

(3.1) a)

$$\rightarrow 49 - 3 \cdot 15 = 4$$

$$i) 7^2 = 49 \equiv 4 \pmod{15}$$

$$7^4 = (7^2)^2 \equiv 4^2 \pmod{15} = 16 \pmod{15} = -1 \pmod{15}$$

$$7^8 = (7^4)^2 \equiv (-1)^2 \pmod{15} = 1 \pmod{15}$$

$$7^{16} = (7^8)^2 \equiv \boxed{1 \pmod{15}}$$

...

$$7^{50} = 7^{32} \cdot 7^{16} \cdot 7^2 = 1 \cdot 1 \cdot 4 \pmod{15} = 4 \pmod{15}$$

$$\rightarrow 64 - 7 \cdot 9 = 1$$

$$ii) 8^2 = 64 \equiv 1 \pmod{9}$$

...

$$8^{2010} = 8^{1024} \cdot 8^{512} \cdot 8^{256} \cdot 8^{128} \cdot 8^{64} \cdot 8^{32} \cdot 8^4 = \boxed{1 \pmod{9}}$$

$$\rightarrow 100 - 3 \cdot 35 = -5$$

$$iii) 10^2 = 100 \equiv -5 \pmod{35}$$

$$10^4 = (10^2)^2 \equiv 25 \pmod{35} = -10 \pmod{35}$$

$$10^8 = (10^4)^2 \equiv 100 \pmod{35} = \boxed{-5 \pmod{35}}$$

b)

$$\boxed{n=15} \quad \phi(15) = \phi(5 \times 3) = \phi(5) \times \phi(3) = 4 \times 2 = \textcircled{8}$$

$$\boxed{n=35} \quad \phi(35) = \phi(5 \times 7) = \phi(5) \times \phi(7) = 4 \times 6 = \textcircled{24}$$

$$\boxed{n=9} \quad \phi(9) = \textcircled{8}$$

c)  $\rightarrow$  calculado em b)

$$(i) \quad 7^{-1} = 7^{\phi(15)-1} \mod 15 = 7^7 \mod 15 = \boxed{13 \mod 15}$$

$$(ii) \quad 8^{-1} = 8^{\phi(9)-1} \mod 9 = 8^7 \mod 9 = \boxed{8 \mod 9}$$

$$(iii) \quad 34^{-1} = 34^{\phi(35)-1} \mod 35 = 34^{23} \mod 35 = \boxed{34 \mod 35}$$

$$(iv) \quad 7^{-50} = (7^{50})^{-1} = (7^{50})^{\phi(15)-1} \mod 15 =$$

$$= (7^{50})^7 \mod 15 = \boxed{4 \mod 15}$$

$$(v) \quad 8^{-2019} = (8^{2019})^{-1} = (8^{2019})^{\phi(9)-1} \mod 9 =$$

$$= (8^{2019})^7 \mod 9 = \boxed{8 \mod 9}$$