Tic-Tac-Tobot

[EECS 149/249A Class Project]

Patrick Scheffe
p.s@berkeley.edu

Nikolas Alberti
nikolas.alberti@berkeley.edu

Department of Electrical Engineering and Computer Science
University of California
Berkeley, CA

ABSTRACT

1. INTRODUCTION

2. HARDWARE

3. ROBOTIC MANIPULATOR

3.1 Choosing the dimensions

For accurate two dimensional actuation, a classical robotic manipulator that consists a chain of dependent joints is not suitable for a low budget approach. The plotclock, a project by Johannes "*Joo*" Heberlein from the Fablab Nuremberg (comparable to a makerspace) impressively proved a way to make low budget 2D actuation work. Therefore, we planed to chose a similar manipulator. However, while the plotclock needs to cover a range whose horizontal dimensions exceed the vertical, it was not possible to blindly adapt the manipulator. For our project, actuation needs to be performed in an area formed like a square. Therefore, we needed to resize the limbs.

This problem can be described as a maximization problem: The desired area of the largest possible square that fits into the reachable range of the actuator is given. Find the dimensions L_1, L_2 and L_3 that minimize the sum L_1+L_2+L3.

Unfortunately, the function of the largest square $A(L_1, L_2, L_3)$ is not linear and therefore the problem can not be solved by partly derive towards L_1, L_2 and L_3 and set the derivations to zero. We therefore chose to model the reachable space in a geometry software called *Geogebra*, an open source software for supporting mathematical education in schools. Then, we empirically derived a near optimal sizing that yields a reachable square of $100cm^2$-

3.2 Manufacturing

3.3 Simplified Kinematic Model

A simplified sketch of the robotic manipulator can be seen in Figure 3.3. As a first approximation it is useful to determine the angles θ_1 to θ_4 from the given position of the joint at which the two arms coincide (x,y). The key for solving this inverse kinematics problem is to divide it into smaller subproblems that each can be solved individually. For that purpose, the line segments a, b and c are introduced.

From the initial information, following values can be di-

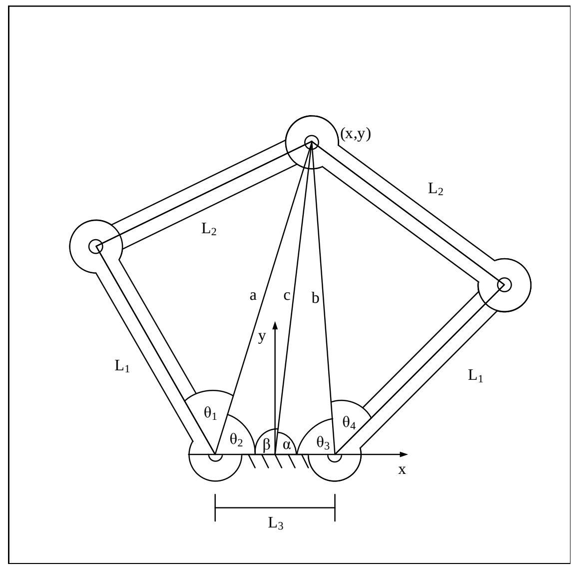

Figure 1: Simplified Version of the Manipulator

rectly computed:

$$\alpha = arctan\left(\frac{y}{x}\right) \tag{1}$$

$$\beta = \pi - \alpha \tag{2}$$

$$c = \sqrt{x^2 + y^2} \tag{3}$$

$$a = \sqrt{\left(x + \frac{L_3}{2}\right)^2 + y^2} \tag{4}$$

$$b = \sqrt{\left(x - \frac{L_3}{2}\right)^2 + y^2} \tag{5}$$

Now, you can solve for θ_2 and θ_3 by either using sine rule or cosine rule. However, the sine rule can be ambiguous in certain setups, which makes case differentiation necessary. Although mathematically steady, in our implementation the domain crossing from one solution to the other resulted in discontinuities of the movement. Therefore, the cosine rule solution is preferred:

$$\theta_2 = arccos\left(\frac{a^2 + (\frac{L_3}{2})^2 - c^2}{aL_3}\right) \tag{6}$$

$$\theta_3 = arccos\left(\frac{b^2 + (\frac{L_3}{2})^2 - c^2}{bL_3}\right) \tag{7}$$

Using cosine rule we can also solve for θ_1 and θ_4:

$$\theta_1 = arccos\left(\frac{a^2 + {L_1}^2 - {L_2}^2}{2aL_1}\right) \tag{8}$$

$$\theta_4 = arccos\left(\frac{b^2 + {L_1}^2 - {L_2}^2}{2bL_1}\right) \tag{9}$$

3.4 Complete Kinematic Model

The simplified version of the kinematic model is good for quickly creating a working implementation. However, any movements executed by the manipulator will suffer from distortion. The pen is not mounted *exactly* at the joint's position but in a small distance. Hence, a precise solution is necessary. For that purpose, new definitions must be made (Figure 3.4).

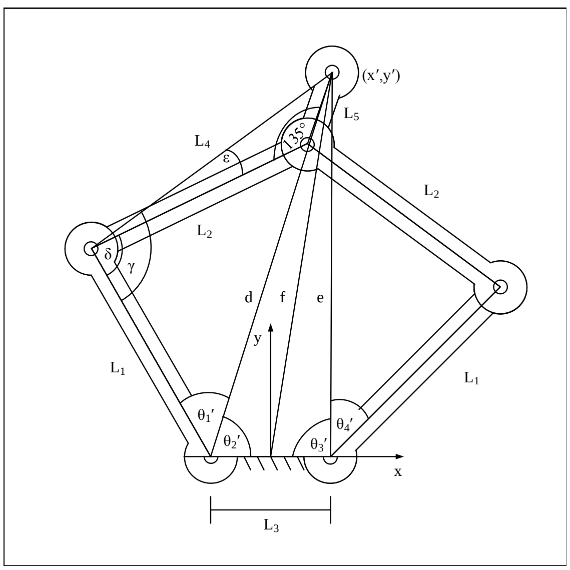

Figure 2: Complete Version of the Manipulator

L_4 and ϵ are fixed measures and not influenced by the position of the manipulator:

$$L_4 = \sqrt{{L_2}^2 + {L_5}^2 - 2L_5L_2cos\left(\frac{3\pi}{4}\right)} \tag{10}$$

$$\epsilon = arccos\left(\frac{{L_4}^2 + {L_2}^2 - {L_5}^2}{2L_4L_2}\right) \tag{11}$$

d, e and f can be yielded by the Pythagorean theorem:

$$f = \sqrt{x'^2 + y'^2} \tag{12}$$

$$d = \sqrt{\left(x' + \frac{L_3}{2}\right)^2 + y'^2} \tag{13}$$

$$e = \sqrt{\left(x' - \frac{L_3}{2}\right)^2 + y'^2} \tag{14}$$

Now, θ_2', θ_3' and δ can be computed using cosine rule:

$$\theta_2' = arccos\left(\frac{d^2 + (\frac{L_3}{2})^2 - f^2}{dL_3}\right) \tag{15}$$

$$\theta_3' = arccos\left(\frac{e^2 + (\frac{L_3}{2})^2 - f^2}{eL_3}\right) \tag{16}$$

$$\delta = arccos\left(\frac{{L_4}^2 + {L_1}^2 - d^2}{2L_4L_1}\right) \tag{17}$$

δ is the sum of ϵ and γ.

$$\gamma = \delta - \epsilon \tag{18}$$

That allows us to calculate some quantities of the simple kinematics model:

$$a = \sqrt{{L_1}^2 + {L_1}^2 - 2L_1L_2cos\left(\gamma\right)} \tag{19}$$

$$\theta_1 = arccos\left(\frac{a^2 + {L_1}^2 - {L_2}^2}{2aL_1}\right) \tag{20}$$

$$\theta_2 = \theta_1' + \theta_2' - \theta_1 \tag{21}$$

Finally, we are able to find the position of the joint at which the two arms coincide:

$$y = a\sin(\theta_2) \tag{22}$$

$$x = a\cos(\theta_2) - \frac{L_3}{2} \tag{23}$$

Now, the methods from section 3.3 can be used to solve for θ_3 and θ_4.

4. KINEMATICS USING THE ARDUINO

4.1 Modelling the Servo Motors

Servo motors are motors that do not support continuous motion but in return precisely can be driven to a desired angle. They have three external cables in different colors. The black cable should be connected to ground and the red one to V_{DD} (approximately 5V). These two cables supply the servo motor with the necessary power. The third cable (white, yellow or orange) carries the control signal. The control is done by pulse width modulation (PWM). The servo motor expects to receive a pulse every 20 ms with a pulse width between 1 ms and 2 ms. By proportional control, the servo motor assumes its most positive position at the the pulse width of 1 ms and the most negative position at a pulse width of 2 ms as defined in the mathematical direction of rotation. The range in between these extrema can be assumed to be linearly covered, i.e. a pulse width of 1.5 ms should yield the position in between these positions. A function can be derived that maps the pulse width to an angular position of the servomotor:

$$\begin{aligned} Angle(t_{pulse}) \quad = \quad & \frac{t_{pulse} - 1\text{ms}}{2\text{ms} - 1\text{ms}} \cdot (Angle_{max} - Angle_{min}) \\ & + Angle_{min}, \qquad 1\text{ ms} \leq t_{pulse} \leq 2\text{ms} \end{aligned}$$

This is just a model of the movement of the servo motors. There are three causes that the actual behavior deviates from the model:

- A high torque forces the servo motors from leaving its desired position.
- The backlash of the gears in the servo motors adds an inaccuracy to the position.
- Nonlinearities make the servomotor cover the range of movement not evenly.

Furthermore, when the pulse width modulation signal is created by a digital signal, a quantization error occurs. As you can see, the model is making some approximations. The inner of the servo motor is treated as black box. However, adding the details would bloat the model and the gain is questionable. The proposed model is precise enough to be useful but not so complex that it becomes cumbersome.

4.2 Software on the Arduino

The Arduino Uno has six PWM pins available. Very conveniently, the Arduino IDE already is equipped with a library `Servo`. This library allows us to instantiate objects of the type `Servo`. The most important functions on this object are `attach()` and `write()`. The `attach()` function assigns the Servo object to a GPIO pin. The `write()` maps an angle in the range of 0° to 180° to a pulse width and makes the attached GPIO pin assume the according PWM. Inherently, a reachable range of 180° is assumed for the servo motor. However, our servo motors only are capable of spinning 150°. A mapping between an angle from 0° to 150° to an angle between 0° to 180° has to be performed.

Furthermore Zahnrad!

5. COMPUTER VISION

6. THE TIC-TAC-TOE AI

ABSTRACT

This paper provides a sample of a LaTeX document which conforms, somewhat loosely, to the formatting guidelines for ACM SIG Proceedings. It is an *alternate* style which produces a *tighter-looking* paper and was designed in response to concerns expressed, by authors, over page-budgets. It complements the document *Author's (Alternate) Guide to Preparing ACM SIG Proceedings Using LaTeX2$_\epsilon$ and BibTeX*. This source file has been written with the intention of being compiled under LaTeX2$_\epsilon$ and BibTeX.

The developers have tried to include every imaginable sort of "bells and whistles", such as a subtitle, footnotes on title, subtitle and authors, as well as in the text, and every optional component (e.g. Acknowledgments, Additional Authors, Appendices), not to mention examples of equations, theorems, tables and figures.

To make best use of this sample document, run it through LaTeX and BibTeX, and compare this source code with the printed output produced by the dvi file. A compiled PDF version is available on the web page to help you with the 'look and feel'.

7. INTRODUCTION

The *proceedings* are the records of a conference. ACM seeks to give these conference by-products a uniform, high-quality appearance. To do this, ACM has some rigid requirements for the format of the proceedings documents: there is a specified format (balanced double columns), a specified set of fonts (Arial or Helvetica and Times Roman) in certain specified sizes (for instance, 9 point for body copy), a specified live area (18 × 23.5 cm [7" × 9.25"]) centered on the page, specified size of margins (1.9 cm [0.75"]) top, (2.54 cm [1"]) bottom and (1.9 cm [.75"]) left and right; specified column width (8.45 cm [3.33"]) and gutter size (.83 cm [.33"]).

The good news is, with only a handful of manual settings[1], the LaTeX document class file handles all of this for you.

The remainder of this document is concerned with showing, in the context of an "actual" document, the LaTeX commands specifically available for denoting the structure of a proceedings paper, rather than with giving rigorous descriptions or explanations of such commands.

8. THE *BODY* OF THE PAPER

Typically, the body of a paper is organized into a hierarchical structure, with numbered or unnumbered headings for sections, subsections, sub-subsections, and even smaller sections. The command `\section` that precedes this paragraph is part of such a hierarchy.[2] LaTeX handles the numbering and placement of these headings for you, when you use the appropriate heading commands around the titles of the headings. If you want a sub-subsection or smaller part to be unnumbered in your output, simply append an asterisk to the command name. Examples of both numbered and

[1]Two of these, the `\numberofauthors` and `\alignauthor` commands, you have already used; another, `\balancecolumns`, will be used in your very last run of LaTeX to ensure balanced column heights on the last page.

[2]This is the second footnote. It starts a series of three footnotes that add nothing informational, but just give an idea of how footnotes work and look. It is a wordy one, just so you see how a longish one plays out.

unnumbered headings will appear throughout the balance of this sample document.

Because the entire article is contained in the **document** environment, you can indicate the start of a new paragraph with a blank line in your input file; that is why this sentence forms a separate paragraph.

8.1 Type Changes and *Special* Characters

We have already seen several typeface changes in this sample. You can indicate italicized words or phrases in your text with the command `\textit`; emboldening with the command `\textbf` and typewriter-style (for instance, for computer code) with `\texttt`. But remember, you do not have to indicate typestyle changes when such changes are part of the *structural* elements of your article; for instance, the heading of this subsection will be in a sans serif[3] typeface, but that is handled by the document class file. Take care with the use of[4] the curly braces in typeface changes; they mark the beginning and end of the text that is to be in the different typeface.

You can use whatever symbols, accented characters, or non-English characters you need anywhere in your document; you can find a complete list of what is available in the *LaTeX User's Guide*[5].

8.2 Math Equations

You may want to display math equations in three distinct styles: inline, numbered or non-numbered display. Each of the three are discussed in the next sections.

8.2.1 Inline (In-text) Equations

A formula that appears in the running text is called an inline or in-text formula. It is produced by the **math** environment, which can be invoked with the usual `\begin. . .\end` construction or with the short form `$. . .$`. You can use any of the symbols and structures, from α to ω, available in LaTeX[5]; this section will simply show a few examples of in-text equations in context. Notice how this equation: $\lim_{n\rightarrow \infty}x = 0$, set here in in-line math style, looks slightly different when set in display style. (See next section).

8.2.2 Display Equations

A numbered display equation – one set off by vertical space from the text and centered horizontally – is produced by the **equation** environment. An unnumbered display equation is produced by the **displaymath** environment.

Again, in either environment, you can use any of the symbols and structures available in LaTeX; this section will just give a couple of examples of display equations in context. First, consider the equation, shown as an inline equation above:

$$\lim_{n\rightarrow \infty}x = 0 \tag{24}$$

Notice how it is formatted somewhat differently in the **displaymath** environment. Now, we'll enter an unnumbered equation:

$$\sum_{i=0}^{\infty} x + 1$$

[3]A third footnote, here. Let's make this a rather short one to see how it looks.

[4]A fourth, and last, footnote.

Table 1: Frequency of Special Characters

Non-English or Math	Frequency	Comments
Ø	1 in 1,000	For Swedish names
π	1 in 5	Common in math
\$	4 in 5	Used in business
Ψ^2_1	1 in 40,000	Unexplained usage

and follow it with another numbered equation:

$$\sum_{i=0}^{\infty}x_i=\int_{0}^{\pi+2} f \tag{25}$$

just to demonstrate LaTeX's able handling of numbering.

8.3 Citations

Citations to articles [1, 3, 2, 4], conference proceedings [3] or books [6, 5] listed in the Bibliography section of your article will occur throughout the text of your article. You should use BibTeX to automatically produce this bibliography; you simply need to insert one of several citation commands with a key of the item cited in the proper location in the `.tex` file [5]. The key is a short reference you invent to uniquely identify each work; in this sample document, the key is the first author's surname and a word from the title. This identifying key is included with each item in the `.bib` file for your article.

The details of the construction of the `.bib` file are beyond the scope of this sample document, but more information can be found in the *Author's Guide*, and exhaustive details in the *LaTeX User's Guide*[5].

This article shows only the plainest form of the citation command, using `\cite`. This is what is stipulated in the SIGS style specifications. No other citation format is endorsed or supported.

8.4 Tables

Because tables cannot be split across pages, the best placement for them is typically the top of the page nearest their initial cite. To ensure this proper "floating" placement of tables, use the environment **table** to enclose the table's contents and the table caption. The contents of the table itself must go in the **tabular** environment, to be aligned properly in rows and columns, with the desired horizontal and vertical rules. Again, detailed instructions on **tabular** material is found in the *LaTeX User's Guide*.

Immediately following this sentence is the point at which Table 1 is included in the input file; compare the placement of the table here with the table in the printed dvi output of this document.

To set a wider table, which takes up the whole width of the page's live area, use the environment **table*** to enclose the table's contents and the table caption. As with a single-column table, this wide table will "float" to a location deemed more desirable. Immediately following this sentence is the point at which Table 2 is included in the input file; again, it is instructive to compare the placement of the table here with the table in the printed dvi output of this document.

8.5 Figures

Like tables, figures cannot be split across pages; the best placement for them is typically the top or the bottom of

Table 2: **Some Typical Commands**

Command	A Number	Comments
`\alignauthor`	100	Author alignment
`\numberofauthors`	200	Author enumeration
`\table`	300	For tables
`\table*`	400	For wider tables

Figure 3: A sample black and white graphic.

Figure 4: A sample black and white graphic that has been resized with the `includegraphics` command.

the page nearest their initial cite. To ensure this proper "floating" placement of figures, use the environment **figure** to enclose the figure and its caption.

This sample document contains examples of **.eps** files to be displayable with LaTeX. If you work with pdfLaTeX, use files in the **.pdf** format. Note that most modern TeX system will convert **.eps** to **.pdf** for you on the fly. More details on each of these is found in the *Author's Guide*.

As was the case with tables, you may want a figure that spans two columns. To do this, and still to ensure proper "floating" placement of tables, use the environment **figure*** to enclose the figure and its caption. and don't forget to end the environment with figure*, not figure!

8.6 Theorem-like Constructs

Other common constructs that may occur in your article are the forms for logical constructs like theorems, axioms, corollaries and proofs. There are two forms, one produced by the command `\newtheorem` and the other by the command `\newdef`; perhaps the clearest and easiest way to distinguish them is to compare the two in the output of this sample document:

This uses the **theorem** environment, created by the `\newtheorem` command:

THEOREM 1. *Let f be continuous on $[a,b]$. If G is an antiderivative for f on $[a,b]$, then*

$$\int_a^b f(t)dt = G(b) - G(a).$$

The other uses the **definition** environment, created by the `\newdef` command:

Definition 1. If z is irrational, then by e^z we mean the unique number which has logarithm z:

$$\log e^z = z$$

Two lists of constructs that use one of these forms is given in the *Author's Guidelines.*

There is one other similar construct environment, which is already set up for you; i.e. you must *not* use a `\newdef` command to create it: the **proof** environment. Here is a example of its use:

PROOF. Suppose on the contrary there exists a real number L such that

$$\lim_{x\to\infty} \frac{f(x)}{g(x)} = L.$$

Then

$$l = \lim_{x\to c} f(x) = \lim_{x\to c} \left[gx \cdot \frac{f(x)}{g(x)}\right] = \lim_{x\to c} g(x) \cdot \lim_{x\to c} \frac{f(x)}{g(x)} = 0 \cdot L = 0,$$

which contradicts our assumption that $l \neq 0$. $\square$

Complete rules about using these environments and using the two different creation commands are in the *Author's Guide*; please consult it for more detailed instructions. If you need to use another construct, not listed therein, which you want to have the same formatting as the Theorem or the Definition[6] shown above, use the `\newtheorem` or the `\newdef` command, respectively, to create it.

A *Caveat* for the TeX Expert

Because you have just been given permission to use the `\newdef` command to create a new form, you might think you can use TeX's `\def` to create a new command: *Please refrain from doing this!* Remember that your LaTeX source code is primarily intended to create camera-ready copy, but may be converted to other forms – e.g. HTML. If you inadvertently omit some or all of the `\def`s recompilation will be, to say the least, problematic.

9. CONCLUSIONS

This paragraph will end the body of this sample document. Remember that you might still have Acknowledgments or Appendices; brief samples of these follow. There is still the Bibliography to deal with; and we will make a disclaimer about that here: with the exception of the reference to the LaTeX book, the citations in this paper are to articles which have nothing to do with the present subject and are used as examples only.

10. ACKNOWLEDGMENTS

This section is optional; it is a location for you to acknowledge grants, funding, editing assistance and what have you. In the present case, for example, the authors would like to thank Gerald Murray of ACM for his help in codifying this *Author's Guide* and the **.cls** and **.tex** files that it describes.

11. REFERENCES

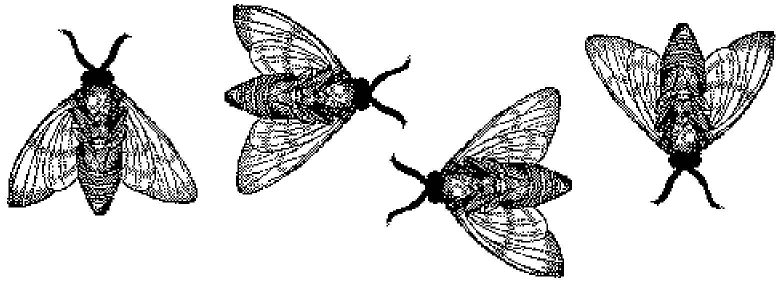

Figure 5: A sample black and white graphic that needs to span two columns of text.

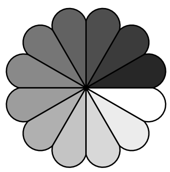

Figure 6: A sample black and white graphic that has been resized with the `includegraphics` command.

[1] M. Bowman, S. K. Debray, and L. L. Peterson. Reasoning about naming systems. *ACM Trans. Program. Lang. Syst.*, 15(5):795–825, November 1993.

[2] J. Braams. Babel, a multilingual style-option system for use with latex's standard document styles. *TUGboat*, 12(2):291–301, June 1991.

[3] M. Clark. Post congress tristesse. In *TeX90 Conference Proceedings*, pages 84–89. TeX Users Group, March 1991.

[4] M. Herlihy. A methodology for implementing highly concurrent data objects. *ACM Trans. Program. Lang. Syst.*, 15(5):745–770, November 1993.

[5] L. Lamport. *LaTeX User's Guide and Document Reference Manual*. Addison-Wesley Publishing Company, Reading, Massachusetts, 1986.

[6] S. Salas and E. Hille. *Calculus: One and Several Variable*. John Wiley and Sons, New York, 1978.

APPENDIX

A. HEADINGS IN APPENDICES

The rules about hierarchical headings discussed above for the body of the article are different in the appendices. In the **appendix** environment, the command **section** is used to indicate the start of each Appendix, with alphabetic order designation (i.e. the first is A, the second B, etc.) and a title (if you include one). So, if you need hierarchical structure *within* an Appendix, start with **subsection** as the highest level. Here is an outline of the body of this document in Appendix-appropriate form:

A.1 Introduction

A.2 The Body of the Paper

A.2.1 Type Changes and Special Characters

A.2.2 Math Equations

Inline (In-text) Equations.

Display Equations.

A.2.3 Citations

A.2.4 Tables

A.2.5 Figures

A.2.6 Theorem-like Constructs

A Caveat for the TEX Expert

A.3 Conclusions

A.4 Acknowledgments

A.5 Additional Authors

This section is inserted by LATEX; you do not insert it. You just add the names and information in the `\additionalauthors` command at the start of the document.

A.6 References

Generated by bibtex from your .bib file. Run latex, then bibtex, then latex twice (to resolve references) to create the .bbl file. Insert that .bbl file into the .tex source file and comment out the command `\thebibliography`.

B. MORE HELP FOR THE HARDY

The sig-alternate.cls file itself is chock-full of succinct and helpful comments. If you consider yourself a moderately experienced to expert user of LATEX, you may find reading it useful but please remember not to change it.