

A League Table

Step 1:

We know Team C has played two games, and we know they lost one of the two, hence the other game must either be a win or a draw. However, Team C has more points for them than against therefore they must have won at least one game, hence the other game must be a win for Team C.

Team	Played	Won	Lost	Drawn	For	Against
A			1	0	1	2
B			1		0	5
C	2	1	1	0	7	4
D		1		0	2	2
E		2		0	5	2

Step 2:

If Team B drew one or more games, the opposing team must also have a drawn game, but since all other teams have zero draws, we can conclude that Team B also has zero draws. In other words, a drawn game requires two teams to have a drawn result.

The Team	Played	Won	Lost	Drawn	For	Against
A			1	0	1	2
B			1	0	0	5
C	2	1	1	0	7	4
D		1		0	2	2
E		2		0	5	2

Step 3:

To win a game, you have to score more than 0. Team B has 0 draws, hence we can conclude that Team B has only played one game which they lost 0-5.

Team	Played	Won	Lost	Drawn	For	Against
A			1	0	1	2
B	1	0	1	0	0	5
C	2	1	1	0	7	4
D		1		0	2	2
E		2		0	5	2

Step 4:

In step three, we found that one team won against Team B with a score of 5-0. Only Team C and E has scored 5 or more points.

Let us assume that it was Team E, and they scored 5-0 against Team B. This means that they scored 0 points in all their other games. If they scored 0 points in all their other games, they couldn't have won another game. But they won two games, so it cannot be Team E.

So it was Team C that scored 5-0 against Team B. Now that we know the score of the game that Team C won, we can deduce the score of the game that they lost.

Games of Team C	For	Against
Game 1 (Lost)	2	4
Game 2 (Won) (vs Team B)	5	0

Step 5:

From step 4, we know that one team scored 4-2 against Team C. Team E is the only other team that has scored at-least 4 points. So we can conclude that Team E scored 4-2 against Team C.

This leaves 1 points 'For' and 0 points 'Against' Team E, hence, Team E must have won against another team with a score of 1-0.

This leaves us with 0 points 'For' and 'Against' Team E which could only result in drawn games and Team E has 0 draws, so we can conclude that Team E has lost 0 games, and have only played 2 games so far, both of which they won.

Games of Team E	For	Against
Game 1 (Won) (vs Team C)	4	2
Game 2 (Won)	1	0

Team	Played	Won	Lost	Drawn	For	Against
A			1	0	1	2
B	1	0	1	0	0	5
C	2	1	1	0	7	4
D		1		0	2	2
E	2	2	0	0	5	2

Step 6:

From step 5 we know that one team lost against Team E with a score of 0-1. It cannot be Team C or B (we know their games from previous steps) so it is either against Team A or D.

Let us assume that it was Team A, and they lost 0-1 against Team E. This leaves us with 1 point 'For' and 1 point 'Against' Team A. We know Team A have 0 drawn games which leaves us with the only other option of one game won (1-0), and another game lost (0-1). However, Team A has only lost one game, hence it couldn't be Team A.

This means it was Team D that lost 0-1 against Team E.

Team	Played	Won	Lost	Drawn	For	Against
A			1	0	1	2
B	1	0	1	0	0	5
C	2	1	1	0	7	4
D	2	1	1	0	2	2
E	2	2	0	0	5	2

Who Step 7:

This leaves Team D with 2 points 'For' and 1 point 'Against' them, and this score could only have been played with Team A since we have already found the games of the other teams. And since each team plays another team only once, we can deduce that the game between Team D and Team A had a score of 2-1, with Team D winning. Therefore, Team A has only played one game and lost it 1-2 against Team D.

Therefore, Team A has only played one game and lost it 1-2 against Team D.

Team	Played	Won	Lost	Drawn	For	Against
A	1	0	1	0	1	2
B	1	0	1	0	0	5
C	2	1	1	0	7	4
D	2	1	1	0	2	2
E	2	2	0	0	5	2

The League Commissioner's claim is true. We have reconstructed the table based on the initial values recovered from memory.

Hats, Hats, Hats!!!

To explain this easier, I will have two variables, x and y , each representing a different colour (either black or white in this case). The chair positioning will go from E (left) to A (right), so when I present an expression such as 'xyxyy', the person in chair E has a hat of the colour x , and the person in chair A has a hat of the colour y . And the person in chair E can see everyone else's hat, while the person in chair A can't see any hats at all. Also, whenever a person in a chair says "I do", then they claim to know what colour hat they are wearing.

There are only three hats for each colour, hence if a person sees three hats of the same colour, they would know that they are wearing a hat of the opposite colour. So if we had a scenario: 'xyyyy', then those seated in chairs E and D will instantly know. However, C and D simultaneously said "I do" only after a pause. The only possible explanation is if they were waiting for the person in chair E to say "I do". If the person in chair E doesn't say anything, then there are two hats for each colour in front of chair E, and if C and D could see two hats of the same colour in front of them, then they could both conclude that they are not wearing a hat of the same colour, otherwise the friend in chair E would say "I do". And for C and D to see two hats in front of them, they both must be sitting at chairs D and C since chairs A or B don't have two chairs in front of them. So now, if the person in Chair E doesn't say "I do", C and D could simultaneously (independently) conclude what colour hat they are wearing. Now, the current scenario we are working with is: '?xyxy'.

The friend seated in chair B, who we will assume is friend B, upon hearing friends D and C say "I do", could also say "I do". This is because B, having the same intelligence as the others, know that what we explained above is the only way that D and C could say "I do". So B knows that they and the person in chair A are wearing the same colour hat, and B could see that the person in chair A has a hat of colour y , so B knows that they must also have a hat of colour y . Therefore, friend B is indeed sitting at chair B and says "So do I" after C and D say "I do". The friend in chair A will have the representation '?xyxy' in his head, but A wouldn't know what colour y is, since A cannot see any hats at all.

So far, these series of events will work for both scenarios: 'xxxxy' and 'yxyxy'. However, recall that A's hat was the same colour as the one left in the bag. If A was seated in chair E, in both scenarios, the three hats, of the colour that A is wearing, are all outside the bag. If A was seated in chair A, same thing occurs with scenario 'yxyxy', but, in scenario 'xxxxy', there are only two hats of colour y , hence the other one is inside the bag. Therefore A must be seated at chair A, and friend E at chair E.

We know that C and D are in chairs C and D, but we do not know in which order. Now, so far, A is seated at chair A, B at chair B, and E at chair E. Hence if C is at chair C and D is at chair D, then all chair labels match the friend sitting on them. But it is stated that 'Not all the chair labels matched the names of the persons sitting in them'. Therefore friend C is in chair D and friend D is in chair C.

Chair	E	D	C	B	A
Friend	E	C	D	B	A

All facing this direction

