Sample title

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American options pricing problem

Definitions

- **Options** are contracts that give **the holders** the right to buy/sell an **underlying asset** at a pre-established at some point in the future before or at the expiration date.
- The **strike** price is the pre-established price at which the holder will buy/sell the **option**.
- The **maturity** date is the expiration date of the contract.
- **Exercising** the option refers to the act of buying/selling the underlying asset.
- The **premium** is the money charged by the writer to the holder for entering the contract.

American options pricing problem

Definitions

- **Options** are classified based on the holder's **position**.
  - **Call options** when the holder's position is to buy.
  - **Put options** when the holder's position is to sell.
- **Options** are also classified based on their **exercise style**.
  - **European options** when the holders can only exercise the option **at** the maturity date.
  - **American options** when the holders can exercise the option at any time **before and at** the maturity date.

American options pricing problem

Definitions

	At time maturity	Before and at maturity
Buy	European call options	American call options
Sell	European put options	American put options

American options pricing problem

Definitions

Goal: Given an American call/put option with a **strike price** (K), and **maturity date** (T), find the **premium** $(V(S))$ charged by the **writer** to the holder such as there is no **arbitrage** opportunity. Arbitrage refers to the possibility that either the writer or holder to make a risk-free profit.

American options pricing problem

Mathematical model

- $\mathcal{T} : [0, T]$
- $\mathcal{X} : [0, \infty)$
- $\mathcal{D} : \mathcal{X} \times \mathcal{T}$
- $V : \mathcal{D} \to \mathbb{R}$

American options pricing problem

Mathematical model

The payoff of function is defined as

$$H(S,t) = \max(S - K, 0) \tag{1a}$$

$$H(S,t) = \max(K - S, 0) \tag{1b}$$

American options pricing problem

Black-Scholes model

- American options is bounded from below by the payoff $V(S,t) \geq H(S,t)$
- $\mathcal{D}$ is divided in two exclusive regions: the exercise region $\mathcal{S}$ and continuation region $\mathcal{C}$.
- $\bar{S}(t)$ is the optimal exercise price.
- The price $V(S,t)$ behaves similar to the price of a European option in the continuation region.
- $\mathcal{S} : \{(S,t) : V(S,t) = H(S,t)\}$
- $\mathcal{C} : \{(S,t) : V(S,t) > H(S,t)\}$
- $\partial\mathcal{C} : \{(S,t) : \bar{S}(t) = S\}$

American options pricing problem

Free boundary problem

$$\begin{cases} \dfrac{\partial V}{\partial t} + \dfrac{1}{2}\sigma^2 \dfrac{\partial^2 V}{\partial S^2} + (r-\delta)\dfrac{\partial V}{\partial S} - rV = 0 & \text{for } (S,t) \in \mathcal{C} \\ V(S,t) = H(S,t) & \text{for } (S,t) \in \partial\mathcal{C} \end{cases} \tag{2}$$

American options pricing problem

Free boundary problem

$$
\begin{cases}
\dfrac{\partial V}{\partial t} + \dfrac{1}{2}\sigma^2 \dfrac{\partial^2 V}{\partial S^2} + (r-\delta)\dfrac{\partial V}{\partial S} - rV = 0 & \text{for } (S,t) \in \mathcal{C} \\
V(S,t) = H(S,t) & \text{for } (S,t) \in \partial\mathcal{C}
\end{cases}
\tag{3}
$$