

Sistemi Operativi

Corso di Laurea in Informatica

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SAPIENZA
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Paging + Segmentation

- Paging (OS' view of memory)
 - Divide memory into fixed-size pages and map them to physical frames
- Segmentation (compiler's view of memory)
 - Divide process into logical segments (e.g., code, data, stack, heap)
- Combine paging with segmentation to get the best of both worlds
 - Segmented Paging

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Virtual Memory uses backing storage (i.e., disk) to store unused pages and give the illusion of infinite virtual address space

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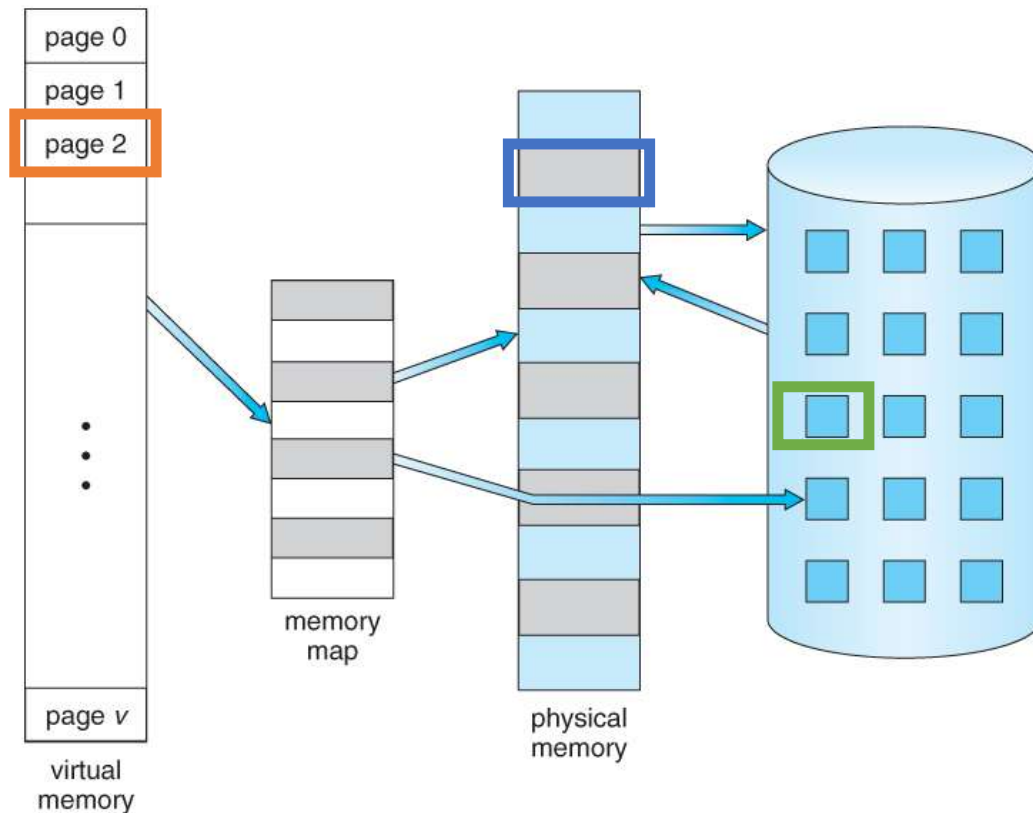
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 - Less I/O is needed for swapping processes in and out of memory, speeding things up

Virtual Memory: The Big Picture

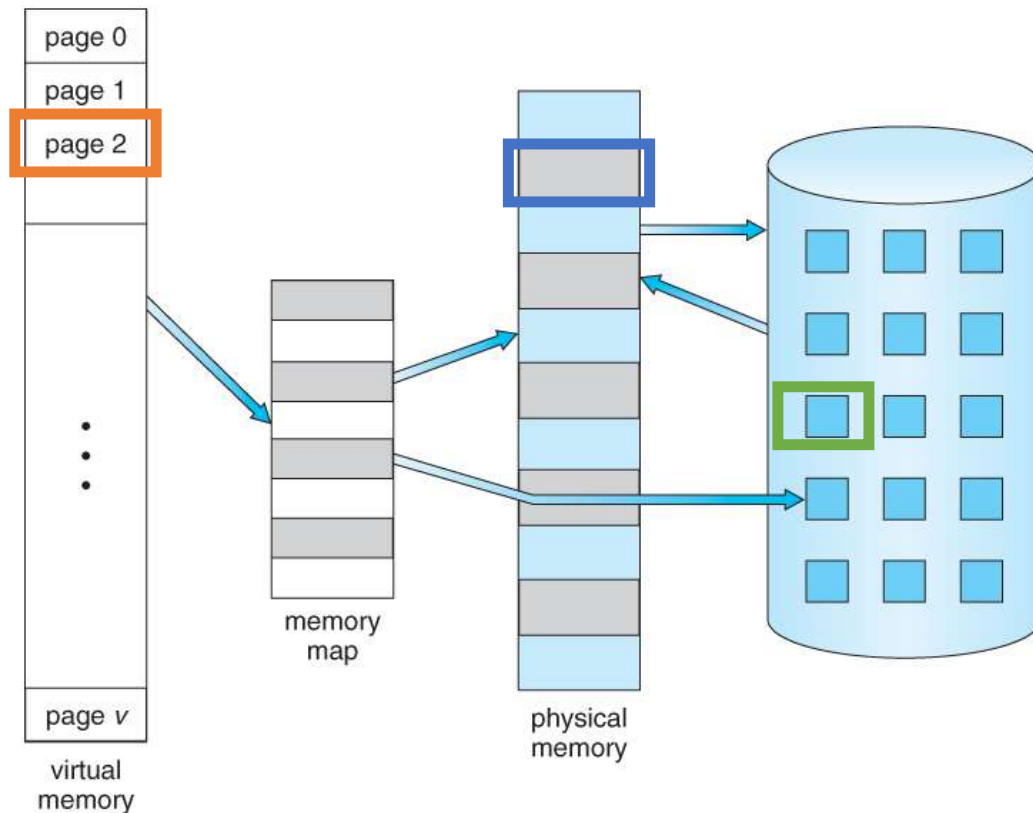


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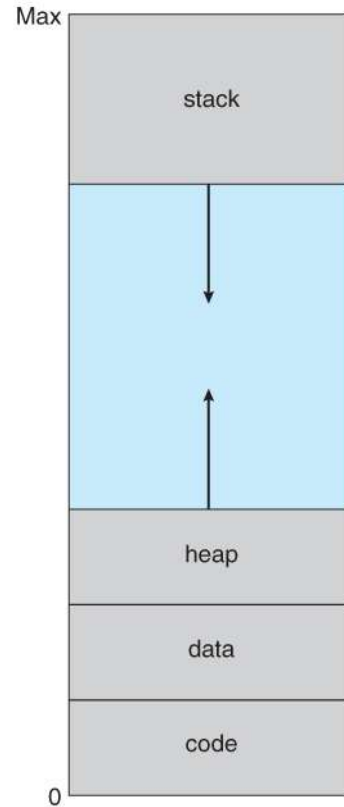
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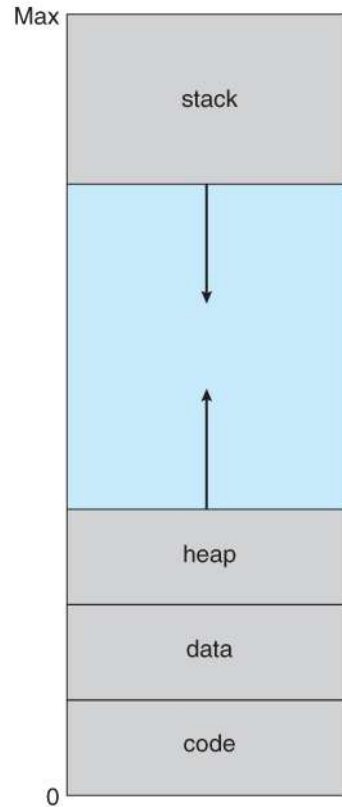
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The Sparseness of Virtual Address Space



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A lot of virtual memory addresses remain
unreferenced

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- Remember: access to disk is extremely slower than access to memory
- Therefore, memory accesses must reference pages that are in memory
with high probability

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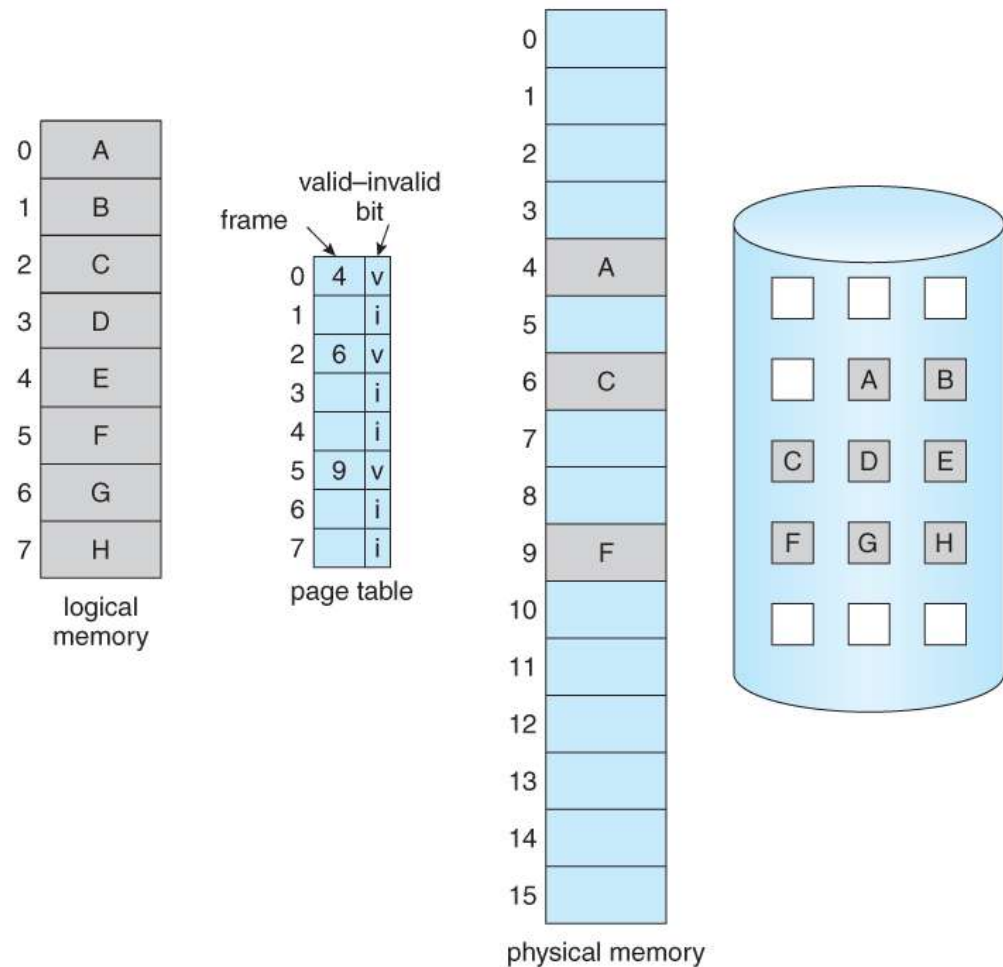
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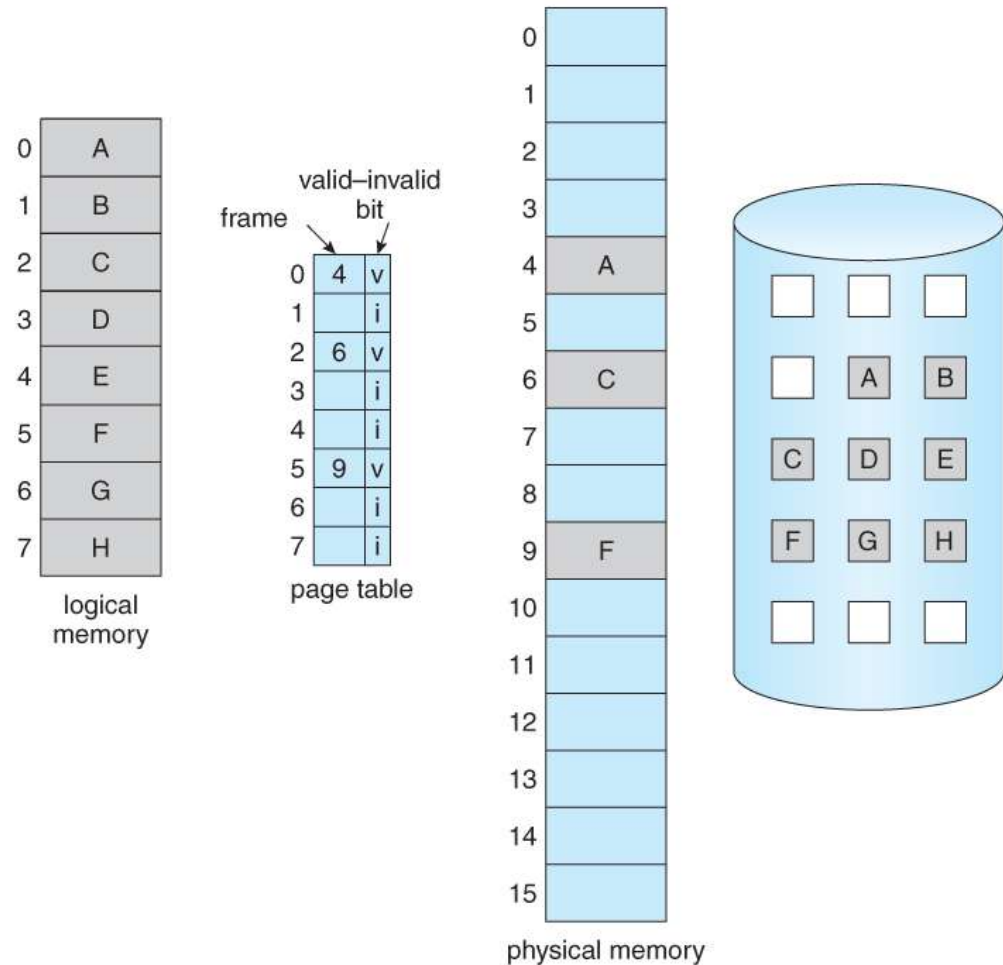
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- But in a reasonably small time frame, the working set stays the same

Virtual Memory: Basic Concepts



At each logical memory reference, a page table lookup is performed as usual

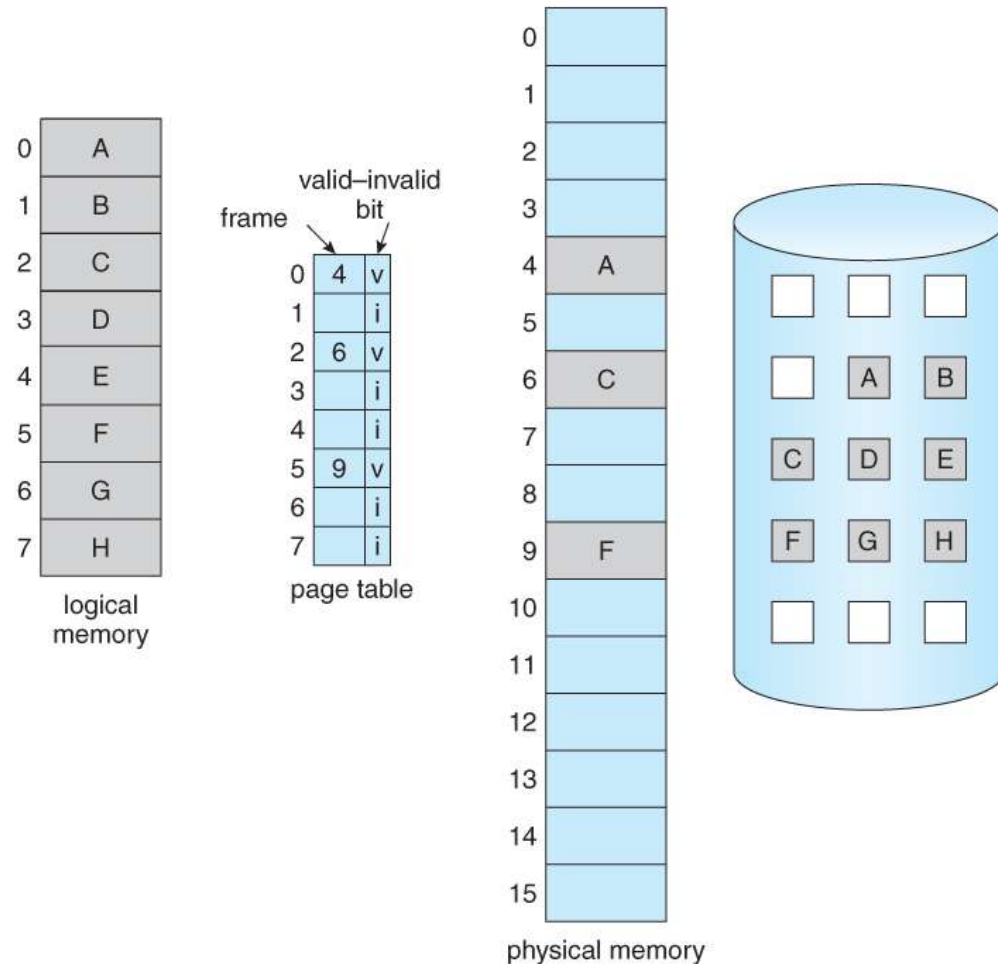
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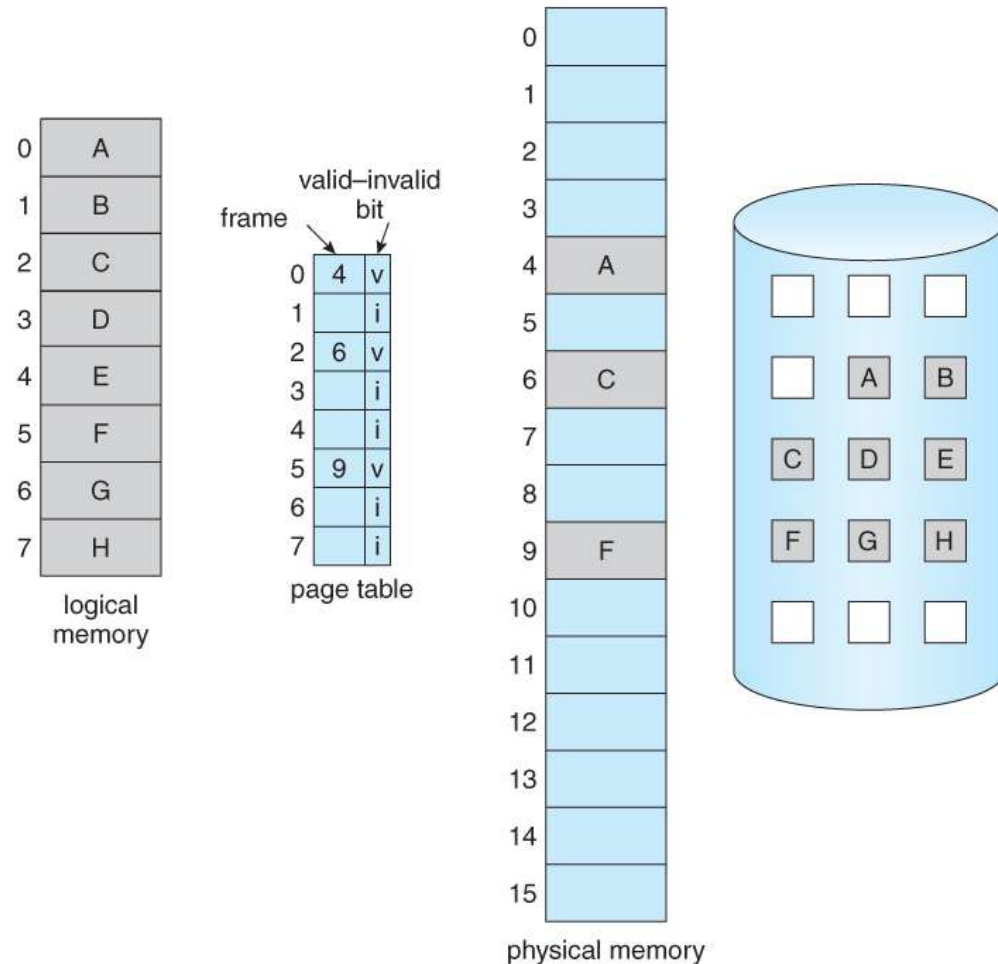


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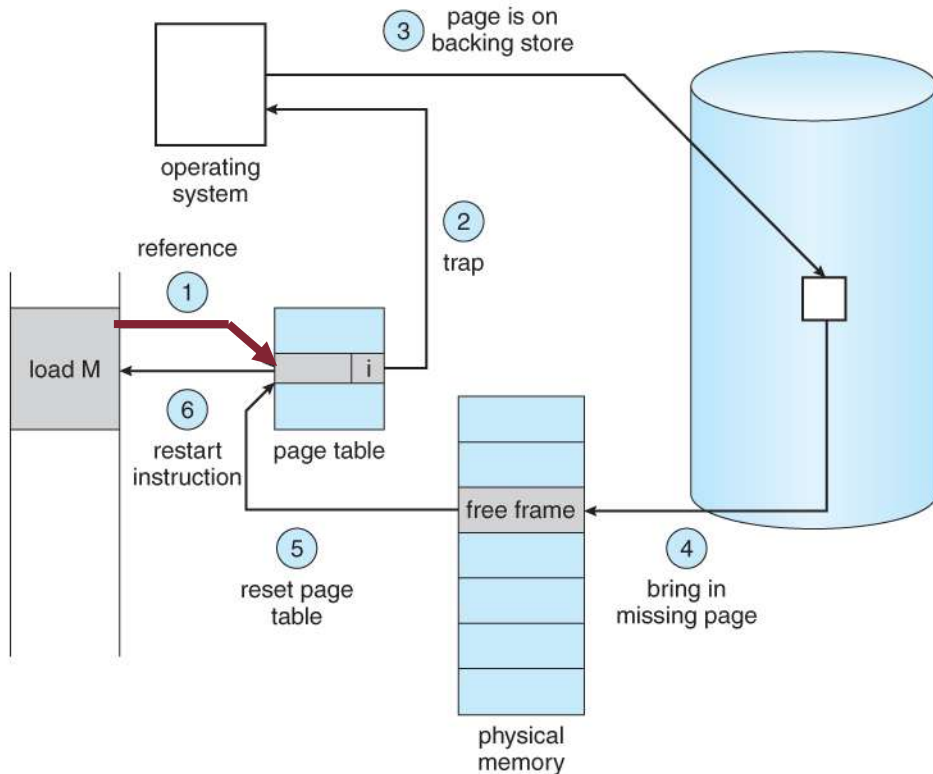
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Otherwise, a **page fault trap** occurs, and the page has to be loaded (i.e., fetched) from disk

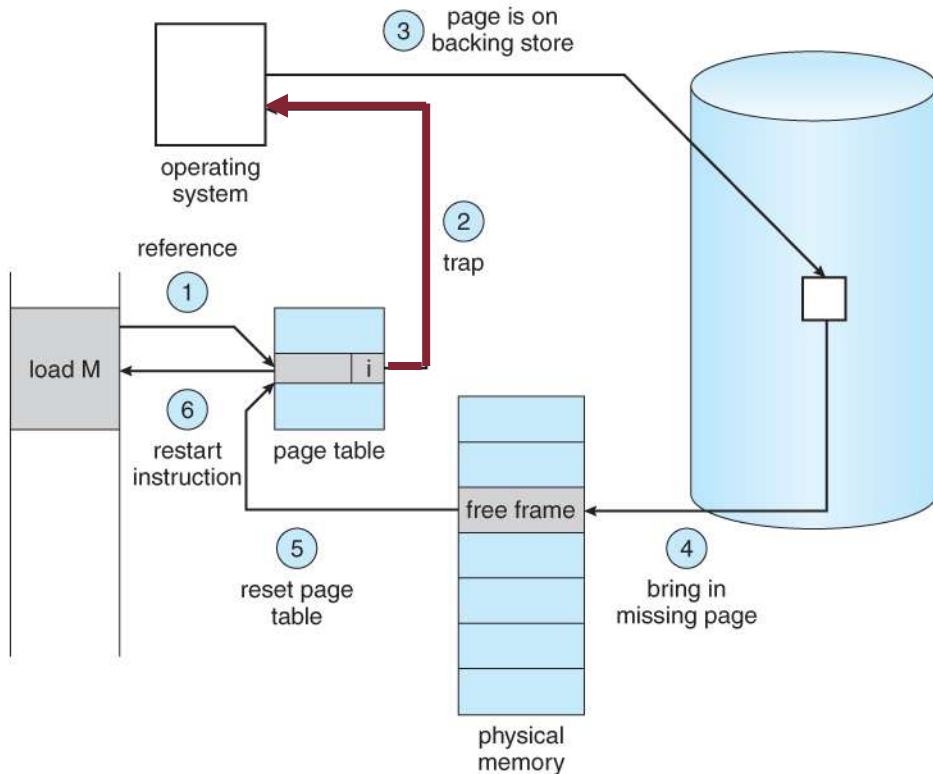
Page Fault Handling

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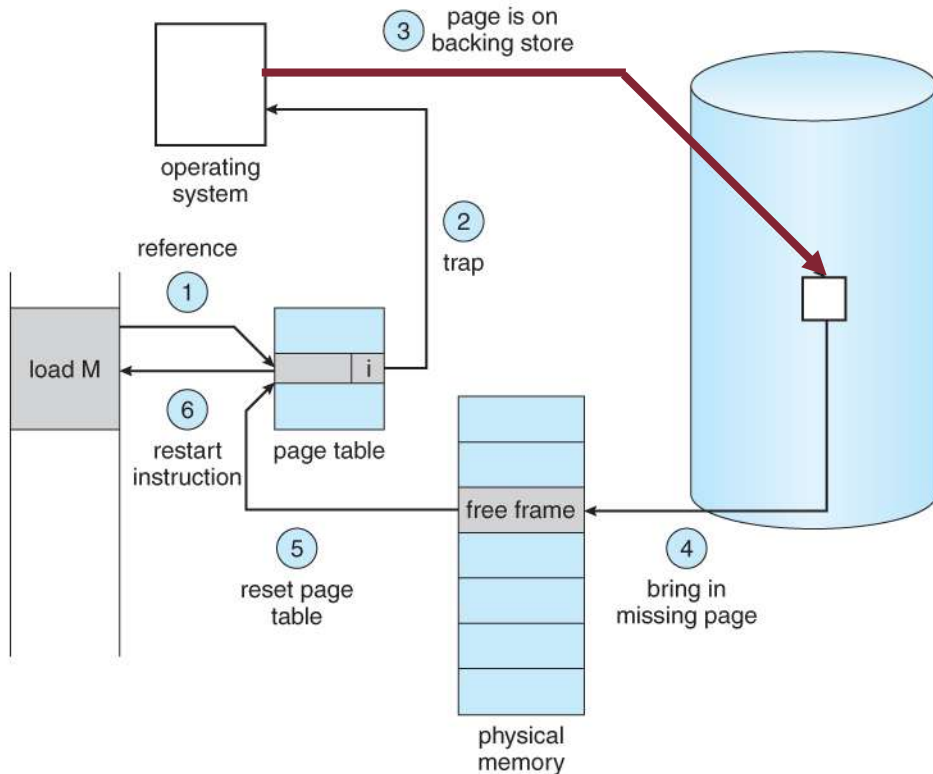


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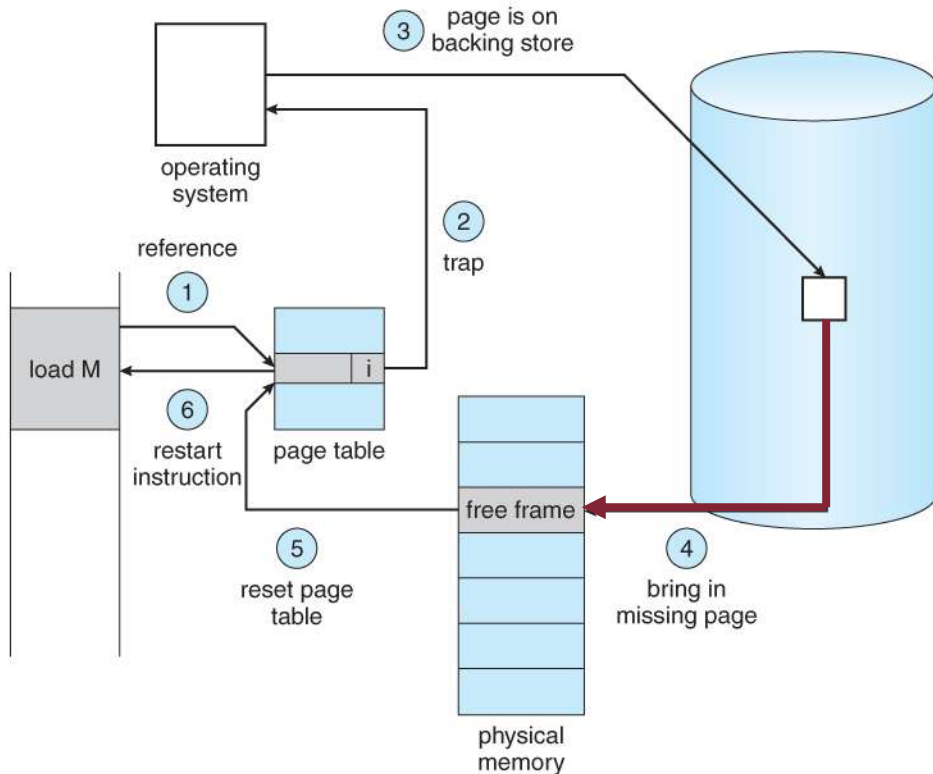


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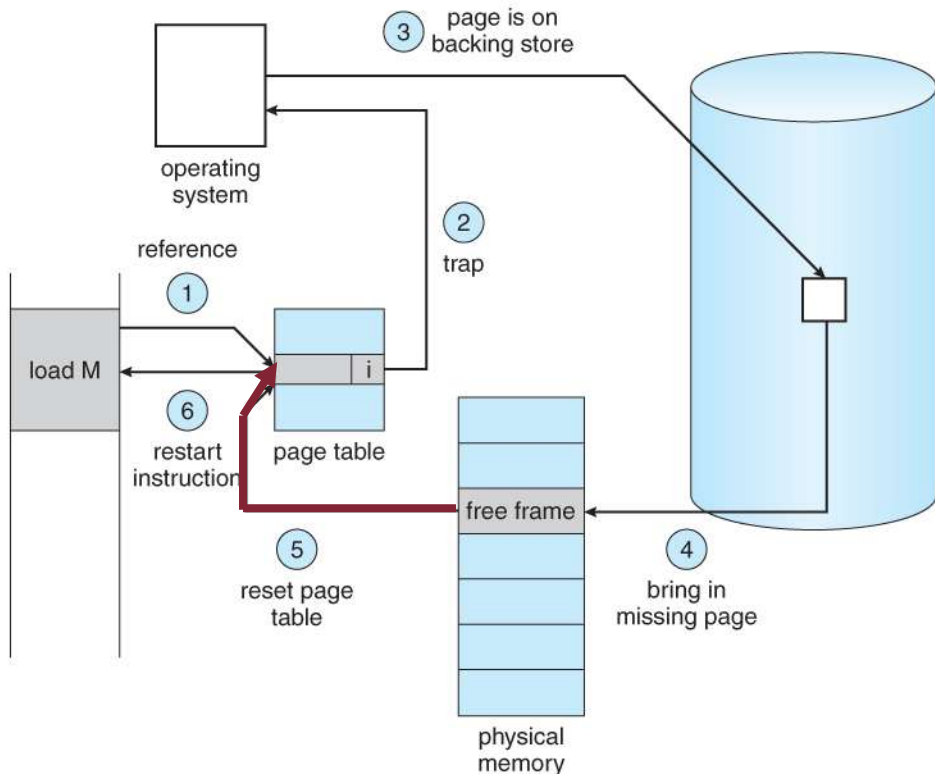
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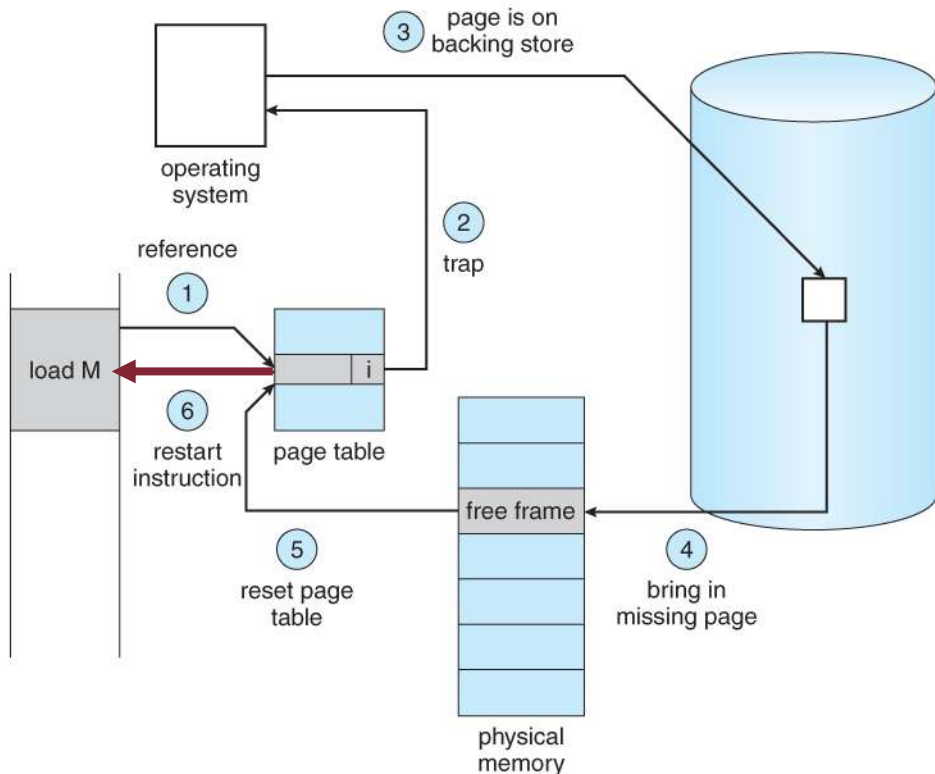
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6. The current process gets interrupted and the instruction that caused the page fault must be restarted from the beginning

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- If we get a TLB hit but the frame is not actually in main memory, we have to go fetch the page from disk anyway!
- TLB hit means the requested page entry is in the cache **and** the referenced frame is also in memory

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- If the requested page is not in the cache (TLB miss) and it is not even in memory (i.e., it is sitting on disk):
 - The OS picks a TLB entry to replace and fills it with the new entry as follows
 - invalidates the TLB entry
 - performs page fault trap operations
 - updates the TLB entry
 - restarts the faulting instruction

Page Fault Handling: Faulty Address

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- Architecture-dependant:
 - x86: hardware saves the virtual address that caused the fault (CR2 register)
 - On some platforms OS gets only address of faulting instruction, must simulate the instruction and try every address to find the one that generated the fault

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- To restart (from scratch) a faulty instruction the OS needs hardware support for saving:
 - The faulting instruction
 - The CPU state

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- idempotent → just restart the faulting instruction (hardware saves instruction address during page fault)
- non-idempotent → much more difficult to restart
 - `MOV [%R1], + (%R2)` → increment the value of R2 and store it to memory address in R1
 - What if memory address [%R1] causes the page fault?
 - Cannot naively redo the instruction from scratch otherwise R2 gets incremented twice

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- Even harder when using instructions that are not easily undoable
 - E.g., instructions that are used to move a block of memory at once
 - The block may span multiple pages: some of them can be in memory while some others not
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How to unwind those complicated side-effects?

Make sure all the addresses within the block to be transferred are in memory before starting executing the instruction

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- Still, an overhead must be paid every time a page fault occurs as the OS needs to interact with slower disk

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t_{MA} = physical memory access time

t_{FAULT} = time to handle a page fault

$p \in [0, 1]$ = probability of page fault

t_{ACCESS} = effective time for each memory reference

$$t_{ACCESS} = (1 - p) * t_{MA} + p * t_{FAULT}$$

Let's assume: $t_{MA} = 100$ nsec and $t_{FAULT} = 20$ msec = 20,000,000 nsec

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This heavily depends on p !

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The access time increases from just 100 nsec up to ~20.1 microsec

200 times slowdown factor

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$$\begin{aligned} 1.1 * 100 &= 100 - 100p + 20,000,000p = \\ 19,999,900p &= 110 - 100 = \end{aligned}$$

To achieve that goal, we can tolerate at most 1 page fault every about 2 million accesses!

$$p = \frac{10}{19,999,900} = \frac{1}{1,999,990} \approx 0,0000005 = 5 * 10^{-7}$$

Virtual Memory: Performance Example

More generally, given t_{MA} , t_{FAULT} , and a threshold $\epsilon > 0$ if we want to find p s.t.:

$$t_{ACCESS} = (1 + \epsilon) * t_{MA}$$

We substitute t_{ACCESS} and solve for p the resulting equation:

$$\begin{aligned}(1 - p) * t_{MA} + p * t_{FAULT} &= (1 + \epsilon) * t_{MA} = \\ t_{MA} - p * t_{MA} + p * t_{FAULT} &= t_{MA} + \epsilon * t_{MA} \\ p(t_{FAULT} - t_{MA}) &= \epsilon * t_{MA} =\end{aligned}$$

$$p = \frac{\epsilon * t_{MA}}{t_{FAULT} - t_{MA}}$$

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- So far, we have described how the OS (with the support of HW) manages page faults
- Still, the OS has to answer 2 fundamental questions:
 - When to load process' pages into main memory (**page fetching**)
 - Which page to remove from memory if this gets filled (**page replacement**)

Page Fetching Goals

- The overall goal is still to make physical memory look larger than it is
- Exploiting the locality reference of programs
- Keep in memory only those pages that is being used
- Keep on disk those pages that are unused
- Ideally, producing a memory system with the performance of main memory and the cost/capacity of disk!

Page Fetching Strategies

3 page fetching strategies

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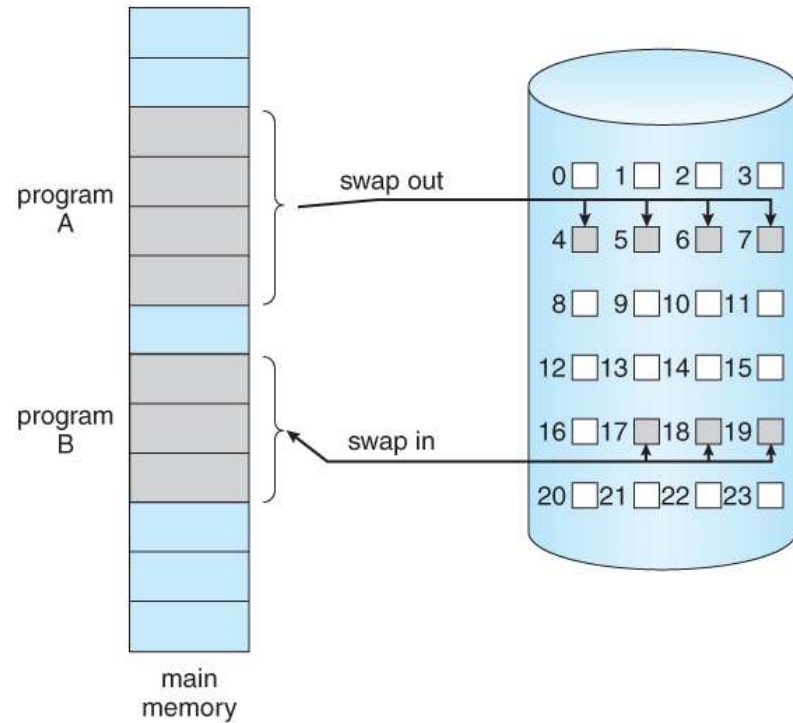
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Most modern OSs use **demand fetching**

(Pure) Demand Paging

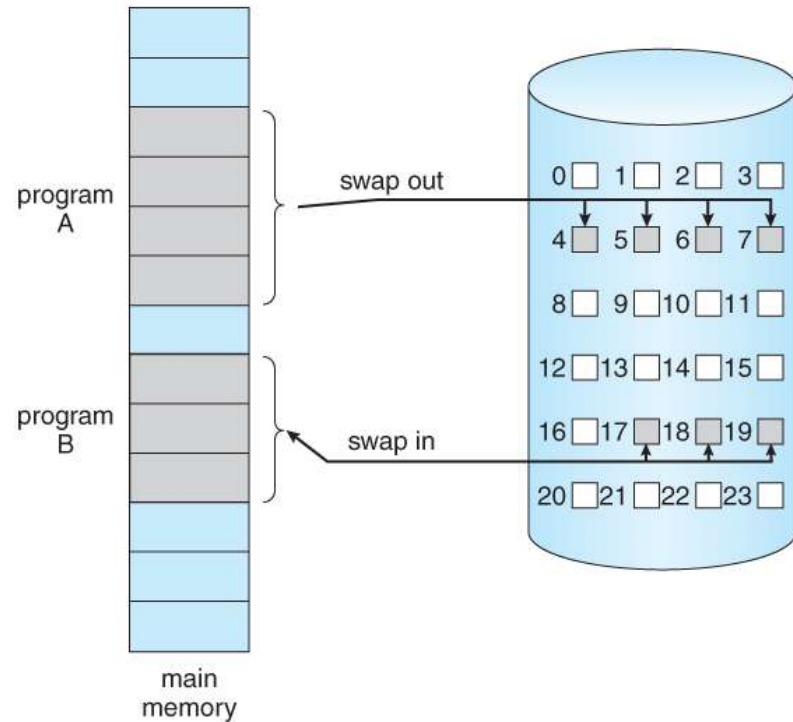
- When a process starts up, **none** of its pages are loaded
- Rather, a page is swapped in only when the process references it (upon a page fault)
- This is termed a **lazy swapper** or **pager**
- Opposite of loading all the pages at process startup!

Prefetching



The pager guesses when pages will be needed and load them ahead of time

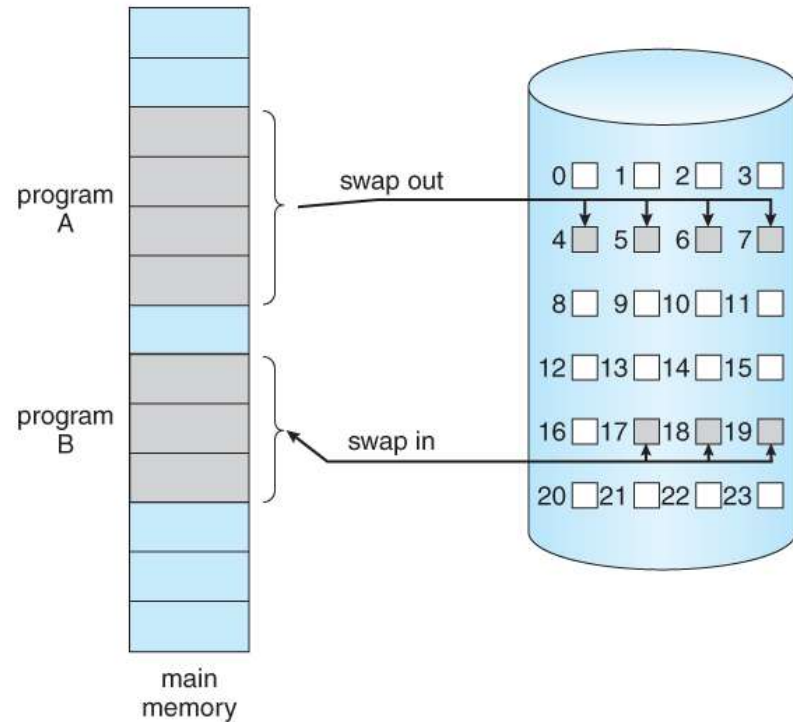
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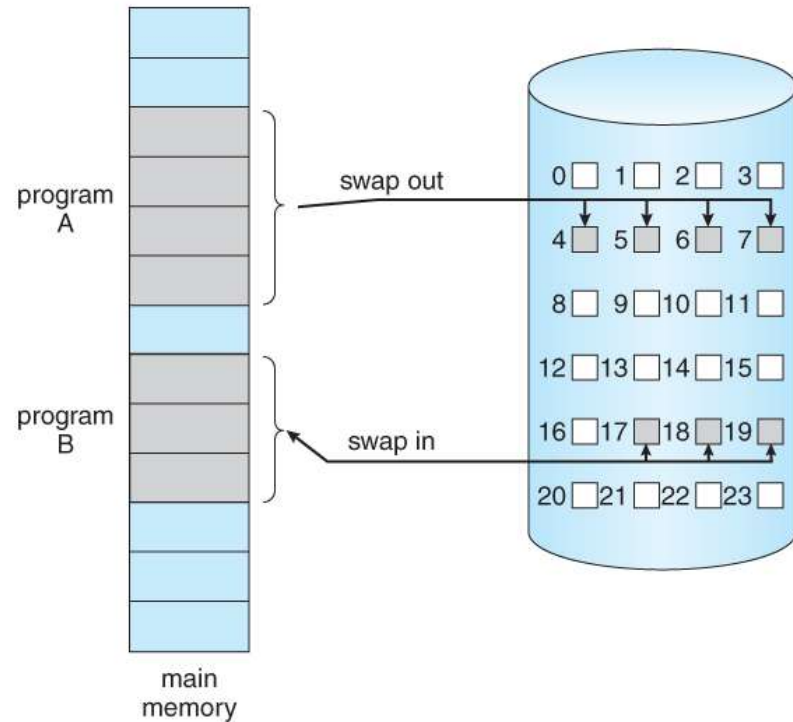


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Possible approach: upon page fault, load many pages instead of only the faulty one

works if program accesses memory sequentially

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- On Mac, instead, swap space is part of the file system
 - swapfiles

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- Depending on which kind of page is removed, different optimizations may apply upon page swap-out

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- **Data** page:
 - Data content does actually change!
 - Save it to a separate paging file, so that no changes are lost when it will be loaded in the future
 - Need to use the dedicated swap space

Page Replacement: Motivation

- On a page fault, we need to load a page from disk into memory
- If physical memory has still free frames, the page can be safely loaded into one of those
- However, if physical memory is full (i.e., all of its frames are loaded) a frame must be swapped out to make room for the swap-in page
- Several algorithms to select the page to evict from memory

Page Replacement Algorithms

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 - Needs to predict the future → very hard!

Page Replacement Algorithms

- **Random**: pick any page at random (works surprisingly well!)
- **FIFO (First-In-First-Out)**: throw out the page that has been in memory for longest time (i.e., the oldest)
 - Easy to implement but may remove frequently accessed pages
- **MIN (OPT)**: remove the page that will not be accessed for the longest time (provably optimal [Belady 1966])
 - Needs to predict the future → very hard!
- **LRU (Least Recently Used)**: approximation of MIN, remove the page that has not been used in the longest time
 - Assumes the past is a good predictor of the future (not always true!)

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1											
F_2											
F_3											

How many page faults (denoted by *)?

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1											
F_2											
F_3											

Initially, no frame is loaded in memory at all
(pure demand paging)

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1											
F_2											
F_3											

Virtual address within page A is referenced

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1											
F_2											
F_3											

Virtual address within page A is referenced



page fault

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: $A, B, C, A, B, D, A, D, B, C, A$

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*										
F_2											
F_3											

Virtual address within page A is referenced



page fault



A loaded

FIFO = A

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: $A, B, C, A, B, D, A, D, B, C, A$

	A	B	C	A	B	D	A	D	B	C	A
F_1	A^*	A									
F_2											
F_3											

Virtual address within page B is referenced

FIFO = A

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: $A, B, C, A, B, D, A, D, B, C, A$

	A	B	C	A	B	D	A	D	B	C	A
F_1	A^*	A									
F_2											
F_3											

Virtual address within page B is referenced



page fault

FIFO = A

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A									
F_2		B*									
F_3											

Virtual address within page B is referenced



page fault



B loaded

FIFO = A → B

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A								
F_2		B*	B								
F_3											

Virtual address within page C is referenced

FIFO = A \rightarrow B

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: $A, B, C, A, B, D, A, D, B, C, A$

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A								
F_2		B*	B								
F_3											

Virtual address within page C is referenced



page fault

FIFO = $A \rightarrow B$

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: $A, B, C, A, B, D, A, D, B, C, A$

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A								
F_2		B*	B								
F_3			C*								

Virtual address within page C is referenced



page fault



C loaded

FIFO = $A \rightarrow B \rightarrow C$

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: $A, B, C, A, B, D, A, D, B, C, A$

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A							
F_2		B*	B	B							
F_3			C*	C							

Virtual address within page A is referenced



A is already loaded

FIFO = $A \rightarrow B \rightarrow C$

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: $A, B, C, A, B, D, A, D, B, C, A$

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A						
F_2		B*	B	B	B						
F_3			C*	C	C						

Virtual address within page B is referenced



B is already loaded

FIFO = $A \rightarrow B \rightarrow C$

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A					
F_2		B*	B	B	B	B					
F_3			C*	C	C	C					

Virtual address within page D is referenced

FIFO = A → B → C

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A					
F_2		B*	B	B	B	B					
F_3			C*	C	C	C					

Virtual address within page D is referenced



page fault

FIFO = A → B → C

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	D*					
F_2		B*	B	B	B	B					
F_3			C*	C	C	C					

Virtual address within page D is referenced



page fault



A replaced
D loaded

FIFO = B → C → D

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	D*	D				
F_2		B*	B	B	B	B	B				
F_3			C*	C	C	C	C				

Virtual address within page A is referenced

FIFO = B → C → D

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: $A, B, C, A, B, D, A, D, B, C, A$

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	D*	D				
F_2		B*	B	B	B	B	B				
F_3			C*	C	C	C	C				

Virtual address within page A is referenced



page fault

FIFO = $B \rightarrow C \rightarrow D$

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: $A, B, C, A, B, D, A, D, B, C, A$

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	D*	D				
F_2		B*	B	B	B	B	A*				
F_3			C*	C	C	C	C				

Virtual address within page A is referenced



page fault



B replaced
 A loaded

FIFO = $C \rightarrow D \rightarrow A$

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: $A, B, C, A, B, D, A, D, B, C, A$

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	D*	D	D			
F_2		B*	B	B	B	B	A*	A			
F_3			C*	C	C	C	C	C			

Virtual address within page D is referenced



D is already loaded

FIFO = $C \rightarrow D \rightarrow A$

FIFO Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	D*	D	D	D	C*	C
F_2		B*	B	B	B	B	A*	A	A	A	A
F_3			C*	C	C	C	C	C	B*	B	B

Eventually, we get a total of 7 page faults

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1											
F_2											
F_3											

How many page faults (denoted by *)?

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1											
F_2											
F_3											

Initially, no frame is loaded in memory at all
(pure demand paging)

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A						
F_2		B*	B	B	B						
F_3			C*	C	C						

Up to this point, the same as FIFO

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A					
F_2		B*	B	B	B	B					
F_3			C*	C	C	C					

Virtual address within page D is referenced

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A					
F_2		B*	B	B	B	B					
F_3			C*	C	C	C					

Virtual address within page D is referenced



page fault

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A					
F_2		B*	B	B	B	B					
F_3			C*	C	C	C					

Virtual address within page D is referenced



page fault

What's the page that
will be requested the
furthest away?

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A					
F_2		B*	B	B	B	B					
F_3			C*	C	C	D*					

Virtual address within page D is referenced



page fault



C replaced
D loaded

C is the page that will be requested the furthest away

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A		
F_2		B*	B	B	B	B	B	B	B		
F_3			C*	C	C	D*	D	D	D		

Up to this point, no more page faults

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	A	
F_2		B*	B	B	B	B	B	B	B	B	
F_3			C*	C	C	D*	D	D	D	D	

Virtual address within page C is referenced

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	A	
F_2		B*	B	B	B	B	B	B	B	B	
F_3			C*	C	C	D*	D	D	D	D	

Virtual address within page C is referenced



page fault

What's the page that
will be requested the
furthest away?

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	A	
F_2		B*	B	B	B	B	B	B	B	C*	
F_3			C*	C	C	D*	D	D	D	D	

Virtual address within page C is referenced



page fault



B replaced
C loaded

B or D will be requested the furthest away (surely not A): pick one (e.g., B)

MIN Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	A	A
F_2		B*	B	B	B	B	B	B	B	C*	C
F_3			C*	C	C	D*	D	D	D	D	D

Eventually, we get a total of 5 page faults

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1											
F_2											
F_3											

How many page faults (denoted by *)?

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1											
F_2											
F_3											

Initially, no frame is loaded in memory at all
(pure demand paging)

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A						
F_2		B*	B	B	B						
F_3			C*	C	C						

Up to this point, the same as FIFO

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: $A, B, C, A, B, D, A, D, B, C, A$

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A					
F_2		B*	B	B	B	B					
F_3			C*	C	C	C					

Virtual address within page D is referenced



page fault

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A					
F_2		B*	B	B	B	B					
F_3			C*	C	C	C					

Virtual address within page D is referenced



page fault

We can't look forward anymore!

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A					
F_2		B*	B	B	B	B					
F_3			C*	C	C	D*					

Virtual address within page D is referenced



page fault



C replaced
D loaded

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A		
F_2		B*	B	B	B	B	B	B	B		
F_3			C*	C	C	D*	D	D	D		

Up to this point, no more page faults

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	A	
F_2		B*	B	B	B	B	B	B	B	B	
F_3			C*	C	C	D*	D	D	D	D	

Virtual address within page C is referenced

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	A	
F_2		B*	B	B	B	B	B	B	B	B	
F_3			C*	C	C	D*	D	D	D	D	

Virtual address within page C is referenced



page fault

We can't look forward anymore!

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	C*	
F_2		B*	B	B	B	B	B	B	B	B	
F_3			C*	C	C	D*	D	D	D	D	

Virtual address within page C is referenced



page fault



A replaced
C loaded

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	C*	C
F_2		B*	B	B	B	B	B	B	B	B	B
F_3			C*	C	C	D*	D	D	D	D	D

Virtual address within page A is referenced

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: $A, B, C, A, B, D, A, D, B, C, A$

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	C*	C
F_2		B*	B	B	B	B	B	B	B	B	B
F_3			C*	C	C	D*	D	D	D	D	D

Virtual address within page A is referenced



page fault

We can't look forward anymore!

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: $A, B, C, A, B, D, A, D, B, C, A$

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	C*	C
F_2		B*	B	B	B	B	B	B	B	B	B
F_3			C*	C	C	D*	D	D	D	D	A*

Virtual address within page A is referenced



page fault



D replaced
 A loaded

LRU Page Replacement: Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, A, B, D, A, D, B, C, A

	A	B	C	A	B	D	A	D	B	C	A
F_1	A*	A	A	A	A	A	A	A	A	C*	C
F_2		B*	B	B	B	B	B	B	B	B	B
F_3			C*	C	C	D*	D	D	D	D	A*

Eventually, we get a total of 6 page faults

LRU Page Replacement: (An Unlucky) Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, D, A, B, C, D, A, B, C

	A	B	C	D	A	B	C	D	A	B	C
F_1											
F_2											
F_3											

How many page faults (denoted by *)?

LRU Page Replacement: (An Unlucky) Example

3 physical frames: F_1, F_2, F_3

4 virtual pages: A, B, C, D

Reference sequence of pages: A, B, C, D, A, B, C, D, A, B, C

	A	B	C	D	A	B	C	D	A	B	C
F_1	A*	A	A								
F_2		B*	B								
F_3			C*								

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	A	B	C	D	A	B	C	D	A	B	C
F_1	A*	A	A	D*							
F_2		B*	B	B							
F_3			C*	C							

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	A	B	C	D	A	B	C	D	A	B	C
F_1	A*	A	A	D*	D						
F_2		B*	B	B	A*						
F_3			C*	C	C						

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	A	B	C	D	A	B	C	D	A	B	C
F_1	A*	A	A	D*	D	D					
F_2		B*	B	B	A*	A					
F_3			C*	C	C	B*					

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	A	B	C	D	A	B	C	D	A	B	C
F_1	A*	A	A	D*	D	D	C*				
F_2		B*	B	B	A*	A	A				
F_3			C*	C	C	B*	B				

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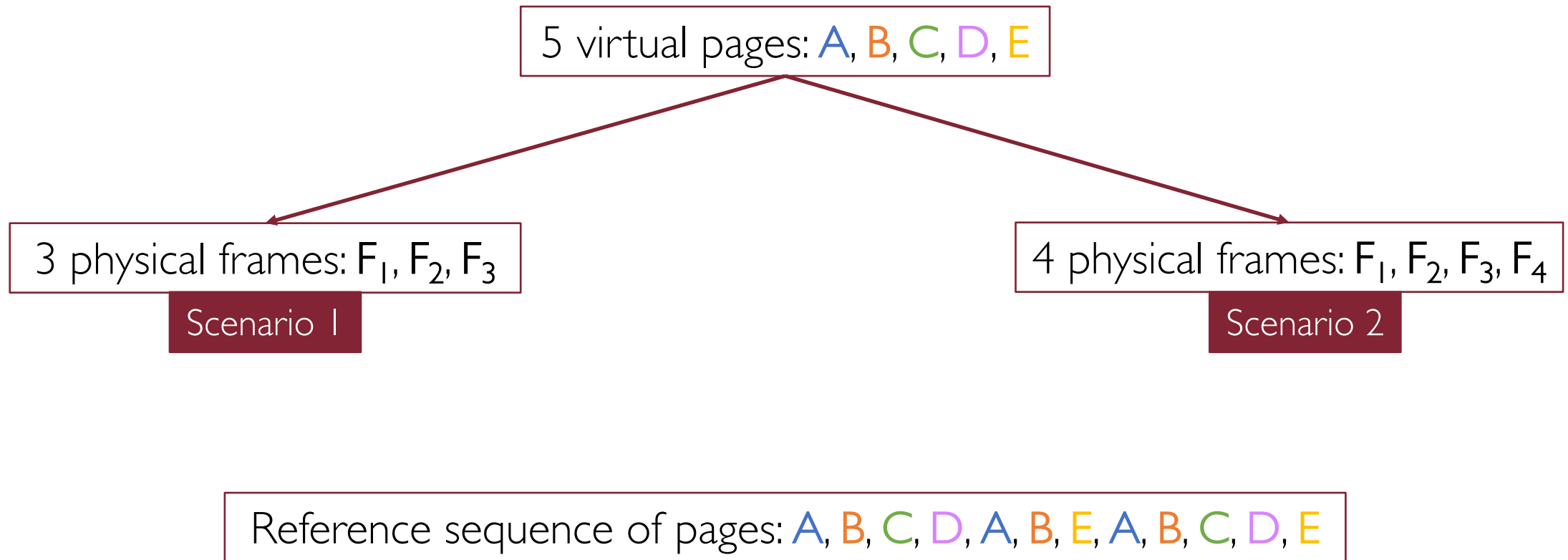
	A	B	C	D	A	B	C	D	A	B	C
F_1	A*	A	A	D*	D	D	C*	C	C	B*	B
F_2		B*	B	B	A*	A	A	D*	D	D	C*
F_3			C*	C	C	B*	B	B	A*	A	A

Eventually, we get a total of 11 page faults

Page Replacement: What If We Add Memory?

- Does adding memory always reduce the number of page faults?
- Intuitively, it would seem so...
- The answer, in fact, depends on the page replacement algorithm
- Let's see this with an example, using FIFO page replacement

FIFO Page Replacement: Example



FIFO Page Replacement: Example

	A	B	C	D	A	B	E	A	B	C	D	E
F ₁	A*	A	A	D*	D	D	E*	E	E	E	E	E
F ₂		B*	B	B	A*	A	A	A	A	C*	C	C
F ₃			C*	C	C	B*	B	B	B	B	D*	D
F ₁	A*	A	A	A	A	A	E*	E	E	E	D*	D
F ₂		B*	B	B	B	B	B	A*	A	A	A	E*
F ₃			C*	C	C	C	C	C	B*	B	B	B
F ₄				D*	D	D	D	D	D	C*	C	C

FIFO Page Replacement: Example

	A	B	C	D	A	B	E	A	B	C	D	E
--	---	---	---	---	---	---	---	---	---	---	---	---

F ₁	A*	A	A	D*	D	D	E*	E	E	E	E	E
F ₂		B*	B	B	A*	A	A	A	A	C*	C	C
F ₃			C*	C	C	B*	B	B	B	B	D*	D

10 page faults

F ₁	A*	A	A	A	A	A	E*	E	E	E	D*	D
F ₂		B*	B	B	B	B	B	A*	A	A	A	E*
F ₃			C*	C	C	C	C	C	B*	B	B	B
F ₄				D*	D	D	D	D	D	C*	C	C

11 page faults

Belady's Anomaly

Adding page frames may cause more page faults with some algorithms

LRU Page Replacement: Example

	A	B	C	D	A	B	E	A	B	C	D	E
F ₁	A*	A	A	D*	D	D	E*	E	E	C*	C	C
F ₂		B*	B	B	A*	A	A	A	A	A	D*	D
F ₃			C*	C	C	B*	B	B	B	B	B	B
F ₁	A*	A	A	A	A	A	A	A	A	A	A	E*
F ₂		B*	B	B	B	B	B	B	B	B	B	B
F ₃			C*	C	C	C	E*	E	E	E	D*	D
F ₄				D*	D	D	D	D	D	C*	C	C

9 page faults

8 page faults

With **LRU**, adding page frames **always** decreases the number of page faults

LRU Page Replacement: Example

	A	B	C	D	A	B	E	A	B	C	D	E
F ₁	A*	A	A	D*	D	D	E*	E	E	C*	C	C
F ₂		B*	B	B	A*	A	A	A	A	A	D*	D
F ₃			C*	C	C	B*	B	B	B	B	B	B
F ₁	A*	A	A	A	A	A	A	A	A	A	A	E*
F ₂		B*	B	B	B	B	B	B	B	B	B	B
F ₃			C*	C	C	C	E*	E	E	E	D*	D
F ₄				D*	D	D	D	D	D	C*	C	C

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Why?

LRU Page Replacement: Example

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F ₂		B*	B	B	A*	A	A	A	A	A	D*	D
F ₃			C*	C	C	B*	B	B	B	B	B	B
F ₁	A*	A	A	A	A	A	A	A	A	A	A	E*
F ₂		B*	B	B	B	B	B	B	B	B	B	B
F ₃			C*	C	C	C	E*	E	E	E	D*	D
F ₄				D*	D	D	D	D	D	C*	C	C

At each point in time 4-frame memory contains a subset of 3-frame

Can't do any worst!

Page Replacement: Summary

- FIFO is easy to implement but may lead to too many page faults
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- **FIFO** is easy to implement but may lead to too many page faults
 - May suffer from Belady's Anomaly
- **MIN** is the optimal choice but cannot be used in practice since future memory references are never known in advance
- **LRU** is a fair approximation of MIN assuming the past is a good predictor of the future
 - Exploits the locality reference (small working set that fits in memory)
 - Works poorly when the locality reference doesn't hold (large working set)

LRU: Implementation Details

How could we implement LRU page replacement algorithm?

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First Idea

Keep a timestamp for each page with the time it has been last accessed
Remove the page with the highest difference w.r.t. current timestamp

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Problems?

Every time a page is accessed its timestamp must be updated

Linear scan of all the pages to select the one to be removed

LRU: Implementation Details

How could we implement LRU page replacement algorithm?



Second Idea

Keep a list of pages with the most recently used in front and the least recently used at the end: every time a page is accessed move it to front

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Still too expensive as the OS must change multiple pointers on each memory access

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- **Single-Reference Bit** → Maintain 1 bit for each page table entry
 - Initially, all bits for all pages are set to 0
 - On each access to a page, the HW sets the reference bit to 1
 - Enough to distinguish pages that have been accessed since the last clear
 - No total order of page access

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- The specific number of bits used and the frequency with which the reference byte is updated are adjustable

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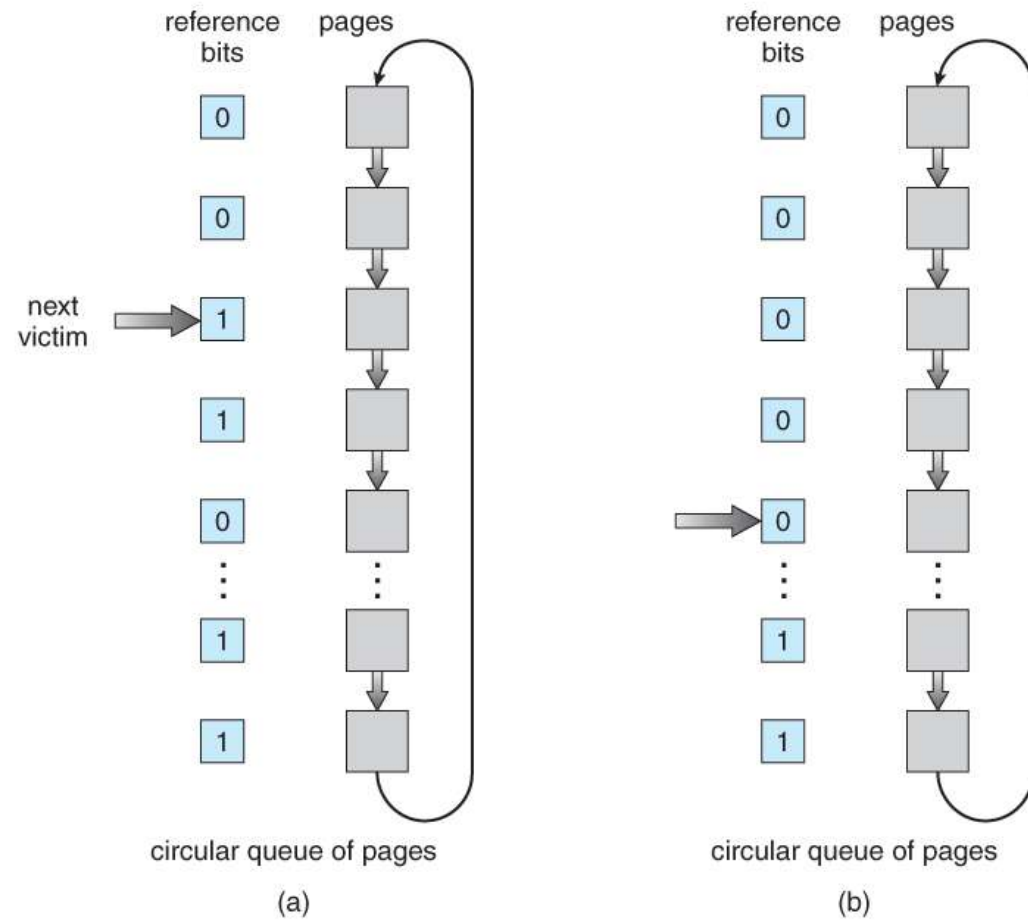
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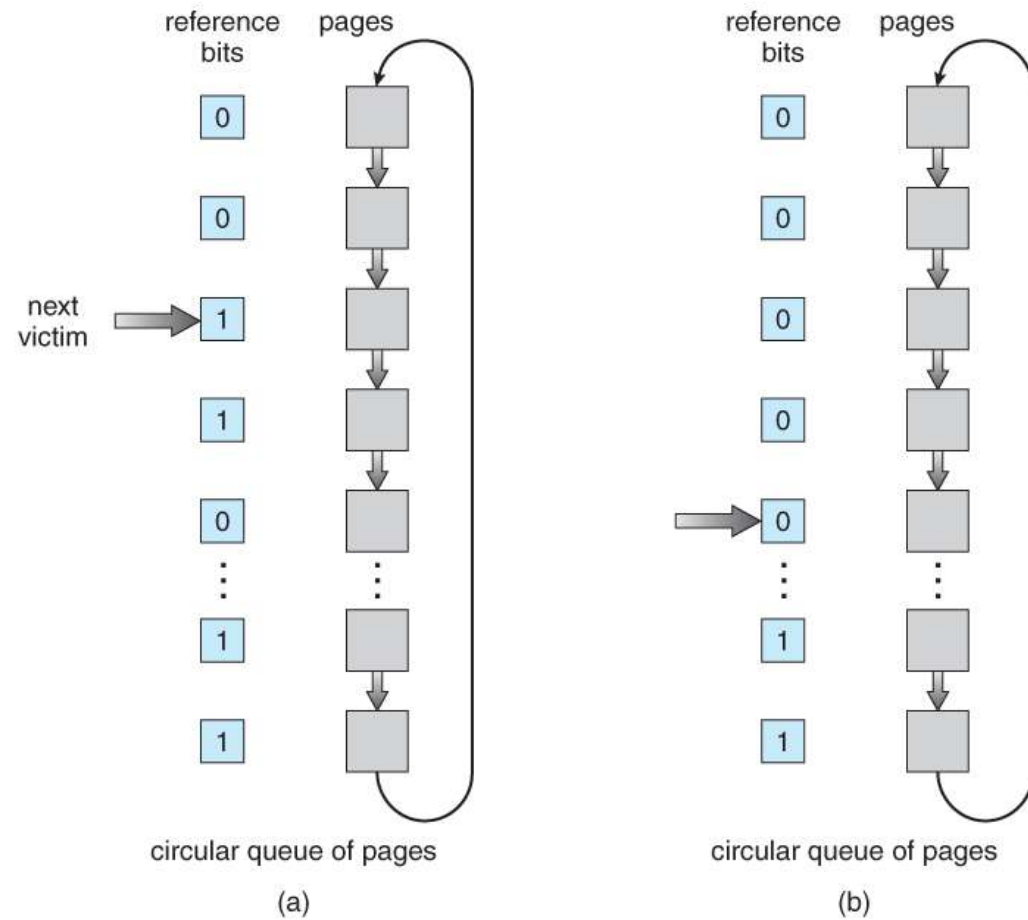
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- OS keeps frames in a FIFO circular list
- On every memory access, the reference bit is set to 1
- On a page fault, the OS scans the list of page table, checking the reference bit of the frame:
 - If this is 0, it replaces the page and sets it to 1
 - If this is 1, it sets it to 0 (second chance) and move to the next frame

Second Chance Algorithm (Clock)



A raw partitioning into: young vs. old frames

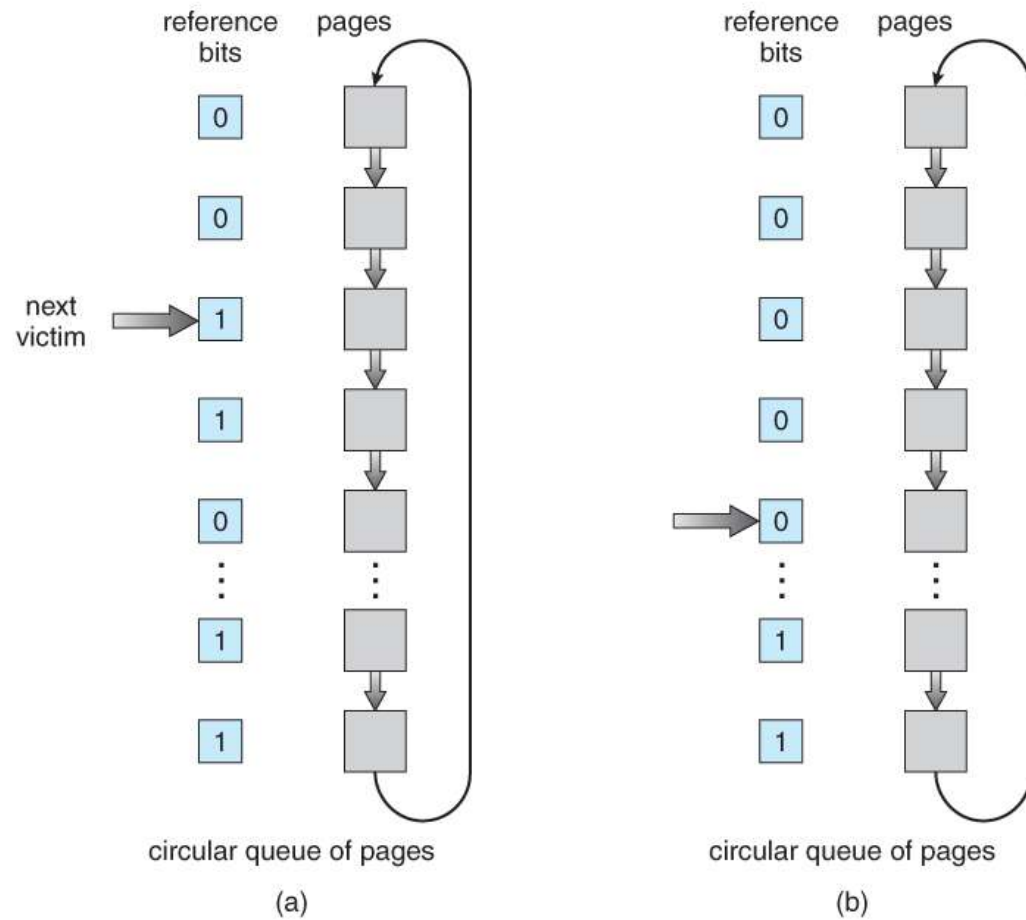
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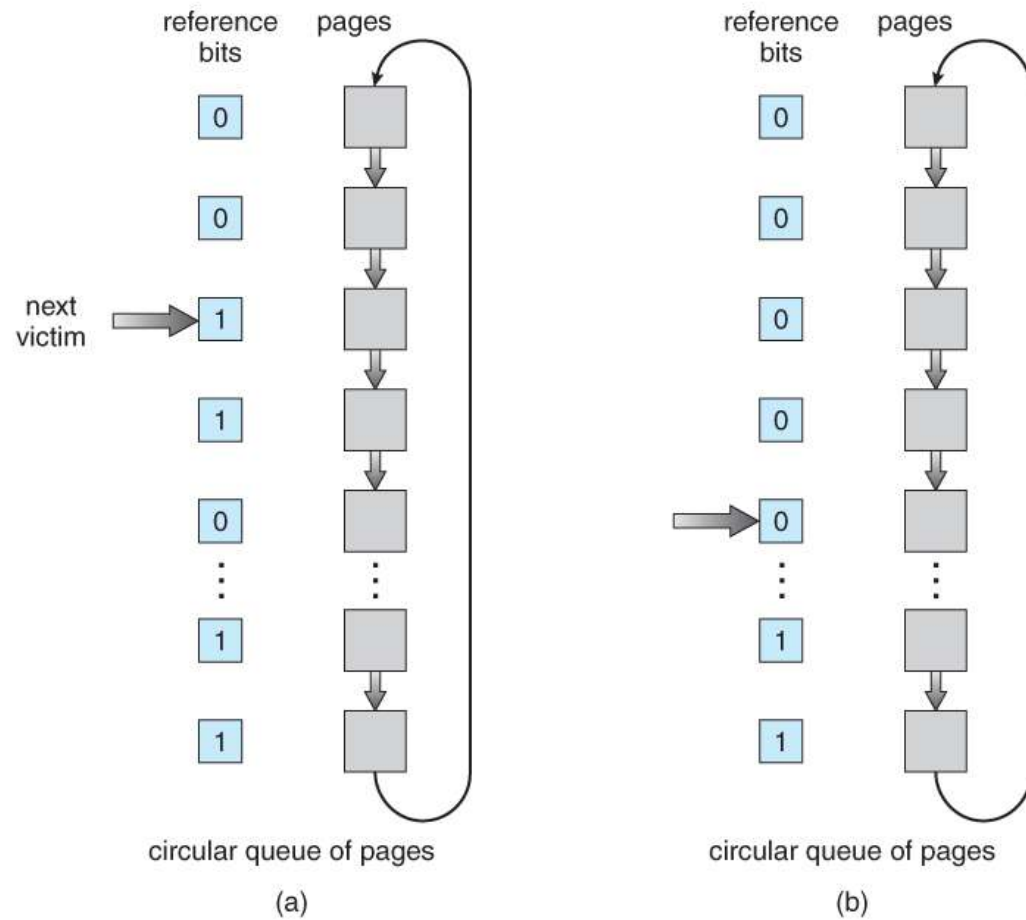


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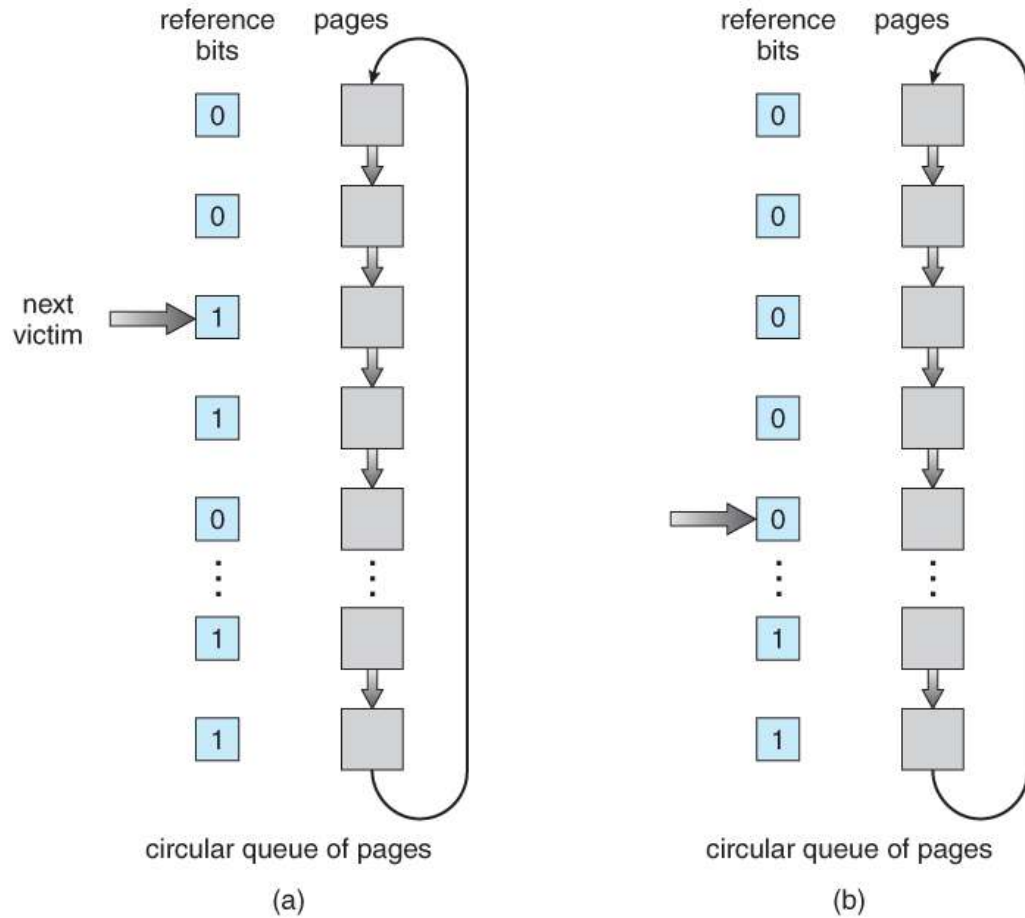
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This algorithm is also known as **clock** because it mimics the hands of a clock

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 - write the evicted page back to disk
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- **Intuition:** It is cheaper to replace a page which has not been modified, since the OS does not need to write this back to disk
- OS should give preference to paging-out un-modified frames
 - Yet, it can proactively write to disk modified frames for later

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- Use both the reference and modify bits (r, m) to classify pages into:
 - $(0, 0)$: neither recently used nor modified;
 - $(0, 1)$: not recently used, but modified;
 - $(1, 0)$: recently used, but clean
 - $(1, 1)$: recently used and modified

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- The main difference between this algorithm and the standard clock is the preference for replacing clean pages if possible

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- Multiple processes can however run concurrently on a single-CPU system
- The degree of multiprogramming is not fixed apriori, yet it is driven by the locality reference (a.k.a. 90÷10 rule)
- This allows a system to load the **working set** (i.e., few pages) of many processes, thereby increasing the degree of multiprogramming

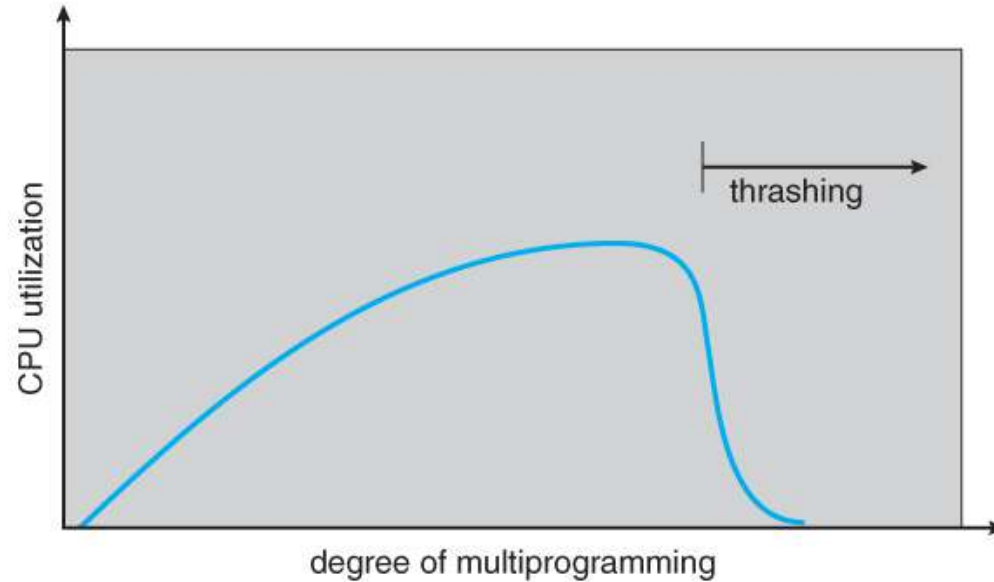
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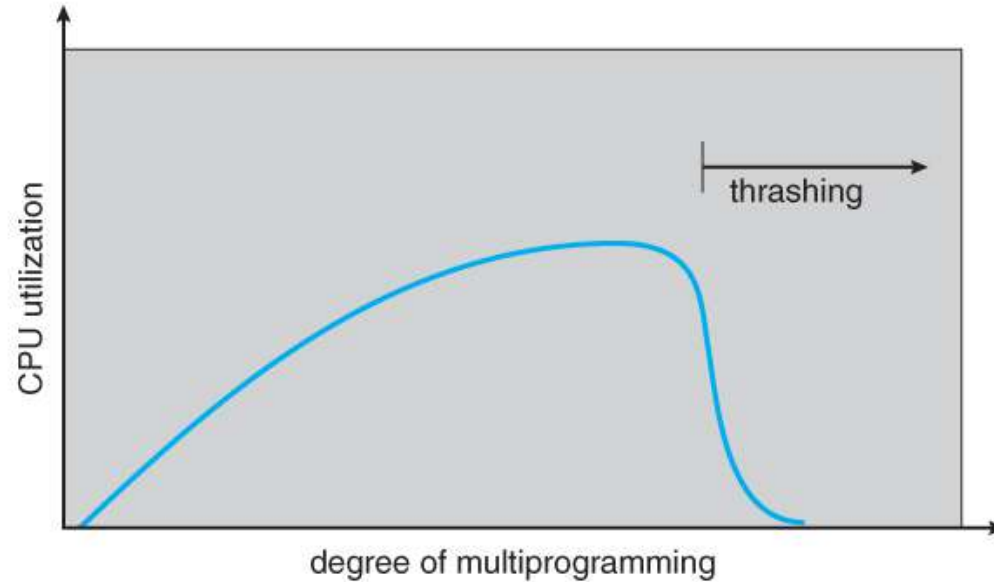
- When the degree of multiprogramming is too high, active working sets of running processes may saturate the whole memory capacity
- **Thrashing** → Memory is over-committed and pages are continuously tossed out while they are still in use
 - Memory access time approaches disk access time due to many page faults
 - Drastic degradation of performance

Multiprogramming and Thrashing



CPU utilization drops after a certain degree of multiprogramming

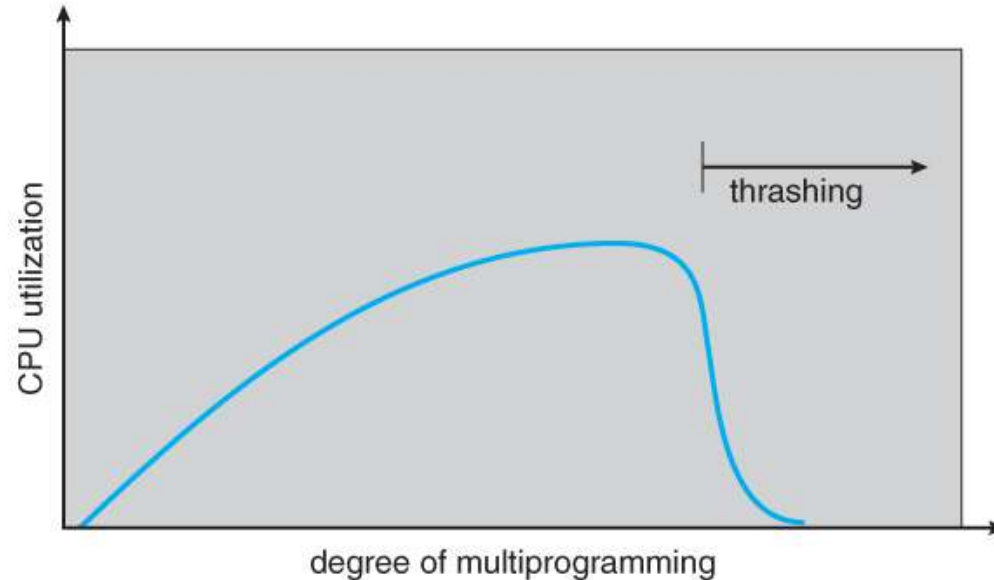
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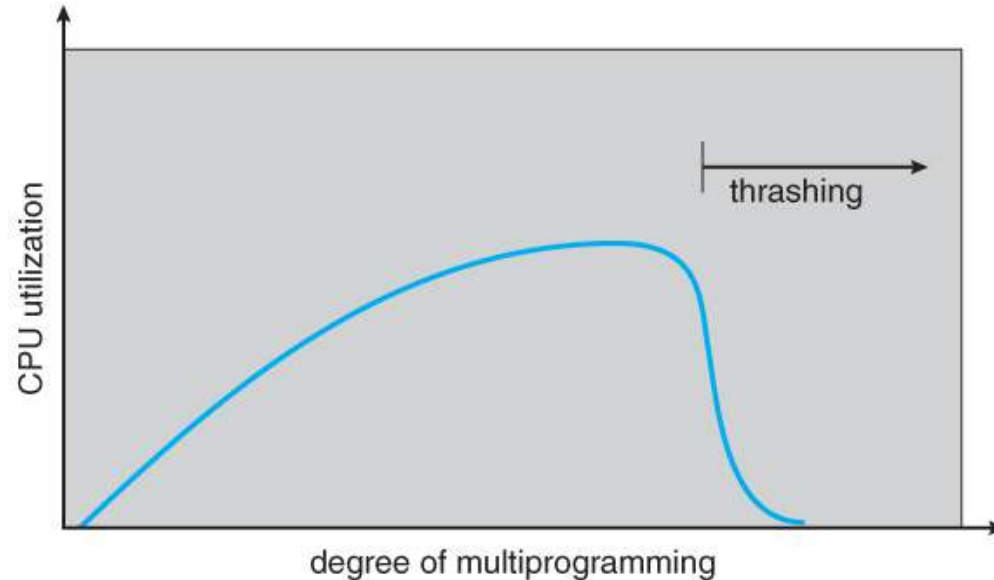


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What can we do to limit thrashing in a multiprogrammed system?

Fixing the degree of multi-programming apriori may be a too inflexible option

Allocation/Replacement Policies

Ultimately, we want to give each process enough memory so as to avoid thrashing

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Global Allocation/Replacement

- All pages from all processes are in a single pool (single LRU queue)
- Upon page replacement, any page may be a potential victim, whether it currently belongs to the process seeking a free frame or not
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Local Allocation/Replacement

- Each process has its own fixed pool of frames
- Run only group of processes that fits in memory
- LRU replacement affects only each process' frames
- **PRO:** isolation
- **CON:** performance (a process may not be given enough memory)

Local Allocation/Replacement

m = number of available physical page frames

n = number of processes

S_i = size of the i -th process; $S = \sum_{i=1}^n S_i$ = total size of all processes

Equal Allocation/Replacement: $\frac{m}{n}$

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As allocations fluctuate over time, so does m
(processes must either be swapped out or not allowed to start if not enough frames)

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- However, there might be cases where this is not true
 - e.g., a process allocates a 1 GB array but then only uses a small portion of it
- In other words, the working set of a process may not be correlated with its memory footprint

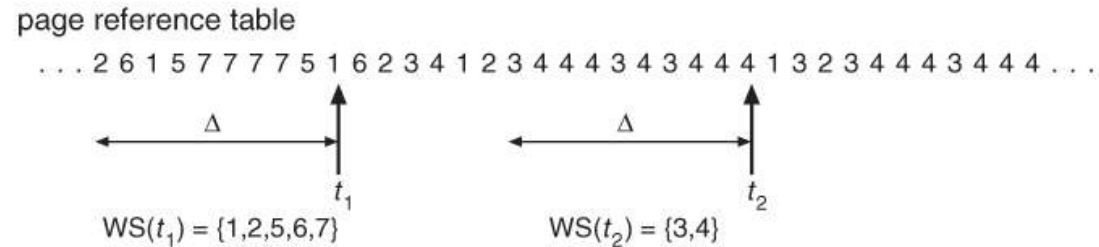
Matching the Working Set

- **Goal** → Give each process enough frames to contain its working set
 - Informally, the working set is the set of pages the process is using "right now"
 - More formally, it is the set of all pages that the process has referenced during the past T units of time (e.g., seconds)

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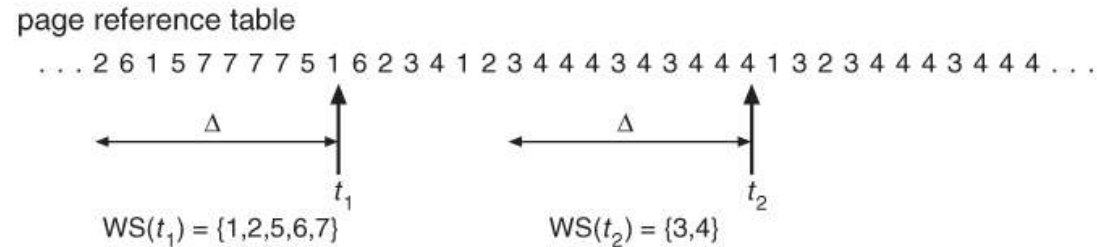
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- How does the OS pick T ?
 - 1 page fault takes order of 10 msec to be served
 - 10 msec \sim 10 million instructions
 - T needs to account for a lot more than 10 million instructions

Determining the Working Set



The selection of Δ is critical to the success of the working set model

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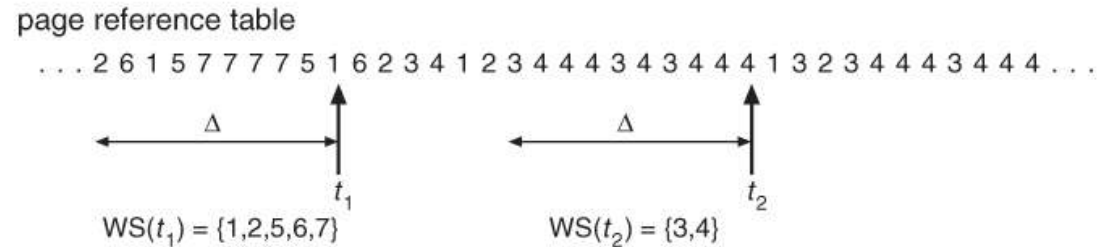


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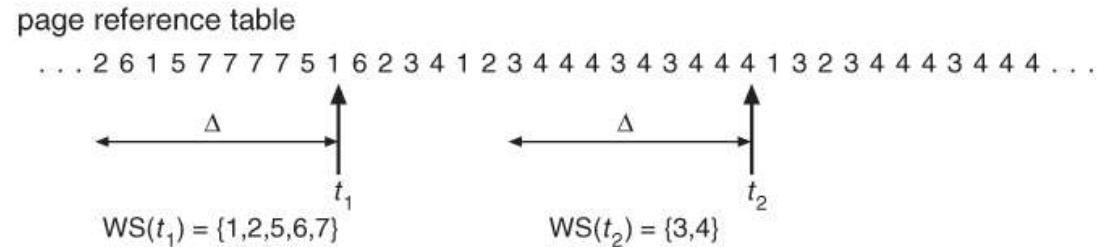
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The selection of Δ is critical to the success of the working set model

Δ too small

it does not encompass all of the pages
of the current locality

Δ too large

it includes pages that are no longer
frequently accessed

Exact tracking is expensive: update the working set at each memory access

Approximating the Working Set

- Computing the working set exactly requires to keep track of a moving window of size Δ

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Approximating the Working Set

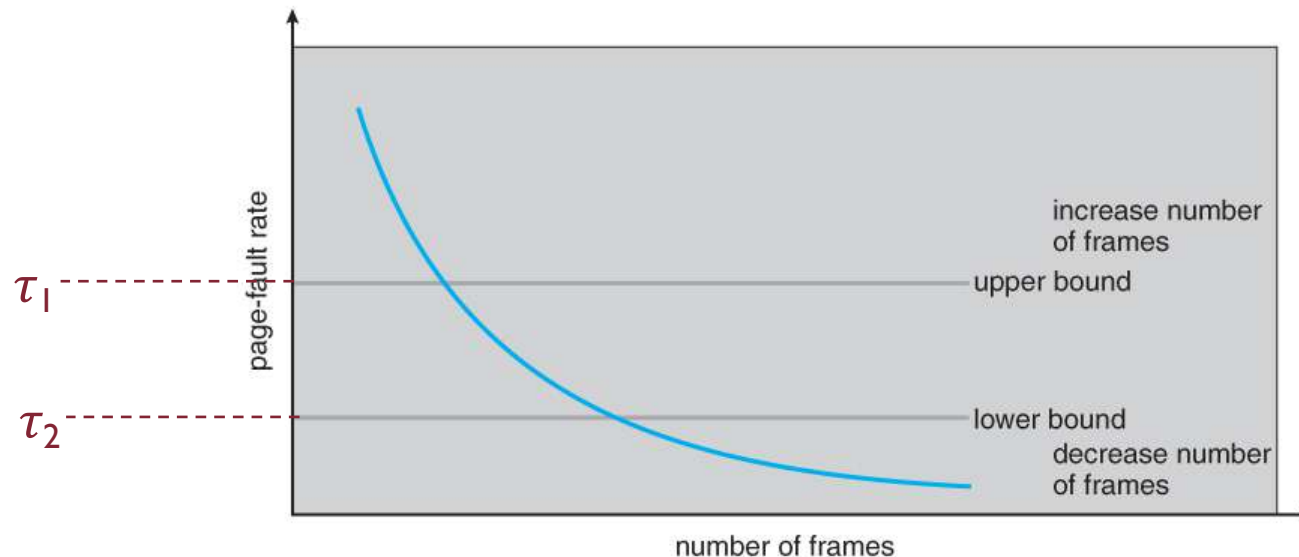
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- At each memory reference a new reference appears at one end and the oldest reference drops off the other end
- To avoid the overhead of keeping a list of the last Δ referenced pages, the working set is often implemented with **sampling**
- Every k memory references (e.g., $k = 1,000$), consider the working set to be all pages referenced within *that* period of time

Tracking Page Fault Rate

- Ultimately, our goal is to minimize the **page fault rate**

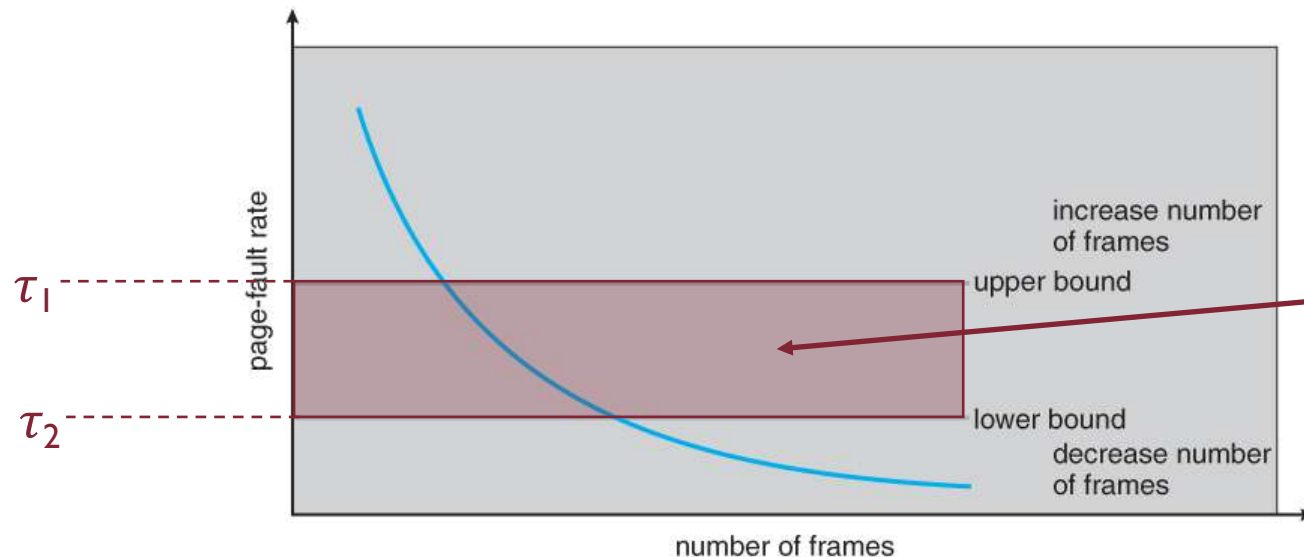
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- A more direct approach is to track page fault rate of each process:
 - If the page fault rate is above a given threshold $\tau_1 \rightarrow$ give it more frames
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Dynamically adjust allocated frames so as to keep processes in this area

Kernel Memory

- So far, we only considered memory allocation for user processes
- But kernel needs memory to store things too: code and data structures like PCB, page tables, etc.
- Kernel does not use any of the advanced mechanisms seen so far
 - No paging → what if a page fault occurs for the kernel?

Kernel Memory: Buddy Allocator

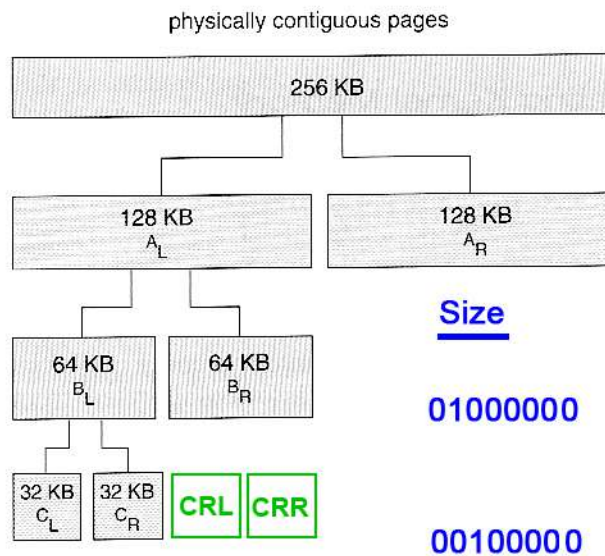


Figure 9.27 Buddy system allocation.

Buddy Addresses

00000000

00000000 10000000

Size

01000000

00000000 01000000

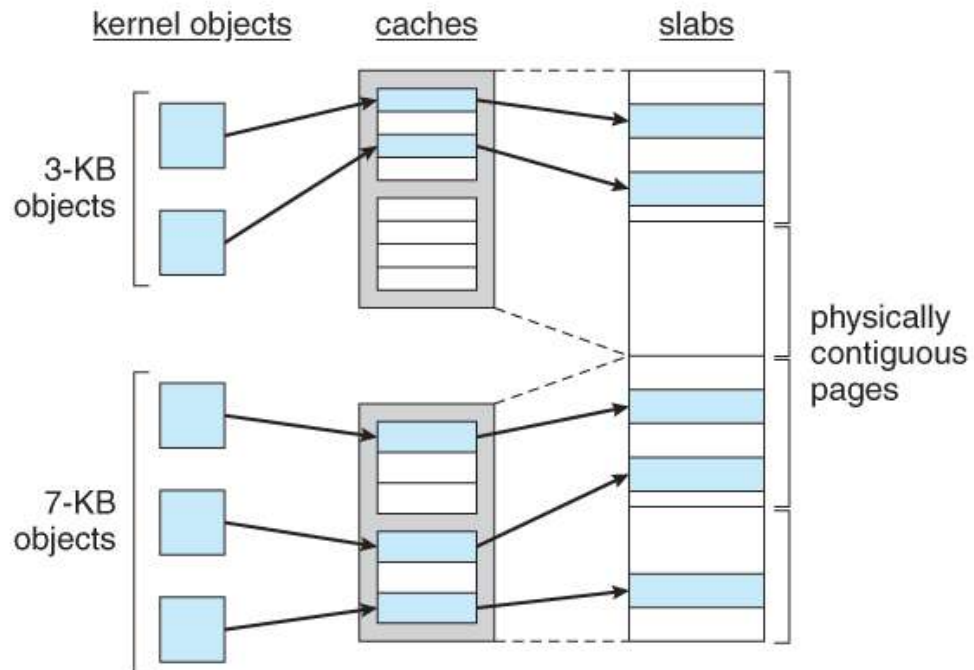
00100000

00000000 00100000

01000000 01100000

- Allocates memory using a power of 2 allocator (e.g., 4K, 8K, 16K), rounding up to the next nearest power of two if necessary
- If a block of the correct size is not available, then one is formed by (repeatedly) splitting the next larger block in two
- Can lead to internal fragmentation

Kernel Memory: Slab Allocator



- Group of objects of the same size in a **slab**
- Object cache points to one or more slabs
- Separate cache for each kernel data structure (e.g., PCB)
- No internal fragmentation
- Used in Solaris and Linux

Page Sizes

- Reasons for **small** pages?

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Page Sizes

- Reasons for **small** pages?
 - Decreasing internal fragmentation
 - Higher degree of multiprogramming
- Reasons for **large** pages?
 - Smaller page table size (i.e., smaller number of page table entries)
 - Fewer page faults (locality reference)
 - Amortizes disk overhead (reading a 1 KB page from disk takes approximately the same as reading an 8KB one)

Summary of Page Replacement

- The choice of page replacement algorithm is crucial when physical memory is limited
 - All algorithms approach to the optimum as the physical memory allocated to a process approaches to the virtual memory size

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- The choice of page replacement algorithm is crucial when physical memory is limited
 - All algorithms approach to the optimum as the physical memory allocated to a process approaches to the virtual memory size
- The more processes running concurrently, the less physical memory each one can have
- The OS must choose how many processes (and the number of frames per process) can share memory simultaneously