse Responses to a Foreign Interest Rate Shock (R^*).



Note: The blue solid lines correspond to the model with search frictions, the red dash-dotted lines to the model with a standard labor market specification, and the green dashed lines to the model lower endogenous separations. In all cases the parameters associated to the shock process (persistence and volatility) are fixed at the prior mean, which is common across models. The variables are real non-mining GDP (Y^{NM}), household consumption (C), investment (I), the real exchange rate (rer), hours per worker (h), the separation rate (ρ), annualized CPI inflation (4π), annualized real wage growth ($4\pi^W$), the unemployment rate (u), vacancies (v), and labor market tightness (θ). All variables are expressed as percentage deviations from steady state.

Figure 2: Impulse Responses to a Domestic Monetary Policy Shock (e^R).



Note: See Figure 1.