



## Computer Science Presentation

# $\mu$ - recursive functions

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Jena, 17th of November

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## Structure

1. Computability
2.  $\mu$  - recursive functions
  - Explanation of terms
  - Initial functions
  - recursion / composition /  $\mu$ -operator
3. Examples for primitive and  $\mu$  recursion functions

## Theory of Computability

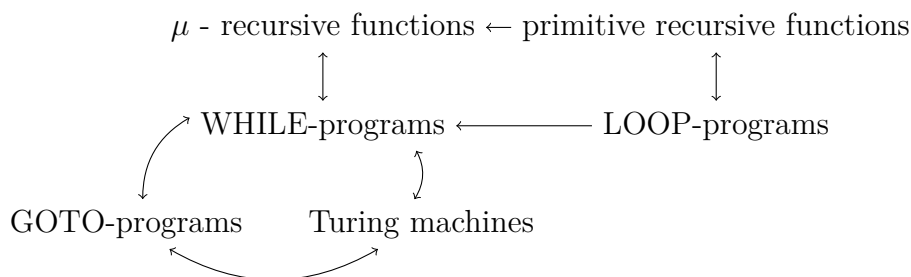
### Definition of computability

A function  $f(x_1, x_2, \dots, x_n)$  is computable if there exists an algorithm  $A(x_1, x_2, \dots, x_n)$  following given rules, that solves  $f$  in its domain and gives no output otherwise.

$$\begin{aligned} (x_1, x_2, \dots, x_n) \in D_f &\implies A(x_1, x_2, \dots, x_n) = f(x_1, x_2, \dots, x_n) \\ (x_1, x_2, \dots, x_n) \notin D_f &\implies A(x_1, x_2, \dots, x_n) = \text{ENDLESS LOOP} \end{aligned}$$

### Theories of computability

- turing machines
- LOOP-programs (incomplete)
- GOTO-programs
- WHILE-programs
- $\lambda$ -calculus
- primitive recursive functions (incomplete)
- $\mu$ -recursive functions



# Building-Block functions

## Constant function

$$N(X_0, x_1, x_2, \dots, x_n):$$
$$N(x_1, x_2, \dots, x_n) = \text{const.}$$

## Successor function

$$S(x_1):$$
$$S(x_1) = x_1 + 1$$

## Projection function

$$P_m^n(x_1, x_2, \dots, x_n):$$
$$P_m^n(x_1, x_2, \dots, x_m, \dots, x_n) = x_m$$

## Composition function

$$C(x_1, x_2, \dots, x_n):$$
$$f_n = f_n(x_1, x_2, \dots, x_n)$$
$$f_{m,k} = f_{m,k}(x_1, x_2, \dots, x_m)$$
$$C = f_n \circ (f_{m,1}, f_{m,2}, \dots, f_{m,n})$$
$$C = f_n(f_{m,1}(x_{1,1}, x_{1,2}, \dots, x_{1,m}), \dots, f_{m,n}(x_{n,1}, x_{n,2}, \dots, x_{n,m}))$$

## Primitive recursion function

$$R(X_0, x_1, x_2, \dots, x_n):$$
$$X_0 = 0 \implies R(X_0, x_1, x_2, \dots, x_n) = f(y_1, \dots, y_m)$$
$$X_0 \neq 0 \implies R(X_0, x_1, x_2, \dots, x_n) = g(R(X_0 - 1, x_1, \dots, x_n), z_1, \dots, z_l)$$

## $\mu$ minimisation function

$$\mu(X_0, x_1, x_2, \dots, x_n):$$
$$0 = f(z, y_1, y_2, \dots, y_n)$$
$$\wedge 0 < f(i, y_1, y_2, \dots, y_n)$$
$$\iff z = \mu(f(Y_0, y_1, y_2, \dots, y_n), x_1, x_2, \dots, x_n)$$

# Bibliography

[Access Date] December 8, 2017

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