### Bollinger Resonance Field Ansatz

We introduce a massless scalar test field as the driver of the resonance:

$$\phi(r,\theta,\phi,t) = \Re\left[A\, e^{i(\omega t - k r_*)}\, Y_{\ell m}(\theta,\phi)\right]$$

where
• $r_*$ is the tortoise coordinate

$$dr_* = \frac{dr}{\Delta/\Sigma} = dr\Big/\left(1 - \frac{2Mr}{\Sigma} + \frac{a^2}{\Sigma}\right)$$

• $Y_{\ell m}$ are spheroidal harmonics

• the mode is tuned just below the superradiant threshold

$$\omega = m\,\Omega_H - \delta \qquad (\delta \ll \Omega_H)$$

with horizon angular velocity

$$\Omega_H = \frac{a}{2Mr_+} = \frac{a}{2M\left(M + \sqrt{M^2 - a^2}\right)}$$

The field couples to the Kerr geometry via an effective stress-energy that sources an **additional frame-dragging term**:

$$\delta g_{t\phi} \propto (\partial_t \phi)^2 - (\partial_\phi \phi)^2 \quad \text{(quadrupole-like torque)}$$

This perturbation remains **everywhere timelike or null** (no exotic matter, no negative energy outside the ergosphere) while resonantly pumping rotational energy out of the black hole spin $a$, ultimately driving the inner Cauchy horizon toward a transient, stabilised closed-timelike region.

(Full linearised Einstein–Klein-Gordon equations + stability analysis in `theory/bollinger_oscillations_derivation.tex`.)