

# Long-Run Carbon Consumption Risk and Asset Prices

Stephane N'Dri

UdeM

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## FIRST

- ▶ Environnemental issues
- ▶ **Goods and services consumption** pollutes the environnement ;
- ▶ Production vs. consumption-based CO2 emissions (GCP). ▶ Fact
- ▶ Despite that, most papers and climate policies focus on the production side.

## SECOND

- ▶ Climate change is a long horizon phenomenon ;
- ▶ Need a long-run risk model to assess it ;
- ▶ Yet, the effect of the canonical long-run risks on the assets depends on investors detecting it ;
  - ▶ Require a new LRR model by considering emission/carbon consumption.
- ▶ However, emission does have long-run risk and is more detectable :
  - ▶ Curbing carbon emission at the pre-industrial level ;
  - ▶ Emission → Damages → affects aggregate consumption.

- ▶ Does the carbon-consumption based long-run risk model explain :
  - ▶ equity premium, risk-free rate and volatility puzzles?
  - ▶ cross section of industries' expected returns?
- ▶ Does carbon consumption help classifying the industries based on their exposures?

- ▶ Provide a new consumption-based carbon emission measure :
  - ▶ By treating differently the consumption categories ;
  - ▶ Since they affect differently the environment.
- ▶ Shed a new light on the long-run risk model :
  - ▶ Propose a carbon-consumption based long-run risk model ;

- ▶ Solve equity premium, volatility and risk-free rate puzzles :
  - ▶ With a simple decomposition of the consumption growth ;
  - ▶ Assuming a long-run risk in volatility and mean ;
- ▶ Our long-run risk is more detectable during the period of high climate events.

- ▶ **Long-run risk models** : Bansal and Yaron (2004), Bansal et al. (2007a), Bansal et al. (2007b), Bonomo et al. (2011), Constantinides and Ghosh (2011).
- ▶ **Climate finance** : Bansal et al. (2016), Chen et al. (2019), Daniel et al. (2016).

# Households

- ▶ Representative agent economy ;
- ▶ Can invest in  $n+1$  assets : one riskless asset ( $i = 0$ ) and  $n$  risky assets ( $i = 1, \dots, n$ ) ;
- ▶ Has the following budget constraint :

$$C_t + \sum_{i=1}^{n+1} P_{it} X_{i,t+1} = \sum_{i=1}^{n+1} (P_{it} + D_{it}) X_{it} = W_t \quad (1)$$

- ▶ Maximizes an Epstein-Zin utility function :

$$V_t = \left[ (1 - \delta) C_t^{\frac{1-\gamma}{\theta}} + \delta (E_t[V_{t+1}]^{1-\gamma})^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}} \quad (2)$$

$\gamma$  is the CRA and  $\psi = 1 - \frac{1-\gamma}{\theta}$  is the EIS.

- ▶ We decompose the total consumption as follow (CC : carbon consumption + GC : green consumption) :

$$C_t = CC_t + GC_t \quad (3)$$

# Long-run carbon consumption risk model

$$\Delta c_{t+1} = \underbrace{\Delta cc_{t+1}}_{\text{Carbon}} - \underbrace{\Delta \alpha_{cc,t+1}}_{\text{share of cc}} \quad \text{► Proof} \quad (4)$$

$$\Delta cc_{t+1} = \nu_{cc} + x_t + \sigma_t \epsilon_{cc,t+1} \quad (5)$$

$$x_{t+1} = \rho_x x_t + \psi_x \sigma_t \epsilon_{x,t+1} \quad (6)$$

$$\sigma_{t+1}^2 = (1 - \nu) \sigma^2 + \nu \sigma_t^2 + \sigma_w \epsilon_{\sigma,t+1} \quad (7)$$

$$\Delta \alpha_{cc,t+1} = \nu_\alpha (1 - \rho_\alpha) + \rho_\alpha \Delta \alpha_{cc,t} + \sigma_\alpha \epsilon_{\alpha,t+1} + \pi \sigma_t \epsilon_{cc,t+1} \quad (8)$$

Finally, the dividend of any asset  $i$  growth rate is as follow

$$\Delta d_{i,t+1} = \nu_i + \phi_i x_t + \phi_{\alpha,i} \Delta \alpha_{cc,t} + \psi_i \sigma_t \epsilon_{i,t+1} \quad (9)$$

$\epsilon_{x,t+1}, \epsilon_{cc,t+1}, \epsilon_{\alpha,t+1}, \epsilon_{i,t+1}$  and  $\epsilon_{\sigma,t+1}$  are i.i.d.



# Resolution

- ▶ For any asset  $i$ , the Euler equation is given by :

$$\mathbb{E}_t(\exp(m_{t+1} + r_{i,t+1})) = 1 \quad (10)$$

where

$$m_{t+1} = \theta \log(\delta) - \frac{\theta}{\psi} \Delta c c_{t+1} + \frac{\theta}{\psi} \Delta \alpha_{cc,t+1} + (\theta - 1) r_{c,t+1} \quad (11)$$

is the natural logarithm of the stochastic discount factor (SDF) ;

- ▶ Innovation in the SDF :

$$m_{t+1} - \mathbb{E}_t[m_{t+1}] = -\lambda_{m,\alpha} \underbrace{\sigma_\alpha \epsilon_{\alpha,t+1}}_{\text{g.r}} - \lambda_{m,cc} \underbrace{\sigma_t \epsilon_{cc,t+1}}_{\text{c.r}} - \lambda_{m,x} \underbrace{\sigma_t \epsilon_{x,t+1}}_{\text{l.r.r}} - \lambda_{m,w} \underbrace{\sigma_w \epsilon_{\sigma,t+1}}_{\text{v.r}}$$

- ▶ Four risks : green risk, carbon risk, long-run risk and the volatility risk ;
- ▶ Expected equity premium of any asset  $i$  :

$$\mathbb{E}_t(r_{i,t+1} - r_{f,t}) = \lambda_{m,x} \beta_{i,x} \sigma_t^2 + \lambda_{m,w} \beta_{i,w} \sigma_w^2 + \lambda_{m,\alpha} \beta_{i,\alpha} \sigma_\alpha^2 + \lambda_{m,cc} \beta_{i,cc} \sigma_t^2$$

# Empirical evaluation of the model

- ▶ Period where no effect on firm-level nor industry-level returns.
- ▶ Textual analysis to identify key period where climate change matters become a public debate ;
  - ▶ Goal : as of which date/period asset prices start reflecting carbon-consumption risk.
- ▶ We split our sample into three sub-periods for that reason :
  - ▶ 1930-1955 (Great economic uncertainty period) ;
  - ▶ 1956-1980 (President Reagan election) ;
  - ▶ 1981-2018 (More climate action period).

# Empirical evaluation of the model

1. Calibrate  $\Theta = [\rho_x \ \psi_x \ \psi_i \ \nu_{cc} \ \nu \ \nu_i \ \sigma_w \ \sigma \ \phi_i \ \delta \ \gamma \ \psi \ \nu_\alpha \ \rho_\alpha \ \sigma_\alpha \ \pi \ \phi_{\alpha,i}]$  to match key data moments
2. Asset pricing implications :
  - ▶ Testing the equity premium, volatility and risk-free rate puzzles :
    - ▶  $r_{f,t} = A_{0,f} + A_{1,f}x_t + A_{2,f}\sigma_t^2 + A_{3,f}\Delta\alpha_{cc,t}$
    - ▶  $\mathbb{E}_t r_{m,t+1} = B_0 + B_1x_t + B_2\sigma_t^2 + B_3\Delta\alpha_{cc,t}$
  - ▶ Pricing portfolios (Cross-section of assets) :
    - ▶  $r_{i,t} = c_i + \beta_{sh,i}\Delta\alpha_{cc,t} + \beta_{cc,i}\Delta c_{ct} \quad i = 1, \dots, 42$
    - ▶ Categorize the industries into positively versus negatively exposed to climate risk;
    - ▶ Recalibrate the model using industries portfolios.

## Measure of $\Delta cc_t$ and $\Delta \alpha_{cc,t}$

- ▶ The central challenge of the climate change is to capture its impact on real economy ;
  - ▶ This paper uses consumption and carbon footprint to capture that.
- ▶ This paper uses :
  - ▶ National Income and Product Accounts (NIPA)
  - ▶ Economic Input-Output Life Cycle Assessment (EIO-LCA)
- ▶ All consumption data are on annual basis and span the period 1930-2018.

## Measure of $\Delta cc_t$ and $\Delta \alpha_{cc,t}$

- ▶ Collect aggregate information on 12 consumption categories using NIPA ;
- ▶ Carbon footprints (CFs) on those 12 categories from EIO-LCA.  
    ▶ [Footprint](#) and details ( ▶ [Details](#) )

## Measure of $\Delta cc_t$ and $\Delta \alpha_{cc,t}$

- ▶ Classify the consumption categories into two groups using the CFs :

- ▶ Carbon consumption :

$$CC_t = \sum_{i=1}^5 C_{i,t} \quad (12)$$

- ▶ Green consumption :

$$GC_t = \sum_{i=6}^{12} C_{i,t} \quad (13)$$

- ▶ Carbon share in the total consumption is :

$$A_{CC,t} = \frac{CC_t}{CC_t + GC_t} = \frac{CC_t}{C_t} \quad (14)$$

- ▶ Then  $\Delta cc_t = \Delta \log CC_t$  and  $\Delta \alpha_{cc,t} = \Delta \log A_{CC,t}$

- ▶ See [▶ Figure](#)

## FIGURE – Carbon footprint by household expenditure category.

The x-axis captures the tCO<sub>2</sub>e per million of US dollars spend. [▶ Back](#)

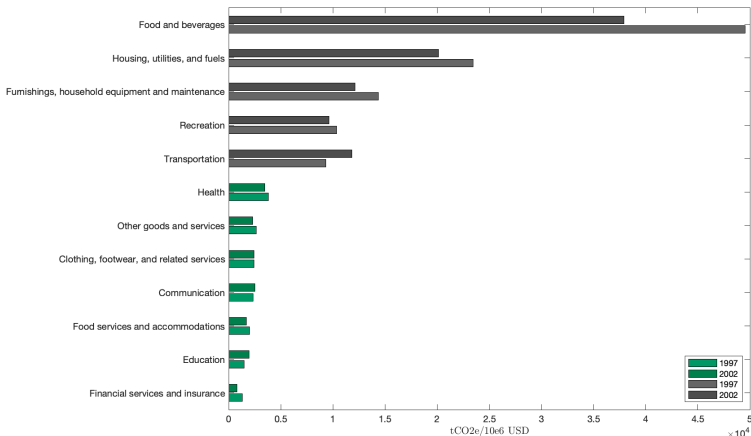
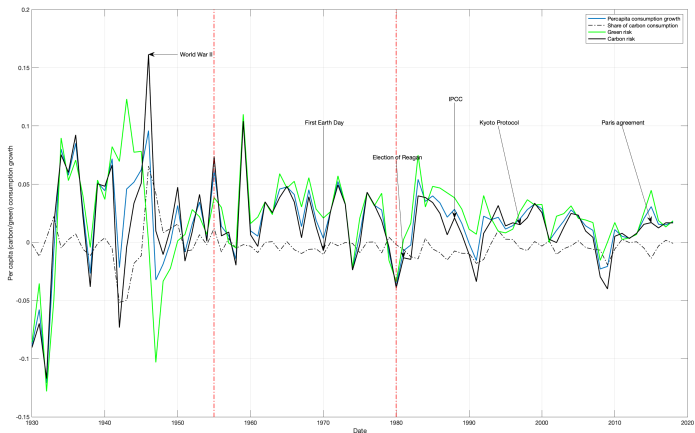


TABLE – Example of footprint at sub-category level. [▶ Back](#)

<b>Health</b>	<b>3825</b>	<b>3480</b>
Medical products, appliances, and equipment	2058	2083
Pharmaceutical and other medical products <sup>9</sup>	799	592
Pharmaceutical products	420	304
Other medical products	379	288
Therapeutic appliances and equipment	1259	1491
Outpatient services	924	665
Physician services <sup>10</sup>	169	157
Dental services	169	157
Paramedical services	755	508
Home health care	197	235
Medical laboratories	558	273
Other professional medical services <sup>11</sup>	na	na
Hospital and nursing home services	843	732
Hospitals <sup>12</sup>	400	366
Nursing homes	443	366
<b>Transportation</b>	<b>9344</b>	<b>11810</b>
Motor vehicles	1004	783
New motor vehicles	382	265
Net purchases of used motor vehicles	622	518
Motor vehicle operation	5130	4397
Motor vehicle parts and accessories	769	710
Motor vehicle fuels, lubricants, and fluids	3540	2790
Motor vehicle maintenance and repair	423	328
Other motor vehicle services	398	569
Public transportation	3210	6630
Ground transportation <sup>13</sup>	0	1870
Air transportation	1780	1980
Water transportation	1430	2780



FIGURE – Carbon risk, Green risk. [▶ Back](#)

# Returns

- ▶ Use the dividend-yield and the dividend growth of the CRSP value-weighted index ;
- ▶ Use Fama-French 49 industries portfolios to build their exposures to carbon risks.

TABLE – Descriptive statistics : 1930-2018. ► Subperiod stats

	$E(.)$	$\sigma(.)$	$\rho(1)$	$\rho(2)$	$\rho(3)$	$\rho(4)$	$\rho(5)$	$CV$
$\Delta d$	0.0176	0.1223	0.1075	-0.1832	-0.1502	-0.0930	0.0459	6.9325
$\Delta c$	0.0178	0.0343	0.3150	0.0608	-0.1508	-0.1491	0.0098	1.9277
$\Delta \alpha_{cc}$	-0.0030	0.0134	0.4545	0.0574	-0.1233	-0.2765	-0.1730	-4.4179
$\Delta cc$	0.0147	0.0382	0.2717	-0.0223	-0.2057	-0.1549	0.0176	2.5893
$\Delta \alpha_{gc}$	0.0042	0.0215	0.4469	0.0640	-0.0749	-0.2692	-0.2105	5.0928
$\Delta gc$	0.0220	0.0385	0.4647	0.2063	-0.0167	-0.2016	-0.1006	1.7505
$z_m$	3.3878	0.5123	0.9276	0.8524	0.7992	0.7605	0.7163	0.1512
$r_m$	0.0694	0.1929	0.0077	-0.2202	0.0181	-0.0053	-0.1215	2.7802
$r_f$	0.0025	0.0351	0.6852	0.3059	0.2040	0.2336	0.2788	14.1068

TABLE – Calibrated parameters

	1930-2018		1930-1955		1956-1980		1981-2018	
	BY04	LRCCR	BY04	LRCCR	BY04	LRCCR	BY04	LRCCR
$\rho_x$	0.932	0.978	0.937	0.979	0.920	0.900	0.976	0.900
$\psi_x$	0.259	0.150	0.278	0.119	0.010	0.204	0.206	0.514
$\psi_d$	4.540	4.340	4.789	4.488	13.361	0.000	10.122	4.288
$\nu_x$	9E-04	1E-03	1E-04	1E-03	-6E-05	2E-03	-2E-04	9E-04
$\nu$	0.999	0.979	0.573	0.985	0.577	0.691	0.988	0.995
$\nu_d$	0.001	-0.011	0.000	-0.025	-0.002	0.001	0.005	0.003
$\sigma_w$	5E-07	2E-08	1E-04	2E-07	7E-08	1E-05	4E-06	4E-06
$\sigma$	8E-03	3E-03	5E-04	4E-03	1E-03	1E-03	7E-04	9E-03
$\phi$	2.294	3.378	2.354	3.734	321.850	10.056	0.792	1.019
$\delta$	0.956	0.998	0.999	0.999	0.998	0.998	0.998	0.997
$\gamma$	7.074	12.290	9.878	10.084	15.940	23.016	6.063	8.732
$\psi$	1.379	1.487	3.018	1.495	1.574	1.235	1.503	1.486
$\bar{z}$	3.088	6.164	6.054	6.602	6.201	6.285	5.720	5.060
$\bar{z}_m$	5.344	3.981	5.153	3.522	4.754	5.696	12.820	5.548
$\nu_a$		-3E-04		4E-05		-3E-04		-4E-04
$\rho_a$		0.455		0.480		-0.281		0.360
$\sigma_a$		0.006		0.006		0.014		0.004
$\pi$		1.344		0.897		3.328		0.626
$\phi_a$		0.590		0.877		-0.294		1.305

# Interpretation

- ▶  $\Psi_x$  tells us how detectable the long-run variable is. The results show that our long-run variable is more detectable than BY during 1956-2018 near climate change events period.
- ▶ From results above, there is long-run risks in the volatility and the carbon consumption growth :
  - ▶  $\nu$  smaller and close to one ;
  - ▶ while  $\rho_x$  smaller and close to one.
- ▶ The risk aversion in our model is higher than the one in BY04 and is in a reasonable range.
  - ▶ This is due to the nature of the risk we are talking about : carbon risk ;
  - ▶ Agents fear more carbon risk than consumption risk.
- ▶  $\phi_\alpha$  functions as a leverage ratio on the share of carbon consumption growth during the period 1981-2018.

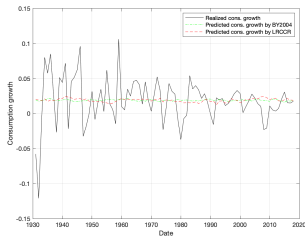
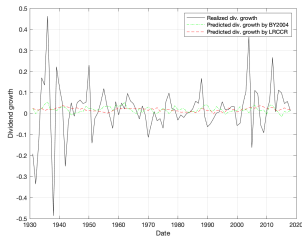
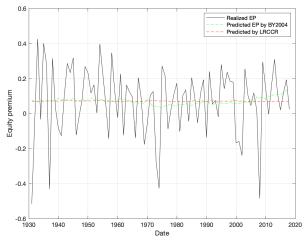
TABLE – Model-implied moments. [► More](#)

		$\sigma(z_m)$	$EP$	$E(R_f)$	$\sigma(r_{m,a})$	$\sigma(r_{f,a})$	$\rho(z_m)$
1930-2018							
BY2004	Data	0.512	0.067	0.002	0.193	0.002	0.928
	Mean	0.189	0.096	0.007	0.212	0.068	0.454
	5%	0.158	-0.029	-0.090	0.180	0.050	0.286
	50%	0.188	0.097	0.005	0.212	0.065	0.458
	95%	0.225	0.220	0.111	0.246	0.095	0.607
LRCCR	Mean	0.197	0.078	0.010	0.134	0.022	0.735
	5%	0.154	0.053	0.001	0.118	0.019	0.601
	50%	0.195	0.078	0.010	0.134	0.022	0.743
	95%	0.248	0.104	0.019	0.151	0.027	0.841
1981-2018							
BY2004	Data	0.415	0.072	0.011	0.162	0.011	0.890
	Mean	0.078	0.066	0.010	0.128	0.002	0.846
	5%	0.042	0.028	0.005	0.103	0.001	0.642
	50%	0.073	0.066	0.010	0.127	0.002	0.871
	95%	0.133	0.106	0.015	0.154	0.004	0.961
LRCCR	Mean	0.204	0.118	0.015	0.183	0.006	0.809
	5%	0.118	0.061	0.003	0.148	0.003	0.586
	50%	0.191	0.117	0.015	0.182	0.005	0.835
	95%	0.330	0.178	0.027	0.220	0.009	0.946

TABLE – Market prices of risks and effects on the risk premium

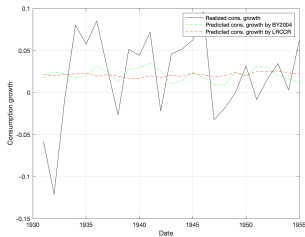
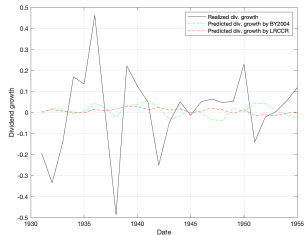
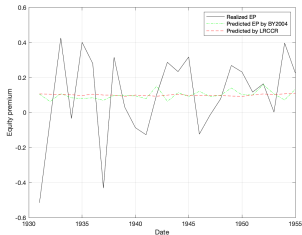
		$\lambda$	$\beta$	effect	$\lambda$	$\beta$	effect
1930-2018					1930-1955		
BY04	srr	7.07	0.00	0	9.88	0.00	0
	lrr	14.47	5.58	+	40.61	8.17	+
	vr	-2564.24	-7723.52	+	-1884.32	-515.00	+
LRCCR	$\alpha risk$	-21.93	1.59	-	-18.75	2.18	-
	srr	-17.19	2.13	-	-6.74	1.96	-
	lrr	73.78	10.16	+	50.61	7.28	+
	vr	-122914.44	-14400.98	+	-79112.00	-6155.20	+
1956-1980					1981-2018		
BY04	srr	15.94	0.00	0	6.06	0.00	0
	lrr	1.92	37.37	+	40.87	1.09	+
	vr	-273.13	1687.23	-	-51947.16	5284.64	-
LRCCR	$\alpha risk$	-18.15	-0.41	+	-13.22	2.40	-
	srr	-37.37	-1.35	+	0.46	1.50	+
	lrr	44.47	18.41	+	38.93	2.04	+
	vr	-5168.90	-1975.90	+	-66622.82	-4374.95	+

## Realized versus predicted macro-financial variables : 1930-2018

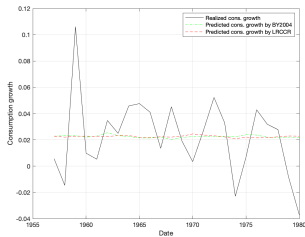
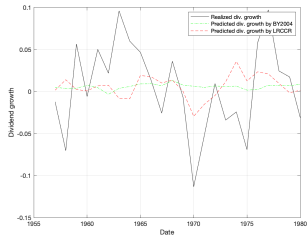
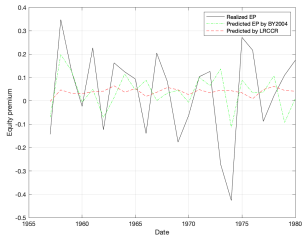




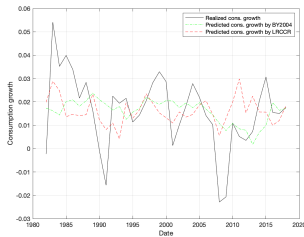
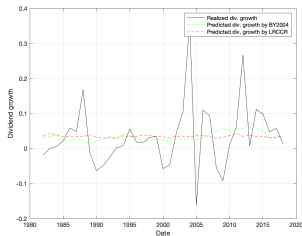
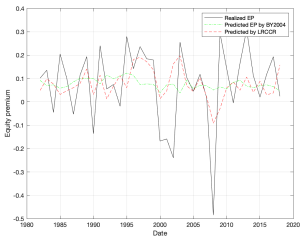
## Realized versus predicted macro-financial variables : 1930-1955



## Realized versus predicted macro-financial variables : 1956-1980



## Realized versus predicted macro-financial variables : 1981-2018



- ▶ Most of the consumption capital asset pricing models find a constant risk premium : approximately constant predicted risk premium.
  - ▶ However, during period of high carbon emission risk, a long-run carbon consumption risks model finds a time-varying risk premium.
  - ▶ Much better than the usual LRR model. (see slides [27](#) versus [25](#) )

## Industries exposures to carbon consumption risks : $\beta$ 's

- ▶ In this paper, we decompose the consumption growth risk  $\Delta c_t$  into two :
  - ▶ Carbon consumption (cc) growth risk ( $\Delta cc_t$ ) ;
  - ▶ Share of carbon consumption growth risk ( $\Delta \alpha_t$ ).
- ▶ Next, we compute the industry-level portfolios exposures to those risks :
  - ▶  $r_{i,t} = c_i + \beta_{sh,i} \Delta \alpha_{cc,t} + \beta_{cc,i} \Delta cc_t + \epsilon_{i,t} \quad i = 1, \dots, 42.$
  - ▶  $\beta_{sh,i}$ 's are in blue and  $\beta_{cc,i}$ 's are in red.
- ▶ As one can see, we cannot classify the industries based on their betas until 1955.
  - ▶ All industries are positively and negatively exposed to cc growth risk and share of cc growth risk respectively.
  - ▶ Except Rubber and Plastic Products industry, which is negatively and positively exposed to cc growth risk and share of cc growth risk respectively.
- ▶ As of 1956, the risk factors start to affect differently the industries.

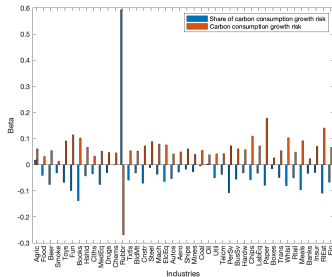
Industries exposures to carbon consumption risks :  $\beta$ 's

FIGURE – Full sample

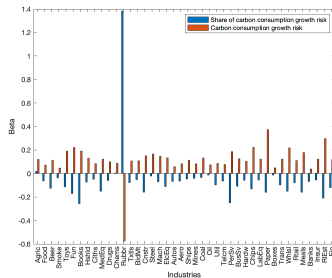


FIGURE – 1930-1955

# Industries exposures to carbon consumption risks : $\beta$ 's

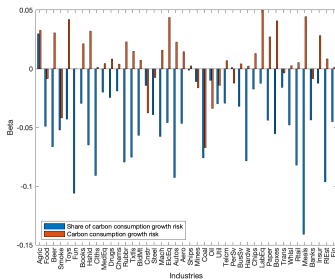


FIGURE – 1956-1980

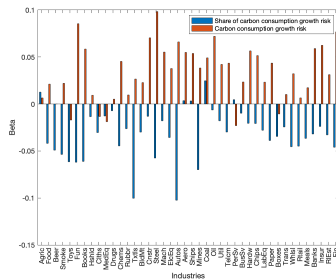
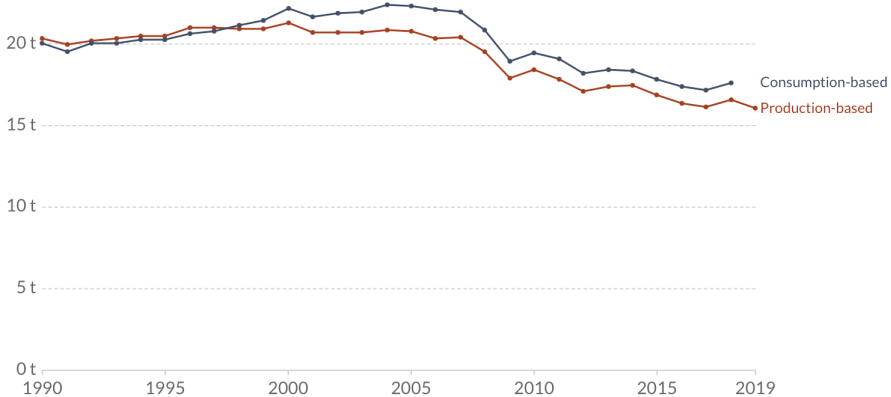


FIGURE – 1981-2018

# Conclusion

- ▶ LRCCR model solves the equity premium, volatility and risk-free rate puzzles :
  - ▶ By decomposing consumption growth into two components ;
  - ▶ Long-run risks in both expected carbon consumption and volatility.
- ▶ Our LRCCR model increases the capacity to detect the long-run risk during the period 1956-2018. But during the period 1930-1955, it was less detectable :
  - ▶ Investors can profit from it using climate change news ;
- ▶ The long-run risk variable  $x_t$  and its conditional variance  $\sigma_t^2$  help improving the predictability of the equity premium and the consumption growth.





Source: Our World in Data based on the Global Carbon Project and UN Population  
OurWorldInData.org/co2-and-other-greenhouse-gas-emissions • CC BY

Let us consider  $I$  categories of consumption among which  $J$  carbon consumption categories and  $I - J$  green consumption categories.

$$C_t = \sum_{i=1}^I C_{i,t} \quad (15)$$

$$C_t = \sum_{i=1}^J C_{i,t} + \sum_{i=J+1}^I C_{i,t} \quad (16)$$

$$C_t = CC_t + GC_t \quad (17)$$

Growth rate decomposition :

$$\Delta c_{t+1} = \log(CC_{t+1} + GC_{t+1}) - \log(CC_t + GC_t) \quad (18)$$

$$\Delta c_{t+1} = \log(CC_{t+1}) + \log \frac{CC_{t+1} + GC_{t+1}}{CC_{t+1}} - \log(CC_t) - \log \frac{CC_t + GC_t}{CC_t} \quad (19)$$

$$\Delta c_{t+1} = \Delta cc_{t+1} - \left( \log \frac{CC_{t+1}}{CC_{t+1} + GC_{t+1}} - \log \frac{CC_t}{CC_t + GC_t} \right) \quad (20)$$

$$\Delta c_{t+1} = \Delta cc_{t+1} - \Delta \alpha_{CC,t+1} \quad (21)$$

where  $\Delta c_{t+1}$ ,  $\Delta cc_{t+1}$  and  $\Delta \alpha_{CC,t+1}$  are consumption, carbon consumption and carbon consumption share growth rates (log-difference) respectively. ▶ LRCCR

For this decomposition, we weight each consumption category by its impact on the environment measured by their carbon footprints.

$$C_t = \sum_{i=1}^I CF_i \times C_{i,t} \quad (22)$$

$$C_t = \sum_{i=1}^J CF_i \times C_{i,t} + \sum_{i=J+1}^I CF_j \times C_{i,t} \quad (23)$$

$$C_t = CC_t + GC_t \quad (24)$$

Growth rate decomposition :

$$\Delta c_{t+1} = \log(CC_{t+1} + GC_{t+1}) - \log(CC_t + GC_t) \quad (25)$$

$$\Delta c_{t+1} = \log(CC_{t+1}) + \log \frac{CC_{t+1} + GC_{t+1}}{CC_{t+1}} - \log(CC_t) - \log \frac{CC_t + GC_t}{CC_t} \quad (26)$$

$$\Delta c_{t+1} = \Delta cc_{t+1} - \left( \log \frac{CC_{t+1}}{CC_{t+1} + GC_{t+1}} - \log \frac{CC_t}{CC_t + GC_t} \right) \quad (27)$$

$$\Delta c_{t+1} = \Delta cc_{t+1} - \Delta \alpha_{CC,t+1} \quad (28)$$

where  $\Delta c_{t+1}$ ,  $\Delta cc_{t+1}$  and  $\Delta \alpha_{CC,t+1}$  are consumption, carbon consumption and carbon consumption share growth rates (log-difference) respectively. ▶ LRCCR

TABLE – Descriptive statistics : 1930-1955.

► Descriptive statistics

	$E(.)$	$\sigma(.)$	$\rho(1)$	$\rho(2)$	$\rho(3)$	$\rho(4)$	$\rho(5)$	$CV$
$\Delta d$	0.0038	0.1914	0.1244	-0.2814	-0.1927	-0.1018	0.0954	50.6037
$\Delta c$	0.0169	0.0540	0.3633	0.1214	-0.1689	-0.2447	0.0208	3.1977
$\Delta\alpha_{cc}$	0.0005	0.0231	0.4800	0.0383	-0.1503	-0.3388	-0.2569	50.1958
$\Delta_{cc}$	0.0173	0.0611	0.2899	-0.0096	-0.2380	-0.2580	0.0343	3.5250
$\Delta\alpha_{gc}$	-0.0009	0.0378	0.4771	0.0408	-0.0949	-0.3182	-0.2775	-42.0192
$\Delta_{gc}$	0.0160	0.0621	0.5495	0.2875	-0.0174	-0.2748	-0.1701	3.8876
$z_m$	2.8535	0.2265	0.4185	-0.0828	-0.2764	-0.4030	-0.2802	0.0794
$r_m$	0.0732	0.2468	0.0904	-0.2068	-0.0779	-0.2504	-0.0288	3.3692
$r_f$	-0.0103	0.0558	0.6365	0.1463	0.0169	0.1026	0.2074	-5.4163

TABLE – Descriptive statistics : 1956-1980.

► Descriptive statistics

	$E(.)$	$\sigma(.)$	$\rho(1)$	$\rho(2)$	$\rho(3)$	$\rho(4)$	$\rho(5)$	$CV$
$\Delta d$	0.0074	0.0523	0.2667	0.0359	0.0091	0.0667	0.0053	7.0994
$\Delta c$	0.0222	0.0290	-0.0204	-0.2613	-0.0383	0.0427	0.1899	1.3081
$\Delta\alpha_{cc}$	-0.0035	0.0043	-0.2815	-0.0210	0.3359	-0.3975	0.0456	-1.2429
$\Delta_{cc}$	0.0187	0.0295	-0.0577	-0.2755	-0.0490	0.0381	0.1629	1.5790
$\Delta\alpha_{gc}$	0.0061	0.0076	-0.2780	-0.0079	0.3276	-0.3867	0.0584	1.2326
$\Delta_{gc}$	0.0283	0.0296	0.0243	-0.2123	0.0213	0.0111	0.2276	1.0473
$z_m$	3.2918	0.1822	0.6555	0.3480	0.2547	0.3035	0.1185	0.0553
$r_m$	0.0445	0.1779	-0.0762	-0.3736	0.1056	0.3032	0.0922	4.0023
$r_f$	0.0030	0.0152	0.5917	0.4075	0.4660	0.3126	0.2736	5.0496

TABLE – Descriptive statistics : 1981-2018.

► Descriptive statistics

	$E(.)$	$\sigma(.)$	$\rho(1)$	$\rho(2)$	$\rho(3)$	$\rho(4)$	$\rho(5)$	$CV$
$\Delta d$	0.0339	0.0926	-0.0365	-0.0047	-0.0714	-0.1272	-0.0885	2.7331
$\Delta c$	0.0155	0.0162	0.4603	0.0530	-0.0526	-0.0848	-0.0747	1.0438
$\Delta \alpha_{cc}$	-0.0052	0.0066	0.3599	-0.0832	-0.2421	-0.0687	0.1352	-1.2812
$\Delta_{cc}$	0.0104	0.0185	0.5090	0.0042	-0.1916	-0.1590	-0.1158	1.7807
$\Delta \alpha_{gc}$	0.0065	0.0083	0.3914	-0.0141	-0.1654	0.0388	0.1809	1.2843
$\Delta_{gc}$	0.0220	0.0172	0.3744	0.1211	0.1178	0.0675	0.0877	0.7844
$z_m$	3.8166	0.4151	0.8895	0.7548	0.6669	0.5546	0.4293	0.1088
$r_m$	0.0831	0.1618	-0.0695	-0.1204	0.0703	-0.0356	-0.4177	1.9464
$r_f$	0.0109	0.0221	0.7975	0.6500	0.5337	0.3759	0.3189	2.0303

TABLE – Model-implied moments. [▶ Back](#)

		$\sigma(z_m)$	$EP$	$E(R_f)$	$\sigma(r_{m,a})$	$\sigma(r_{f,a})$	$\rho(z_m)$
1930-1955							
BY2004	Data	0.227	0.084	-0.010	0.247	0.010	0.418
	Mean	0.238	0.109	0.007	0.264	0.159	0.354
	5%	0.166	-0.019	-0.051	0.193	0.122	0.019
	50%	0.234	0.105	0.006	0.261	0.158	0.368
	95%	0.325	0.251	0.065	0.346	0.198	0.637
LRCCR	Mean	0.150	0.040	0.010	0.116	0.019	0.608
	5%	0.098	0.000	-0.005	0.089	0.014	0.291
	50%	0.145	0.039	0.010	0.115	0.018	0.635
	95%	0.216	0.081	0.024	0.144	0.025	0.834
1956-1980							
BY2004	Data	0.182	0.041	0.003	0.178	0.003	0.656
	Mean	0.125	0.093	-0.002	0.134	0.001	0.345
	5%	0.093	0.045	-0.002	0.102	0.001	0.019
	50%	0.124	0.093	-0.002	0.133	0.001	0.360
	95%	0.162	0.143	-0.001	0.167	0.002	0.620
LRCCR	Mean	0.175	0.068	0.003	0.197	0.019	0.282
	5%	0.119	0.003	-0.005	0.138	0.014	-0.041
	50%	0.172	0.066	0.003	0.194	0.019	0.293
	95%	0.240	0.141	0.010	0.263	0.024	0.569