Main Contributions of the Thesis “Leadership Games: Multiple Follower, Multiple Leaders, and Perfection”

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Economic paradigms and multi-agent systems are two crucial areas of *artificial intelligence* (AI) that are receiving a growing attention in the scientific community. These address competitive interactions that involve multiple actors, either humans beings or artificial bots, and, thus, their techniques can be naturally adopted in many application domains, either on the Internet, such as, *e.g.*, in e-commerce, web advertising, and social networks, or in the real world, such as in physical security and poaching prevention, among the others.

Stackelberg games model settings in which some players act as *leaders* with the ability to commit to a strategy beforehand, whereas the others are followers who decide how to play after observing the commitment. The study of Stackelberg games has received a terrific attention in AI, leading to the deployment of ground-breaking applications in physical and cyber security (Tambe, 2011). Most of these models focus on strategic interactions involving two players, a leader and a follower, who play the role of defender and attacker, respectively. However, Stackelberg games are suitable in all the scenarios involving players with a competitive advantage, such as, *e.g.*, firms forming a price-determining dominant cartel and owners of resource-sharing platforms, where there might be multiple (more than one) players playing the role of leader and/or follower.

In this thesis, we significantly extend the state of the art by providing the first comprehensive computational study of general Stackelberg games, along three main directions: games with *multiple followers*, games with *multiple leaders*, and *equilibrium refinement* in Stackelberg games with a sequential structure. Our results enable the adoption of Stackelberg games in a much wider spectrum of applications, beyond simple single-leader single-follower cases.

In the first part the thesis, we study games with a single leader and multiple followers, providing a fine-grained taxonomy of which game classes are computationally tractable and which are not (Marchesi et al., 2018; De Nittis et al., 2018; Marchesi et al., 2019a). This taxonomy is crucial to investigate the scalability of the algorithms. Remarkably, we also address the yet widely unexplored problem of finding equilibria in games where the followers break ties against the leader, which begets additional computational challenges (Coniglio et al., 2017; Basilico et al., 2017; Coniglio et al., 2020; Basilico et al., 2020). Our central result is the identification of the first class of Stackelberg games with many followers in which computing equilibria is computationally tractable. These are congestion games modeling resource-sharing platforms in which a leader (*e.g.* the owner) has a prioritized access to the platform (Castiglioni et al., 2019b).

In the second part, we propose a novel framework for Stackelberg games with multiple leaders, extending the idea of *commitment to correlated strategies* beyond the single-leader case (Conitzer and Korzhyk, 2011). Despite the several attempts to address Stackelberg games with multiple leaders in the literature, even designing a satisfactory model was elusive. The core idea of our work is to let the leaders decide whether they want to participate in the commitment or defect from it by becoming followers, so that the externalities due to each leader determine her value in the equilibrium. This is orchestrated by a suitably defined agreement protocol represented as a sequential game, which allows us to introduce interesting properties for the commitments (Castiglioni et al., 2019a).

In the last part of the thesis, we introduce, for the first time, equilibrium refinements in sequential Stackelberg games, so as to amend weaknesses off the equilibrium path, which may prescribe the players sub-optimal actions. We first introduce the general framework of *trembling-hand perfection* in Stackelberg games (Farina et al., 2018). Then, we focus on *quasi perfection*, providing a computational method for finding the resulting equilibria (Marchesi et al., 2019b), which partially relies on a recent characterization result on (non-Stackelberg) quasi-perfect equilibria (Gatti et al., 2020).

The thesis contributions are organized as follows:

- Chapters 1 , 2, and 3 introduce the work and the preliminaries needed.
- Chapters 4, 5, 6, and 7 deal with games with a single leader and multiple followers; the relevant publications are (Coniglio et al., 2017; Basilico et al., 2017; De Nittis et al., 2018; Marchesi et al., 2018, 2019a; Castiglioni et al., 2019b; Coniglio et al., 2020; Basilico et al., 2020).
- Chapters 8 and 9 address games with multiple leaders and followers; the relevant publication is (Castiglioni et al., 2019a).
- Chapters 10, 11, and 12 are about equilibrium refinements; the relevant publications are (Farina et al., 2018; Marchesi et al., 2019b).
- Chapter 13 concludes the work.

References

N. Basilico, S. Coniglio, N. Gatti, and A. Marchesi. Bilevel programming approaches to the computation of optimistic and pessimistic single-leader-multi-follower equilibria. In *SEA*, volume 75, pages 1–14. Schloss Dagstuhl-Leibniz-Zentrum fur Informatik GmbH, Dagstuhl Publishing, 2017.

N. Basilico, S. Coniglio, N. Gatti, and A. Marchesi. Bilevel programming methods for computing single-leader-multi-follower equilibria in normal-form and polymatrix games. *EURO Journal on Computational Optimization*, pages 1–29, 2020.

M. Castiglioni, A. Marchesi, and N. Gatti. Be a leader or become a follower: The strategy to commit to with multiple leaders. In *Proceedings of the Twenty-Eighth International Joint Conference on Artificial Intelligence, IJCAI 2019, Macao, China, August 10-16, 2019*, pages 123–129, 2019a.

M. Castiglioni, A. Marchesi, N. Gatti, and S. Coniglio. Leadership in singleton congestion games: What is hard and what is easy. *Artificial Intelligence*, 277, 2019b.

S. Coniglio, N. Gatti, and A. Marchesi. Pessimistic leader-follower equilibria with multiple followers. In *Proceedings of the Twenty-Sixth International Joint Conference on Artificial Intelligence, IJCAI-17*, pages 171–177, 2017.

S. Coniglio, N. Gatti, and A. Marchesi. Computing a pessimistic Stackelberg equilibrium with multiple followers: the mixed-pure case. *Algorithmica*, 82 (5):1189–1238, 2020.

V. Conitzer and D. Korzhyk. Commitment to correlated strategies. In *Proceedings of the Twenty-Fifth AAAI Conference on Artificial Intelligence (AAAI 2011)*, pages 632–637, 2011.

G. De Nittis, A. Marchesi, and N. Gatti. Computing the strategy to commit to in polymatrix games. In *Proceedings of the Thirty-Second AAAI Conference on Artificial Intelligence (AAAI-18)*, pages 989–996, 2018.

G. Farina, A. Marchesi, C. Kroer, N. Gatti, and T. Sandholm. Trembling-hand perfection in extensive-form games with commitment. In *Proceedings of the Twenty-Seventh International Joint Conference on Artificial Intelligence, IJCAI 2018, July 13-19, 2018, Stockholm, Sweden.*, pages 233–239, 2018.

N. Gatti, M. Gilli, and A. Marchesi. A characterization of quasi-perfect equilibria. *Games Econ. Behav.*, 122:240–255, 2020. doi: 10.1016/j.geb.2020.04.012. URL `https://doi.org/10.1016/j.geb.2020.04.012`.

A. Marchesi, S. Coniglio, and N. Gatti. Leadership in singleton congestion games. In *Proceedings of the Twenty-Seventh International Joint Conference on Artificial Intelligence, IJCAI 2018, July 13-19, 2018, Stockholm, Sweden.*, pages 447–453, 2018.

A. Marchesi, M. Castiglioni, and N. Gatti. Leadership in congestion games: Multiple user classes and non-singleton actions. In *Proceedings of the Twenty-Eighth International Joint Conference on Artificial Intelligence, IJCAI 2019, Macao, China, August 10-16, 2019*, pages 485–491, 2019a.

A. Marchesi, G. Farina, C. Kroer, N. Gatti, and T. Sandholm. Quasi-perfect Stackelberg equilibrium. In *The Thirty-Third AAAI Conference on Artificial Intelligence, AAAI 2019, Honolulu, Hawaii, USA, January 27 - February 1, 2019.*, pages 2117–2124, 2019b.

M. Tambe. *Security and game theory: algorithms, deployed systems, lessons learned*. Cambridge university press, 2011.