CL202: Fluid Mechanics

Fluid Statics

Raghvendra Gupta
Department of Chemical Engineering
Indian Institute of Technology Guwahati

What has been covered till now?

Let us list down

- Chapter 1 and 2: Introductory concepts
- Chapter 7: Dimensional analysis and similitude
- Chapter 3: Fluid Statics
  - Pressure distribution
    - Manometers
  - Force on a plane submerged surface

- Tutorial 2 tomorrow (26.8.2021) on Moodle
- Syllabus: Dimensional analysis, similitude, pressure distribution in a static fluid, manometers, force on a plane submerged surface

Hydrostatic Force on a Plane Submerged Surface

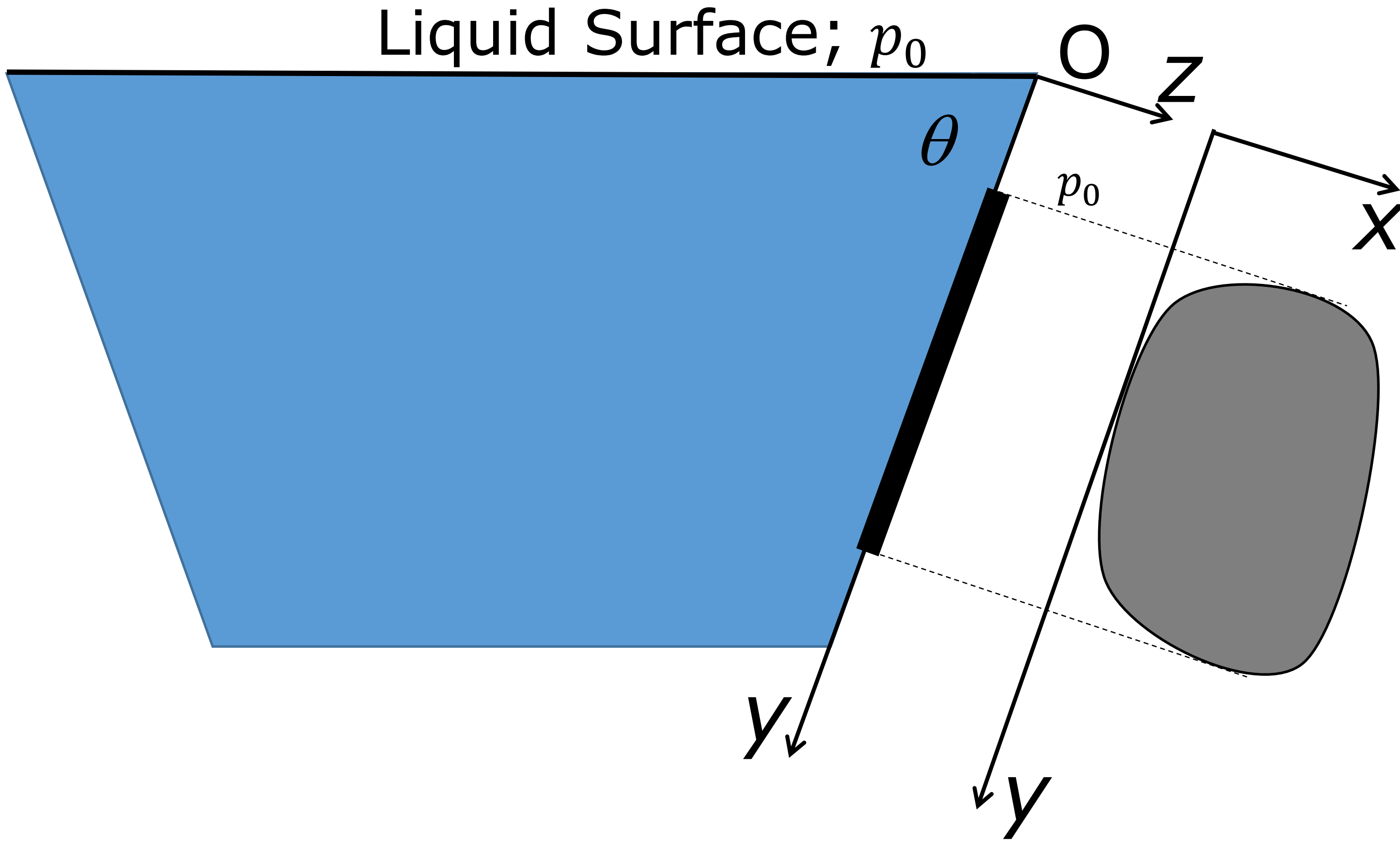
- The magnitude and direction of pressure force on the surface

$$\boxed{\boldsymbol{F}_R = (p_0 + \rho g h_C)A}$$

- The line of action of force

$$\boxed{x_L = x_C + \frac{\rho g\ sin\theta I_{xy,centroid}}{\boldsymbol{F}_R}}$$

$$\boxed{y_L = y_C + \frac{\rho g sin\theta\ I_{xx,centroid}}{\boldsymbol{F}_R}}$$

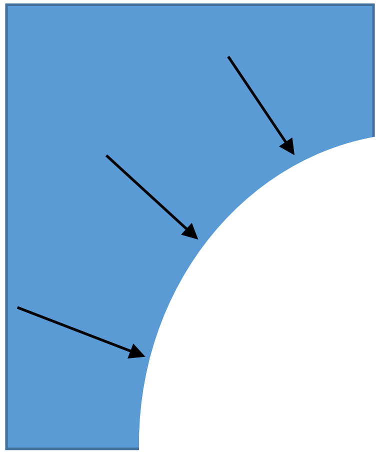


$\boldsymbol{F}_{net} = \rho g h_C A$

Hydrostatic Force on a Curved Submerged Surface

- For a planar surface, the direction of force is same everywhere
- For a curved surface, the direction of pressure forces varies at different points
    - We will need to consider vector form of the equation

Force on a surface element dA



$$d\boldsymbol{F} = -p d\boldsymbol{A}$$

The resultant force $\boldsymbol{F}_R = \int_A -p d\boldsymbol{A}$

Component of resultant force in *x*-direction

$$\boldsymbol{F}_{R,x} = \boldsymbol{F}_R . \hat{\boldsymbol{\imath}}$$

$$= \int_A -p d\boldsymbol{A} . \hat{\boldsymbol{\imath}}$$

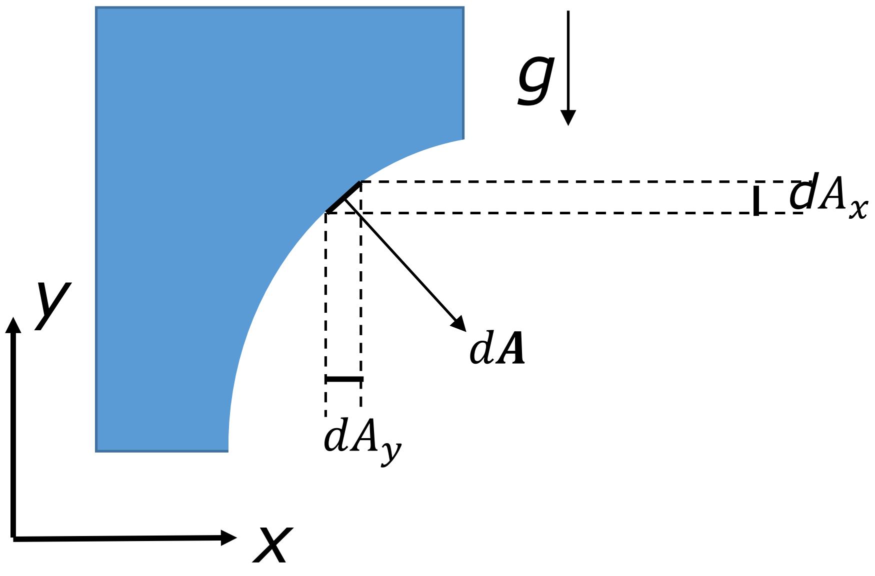
Hydrostatic Force on a Curved Submerged Surface

$$\boldsymbol{F}_{R,x} = \int_A -p d\boldsymbol{A}.\hat{\boldsymbol{\imath}}$$

$$\boldsymbol{F}_{R,x} = \int_{A_x} -p dA_x$$

In the same manner, another horizontal force component can be found

$$\boldsymbol{F}_{R,\boldsymbol{z}} = \int_{A_z} -p dA_z$$



Horizontal force and its point of action are same as that for a vertical plane surface of same projected area.

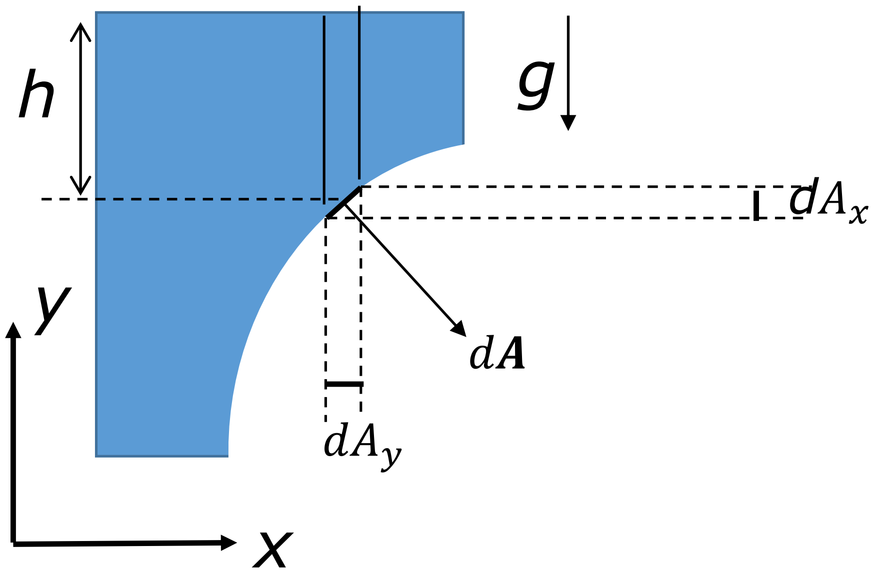
Hydrostatic Force on a Curved Submerged Surface

Vertical force component due to the fluid only

$$\boldsymbol{F}_{R,\boldsymbol{y}} = \int_{A_y} -p \, dA_y$$

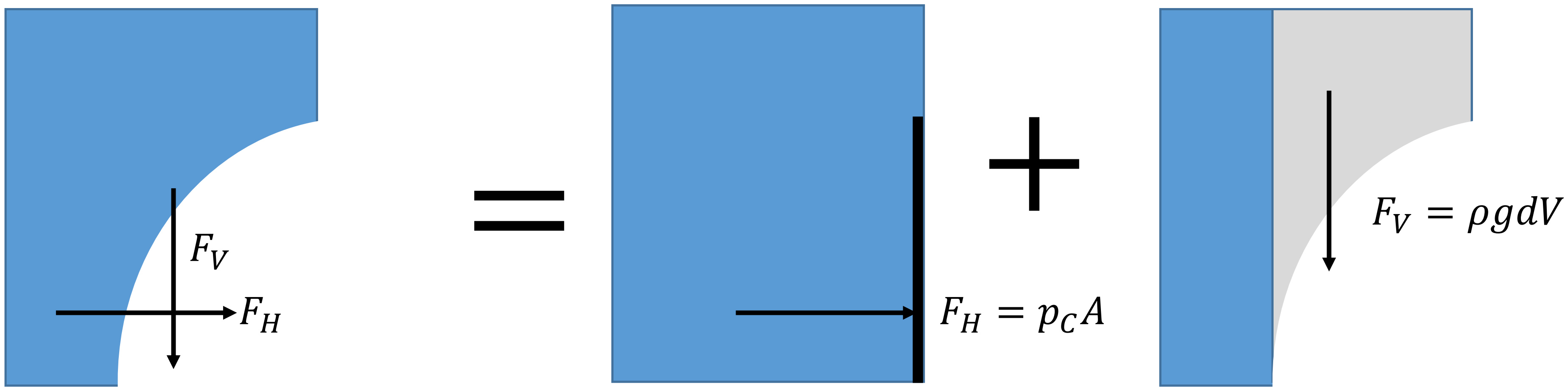
$$\boldsymbol{F}_{R,\boldsymbol{y}} = \int_{y} \rho g h \; dA_y$$

$$\boldsymbol{F}_{R,\boldsymbol{y}} = \int_{\forall} \rho g d\forall$$



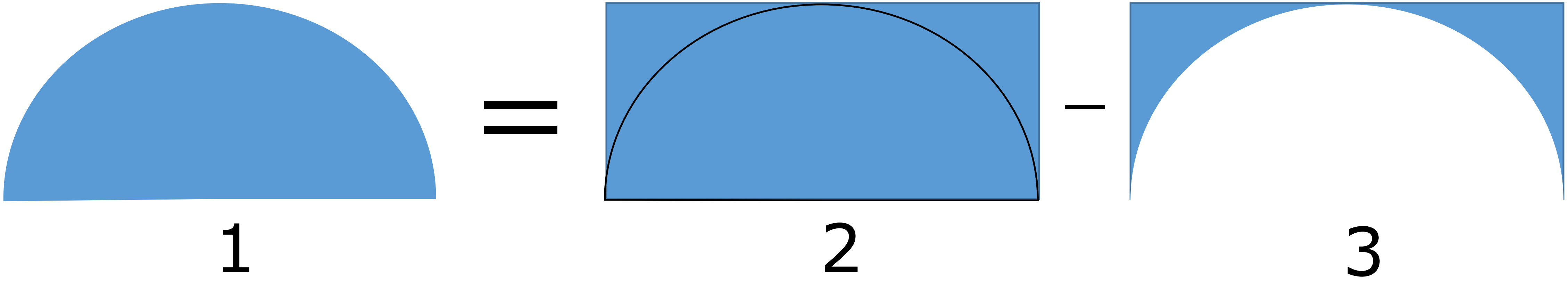
- Vertical force is the weight of fluid above the surface.
- Line of action of vertical force passes through the centre of gravity of the volume directly above the surface.

Hydrostatic Force on a Curved Submerged Surface



Hydrostatic Force on a Curved Submerged Surface

- To find vertical force when there is no liquid above the surface.



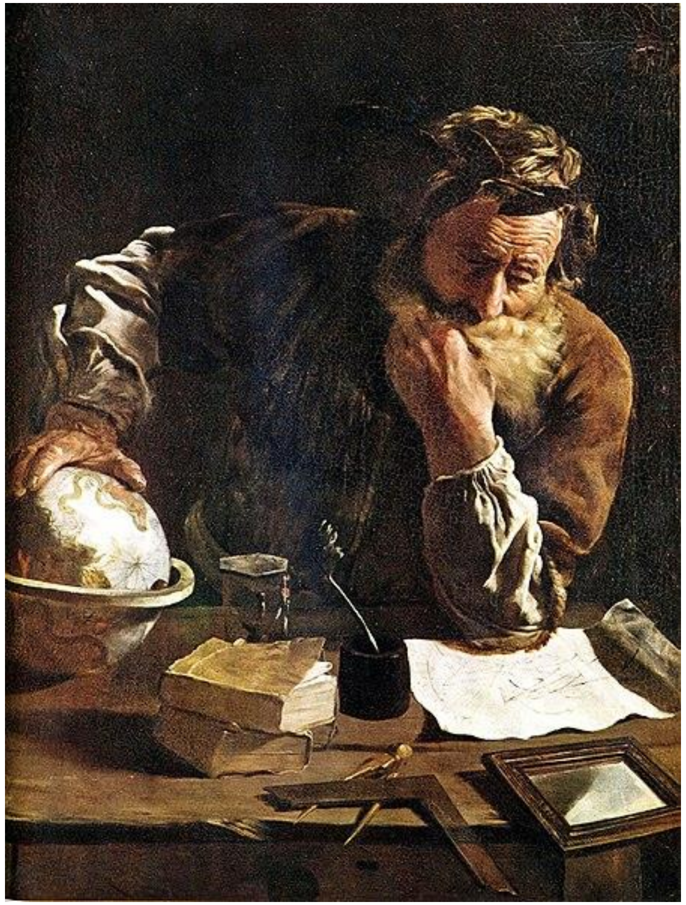
- In 2, the net pressure force on the body will be zero.
- In 3, the vertical force will be equal and opposite to that in 1.
- Of course, one can also calculate the vertical force directly by taking an elemental surface and integrating it over the entire surface.

Archimedes' Principle

Any body when submerged partially or fully in a liquid experiences an upward buoyant force on it and this force is equal to the weight of the fluid the body displaces.

Is the crown made of pure gold?

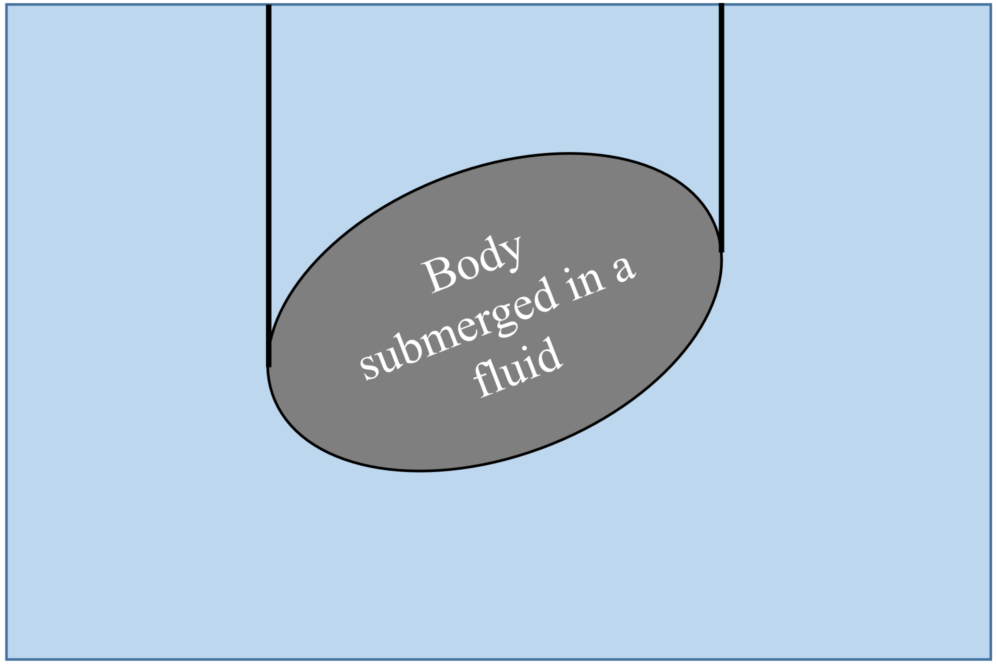
The principle can be derived from hydrostatics.



Archimedes: A Greek mathematician, physicist, engineer, inventor and astronomer

Image Source: http://archimedes2.mpiwg-berlin.mpg.de/archimedes_templates/popup.htm

Archimedes' Principle



The body is made up of two curved surfaces:

Vertical force on the upper curved surface

= weight of liquid above the surface

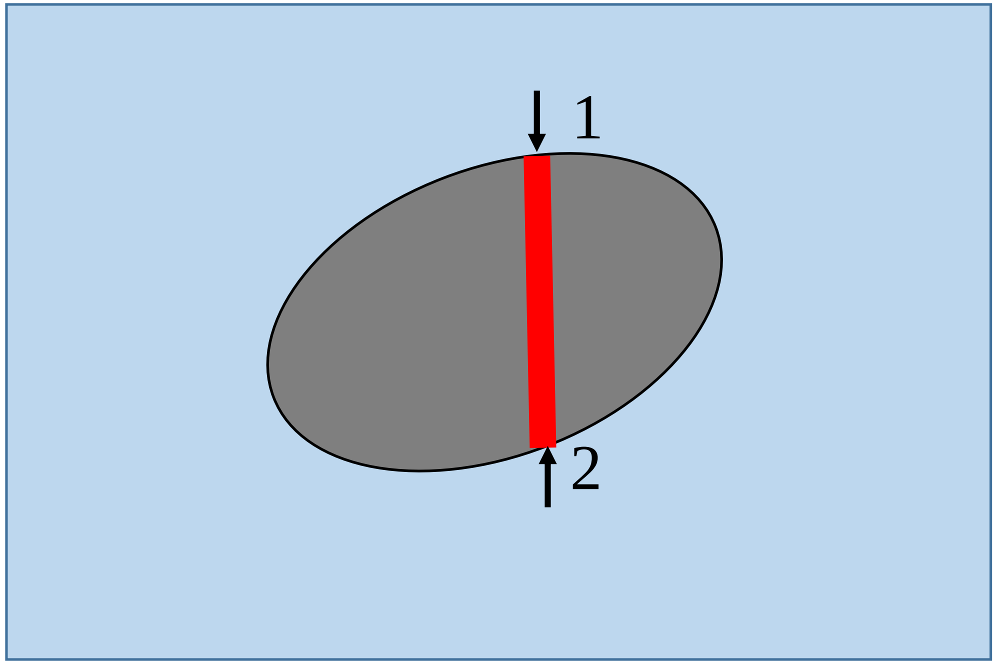
Vertical force on the lower curved surface

= - weight of liquid above the surface

Net vertical force on the body caused by the fluid = weight of liquid above the upper curved surface - weight of liquid above the lower curved surface

= -weight of liquid equal to the volume of the body

Archimedes' Principle



Consider a vertical elemental slice of the body having area dA

Upward force on the element $= (p_0 + \rho g h_2)dA - (p_0 + \rho g h_1)dA$

$$= \rho g(h_2 - h_1)dA$$

$$= \rho g dV$$

Force on the body $= \int_{\forall} \rho g dV$

Problem: Force on a curved surface

A glass observation room is to be installed at the corner of the bottom of an aquarium. The aquarium is filled with seawater up to a depth H. The glass wall of the observation room is part of a spherical shell of radius R mounted symmetrically in the corner. You can assume the density of seawater to be ρ. The pressure at the surface of the liquid and the pressure inside the observation room are the same. Find the resultant force on the glass surface.

Centroid of quarter of a circle = $4R/3\pi$

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$Horizontal\ components\ F_x = F_y = \rho g h_C \frac{\pi}{4} R^2$ $\quad$ $where\ h_C = H - \frac{4R}{3\pi}$

$F_z = \frac{1}{4}\pi R^2 H - \frac{1}{8}\frac{4}{3}\pi R^3$