CL202: Fluid Mechanics

Dimensional Analysis and Similitude

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What has been covered till now?

Let us list down

- Introductory concepts
- Dimensional analysis
  - Buckingham Pi theorem
  - Classification of flows
    - Mach number
    - Boundary layer
- Similitude/ Similarity/ Scale-up

Flow Similarity

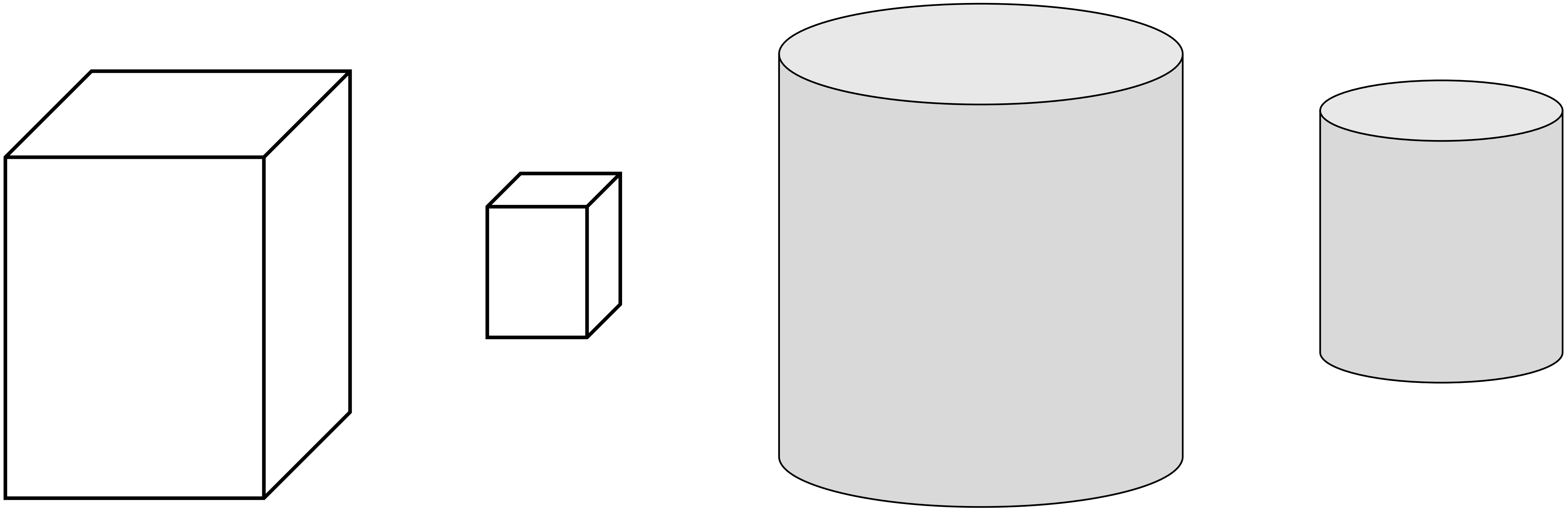
- A chemical engineer develops processes for the production of chemicals / materials at the industrial scale
- The job is to scale up the process from lab scale to plant scale
- The processes involved fluid flow, heat and mass transfer are scale-dependent
- He often needs to deal with ‘scale-up’ and sometimes with ‘scale-down’ problem
- The process is often simulated at the small scale (lab or pilot)
    - Is it possible to achieve full similarity between the model and full-scale plant?
    - If yes, what rules govern the adaptation?
    - If not, then what?
    - How small should the model be?

Flow Similarity

- From the fluid mechanics perspective, the model and equipment should have:
  - Geometric Similarity
  - Kinematic Similarity
  - Dynamic Similarity

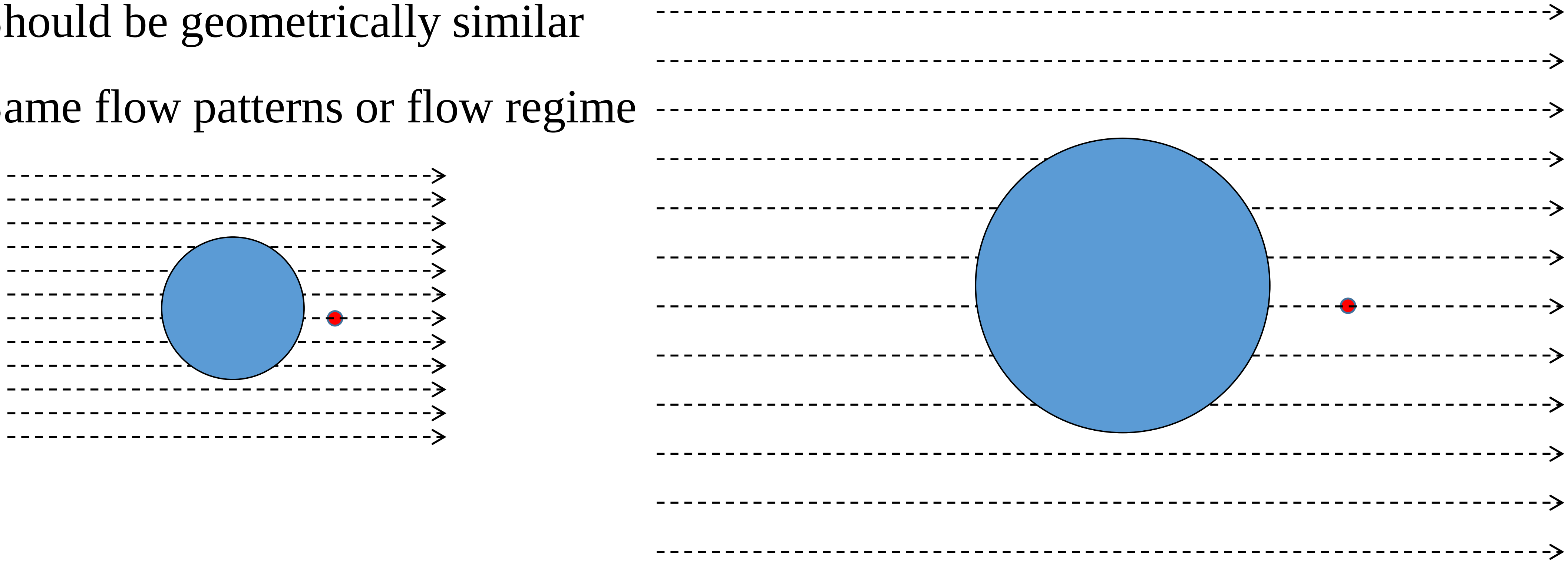
Geometric Similarity

- Shape of model and prototype (full scale equipment) should be same
- Corresponding dimensions of the two should have same ratio

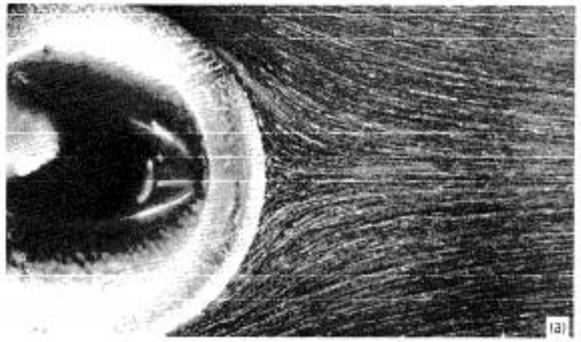


Kinematic Similarity

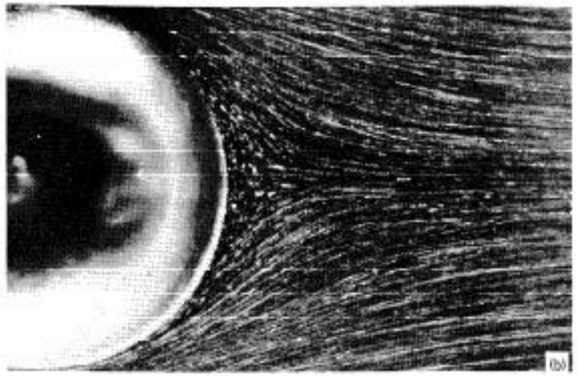
- Velocity at corresponding points should be in the same direction
- Velocity at corresponding points should have a constant scaling factor
- Similar streamline patterns for both the flows e.g. flow around a sphere
  - Should be geometrically similar
  - Same flow patterns or flow regime



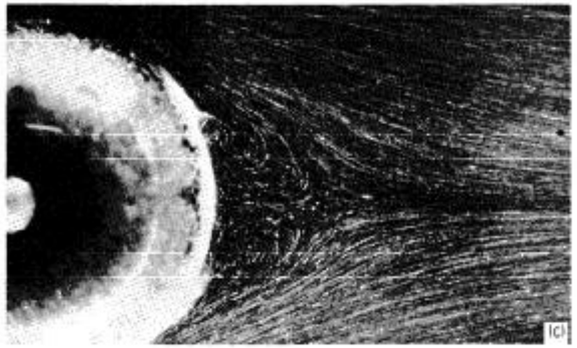
Flow Patterns around a Sphere



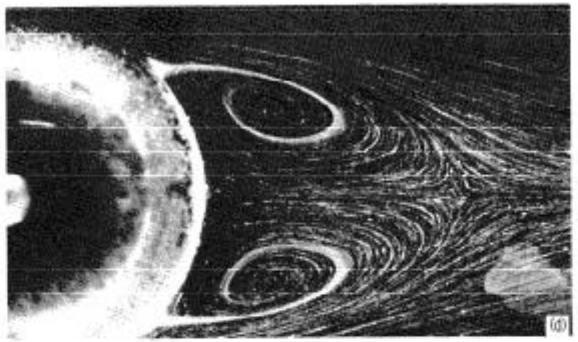
Re = 17.9



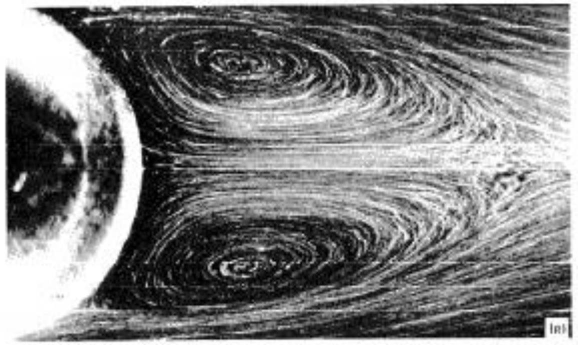
Re = 26.8



Re = 37.7



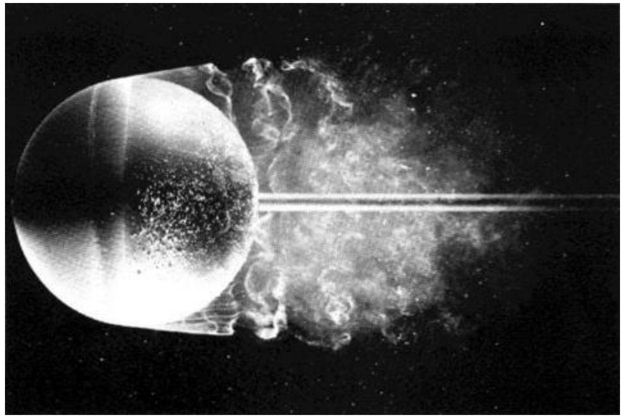
Re = 73.6



Re = 118

Re = 133

Images from Clift et al., 1978



Re = 30,000
Image from
http://media.efluids.com/galleries/boundary?medium=268

You can try creating some images using incense stick smoke as a tracer, ball(s) as a sphere and fan to change air speed

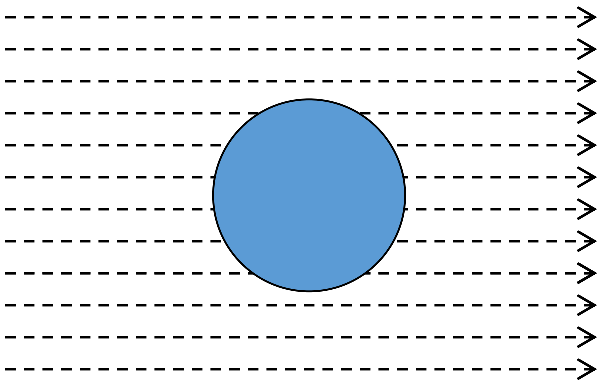
Dynamic Similarity

- Identical force distribution in the model and prototype
  - Forces are parallel for the two cases
  - Their magnitude have a constant scaling factor
- Should also have kinematic similarity
  - Should have geometric similarity
  - Necessary but not sufficient condition

Condition for Complete Similarity

- Two processes can be considered similar if:
  - They have geometric similarity
  - All relevant non-dimensional numbers describing them have same numerical value

Example: Flow around a sphere



- We want to predict force on a bigger sphere
- When there is dynamic similarity:
  - $\Pi_1 = \frac{\mu}{\rho V d}$, $\Pi_2 = \frac{F}{\rho V^2 d^2}$ are same in the two cases
- Experiments with the model should be such that Re is same in both the cases
- For dynamically similar case- drag coefficient will be same
  - We will be able to find drag on bigger sphere by measuring it for smaller sphere

Example: Flow in a pipe

Oil (ρ_O, μ_O) flows in a pipe of diameter d at an average speed U_o and produces a pressure drop per unit length of p_o. Water (ρ_w, μ_w) is flown in the same pipe under dynamically similar conditions. What should be the average speed of water flow and the corresponding pressure drop per unit length.

$$p(Pressure\ drop\ per\ unit\ length) = f(\rho, \mu, d, U)$$

$$Repeating\ variables = \rho, d, U$$

$$Two\ non\text{-}dimensional\ groups = \frac{\mu}{\rho d U};\ \frac{pd}{\rho U^2}$$

For dynamic similarity:
Both the dimensionless groups should be same in case of flow of water and flow of oil.

$$\frac{\mu_w}{\rho_w d U_w} = \frac{\mu_o}{\rho_o d U_o} \quad \Rightarrow \quad U_w = U_o \frac{\rho_o}{\rho_w}\frac{\mu_w}{\mu_o}$$

$$\frac{p_w d}{\rho_w {U_w}^2} = \frac{p_o d}{\rho_o {U_o}^2} \quad \Rightarrow \quad p_w = p_o \frac{\rho_w}{\rho_o}\frac{{U_w}^2}{{U_o}^2} = p_o \frac{\rho_o}{\rho_w}\frac{{\mu_w}^2}{{\mu_o}^2}$$

Incomplete Similarity

For some cases, it is not possible to all dimensionless groups to be same.

Example: Drag force on a ship: Consider 1/100 scale model (m) of full scale prototype (p)

- Important parameters: ρ, μ, L, g, V, F
- Find dimensionless groups: Reynolds and Froude numbers; drag coefficient
- For same drag: Re and Fr should be equal
- Fr same: $\left(\frac{V_m}{\sqrt{gL_m}}\right) = \left(\frac{V_P}{\sqrt{gL_P}}\right)$: $\frac{V_m}{V_P} = \frac{\sqrt{L_m}}{\sqrt{L_P}} = \frac{1}{10}$
- For Re to be same: $\left(\frac{V_m L_m}{\nu_m}\right) = \left(\frac{V_P L_P}{\nu_P}\right)$ (Note $\nu = \mu/\rho$)
- $\frac{\nu_m}{\nu_P} = \frac{V_m}{V_P}\frac{L_m}{L_P} = 0.1 \times 0.01 = 0.001$
- Fluid will need to changed but for realistic numbers such fluids do not exist

Governing Equations for Fluid Flow

Mass conservation: $$\boxed{\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}.\rho\boldsymbol{v} = 0}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

Momentum conservation:
Newtonian fluid (**Navier-Stokes Equations**)

$$\boxed{\frac{\partial(\rho\boldsymbol{v})}{\partial t} + \boldsymbol{v}.\boldsymbol{\nabla}(\rho\boldsymbol{v}) = -\boldsymbol{\nabla}p + \mu\boldsymbol{\nabla}^2\boldsymbol{v} + \rho\boldsymbol{g}}$$

Transient/ unsteady term | Convective term | Pressure term | Viscous term | Gravity/ other body force term

x: $$\frac{\partial(\rho u)}{\partial t} + u\frac{\partial(\rho u)}{\partial x} + v\frac{\partial(\rho u)}{\partial y} + w\frac{\partial(\rho u)}{\partial z} = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + \rho g_x$$

y: $$\frac{\partial(\rho v)}{\partial t} + u\frac{\partial(\rho v)}{\partial x} + v\frac{\partial(\rho v)}{\partial y} + w\frac{\partial(\rho v)}{\partial z} = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) + \rho g_y$$

z: $$\frac{\partial(\rho w)}{\partial t} + u\frac{\partial(\rho w)}{\partial x} + v\frac{\partial(\rho w)}{\partial y} + w\frac{\partial(\rho w)}{\partial z} = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) + \rho g_z$$

Non-linear partial differential equations

Computational Fluid Dynamics (CFD)

You can learn it along with Python

https://github.com/barbagroup/CFDPython

Extra class tomorrow 11 AM to 12 Noon