Long-Run Carbon Consumption Risks Model and Asset Prices

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Abstract

This paper analyzes how environmental policies that aim to reduce carbon emissions affect asset prices and household consumption. Using novel data, I propose a measure of carbon emissions from a consumer point of view and a carbon consumption growth risk measure. The measures are based on information on aggregate consumption and the carbon footprint for each good and service. To analyze the effects of environmental policies, a long-run risks model is developed where consumption growth is decomposed into two components: the growth rate of carbon consumption and the growth rate of the share of carbon consumption out of total consumption. This paper argues that the long-run risk in consumption growth comes mainly from the carbon consumption growth arising from policies and actions to curb emissions, such as the Paris Agreement and the U.N. Climate Change Conference (COP26). My model helps to detect long-run risk in consumption from climate policies while simultaneously solving the equity premium and volatility puzzles. The decomposition of consumption could lead to identifying the most polluting consumption items and to constructing an investment strategy that minimizes or maximizes a long-term environmental criterion.

Keywords: Carbon Emissions, Carbon Risk, Long-Run Risk, Asset Pricing.

JEL Codes: G11, G12, Q54

Introduction

Regulators are increasingly worried about the extent to which stock markets efficiently price climate change risks and the discount rate that should be used to evaluate investments' uncertain future benefits. In fact, part of these risks stems from the transition to a low-carbon economy. More precisely, to curb carbon dioxide (CO2) emissions, climate policy aims to hold the increase in the average global temperature to within $2°C$ of pre-industrial levels. A burgeoning climate finance literature examines the efficiency of capital markets in pricing risks associated with climate change. For a detailed recent literature review, see Hong et al. (2020) and Giglio et al. (2020).

This market price related to carbon emissions, however, is narrowly confined to the production level and neglects carbon leakage inside and outside a given boundary[1]. Carbon leakage alludes to the situation that may take place if, for any cost-related reasons for climate actions, firms were to transfer production to other countries with fewer pollution-related constraints. This delocalization of production activities could lead to an increase in the total emissions of the corresponding firm. Thus, the production-based market price of carbon emissions incorrectly measures the actual impact of the carbon emissions. Moreover, as depicted in Figure 1, since 1998, carbon emissions in the United States, as measured with the consumption-based approach, are consistently larger than those measured with the production-based approach. Hence, the use of production-based emissions minimizes the real CO2 emissions in the atmosphere. Despite this fact, most papers and climate policies focus on the production side. This

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[1] Production-based carbon emissions exclusively refer to emissions generated at the point of production—that is, emissions physically produced

paper addresses this issue by providing a consumption-based carbon emissions measure using 12 consumption categories. Using the consumption-based carbon emissions approach has two benefits. First, it captures carbon leakage. Second, it captures the life cycle of greenhouse gases (GHG) emissions expressed in a CO2 equivalent. The life cycle assessment gives a more complete picture of a product's environmental impact. It tells us about the parts of its life cycle period during which the product most negatively affects the environment.

Climate change is a long-horizon phenomenon. But our actions today can help mitigate and adapt to that forthcoming risk. Our mitigation and adaptation actions will be efficient if we have a deep understanding of that long-run risk. To better understand the climate change effects on the economy, we need a long-run risks model. However, the canonical long-run risks (LRR) model studied by Bansal and Yaron (2004) is not suitable for analyzing climate risk, nor is it suitable for analyzing the effects of environmental policies on asset prices and household consumption. The reasons are twofold. First, the dynamics of the consumption growth rate in the canonical LRR model are not directly affected by any climate-related variable. Second, it is difficult to detect long-run risk in consumption coming from the canonical model (Bansal et al. (2007a), Bansal et al. (2007b), Pohl et al. (2018), Schorfheide et al. (2018) among others). Therefore, we need a new long-run risks model that specifies the consumption growth rate dynamics such that long-run risk is easily detectable and consumption growth is directly affected by climate-related shocks. This paper proposes such a model based on insight from Bansal et al. (2016b), Bansal et al. (2017), and Giglio et al. (2021).

This paper adds two contributions to the existing literature. First, I use novel data to provide a consumption-based carbon emissions measure. Second, I introduce a long-run carbon consumption risks model that departs from the existing long-run risks model through its decomposition of consumption growth into two components: carbon consumption growth and growth in the share of carbon consumption. The first component captures the effect of carbon consumption on aggregate consumption, and the second component captures the effect of green consumption. In addition, my model differs from the current literature in its ability to study the effects of environmental policies on asset prices and household consumption.

To that end, this paper links the carbon footprint information of each good and service from the Economic Input-Output Life Cycle Assessment (EIO-LCA) database and the aggregate consumption information of the same goods and services from the national income and product accounts (NIPA) consumption data to construct my consumption-based carbon dioxide emissions measure. To the best of my knowledge, this paper is the first to link the NIPA and EIO-LCA data to provide a consumption-based carbon emissions measure. Then, I use industry-level returns data from Kenneth R. French's website to empirically test the long-run carbon consumption risks model.

To assess the impact of climate change on macro-financial variables such as dividend growth, the equity premium, and consumption growth, this paper decomposes the consumption growth rate into two components: the carbon consumption growth component and the share of the carbon consumption growth component. In our setting, "carbon consumption risk" occurs for two main reasons. First, carbon risk stems from regulators' willingness to curb carbon emissions at the pre-industrial level, which in turn may affect future household consumption that heavily depends on carbon consumption. Second, carbon consumption creates damage through the lens of climate change. As a result, carbon-based consumption carries potential long-run risks in both cases. Building on this insight, I theoretically characterize and then quantify carbon price risk in an asset pricing model with long-run risks in carbon consumption. This paper argues that the long-run risk in consumption growth comes mainly from the carbon consumption growth arising from policies and actions to curb emissions, such as the Paris Agreement and the U.N. Climate Change Conference (COP26). I hypothesize that the growth rate of the share of carbon consumption out of total consumption does not carry any long-run risks.

Turning to the findings, we see that the decomposition of consumption growth gives more flexibility to policymakers in their efforts to stimulate consumption. They can target either carbon consumption or the share of carbon consumption out of total consumption. For example, a one standard deviation decrease in expected carbon consumption growth leads to a 27.12% decrease in consumption growth, whereas a one standard deviation decrease in expected carbon consumption growth, combined with a one standard deviation decrease in the growth of the share of carbon consumption, leads to a 26.8% decrease in consumption growth. This means that a policy aiming to reduce the consumption of goods and services that most pollute the environment without also encouraging green consumption is weak in terms of the impact of household consumption. Further analysis over different subperiods shows that the impact of environmental policies on asset prices and household consumption is bigger during periods of high climate change uncertainty. In fact, the decrease in consumption growth is approximately 9.5%, 0.55%, and 23% during the periods 1930–1955, 1956–1980, and 1981–2018, respectively. In addition, my model helps to detect future persistent fluctuations in the mean and volatility of carbon consumption growth arising from environmental policies. It also doubles the ability to detect long-run risks, as compared to the canonical model, while also solving the equity premium and volatility puzzles. My model is especially useful during periods of high climate change uncertainty, such as the period after the election of President Ronald Reagan in the US.

This article is related to the strand of literature on the long-run risks model. Papers here include Bansal and Yaron (2004), Bansal et al. (2007a), Bansal et al. (2007b), Koijen et al. (2010), Bonomo et al. (2011), Constantinides and Ghosh (2011), Schorfheide et al. (2018), Pohl et al. (2018), and Pohl et al. (2021). These papers model consumption growth dynamics as containing a small, predictable component. In these papers, long-run risks come from the aggregate consumption growth rate, and the economy is governed by two state variables: expected consumption growth and the conditional volatility of consumption growth. They find that expected consumption growth is highly persistent and that long-run risks are difficult to detect. To depart from this literature, I consider new consumption growth dynamics that allow me to study the effects of environmental policies on asset prices and household consumption beyond the usual effects analyzed by the canonical LRR model. I decompose the consumption growth rate into two parts: a carbon growth component, which creates long-run growth risk, and a share of the carbon growth component, which does not create any long-run risk but does affect the dynamics of consumption growth and acts as a hedge against carbon risk. In the previously cited papers, long-run risk comes directly from aggregate consumption growth, which is barely detectable. This decomposition allows me to study the effects of climate change on macro-financial variables such as consumption growth, dividend growth, and the equity premium.

My study is also related to the strand of literature on climate finance. Papers here include Daniel et al. (2016), Bansal et al. (2016b), Bansal et al. (2017), Chen et al. (2019), Giglio et al. (2021), and Stroebel and Wurgler (2021), Avramov et al. (2021). This paper focuses on carbon or transition risk,[2] whereas the previously cited papers study the physical risk[3] side of climate risk. In particular, since climate change is a long-horizon phenomenon, we need to assess it using a long-run risks model by looking at the consumption or household side.

The rest of the paper is organized as followed. Section 1 describes the methodology used to build a new measure of consumption-based carbon risk. Section 2 sets up the theoretical model. Section 3 presents the results and the asset pricing implications of the model, and section 4 concludes.

[2] Carbon or transition risk is that which is inherent to the process of transitioning to a lower-carbon economy. Examples include policy and legal risks, technology risk, market risk, and reputation risk.

[3] Physical risk includes event-driven risks that damage assets and disrupt the supply chain (examples include hurricanes, floods, and fires), and long-term shifts in climate patterns (for example, increasing temperatures or rising sea levels).

1 A measure of consumption-based carbon risk

The central challenge of climate finance is to capture the actual impact of carbon emissions. To address that challenge, this paper uses aggregate consumption data and the carbon footprint to identify a carbon/green risk measure. All data are on an annual basis and span the period 1930–2018. I describe below how I constructed the carbon consumption, green consumption measures to assess the empirical implications of a long-run carbon consumption risks model. Summary statistics are presented in table 1.

To construct the carbon consumption measure, I consider carbon dioxide emissions from a household consumption perspective. These carbon dioxide emissions indicators provide an alternative view of carbon dioxide emissions, where the emissions are tied to the consumption of durable goods, non-durable goods, and services in the United States. This approach allows for accounting for a potential carbon leakage and the actual impact of carbon emissions within a given boundary. In fact, under the assumption of a linear life cycle progression of a product, households stand at the usage stage where they have control of the product. Using NIPA data, [4] I collect aggregate information on 11 consumption categories (food, clothing, housing, furniture, health, transportation, communication, recreation, education, food services and accommodation, and financial services and insurance). Then, I match this aggregate information to the carbon footprint information provided by the Economic Input-Output Life Cycle Assessment (EIO-LCA) database using the purchaser (retail) price model[5]. The Life Cycle Assessment (LCA) investigates, estimates, and evaluates the environmental

[4] NIPA is an abbreviation for the national income and product accounts from the Bureau of Economic Analysis. I use annual aggregate consumption data for US households from the period 1930 to 2018.

[5] The purchaser (retail) price model is a commodity-based model. The purchaser model is designed to adjust for retail to producer prices and thus models the delivery and retailing stages of the supply chain. It also allows for the modeling of commodities as opposed to industrial activity (e.g., a car instead of "automobile production").

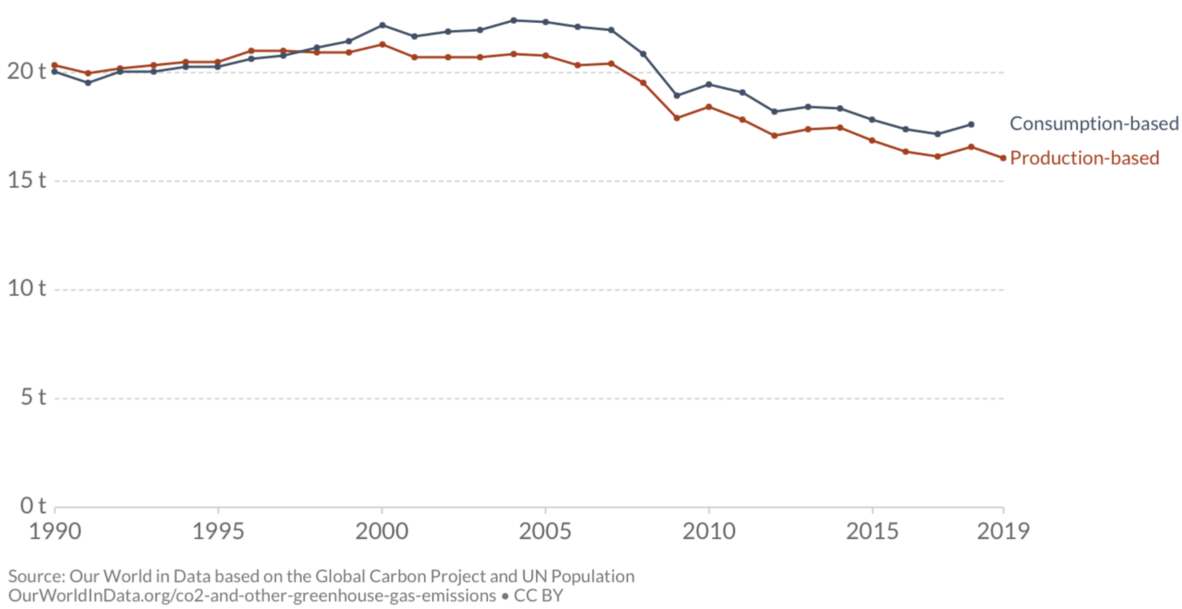
burdens imposed by a material, product, process, or service throughout its life span. Environmental burdens include the materials and energy resources required to create the product, as well as the wastes and emissions generated during the process. The EIO-LCA is developed by Carnegie Mellon University (Institute. (2021)) and provides an estimate of economy-wide cradle-to-gate GHG emissions per dollar of producer output for 428 sectors of the US economy. This paper uses the US 2002 benchmark model - purchaser price[6] to collect the carbon footprint for the 11 household expenditure categories identified in the NIPA data. I identify in the EIO-LCA database a total of 50 sectors representing household consumption good production. A sample of such carbon emissions is given in figure 2 for power generation and supply (electricity), and soft drinks and ice manufacturing. It depicts the direct and indirect emissions related to the purchase of $US 1 million of electricity (top panel) and soft drinks (bottom panel). It amounts to 9,370 and 651 tons of CO2 emissions (t CO2e), respectively. Note that electricity places a higher burden on the environment than soft drinks.

Using these 50 sectors, we covered 96% of total household consumption expenditure, which represents a total of 29 consumption goods out of 44 in the NIPA data table. In figure 3, the x-axis captures the tons of CO2 emissions (tCO2e) per million US dollars. Figure 3 displays the total carbon footprint in each consumption category. As shown, transportation, food, and housing account for a large part of the carbon footprint in household consumption expenditure, representing a total of 77% of US household CO2 emissions. In household consumption baskets, food and beverages contribute the most to the carbon footprint, followed by housing, household utilities, furnishings, recreation, and transportation.

The direct carbon dioxide emissions, which include natural gas, motor oil, and lubricant grass, represent only 11% of the total emissions in the household consumption basket. In this paper, we use both direct and indirect burdens to compute the total

[6] The US 2002 model uses information on the 2002 US economy.

Figure 1: Fact



carbon emissions.

Next, I map the NIPA expenditure category to the carbon footprint information to compute the consumption-based carbon measure. Since the carbon footprint information is related to the 2002 consumer price purchase, all of the NIPA data are deflated using the 2002 reference base period for the consumer price index (CPI). Figure 2 shows that all of the consumption categories do not affect the environment equally. Therefore, this paper weights the aggregate consumption of each good and service by its burden on the atmosphere to compute a new total consumption measure. The total US household consumption-based carbon emissions can be expressed simply as the product of consumption, denoted C, in dollars, and carbon emissions per unit of consumption, denoted CE, summed over each carbon footprint activity (i) included in the model. Put simply, when it comes to analyzing the effect of carbon emissions on the economy and environment, consumption categories should not be treated the same. Each category affects the environment differently , so I compute the aggregate consumption by weighting each category consumption by its footprint. Alternatively, I classify the

Figure 2: Carbon footprint of electricity versus soft drinks

Sector #221100: Power generation and supply
Economic Activity: $1 Million Dollars
Displaying: Greenhouse Gases
Number of Sectors: Top 10

Change Inputs (Click here to view greenhouse gases, air pollutants, etc...)

Documentation:
The sectors of the economy used in this model.
The environmental, energy, and other data used and their sources.

Frequently asked questions about IO-LCA (or EEIO) models.

This EIO-LCA data model was contributed by Green Design Institute.

	Sector	Total t CO2e	CO2 Fossil t CO2e	CO2 Process t CO2e	CH4 t CO2e	N2O t CO2e	HFC/PFCs t CO2e
	Total for all sectors	*9370*	*8880*	*31.3*	*346.*	*56.3*	*57.5*
221100	Power generation and supply	8820	8690	0.000	23.9	54.0	55.9
212100	Coal mining	230	25.9	0.000	204.0	0.000	0.000
211000	Oil and gas extraction	129.0	36.3	23.6	69.0	0.000	0.000
486000	Pipeline transportation	67.1	30.7	0.084	36.3	0.000	0.000
482000	Rail transportation	25.9	25.9	0.000	0.000	0.000	0.000
324110	Petroleum refineries	19.8	19.8	0.000	0.061	0.000	0.000
484000	Truck transportation	9.17	9.17	0.000	0.000	0.000	0.000
230301	Nonresidential maintenance and repair	8.77	8.77	0.000	0.000	0.000	0.000
331110	Iron and steel mills	7.54	2.85	4.65	0.046	0.000	0.000
221200	Natural gas distribution	7.28	0.658	0.000	6.63	0.000	0.000

(a) Power generation and supply

Sector #312110: Soft drink and ice manufacturing
Economic Activity: $1 Million Dollars
Displaying: Greenhouse Gases
Number of Sectors: Top 10

Change Inputs (Click here to view greenhouse gases, air pollutants, etc...)

Documentation:
The sectors of the economy used in this model.
The environmental, energy, and other data used and their sources.

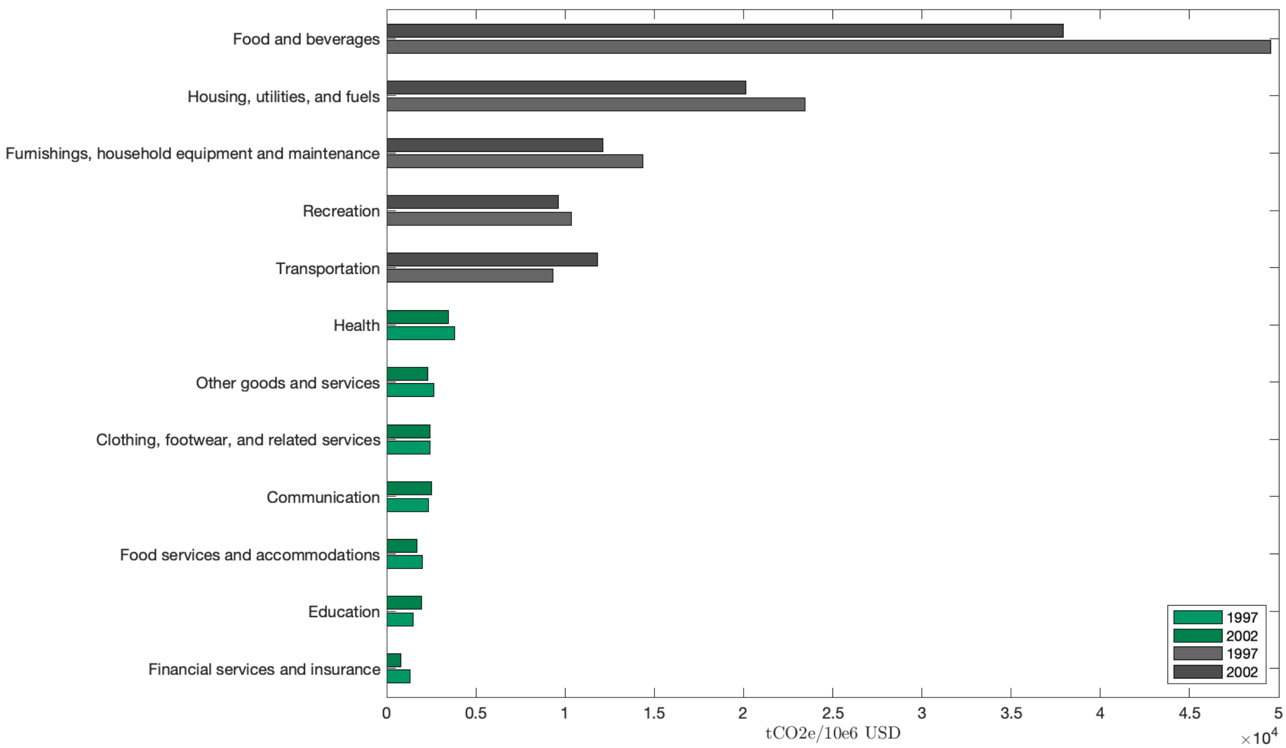
Frequently asked questions about IO-LCA (or EEIO) models.

This EIO-LCA data model was contributed by Green Design Institute.

	Sector	Total t CO2e	CO2 Fossil t CO2e	CO2 Process t CO2e	CH4 t CO2e	N2O t CO2e	HFC/PFCs t CO2e
	Total for all sectors	*651.*	*513.*	*30.6*	*50.9*	*37.3*	*19.4*
221100	Power generation and supply	234.0	230	0.000	0.633	1.43	1.48
311221	Wet corn milling	33.1	33.1	0.000	0.000	0.000	0.000
33131A	Alumina refining and primary aluminum production	31.7	7.18	11.2	0.000	0.000	13.2
484000	Truck transportation	30.7	30.7	0.000	0.000	0.000	0.000
1111B0	Grain farming	26.6	3.92	0.000	2.17	20.5	0.000
211000	Oil and gas extraction	26.4	7.44	4.84	14.1	0.000	0.000
312110	Soft drink and ice manufacturing	20.4	20.4	0.000	0.000	0.000	0.000
325190	Other basic organic chemical manufacturing	20.0	18.0	0.000	0.000	2.06	0.000
324110	Petroleum refineries	15.0	15.0	0.000	0.047	0.000	0.000
33131B	Aluminum product manufacturing from purchased aluminum	14.6	14.6	0.000	0.000	0.000	0.000

(b) Soft drinks and ice manufacturing

Figure 3: **Carbon footprint by household expenditure category**.



Note: The x-axis captures the tons of CO2 emissions (tCO2e) per million US dollars

carbon footprint in decreasing order and use the five categories with the highest carbon footprints to compute what I call "carbon consumption" and whatever is left over to compute "green consumption." Overall, the total carbon emissions at any time t are calculated as follows:

$$TC_t = \sum_{i=1}^{11} C_{i,t} * CE_i \tag{1}$$

However, I subdivide all of the consumption categories into two parts in order to separate the usual consumption risk into two risks. The first risk measures the carbon consumption risk—including the consumption categories that pollute the most based on their carbon footprint; see equation 2). The second risk measures the green consumption risk—including the consumption categories that pollute the least based on their carbon footprint; see equation 3). Henceforth, I will call the risk associated with green consumption "*green risk*" and the risk associated with carbon consumption "*carbon risk*".

$$\boldsymbol{CC_t} = \sum_{\boldsymbol{i=1}}^{\boldsymbol{5}} \boldsymbol{C_{i,t} * CF_i} \quad or \quad \sum_{\boldsymbol{i=1}}^{\boldsymbol{5}} \boldsymbol{C_{i,t}} \tag{2}$$

$$\boldsymbol{GC_t} = \sum_{\boldsymbol{i=6}}^{\boldsymbol{11}} \boldsymbol{C_{i,t} * CF_i} \quad \boldsymbol{or} \quad \sum_{\boldsymbol{i=6}}^{\boldsymbol{11}} \boldsymbol{C_{i,t}} \tag{3}$$

The pattern of those components is shown in figure 1 in terms of log difference:

$$\Delta cc_{t+1} = log\left(\frac{CC_{t+1}}{CC_t}\right) \tag{4}$$

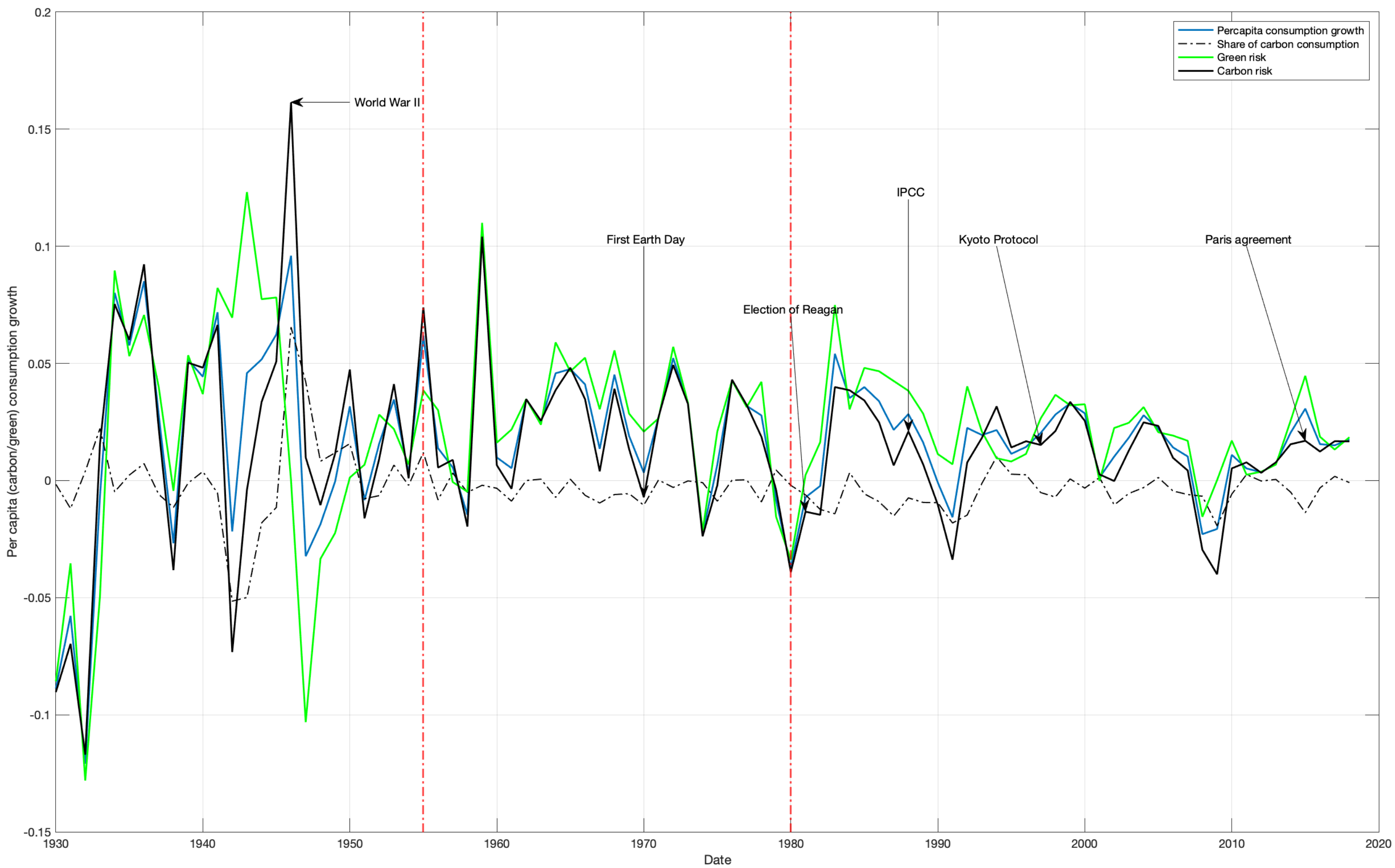
$$\Delta gc_{t+1} = log\left(\frac{GC_{t+1}}{GC_t}\right) \tag{5}$$

$$\Delta \alpha_{cc,t+1} = log\left(\frac{\alpha_{cc,t+1}}{\alpha_{cc,t}}\right) \tag{6}$$

where $\alpha_{cc,t} = \dfrac{CC_t}{C_t}$. I call Δcc_{t+1} carbon consumption growth risk, Δgc_{t+1} green consumption growth risk, and $\Delta \alpha_{cc,t}$ the share of carbon consumption growth risk. I could have defined the share of green consumption growth risk ($\Delta \alpha_{gc,t}$) in the same manner.

Figure 4 displays the time series of the key variables of this paper. One can clearly see that the series replicate some business cycles and climate change events.

Figure 4: **Carbon risk, green risk, and household expenditure growth**



To link the real economy to the financial market, I also use data on industry, small, large, value, and growth portfolio returns from Kenneth R. French's website. I use value-weighted portfolios including and excluding dividends to compute the dividend and price series on a per-share basis (Campbell and Shiller (1988), Hansen et al. (2008)). Table 1 presents some descriptive statistics. All returns and dividend growth series have been deflated using CPI growth.

Table 1: Summary statistics

	$E(.)$	$\sigma(.)$	AC(1)	AC(2)	AC(3)	AC(4)	AC(5)
			Macro variables				
Δc	0.0178	0.0343	0.3150	0.0608	-0.1508	-0.1491	0.0098
$\Delta \alpha_{cc}$	-0.0030	0.0134	0.4545	0.0574	-0.1233	-0.2765	-0.1730
Δcc	0.0147	0.0382	0.2717	-0.0223	-0.2057	-0.1549	0.0176
$\Delta \alpha_{gc}$	0.0042	0.0215	0.4469	0.0640	-0.0749	-0.2692	-0.2105
Δgc	0.0220	0.0385	0.4647	0.2063	-0.0167	-0.2016	-0.1006
			Financial variables				
Δd	0.0176	0.1223	0.1075	-0.1832	-0.1502	-0.0930	0.0459
z_m	3.3878	0.5123	0.9276	0.8524	0.7992	0.7605	0.7163
r_m	0.0694	0.1929	0.0077	-0.2202	0.0181	-0.0053	-0.1215
r_f	0.0025	0.0351	0.6852	0.3059	0.2040	0.2336	0.2788

The table reports the sample mean, standard deviation, and first-order to fifth-order autocorrelation of the marketwide log price-dividend ratio, the log dividend, consumption, and the (share of) carbon/green consumption growth rates.

2 Model

The model builds on the Bansal and Yaron (2004) LRR model and uses insight from Giglio et al. (2021). My model introduces three state variables: a long-run risk variable, the variance of the innovation of carbon consumption growth alongside the growth rate of the share of carbon consumption out of total consumption that jointly drive the conditional mean of aggregate consumption growth, and dividend growth.

2.1 Preferences

In this economy, there is a representative household with recursive preferences (Kreps and Porteus (1978), Epstein and Zin (1989)). This paper chooses these types of preferences for two main reasons: First, they allow for separation between the coefficient of risk aversion and the elasticity of intertemporal substitution. Second, an Epstein–Zin (EZ) investor's marginal utility depends on both the one-period innovation in the consumption growth rate and news about consumption growth at future horizons. This feature is important for the climate change thematic as news about future global warming will affect consumers' consumption behaviors. Hence, consumption growth will incur a proportional shock. One would like a utility function specification that affects the level of the climate risk premium and the term structure of the discount rate. Epstein-Zin utility specified in equation 7 does what I just described:

$$V_t = \left[(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta\left(E_t[V_{t+1}]^{1-\gamma}\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}} \tag{7}$$

where δ is the subjective discount factor parameter, $\gamma > 0$ is the coefficient of risk aversion, and $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$ with $\psi > 0$ represents the elasticity of intertemporal substitution (EIS). The standard time-separable power utility model is a special case of the EZ utility when $\gamma = \frac{1}{\psi}$. The agent prefers early resolution of the risk if $\gamma > \frac{1}{\psi}$ and late resolution if $\gamma < \frac{1}{\psi}$.

In this formulation, the household evaluates her consumption plan recursively. She consumes at time t and receives a continuation value of her consumption, which can bear a long-run risk component through its carbon consumption. Indeed, with a canonical expected utility risk, only short-run risks are compensated, whereas long-run risks do not carry a separate risk premium. With the above preference, long-run risks earn a positive risk premium if households prefer an early resolution of uncertainty.

Furthermore, there are N+1 tradable assets in the economy: one risk-free asset ($i = 0$) and N risky assets ($i = 1, \ldots, N$). In each period t, the representative household invests X_{it} unit of its discretionary wealth in asset i. The tradable asset i has a price of P_{it} and a future dividend of D_{it}, with a gross return of $R_{it+1} = \frac{D_{it+1}+P_{it+1}}{P_{it}}$. The intertemporal budget constraint faced by the household is given by

$$C_t + \sum_{i=1}^{N+1} P_{it} X_{i,t+1} = \sum_{i=1}^{N+1} (P_{it} + D_{it}) X_{it} = W_t \tag{8}$$

where

$$C_t = CC_t + GC_t$$

is total consumption and the sum of the consumption considered as carbon consumption (CC_t) and the consumption considered as green consumption (GC_t).

2.2 A long-run carbon consumption risks (LRCCR) model

This paper assumes that consumption growth in the economy depends on two components. One carries a long-run risk, and the other does not carry any long-run risks. In particular, we assume that aggregate consumption growth is given by (the proof can be found in appendix A):

$$\Delta c_{t+1} = \underbrace{\Delta cc_{t+1}}_{Carbon} - \underbrace{\Delta \alpha_{cc,t+1}}_{share\ of\ cc} \tag{9}$$

where $\Delta c_{t+1} = log\left(\frac{C_{t+1}}{C_t}\right)$ is the log consumption growth rate. The expression Δcc_{t+1} is the growth rate of carbon consumption and $\Delta\alpha_{cc,t+1}$ is the growth rate of the share of carbon consumption in total consumption as defined in section 1, equations 4–6. Note that the conditional mean of cc_{t+1} and its conditional volatility are a source of carbon consumption risk. In fact, the transition to a low-carbon economy raises the future likelihood of carbon consumption risk, which, if realized, leads to a consumption risk. For instance, the nationally determined contributions (NDC) policy scenario aims to reduce carbon consumption by 32(15) gigatons of CO2 to stay within the 1.5°C(2°C) limit by 2030. The dynamics of the other variables are described as follows:

$$\Delta cc_{t+1} = \nu_{cc} + x_t + \sigma_t \epsilon_{cc,t+1} \tag{10}$$

$$x_{t+1} = \rho_x x_t + \psi_x \sigma_t \epsilon_{x,t+1} \tag{11}$$

$$\sigma^2_{t+1} = (1-\nu)\sigma^2 + \nu\sigma^2_t + \sigma_w \epsilon_{\sigma,t+1} \tag{12}$$

Thus, in our model x_t, $\Delta\alpha_{cc,t}$, and σ^2_t are the state variables. In particular, x_t captures the conditional mean of the carbon consumption growth rate, while σ^2_t captures the uncertainty associated with the transition to a lower-carbon economy. The growth of the share of carbon consumption out of total consumption component doesn't carry any long-run risk and evolves, as given by

$$\Delta\alpha_{cc,t+1} = \nu_\alpha(1-\rho_\alpha) + \rho_\alpha \Delta\alpha_{cc,t} + \sigma_\alpha \epsilon_{\alpha,t+1} + \pi\sigma_t\epsilon_{cc,t+1} \tag{13}$$

This paper assumes that innovations in the share of carbon consumption out of total consumption and carbon consumption are correlated. That correlation depends on the parameters π[7] and σ_α. Finally, the dividend growth rate of any dividend-paying asset i is as follows:

$$\Delta d_{i,t+1} = \nu_i + \phi_i x_t + \phi_{\alpha,i}\Delta\alpha_{cc,t} + \psi_i \sigma_t \epsilon_{i,t+1} \tag{14}$$

[7] Alternative specification : $\epsilon_{\alpha,t}$ and $\epsilon_{cc,t}$ are correlated instead of i.i.d. and set $\pi = 0$

where ϕ_i, $\phi_{\alpha,i}$, ψ_i determine asset i's exposure to the long-run share of carbon consumption and volatility risks, respectively. The shocks $\epsilon_{x,t+1}$, $\epsilon_{\alpha,t+1}$, $\epsilon_{i,t+1}$, $\epsilon_{cc,t+1}$, and $\epsilon_{\sigma,t+1}$ are assumed to be i.i.d. N(0,1) and mutually independent. Equations (9)–(14) represent the building blocks of our long-run carbon consumption risks model, henceforth LRCCR model. The dynamics of the variables and the utility function involve 17 parameters Θ=[ρ_x ψ_x ψ_i ν_{cc} ν ν_i σ_w σ ϕ_i δ γ ψ ν_α ρ_α σ_α π $\phi_{\alpha,i}$]. I calibrate the model parameters to match key sample moments. I derive some moments condition for carbon consumption, the share of carbon consumption, and asset i dividend growth rates as functions of the time series and the preferences parameters. See the appendices section 4 for more details.

2.3 Solving the model

For any asset i, the corresponding Euler equation regarding the consumer's utility maximization is given by

$$\mathbb{E}_t[e^{m_{t+1}+r_{i,t+1}}] = 1 \tag{15}$$

where

$$m_{t+1} = \theta log(\delta) - \frac{\theta}{\psi}\Delta cc_{t+1} + \frac{\theta}{\psi}\Delta\alpha_{cc,t+1} + (\theta-1)r_{c,t+1} \tag{16}$$

is the natural logarithm of the stochastic discount factor; $\mathbb{E}_t[.]$ denotes expectation conditional on time t information; $r_{i,t+1}$ is the continuously compounded return on asset i; and $r_{c,t+1}$ is the unobservable continuously compounded return on an asset that delivers aggregate consumption as its dividend each period.

Following Campbell and Shiller (1988), the log return on the consumption claim, namely $r_{c,t+1}$, and the log return of the asset i $r_{i,t+1}$ are approximated as follows:

$$r_{c,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + \Delta cc_{t+1} - \Delta\alpha_{cc,t+1} \tag{17}$$

$$r_{i,t+1} = \kappa_{0,i} + \kappa_{1,i} z_{i,t+1} - z_{i,t} + \Delta d_{i,t+1} \tag{18}$$

where $z_t = \log\left(\frac{P_{m,t}}{C_t}\right)$ and $P_{m,t}$ stands for the market portfolio price, $z_{i,t} = \log\left(\frac{P_{i,t}}{D_{i,t}}\right)$. $\kappa_1 = \frac{e^{\bar{z}}}{1+e^{\bar{z}}}$ and $\kappa_0 = \log\left(1 + e^{\bar{z}}\right) - \kappa_1 \bar{z}$ are log-linearization constants. The term $\bar{z}$ denotes the long-run mean of the log price-consumption ratio (z). Regarding equation (18), $\kappa_{1,i} = \frac{e^{\bar{z}_i}}{1+e^{\bar{z}_i}}$ and $\kappa_{0,i} = \log(1 + e^{\bar{z}_i}) - \kappa_1 \bar{z}_i$ where $\bar{z}_i$ denotes the long-run mean of the log price-dividend ratio (z_i). Throughout this paper, subscript m refers to the market portfolio, and subscript i refers to any asset.

As in Bansal and Yaron (2004), I conjecture that z_t and $z_{i,t}$ are affine functions of the state variables x_t (LRR variable or conditional expected carbon consumption growth), σ_t^2 (conditional volatility of the carbon consumption growth), and $\Delta\alpha_{cc,t}$ (share of carbon consumption growth):

$$z_t = A_0 + A_1 x_t + A_2 \sigma_t^2 + A_3 \Delta\alpha_{cc,t} \tag{19}$$

$$z_{i,t} = A_{0,i} + A_{1,i} x_t + A_{2,i} \sigma_t^2 + A_{3,i} \Delta\alpha_{cc,t} \tag{20}$$

The functions A_0, A_1, A_2, A_3, $A_{0,i}$, $A_{1,i}$, $A_{2,i}$, and $A_{3,i}$ are functions of parameters in Θ and the linearization parameters. Their expressions are given in the appendix. An increase in the expected carbon consumption growth rate will raise the price-consumption ratio if the intertemporal substitution effect dominates the wealth effect. However, a higher share of carbon consumption out of total consumption implies a lower price-consumption ratio when $\psi > 1$. Turning now to the price-dividend ratio, we see that the conclusions are different for the share of the carbon consumption growth effect. While the expected carbon consumption growth measure still raises the price-dividend ratio but is much higher under the conditions that $\psi > 1$ and $\phi_i > 1$ (the LRR variable acts as a leverage), the share of carbon consumption now positively affects the price-dividend ratio, hypothesizing $\phi_{\alpha,i} > 0$.

Using equation 15, I show that the log risk-free rate can be written as a function of the carbon consumption measure and its volatility (state variables) as follows:

$$\begin{aligned} r_{f,t} &= -log\mathbb{E}_t[e^{m_{t+1}}] \\ &= A_{0,f} + A_{1,f}x_t + A_{2,f}\sigma_t^2 + A_{3,f}\Delta\alpha_{cc,t} \end{aligned} \tag{21}$$

Once again $A_{0,f}$, $A_{1,f}$, $A_{2,f}$, $A_{3,f}$ are functions of parameters in Θ and the linearization parameters, and their expressions are given in the appendix.

2.4 Asset pricing implications

To test the implications of the model for the equity premium and the cross section of returns, I combine equations (16), (17), and (19) to get the expression of the stochastic discount factor in terms of state variables:

$$\begin{aligned} m_{t+1} = &(\theta log(\delta) + (\theta-1)[\kappa_0 + (\kappa_1 - 1)A_0]) + \left(-\frac{\theta}{\psi} + (\theta-1)\right)\Delta cc_{t+1} \\ &+ \left(\frac{\theta}{\psi} - (\theta-1) + (\theta-1)\kappa_1 A_3\right)\Delta\alpha_{cc,t+1} \\ &+ (\theta-1)\kappa_1 A_1 x_{t+1} + (\theta-1)\kappa_1 A_2 \sigma_{t+1}^2 \\ &- (\theta-1)A_1 x_t - (\theta-1)A_2\sigma_t^2 - (\theta-1)A_3\Delta\alpha_{cc,t} \end{aligned} \tag{22}$$

The innovation in the m_{t+1} conditional on time-t information is given by

$$m_{t+1} - \mathbb{E}_t[m_{t+1}] = -\lambda_{m,\alpha}\sigma_\alpha\epsilon_{\alpha,t+1} - \lambda_{m,cc}\sigma_t\epsilon_{cc,t+1} - \lambda_{m,x}\sigma_t\epsilon_{x,t+1} - \lambda_{m,w}\sigma_w\epsilon_{\sigma,t+1} \tag{23}$$

Therefore, the equity risk premium for any asset i is

$$\mathbb{E}_t(r_{i,t+1} - r_{f,t}) + 0.5\mathbb{V}_t(r_{i,t+1}) = \lambda_{m,x}\beta_{i,x}\sigma_t^2 + \lambda_{m,w}\beta_{i,w}\sigma_w^2 + \lambda_{m,\alpha}\beta_{i,\alpha}\sigma_\alpha^2 + \lambda_{m,cc}\beta_{i,cc}\sigma_t^2 \tag{24}$$

where the β's are asset i exposure to the long-run risk, the volatility risk, the share of the carbon consumption risk, and the short-run risk, and the λ's are the respective risk prices. The β's and λ's are functions of the preference parameters, the linearization parameters, and the dynamics of the parameters of the macro-financial variables (see appendix 4). The price of the short-run carbon consumption risk and the exposure of any asset to this risk rise with the correlation between the share of carbon growth and the carbon growth. The XXX $\lambda_{m,x}$, and $\beta_{i,x}$ increase with the persistence of the expected carbon growth. In the same way, $\lambda_{m,cc}$, $\lambda_{m,\alpha}$, $\beta_{i,cc}$, and $\beta_{i,\alpha}$ increase with the persistence of the share of carbon consumption out of total consumption growth.

I substitute equation (22) into the set of Euler equations (15) to have moment conditions that are expressed entirely in terms of observables. Then I examine the empirical plausibility of the model when the set of assets in the economy consists of the market portfolio and the risk-free rate, thereby focusing on the equity premium and risk-free rate puzzles. In particular, I consider a set of moments, namely, the expected value and the standard deviation of the equity premium, the real risk-free rate, and the price-dividend ratio, and I calibrate the parameters Θ to match those moments.

Next, this paper examines whether the model can explain the cross section of returns in different asset classes including "carbon-intensive" (high heat-exposed and low heat-exposed) portfolios and Fama-French 25 portfolios. In total, I use the 42 Fama-French industry portfolios and 25 Fama-French portfolios. I adopt the two-pass regression methodology of Fama and MacBeth (1973) to estimate the risk premia on each risk factor (see also Kan et al. (2013), Bai and Zhou (2015)). I consider the two risks I built, namely, the carbon consumption (cc) growth risk (Δcc_t) and the share of carbon consumption (shcc) growth risk ($\Delta\alpha_{cc,t}$). In the first stage, I compute the portfolio's exposures to the risk factors by regressing each portfolio's excess return ($r_{i,t}$) on Δcc_t and $\Delta\alpha_t$:

$$r_{i,t} = c_i + \beta_{sh,i}\Delta\alpha_{cc,t} + \beta_{cc,i}\Delta cc_t + \epsilon_{i,t} \quad i = 1, ..., N \tag{25}$$

In the second stage, I run a cross-sectional regression of $r_{i,t}$ on the betas from the first-stage time-series regression for each period t (see equation 27):

$$r_{i,t} = a_t + \gamma_{sh,t}\hat{\hat{\beta}}_{sh,i} + \gamma_{cc,t}\hat{\hat{\beta}}_{cc,i} + \epsilon_{i,t} \quad t = 1, ..., T \tag{26}$$

Rather than running a T cross-sectional regression (CSR), I run a single CSR of the sample excess return mean μ_r a constant and the betas estimated from the first stage. Put in equation terms, this gives the following:

$$\mu_{r,i} = \gamma_0 + \gamma_{sh}\hat{\beta}_{sh,i} + \gamma_{cc}\hat{\beta}_{cc,i} + \epsilon_i \quad i = 1, ..., N \tag{27}$$

For comparison purposes, this paper also applies the two-pass regression to the case of one consumption growth risk factor. I plot the exposures $\beta_{sh,i}$'s and $\beta_{cc,i}$'s in Figure 6. Once we know which portfolio (industry) is significantly positively and negatively exposed to the carbon risk, I calibrate the LRCCR model to match key moments of those portfolios (industries). The main goal is to explain the cross section of the portfolios' (industries') expected returns. My parameters of interest are the leverage parameters—that is, the dividend exposure to the long-run risk variable and to the share of the carbon consumption growth rate for each portfolio (industry) ϕ_i and $\phi_{\alpha,i}$—and the dividend exposure to volatility risks ψ_i. To test the effectiveness of the LRCCR model, I start by looking at the cross-sectional properties of the well-known portfolios, in particular, the value, growth, small size, and large portfolios. Based on empirical evidence (see Bansal et al. (2005) and Hansen et al. (2008), Bansal et al. (2016a)), the value portfolio presents a much higher exposure to low-frequency risks in consumption relative to the growth portfolio. Likewise, the long-run risk exposure of the small-size portfolio exceeds that of the large-size portfolio. Then, I look at the cross-sectional properties of the Fama-French industry portfolios.

3 Model estimation and comparative statistics

3.1 Equity premium, volatility, and the risk-free rate: BY2004 versus LRCCR

This part of the paper compares the LRCCR and LRR models in terms of replicating the observed equity premium, the volatility of the equity premium, and the risk-free rate. Here I calibrate the 17 parameters Θ=[ρ_x ψ_x ψ_d ν_{cc} ν ν_d σ_w σ ϕ_d δ γ ψ ν_α ρ_α σ_α π $\phi_{\alpha,d}$] to match the (share of) carbon consumption growth, the (share of) green consumption growth, dividend growth, the market return, and the risk-free rate means, variances, and (auto)correlations observed in the data. The calibration results are displayed in table 2 for both my setting and Bansal and Yaron (2004)'s setting (BY model). I present four sets of results: the full sample, the period around World War II, the period around the First Earth Day, and finally the post-Reagan election sample. This paper splits the results into four sets because returns react to news, and in earlier times, climate change or global warming was not a prominent issue. Therefore, we hypothesize that there is probably no big effect during the pre-Reagan election period. The paper uses President Reagan's election as a reference day because global public awareness of energy conservation and improvements in energy efficiency start around this time period.

The term Ψ_x tells us how detectable the long-run variable is. The results show that the long-run risk variable is more detectable than it is in the BY model during the 1956–2018 period, which is near the climate change events. The results in the table 2 show that there are long-run risks in volatility and carbon consumption growth: ν smaller and close to one, and ρ_x smaller and close to one. Overall, the risk aversion in our model is higher than the one in Bansal and Yaron (2004)'s setting but is within a reasonable range. This high value is related to the nature of the risk discussed in this paper (carbon risk). Furthermore, agents are more fearful of carbon risk than consumption risk because carbon risk will increase (amplify) consumption risk even more. The expression $\phi_{\alpha,d}$ functions as a leverage ratio on the share of carbon consumption growth during the period 1981–2018.

Table 2: Calibrated parameters

	1930-2018		1930-1955		1956-1980		1981-2018	
	BY04	LRCCR	BY04	LRCCR	BY04	LRCCR	BY04	LRCCR
ρ_x	0.932	0.978	0.937	0.979	0.920	0.900	0.976	0.900
ψ_x	0.259	0.150	0.278	0.119	0.010	0.204	0.206	0.514
ψ_d	4.540	4.340	4.789	4.488	13.361	0.000	10.122	4.288
ν_x	9E-04	1E-03	1E-04	1E-03	-6E-05	2E-03	-2E-04	9E-04
ν	0.999	0.979	0.573	0.985	0.577	0.691	0.988	0.995
ν_d	0.001	-0.011	0.000	-0.025	-0.002	0.001	0.005	0.003
σ_w	5E-07	2E-08	1E-04	2E-07	7E-08	1E-05	4E-06	4E-06
σ	8E-03	3E-03	5E-04	4E-03	1E-03	1E-03	7E-04	9E-03
ϕ	2.294	3.378	2.354	3.734	321.850	10.056	0.792	1.019
δ	0.956	0.998	0.999	0.999	0.998	0.998	0.998	0.997
γ	7.074	12.290	9.878	10.084	15.940	23.016	6.063	8.732
ψ	1.379	1.487	3.018	1.495	1.574	1.235	1.503	1.486
$\bar{z}$	3.088	6.164	6.054	6.602	6.201	6.285	5.720	5.060
$\bar{z}_m$	5.344	3.981	5.153	3.522	4.754	5.696	12.820	5.548
ν_a		-3E-04		4E-05		-3E-04		-4E-04
ρ_a		0.455		0.480		-0.281		0.360
σ_a		0.006		0.006		0.014		0.004
π		1.344		0.897		3.328		0.626
ϕ_a		0.590		0.877		-0.294		1.305

The table reports the calibrated parameters for the different subsamples for both our setting (LRCCR) and the Bansal and Yaron (2004) setting (BY04).

This paper simulates the time series of the model-implied carbon consumption growth, the share of carbon consumption growth, dividend growth, market return, and the risk-free rate. I present some quantiles of those series in tables 3 and 9 for the four samples. The quantiles 5% and 95% serve as the confident interval, and overall, the sample moments are within those intervals generated by our simulation in the preferred subsamples.

Now let us turn to the predictability implication of my model versus the one of Bansal and Yaron (2004) by comparing the predicted equity premium, consumption growth, and dividend growth rates and their realized counterparts. Most of the consumption capital asset pricing models find a constant risk premium: approximately constant predicted risk premium. However, during 1980–2018, a period of high carbon emissions risk, a long-run carbon consumption risks model finds a time-varying risk premium. My model performs much better than the usual long-run risk model. (See Figure 5 and figure 7 in the appendix.) The difference is quite clear when I predict the macro-financial variables in the subsample, especially during the period 1981–2018, a period starting around the election of President Reagan, at time when climate actions began.

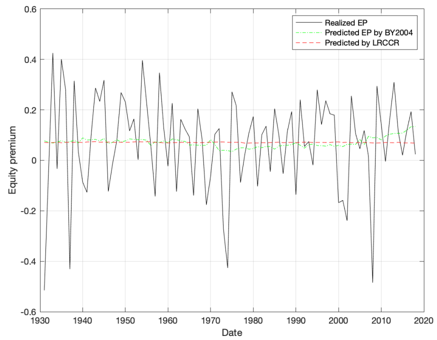
This paper conducts comparative statistics by simulating the model following two policies : (i) reduce carbon consumption growth by one standard deviation, (ii) reduce carbon consumption growth by one standard deviation, combined with a decrease in the growth of the share of carbon consumption by one standard deviation. As a result, the first policy leads to a decrease in consumption growth by 27.12%, and the second policy leads to a decrease in consumption growth by 26.8%. These findings mean that a policy that aims to reduce the consumption of goods and services that contribute the most to environmental pollution without also encouraging green consumption is bad in terms of the impact on household consumption. Further analysis over different subperiods shows that the impacts of environmental policies on asset prices and household consumption are bigger during periods of high climate change uncertainty (see table 4). In fact, the decrease in consumption growth is approximately 9.5%, 0.55%, and 23% during the periods 1930–1955, 1956–1980, and 1981–2018, respectively.

Table 3: Model-implied moments.

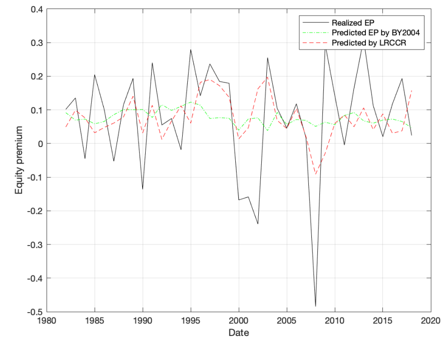
		$\sigma(z_m)$	EP	$E(R_f)$	$\sigma(r_{m,a})$	$\sigma(r_{f,a})$	$\rho(z_m)$
1930-2018							
	Data	0.512	0.067	0.002	0.193	0.002	0.928
BY2004	Mean	0.189	0.096	0.007	0.212	0.068	0.454
	5%	0.158	-0.029	-0.090	0.180	0.050	0.286
	50%	0.188	0.097	0.005	0.212	0.065	0.458
	95%	0.225	0.220	0.111	0.246	0.095	0.607
LRCCR	Mean	0.197	0.078	0.010	0.134	0.022	0.735
	5%	0.154	0.053	0.001	0.118	0.019	0.601
	50%	0.195	0.078	0.010	0.134	0.022	0.743
	95%	0.248	0.104	0.019	0.151	0.027	0.841
1981-2018							
	Data	0.415	0.072	0.011	0.162	0.011	0.890
BY2004	Mean	0.078	0.066	0.010	0.128	0.002	0.846
	5%	0.042	0.028	0.005	0.103	0.001	0.642
	50%	0.073	0.066	0.010	0.127	0.002	0.871
	95%	0.133	0.106	0.015	0.154	0.004	0.961
LRCCR	Mean	0.204	0.118	0.015	0.183	0.006	0.809
	5%	0.118	0.061	0.003	0.148	0.003	0.586
	50%	0.191	0.117	0.015	0.182	0.005	0.835
	95%	0.330	0.178	0.027	0.220	0.009	0.946

The table reports the model-implied moments (the equity premium (EP), the mean of the risk-free rate, the standard deviations of the log price-dividend ratio, the market return, and the risk-free rate, and the first-order autocorrelation of the log price-dividend ratio), alongside some-20 quantiles.

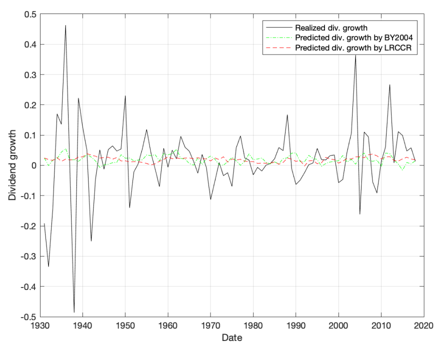
Figure 5: **Realized versus predicted equity premium, consumption growth, and dividend growth.** In this figure,



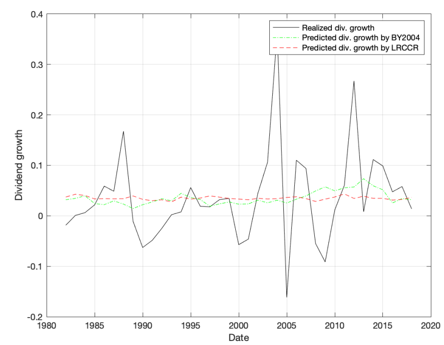
(a) Full sample: 1930-2018



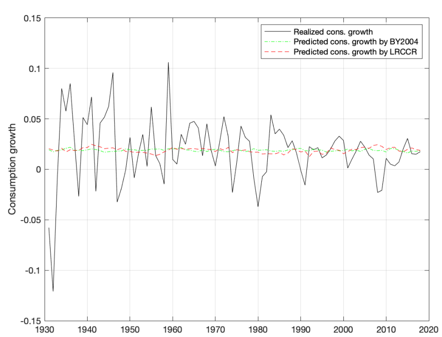
(b) 1981-2018



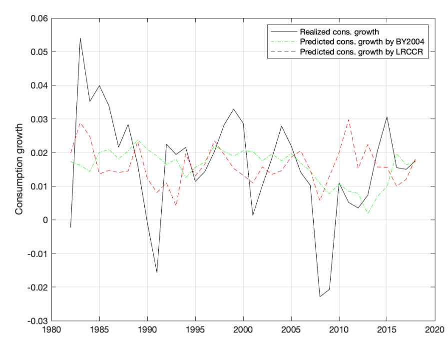
(c) Full sample: 1930-2018



(d) 1981-2018



(e) Full sample: 1930-2018



(f) 1981-2018

Table 4: Comparative statistics

	Δc	Δd	Δcc
Without controlling for the share of carbon consumption			
1930-2018	-27.12	-39.34	-32.72
1930-1955	-9.74	2.03	-9.48
1956-1980	-0.58	-7.87	-0.69
1981-2018	-23.26	-11.40	-34.83
Controlling for the share of carbon consumption			
1930-2018	-26.79	-39.93	-32.72
1930-1955	-9.48	2.06	-9.48
1956-1980	-0.56	-7.89	-0.69
1981-2018	-23.14	-11.54	-34.83

3.2 Risks and Price of risks

In this section, I compute the price of the four risk sources discussed in the model section: carbon consumption growth risk, the share of carbon consumption growth risk—which is correlated with the share of green consumption growth risk–long-run risk, and volatility risk. The most important result from table 5 is the consistent sign of the contribution of volatility risk in the equity premium under my model. The market is negatively exposed to volatility risk in every sample I considered.

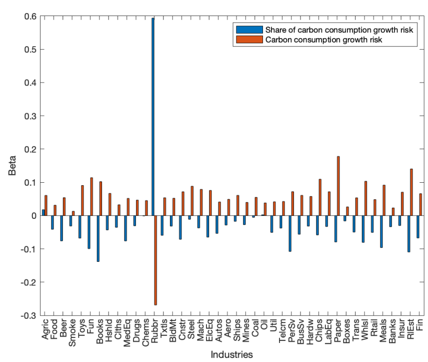
Table 5: Market prices of risks and effects on the risk premium

		λ	β	effect	λ	β	effect
		1930-2018			1930-1955		
BY04	srr	7.07	0.00	0	9.88	0.00	0
	lrr	14.47	5.58	$+$	40.61	8.17	$+$
	vr	-2564.24	-7723.52	$+$	-1884.32	-515.00	$+$
LRCCR	$\alpha risk$	-21.93	1.59	$-$	-18.75	2.18	$-$
	srr	-17.19	2.13	$-$	-6.74	1.96	$-$
	lrr	73.78	10.16	$+$	50.61	7.28	$+$
	vr	-122914.44	-14400.98	$+$	-79112.00	-6155.20	$+$
		1956-1980			1981-2018		
BY04	srr	15.94	0.00	0	6.06	0.00	0
	lrr	1.92	37.37	$+$	40.87	1.09	$+$
	vr	-273.13	1687.23	$-$	-51947.16	5284.64	$-$
LRCCR	$\alpha risk$	-18.15	-0.41	$+$	-13.22	2.40	$-$
	srr	-37.37	-1.35	$+$	0.46	1.50	$+$
	lrr	44.47	18.41	$+$	38.93	2.04	$+$
	vr	-5168.90	-1975.90	$+$	-66622.82	-4374.95	$+$

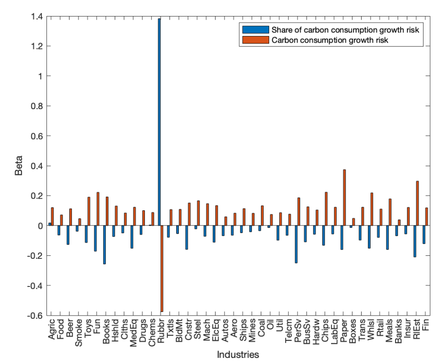
3.3 Cross-sectional implications

As evident in figure 6, I cannot classify the industries based on their betas until 1955. All of the industries are positively and negatively exposed to the carbon consumption growth risk and the share of the carbon consumption growth risk, respectively. One exception is the rubber and plastic products industry, which is negatively and positively exposed to the carbon consumption growth risk and the share of the carbon consumption growth risk, respectively. Starting in 1956, the risk factors start to affect the industries differently. This result is interesting because it tells us that our risk factors are eventually able to identify industries that pollute the most based on their levels of risk exposure at a time when it matters the most.

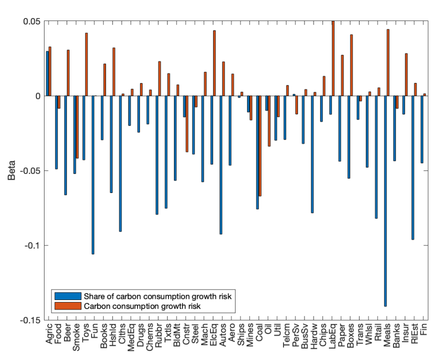
Figure 6: Exposure of industries to carbon consumption risks: β's



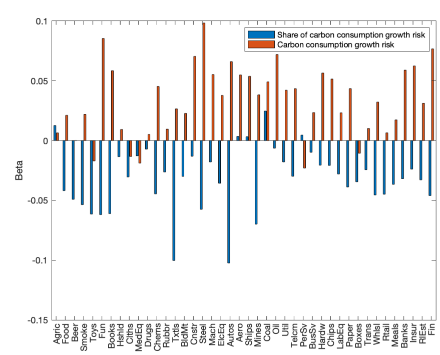
(a) Full sample



(b) 1930-1955



(c) 1956-1980



(d) 1981-2018

4 Conclusion

This article tackles the long-run carbon consumption risks model by allowing both long-run risks in both mean and volatility. We use an Epstein-Zin utility function to disentangle the risk aversion coefficient and because of its ability to deal with the climate change thematic. This paper finds empirical support for the long-run risks model in the context of carbon-green consumption. Three state variables completely define the other variables in the economy. To sum up, our long-run carbon consumption model solves the equity premium, volatility, and risk-free rate puzzles by decomposing consumption growth into two components: the growth rate of the carbon consumption component and the growth rate of the share of green consumption out of total consumption. Our model setting increases the ability of investors to detect long-run risk; namely, investors can profit from this risk by using climate change news. Also, our long-run risk variable explains the cross section of industries and firms. Thus, this paper recommends using the carbon risk measures we computed to identify industries or firms that pollute the environment the most and to construct an investment strategy that minimizes/maximizes a long-term environmental criterion.

Further research can use other proxies for the green component in the consumption decomposition and conduct the same analysis. One such proxy could be R&D expenses of carbon-intensive firms allocated to green technology or the revenue from selling Solar Renewable Energy Certificates (SRECs).

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Appendices

A Consumption growth decomposition

Let us consider I categories of consumption among which J carbon consumption categories and $I-J$ green consumption categories.

$$C_t = \sum_{i=1}^{I} C_{i,t} \tag{28}$$

$$C_t = \sum_{i=1}^{J} C_{i,t} + \sum_{i=J+1}^{I} C_{i,t} \tag{29}$$

$$C_t = CC_t + GC_t \tag{30}$$

Growth rate decomposition :

$$\Delta c_{t+1} = \log(CC_{t+1} + GC_{t+1}) - \log(CC_t + GC_t) \tag{31}$$

$$\Delta c_{t+1} = \log(CC_{t+1}) + \log \frac{CC_{t+1} + GC_{t+1}}{CC_{t+1}} - \log(CC_t) - \log \frac{CC_t + GC_t}{CC_t} \tag{32}$$

$$\Delta c_{t+1} = \Delta cc_{t+1} - \left(\log \frac{CC_{t+1}}{CC_{t+1} + GC_{t+1}} - \log \frac{CC_t}{CC_t + GC_t} \right) \tag{33}$$

$$\Delta c_{t+1} = \Delta cc_{t+1} - \Delta \alpha_{CC,t+1} \tag{34}$$

where Δc_{t+1}, Δcc_{t+1} and $\Delta \alpha_{CC,t+1}$ are consumption, carbon consumption and carbon consumption share growth rates (log-difference) respectively.

B Price of risks

$$\lambda_{m,\alpha} = (-\gamma + (1-\theta)\kappa_1 A_3) \tag{35}$$

$$\lambda_{m,cc} = \gamma + (-\gamma + (1-\theta)\kappa_1 A_3)\,\pi \tag{36}$$

$$\lambda_{m,x} = (1-\theta)\kappa_1 A_1 \psi_x \tag{37}$$

$$\lambda_{m,w} = (1-\theta)\kappa_1 A_2 \tag{38}$$

are prices of risk that correspond to the four sources of risk $\epsilon_{\alpha,t+1},\ \epsilon_{cc,t+1},\ \epsilon_{x,t+1}, \epsilon_{\sigma,t+1}$.

C Theoretical moments calculation

From the carbon/green consumption growth rate processes, we have :

$$\mathbb{E}[\Delta cc_{t+1}] = \nu_{cc} \tag{39}$$

$$\mathbb{E}[\Delta \alpha_{cc,t+1}] = \nu_{\alpha} \tag{40}$$

$$\begin{aligned}
\mathbb{V}[\Delta cc_{t+1}] &= \mathbb{V}[\nu_{cc} + x_t + \sigma_t \epsilon_{cc,t+1}] \\
&= \mathbb{V}[x_t] + \mathbb{V}[\sigma_t \epsilon_{cc,t+1}] \\
&= \frac{\psi_x^2}{1-\rho_x^2}\sigma^2 + \sigma^2
\end{aligned} \tag{41}$$

$$\begin{aligned}
\mathbb{V}[\Delta \alpha_{cc,t+1}] &= \mathbb{V}[\nu_{\alpha}(1-\rho_{\alpha}) + \rho_{\alpha}\Delta\alpha_{cc,t} + \sigma_{\alpha}\epsilon_{\alpha,t+1} + \pi\sigma_t\epsilon_{cc,t+1}] \\
(1-\rho_{\alpha}^2)\mathbb{V}[\Delta \alpha_{cc,t+1}] &= \sigma_{\alpha}^2 + \pi^2\sigma^2 \\
\mathbb{V}[\Delta \alpha_{cc,t+1}] &= \frac{\sigma_{\alpha}^2 + \pi^2\sigma^2}{1-\rho_{\alpha}^2}
\end{aligned} \tag{42}$$

$$Cov[\Delta cc_{t+1}, \Delta cc_{t+2}] = \rho_x \frac{\psi_x^2}{1-\rho_x^2}\sigma^2 \tag{43}$$

$$\begin{aligned}
Cov[\Delta\alpha_{cc,t+1}, \Delta\alpha_{cc,t+2}] &= Cov[\Delta\alpha_{cc,t+1}, \rho_\alpha \Delta\alpha_{cc,t+1} + \sigma_\alpha \epsilon_{\alpha,t+2} + \pi\sigma_{t+1}\epsilon_{cc,t+2}] \\
&= \rho_\alpha \mathbb{V}[\Delta\alpha_{cc,t+1}] \\
&= \rho_\alpha \frac{\sigma_\alpha^2 + \pi^2\sigma^2}{1-\rho_\alpha^2}
\end{aligned} \tag{44}$$

From the dividend growth rate process, we can get :

$$\mathbb{E}[\Delta d_{t+1}] = \nu_i + \phi_{\alpha,i}\nu_\alpha \tag{45}$$

$$\begin{aligned}
\mathbb{V}[\Delta d_{i,t+1}] &= \phi_i^2 \mathbb{V}[x_t] + \phi_{\alpha,i}^2 \mathbb{V}[\Delta\alpha_{cc,t}] + \psi_i^2 \mathbb{V}[\sigma_t \epsilon_{i,t+1}] \\
&= \phi_i^2 \frac{\psi_x^2}{1-\rho_x^2}\sigma^2 + \phi_{\alpha,i}^2 \frac{\sigma_\alpha^2 + \pi^2\sigma^2}{1-\rho_\alpha^2} + \psi_i^2\sigma^2
\end{aligned} \tag{46}$$

$$\begin{aligned}
Cov[\Delta d_{i,t+1}, \Delta d_{i,t+2}] &= Cov[\phi_i x_t + \phi_{\alpha,i}\Delta\alpha_{cc,t} + \psi_i\sigma_t\epsilon_{i,t+1}, \phi_i x_{t+1} + \phi_{\alpha,i}\Delta\alpha_{cc,t+1} + \psi_i\sigma_{t+1}\epsilon_{i,t+2}] \\
&= \phi_i^2 Cov[x_t, x_{t+1}] + \phi_{\alpha,i}^2 Cov[\Delta\alpha_{cc,t}, \Delta\alpha_{cc,t+1}] \\
&= \phi_i^2 \rho_x \mathbb{V}[x_t] + \phi_{\alpha,i}^2 \rho_\alpha \frac{\sigma_\alpha^2 + \pi^2\sigma^2}{1-\rho_\alpha^2} \\
&= \phi_i^2 \rho_x \frac{\psi_x^2}{1-\rho_x^2}\sigma^2 + \phi_{\alpha,i}^2 \rho_\alpha \frac{\sigma_\alpha^2 + \pi^2\sigma^2}{1-\rho_\alpha^2}
\end{aligned} \tag{47}$$

From both carbon/green consumption and dividend growth rates, we get the cross moments :

$$
\begin{aligned}
Cov[\Delta\alpha_{cc,t+1}, \Delta cc_{t+1}] &= Cov[\rho_\alpha \Delta\alpha_{cc,t} + \sigma_\alpha \epsilon_{\alpha,t+1} + \pi\sigma_t \epsilon_{cc,t+1}, x_t + \sigma_t \epsilon_{cc,t+1}] \\
&= \pi \mathbb{V}[\sigma_t \epsilon_{cc,t+1}] \\
&= \pi\sigma^2
\end{aligned} \tag{48}
$$

$$
\begin{aligned}
Cov[\Delta d_{i,t+1}, \Delta cc_{t+1}] &= Cov[\phi_i x_t + \phi_{\alpha,i}\Delta\alpha_{cc,t} + \psi_i \sigma_t \epsilon_{i,t+1}, x_t + \sigma_t \epsilon_{cc,t+1}] \\
&= \phi_i \mathbb{V}[x_t] \\
&= \phi_i \frac{\psi_x^2}{1-\rho_x^2}\sigma^2
\end{aligned} \tag{49}
$$

$$
\begin{aligned}
Cov[\Delta d_{i,t+1}, \Delta\alpha_{cc,t+1}] &= Cov[\phi_i x_t + \phi_{\alpha,i}\Delta\alpha_{cc,t} + \psi_i \sigma_t \epsilon_{i,t+1}, \rho_\alpha \Delta\alpha_{cc,t} + \sigma_\alpha \epsilon_{\alpha,t+1} + \pi\sigma_t \epsilon_{cc,t+1}] \\
&= \phi_{\alpha,i}\rho_\alpha \mathbb{V}[\Delta\alpha_{cc,t}] \\
&= \phi_{\alpha,i}\rho_\alpha \frac{\sigma_\alpha^2 + \pi^2\sigma^2}{1-\rho_\alpha^2}
\end{aligned} \tag{50}
$$

From the log price dividend process:

$$
\mathbb{E}[z_{i,t}] = A_{0,i} + A_{2,i}\sigma^2 + A_{3,i}\nu_\alpha \tag{51}
$$

$$
\mathbb{V}[z_{i,t}] = A_{1,i}^2 \frac{\psi_x^2}{1-\rho_x^2}\sigma^2 + A_{2,i}^2 \frac{\sigma_w^2}{1-\nu^2} + A_{3,i}^2 \frac{\sigma_\alpha^2 + \pi^2\sigma^2}{1-\rho_\alpha^2} \tag{52}
$$

$$
\begin{aligned}
Cov[\Delta d_{i,t+1}, z_{i,t}] &= Cov[\phi_i x_t + \phi_{\alpha,i}\Delta\alpha_{cc,t} + \psi_i \sigma_t \epsilon_{i,t+1}, A_{1,i}x_t + A_{2,i}\sigma_t^2 + A_{3,i}\Delta\alpha_{cc,t}] \\
&= \phi_i A_{1,i} \frac{\psi_x^2}{1-\rho_x^2}\sigma^2 + \phi_{\alpha,i}A_{3,i}\frac{\sigma_\alpha^2 + \pi^2\sigma^2}{1-\rho_\alpha^2}
\end{aligned} \tag{53}
$$

$$
\begin{aligned}
Cov[\Delta c_{t+1}, z_{i,t}] &= Cov[\Delta cc_{t+1} - \Delta\alpha_{cc,t+1}, A_{1,i}x_t + A_{2,i}\sigma_t^2 + A_{3,i}\Delta\alpha_{cc,t}] \\
&= Cov[x_t + \sigma_t\epsilon_{cc,t+1}, A_{1,i}x_t + A_{2,i}\sigma_t^2 + A_{3,i}\Delta\alpha_{cc,t}] \\
&- Cov[\rho_\alpha\Delta\alpha_{cc,t} + \sigma_\alpha\epsilon_{\alpha,t+1} + \pi\sigma_t\epsilon_{cc,t+1}, A_{1,i}x_t + A_{2,i}\sigma_t^2 + A_{3,i}\Delta\alpha_{cc,t}] \\
&= A_{1,i}\frac{\psi_x^2}{1-\rho_x^2}\sigma^2 - \rho_\alpha A_{3,i}\frac{\sigma_\alpha^2 + \pi^2\sigma^2}{1-\rho_\alpha^2} \qquad (54)
\end{aligned}
$$

Return on consumption claim $r_{c,t+1}$, on dividend paying asset $r_{i,t+1}$ and risk-free rate $r_{f,t}$

Let us determine $z_t = A_0 + A_1x_t + A_2\sigma_t^2 + A_3\Delta\alpha_{cc,t}$. From the Euler equation 15, we have :

$$
\begin{aligned}
1 &= \mathbb{E}_t e^{\theta log\delta - \frac{\theta}{\psi}\Delta c_{t+1} + \theta r_{c,t+1}} \\
&= e^{\mathbb{E}_t(\theta log\delta - \frac{\theta}{\psi}\Delta c_{t+1} + \theta r_{c,t+1}) + 0.5\mathbb{V}_t(\theta log\delta - \frac{\theta}{\psi}\Delta c_{t+1} + \theta r_{c,t+1})} \\
&= e^{\theta log\delta - \frac{\theta}{\psi}\mathbb{E}_t(\Delta c_{t+1}) + \theta\mathbb{E}_t r_{c,t+1} + 0.5\mathbb{V}_t((-\frac{\theta}{\psi}+\theta)\Delta c_{t+1} + \theta\kappa_1 z_{t+1})} \\
&= exp(\theta log\delta + (1-\gamma)(\nu_{cc} + x_t - \nu_\alpha(1-\rho_\alpha) - \rho_\alpha\Delta\alpha_{cc,t}) + \theta(\kappa_0 - A_0 - A_1x_t - A_2\sigma_t^2 - A_3\Delta\alpha_{cc,t}) \\
&+ \theta\kappa_1(A_0 + A_1\rho_x x_t + A_2((1-\nu)\sigma^2 + \nu\sigma_t^2) + A_3(\nu_\alpha(1-\rho_\alpha) + \rho_\alpha\Delta\alpha_{cc,t})) \\
&+ 0.5\left\{\left((1-\gamma+\pi(-1+\gamma+\theta\kappa_1A_3))^2 + (\theta\kappa_1A_1)^2\psi_x^2\right)\sigma_t^2 + (-1+\gamma+\theta\kappa_1A_3)^2\sigma_\alpha^2 + \theta^2\kappa_1^2A_2^2\sigma_w^2\right\}) \\
0 &= \theta log\delta + (1-\gamma)(\nu_{cc} - \nu_\alpha(1-\rho_\alpha)) + \theta(\kappa_0 - A_0) + \theta\kappa_1(A_0 + A_2(1-\nu)\sigma^2 + A_3\nu_\alpha(1-\rho_\alpha)) \\
&+ 0.5\left\{(-1+\gamma+\theta\kappa_1A_3)^2\sigma_\alpha^2 + \theta^2\kappa_1^2A_2^2\sigma_w^2\right\} \\
&+ (1-\gamma-\theta A_1 + \theta\kappa_1A_1\rho_x)\,x_t \\
&+ \left(-\theta A_2 + \theta\kappa_1A_2\nu + 0.5\,(1-\gamma+\pi(-1+\gamma+\theta\kappa_1A_3))^2 + 0.5(\theta\kappa_1A_1)^2\psi_x^2\right)\sigma_t^2 \\
&+ (-(1-\gamma)\rho_\alpha - \theta A_3 + \theta\kappa_1A_3\rho_\alpha)\,\Delta\alpha_{cc,t}
\end{aligned}
$$

By identification :

$$A_1 = \frac{1 - \dfrac{1}{\psi}}{1 - \kappa_1 \rho_x}$$

$$A_3 = -\frac{\left(1 - \dfrac{1}{\psi}\right)\rho_\alpha}{1 - \kappa_1 \rho_\alpha}$$

$$A_2 = 0.5\theta \frac{\left(1 - \dfrac{1}{\psi} + \pi(-1 + \dfrac{1}{\psi} + \kappa_1 A_3)\right)^2 + (\kappa_1 A_1)^2 \psi_x^2}{1 - \kappa_1 \nu}$$

$$A_0 = \frac{log\delta + (1 - \dfrac{1}{\psi})(\nu_{cc} - \nu_\alpha(1 - \rho_\alpha)) + \kappa_0 + \kappa_1(A_2(1 - \nu)\sigma^2 + A_3\nu_\alpha(1 - \rho_\alpha))}{1 - \kappa_1}$$

$$+ \frac{0.5\theta \left\{ \left(-1 + \dfrac{1}{\psi} + \kappa_1 A_3\right)^2 \sigma_\alpha^2 + \kappa_1^2 A_2^2 \sigma_w^2 \right\}}{1 - \kappa_1}$$

Let us determine :

$$z_{i,t} = A_{0,i} + A_{1,i} x_t + A_{2,i} \sigma_t^2 + A_{3,i} \Delta\alpha_{cc,t}$$

Let us remind that one can rewrite the return on any asset and its dividend growth process as follow :

$$r_{i,t+1} = \kappa_{0,i} + \kappa_{1,i} z_{i,t+1} - z_{i,t} + \Delta d_{i,t}$$

$$\Delta d_{i,t+1} = \nu_i + \phi_i x_t + \phi_{\alpha,i} \Delta\alpha_{cc,t} + \psi_i \sigma_t \epsilon_{i,t+1}$$

From the Euler equation 15, we have :

$$
\begin{aligned}
1 &= \mathbb{E}_t e^{\theta log\delta - \dfrac{\theta}{\psi}\Delta c_{t+1} + (\theta-1) r_{c,t+1} + r_{i,t+1}} \\
&= e^{\mathbb{E}_t(\theta log\delta - \dfrac{\theta}{\psi}\Delta c_{t+1} + (\theta-1) r_{c,t+1} + r_{i,t+1}) + 0.5\mathbb{V}_t(\theta log\delta - \dfrac{\theta}{\psi}\Delta c_{t+1} + (\theta-1) r_{c,t+1} + r_{i,t+1})} \\
&= e^{\theta log\delta - \dfrac{\theta}{\psi}\mathbb{E}_t(\Delta c_{t+1}) + (\theta-1)\mathbb{E}_t r_{c,t+1} + \mathbb{E}_t r_{i,t+1} + 0.5\mathbb{V}_t((-\dfrac{\theta}{\psi}\Delta c_{t+1} + (\theta-1) r_{c,t+1} + r_{i,t+1})}
\end{aligned}
$$

$$
\begin{aligned}
0 &= \theta log\delta - \gamma(\nu_{cc} + x_t - \nu_\alpha(1-\rho_\alpha) - \rho_\alpha \Delta\alpha_{cc,t}) + (\theta-1)(\kappa_0 - A_0 - A_1 x_t - A_2\sigma_t^2 - A_3\Delta\alpha_{cc,t}) \\
&+ (\theta-1)\kappa_1(A_0 + A_1\rho_x x_t + A_2((1-\nu)\sigma^2 + \nu\sigma_t^2) + A_3(\nu_\alpha(1-\rho_\alpha) + \rho_\alpha\Delta\alpha_{cc,t})) + \kappa_{0,i} \\
&+ \kappa_{1,i}(A_{0,i} + A_{1,i}\rho_x x_t + A_{2,i}((1-\nu)\sigma^2 + \nu\sigma_t^2) + A_{3,i}(\nu_\alpha(1-\rho_\alpha) + \rho_\alpha\Delta\alpha_{cc,t})) + \nu_i + \phi_i x_t + \phi_{\alpha,i}\Delta\alpha_{cc,t} \\
&+ 0.5\left\{(-\gamma + \pi(\gamma + (\theta-1)\kappa_1 A_3 + \kappa_{1,i}A_{3,i}))^2 + ((\theta-1)\kappa_1 A_1 + \kappa_{1,i}A_{1,i})^2\psi_x^2 + \psi_i^2\right\}\sigma_t^2 \\
&+ 0.5\left(\gamma + (\theta-1)\kappa_1 A_3 + \kappa_{1,i}A_{3,i}\right)^2\sigma_\alpha^2 \\
&+ 0.5\left((\theta-1)\kappa_1 A_2 + \kappa_{1,i}A_{2,i}\right)^2\sigma_w^2 - A_{0,i} - A_{1,i}x_t - A_{2,i}\sigma_t^2 - A_{3,i}\Delta\alpha_{cc,t} \\
0 &= \theta log\delta - \gamma(\nu_{cc} - \nu_\alpha(1-\rho_\alpha)) + (\theta-1)(\kappa_0 - A_0) + (\theta-1)\kappa_1(A_0 + A_2(1-\nu)\sigma^2 + A_3\nu_\alpha(1-\rho_\alpha)) + \kappa_{0,i} \\
&+ \kappa_{1,i}(A_{0,i} + A_{2,i}(1-\nu)\sigma^2 + A_{3,i}\nu_\alpha(1-\rho_\alpha)) + \nu_i \\
&+ 0.5\left(\gamma + (\theta-1)\kappa_1 A_3 + \kappa_{1,i}A_{3,i}\right)^2\sigma_\alpha^2 + 0.5\left((\theta-1)\kappa_1 A_2 + \kappa_{1,i}A_{2,i}\right)^2\sigma_w^2 - A_{0,i} \\
&+ \left(-\gamma - (\theta-1)A_1 + (\theta-1)\kappa_1 A_1\rho_x + \kappa_{1,i}A_{1,i}\rho_x + \phi_i - A_{1,i}\right)x_t \\
&+ \left(0.5((-\gamma + \pi(\gamma + (\theta-1)\kappa_1 A_3 + \kappa_{1,i}A_{3,i}))^2 + ((\theta-1)\kappa_1 A_1 + \kappa_{1,i}A_{1,i})^2\psi_x^2 + \psi_i^2)\right)\sigma_t^2 \\
&+ \left(-(\theta-1)A_2 + (\theta-1)\kappa_1 A_2\nu + \kappa_{1,i}A_{2,i}\nu - A_{2,i}\right)\sigma_t^2 \\
&+ \left(\gamma\rho_\alpha - (\theta-1)A_3 + (\theta-1)\kappa_1 A_3\rho_\alpha + \kappa_{1,i}A_{3,i}\rho_\alpha + \phi_{\alpha,i} - A_{3,i}\right)\Delta\alpha_{cc,t}
\end{aligned}
$$

By identification :

$$A_{1,i} = \frac{\phi_i - \dfrac{1}{\psi}}{1 - \kappa_{1,i}\rho_x}$$

$$A_{3,i} = \frac{\phi_{\alpha,i} + \dfrac{\rho_\alpha}{\psi}}{1 - \kappa_{1,i}\rho_\alpha}$$

$$A_{2,i} = \frac{(1-\theta)A_2(1-\kappa_1\nu) + 0.5\left\{(-\gamma + \pi(\gamma + (\theta-1)\kappa_1 A_3 + \kappa_{1,i}A_{3,i}))^2 + ((\theta-1)\kappa_1 A_1 + \kappa_{1,i}A_{1,i})^2\psi_x^2 + \psi_i^2\right\}}{1 - \kappa_{1,i}\nu}$$

$$\begin{aligned} A_{0,i} = & \frac{\theta log\delta - \gamma(\nu_{cc} - \nu_\alpha(1-\rho_\alpha)) + (\theta-1)(\kappa_0 - A_0) + (\theta-1)\kappa_1(A_0 + A_2(1-\nu)\sigma^2 + A_3\nu_\alpha(1-\rho_\alpha)) + \kappa_{0,i}}{1-\kappa_{1,i}} \\ & + \frac{\kappa_{1,i}(A_{2,i}(1-\nu)\sigma^2 + A_{3,i}\nu_\alpha(1-\rho_\alpha)) + \nu_i}{1-\kappa_{1,i}} \\ & + \frac{0.5\left(\gamma + (\theta-1)\kappa_1 A_3 + \kappa_{1,i}A_{3,i}\right)^2\sigma_\alpha^2 + 0.5\left((\theta-1)\kappa_1 A_2 + \kappa_{1,i}A_{2,i}\right)^2\sigma_w^2}{1-\kappa_{1,i}} \end{aligned}$$

Deriving $r_{f,t}$:

$$\mathbb{E}_t e^{\theta log\delta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta-1)r_{c,t+1} + r_{f,t}} = 1$$

So

$$
\begin{aligned}
e^{-r_{f,t}} &= \mathbb{E}_t e^{\theta log\delta - \dfrac{\theta}{\psi}\Delta c_{t+1} + (\theta-1) r_{c,t+1}} \\
&= e^{\mathbb{E}_t(\theta log\delta - \dfrac{\theta}{\psi}\Delta c_{t+1} + (\theta-1) r_{c,t+1}) + 0.5\mathbb{V}_t(\theta log\delta - \dfrac{\theta}{\psi}\Delta c_{t+1} + (\theta-1) r_{c,t+1})} \\
&= e^{\theta log\delta - \dfrac{\theta}{\psi}\mathbb{E}_t(\Delta c_{t+1}) + (\theta-1)\mathbb{E}_t r_{c,t+1} + 0.5\mathbb{V}_t((-\dfrac{\theta}{\psi}\Delta c_{t+1} + (\theta-1) r_{c,t+1})} \\
-r_{f,t} &= \theta log\delta - \gamma(\nu_{cc} + x_t - \nu_\alpha(1-\rho_\alpha) - \rho_\alpha \Delta\alpha_{cc,t}) + (\theta-1)(\kappa_0 - A_0 - A_1 x_t - A_2\sigma_t^2 - A_3\Delta\alpha_{cc,t}) \\
&+ (\theta-1)\kappa_1(A_0 + A_1\rho_x x_t + A_2((1-\nu)\sigma^2 + \nu\sigma_t^2) + A_3(\nu_\alpha(1-\rho_\alpha) + \rho_\alpha\Delta\alpha_{cc,t})) \\
&+ 0.5\left\{(-\gamma + \pi(\gamma + (\theta-1)\kappa_1 A_3))^2 + ((\theta-1)\kappa_1 A_1)^2\psi_x^2\right\}\sigma_t^2 \\
&+ 0.5\left(\gamma + (\theta-1)\kappa_1 A_3\right)^2\sigma_\alpha^2 + 0.5\left((\theta-1)\kappa_1 A_2\right)^2\sigma_w^2 \\
-r_{f,t} &= \theta log\delta - \gamma(\nu_{cc} - \nu_\alpha(1-\rho_\alpha)) + (\theta-1)(\kappa_0 - A_0) + (\theta-1)\kappa_1(A_0 + A_2(1-\nu)\sigma^2 + A_3\nu_\alpha(1-\rho_\alpha)) \\
&+ 0.5\left(\gamma + (\theta-1)\kappa_1 A_3\right)^2\sigma_\alpha^2 + 0.5\left((\theta-1)\kappa_1 A_2\right)^2\sigma_w^2 \\
&+ \left(-\gamma - (\theta-1)A_1 + (\theta-1)\kappa_1 A_1\rho_x\right)x_t \\
&+ \left(-(\theta-1)A_2 + (\theta-1)\kappa_1 A_2\nu + 0.5((-\gamma + \pi(\gamma + (\theta-1)\kappa_1 A_3))^2 + ((\theta-1)\kappa_1 A_1)^2\psi_x^2)\right)\sigma_t^2 \\
&+ \left(\gamma\rho_\alpha - (\theta-1)A_3 + (\theta-1)\kappa_1 A_3\rho_\alpha\right)\Delta\alpha_{cc,t}
\end{aligned}
$$

Therefore :

$$
r_{f,t} = A_{0,f} + A_{1,f}x_t + A_{2,f}\sigma_t^2 + A_{3,f}\Delta\alpha_{cc,t}
$$

Deriving $\mathbb{E}_t r_{i,t+1}$:

$$
\begin{aligned}
\mathbb{E}_t r_{i,t+1} =& \kappa_{0,i} + \kappa_{1,i}\mathbb{E}_t z_{i,t+1} - A_{0,i} - A_{1,i}x_t - A_{2,i}\sigma_t^2 - A_{3,i}\Delta\alpha_{cc,t} + \nu_i + \phi x_t + \phi_{\alpha,i}\Delta\alpha_{cc,t} \\
=& \kappa_{0,i} + \kappa_{1,i}\mathbb{E}_t(A_{0,i} + A_{1,i}x_{t+1} + A_{2,i}\sigma_{t+1}^2 + A_{3,i}\Delta\alpha_{cc,t+1}) \\
& - A_{0,i} - A_{1,i}x_t - A_{2,i}\sigma_t^2 - A_{3,i}\Delta\alpha_{cc,t} + \nu_i + \phi_i x_t + \phi_{\alpha,i}\Delta\alpha_{cc,t} \\
=& \kappa_{0,i} + \kappa_{1,i}A_{0,i} + \kappa_{1,i}A_{1,i}\rho_x x_t + \kappa_{1,i}A_{2,i}((1-\nu^2)\sigma^2 + \nu\sigma_t^2) \\
& + \kappa_{1,i}A_{3,i}(\nu_\alpha(1-\rho_\alpha) + \rho_\alpha\Delta\alpha_{cc,t}) - A_{0,i} - A_{1,i}x_t - A_{2,i}\sigma_t^2 - A_{3,i}\Delta\alpha_{cc,t} + \nu_i + \phi_i x_t + \phi_{\alpha,i}\Delta\alpha_{cc,t} \\
=& \kappa_{0,i} + \kappa_{1,i}A_{0,i} + \kappa_{1,i}A_{2,i}(1-\nu)\sigma^2 + \kappa_{1,i}A_{3,i}\nu_\alpha(1-\rho_\alpha) - A_{0,i} + \nu_i \\
& + (-A_{1,i} + \phi_i + \kappa_{1,i}A_{1,i}\rho_x)x_t + (-A_{2,i} + \kappa_{1,i}A_{2,i}\nu)\sigma_t^2 + (-A_{3,i} + \phi_{\alpha,i} + \kappa_{1,i}A_{3,i}\rho_\alpha)\Delta\alpha_{cc,t} \\
=& B_0 + B_1 x_t + B_2\sigma_t^2 + B_3\Delta\alpha_{cc,t}
\end{aligned}
$$

The innovation in the market return is :

$$r_{i,t+1} - \mathbb{E}_t r_{i,t+1} = \kappa_{1,i} A_{1,i} \psi_x \sigma_t \epsilon_{x,t+1} + \kappa_{1,i} A_{2,i} \sigma_w \epsilon_{\sigma,t+1} + \kappa_{1,i} A_{3,i} \sigma_\alpha \epsilon_{\alpha,t+1} + \kappa_{1,i} A_{3,i} \pi \sigma_t \epsilon_{cc,t+1} + \psi_i \sigma_t \epsilon_{i,t+1}$$

So the expected equity premium on any dividend paying asset i is given by :

$$\begin{aligned}
\mathbb{E}_t(r_{i,t+1} - r_{f,t}) &= -Cov(r_{i,t+1} - \mathbb{E}_t r_{i,t+1}, m_{t+1} - \mathbb{E}_t m_{t+1}) - 0.5\mathbb{V}_t(r_{i,t+1}) \\
&= \lambda_{m,x} \underbrace{\kappa_{1,i} A_{1,i} \psi_x}_{\beta_{i,x}} \sigma_t^2 + \lambda_{m,w} \underbrace{\kappa_{1,i} A_{2,i}}_{\beta_{i,w}} \sigma_w^2 + \lambda_{m,\alpha} \underbrace{\kappa_{1,i} A_{3,i}}_{\beta_{i,\alpha}} \sigma_\alpha^2 + \lambda_{m,cc} \underbrace{\kappa_{1,i} A_{3,i} \pi}_{\beta_{i,cc}} \sigma_t^2 - 0.5\mathbb{V}_t(r_{i,t+1})
\end{aligned}$$

And

$$\begin{aligned}
\mathbb{V}_t(r_{i,t+1}) =& \mathbb{V}_t(\kappa_{1,i} z_{i,t+1} + \Delta d_{i,t+1}) \\
=& \left(\kappa_{1,i}^2 A_{1,i}^2 \psi_x^2 + \pi^2 \kappa_{1,i}^2 A_{3,i}^2 + \psi_i^2\right) \sigma_t^2 + \kappa_{1,i}^2 A_{2,i}^2 \sigma_w^2 + \kappa_{1,i}^2 A_{3,i}^2 \sigma_\alpha^2 \\
=& (\beta_{i,x}^2 + \beta_{i,cc}^2 + \psi_i^2) \sigma_t^2 + \beta_{i,w}^2 \sigma_w^2 + \beta_{i,\alpha}^2 \sigma_\alpha^2
\end{aligned}$$

D Tables

Table 6: Descriptive statistics : 1930-1955.

	$E(.)$	$\sigma(.)$	$\rho(1)$	$\rho(2)$	$\rho(3)$	$\rho(4)$	$\rho(5)$	CV
Δd	0.0038	0.1914	0.1244	-0.2814	-0.1927	-0.1018	0.0954	50.6037
Δc	0.0169	0.0540	0.3633	0.1214	-0.1689	-0.2447	0.0208	3.1977
$\Delta \alpha_{cc}$	0.0005	0.0231	0.4800	0.0383	-0.1503	-0.3388	-0.2569	50.1958
Δcc	0.0173	0.0611	0.2899	-0.0096	-0.2380	-0.2580	0.0343	3.5250
$\Delta \alpha_{gc}$	-0.0009	0.0378	0.4771	0.0408	-0.0949	-0.3182	-0.2775	-42.0192
Δgc	0.0160	0.0621	0.5495	0.2875	-0.0174	-0.2748	-0.1701	3.8876
z_m	2.8535	0.2265	0.4185	-0.0828	-0.2764	-0.4030	-0.2802	0.0794
r_m	0.0732	0.2468	0.0904	-0.2068	-0.0779	-0.2504	-0.0288	3.3692
r_f	-0.0103	0.0558	0.6365	0.1463	0.0169	0.1026	0.2074	-5.4163

Table 7: Descriptive statistics : 1956-1980.

	$E(.)$	$\sigma(.)$	$\rho(1)$	$\rho(2)$	$\rho(3)$	$\rho(4)$	$\rho(5)$	CV
Δd	0.0074	0.0523	0.2667	0.0359	0.0091	0.0667	0.0053	7.0994
Δc	0.0222	0.0290	-0.0204	-0.2613	-0.0383	0.0427	0.1899	1.3081
$\Delta\alpha_{cc}$	-0.0035	0.0043	-0.2815	-0.0210	0.3359	-0.3975	0.0456	-1.2429
Δcc	0.0187	0.0295	-0.0577	-0.2755	-0.0490	0.0381	0.1629	1.5790
$\Delta\alpha_{gc}$	0.0061	0.0076	-0.2780	-0.0079	0.3276	-0.3867	0.0584	1.2326
Δgc	0.0283	0.0296	0.0243	-0.2123	0.0213	0.0111	0.2276	1.0473
z_m	3.2918	0.1822	0.6555	0.3480	0.2547	0.3035	0.1185	0.0553
r_m	0.0445	0.1779	-0.0762	-0.3736	0.1056	0.3032	0.0922	4.0023
r_f	0.0030	0.0152	0.5917	0.4075	0.4660	0.3126	0.2736	5.0496

Table 8: Descriptive statistics : 1981-2018.

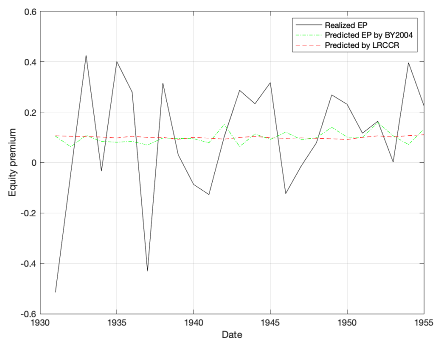
	$E(.)$	$\sigma(.)$	$\rho(1)$	$\rho(2)$	$\rho(3)$	$\rho(4)$	$\rho(5)$	CV
Δd	0.0339	0.0926	-0.0365	-0.0047	-0.0714	-0.1272	-0.0885	2.7331
Δc	0.0155	0.0162	0.4603	0.0530	-0.0526	-0.0848	-0.0747	1.0438
$\Delta\alpha_{cc}$	-0.0052	0.0066	0.3599	-0.0832	-0.2421	-0.0687	0.1352	-1.2812
Δcc	0.0104	0.0185	0.5090	0.0042	-0.1916	-0.1590	-0.1158	1.7807
$\Delta\alpha_{gc}$	0.0065	0.0083	0.3914	-0.0141	-0.1654	0.0388	0.1809	1.2843
Δgc	0.0220	0.0172	0.3744	0.1211	0.1178	0.0675	0.0877	0.7844
z_m	3.8166	0.4151	0.8895	0.7548	0.6669	0.5546	0.4293	0.1088
r_m	0.0831	0.1618	-0.0695	-0.1204	0.0703	-0.0356	-0.4177	1.9464
r_f	0.0109	0.0221	0.7975	0.6500	0.5337	0.3759	0.3189	2.0303

Table 9: Model-implied moments.

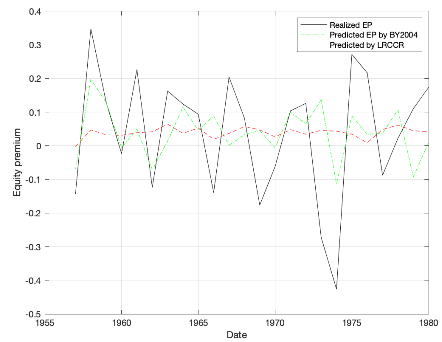
		$\sigma(z_m)$	EP	$E(R_f)$	$\sigma(r_{m,a})$	$\sigma(r_{f,a})$	$\rho(z_m)$
1930-1955							
	Data	0.227	0.084	-0.010	0.247	0.010	0.418
BY2004	Mean	0.238	0.109	0.007	0.264	0.159	0.354
	5%	0.166	-0.019	-0.051	0.193	0.122	0.019
	50%	0.234	0.105	0.006	0.261	0.158	0.368
	95%	0.325	0.251	0.065	0.346	0.198	0.637
LRCCR	Mean	0.150	0.040	0.010	0.116	0.019	0.608
	5%	0.098	0.000	-0.005	0.089	0.014	0.291
	50%	0.145	0.039	0.010	0.115	0.018	0.635
	95%	0.216	0.081	0.024	0.144	0.025	0.834
1956-1980							
	Data	0.182	0.041	0.003	0.178	0.003	0.656
BY2004	Mean	0.125	0.093	-0.002	0.134	0.001	0.345
	5%	0.093	0.045	-0.002	0.102	0.001	0.019
	50%	0.124	0.093	-0.002	0.133	0.001	0.360
	95%	0.162	0.143	-0.001	0.167	0.002	0.620
LRCCR	Mean	0.175	0.068	0.003	0.197	0.019	0.282
	5%	0.119	0.003	-0.005	0.138	0.014	-0.041
	50%	0.172	0.066	0.003	0.194	0.019	0.293
	95%	0.240	0.141	0.010	0.263	0.024	0.569

E Figures

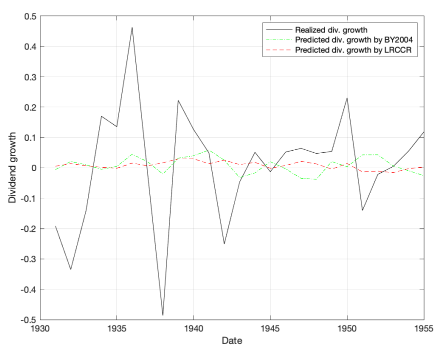
Figure 7: Realized versus predicted equity premium, consumption growth and dividend growth



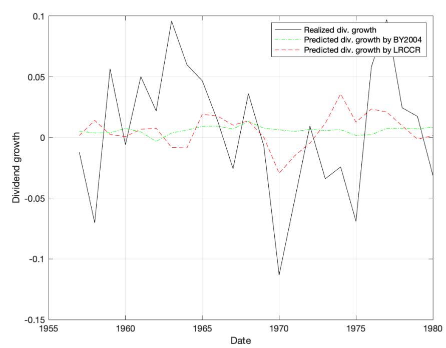
(a) 1930-1955



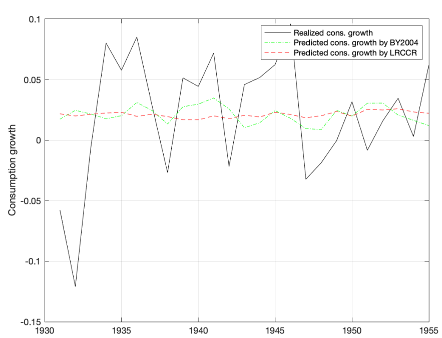
(b) 1956-1980



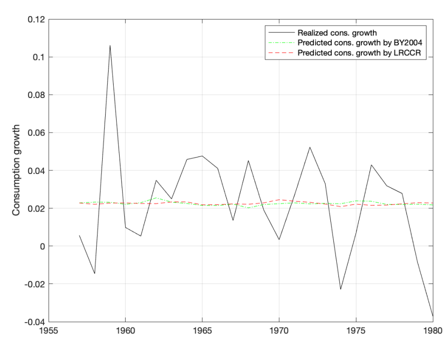
(c) 1930-1955



(d) 1956-1980



(e) 1930-1955



(f) 1956-1980

F Data construction details

Table 10: **Carbon footprint covered from the NIPA expenditure data**

Household consumption expenditures category (2 digit level)	Footprint Covererage E
1-Food and beverages purchased for off-premises consumption	
Food and nonalcoholic beverages purchased for off-premises consumption	√
Alcoholic beverages purchased for off-premises consumption	x
Food produced and consumed on farms	√
2-Clothing, footwear, and related services	
Clothing	√
Footwear	√
3-Housing, utilities, and fuels	
Housing	√
Household utilities and fuels	
Water supply and sanitation	√
Electricity, gas, and other fuels	x
Electricity	√
Natural gas	√
Fuel oil and other fuels	
4-Furnishings, household equipment, and routine household maintenance	
Furniture, furnishings, and floor coverings	√
Household textiles	√
Household appliances	√
Glassware, tableware, and household utensils	√
Tools and equipment for house and garden	x
5-Health	
Medical products, appliances, and equipment	x
Outpatient services	√
Hospital and nursing home services	
Hospital	√
Nursing home services	√
6-Transportation	
Motor vehicles	√
Motor vehicle operation	√
Public transportation	
Ground transportation	√
Air transportation	√
Water transportation	√
7-Communication	
Telephone and related communication equipment	√
Postal and delivery services	√
Telecommunication services	√
Internet access	x
8-Recreation	
Video and audio equipment, computers, and related services	√
Sports and recreational goods and related services	√
Membership clubs, sports centers, parks, theaters, and museums	√
Magazines, newspapers, books, and stationery	√
Gambling	x
Pets, pet products, and related services	x
Photographic goods and services	x

Continued on next page

Table 10 – continued from previous page

Household consumption expenditures category (2 digit level)	**Footprint Covereráge E**
Package tours	x
9-Education	
Educational books	x
Higher education	$\surd$
Nursery, elementary, and secondary schools	$\surd$
Commercial and vocational schools	$\surd$
10-Food services and accommodations	
Food services	$\surd$
Accommodations	$\surd$
11-Financial services and insurance	
Financial services	$\surd$
Insurance	$\surd$
12-Other goods and services	
Personal care	$\surd$
Personal items	$\surd$
Social services and religious activities	$\surd$
Professional and other services	$\surd$
Tobacco	$\surd$