Avoiding trig calculations

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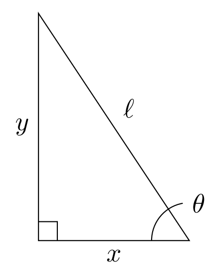
Very often, we can simplify expressions involving trig functions so that they no longer include trig functions. This is a common thing to need to do when doing trig substitution in calculus. It can also give formulas which are more accurate to compute using a computer. Let's consider an example. Let's say I have a ladder of a known length ℓ leaned up against a wall, sitting x feet away at its base. I want to know the height y where it rests on the wall. Since I have an adjacent side and a hyportenuse, I can find the angle θ using an inverse cosine.



$$\begin{aligned} \cos\theta &= x/\ell \\ \theta &= \arccos(x/\ell) \end{aligned}$$

Now it is easy to find y. I could use, for example, the tangent function to relate it to x:

$$\begin{aligned} y/x &= \tan\theta \\ y &= x\tan\theta \\ &= x\tan\arccos(x/\ell) \end{aligned}$$

This works, the answer is correct. However, you probably know a much quicker, more accurate way to get the answer using Pythagoras:

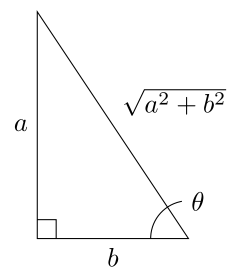
$$\begin{aligned} x^2 + y^2 &= \ell^2 \\ y &= \sqrt{\ell^2 - x^2} \end{aligned}$$

In fact we have found a general identity:

$$x \tan \arccos(x/\ell) = \sqrt{\ell^2 - x^2}.$$

Whenever possible, it would be better to use the square-root method.

In general, if you have a trig function of an inverse trig function, you can find a better formula which gives the same answer, using Pythagoras. You don't have to memorize all the possible combinations to be able to do this. For example, suppose I have an angle θ and I know $\tan\theta = a/b$, and I want $\sec\theta$. I could just say $\sec\theta = \sec(\arctan(a/b))$, but that is messy and complicated. Instead, I will just make up a triangle with some convenient side lengths.



I carefully chose the triangle so that $\tan\theta = a/b$. The triangle doesn't have to correspond to anythign physical, it is just a convenient way to organize my information. Since the secant is always the hypotenuse over the adjacent side, I can just read the answer from the triangle:

$$\sec(\theta) = \frac{\sqrt{a^2+b^2}}{b}.$$

This gives us a general formula, $\sec(\arctan(a/b)) = \frac{\sqrt{a^2+b^2}}{b}$.