Computer Science Presentation

μ - recursive functions

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Structure

1. Computability
2. μ - recursive functions
   - Explanation of terms
   - Initial functions
   - recursion / composition / μ-operator
3. Examples for primitive and μ recursion functions

Theory of Computability

Definition of computability

A function $f(x_1, x_2, \cdots, x_n)$ is computable if there exists an algoritm $A(x_1, x_2, \cdots, x_n)$ following given rules, that solves f in its domain and gives no output otherwise.

$(x_1, x_2, \cdots, x_n) \in D_f \Longrightarrow A(x_1, x_2, \cdots, x_n) = f(x_1, x_2, \cdots, x_n)$
$(x_1, x_2, \cdots, x_n) \notin D_f \Longrightarrow A(x_1, x_2, \cdots, x_n) =$ **ENDLESS LOOP**

Theories of computability

- turing machines
- LOOP-programs (incomplete)
- GOTO-programs
- WHILE-programs
- λ-calculus
- primitive recursive functions (incomplete)
- μ-recursive functions

$$\begin{tikzcd}
 & \mu\text{ - recursive functions} \arrow[d, leftrightarrow] & \text{primitive recursive functions} \arrow[l] \arrow[d, leftrightarrow] \\
 & \text{WHILE-programs} \arrow[dl, leftrightarrow, bend right] \arrow[d, leftrightarrow] & \text{LOOP-programs} \arrow[l] \\
\text{GOTO-programs} \arrow[r, leftrightarrow, bend right] & \text{Turing machines} &
\end{tikzcd}$$

Building-Block functions

Constant function

$N(X_0, x_1, x_2, \cdots, x_n)$:
$N(x_1, x_2, \cdots, x_n) = const.$

Successor function

$S(x_1)$:
$S(x_1) = x_1 + 1$

Projection function

$P^n_m(x_1, x_2, \cdots, x_n)$:
$P^n_m(x_1, x_2, \cdots, x_m, \cdots, x_n) = x_m$

Composition function

$C(x_1, x_2, \cdots, x_n)$:
$f_n = f_n(x_1, x_2, \cdots, x_n)$
$f_{m,k} = f_{m,k}(x_1, x_2, \cdots, x_m)$
$C = f_n \circ (f_{m,1}, f_{m,2}, \cdots, f_{m,n})$
$C = f_n(f_{m,1}(x_{1,1}, x_{1,2}, \cdots, x_{1,m}), \cdots, f_{m,n}(x_{n,1}, x_{n,2}, \cdots, x_{n,m}))$

Primitive recursion function

$R(X_0, x_1, x_2, \cdots, x_n)$:
$X_0 = 0 \Longrightarrow\ R(X_0, x_1, x_2, \cdots, x_n) = f(y_1, \cdots, y_m)$
$X_0 \neq 0 \Longrightarrow\ R(X_0, x_1, x_2, \cdots, x_n) = g(R(X_0 - 1, x_1, \cdots, x_n), z_1, \cdots, z_l)$

μ minimisation function

$\mu(X_0, x_1, x_2, \cdots, x_n)$:
$0 = f(z, y_1, y_2, \cdots, y_n)$
$\wedge\ 0 < f(i, y_1, y_2, \cdots, y_n)$
$\Longleftrightarrow z = \mu(f(Y_0, y_1, y_2, \cdots, y_n), x_1, x_2, \cdots, x_n)$

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