ALCHEMY AOE ALLIANCE

TYPE: [INSERT SPEC TYPE] (MTHD)		PROJECT: N/A
MTHD--001		**REVISION**: R1
Prize Distribution, Round Robin		

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1 - Definitions

The following mathematical definitions apply to player statistics within the same Round-Robin tournament:

- r – Win Rate. A value from 0 to 1, calculated as the quotient of games won and total games played. For transparency, win rate can never be raised to any power other than 1.
- p – Participation Rate. A value from 0 to 1, calculated by subtracting administrative penalties from the total planned games, and dividing by total planned games. For example, a single administrative penalty in a scoring period of ten games results in a 90% participation rate, $p=0.9$.
- α – Participation Bias. A value optionally defined by tournament host in a handbook or equivalent, used to magnify or shrink the importance of Participation Rate p in prize money calculation. If this factor is not defined, then it may be assumed to be 1.
- c – Challenge Factor. A value calculated by dividing a player's tournament elo into the average of their opponents. Participants who play against higher (on average) elo opponents will have Challenge Factor greater than 1, and vice versa.
- β – Challenge Bias. A value optionally defined by tournament host in a handbook or equivalent, used to magnify or shrink the importance of Challenge Factor c in prize money calculation. If this factor is not defined, then it may be assumed to be 1.
- s – Skill Factor. A value from 0 to 1, calculated as the quotient of a player's tournament elo and the average tournament elo that was calculated among players of their division.
- γ – Skill Bias. A value defined by tournament host to weigh the importance of Skill Factor s over other scoring criteria. Typically exceeds 1 and is used to attract exceptional players who are presumed to invent/inspire effective and entertaining strategies. Equal to 1.25 unless otherwise specified by handbook or equivalent.

2 - Overview

This document specifies a method for distributing prizes in a division-less round robin style tournament, factoring in all variables common to an eEports distribution.

3 - Background

This specification was originally developed for Alchemy League Season 2, but written as a general reference. Lessons learned from Seasons 2 through 4 introduced an opportunity refine the mathematical prize model, to account for additional variables, and address player concerns about penalization for the failure of an opponent to play. In the Original Issue, a 50% win rate was averaged into each participant of an un-played set, regardless of who was at fault, and who was higher or lower seed, and may have benefited from the games being played.

4 - Formulation

4.A - Personal Tournament Score

Personal tournament score f is an amalgamation of the contributing factors defined in §1, satisfying Equation 4.A below:

$$f = r(p)^{\alpha}(c)^{\beta}(s)^{\gamma}$$

The relationship of any one player's personal tournament score to the personal scores of all other players determines their fraction of the prize pool.

The factors on the right hand side of Equation 4.A are not exhaustive, but if a tournament would include a more complex score calculation, then its handbook would define additional factors to multiply in.

4.A.1 - Example

A Round-Robin player has tournament elo 1512, a 42% win rate, and participation score of 92%. Calculate Personal Tournament Score if *average opponent tournament elo was 1550 and average tournament elo was 1297.*

Since bias factors were not mentioned, they are assumed to be the default values defined in §1:

Participation Bias (α) = 1

Challenge Bias (β) = 1

Skill Bias (γ) = 1.25

From the problem statement, additional variables are identified:

Win Rate (r) = 0.42

Participation Rate (p) = 0.92

Challenge Factor (c) = $\frac{1550}{1512}$

Skill Factor (s) = $\frac{1550}{1297}$

The remaining step is to calculate Personal Tournament Score:

$$f = r(p)^{\alpha}(c)^{\beta}(s)^{\gamma} = (0.62)(0.92)^{1}\left(\frac{1550}{1512}\right)^{1}\left(\frac{1550}{1297}\right)^{1.25} = 0.731$$

4.A.2 - Example

Player1 is in a close contest for achieving highest personal score against a friend participating in the same Round-Robin (Player2), but incurred one administrative penalty in the first week. Assuming no additional penalties, determine the win rate Player1 would need in order to match Player2's score if Player1 and Player2 have tournament elos of 2011, 1987, respectively, and the tournament challenge and skill biases are 0 and 1.5, respectively. The tournament consists of eight games, and Player2 has a 50% win rate.

Since the challenge bias β is equal to zero and participation bias is presumed 1, Equation 4.A is simplified thusly:

$$f=r(p)^{\alpha}(c)^{\beta}(s)^{\gamma}=r(p)^{1}(c)^{0}(s)^{\gamma}=r\,p(s)^{\gamma}$$

Each player gets their own version:

$$f_1=r_1 p_1(s_1)^{\gamma}$$

$$f_2=r_2 p_2(s_2)^{\gamma}$$

To achieve the same Personal Tournament Score, $f_1=f_2$:

$$r_1 p_1(s_1)^{\gamma}=r_2 p_2(s_2)^{\gamma}$$

Rewritten:

$$r_1=r_2\frac{p_2(s_2)^{\gamma}}{p_1(s_1)^{\gamma}}=r_2\frac{p_2}{p_1}\left(\frac{s_2}{s_1}\right)^{\gamma}$$

Recognizing that the skill factor is the tournament elo of a player divided by the average of the group ($\bar{E}$):

$$s_1=\frac{E_1}{\bar{E}} \qquad s_2=\frac{E_2}{\bar{E}}$$

Such that:

$$\frac{s_2}{s_1}=\left(\frac{E_2}{\bar{E}}\right)\left(\frac{\bar{E}}{E_1}\right)=\frac{E_2}{E_1}$$

And it does not matter the average tournament elo of the division, as long as they are in the same division. Then finally:

$$r_1=r_2\frac{p_2}{p_1}\left(\frac{s_2}{s_1}\right)^{\gamma}=r_2\frac{p_2}{p_1}\left(\frac{E_2}{E_1}\right)^{\gamma}=0.5\left(\frac{8}{8-1}\right)\left(\frac{1987}{2011}\right)^{1.5}=0.5612$$

Thus, in order to achieve the same proportion of tournament earnings, Player1 will require a 56% win rate to compensate for the administrative penalty. With the same tournament elos, the required win rate from this example would increase to 57%.

4.B - Aggregate Tournament Score

Aggregate tournament score F is determined per Equation 4.B:

$$F = \sum_{i=1}^{N} f_i$$

Where N is the number of players in the division and f_i is the personal tournament score for each player, calculated using Equation 4.A.

4.B.1 - Example

A Round-Robin tournament has the following personal scores for each of its five players: 0.5, 0.6, 0.7, 0.8, 0.9. Calculate the Aggregate Tournament Score.

Applying Equation 4.B, it is straightforward to find the sum:

$$F = \sum_{i=1}^{N} f_i = f_1 + f_2 + f_3 + f_4 + f_5 = 0.5 + 0.6 + 0.7 + 0.8 + 0.9 = 3.5$$

4.C - Prize Fraction

The Prize Fraction awarded to any player is the same as their *Score Fraction*, which is calculated by dividing Aggregate Tournament Score (per Equation 4.B) into Personal Tournament Score (per Equation 4.A).

The sum of these fractions across all players must equal 1.

4.C.1 - Example

Calculate the prize fraction of the fourth player in Example 4.B.1:

Obtained simply by dividing 3.5 into 0.8, and getting 0.2286.

4.D - Prize

The Prize awarded to any player is the product of their Prize Fraction and the total prize pool for their division.

The sum of prizes across all players must equal the total prize pool.

4.D.1 - Example

Calculate player winnings from Example 4.C.1, if the tournament has total prize pool of $300 USD.

Obtained by simply multiplying 0.2286 and $300 USD, obtaining $68.57 USD.

5 - Cutoff & Rounding

Tournaments reliant on this specification will define a threshold, below which prizes will not be awarded, or will state that prizes will be awarded to the top X players, etc. Practical limitations in online banking include transaction fees that might exceed the value of the prize, or represent an unacceptable proportion of it; E. G. a $3 transaction fee for a $5 prize.

In the first case, players not meeting the threshold are eliminated one-by-one (starting with lowest earners) from the calculation, with their earnings distributed among remaining participants, until each is above the threshold.

In the second case, no iteration is required: the Prize Fraction is calculated only for the top X players.

5.A - Example

A tournament with $100 USD total prize pool will not award winnings to anyone who would earn less than $20 USD. Determine earnings for the player distribution from Example 4.B.1.

The Personal Tournament Scores from this example are "vectorized" below:

$$\vec{F}=[F_1,F_2,F_3,F_4,F_5]=[0.5,0.6,0.7,0.8,0.9]$$

Winnings for each player, denoted by $\vec{W}$ are calculated thusly:

$$\vec{W}=W_T\frac{[F_1,F_2,F_3,F_4,F_5]}{\sum\vec{F}}=\$100\,USD\frac{[0.5,0.6,0.7,0.8,0.9]}{3.5}$$

$$\vec{W}=\$[14.28,17.14,20.00,22.86,25.71]\,USD$$

This winnings breakdown correctly adds to $100 USD (the original total prize pool), but incorrectly contains two values below the threshold of $20, which makes iteration necessary.

Even though two entries are below the threshold, it is necessary to give the second a chance to meet the threshold, once some of the first's winnings are distributed to it. Thus, the player with Personal Tournament Score of 0.5 is eliminated from the earnings model and the calculation repeated, the new vector becomes:

$$\vec{F}=[F_2,F_3,F_4,F_5]=[0.6,0.7,0.8,0.9]$$

$$\vec{W}=W_T\frac{[F_2,F_3,F_4,F_5]}{\sum\vec{F}}=\$100\,USD\frac{[0.6,0.7,0.8,0.9]}{3}$$

$$\vec{W}=\$[20.00,23.33,26.67,30.00]\,USD$$

The second-to-last player, having now achieved the threshold, is eligible for prize in a way that could not be proven on the first iteration.

6 - Conclusion

A model was presented for compensating participants of a round-robin style tournament that can account for their performance (win rate), administrative burden (participation rate), unlucky matchups (challenge factor), and skill level (elo). The model contains parameters to bias for criteria to the tournament, and may be iterated to ensure that winnings are large compared with banking transaction fees.

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Revision History:

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OI	------ Original Issue ------	N/A	2024-06-02
R1	Substantial re-write (see archived version). Changes include update to new format, and more complex system that accounts for win rate, admin penalties, opponent difficulty, etc.	N/A	2024-12-14