CL202: Fluid Mechanics

Dimensional Analysis

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What has been covered till now?

Let us list down

- Definition of a fluid
- Applications
- Three approaches to analyse fluid flow problems
- Continuum hypothesis
- No-slip condition
  - Zero shear condition at the free surface
- Eulerian and Lagrangian descriptions
- System and control volume
- Velocity and stress fields (stress is a second order tensor)
- Strain rate, Newton’s law of viscosity
- Non-Newtonian fluids
- Flow visualization: Streamlines, pathlines and streaklines

Tutorial 1

- Tutorial 1 will be on Thursday, August 12 during the class hours (11:00-11:45AM).
- Will be conducted on Moodle
    - Please enroll yourself in the course if you have not done so already
- Syllabus: Introductory concepts (listed on Slide 2)
- Next Thursday August 19 is a holiday.
- August 26: Tutorial 2 will be conducted.
- Quiz 1: September 2

Base and Derived Quantities

Basic or fundamental quantities and SI units:

All other physical quantities can be represented in terms of these basic quantities.

- Time (second)
- Length (meter)
- Mass (kilogram)
  - Earlier Force was considered a base quantity
- Temperature (Kelvin)
- Electric current (Ampere)
- Amount of substance (Mole)
- Luminous intensity (Candela)

Derived quantities:

Can be derived from base quantities.

Examples: Velocity, acceleration, force, energy, power

Dimensional Homogeneity

A mathematical formulation that expresses relationship between variables in a physical process will be dimensionally homogeneous i.e. each of its additive terms will have the same dimension.

Example:

Displacement of a body moving with a constant acceleration:

Bernoulli's Theorem:

$$P + \frac{1}{2}\rho V^2 + \rho g H = Constant$$

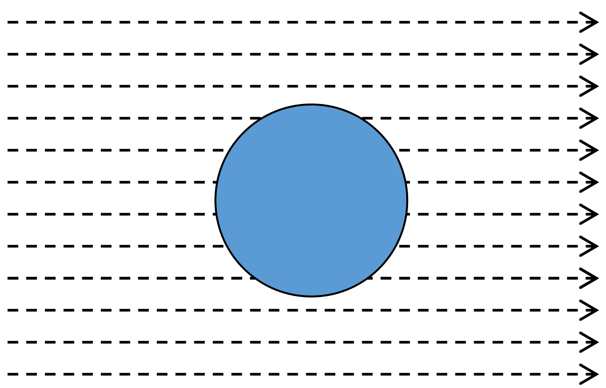
$$\frac{P}{\rho} + \frac{1}{2}V^2 + gH = Constant$$

A dimensionless expression is dimensionally homogeneous.

Why Dimensional Analysis

- Reduces the number and complexity of experimental variables

- Example: Force on a sphere immersed in a fluid stream
    - Drag force F
    - Sphere diameter d
    - Fluid density ρ
    - Fluid viscosity μ
    - Sphere speed V



Why dimensional Analysis

- Reduces the number and complexity of experimental variables
- Can save time, effort and money
- Plan experiments or numerical simulations
- Provides scaling laws: lab-scale model to large prototype

Π Theorem

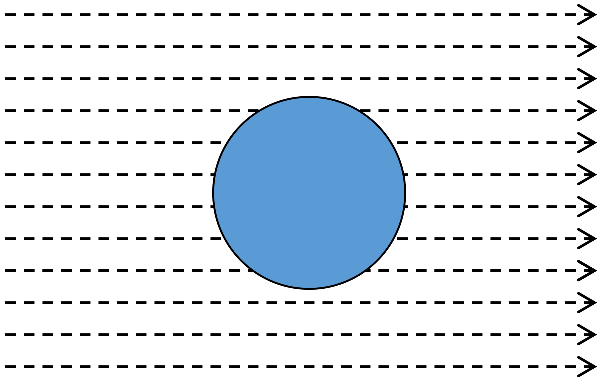
If a physical process depends on *n* physical parameters, it can be reduced to a relation between *n*-*m* dimensionless variables.

The reduction *m* is equal to the maximum number of variables that do not form a dimensionless group among themselves.

The reduction *m* is always less than or equal to the number of fundamental dimensions describing the variables.

An Example

Consider the problem of drag on a sphere



1. List all the dimensional parameters involved (n = 5)
   - Drag force F
   - Sphere diameter d
   - Fluid density ρ
   - Fluid viscosity μ
   - Sphere speed V
2. Select a set of fundamental primary dimensions
   - M, L, T (r = 3)

An Example

3. List the dimensions of all the parameters in terms of primary dimensions

- Drag force F:
- Sphere diameter d :
- Fluid density ρ:
- Fluid viscosity μ:
- Sphere speed V:

An Example

4. Select a set of $m = r$ (in most of the cases) dimensional parameters that include all the primary dimensions

- F: MLT^{-2} ; d : L; ρ: ML^{-3} ; μ: $ML^{-1}T^{-1}$; V: LT^{-1}

<u>Thumb rules:</u>

- They must not form a dimensionless group among themselves
- Dependent variable (F in this example) should not be selected
- The selected variables should not be power of another variable e.g. length and volume
- Try not to choose viscosity or surface tension as a repeating variable
- Choose density, speed and characteristic dimensions whenever possible
  - d, ρ, V

An Example

5. Set up dimensional equations, combining the parameters selected in step 4 with each of the remaining parameters to form dimensionless group

- There will be n-m= 5-3 =2 **independent** dimensionless group
- Combine repeating variables with μ and F
- $\Pi_1 = \mu V^a d^b \rho^c$
- $\Pi_2 = F V^x d^y \rho^z$
- Obtain the exponents for each so that each group is dimensionless

An Example

$\Pi_1 = \mu V^a d^b \rho^c$: Obtain a, b and c

$$M^0 L^0 T^0 = M^1 L^{-1} T^{-1} \; M^0 L^a T^{-a} M^0 L^b T^0 M^c L^{-3c} T^0$$

Now, you can make 3 equations in a, b and c and find a, b and c.

An Example

$\Pi_2 = FV^x d^y \rho^z$: Obtain *x*, *y* and *z*

An Example

6. Check to see that each obtained group is dimensionless

$$\Pi_1 = \frac{\mu}{\rho V d}$$

$$\Pi_2 = \frac{F}{\rho V^2 d^2}$$

The two (***n-m***) parameters are independent i.e. any parameter cannot be obtained by combining other two.

Obtaining Independent Dimensionless Groups

1. List all the dimensional parameters, say *n* in number, involved
2. Select a set of fundamental primary dimensions- M (or F), L, T, θ(Temperature)
3. List the dimensions of all the parameters in terms of primary dimensions (*r* in number)
4. Select a set of *m* repeating dimensional parameters that includes all the primary dimensions
5. Set up dimensional equations, combining the repeating dimensional parameters with the remaining parameters to form dimensionless group
6. Check to see that each obtained group is dimensionless

Obtaining Independent Dimensionless Groups

- Reynolds number = $\frac{Inertia}{Viscous} = \frac{DV\rho}{\mu}$

- Mach number $\frac{V}{c}$

- Drag $coefficient = \frac{F_{Drag}}{\rho V^2 A}$

- Pressure $coefficient = \frac{p - p_\infty}{\rho V^2}$

- Froude number = $\frac{Inertia}{Gravity} = \frac{V}{\sqrt{gL}}$

Important Dimensionless Groups

- Capillary number = $\frac{Viscous}{Surface\ tension} = \frac{\mu V}{\sigma}$

- Weber number = $\frac{Inertia}{Surface\ tension} = \frac{\rho L V^2}{\sigma}$

- Strouhal number $\frac{Oscillation}{Mean\ speed} = \frac{\omega L}{V}$