Inverse Kinematics of the Robotic Manipulator

Patrick Scheffe, Nikolas Alberti

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1 Simplified Kinematic Model

A simplified sketch of the robotic manipulator can be seen in Figure 1. As a first approximation it is useful to determine the angles θ_1 to θ_4 from the given position of the joint at which the two arms coincide (x,y). The key for solving this inverse kinematics problem is to divide it into smaller subproblems that each can be solved individually. For that purpose, the line segments a, b and c are introduced.

From the initial information, following values can be directly computed:

$$\alpha = arctan\left(\frac{y}{x}\right) \tag{1}$$

$$\beta = \pi - \alpha \tag{2}$$

$$c = \sqrt{x^2 + y^2} \tag{3}$$

$$a = \sqrt{\left(x + \frac{L_3}{2}\right)^2 + y^2} \tag{4}$$

$$b = \sqrt{\left(x - \frac{L_3}{2}\right)^2 + y^2} \tag{5}$$

Now, you can solve for θ_2 and θ_3 by either using sine rule or cosine rule. However, the sine rule can be ambiguous in certain setups, which makes case differentiation necessary. Although mathematically steady, in our implementation the domain crossing from one solution to the other resulted in discontinuities of the movement. Therefore, the cosine rule solution is preferred:

$$\theta_2 = arccos\left(\frac{a^2 + (\frac{L_3}{2})^2 - c^2}{aL_3}\right) \tag{6}$$

$$\theta_3 = arccos\left(\frac{b^2 + (\frac{L_3}{2})^2 - c^2}{bL_3}\right) \tag{7}$$

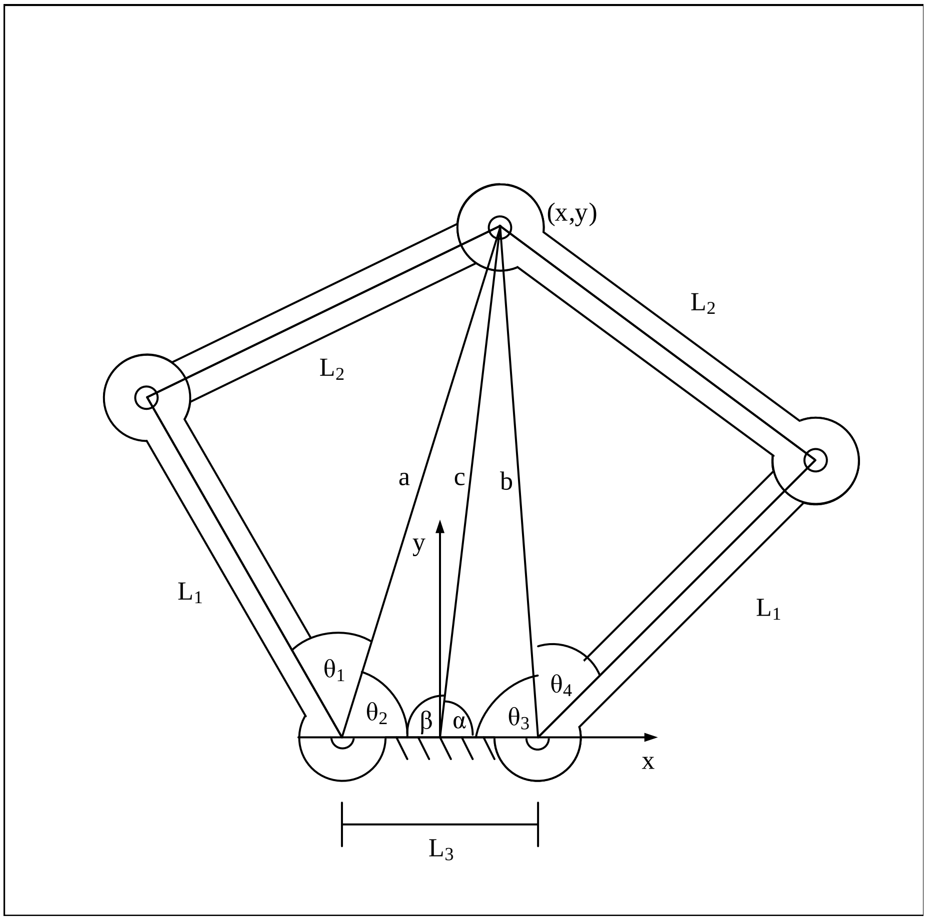

Figure 1: Simplified Version of the Manipulator

Using cosine rule we can also solve for θ_1 and θ_4:

$$\theta_1 = arccos\left(\frac{a^2 + {L_1}^2 - {L_2}^2}{2aL_1}\right) \tag{8}$$

$$\theta_4 = arccos\left(\frac{b^2 + {L_1}^2 - {L_2}^2}{2bL_1}\right) \tag{9}$$

2 Complete Kinematic Model

The simplified version of the kinematic model is good for quickly creating a working implementation. However, any movements executed by the manipulator will suffer from distortion because the pen is not mounted *exactly* at the joint's position but in a small distance. Hence, a precise solution is necessary. For that purpose, new definitions must be made (Figure 2).

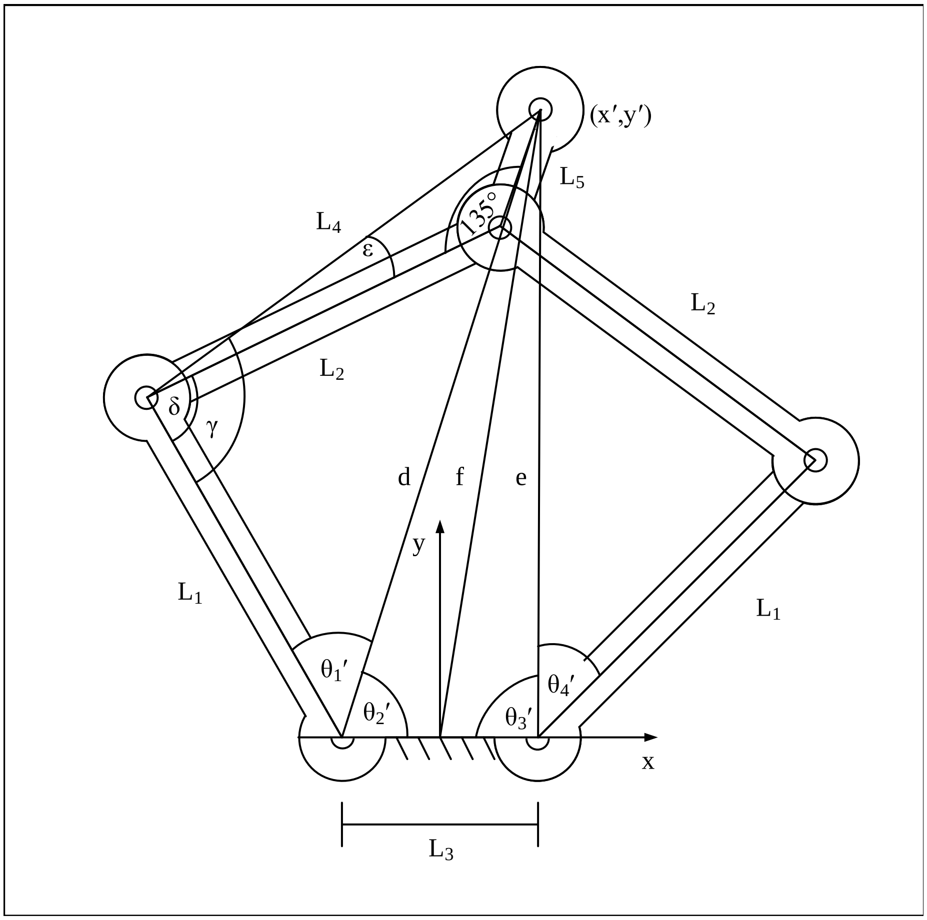

Figure 2: Simplified Version of the Manipulator

L_4 and ϵ are fixed measures and not influenced by the position of the manipulator:

$$L_4 = \sqrt{{L_2}^2 + {L_5}^2 - 2L_5L_2cos\left(\frac{3\pi}{4}\right)} \tag{10}$$

$$\epsilon = arccos\left(\frac{{L_4}^2 + {L_2}^2 - {L_5}^2}{2L_4L_2}\right) \tag{11}$$

d, e and f can be yielded by the Pythagorean theorem:

$$f = \sqrt{x'^2 + y'^2} \tag{12}$$

$$d = \sqrt{\left(x' + \frac{L_3}{2}\right)^2 + y'^2} \tag{13}$$

$$e = \sqrt{\left(x' - \frac{L_3}{2}\right)^2 + y'^2} \tag{14}$$

Now, θ_2', θ_3' and δ can be computed:

$$\theta_2' = arccos\left(\frac{d^2 + (\frac{L_3}{2})^2 - f^2}{dL_3}\right) \tag{15}$$

$$\theta_3' = arccos\left(\frac{e^2 + (\frac{L_3}{2})^2 - f^2}{eL_3}\right) \tag{16}$$

$$\delta = arccos\left(\frac{{L_4}^2 + {L_1}^2 - d^2}{2L_4L_1}\right) \tag{17}$$

δ is the sum of ϵ and γ.

$$\gamma = \delta - \epsilon \tag{18}$$

That allows us to calculate some quantities of the simple kinematics model:

$$a = \sqrt{{L_1}^2 + {L_1}^2 - 2L_1L_2cos\left(\gamma\right)} \tag{19}$$

$$\theta_1 = arccos\left(\frac{a^2 + {L_1}^2 - {L_2}^2}{2aL_1}\right) \tag{20}$$

$$\theta_2 = \theta_1' + \theta_2' - \theta_1 \tag{21}$$

Finally, we are able to find the position of the joint at which the two arms coincide:

$$y = a\sin(\theta_2) \tag{22}$$

$$x = a\cos(\theta_2) - \frac{L_3}{2} \tag{23}$$

Now, the methods from section 1 can be used to solve for θ_3 and θ_4.