Long-Run Carbon Consumption Risks Model And Asset Prices

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Abstract

This paper analyzes how carbon risks affect the stochastic discount factor (SDF). The consumption-based carbon risk measure is based on information on aggregate consumption and carbon footprint for each good and service. The consumption growth is decomposed into two components- the growth rate of carbon consumption component and the growth rate of the share of carbon consumption out of total consumption. A long run risks model is developed where the SDF is affected by the green and carbon shocks through the consumption. This paper argues that the long-run risk in the consumption growth comes mainly from the carbon consumption growth due to policies and actions to curb the emission such as Paris Agreement and COP26. The presence of carbon consumption growth in the model helps detect the long run risk in consumption. The model explains the cross section of industry level returns based on their exposure to the two risk factors I constructed, carbon consumption growth risk and the share of carbon consumption out of total consumption growth risk. This disentangling could lead to identify industries or firms which pollute the most and construct an investment strategy that minimize/maximize a long term environmental criteria.

Keywords: Climate Finance, Consumption-Based Carbon Emission, Carbon Risk, Long-Run Risk, Asset Pricing.

JEL Codes: G11, G12, Q54

Introduction

Regulators are increasingly worried about the extent to which stock markets efficiently price climate change risks and the discount rate that should be used to evaluate the investment's uncertain future benefits. In fact, part of these risks are stemming from the transition process to a low-carbon economy. More precisely, to curb the carbon dioxide(CO2) emissions, climate policy aims to hold the increase of the average global temperatures within $2°C$ of pre-industrial levels. As such there is a burgeoning climate finance literature that examines the efficiency of capital markets in pricing risks associated with climate change. For a detailed recent literature review, see Hong et al. (2020) and Giglio et al. (2020).

However, this market price related to carbon emissions is narrowly confined at the production[1] level and neglect carbon leakage inside and outside a given boundary. Carbon leakage alludes to the situations that may take place if, for any reasons of costs related to climate actions, firms were to transfer production to other countries with less constraints in terms of pollution. This could lead to an increase in their total emissions. Thus, it measures incorrectly the actual impact of the carbon emission. Moreover, as one can see on Figure 4, the consumption-based carbon emission in USA is over the production-based emission. First, this paper addresses this issue by proving an alternative measure namely the consumption-based carbon emission(CCE). The main benefits of the CCE are that not only it captures carbon leakage but also it captures the life-cycle of the greenhouse gases (GHG) emissions expressed in carbon dioxide equivalent. The life-cycle assessment gives a complete picture of a product's environmental impacts. It tells us about which parts of its life cycle period, the product most negatively impacts the environment. Second, this paper sheds a new light on the long-run risks model presented for the first time in the canonical paper by Bansal and Yaron (2004) stating

[1] The production-based carbon emission(PCE) is exclusively referring to emissions generated at the point of production, that is, emissions physically produced

that there is a small long-run predictable component driving consumption growth rate (long-run risk in expected growth) and a fluctuating economic uncertainty measured by the volatility of consumption growth (long-run volatility risk). Despite the interesting implication of this model for explaining stylised facts on the asset market, it has some shortcomings. Because not everyone believe in the existence of a long-run growth risk component for at least two reasons. First of all, it is hard to detect statistically by univariate methods that the consumption growth rate has a predictable component. As a matter of fact, the consumption process is close to a random walk. Second of all, the effect on asset prices depends on investors detecting it. In front of these shortcomings, authors tested empirically the long-run risks model (Bansal et al. (2007a), Bansal et al. (2007b), Pohl et al. (2018), Schorfheide et al. (2018) among others). Bonomo et al. (2011) proposes a long-run volatility-risk-only model to deal with the critics of the canonical long-run risks model. Along the same line, this paper revisits this literature by proposing a long-run carbon consumption risks model. Even though the consumption growth might not admit both long-run risks in mean and volatility, there is more reasons to hypothesize that carbon emission does have both long-run risks. One reason is the willingness of policy makers to reduce at the pre-industrial level the emission in the atmosphere.

This paper adds two contributions to the existing literature. First, we provide a new consumption-based carbon emission measure. Second, we introduce a long-run carbon-consumption risks model, which allow us to explain the equity premium, volatility and risk-free rate puzzles and explains the cross-sectional difference of industries returns.

To that end, we use the Economic Input-Output Life Cycle Assessment (EIO-LCA) database, the NIPA consumption data to construct our carbon consumption measure and we use returns data from Kenneth R. French website to test empirically our long-run carbon consumption risks model.

To test the existence of long-run risks in carbon consumption, this paper starts by decomposing the consumption growth into two components : carbon consumption growth component and share of carbon consumption growth component. In our setting, "carbon consumption risk" occurs for two main reasons. First, carbon risk stems from regulator willingness to curb carbon emission at the pre-industrial level which in turn may affect future household consumption that heavily depends on carbon consumption. Second, carbon consumption creates damages through the lens of climate changes. As results, there is potential long-run risks for carbon-based consumption in both case. Building on this insight, we theoretically characterizes and then quantifies carbon price risk in an integrated assessment asset pricing-climate model with long run risks in carbon consumption. We hypothesize that the share of carbon consumption component does not contain any long-run risks while the carbon consumption component does.

First, this article is related to the strand of literature on long-run risks model. Papers here include Bansal and Yaron (2004), Bansal et al. (2007a), Bansal et al. (2007b), Koijen et al. (2010), Bonomo et al. (2011), Constantinides and Ghosh (2011), Schorfheide et al. (2018), Pohl et al. (2018), Pohl et al. (2021), etc. Compared to these, the novelty here is the consumption growth dynamic we considered. We decompose the consumption growth rate into two parts : a carbon growth component which creates the long-run growth risk, and a share of carbon growth component which does not create any long-run risk but affects the dynamic of the consumption growth and acts like a hedge against the carbon risk while in the cited papers, their long-run risks come directly from the aggregate consumption which are hardly detectable.

Second, this paper is also related to the strand of literature on climate finance. Papers here include Daniel et al. (2016), Bansal et al. (2016b), Bansal et al. (2017), Chen et al. (2019), Giglio et al. (2021), Stroebel and Wurgler (2021), Avramov et al. (2021), etc. Our paper is focus on the carbon or transition risk while those papers study the physical risk side of the climate risk. Especially, since the climate change is a long horizon phenomenon, one need to asset it using a long-run risk model.

The rest of the paper is organized as followed. Section 2 sets up the theoretical model. In section 1, we describe the data we use to test empirically our long-run risks model. Section 3 presents the results and the asset pricing implication of the model. Section 4 concludes.

1 A new measure of consumption-based carbon risk

The central challenge of the climate finance is to capture the actual impact of carbon emission. To deal with that, this paper uses aggregate consumption data and the carbon footprint to identify a simple but rich carbon/green risk measure. All data are on annual basis and span the period 1930-2018. I describe below how I constructed the carbon consumption, green consumption measures to access the empirical implication of a long-run carbon consumption risks model. Summary statistics are presented in the table 1.

To construct the carbon consumption, I consider the carbon dioxide emissions from household consumption perspective. The carbon dioxide emissions from a consumption perspective indicators provide an alternative view of carbon dioxide emissions, where the emissions are tied to the consumption of durable goods, non-durable goods and services in each state of US. This allows to account for a potential carbon leakage and the actual impact of carbon emission in a given boundary. In fact, under the assumption of linear life cycle progression of product, household stand at the usage stage where they have control of the product. Using the NIPA [2] data, I collect aggregate information on 11 consumption categories (Food, clothing, housing, furniture, health, transportation, communication, recreation, education, food services and accommodation, financial Services and insurances). Then I match these aggregate information to the carbon footprint information provided by the Economic Input-Output Life Cycle

[2] National Income and Product Accounts

Assessment (EIO-LCA) database using the purchaser(retail) price model. The Life cycle assessment (LCA) investigates, estimates, and evaluates the environmental burdens caused by a material, product, process, or service throughout its life span. Environmental burdens include the materials and energy resources required to create the product, as well as the wastes and emissions generated during the process. The EIO-LCA is developed by Carnegie Mellon University (Institute. (2021)) and it provides an estimate of economy-wide cradle-to-gate GHG emissions per dollar of producer output for 428 sectors of the U.S. economy. This paper uses the US 2002 Benchmark Model - purchaser price to collect the carbon footprint for the 11 household expenditure categories identified in the NIPA data. I identify in the Economic Input-Output Life Cycle Assessment data base a total of 50 sectors representing household consumption good productions. A sample of such carbon emission is given in the figure 2 for power generation and supply (electricity), and soft drink and ice manufacturing. It tells us about the direct and indirect emission due to the purchase of $US 1 million of electricity for the left panel and soft drink for the right panel. It amounts to 9,370 and 651 tons of CO2 emission (t CO2e) respectively. One can see that electricity has a higher burden on the environment than soft drink.

Using these 50 sectors, we covered 96% of total household consumption expenditure which represent a total of 29 consumption goods out of 44 in the NIPA table. Figure 3 displays the total carbon footprint in each consumption category. As shown, transport, food and housing account for large part of carbon footprint in household consumption expenditure representing a total of 77% of US household dioxide carbon emission. Among household consumption basket, food and beverages contribute the most in carbon footprint followed by housing, household utilities, furnishings, recreation and transportation.

As one can see on the figure 4, direct carbon footprint represents only 11% of the total emission in household's consumption basket. So this paper uses both direct

Figure 1: Fact

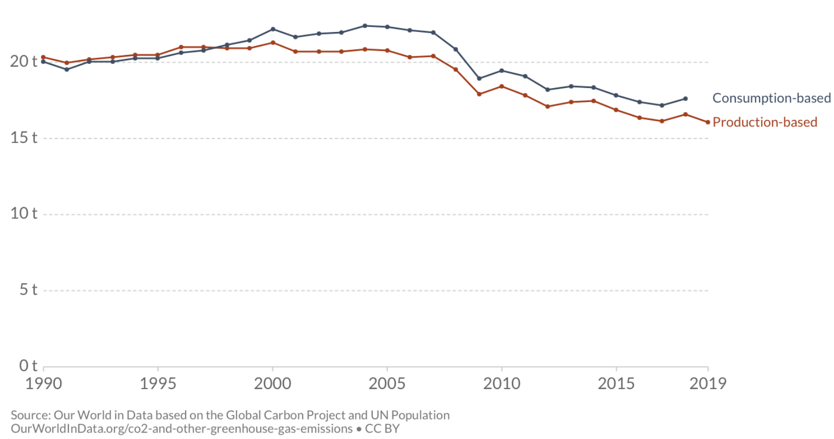
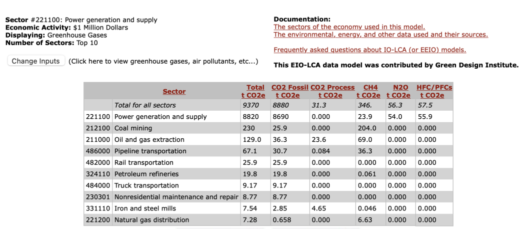


Figure 2: Carbon footprint of electricity versus soft drink



Sector #221100: Power generation and supply
Economic Activity: $1 Million Dollars
Displaying: Greenhouse Gases
Number of Sectors: Top 10

Change Inputs (Click here to view greenhouse gases, air pollutants, etc...)

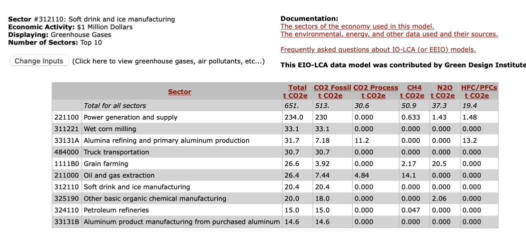
Documentation:
The sectors of the economy used in this model.
The environmental, energy, and other data used and their sources.

Frequently asked questions about IO-LCA (or EEIO) models.

This EIO-LCA data model was contributed by Green Design Institute.

	Sector	Total t CO2e	CO2 Fossil t CO2e	CO2 Process t CO2e	CH4 t CO2e	N2O t CO2e	HFC/PFCs t CO2e
	Total for all sectors	*9370*	*8880*	*31.3*	*346.*	*56.3*	*57.5*
221100	Power generation and supply	8820	8690	0.000	23.9	54.0	55.9
212100	Coal mining	230	25.9	0.000	204.0	0.000	0.000
211000	Oil and gas extraction	129.0	36.3	23.6	69.0	0.000	0.000
486000	Pipeline transportation	67.1	30.7	0.084	36.3	0.000	0.000
482000	Rail transportation	25.9	25.9	0.000	0.000	0.000	0.000
324110	Petroleum refineries	19.8	19.8	0.000	0.061	0.000	0.000
484000	Truck transportation	9.17	9.17	0.000	0.000	0.000	0.000
230301	Nonresidential maintenance and repair	8.77	8.77	0.000	0.000	0.000	0.000
331110	Iron and steel mills	7.54	2.85	4.65	0.046	0.000	0.000
221200	Natural gas distribution	7.28	0.658	0.000	6.63	0.000	0.000

(a) Power generation and supply



Sector #312110: Soft drink and ice manufacturing
Economic Activity: $1 Million Dollars
Displaying: Greenhouse Gases
Number of Sectors: Top 10

Change Inputs (Click here to view greenhouse gases, air pollutants, etc...)

Documentation:
The sectors of the economy used in this model.
The environmental, energy, and other data used and their sources.

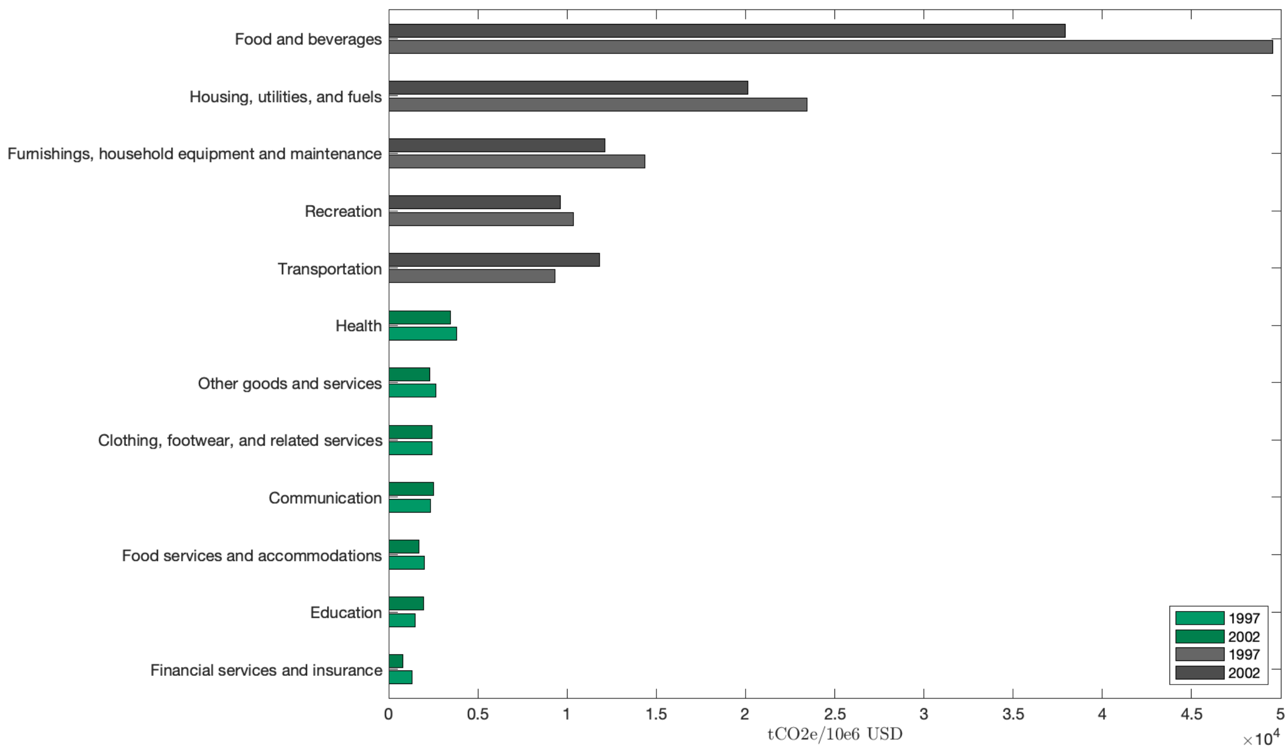
Frequently asked questions about IO-LCA (or EEIO) models.

This EIO-LCA data model was contributed by Green Design Institute.

	Sector	Total t CO2e	CO2 Fossil t CO2e	CO2 Process t CO2e	CH4 t CO2e	N2O t CO2e	HFC/PFCs t CO2e
	Total for all sectors	*651.*	*513.*	*30.6*	*50.9*	*37.3*	*19.4*
221100	Power generation and supply	234.0	230	0.000	0.633	1.43	1.48
311221	Wet corn milling	33.1	33.1	0.000	0.000	0.000	0.000
33131A	Alumina refining and primary aluminum production	31.7	7.18	11.2	0.000	0.000	13.2
484000	Truck transportation	30.7	30.7	0.000	0.000	0.000	0.000
1111B0	Grain farming	26.6	3.92	0.000	2.17	20.5	0.000
211000	Oil and gas extraction	26.4	7.44	4.84	14.1	0.000	0.000
312110	Soft drink and ice manufacturing	20.4	20.4	0.000	0.000	0.000	0.000
325190	Other basic organic chemical manufacturing	20.0	18.0	0.000	0.000	2.06	0.000
324110	Petroleum refineries	15.0	15.0	0.000	0.047	0.000	0.000
33131B	Aluminum product manufacturing from purchased aluminum	14.6	14.6	0.000	0.000	0.000	0.000

(b) Soft drink and ice manufacturing

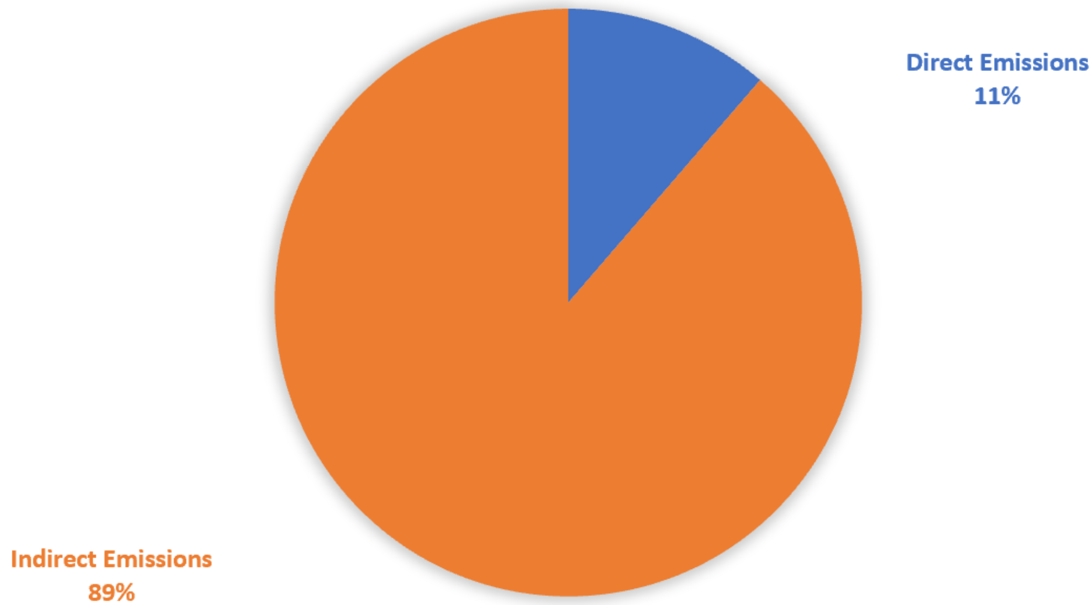
Figure 3: **Carbon footprint by household expenditure category**. The x-axis captures the tons of CO2 emission(tCO2e) per million of US dollars



and indirect burdens to compute the total carbon emission.

Next, I map the NIPA expenditure category to the carbon footprint information in order to compute the consumption-based carbon. Since the carbon footprint information is related to the 2002 consumer price purchase, all the NIPA data are deflated using the 2002 reference base period for the consumer price index (CPI). The figure 2 shows that all the consumption categories do not affect equally the environment. Therefore, this paper weights the aggregate consumption of each good and service by its burden on the atmosphere to compute a new total consumption measure. The total US-household consumption-based carbon emission can be expressed simply as the product of consumption, C, in dollars , and carbon emissions per unit of consump-

Figure 4: **Percentage Carbon footprint**. The direct emissions include natural gaz, motor oil , lubricant grass and the indirect emissions represent the remaining goods in the consumption basket.



tion, CE, summed over each carbon footprint activity (i) included in the model. Put it simply, when it comes to analyze the effect of the carbon emission on the economy, environment, consumption categories should not be treated as the same. Each category affects differently the environment, so I compute the aggregate consumption by weighting each category consumption by its footprint. Alternatively, I classify the carbon footprint in a decreasing order and use the five categories with the highest carbon footprints to compute what I call carbon consumption and the leftover to compute the green consumption. Overall, the total carbon emission at any time t is calculated as follow :

$$TC_t = \sum_{i=1}^{11} C_{i,t} * CE_i \tag{1}$$

However, I subdivide all the consumption categories into two parts to be able to separate the usual consumption risk into two risks. One to measure the carbon consumption (the consumption category which pollute the most based on their footprint. See equation 2) and one to measure the green consumption (those who pollute the least. See equation 3). Henceforth, I will call the risk associated to the green consumption, green risk and the risk associated to the carbon consumption, carbon risk.

$$\boldsymbol{CC_t} = \sum_{\boldsymbol{i=1}}^{\boldsymbol{5}} \boldsymbol{C_{i,t} * CF_i} \quad or \quad \sum_{\boldsymbol{i=1}}^{\boldsymbol{5}} \boldsymbol{C_{i,t}} \tag{2}$$

$$\boldsymbol{GC_t} = \sum_{\boldsymbol{i=6}}^{\boldsymbol{11}} \boldsymbol{C_{i,t} * CF_i} \quad \boldsymbol{or} \quad \sum_{\boldsymbol{i=6}}^{\boldsymbol{11}} \boldsymbol{C_{i,t}} \tag{3}$$

The pattern of those components are shown on the figure 1 in term of log difference :

$$\Delta cc_{t+1} = log\left(\frac{CC_{t+1}}{CC_t}\right) \tag{4}$$

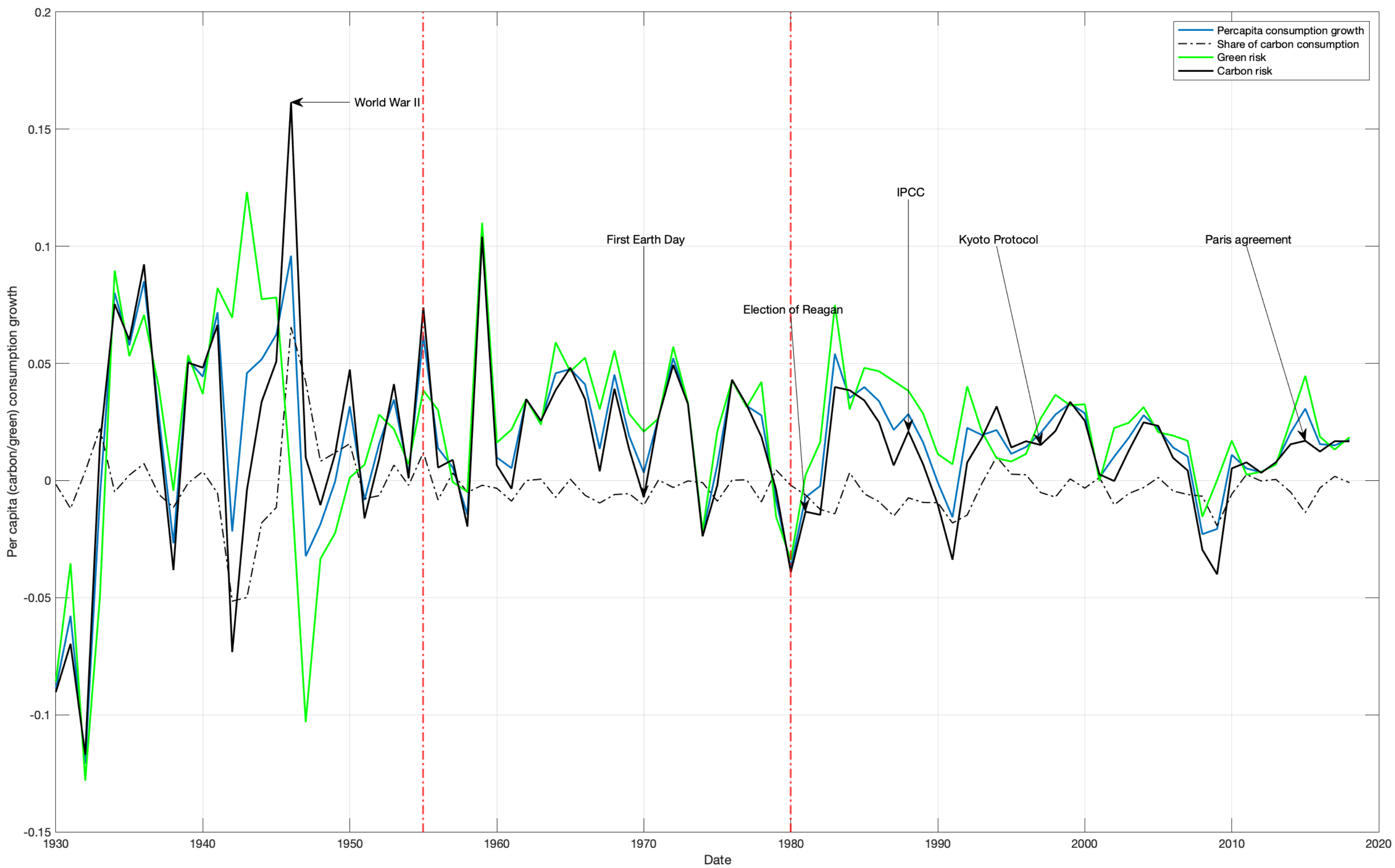
$$\Delta gc_{t+1} = log\left(\frac{GC_{t+1}}{GC_t}\right) \tag{5}$$

$$\Delta \alpha_{cc,t+1} = log\left(\frac{\alpha_{cc,t+1}}{\alpha_{cc,t}}\right) \tag{6}$$

where $\alpha_{cc,t} = \frac{CC_t}{C_t}$. I call Δcc_{t+1} carbon consumption growth risk, Δgc_{t+1} green consumption growth risk, $\Delta \alpha_{cc,t}$ share of carbon consumption growth risk. I could have defined in the same manner the share of green consumption growth risk ($\Delta \alpha_{gc,t}$).

Figure 5 displays the time-series of the key variables of this paper. One can clearly see that the series replicate some business cycles and climate change events.

Figure 5: **Carbon risk, green risk and Household expenditure growth**



In order to link the real economy to the financial market, I also use data on industry, small, large, value and growth portfolios returns from Kenneth R. French website. I use value weighted portfolios including and excluding dividend to compute the dividend and price series on the per-share basis (Campbell and Shiller (1988), Hansen et al. (2008)). The table 1 presents some descriptive statistics. All returns and dividend growth series have been deflated using the consumer price index growth.

Table 1: Summary statistics

	$E(.)$	$\sigma(.)$	AC(1)	AC(2)	AC(3)	AC(4)	AC(5)
				Macro variables			
Δc	0.0178	0.0343	0.3150	0.0608	-0.1508	-0.1491	0.0098
$\Delta\alpha_{cc}$	-0.0030	0.0134	0.4545	0.0574	-0.1233	-0.2765	-0.1730
Δcc	0.0147	0.0382	0.2717	-0.0223	-0.2057	-0.1549	0.0176
$\Delta\alpha_{gc}$	0.0042	0.0215	0.4469	0.0640	-0.0749	-0.2692	-0.2105
Δgc	0.0220	0.0385	0.4647	0.2063	-0.0167	-0.2016	-0.1006
				Financial variables			
Δd	0.0176	0.1223	0.1075	-0.1832	-0.1502	-0.0930	0.0459
z_m	3.3878	0.5123	0.9276	0.8524	0.7992	0.7605	0.7163
r_m	0.0694	0.1929	0.0077	-0.2202	0.0181	-0.0053	-0.1215
r_f	0.0025	0.0351	0.6852	0.3059	0.2040	0.2336	0.2788

The table reports the sample mean, standard deviation, and first-order to fifth order autocorrelation of the marketwide log price/dividend ratio, the log dividend, consumption and (share of) carbon/green consumption growth rates.

2 Model

The model builds on Bansal and Yaron (2004) LRR model and use insight from Giglio et al. (2021). My model introduces three state variables : a long-run risk variable, the variance of the innovation of the carbon consumption growth alongside with the share of carbon consumption out of total consumption growth rate that jointly drive the conditional mean of the aggregate consumption and dividend growth rates.

2.1 Preferences

In this economy, there is a representative household with recursive preferences (Kreps and Porteus (1978), Epstein and Zin (1989)). This paper chooses these type of preferences for two main reasons : First, they allow for separation between the coefficient of risk aversion and the elasticity of inter-temporal substitution. Second, an Epstein–Zin (EZ) investor's marginal utility depends both on the one-period innovation in consumption growth rate and on news about consumption growth at future horizons. Which is an important feature for the climate change thematic as the news about future global warming will affect consumers consumption behaviors. Hence, the consumption growth will incur a proportional shock. One would like an utility function specification that affects the level of climate risk premium and the term structure of the discount rate. Epstein-Zin utility specified in equation 7 does what I just described.

$$V_t = \left[(1-\delta)C_t^{\frac{1-\gamma}{\theta}} + \delta\left(E_t[V_{t+1}]^{1-\gamma}\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}} \tag{7}$$

where δ is the subjective discount factor parameter, $\gamma > 0$ is the coefficient of risk aversion, $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$ with $\psi > 0$ represents the elasticity of inter-temporal substitution (EIS). The standard time-separable power utility model is a special case for the EZ utility when $\gamma = \frac{1}{\psi}$. The agent prefers early resolution of the risk if $\gamma > \frac{1}{\psi}$ and late resolution if $\gamma < \frac{1}{\psi}$.

In this formulation, the household evaluates her consumption plan recursively. She consumes at time t and receives a continuation value of her consumption which can bear a long-run risk component through it carbon consumption. Indeed, with a canonical expected utility risk, only short-run risks are compensated, while long-run risks do not carry separate risk premium. With the above preference, long-run risks earn a positive risk premium as long as households prefer early resolution of uncertainty.

Furthermore, there are N+1 tradable assets in the economy: one risk-free asset ($i = 0$) and N risky assets ($i = 1, \ldots, N$). In each period t, the representative household invests X_{it} unit of its discretionary wealth in asset i. The tradable asset i has a price of P_{it} and a future dividend of D_{it}, with a gross return of $R_{it+1} = \frac{D_{it+1}+P_{it+1}}{P_{it}}$. The inter-temporal budget constraint faced by the household is given by

$$C_t + \sum_{i=1}^{N+1} P_{it} X_{i,t+1} = \sum_{i=1}^{N+1} (P_{it} + D_{it}) X_{it} = W_t \tag{8}$$

where

$$C_t = CC_t + GC_t$$

is the total consumption and the sum of the consumption considered as carbon consumption (CC_t) and the consumption considered as green consumption (GC_t).

2.2 A long-run carbon consumption risks (LRCCR) model

This paper assumes that consumption growth rate in the economy depends on two components. One carry a long-run risk and the other does not carry any long-run risks. In particular, we assume that aggregate consumption growth is given by (the proof can be found in the appendix A) :

$$\Delta c_{t+1} = \underbrace{\Delta cc_{t+1}}_{Carbon} - \underbrace{\Delta\alpha_{cc,t+1}}_{share\ of\ cc} \tag{9}$$

where $\Delta c_{t+1} = log\left(\frac{C_{t+1}}{C_t}\right)$ is the log consumption growth rate. Δcc_{t+1} is the growth rate of the carbon consumption and $\Delta\alpha_{cc,t+1}$ is the growth rate of the share of carbon consumption in the total consumption as defined in section 1 equations 4-6. Note that the conditional mean of cc_{t+1} and its conditional volatility are a source of carbon consumption risk. In fact, transition to low-carbon economy raises the future likelihood of carbon consumption risk, which if realized, lead to a consumption risk. For instance, the natural determined contribution(NDC) policy scenario aims to reduce carbon consumption by 32(15) giga tons of CO2 to stay within the 1.5° C(2° C) by 2030. The dynamic of the other variables are described as follows :

$$\Delta cc_{t+1} = \nu_{cc} + x_t + \sigma_t \epsilon_{cc,t+1} \tag{10}$$

$$x_{t+1} = \rho_x x_t + \psi_x \sigma_t \epsilon_{x,t+1} \tag{11}$$

$$\sigma^2_{t+1} = (1-\nu)\sigma^2 + \nu\sigma^2_t + \sigma_w \epsilon_{\sigma,t+1} \tag{12}$$

Thus, in our model x_t, $\Delta\alpha_{cc,t}$, and σ^2_t are the state variables. In particular, x_t captures the conditional mean of the carbon consumption growth rate, while σ^2_t captures the uncertainty due to the transition to lower carbon economy. The share of carbon consumption out of total consumption growth rate component doesn't carry any long-run risk and evolves as given by

$$\Delta\alpha_{cc,t+1} = \nu_\alpha(1-\rho_\alpha) + \rho_\alpha \Delta\alpha_{cc,t} + \sigma_\alpha \epsilon_{\alpha,t+1} + \pi\sigma_t \epsilon_{cc,t+1} \tag{13}$$

This paper assumes that the innovation of the share and the one of the carbon consumption growth are correlated. That correlation depends on the parameters π[3] and σ_α. Finally, the dividend growth rate of any dividend paying asset i is as follow

$$\Delta d_{i,t+1} = \nu_i + \phi_i x_t + \phi_{\alpha,i}\Delta\alpha_{cc,t} + \psi_i \sigma_t \epsilon_{i,t+1} \tag{14}$$

[3] Alternative specification : $\epsilon_{\alpha,t}$ and $\epsilon_{cc,t}$ are correlated instead of iid and set $\pi = 0$

Where $\phi_i,\ \phi_{\alpha,i},\ \psi_i$ determine asset i's exposure to the long-run, share of carbon consumption and volatility risks, respectively. The shocks $\epsilon_{x,t+1}$, $\epsilon_{\alpha,t+1}$, $\epsilon_{i,t+1}$, $\epsilon_{cc,t+1}$ and $\epsilon_{\sigma,t+1}$ are assumed to be i.i.d. N(0,1) and mutually independent. Equations (9)-(14) represent the building block of our long-run carbon consumption risks model, henceforth LRCCR model. The dynamic of the variables and the utility function involve seventeen parameters Θ=[$\rho_x\ \psi_x\ \psi_i\ \nu_{cc}\ \nu\ \nu_i\ \sigma_w\ \sigma\ \phi_i\ \delta\ \gamma\ \psi\ \nu_\alpha\ \rho_\alpha\ \sigma_\alpha\ \pi\ \phi_{\alpha,i}$]. I calibrate the model parameters to match key sample moments. I derive some moments condition for the carbon consumption, the share of carbon consumption and asset i dividend growth rates as functions of the time-series and the preferences parameters. See the appendices section 4 for more details.

2.3 Solving the model

For any asset i, the corresponding Euler equation regarding from the consumer's utility maximization is given by:

$$\mathbb{E}_t[e^{m_{t+1}+r_{i,t+1}}] = 1 \tag{15}$$

where

$$m_{t+1} = \theta log(\delta) - \frac{\theta}{\psi}\Delta cc_{t+1} + \frac{\theta}{\psi}\Delta\alpha_{cc,t+1} + (\theta-1)r_{c,t+1} \tag{16}$$

is the natural logarithm of the stochastic discount factor; $\mathbb{E}_t[.]$ denotes expectation conditional on time t information; $r_{i,t+1}$ is the continuously compounded return on asset i; and $r_{c,t+1}$ is the unobservable continuously compounded return on an asset that delivers aggregate consumption as its dividend each period.

Following Campbell and Shiller (1988), the log return on the consumption claim, namely $r_{c,t+1}$, and the log return of the asset i $r_{i,t+1}$ are approximated as follow :

$$r_{c,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + \Delta cc_{t+1} - \Delta\alpha_{cc,t+1} \tag{17}$$

$$r_{i,t+1} = \kappa_{0,i} + \kappa_{1,i} z_{i,t+1} - z_{i,t} + \Delta d_{i,t+1} \tag{18}$$

where $z_t = \log\left(\frac{P_{m,t}}{C_t}\right)$ and $P_{m,t}$ stands for the market portfolio price, $z_{i,t} = \log\left(\frac{P_{i,t}}{D_{i,t}}\right)$. $\kappa_1 = \frac{e^{\bar{z}}}{1+e^{\bar{z}}}$ and $\kappa_0 = \log\left(1+e^{\bar{z}}\right) - \kappa_1 \bar{z}$ are log-linearization constants. $\bar{z}$ denotes the long-run mean of the log price/consumption ratio (z). Regarding equation (18), $\kappa_{1,i} = \frac{e^{\bar{z}_i}}{1+e^{\bar{z}_i}}$ and $\kappa_{0,i} = \log(1+e^{\bar{z}_i}) - \kappa_1 \bar{z}_i$ where $\bar{z}_i$ denotes the long-run mean of the log price/dividend ratio (z_i). Whenever this paper uses a subscript m, it refers to the market portfolio and whenever, I use a subscript i, it refers to any asset.

As in Bansal and Yaron (2004), I conjecture that z_t and $z_{i,t}$ are affine functions of the state variables x_t (LRR variable or conditional expected carbon consumption growth), σ_t^2 (conditional volatility of the carbon consumption growth), and $\Delta\alpha_{cc,t}$ (share of carbon consumption growth).

$$z_t = A_0 + A_1 x_t + A_2 \sigma_t^2 + A_3 \Delta\alpha_{cc,t} \tag{19}$$

$$z_{i,t} = A_{0,i} + A_{1,i} x_t + A_{2,i} \sigma_t^2 + A_{3,i} \Delta\alpha_{cc,t} \tag{20}$$

A_0, A_1, A_2, A_3, $A_{0,i}$, $A_{1,i}$, $A_{2,i}$, and $A_{3,i}$ are function of parameters in Θ and the linearization parameters. Their expressions are given in the Appendix. An increase in the expected carbon consumption growth rate will raise the price-consumption ratio as long as the intertemporal substitution effect dominates the wealth effect. However, the a higher share of carbon consumption out of total consumption implies a lower price-consumption ratio when $\psi > 1$. Turning now into the price-dividend ratio,the conclusions are different for the share of carbon consumption growth effect. While the expected carbon consumption growth still raise the price dividend ratio but much higher under the conditions that $\psi > 1$ and $\phi_i > 1$ (the LRR variable acts as a leverage), the share of carbon consumption now positively affects the price-dividend ratio hypothesizing $\phi_{\alpha,i} > 0$.

Using equation 15, I show that the log-risk free rate can be written in function of the carbon-consumption and its volatility (state variables) as follows :

$$\begin{aligned} r_{f,t} &= -log\mathbb{E}_t[e^{m_{t+1}}] \\ &= A_{0,f} + A_{1,f}x_t + A_{2,f}\sigma_t^2 + A_{3,f}\Delta\alpha_{cc,t} \end{aligned} \tag{21}$$

Once again $A_{0,f}$, $A_{1,f}$, $A_{2,f}$,$A_{3,f}$ are function of parameters in Θ and the linearization parameters and their expression are given in the Appendix.

2.4 Testable implications for Asset prices: m_{t+1}

To test the implication of the model for equity premium and the cross-section of returns, I combine equations (16), (17), (19) to get the expression of the stochastic discount factor in terms of state variables :

$$\begin{aligned} m_{t+1} = &(\theta log(\delta) + (\theta-1)[\kappa_0 + (\kappa_1 - 1)A_0]) + \left(-\frac{\theta}{\psi} + (\theta-1)\right)\Delta cc_{t+1} \\ &+ \left(\frac{\theta}{\psi} - (\theta-1) + (\theta-1)\kappa_1 A_3\right)\Delta\alpha_{cc,t+1} \\ &+ (\theta-1)\kappa_1 A_1 x_{t+1} + (\theta-1)\kappa_1 A_2 \sigma_{t+1}^2 \\ &- (\theta-1)A_1 x_t - (\theta-1)A_2\sigma_t^2 - (\theta-1)A_3\Delta\alpha_{cc,t} \end{aligned} \tag{22}$$

The innovation in the m_{t+1} conditional on time-t information is given by:

$$m_{t+1} - \mathbb{E}_t[m_{t+1}] = -\lambda_{m,\alpha}\sigma_\alpha\epsilon_{\alpha,t+1} - \lambda_{m,cc}\sigma_t\epsilon_{cc,t+1} - \lambda_{m,x}\sigma_t\epsilon_{x,t+1} - \lambda_{m,w}\sigma_w\epsilon_{\sigma,t+1} \tag{23}$$

Therefore, the equity risk premium for any asset i is :

$$\mathbb{E}_t(r_{i,t+1} - r_{f,t}) + 0.5\mathbb{V}_t(r_{i,t+1}) = \lambda_{m,x}\beta_{i,x}\sigma_t^2 + \lambda_{m,w}\beta_{i,w}\sigma_w^2 + \lambda_{m,\alpha}\beta_{i,\alpha}\sigma_\alpha^2 + \lambda_{m,cc}\beta_{i,cc}\sigma_t^2 \tag{24}$$

where the β's are the asset i exposures to the long-run risk, the volatility risk, the share of carbon consumption risk and the short run risk, and the λ's are the respective risk prices. β's and λ's are functions of the preference parameters, the linearization parameters and the dynamics of macro-financial variables dynamic parameters (see the appendix 4). The price of the short-run carbon consumption risk and the exposure of any asset on this risk rise with the the correlation between the share of carbon growth and the carbon growth. $\lambda_{m,x}$, and $\beta_{i,x}$ increase with the persistence of the expected carbon growth. In the same way, $\lambda_{m,cc}$, $\lambda_{m,\alpha}$, $\beta_{i,cc}$, and $\beta_{i,\alpha}$ increase with the persistence of the share of carbon consumption out of total consumption growth.

I substitute equation (22) into the set of Euler equations (15) to have moment conditions that are expressed entirely in terms of observables. Then I examine the empirical plausibility of the model when the set of assets in the economy consists of the market portfolio and the risk-free rate, thereby focusing on the equity premium and risk-free rate puzzles. In particular, I consider a set of moments, namely the expected value and the standard deviation of the equity premium, the real risk-free rate, and the price-dividend ratio and calibrate the parameters Θ to match those moments. Next, this paper examines whether the model can explain the the cross-section of returns in different asset classes including "carbon intensive" (high heat-exposed and low heat-exposed) portfolios and Fama-French twenty-five portfolios. In total, I use the 42 Fama-French industry portfolios and 25 FFPs.

2.5 Cross-sectional implication of the LRCCR model

This paper decomposes the consumption growth risk Δc_t into two new risks. Namely, the carbon consumption (cc) growth risk (Δcc_t) and the share of carbon consumption (shcc) growth risk ($\Delta\alpha_{cc,t}$). I adopt Fama and MacBeth (1973) two-pass regression methodology to estimate the risk premia on each risk factors (see also Kan et al. (2013), Bai and Zhou (2015)). In the first stage, I compute the portfolios exposures to the risk factors by regressing the each portfolio excess return ($r_{i,t}$) on Δcc_t and $\Delta\alpha_t$:

$$r_{i,t} = c_i + \beta_{sh,i}\Delta\alpha_{cc,t} + \beta_{cc,i}\Delta cc_t + \epsilon_{i,t} \quad i = 1, ..., N \tag{25}$$

In the second stage, I run a cross-sectional regression of $r_{i,t}$ on the betas from the first stage time-series regression for each period t (see equation 27).

$$r_{i,t} = a_t + \gamma_{sh,t}\hat{\beta}_{sh,i} + \gamma_{cc,t}\hat{\beta}_{cc,i} + \epsilon_{i,t} \quad t = 1, ..., T \tag{26}$$

Rather than running T cross-sectional regression (CSR), I run a single CSR of the sample excess return mean μ_r a constant and the betas estimated from the first stage. Put in equation terms, it gives :

$$\mu_{r,i} = \gamma_0 + \gamma_{sh}\hat{\beta}_{sh,i} + \gamma_{cc}\hat{\beta}_{cc,i} + \epsilon_i \quad i = 1, ..., N \tag{27}$$

For comparison purpose, this paper applies also the two-pass regression to the case of one consumption growth risk factor. I plot the exposures $\beta_{sh,i}$'s and $\beta_{cc,i}$'s in Figure 7. Once we know which portfolio (industry) is significantly positively and negatively exposed to the carbon risk, I calibrate the LRCCR model to match key moments of those portfolios (industries). The main goal is to explain the cross-section of the portfolios (industries) expected returns. My parameters of interest are the leverage parameters, i.e the dividend exposures to the long-run risk variable and to the share of carbon consumption growth rate for each portfolio (industry) ϕ_i , and $\phi_{\alpha,i}$ and the dividend exposure to volatility risks ψ_i. To test the effectiveness of the LRCCR model, I start by looking at the cross-sectional properties of the well-known portfolios. Especially, the value, growth, small size and large portfolios. Based on empirical evidence (see Bansal et al. (2005) and Hansen et al. (2008), Bansal et al. (2016a)), the value portfolio presents much higher exposure to low-frequency risks in consumption relative to the growth portfolio. Likewise, long-run risk exposure of the small-size portfolio exceeds that of the large-size portfolio. Next, I look at the cross-sectional properties of the Fama-French industry portfolios.

3 Model estimation

3.1 Equity premium, volatility and risk-free rate puzzles : BY2004 versus LRCCR

In this part, this paper compares the LRCCR and LRR models in terms of replicating the observed equity premium, volatility of the equity premium and the risk-free rate. This paper calibrates the seventeen parameters Θ=[ρ_x ψ_x ψ_d ν_{cc} ν ν_d σ_w σ ϕ_d δ γ ψ ν_α ρ_α σ_α π $\phi_{\alpha,d}$] to match (share of) carbon consumption growth, (share of) green consumption growth, dividend growth, market return and risk-free rate means, variances, (auto)correlations observed in the data. The calibration results are displayed in the table 2 for both my setting and Bansal and Yaron (2004) setting. I present four set of results : full sample, period around the World War II, period around the First Earth Day and finally post Reagan Election sample. This paper do this split because-as we know-returns react to news and back in the days, climate change or global warming was not a big issue. As a consequences, we hypothesize that there is probably no big effect during the pre-Reagan Election period using the President Reagan Election as reference day because global public awareness of energy conservation and improvements in energy efficiency start around this period.

Ψ_x tells us how detectable the long-run variable is. The results show that the long-run risk variable is more detectable than BY during 1956-2018 near climate change events period. From results in the table 2, there is long-run risks in the volatility and the carbon consumption growth : ν smaller and close to one; and ρ_x smaller and close to one. Overall, the risk aversion in our model is higher than the one in Bansal and Yaron (2004) setting but is in a reasonable range. This high value is due to the nature of the risk I am talking about (carbon risk). Furthermore,agents fear more carbon risk than consumption risk because the carbon risk will increase even more (amplify) the consumption risk. $\phi_{\alpha,d}$ functions as a leverage ratio on the share of carbon consumption growth during the period 1981-2018.

Table 2: Calibrated parameters

	1930-2018		1930-1955		1956-1980		1981-2018	
	BY04	LRCCR	BY04	LRCCR	BY04	LRCCR	BY04	LRCCR
ρ_x	0.932	0.978	0.937	0.979	0.920	0.900	0.976	0.900
ψ_x	0.259	0.150	0.278	0.119	0.010	0.204	0.206	0.514
ψ_d	4.540	4.340	4.789	4.488	13.361	0.000	10.122	4.288
ν_x	9E-04	1E-03	1E-04	1E-03	-6E-05	2E-03	-2E-04	9E-04
ν	0.999	0.979	0.573	0.985	0.577	0.691	0.988	0.995
ν_d	0.001	-0.011	0.000	-0.025	-0.002	0.001	0.005	0.003
σ_w	5E-07	2E-08	1E-04	2E-07	7E-08	1E-05	4E-06	4E-06
σ	8E-03	3E-03	5E-04	4E-03	1E-03	1E-03	7E-04	9E-03
ϕ	2.294	3.378	2.354	3.734	321.850	10.056	0.792	1.019
δ	0.956	0.998	0.999	0.999	0.998	0.998	0.998	0.997
γ	7.074	12.290	9.878	10.084	15.940	23.016	6.063	8.732
ψ	1.379	1.487	3.018	1.495	1.574	1.235	1.503	1.486
$\bar{z}$	3.088	6.164	6.054	6.602	6.201	6.285	5.720	5.060
$\bar{z}_m$	5.344	3.981	5.153	3.522	4.754	5.696	12.820	5.548
ν_a		-3E-04		4E-05		-3E-04		-4E-04
ρ_a		0.455		0.480		-0.281		0.360
σ_a		0.006		0.006		0.014		0.004
π		1.344		0.897		3.328		0.626
ϕ_a		0.590		0.877		-0.294		1.305

The table reports the calibrated parameters for the different sub-samples for both our setting (LRCCR) and Bansal and Yaron (2004) setting (BY04).

This paper simulates the time series of the model-implied carbon consumption growth, share of carbon consumption growth, dividend growth, market return and risk-free rate and present some quantiles of those series in the table 3 and 8 for the four samples. The quantiles 5% and 95% serve as the confident interval and the sample moments are overall within those intervals generated by our simulation in the preferred sub-samples.

Table 3: Model-implied moments.

		$\sigma(z_m)$	EP	$E(R_f)$	$\sigma(r_{m,a})$	$\sigma(r_{f,a})$	$\rho(z_m)$
1930-2018							
	Data	0.512	0.067	0.002	0.193	0.002	0.928
BY2004	Mean	0.189	0.096	0.007	0.212	0.068	0.454
	5%	0.158	-0.029	-0.090	0.180	0.050	0.286
	50%	0.188	0.097	0.005	0.212	0.065	0.458
	95%	0.225	0.220	0.111	0.246	0.095	0.607
LRCCR	Mean	0.197	0.078	0.010	0.134	0.022	0.735
	5%	0.154	0.053	0.001	0.118	0.019	0.601
	50%	0.195	0.078	0.010	0.134	0.022	0.743
	95%	0.248	0.104	0.019	0.151	0.027	0.841
1981-2018							
	Data	0.415	0.072	0.011	0.162	0.011	0.890
BY2004	Mean	0.078	0.066	0.010	0.128	0.002	0.846
	5%	0.042	0.028	0.005	0.103	0.001	0.642
	50%	0.073	0.066	0.010	0.127	0.002	0.871
	95%	0.133	0.106	0.015	0.154	0.004	0.961
LRCCR	Mean	0.204	0.118	0.015	0.183	0.006	0.809
	5%	0.118	0.061	0.003	0.148	0.003	0.586
	50%	0.191	0.117	0.015	0.182	0.005	0.835
	95%	0.330	0.178	0.027	0.220	0.009	0.946

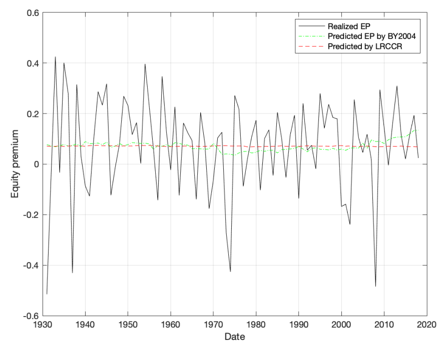
The table reports the model-implied moments (the equity premium (EP), the mean of the risk free rate, the standard deviations of the log price-dividend ratio, the market return, and the risk-free rate, and the first-order autocorrelation of the log price-dividend ratio.) alongside with some 20-quantiles.

Now let us turn into the predictability implication of my model versus the one of Bansal and Yaron (2004) by comparing the predicted equity premium, consumption growth ; and dividend growth rates and their realized counterparts. Most of the consumption capital asset pricing models find a constant risk premium : approximately constant predicted risk premium. However, during period of high carbon emission risk 1980-2018, a long-run carbon consumption risks model finds a time-varying risk premium. My model is much better than the usual Long-run risk model. (see Figures 6, and 8 in the Appendix). The difference is quite clear when I predict the macro-financial variables in the sub-sample, especially during the period 1981-2018, period starting around the election of the United- States President Reagan.

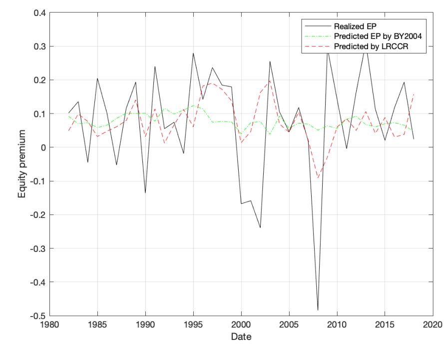
3.2 Risks and Price of risks

In this section, I compute the price of the four risk sources this paper talked about in the model section : carbon consumption growth risk, share of carbon consumption growth risk - which is correlated to the share of green consumption growth risk-, long-run risk and the volatility risk. The most important result from table 4, is the consistent sign of the contribution of the volatility risk in the equity premium under my model. The market is negatively exposed to the volatility risk in every sample I considered.

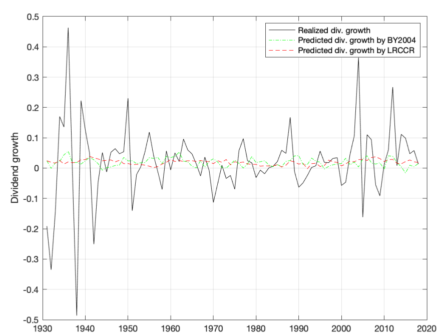
Figure 6: **Realized versus predicted equity premium, consumption growth and dividend growth.** In this figure,



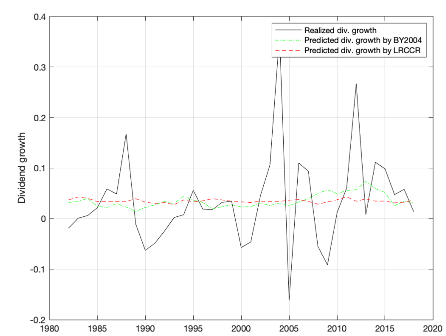
(a) Full sample : 1930-2018



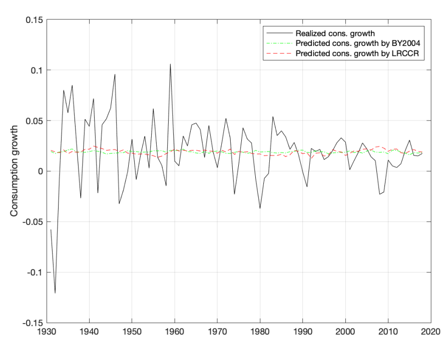
(b) 1930-1955



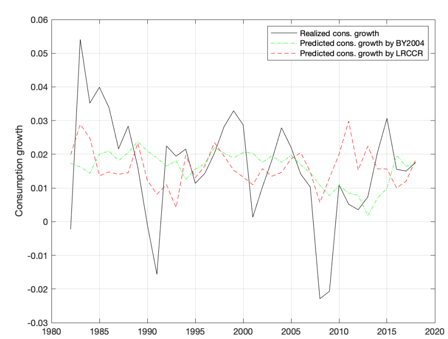
(c) Full sample : 1930-2018



(d) 1981-2018



(e) Full sample : 1930-2018



(f) 1981-2018

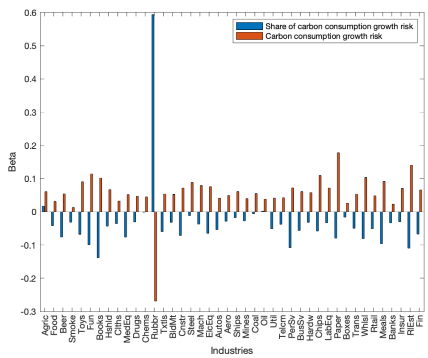
Table 4: Market prices of risks and effects on the risk premium

		λ	β	effect	λ	β	effect
		1930-2018			1930-1955		
BY04	srr	7.07	0.00	0	9.88	0.00	0
	lrr	14.47	5.58	+	40.61	8.17	+
	vr	-2564.24	-7723.52	+	-1884.32	-515.00	+
LRCCR	$\alpha risk$	-21.93	1.59	$-$	-18.75	2.18	$-$
	srr	-17.19	2.13	$-$	-6.74	1.96	$-$
	lrr	73.78	10.16	+	50.61	7.28	+
	vr	-122914.44	-14400.98	+	-79112.00	-6155.20	+
		1956-1980			1981-2018		
BY04	srr	15.94	0.00	0	6.06	0.00	0
	lrr	1.92	37.37	+	40.87	1.09	+
	vr	-273.13	1687.23	$-$	-51947.16	5284.64	$-$
LRCCR	$\alpha risk$	-18.15	-0.41	+	-13.22	2.40	$-$
	srr	-37.37	-1.35	+	0.46	1.50	+
	lrr	44.47	18.41	+	38.93	2.04	+
	vr	-5168.90	-1975.90	+	-66622.82	-4374.95	+

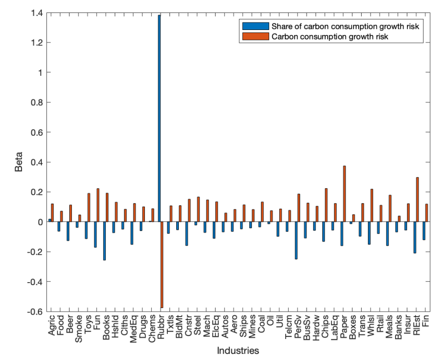
3.3 Cross-sectional implication

As one can see from the figure 7, I cannot classify the industries based on their betas until 1955. All industries are positively and negatively exposed to cc growth risk and share of carbon consumption growth risk respectively. Except Rubber and Plastic Products industry, which is negatively and positively exposed to carbon consumption growth risk and share of cc growth risk respectively. As of 1956, the risk factors start to affect differently the industries. This is an really interesting results, as it tells us our risk factors become able to identify industries which pollute the most based on their risk exposures.

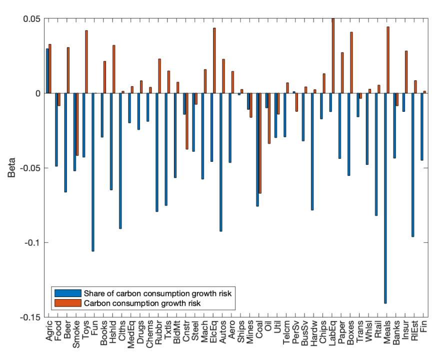
Figure 7: Industries exposures to carbon consumption risks : β's



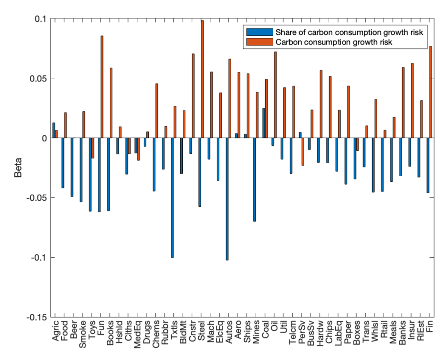
(a) Full sample



(b) 1930-1955



(c) 1956-1980



(d) 1981-2018

4 Conclusion

This article tackles the long-run carbon consumption risks model by allowing both long-run risks in mean and in volatility. We use Epstein-Zin utility function to disentangle the risk aversion coefficient and because of its property to deal with climate change thematic. This paper found empirical support to the long-run risks model in the context of carbon-green consumption. We have three state variables which define completely the other variables in the economy. To sum up, our long-run carbon consumption model solves the equity premium, volatility and risk-free rate puzzles by decomposing the consumption growth into two components-the growth rate of carbon consumption component and the growth rate of the share of green consumption out of total consumption. Our model setting increases the ability of the investors to detect the long-run risk. Namely, investors can profit from it by using climate change news. Also, our long-run risk variable explains the cross section of industries/firms. Thus, this paper recommends to use the two carbon risk measures we computed to identify industries or firms which pollute the most and construct an investment strategy that minimize/maximize a long term environmental criteria.

Further research can use other proxies for the green component in the consumption decomposition and do the same analysis. Such proxy could be R&D expenses of carbon-intensive firms allocated to green technology or the revenue from selling the Solar Renewable Energy Certificates (SRECs).

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Appendices

A Consumption growth decomposition

Let us consider I categories of consumption among which J carbon consumption categories and $I-J$ green consumption categories.

$$C_t = \sum_{i=1}^{I} C_{i,t} \tag{28}$$

$$C_t = \sum_{i=1}^{J} C_{i,t} + \sum_{i=J+1}^{I} C_{i,t} \tag{29}$$

$$C_t = CC_t + GC_t \tag{30}$$

Growth rate decomposition :

$$\Delta c_{t+1} = \log(CC_{t+1} + GC_{t+1}) - \log(CC_t + GC_t) \tag{31}$$

$$\Delta c_{t+1} = \log(CC_{t+1}) + \log\frac{CC_{t+1} + GC_{t+1}}{CC_{t+1}} - \log(CC_t) - \log\frac{CC_t + GC_t}{CC_t} \tag{32}$$

$$\Delta c_{t+1} = \Delta cc_{t+1} - \left(\log\frac{CC_{t+1}}{CC_{t+1} + GC_{t+1}} - \log\frac{CC_t}{CC_t + GC_t}\right) \tag{33}$$

$$\Delta c_{t+1} = \Delta cc_{t+1} - \Delta\alpha_{CC,t+1} \tag{34}$$

where Δc_{t+1}, Δcc_{t+1} and $\Delta\alpha_{CC,t+1}$ are consumption, carbon consumption and carbon consumption share growth rates (log-difference) respectively.

B Price of risks

$$\lambda_{m,\alpha} = (-\gamma + (1-\theta)\kappa_1 A_3) \tag{35}$$

$$\lambda_{m,cc} = \gamma + (-\gamma + (1-\theta)\kappa_1 A_3)\,\pi \tag{36}$$

$$\lambda_{m,x} = (1-\theta)\kappa_1 A_1 \psi_x \tag{37}$$

$$\lambda_{m,w} = (1-\theta)\kappa_1 A_2 \tag{38}$$

are prices of risk that correspond to the four sources of risk $\epsilon_{\alpha,t+1},\ \epsilon_{cc,t+1},\ \epsilon_{x,t+1}, \epsilon_{\sigma,t+1}$.

C Theoretical moments calculation

From the carbon/green consumption growth rate processes, we have :

$$\mathbb{E}[\Delta cc_{t+1}] = \nu_{cc} \tag{39}$$

$$\mathbb{E}[\Delta\alpha_{cc,t+1}] = \nu_{\alpha} \tag{40}$$

$$\begin{aligned}\mathbb{V}[\Delta cc_{t+1}] &= \mathbb{V}[\nu_{cc} + x_t + \sigma_t\epsilon_{cc,t+1}] \\ &= \mathbb{V}[x_t] + \mathbb{V}[\sigma_t\epsilon_{cc,t+1}] \\ &= \frac{\psi_x^2}{1-\rho_x^2}\sigma^2 + \sigma^2 \end{aligned} \tag{41}$$

$$\begin{aligned}\mathbb{V}[\Delta\alpha_{cc,t+1}] &= \mathbb{V}[\nu_\alpha(1-\rho_\alpha) + \rho_\alpha\Delta\alpha_{cc,t} + \sigma_\alpha\epsilon_{\alpha,t+1} + \pi\sigma_t\epsilon_{cc,t+1}] \\ (1-\rho_\alpha^2)\mathbb{V}[\Delta\alpha_{cc,t+1}] &= \sigma_\alpha^2 + \pi^2\sigma^2 \\ \mathbb{V}[\Delta\alpha_{cc,t+1}] &= \frac{\sigma_\alpha^2 + \pi^2\sigma^2}{1-\rho_\alpha^2} \end{aligned} \tag{42}$$

$$Cov[\Delta cc_{t+1}, \Delta cc_{t+2}] = \rho_x \frac{\psi_x^2}{1-\rho_x^2}\sigma^2 \tag{43}$$

$$\begin{aligned}
Cov[\Delta\alpha_{cc,t+1}, \Delta\alpha_{cc,t+2}] &= Cov[\Delta\alpha_{cc,t+1}, \rho_\alpha \Delta\alpha_{cc,t+1} + \sigma_\alpha \epsilon_{\alpha,t+2} + \pi\sigma_{t+1}\epsilon_{cc,t+2}] \\
&= \rho_\alpha \mathbb{V}[\Delta\alpha_{cc,t+1}] \\
&= \rho_\alpha \frac{\sigma_\alpha^2 + \pi^2\sigma^2}{1-\rho_\alpha^2}
\end{aligned} \tag{44}$$

From the dividend growth rate process, we can get :

$$\mathbb{E}[\Delta d_{t+1}] = \nu_i + \phi_{\alpha,i}\nu_\alpha \tag{45}$$

$$\begin{aligned}
\mathbb{V}[\Delta d_{i,t+1}] &= \phi_i^2 \mathbb{V}[x_t] + \phi_{\alpha,i}^2 \mathbb{V}[\Delta\alpha_{cc,t}] + \psi_i^2 \mathbb{V}[\sigma_t \epsilon_{i,t+1}] \\
&= \phi_i^2 \frac{\psi_x^2}{1-\rho_x^2}\sigma^2 + \phi_{\alpha,i}^2 \frac{\sigma_\alpha^2 + \pi^2\sigma^2}{1-\rho_\alpha^2} + \psi_i^2\sigma^2
\end{aligned} \tag{46}$$

$$\begin{aligned}
Cov[\Delta d_{i,t+1}, \Delta d_{i,t+2}] &= Cov[\phi_i x_t + \phi_{\alpha,i}\Delta\alpha_{cc,t} + \psi_i\sigma_t\epsilon_{i,t+1}, \phi_i x_{t+1} + \phi_{\alpha,i}\Delta\alpha_{cc,t+1} + \psi_i\sigma_{t+1}\epsilon_{i,t+2}] \\
&= \phi_i^2 Cov[x_t, x_{t+1}] + \phi_{\alpha,i}^2 Cov[\Delta\alpha_{cc,t}, \Delta\alpha_{cc,t+1}] \\
&= \phi_i^2 \rho_x \mathbb{V}[x_t] + \phi_{\alpha,i}^2 \rho_\alpha \frac{\sigma_\alpha^2 + \pi^2\sigma^2}{1-\rho_\alpha^2} \\
&= \phi_i^2 \rho_x \frac{\psi_x^2}{1-\rho_x^2}\sigma^2 + \phi_{\alpha,i}^2 \rho_\alpha \frac{\sigma_\alpha^2 + \pi^2\sigma^2}{1-\rho_\alpha^2}
\end{aligned} \tag{47}$$

From both carbon/green consumption and dividend growth rates, we get the cross moments :

$$\begin{aligned}
Cov[\Delta\alpha_{cc,t+1}, \Delta cc_{t+1}] &= Cov[\rho_\alpha \Delta\alpha_{cc,t} + \sigma_\alpha \epsilon_{\alpha,t+1} + \pi\sigma_t\epsilon_{cc,t+1}, x_t + \sigma_t\epsilon_{cc,t+1}] \\
&= \pi\mathbb{V}[\sigma_t\epsilon_{cc,t+1}] \\
&= \pi\sigma^2
\end{aligned} \tag{48}$$

$$\begin{aligned}
Cov[\Delta d_{i,t+1}, \Delta cc_{t+1}] &= Cov[\phi_i x_t + \phi_{\alpha,i}\Delta\alpha_{cc,t} + \psi_i\sigma_t\epsilon_{i,t+1}, x_t + \sigma_t\epsilon_{cc,t+1}] \\
&= \phi_i\mathbb{V}[x_t] \\
&= \phi_i\frac{\psi_x^2}{1-\rho_x^2}\sigma^2
\end{aligned} \tag{49}$$

$$\begin{aligned}
Cov[\Delta d_{i,t+1}, \Delta\alpha_{cc,t+1}] &= Cov[\phi_i x_t + \phi_{\alpha,i}\Delta\alpha_{cc,t} + \psi_i\sigma_t\epsilon_{i,t+1}, \rho_\alpha\Delta\alpha_{cc,t} + \sigma_\alpha\epsilon_{\alpha,t+1} + \pi\sigma_t\epsilon_{cc,t+1}] \\
&= \phi_{\alpha,i}\rho_\alpha\mathbb{V}[\Delta\alpha_{cc,t}] \\
&= \phi_{\alpha,i}\rho_\alpha\frac{\sigma_\alpha^2 + \pi^2\sigma^2}{1-\rho_\alpha^2}
\end{aligned} \tag{50}$$

From the log price dividend process:

$$\mathbb{E}[z_{i,t}] = A_{0,i} + A_{2,i}\sigma^2 + A_{3,i}\nu_\alpha \tag{51}$$

$$\mathbb{V}[z_{i,t}] = A_{1,i}^2\frac{\psi_x^2}{1-\rho_x^2}\sigma^2 + A_{2,i}^2\frac{\sigma_w^2}{1-\nu^2} + A_{3,i}^2\frac{\sigma_\alpha^2+\pi^2\sigma^2}{1-\rho_\alpha^2} \tag{52}$$

$$\begin{aligned}
Cov[\Delta d_{i,t+1}, z_{i,t}] &= Cov[\phi_i x_t + \phi_{\alpha,i}\Delta\alpha_{cc,t} + \psi_i\sigma_t\epsilon_{i,t+1}, A_{1,i}x_t + A_{2,i}\sigma_t^2 + A_{3,i}\Delta\alpha_{cc,t}] \\
&= \phi_i A_{1,i}\frac{\psi_x^2}{1-\rho_x^2}\sigma^2 + \phi_{\alpha,i}A_{3,i}\frac{\sigma_\alpha^2+\pi^2\sigma^2}{1-\rho_\alpha^2}
\end{aligned} \tag{53}$$

$$
\begin{aligned}
Cov[\Delta c_{t+1}, z_{i,t}] &= Cov[\Delta cc_{t+1} - \Delta\alpha_{cc,t+1}, A_{1,i}x_t + A_{2,i}\sigma_t^2 + A_{3,i}\Delta\alpha_{cc,t}] \\
&= Cov[x_t + \sigma_t\epsilon_{cc,t+1}, A_{1,i}x_t + A_{2,i}\sigma_t^2 + A_{3,i}\Delta\alpha_{cc,t}] \\
&- Cov[\rho_\alpha\Delta\alpha_{cc,t} + \sigma_\alpha\epsilon_{\alpha,t+1} + \pi\sigma_t\epsilon_{cc,t+1}, A_{1,i}x_t + A_{2,i}\sigma_t^2 + A_{3,i}\Delta\alpha_{cc,t}] \\
&= A_{1,i}\frac{\psi_x^2}{1-\rho_x^2}\sigma^2 - \rho_\alpha A_{3,i}\frac{\sigma_\alpha^2 + \pi^2\sigma^2}{1-\rho_\alpha^2} \qquad (54)
\end{aligned}
$$

Return on consumption claim $r_{c,t+1}$, on dividend paying asset $r_{i,t+1}$ and risk-free rate $r_{f,t}$

Let us determine $z_t = A_0 + A_1x_t + A_2\sigma_t^2 + A_3\Delta\alpha_{cc,t}$. From the Euler equation 15, we have :

$$
\begin{aligned}
1 &= \mathbb{E}_t e^{\theta log\delta - \frac{\theta}{\psi}\Delta c_{t+1} + \theta r_{c,t+1}} \\
&= e^{\mathbb{E}_t(\theta log\delta - \frac{\theta}{\psi}\Delta c_{t+1} + \theta r_{c,t+1}) + 0.5\mathbb{V}_t(\theta log\delta - \frac{\theta}{\psi}\Delta c_{t+1} + \theta r_{c,t+1})} \\
&= e^{\theta log\delta - \frac{\theta}{\psi}\mathbb{E}_t(\Delta c_{t+1}) + \theta\mathbb{E}_t r_{c,t+1} + 0.5\mathbb{V}_t((-\frac{\theta}{\psi}+\theta)\Delta c_{t+1} + \theta\kappa_1 z_{t+1})} \\
&= exp(\theta log\delta + (1-\gamma)(\nu_{cc} + x_t - \nu_\alpha(1-\rho_\alpha) - \rho_\alpha\Delta\alpha_{cc,t}) + \theta(\kappa_0 - A_0 - A_1x_t - A_2\sigma_t^2 - A_3\Delta\alpha_{cc,t}) \\
&+ \theta\kappa_1(A_0 + A_1\rho_x x_t + A_2((1-\nu)\sigma^2 + \nu\sigma_t^2) + A_3(\nu_\alpha(1-\rho_\alpha) + \rho_\alpha\Delta\alpha_{cc,t})) \\
&+ 0.5\left\{\left((1-\gamma+\pi(-1+\gamma+\theta\kappa_1A_3))^2 + (\theta\kappa_1A_1)^2\psi_x^2\right)\sigma_t^2 + (-1+\gamma+\theta\kappa_1A_3)^2\sigma_\alpha^2 + \theta^2\kappa_1^2A_2^2\sigma_w^2\right\}) \\
0 &= \theta log\delta + (1-\gamma)(\nu_{cc} - \nu_\alpha(1-\rho_\alpha)) + \theta(\kappa_0 - A_0) + \theta\kappa_1(A_0 + A_2(1-\nu)\sigma^2 + A_3\nu_\alpha(1-\rho_\alpha)) \\
&+ 0.5\left\{(-1+\gamma+\theta\kappa_1A_3)^2\sigma_\alpha^2 + \theta^2\kappa_1^2A_2^2\sigma_w^2\right\} \\
&+ (1-\gamma-\theta A_1 + \theta\kappa_1A_1\rho_x)\, x_t \\
&+ \left(-\theta A_2 + \theta\kappa_1A_2\nu + 0.5\,(1-\gamma+\pi(-1+\gamma+\theta\kappa_1A_3))^2 + 0.5(\theta\kappa_1A_1)^2\psi_x^2\right)\sigma_t^2 \\
&+ (-(1-\gamma)\rho_\alpha - \theta A_3 + \theta\kappa_1A_3\rho_\alpha)\,\Delta\alpha_{cc,t}
\end{aligned}
$$

By identification :

$$A_1 = \frac{1 - \dfrac{1}{\psi}}{1 - \kappa_1 \rho_x}$$

$$A_3 = -\frac{\left(1 - \dfrac{1}{\psi}\right)\rho_\alpha}{1 - \kappa_1 \rho_\alpha}$$

$$A_2 = 0.5\theta \frac{\left(1 - \dfrac{1}{\psi} + \pi(-1 + \dfrac{1}{\psi} + \kappa_1 A_3)\right)^2 + (\kappa_1 A_1)^2 \psi_x^2}{1 - \kappa_1 \nu}$$

$$A_0 = \frac{log\delta + (1 - \dfrac{1}{\psi})(\nu_{cc} - \nu_\alpha(1 - \rho_\alpha)) + \kappa_0 + \kappa_1(A_2(1 - \nu)\sigma^2 + A_3\nu_\alpha(1 - \rho_\alpha))}{1 - \kappa_1}$$
$$+ \frac{0.5\theta \left\{ \left(-1 + \dfrac{1}{\psi} + \kappa_1 A_3\right)^2 \sigma_\alpha^2 + \kappa_1^2 A_2^2 \sigma_w^2 \right\}}{1 - \kappa_1}$$

Let us determine :

$$z_{i,t} = A_{0,i} + A_{1,i} x_t + A_{2,i} \sigma_t^2 + A_{3,i} \Delta\alpha_{cc,t}$$

Let us remind that one can rewrite the return on any asset and its dividend growth process as follow :

$$r_{i,t+1} = \kappa_{0,i} + \kappa_{1,i} z_{i,t+1} - z_{i,t} + \Delta d_{i,t}$$
$$\Delta d_{i,t+1} = \nu_i + \phi_i x_t + \phi_{\alpha,i} \Delta\alpha_{cc,t} + \psi_i \sigma_t \epsilon_{i,t+1}$$

From the Euler equation 15, we have :

$$
\begin{aligned}
1 &= \mathbb{E}_t e^{\theta log\delta - \dfrac{\theta}{\psi}\Delta c_{t+1} + (\theta-1) r_{c,t+1} + r_{i,t+1}} \\
&= e^{\mathbb{E}_t(\theta log\delta - \dfrac{\theta}{\psi}\Delta c_{t+1} + (\theta-1) r_{c,t+1} + r_{i,t+1}) + 0.5\mathbb{V}_t(\theta log\delta - \dfrac{\theta}{\psi}\Delta c_{t+1} + (\theta-1) r_{c,t+1} + r_{i,t+1})} \\
&= e^{\theta log\delta - \dfrac{\theta}{\psi}\mathbb{E}_t(\Delta c_{t+1}) + (\theta-1)\mathbb{E}_t r_{c,t+1} + \mathbb{E}_t r_{i,t+1} + 0.5\mathbb{V}_t((-\dfrac{\theta}{\psi}\Delta c_{t+1} + (\theta-1) r_{c,t+1} + r_{i,t+1})}
\end{aligned}
$$

$$
\begin{aligned}
0 &= \theta log\delta - \gamma(\nu_{cc} + x_t - \nu_\alpha(1-\rho_\alpha) - \rho_\alpha \Delta\alpha_{cc,t}) + (\theta-1)(\kappa_0 - A_0 - A_1 x_t - A_2\sigma_t^2 - A_3\Delta\alpha_{cc,t}) \\
&+ (\theta-1)\kappa_1(A_0 + A_1\rho_x x_t + A_2((1-\nu)\sigma^2 + \nu\sigma_t^2) + A_3(\nu_\alpha(1-\rho_\alpha) + \rho_\alpha\Delta\alpha_{cc,t})) + \kappa_{0,i} \\
&+ \kappa_{1,i}(A_{0,i} + A_{1,i}\rho_x x_t + A_{2,i}((1-\nu)\sigma^2 + \nu\sigma_t^2) + A_{3,i}(\nu_\alpha(1-\rho_\alpha) + \rho_\alpha\Delta\alpha_{cc,t})) + \nu_i + \phi_i x_t + \phi_{\alpha,i}\Delta\alpha_{cc,t} \\
&+ 0.5\left\{(-\gamma + \pi(\gamma + (\theta-1)\kappa_1 A_3 + \kappa_{1,i}A_{3,i}))^2 + ((\theta-1)\kappa_1 A_1 + \kappa_{1,i}A_{1,i})^2\psi_x^2 + \psi_i^2\right\}\sigma_t^2 \\
&+ 0.5\left(\gamma + (\theta-1)\kappa_1 A_3 + \kappa_{1,i}A_{3,i}\right)^2\sigma_\alpha^2 \\
&+ 0.5\left((\theta-1)\kappa_1 A_2 + \kappa_{1,i}A_{2,i}\right)^2\sigma_w^2 - A_{0,i} - A_{1,i}x_t - A_{2,i}\sigma_t^2 - A_{3,i}\Delta\alpha_{cc,t} \\
0 &= \theta log\delta - \gamma(\nu_{cc} - \nu_\alpha(1-\rho_\alpha)) + (\theta-1)(\kappa_0 - A_0) + (\theta-1)\kappa_1(A_0 + A_2(1-\nu)\sigma^2 + A_3\nu_\alpha(1-\rho_\alpha)) + \kappa_{0,i} \\
&+ \kappa_{1,i}(A_{0,i} + A_{2,i}(1-\nu)\sigma^2 + A_{3,i}\nu_\alpha(1-\rho_\alpha)) + \nu_i \\
&+ 0.5\left(\gamma + (\theta-1)\kappa_1 A_3 + \kappa_{1,i}A_{3,i}\right)^2\sigma_\alpha^2 + 0.5\left((\theta-1)\kappa_1 A_2 + \kappa_{1,i}A_{2,i}\right)^2\sigma_w^2 - A_{0,i} \\
&+ \left(-\gamma - (\theta-1)A_1 + (\theta-1)\kappa_1 A_1\rho_x + \kappa_{1,i}A_{1,i}\rho_x + \phi_i - A_{1,i}\right)x_t \\
&+ \left(0.5((-\gamma + \pi(\gamma + (\theta-1)\kappa_1 A_3 + \kappa_{1,i}A_{3,i}))^2 + ((\theta-1)\kappa_1 A_1 + \kappa_{1,i}A_{1,i})^2\psi_x^2 + \psi_i^2)\right)\sigma_t^2 \\
&+ \left(-(\theta-1)A_2 + (\theta-1)\kappa_1 A_2\nu + \kappa_{1,i}A_{2,i}\nu - A_{2,i}\right)\sigma_t^2 \\
&+ \left(\gamma\rho_\alpha - (\theta-1)A_3 + (\theta-1)\kappa_1 A_3\rho_\alpha + \kappa_{1,i}A_{3,i}\rho_\alpha + \phi_{\alpha,i} - A_{3,i}\right)\Delta\alpha_{cc,t}
\end{aligned}
$$

By identification :

$$A_{1,i} = \frac{\phi_i - \dfrac{1}{\psi}}{1 - \kappa_{1,i}\rho_x}$$

$$A_{3,i} = \frac{\phi_{\alpha,i} + \dfrac{\rho_\alpha}{\psi}}{1 - \kappa_{1,i}\rho_\alpha}$$

$$A_{2,i} = \frac{(1-\theta)A_2(1-\kappa_1\nu) + 0.5\left\{(-\gamma + \pi(\gamma + (\theta-1)\kappa_1 A_3 + \kappa_{1,i}A_{3,i}))^2 + ((\theta-1)\kappa_1 A_1 + \kappa_{1,i}A_{1,i})^2\psi_x^2 + \psi_i^2\right\}}{1 - \kappa_{1,i}\nu}$$

$$\begin{aligned}
A_{0,i} &= \frac{\theta log\delta - \gamma(\nu_{cc} - \nu_\alpha(1-\rho_\alpha)) + (\theta-1)(\kappa_0 - A_0) + (\theta-1)\kappa_1(A_0 + A_2(1-\nu)\sigma^2 + A_3\nu_\alpha(1-\rho_\alpha)) + \kappa_{0,i}}{1-\kappa_{1,i}} \\
&+ \frac{\kappa_{1,i}(A_{2,i}(1-\nu)\sigma^2 + A_{3,i}\nu_\alpha(1-\rho_\alpha)) + \nu_i}{1-\kappa_{1,i}} \\
&+ \frac{0.5\left(\gamma + (\theta-1)\kappa_1 A_3 + \kappa_{1,i}A_{3,i}\right)^2\sigma_\alpha^2 + 0.5\left((\theta-1)\kappa_1 A_2 + \kappa_{1,i}A_{2,i}\right)^2\sigma_w^2}{1-\kappa_{1,i}}
\end{aligned}$$

Deriving $r_{f,t}$:

$$\mathbb{E}_t e^{\theta log\delta - \dfrac{\theta}{\psi}\Delta c_{t+1} + (\theta-1)r_{c,t+1} + r_{f,t}} = 1$$

So

$$
\begin{aligned}
e^{-r_{f,t}} &= \mathbb{E}_t e^{\theta log\delta - \dfrac{\theta}{\psi}\Delta c_{t+1} + (\theta-1) r_{c,t+1}} \\
&= e^{\mathbb{E}_t(\theta log\delta - \dfrac{\theta}{\psi}\Delta c_{t+1} + (\theta-1) r_{c,t+1}) + 0.5\mathbb{V}_t(\theta log\delta - \dfrac{\theta}{\psi}\Delta c_{t+1} + (\theta-1) r_{c,t+1})} \\
&= e^{\theta log\delta - \dfrac{\theta}{\psi}\mathbb{E}_t(\Delta c_{t+1}) + (\theta-1)\mathbb{E}_t r_{c,t+1} + 0.5\mathbb{V}_t((-\dfrac{\theta}{\psi}\Delta c_{t+1} + (\theta-1) r_{c,t+1})} \\
-r_{f,t} &= \theta log\delta - \gamma(\nu_{cc} + x_t - \nu_\alpha(1-\rho_\alpha) - \rho_\alpha \Delta\alpha_{cc,t}) + (\theta-1)(\kappa_0 - A_0 - A_1 x_t - A_2\sigma_t^2 - A_3\Delta\alpha_{cc,t}) \\
&+ (\theta-1)\kappa_1(A_0 + A_1\rho_x x_t + A_2((1-\nu)\sigma^2 + \nu\sigma_t^2) + A_3(\nu_\alpha(1-\rho_\alpha) + \rho_\alpha\Delta\alpha_{cc,t})) \\
&+ 0.5\left\{(-\gamma + \pi(\gamma + (\theta-1)\kappa_1 A_3))^2 + ((\theta-1)\kappa_1 A_1)^2\psi_x^2\right\}\sigma_t^2 \\
&+ 0.5\left(\gamma + (\theta-1)\kappa_1 A_3\right)^2\sigma_\alpha^2 + 0.5\left((\theta-1)\kappa_1 A_2\right)^2\sigma_w^2 \\
-r_{f,t} &= \theta log\delta - \gamma(\nu_{cc} - \nu_\alpha(1-\rho_\alpha)) + (\theta-1)(\kappa_0 - A_0) + (\theta-1)\kappa_1(A_0 + A_2(1-\nu)\sigma^2 + A_3\nu_\alpha(1-\rho_\alpha)) \\
&+ 0.5\left(\gamma + (\theta-1)\kappa_1 A_3\right)^2\sigma_\alpha^2 + 0.5\left((\theta-1)\kappa_1 A_2\right)^2\sigma_w^2 \\
&+ \left(-\gamma - (\theta-1)A_1 + (\theta-1)\kappa_1 A_1\rho_x\right)x_t \\
&+ \left(-(\theta-1)A_2 + (\theta-1)\kappa_1 A_2\nu + 0.5((-\gamma + \pi(\gamma + (\theta-1)\kappa_1 A_3))^2 + ((\theta-1)\kappa_1 A_1)^2\psi_x^2)\right)\sigma_t^2 \\
&+ \left(\gamma\rho_\alpha - (\theta-1)A_3 + (\theta-1)\kappa_1 A_3\rho_\alpha\right)\Delta\alpha_{cc,t}
\end{aligned}
$$

Therefore :

$$
r_{f,t} = A_{0,f} + A_{1,f}x_t + A_{2,f}\sigma_t^2 + A_{3,f}\Delta\alpha_{cc,t}
$$

Deriving $\mathbb{E}_t r_{i,t+1}$:

$$
\begin{aligned}
\mathbb{E}_t r_{i,t+1} =& \kappa_{0,i} + \kappa_{1,i}\mathbb{E}_t z_{i,t+1} - A_{0,i} - A_{1,i}x_t - A_{2,i}\sigma_t^2 - A_{3,i}\Delta\alpha_{cc,t} + \nu_i + \phi x_t + \phi_{\alpha,i}\Delta\alpha_{cc,t} \\
=& \kappa_{0,i} + \kappa_{1,i}\mathbb{E}_t(A_{0,i} + A_{1,i}x_{t+1} + A_{2,i}\sigma_{t+1}^2 + A_{3,i}\Delta\alpha_{cc,t+1}) \\
& - A_{0,i} - A_{1,i}x_t - A_{2,i}\sigma_t^2 - A_{3,i}\Delta\alpha_{cc,t} + \nu_i + \phi_i x_t + \phi_{\alpha,i}\Delta\alpha_{cc,t} \\
=& \kappa_{0,i} + \kappa_{1,i}A_{0,i} + \kappa_{1,i}A_{1,i}\rho_x x_t + \kappa_{1,i}A_{2,i}((1-\nu^2)\sigma^2 + \nu\sigma_t^2) \\
& + \kappa_{1,i}A_{3,i}(\nu_\alpha(1-\rho_\alpha) + \rho_\alpha\Delta\alpha_{cc,t}) - A_{0,i} - A_{1,i}x_t - A_{2,i}\sigma_t^2 - A_{3,i}\Delta\alpha_{cc,t} + \nu_i + \phi_i x_t + \phi_{\alpha,i}\Delta\alpha_{cc,t} \\
=& \kappa_{0,i} + \kappa_{1,i}A_{0,i} + \kappa_{1,i}A_{2,i}(1-\nu)\sigma^2 + \kappa_{1,i}A_{3,i}\nu_\alpha(1-\rho_\alpha) - A_{0,i} + \nu_i \\
& + (-A_{1,i} + \phi_i + \kappa_{1,i}A_{1,i}\rho_x)x_t + (-A_{2,i} + \kappa_{1,i}A_{2,i}\nu)\sigma_t^2 + (-A_{3,i} + \phi_{\alpha,i} + \kappa_{1,i}A_{3,i}\rho_\alpha)\Delta\alpha_{cc,t} \\
=& B_0 + B_1 x_t + B_2\sigma_t^2 + B_3\Delta\alpha_{cc,t}
\end{aligned}
$$

The innovation in the market return is :

$$r_{i,t+1} - \mathbb{E}_t r_{i,t+1} = \kappa_{1,i} A_{1,i} \psi_x \sigma_t \epsilon_{x,t+1} + \kappa_{1,i} A_{2,i} \sigma_w \epsilon_{\sigma,t+1} + \kappa_{1,i} A_{3,i} \sigma_\alpha \epsilon_{\alpha,t+1} + \kappa_{1,i} A_{3,i} \pi \sigma_t \epsilon_{cc,t+1} + \psi_i \sigma_t \epsilon_{i,t+1}$$

So the expected equity premium on any dividend paying asset i is given by :

$$\begin{aligned}\mathbb{E}_t(r_{i,t+1} - r_{f,t}) &= -Cov(r_{i,t+1} - \mathbb{E}_t r_{i,t+1}, m_{t+1} - \mathbb{E}_t m_{t+1}) - 0.5\mathbb{V}_t(r_{i,t+1}) \\ &= \lambda_{m,x} \underbrace{\kappa_{1,i} A_{1,i} \psi_x}_{\beta_{i,x}} \sigma_t^2 + \lambda_{m,w} \underbrace{\kappa_{1,i} A_{2,i}}_{\beta_{i,w}} \sigma_w^2 + \lambda_{m,\alpha} \underbrace{\kappa_{1,i} A_{3,i}}_{\beta_{i,\alpha}} \sigma_\alpha^2 + \lambda_{m,cc} \underbrace{\kappa_{1,i} A_{3,i} \pi}_{\beta_{i,cc}} \sigma_t^2 - 0.5\mathbb{V}_t(r_{i,t+1})\end{aligned}$$

And

$$\begin{aligned}\mathbb{V}_t(r_{i,t+1}) =& \mathbb{V}_t(\kappa_{1,i} z_{i,t+1} + \Delta d_{i,t+1}) \\ =& \left(\kappa_{1,i}^2 A_{1,i}^2 \psi_x^2 + \pi^2 \kappa_{1,i}^2 A_{3,i}^2 + \psi_i^2\right) \sigma_t^2 + \kappa_{1,i}^2 A_{2,i}^2 \sigma_w^2 + \kappa_{1,i}^2 A_{3,i}^2 \sigma_\alpha^2 \\ =& (\beta_{i,x}^2 + \beta_{i,cc}^2 + \psi_i^2)\sigma_t^2 + \beta_{i,w}^2 \sigma_w^2 + \beta_{i,\alpha}^2 \sigma_\alpha^2\end{aligned}$$

D Tables

Table 5: Descriptive statistics : 1930-1955.

	$E(.)$	$\sigma(.)$	$\rho(1)$	$\rho(2)$	$\rho(3)$	$\rho(4)$	$\rho(5)$	CV
Δd	0.0038	0.1914	0.1244	-0.2814	-0.1927	-0.1018	0.0954	50.6037
Δc	0.0169	0.0540	0.3633	0.1214	-0.1689	-0.2447	0.0208	3.1977
$\Delta\alpha_{cc}$	0.0005	0.0231	0.4800	0.0383	-0.1503	-0.3388	-0.2569	50.1958
Δcc	0.0173	0.0611	0.2899	-0.0096	-0.2380	-0.2580	0.0343	3.5250
$\Delta\alpha_{gc}$	-0.0009	0.0378	0.4771	0.0408	-0.0949	-0.3182	-0.2775	-42.0192
Δgc	0.0160	0.0621	0.5495	0.2875	-0.0174	-0.2748	-0.1701	3.8876
z_m	2.8535	0.2265	0.4185	-0.0828	-0.2764	-0.4030	-0.2802	0.0794
r_m	0.0732	0.2468	0.0904	-0.2068	-0.0779	-0.2504	-0.0288	3.3692
r_f	-0.0103	0.0558	0.6365	0.1463	0.0169	0.1026	0.2074	-5.4163

Table 6: Descriptive statistics : 1956-1980.

	$E(.)$	$\sigma(.)$	$\rho(1)$	$\rho(2)$	$\rho(3)$	$\rho(4)$	$\rho(5)$	CV
Δd	0.0074	0.0523	0.2667	0.0359	0.0091	0.0667	0.0053	7.0994
Δc	0.0222	0.0290	-0.0204	-0.2613	-0.0383	0.0427	0.1899	1.3081
$\Delta\alpha_{cc}$	-0.0035	0.0043	-0.2815	-0.0210	0.3359	-0.3975	0.0456	-1.2429
Δcc	0.0187	0.0295	-0.0577	-0.2755	-0.0490	0.0381	0.1629	1.5790
$\Delta\alpha_{gc}$	0.0061	0.0076	-0.2780	-0.0079	0.3276	-0.3867	0.0584	1.2326
Δgc	0.0283	0.0296	0.0243	-0.2123	0.0213	0.0111	0.2276	1.0473
z_m	3.2918	0.1822	0.6555	0.3480	0.2547	0.3035	0.1185	0.0553
r_m	0.0445	0.1779	-0.0762	-0.3736	0.1056	0.3032	0.0922	4.0023
r_f	0.0030	0.0152	0.5917	0.4075	0.4660	0.3126	0.2736	5.0496

Table 7: Descriptive statistics : 1981-2018.

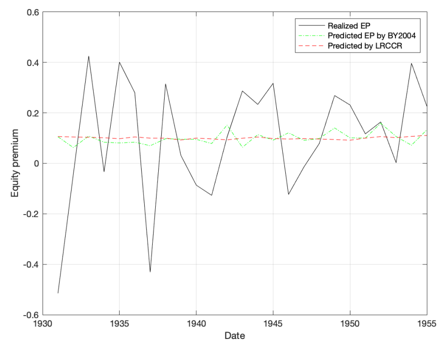
	$E(.)$	$\sigma(.)$	$\rho(1)$	$\rho(2)$	$\rho(3)$	$\rho(4)$	$\rho(5)$	CV
Δd	0.0339	0.0926	-0.0365	-0.0047	-0.0714	-0.1272	-0.0885	2.7331
Δc	0.0155	0.0162	0.4603	0.0530	-0.0526	-0.0848	-0.0747	1.0438
$\Delta\alpha_{cc}$	-0.0052	0.0066	0.3599	-0.0832	-0.2421	-0.0687	0.1352	-1.2812
Δcc	0.0104	0.0185	0.5090	0.0042	-0.1916	-0.1590	-0.1158	1.7807
$\Delta\alpha_{gc}$	0.0065	0.0083	0.3914	-0.0141	-0.1654	0.0388	0.1809	1.2843
Δgc	0.0220	0.0172	0.3744	0.1211	0.1178	0.0675	0.0877	0.7844
z_m	3.8166	0.4151	0.8895	0.7548	0.6669	0.5546	0.4293	0.1088
r_m	0.0831	0.1618	-0.0695	-0.1204	0.0703	-0.0356	-0.4177	1.9464
r_f	0.0109	0.0221	0.7975	0.6500	0.5337	0.3759	0.3189	2.0303

Table 8: Model-implied moments.

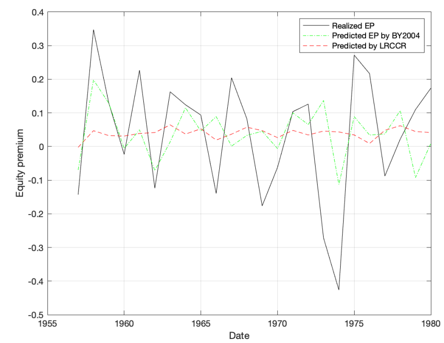
		$\sigma(z_m)$	EP	$E(R_f)$	$\sigma(r_{m,a})$	$\sigma(r_{f,a})$	$\rho(z_m)$
1930-1955							
	Data	0.227	0.084	-0.010	0.247	0.010	0.418
BY2004	Mean	0.238	0.109	0.007	0.264	0.159	0.354
	5%	0.166	-0.019	-0.051	0.193	0.122	0.019
	50%	0.234	0.105	0.006	0.261	0.158	0.368
	95%	0.325	0.251	0.065	0.346	0.198	0.637
LRCCR	Mean	0.150	0.040	0.010	0.116	0.019	0.608
	5%	0.098	0.000	-0.005	0.089	0.014	0.291
	50%	0.145	0.039	0.010	0.115	0.018	0.635
	95%	0.216	0.081	0.024	0.144	0.025	0.834
1956-1980							
	Data	0.182	0.041	0.003	0.178	0.003	0.656
BY2004	Mean	0.125	0.093	-0.002	0.134	0.001	0.345
	5%	0.093	0.045	-0.002	0.102	0.001	0.019
	50%	0.124	0.093	-0.002	0.133	0.001	0.360
	95%	0.162	0.143	-0.001	0.167	0.002	0.620
LRCCR	Mean	0.175	0.068	0.003	0.197	0.019	0.282
	5%	0.119	0.003	-0.005	0.138	0.014	-0.041
	50%	0.172	0.066	0.003	0.194	0.019	0.293
	95%	0.240	0.141	0.010	0.263	0.024	0.569

E Figures

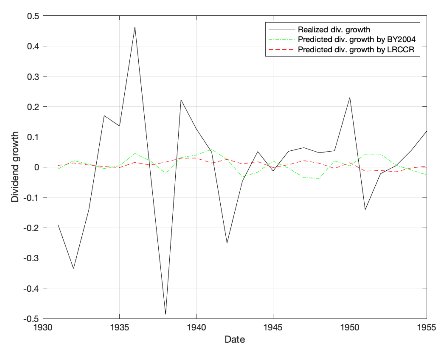
Figure 8: Realized versus predicted equity premium, consumption growth and dividend growth



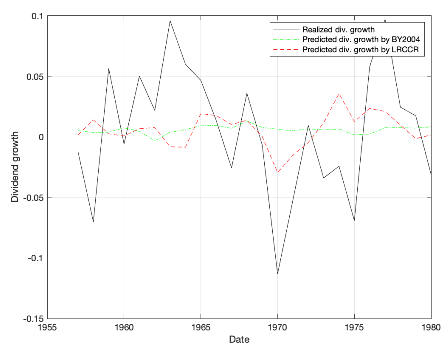
(a) 1930-1955



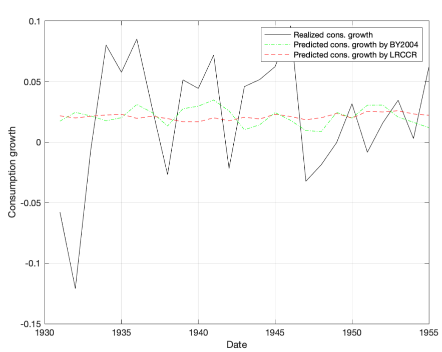
(b) 1956-1980



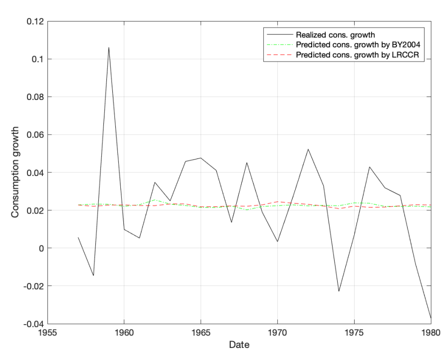
(c) 1930-1955



(d) 1956-1980



(e) 1930-1955



(f) 1956-1980

F Data construction details

Table 9: **Carbon footprint covered from the NIPA expenditure data**

Household consumption expenditures category (2 digit level)	Footprint Covererage E
1-Food and beverages purchased for off-premises consumption	
Food and nonalcoholic beverages purchased for off-premises consumption	√
Alcoholic beverages purchased for off-premises consumption	x
Food produced and consumed on farms	√
2-Clothing, footwear, and related services	
Clothing	√
Footwear	√
3-Housing, utilities, and fuels	
Housing	√
Household utilities and fuels	
Water supply and sanitation	√
Electricity, gas, and other fuels	x
Electricity	√
Natural gas	√
Fuel oil and other fuels	
4-Furnishings, household equipment, and routine household maintenance	
Furniture, furnishings, and floor coverings	√
Household textiles	√
Household appliances	√
Glassware, tableware, and household utensils	√
Tools and equipment for house and garden	x
5-Health	
Medical products, appliances, and equipment	x
Outpatient services	√
Hospital and nursing home services	
Hospital	√
Nursing home services	√
6-Transportation	
Motor vehicles	√
Motor vehicle operation	√
Public transportation	
Ground transportation	√
Air transportation	√
Water transportation	√
7-Communication	
Telephone and related communication equipment	√
Postal and delivery services	√
Telecommunication services	√
Internet access	x
8-Recreation	
Video and audio equipment, computers, and related services	√
Sports and recreational goods and related services	√
Membership clubs, sports centers, parks, theaters, and museums	√
Magazines, newspapers, books, and stationery	√
Gambling	x
Pets, pet products, and related services	x
Photographic goods and services	x

Continued on next page

Table 9 – continued from previous page

Household consumption expenditures category (2 digit level)	Footprint Covererage E
Package tours	x
9-Education	
Educational books	x
Higher education	√
Nursery, elementary, and secondary schools	√
Commercial and vocational schools	√
10-Food services and accommodations	
Food services	√
Accommodations	√
11-Financial services and insurance	
Financial services	√
Insurance	√
12-Other goods and services	
Personal care	√
Personal items	√
Social services and religious activities	√
Professional and other services	√
Tobacco	√