**Nama : Aldino Harun**

**Nim : 19101106017**

**Sistem Informasi A**

# Tugas Desain dan Analisis Algoritma

## Exercise 2.2

1. a. $C_{worst}$(n) = n

   $C_{worst}$(n) ∈ Θ(n).

   b. $C_{best}$(n) = 1

   $C_{best}$(1) ∈ Θ(1).

   c. $C_{avg}(n) = \frac{p(n+1)}{2} + n(1-p) = \left(1 - \frac{p}{2}\right)n + \frac{p}{2}\ where\ 0\ \leq p\ \leq 1$

   $C_{avg}(n) \in \Theta(n)$.

2. a. $n(n+1)/2 \in O(n^3)$ is true.

   b. $n(n+1)/2 \in O(n^2)$ is true.

   c. $n(n+1)/2 \in \Theta(n^3)$ is false.

   d. $n(n+1)/2 \in \Omega(n)$ is true.

3. a. $(n^2+1)^{10} \approx (n^2)^{10} = n^{20} \in \Theta(n^{20})$

   $$\lim_{n\to\infty} \frac{(n^2+1)^{10}}{n^{20}} = \lim_{n\to\infty} \frac{(n^2+1)^{10}}{(n^2)^{10}} = \lim_{n\to\infty} \left(\frac{n^2+1}{n^2}\right)^{10} == \lim_{n\to\infty} \left(1 + \frac{1}{n^2}\right)^{10} = 1.$$

   Hasil = $(n^2+1)^{10} \in \Theta(n^{20})$.

   b. $\sqrt{10n^2+7n+3}\ \approx\ \sqrt{10n^2} =\ \sqrt{10}n \in \Theta(n)$.

$$\lim_{n\to\infty} \frac{\sqrt{10n^2+7n+3}}{n} = \lim_{n\to\infty} \sqrt{\frac{10n^2+7n+3}{n^3}} = \lim_{n\to\infty} \sqrt{10+\frac{7}{n}+\frac{3}{n^2}} = \sqrt{10}$$

Hasil = $\sqrt{10n^2+7n+3} \in \Theta(n)$.

c. $2n\lg(n+2)^2 + (n+2)^2 \lg\frac{n}{2}$

$2n2\lg(n+2) + (n+2)^2(\lg n - 1) \in \Theta(n \lg n) + \Theta(n^2 \lg n)$

Hasil = $\Theta(n^2 \lg n)$.

d. 2n+1 + 3n−1 = 2n2+3n 1 3 ∈ Θ(2n) + Θ(3n)

Hasil = Θ(3n)

e. Informally, $|\log_2 n\rfloor \approx \log_2 n \in \Theta(\log n)$. Formally, by using the in- equalities $x-1 < |x\rfloor \le x$ (see Appendix A), we obtain an upper bound

$$|\log_2 n\rfloor \le \log_2 n$$

and a lower bound

$|\log_2 n$

Hasil = $|\log_2 n\rfloor \in \Theta(\log_2 n) = \Theta(\log n)$.

7. a. The assertion should be correct because it states that if the order of growth of

t(n) is smaller than or equal to the order of growth of g(n), then

the order of growth of $g(n)$ is larger than or equal to the order of growth of $t(n)$. The formal proof is immediate, too:

$$t(n) \le cg(n) \text{ for all } n \ge n_0, \text{ where } c > 0,$$

implies

$$\left(\frac{1}{c}\right) t(n) \le cg(n) for\ all\ n\ \ge n0$$

b. The assertion that $\Theta(\alpha g(n)) = \Theta(g(n))$ should be true because $\alpha g(n)$ and $g(n)$ differ just by a positive constant multiple and, hence, by the definition of $\Theta$, must have the same order of growth. The formal proof has to show that $\Theta(\alpha g(n)) \subseteq \Theta(g(n))$ and $\Theta(g(n)) \subseteq \Theta(\alpha g(n))$. Let $f(n) \in \Theta(\alpha g(n))$; we'll show that $f(n) \in \Theta(g(n))$. Indeed,

$$f(n) \le c\alpha g(n) \text{ for all } n \ge n_0 \text{ (where } c > 0)$$

can be rewritten as

$f(n) \le c_1 g(n)$ for all $n \ge n_0$ (where $c_1 = c\alpha > 0$), i.e., $f(n) \in \Theta(g(n))$.

Let now $f(n) \in \Theta(g(n))$; we'll show that $f(n) \in \Theta(\alpha g(n))$ for $\alpha > 0$. Indeed, if $f(n) \in \Theta(g(n))$,

$$f(n) \le cg(n) \text{ for all } n \ge n_0 \text{ (where } c > 0)$$

and therefore

$$f(n) \le \frac{c}{\alpha}\alpha g(n) = c1\ \alpha g(n)\ for\ all\ n\ \ge n0\ \left(where\ c1 = \frac{c}{\alpha} > 0\right),$$

i.e., $f(n) \in \Theta(\alpha g(n))$.

c. The assertion is obviously correct (similar to the assertion that $a = b$ if and only if $a \le b$ and $a \ge b$). The formal proof should show that $\Theta(g(n)) \subseteq O(g(n)) \cap \Omega(g(n))$ and that $O(g(n)) \cap \Omega(g(n)) \subseteq \Theta(g(n))$, which immediately follow from the definitions of $O$, $\Omega$, and $\Theta$.

d. The assertion is false. The following pair of functions can serve as a counter example

$$t(n) = \begin{cases} n & if\ n\ is\ even \\ n^2 & if\ n\ is\ odd \end{cases} \text{ and } g(n) = \begin{cases} n^2 & if\ n\ is\ odd \\ n & if\ n\ is\ even \end{cases}$$