Title of the report goes here and it may have several lines

G14PJA = MATH4041

Mathematics 3rd Year Project

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School of Mathematical Sciences

University of Nottingham

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Supervisor: Dr. Your Supervisor

Assessment type: Review

I have read and understood the School and University guidelines on plagiarism. I confirm that this work is my own, apart from the acknowledged references.

Abstract

The abstract of the report goes here. The abstract should state the topic(s) under investigation and the main results or conclusions. Methods or approaches should be stated if this is appropriate for the topic. The abstract should be self-contained, concise and clear. The typical length is one paragraph.

Contents

1 Introduction

The introductory section goes here. And remember the introduction is the last thing you write.

The end of the introductory section would typically outline the structure of the report. In this template, section 4 gives the background of the topic, sections 5 and 6 contain the bulk of the work and section 7 summarises and discusses what has been achieved. Appendix A displays the raw data, and certain technical calculations for section 5 are deferred to appendix B.

2 Background

2.1 Finance

A common problem in finance is to price financial derivatives, often referred just as derivative. In essence, a derivatives are contracts set between parties whose value in time derives from the price of their underlying assets. A notorious family of derivatives in financial markets are options. Options are contracts set between two parties in which the buyer of option has the right to sell or buy, commonly referred as, exercise, an stock at a prestablished price, also known as, strike price, in the future. An option is referred as a call option or as a put option if the exercise position is to buy or to sell respectevely. Similarly, options are classified depending on their exercise style. In that regard, the simplest of options are European options. European options gives buyer of the option can only exercise the option at given point in time in the future, often referred as the maturity or expiration date. The payoff of an European option is expressed using the following formula

$$P = \max(S_T - K, 0) \tag{2.1}$$

$$P = \max(K - S_T, 0) \tag{2.2}$$

where T is the time elapsed between the starting and expiration date of the contract, S_T is the stock price at the expiration time, and K is the strike price. Note that formula (2.1) and (2.2) are for an call and a put option respectevely. Obviously, options give greater flexibility to the buyers because is removing their exposure of having a negative payoff. It is because the writers of options charge premiums to the buyers at the time they enter the contracts. The premium is often referred as the price or value of the option and the problem of finding this value is called options pricing or valuing. When pricing options, it is important to find the just price because otherwise the writer or buyer of the option could set some scheme in which option will always be profitable to them. In other words, options pricing must follow the principle of no-arbitrage. Suppose that there exists a risk-free financial instruments with a continous rate of return r. This financial instrument could be a US tresury bill or a certificate of deposit. By principle of non-arbitrage, the price of an option must by

That is we want to gi

3 Front fixing method

3.1 Inverse transformation

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + (r-\delta)S\frac{\partial V}{\partial S} - rV = 0 \quad \text{for } S > \bar{S}(t) \text{ and } 0 \leq t < T \tag{3.1}$$

$$V(S,t) = K - S \quad \text{for } 0 \leq S \leq \bar{S}(t) \text{ and } 0 \leq t < T \tag{3.2}$$

$$V(S,T) = \max(K-S,0) \quad \text{for } S \geq 0 \tag{3.3}$$

$$\frac{\partial V}{\partial S}(\bar{S}(t),t) = -1 \tag{3.4}$$

$$\lim_{S\to\infty} V(S,t) = 0 \tag{3.5}$$

$$\bar{S}(T) = K \tag{3.6}$$

In order for remove the free boundary in the system of equation, the following transformation is used:

$$x = \frac{S}{\bar{S}(t)} \tag{3.7}$$

Next,

$$v(x,t) := V(x\bar{S}(t),t) = V(S,t) \tag{3.8}$$

By computing the partial derivatives of V with respect to S and t

$$\frac{\partial V}{\partial t} = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial x}\frac{\partial x}{\partial t} = \frac{\partial v}{\partial t} - x\frac{\bar{S}'(t)}{\bar{S}(t)}\frac{\partial v}{\partial x} \tag{3.9}$$

$$\frac{\partial V}{\partial S} = \frac{\partial v}{\partial x}\frac{\partial x}{\partial S} = \frac{1}{\bar{S}(t)}\frac{\partial v}{\partial x} \tag{3.10}$$

$$\frac{\partial^2 V}{\partial S^2} = \frac{1}{\bar{S}(t)^2}\frac{\partial^2 v}{\partial x^2} \tag{3.11}$$

an expression for (3.1) with respect to x is derived:

$$\frac{\partial v}{\partial t} + \frac{1}{2}\sigma^2 x^2 \frac{\partial^2 v}{\partial x^2} + \left[(r-\delta) - \frac{\bar{S}'(t)}{\bar{S}(t)}\right] x \frac{\partial v}{\partial x} - rv = 0 \quad \text{for } x > 1 \text{ and } 0 \leq t < T \tag{3.12}$$

Similarly, (3.2) is reformulated in term of x to:

$$v(x,t) = K - x\bar{S}(t) \quad \text{for } 0 \leq x \leq 1 \text{ and } 0 \leq t < T \tag{3.13}$$

Next, the terminal condition (3.3) is re-written with respect of x:

$$v(x,T) = \max(K - x\bar{S}(T), 0) = K\max(1-x, 0) = 0 \quad \text{for } x \geq 1 \tag{3.14}$$

Finally, the left and right boundary conditions are given with respect to x:

$$\frac{\partial v}{\partial x}(x,t) = -\bar{S}(t) \tag{3.15}$$

$$\lim_{x\to\infty} v(x,t) = 0 \tag{3.16}$$

In summary, a non linear system of PDEs is obtained:

$$\frac{\partial v}{\partial t} + \frac{1}{2}\sigma^2 x^2 \frac{\partial^2 v}{\partial x^2} + \left[(r-\delta) - \frac{\bar{S}'(t)}{\bar{S}(t)}\right] x \frac{\partial v}{\partial x} - rv = 0 \quad \text{for } x > 1 \text{ and } 0 \leq t < T \tag{3.17}$$

$$v(x,t) = K - x\bar{S}(t) \quad \text{for } 0 \leq x \leq 1 \text{ and } 0 \leq t < T \tag{3.18}$$

$$v(x,T) = 0 \quad \text{for } x \geq 1 \tag{3.19}$$

$$\frac{\partial v}{\partial x}(x,t) = -\bar{S}(t) \tag{3.20}$$

$$\lim_{x \to \infty} v(x,t) = 0 \tag{3.21}$$

$$\bar{S}(T) = K \tag{3.22}$$

4 A section

References can be for example textbooks [3, 7, 13, 1, 6], conventional journal articles [12, 4], conventional journal articles that are also available at an e-print server [8, 2], electronic journal articles [10], articles in conference proceedings [11], PhD theses [5, 9] or websites [14]. This template orders the references by their first citation, cites them by their number and keeps any footnotes[1] separate from the references. Other citation practices exist: Your supervisor can advise as to what is appropriate for your topic.

5 Another section

5.1 A subsection

Subsections may be used. Use a clear structure in your report.

We denote the set of real numbers by $\mathbb{R}$, the set of integers by $\mathbb{Z}$ and the set of complex numbers by $\mathbb{C}$. Our analysis is based on the equation $e^{\pi i} = -1$ and the relation

$$\frac{2}{4} = \frac{1}{2} \tag{5.1}$$

which we verify in the appendix B. Useful consequences are

$$\frac{4}{8} = \frac{1}{2} \tag{5.2}$$

$$\frac{4}{12} + \frac{1}{\Gamma(s)} \int_0^\infty \frac{t^{s-1}}{e^t - 1} dt = \frac{1}{3} + \sum_{n=1}^{\infty} \frac{1}{n^s} \tag{5.3}$$

$$\frac{2}{10} = \frac{1}{5} \tag{5.4}$$

For any $0 \neq a \in \mathbb{Z}$, the equality

$$\frac{2a}{4a} = \frac{1}{2}$$

follows from equation (5.1).

[1] Such as this.

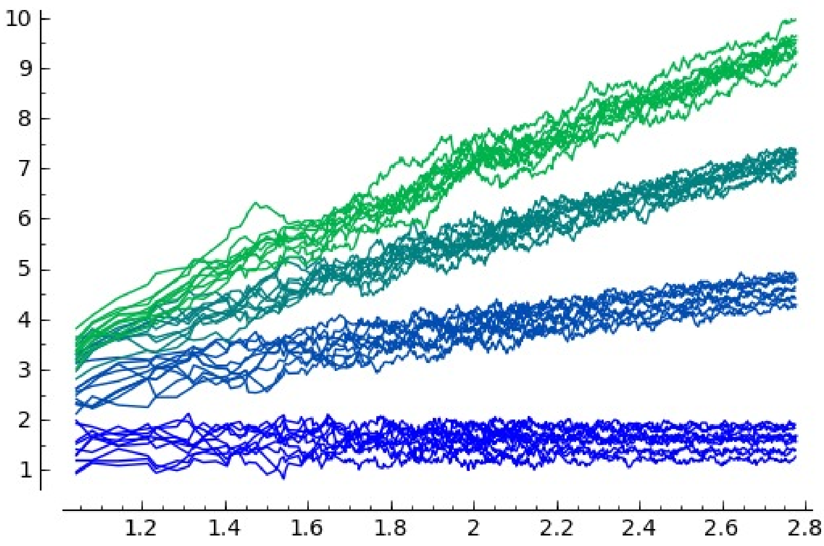

Figure 1: Oh look, something happens here !

5.2 Another subsection

5.2.1 A subsubsection

Sometimes subsubsections may be appropriate.

5.2.2 Another subsubsection

This could contain a table of interesting numbers

n	1	2	3	4	5	6
F_n	1	1	2	3	5	8
B_n	$\frac{1}{2}$	$\frac{1}{6}$	0	$-\frac{1}{30}$	0	$\frac{1}{42}$
p_n	2	3	5	7	11	13

6 Yet another section

Graphics can be included. Figure 1 shows an example. Learn about floats and pictures in the LaTeX wikibook to place the figures at the right place.

7 Conclusions

Further help on LaTeX can be found easily on the internet. The LaTeX wikibook[2] contains a lot. For instance you would find there how to type theorems and proofs nicely. Or how to include source code written in some programming language like python. There are long lists available with all sorts of common mathematical symbols like ξ, ∇, ∞, $\log$, $\iff$, etc.

[2] `http://en.wikibooks.org/wiki/LaTeX`

A Raw data

Material that needs to be included but would distract from the main line of presentation can be put in appendices. Examples of such material are raw data, computing codes and details of calculations.

But note tha the maximal number of pages includes the appendix and the references.

B Calculations for section 5

In this appendix we could verify equation (5.1) or present the code that was used.

```python
def gcd(a,b):
    """
    Return the greatest common divisor
    of a and b
    """
    while b > 0:
        (a, b) = (b, a % b)
    return a
```

References

[1] N. L. Alling and N. Greenleaf, *Foundations of the Theory of Klein Surfaces*, Lecture Notes in Mathematics Vol. 219 (Springer, Berlin, 1971).

[2] J. W. Barrett and R. A. Dawe Martins, "Non-commutative geometry and the standard model vacuum", *J. Math. Phys.* **47**, 052305 (2006). (arXiv:hep-th/0601192)

[3] R. Bott and L. W. Tu, *Differential Forms in Algebraic Topology* (Springer, New York, 1982).

[4] B. S. DeWitt, "Quantum theory of gravity. I. The canonical theory", *Phys. Rev.* **160**, 1113–1148 (1967).

[5] D. Giulini, "3-manifolds in canonical quantum gravity", PhD Thesis, University of Cambridge (1990).

[6] A. Hatcher, *Algebraic Topology* (Cambridge University Press, Cambridge, 2002), Proposition 1.40 and Exercise 1.3.24.

[7] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-Time* (Cambridge University Press, Cambridge, 1973).

[8] K. Krasnov and J. Louko, "$\mathrm{SO}_0(1, d+1)$ Racah coefficients: Type I representations", *J. Math. Phys.* **47**, 033513 (2006). (arXiv:math-ph/0502017)

[9] P. Langlois, "Imprints of spacetime topology in the Hawking-Unruh effect", PhD Thesis, University of Nottingham (2005). (arXiv: gr-qc/0510127)

[10] E. Poisson, "The motion of point particles in curved spacetime", *Living Rev. Relativity* **7** 6 (2004), URL : `http://www.livingreviews.org/lrr-2004-6` (cited on 31 August 2006). (arXiv: gr-qc/0306052)

[11] E. Poisson, "The gravitational self-force", in *Proceedings of the 17th International Conference on General Relativity and Gravitation* (Dublin, Ireland, July 18–23, 2004), edited

by P. Florides, B. Nolan and A. Ottewill (World Scientific, Singapore, 2005) 119–141. (arXiv:gr-qc/0410127)

[12] J. A. Wheeler, "Geons", *Phys. Rev.* **97**, 511–536 (1955).

[13] J. A. Wolf, *Spaces of Constant Curvature*, 5th edition (Publish or Perish, Wilmington, 1984).

[14] Website `http://www.ligo.caltech.edu/`, visited 14 August 2007.