$$A = \begin{pmatrix} 3 & -1 & 1 & -7 \\ 9 & -3 & -7 & -1 \\ 0 & 0 & 4 & -8 \\ 0 & 0 & 2 & -4 \end{pmatrix} \qquad p_A(x) = \det(xI - A)$$

$$= x^4$$

0 é o único autovalor

autovetores (em $\mathbb{R}^4$)

$$\begin{pmatrix} 3 & -1 & 1 & -7 \\ 9 & -3 & -7 & -1 \\ 0 & 0 & 4 & -8 \\ 0 & 0 & 2 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$z + 2w = 0$$

$$\mathrm{Aut}_A(0) = \left[\begin{pmatrix} 5 \\ 0 \\ 6 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 0 \\ 0 \end{pmatrix} \right] \xrightarrow{\dim} 2 \Rightarrow \text{há dois blocos de Jordan}$$

$$A_1 = \left(\begin{array}{cc|cc} 0 & & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \hline 0 & & 0 & \\ & & 1 & 0 \end{array}\right) \quad \text{ou} \quad A_2 = \left(\begin{array}{ccc|c} 0 & & & \\ 1 & 0 & & \\ & 1 & 0 & \\ \hline & & & 0 \end{array}\right)$$

$$v_4 = \begin{pmatrix} 1 \\ 3 \\ 0 \\ 0 \end{pmatrix}. \quad \text{Existe } v_3 \text{ tq } Av_3 = v_4 \,?$$

$$v_3 = \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \quad \begin{pmatrix} 3 & -1 & 1 & -7 \\ 9 & -3 & -7 & -1 \\ 0 & 0 & 4 & -8 \\ 0 & 0 & 2 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -1 & 1 & -7 & 1 \\ 9 & -3 & -7 & -1 & 3 \\ 0 & 0 & 4 & -8 & 0 \\ 0 & 0 & 2 & -4 & 0 \end{pmatrix} \to \begin{pmatrix} 3 & -1 & 1 & -7 & 1 \\ 0 & 0 & -10 & 20 & 0 \\ & - & 0 & - & \\ & - & 0 & - & \end{pmatrix}$$

$$z = 2w$$

$$3x - y + z - 7w = 1$$
$$3x - y + 2w - 7w = 1$$
$$3x - y - 5w = 1 \qquad y = 3x - 5w - 1$$

$$S = \left\{ \begin{pmatrix} x \\ 3x-5w-1 \\ 2w \\ w \end{pmatrix} ; x, w \in \mathbb{R} \right\}$$

$$x = 1, \quad w = 0, \quad v_3 = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 0 \end{pmatrix}.$$

Como
$$\begin{pmatrix} 3 & -1 & 1 & -7 \\ 9 & -3 & -7 & -1 \\ 0 & 0 & 4 & -8 \\ 0 & 0 & 2 & -4 \end{pmatrix}^2 = 0$$

$\Rightarrow m_A(x) = x^2 \Rightarrow$ o maior dos blocos de Jordan é de ordem 2

$$\Rightarrow J(A) = \begin{pmatrix} 0 & 0 & & 0 \\ 1 & 0 & & \\ & & 0 & 0 \\ 0 & & 1 & 0 \end{pmatrix}$$

v_3, v_4 ; $v_2 = \begin{pmatrix} 5 \\ 0 \\ 6 \\ 3 \end{pmatrix}$; v_1 é soluçã de

$$(A - 0I)v_1 = v_2 .$$

—— " ——

(6) $P_T(x) = (x-3)^5 (x-2)^4$, $m_T(x) = (x-3)^3 (x-2)^2$

Um bloco $5\times\cancel{3}5 \leftrightarrow \lambda_1 = 3$

$4\times 4 \leftrightarrow \lambda_2 = 2$

No bloco $5\times 5 \leftrightarrow \lambda_1 = 3$, o maio dos blocos de Jordan é 3×3 $\left(m_T(x) = (x-3)^3 (x-2)^2 \right)$

$\Rightarrow$ há exatamente duas possibilidades

$$\underbrace{\left(\begin{array}{ccc|cc} 3 & & & & \\ 1 & 3 & & & \\ & 1 & 3 & & \\ \hline & & & 3 & \\ & & & 1 & 3 \end{array}\right)}_{\dim \mathrm{Aut}_T(3) = 2} \quad \text{ou} \quad \underbrace{\left(\begin{array}{ccc|cc} 3 & & & & \\ 1 & 3 & & & \\ & 1 & 3 & & \\ \hline & & & 3 & 0 \\ & & & 0 & 3 \end{array}\right)}_{\dim \mathrm{Aut}_T(3) = 3}$$

Para o bloco $4\times 4 \leftrightarrow 2$, o maior dos blocos de Jordan é 2×2. Daí, as possibilidades são

$$\begin{pmatrix} 2 & 0 & & \\ 1 & 2 & & \\ & & 2 & 0 \\ & & 1 & 2 \end{pmatrix} \quad \text{ou} \quad \begin{pmatrix} 2 & 0 & & \\ 1 & 2 & & \\ & & 2 & \\ & & & 2 \end{pmatrix}$$

$\dim \text{Aut}_T(2) = 2 \qquad\qquad = 3$

$$9\times 9: \begin{pmatrix} 5\times 5 & \\ & 4\times 4 \end{pmatrix}$$

———n———

$$S - A = \begin{pmatrix} 0 & -9 & 0 & 0 \\ 1 & 6 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix}$$

$$p_A(x) = \det(xI - A)$$
$$= \left(\det \begin{pmatrix} x & +9 \\ -1 & x-6 \end{pmatrix}\right)(x-3)^2$$

$$(x(x-6)+9)(x-3)^2 =$$
$$(x^2 - 6x + 9)(x-3)^2 = (x-3)^4$$

$$\begin{pmatrix} 3 & 9 \\ -1 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \qquad \begin{pmatrix} 3 & 9 & & \\ -1 & -3 & & \\ & & 0 & \\ & & & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = 0$$

z, w livres $\qquad x = -3y$

$$\text{Aut}_A(3) = S = \left\{ \begin{pmatrix} -3y \\ y \\ z \\ w \end{pmatrix} \right\}, \quad \dim S = 3 \Rightarrow \text{não é}$$

diagonalizável e o nº de blocos de Jordan é 3

$$\Rightarrow \quad J(A) = \left(\begin{array}{cc|c|c} 3 & & & \\ 1 & 3 & & \\ \hline & & 3 & \\ \hline & & & 3 \end{array}\right)$$

$$3I - A' = \begin{pmatrix} 3 & 9 \\ -1 & -3 \end{pmatrix} \; ; \qquad (3I - A')\begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$S = \left\{ \begin{pmatrix} -3y \\ y \end{pmatrix} \right\}$$

$v_2 = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$. v_1 é solução de

$$\begin{pmatrix} 3 & 9 \\ -1 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$$

$$x + 3y = 1 \qquad x = 1 - 3y$$

$S = \left\{ \begin{pmatrix} 1-3y \\ y \end{pmatrix} \right\}$, escolho $v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

v_1 : $\quad (3I - A)v_1 = v_2$

$$3v_1 - Av_1 = v_2$$

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T ; $\mathrm{Nuc}\,T$, $\quad v \in \mathrm{Nuc}\,T \Rightarrow Tv = 0$

$\Rightarrow T(Tv) = T(0) = 0 \Rightarrow v \in \mathrm{Nuc}\, T^2$

$\Rightarrow \mathrm{Nuc}\, T \subset \mathrm{Nuc}\, T^2$

$\Rightarrow \mathrm{Nuc}\, T \subset \mathrm{Nuc}\, T^2 \subset \cdots \subset \mathrm{Nuc}\, T^m \subset \mathrm{Nuc}\, T^{m+1} \subset \cdots$

$\dim V < \infty \Rightarrow \exists\, m$ (o menor) tq

$$\mathrm{Nuc}\, T^{m+i} = \mathrm{Nuc}\, T^m, \ \forall i$$

$\Rightarrow T = T_1 \oplus T_2$, onde

$T_1 = T|_{U_1}$; $U_1 = \mathrm{Nuc}\, T^m$

$T_2 = T|_{U_2}$; $U_2 = \mathrm{Im}\, T^m$.

(8) $T : P_n(\mathbb{R}) \to P_n(\mathbb{R})$, $T(p(x)) = p(x+1)$

(a) Forma de Jordan de T

(b) Uma base p/ $n=4$

$B_0 = \{1, x, \dots, x^n\}$.

$T(1) = 1$, $T(x) = x+1$, $T(x^2) = (x+1)^2$
$= x^2 + 2x + 1$

$T(x^3) = x^3 + 3x^2 + 3x + 1$, ...

$$T_{B_0} = \begin{bmatrix} 1 & 1 & 1 & 1 & \\ 0 & 1 & 2 & 3 & \cdots \\ 0 & 0 & 1 & 3 & \\ \vdots & \vdots & 0 & 1 & \\ & & \vdots & & \ddots \\ 0 & 0 & 0 & & \end{bmatrix}$$

$$\det(xI - T) = (x-1)^n$$

$$(T - I)X = 0 \qquad \begin{bmatrix} 0 & 1 & 1 & 1 & & 1 \\ 0 & 0 & 2 & 3 & \cdots & * \\ 0 & \vdots & 0 & 3 & & * \\ \vdots & & \vdots & 0 & & \vdots \\ \vdots & & \vdots & \vdots & & \boxed{x} \\ 0 & 0 & 0 & 0 & & 0 \end{bmatrix} = \vec{0}$$

$\boxed{x} \to \binom{n}{1}$

x_1 é livre ; da penúltima linha $x_n = 0$

da antepenúltima $x_{n-1} = 0$

$\vdots$

$x_2 = 0$

$$v = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \qquad \left(\dim \operatorname{Aut}_T(1) = 1 \right)$$

$\Rightarrow$ um bloco de Jordan (em particular

$$m_T(x) = P_T(x) \Big)$$

$$J(T) = \begin{pmatrix} 1 & & & & & \\ 1 & 1 & & & 0 & \\ & 1 & 1 & & & \\ 0 & & 1 & 1 & & \\ & & & 1 & \ddots & \\ & & & & 1 & 1 \end{pmatrix}_{n \times n}$$

Se $n = \cancel{4}\,3$ $\quad T_0 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$$T - I = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 3 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \text{e} \quad v = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ gera}$$

$Aut_T(1)$; chame $\quad v_4 = v = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

v_3 é sol de $(T - I)X = v_4$

v_2 $\qquad\qquad\qquad\qquad\qquad\qquad v_3$

v_1 $\qquad\qquad\qquad\qquad\qquad\qquad v_2$

$$P_3(\mathbb{R}) = [v_1, Tv_1, T^2v_1, T^3v_1]$$

↑ autovetor

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$$A \in M_6(\mathbb{R}) : A^4 - 8A^2 + 16I = 0$$

$$q(x) = x^4 - 8x^2 + 16$$
$$= (x^2 - 4)^2$$

$$q(x) = (x+2)^2 (x-2)^2$$

$m_A(x) \mid q(x)$ pois $q(A) \equiv 0 \in L(\mathbb{R}^6, \mathbb{R}^6)$

Os possíveis $m_A(x)$:

$$\left\{ \begin{array}{ll} (x+2)(x-2) & \leftarrow \text{diag} \\ (x+2)^2(x-2) & \\ (x+2)(x-2)^2 & \\ (x+2)^2(x-2)^2 & \\ (x+2) & \leftarrow \text{diag} \\ (x+2)^2 & \\ (x-2) & \leftarrow \text{diag} \\ (x-2)^2 & \end{array} \right.$$

Suponha que

$$P_A(x) = (x-2)^m (x+2)^l, \quad m+l = 6$$

$$m_A(x) = (x-2)^2 (x+2)$$

Um bloco $J(2)_{2\times 2}$

$J(-2)$ só 1×1

$m=2,\ l=4$
$$\begin{pmatrix} 2 & & & & & \\ 1 & 2 & & & & \\ & & -2 & & & \\ & & & -2 & & \\ & & & & -2 & \\ & & & & & -2 \end{pmatrix},$$

$m=3,\ l=3$
$$\begin{pmatrix} 2 & & & & & \\ 1 & 2 & & & & \\ & & 2 & & & \\ & & & -2 & & \\ & & & & -2 & \\ & & & & & -2 \end{pmatrix}$$

$m=4,\ l=2$
$$\begin{pmatrix} 2 & & & & & \\ 1 & 2 & & & & \\ & & 2 & & & \\ & & 1 & 2 & & \\ & & & & -2 & \\ & & & & & -2 \end{pmatrix}$$ ou

$$\begin{pmatrix} 2 & & & & & \\ 1 & 2 & & & & \\ & & 2 & & & \\ & & & 2 & & \\ & & & & -2 & \\ & & & & & -2 \end{pmatrix}$$

$$\vdots$$

$$P_T(x) = (x-\lambda_1)^{m_1} \cdots (x-\lambda_t)^{m_t}$$

$\exists\, U_i$, $\dim U_i = m_i$, $i = 1, \dots, t$, T-invariante

$$T|_{U_i} = \begin{bmatrix} J_1(\lambda_i) & & \\ & \ddots & \\ & & J_k(\lambda_i) \end{bmatrix}$$

ordem $J_{\ell+1} \leq$ or J_ℓ ; ordem J_1 = grau de λ_1 em $m_T(x)$

e, para cada ℓ, $\exists\, v$ e um ciclo que "gera" o bloco correspondente.

$k = \dim \mathrm{Aut}(\lambda_i)$

$\alpha \cdot 2$, $\alpha \in \mathbb{C}$

$\in \mathbb{C}$.

$T \in L(V,V)$, $\mathbb{R}$

$$P_T(x) = (x-\lambda_1)^{m_1} \cdots (x-\lambda_t)^{m_t} \cdot p_1(x)\, p_2(x) \cdots p_s(x)$$

$p_j(x)$ irredutível de grau 2 .

$t, m_1, m_2, \ldots, m_t \longleftrightarrow T$

$\longleftrightarrow S \quad \Rightarrow \exists$ isomorfismo

$\varphi : V \to W$:

$\varphi T \varphi^{-1} = S$

$\varphi T = S \varphi$

$$\begin{tikzcd} V \arrow[r, "T"] \arrow[d, "\varphi"'] & V \arrow[d, "\varphi"] \\ W \arrow[r, "S"] & W \end{tikzcd}$$

$v_1, Tv_1, \ldots, T^{m_1-1} v_1, v_2 \ldots$

$\downarrow \varphi$

$\varphi v_1, \varphi T v_1, \ldots, \varphi T^{m_1-1} v_1, \varphi v_2$

$w_1, \quad w_2 \qquad\qquad w_{m_1},$

$S w_1 = S \varphi v_1$

$= \varphi T v_1 = w_2$

$S^2 w_1 = S^2 \varphi v_1 = S(S\varphi v_1) = S(\varphi T v_1)$

$= \varphi T^2 v_1$

LI $\vdots$

$S^{m_1-1} w_1 = \varphi T^{m_1-1} v_1$

$S^{m_1} w_1 = \varphi T^{m_1} v_1 = 0 \quad \forall$

$m_1 \longleftrightarrow T$

$l_1 \longleftrightarrow S$

$\left(\begin{array}{l} l_1 \leq m_1 \\ l_1 \geq m_1 \end{array} \right)$

$v_1 , Tv_1 , T^2 v_1 , v_2 , Tv_2$

$$\downarrow \varphi$$

$\varphi v_1 = w_1$

$Sw_1 = S\varphi v_1 = \varphi T v_1 \Rightarrow \{ w_1 , Sw_1 , S^2 w_1 \}$ LI

$S^2 w_1 = S^2 \varphi v_1 = \varphi T^2 v_1$ e $S^3 w_1 = 0$

$S^3 w_1 = \varphi T^3 v_1 = 0$

$l_1 = m_1$

Se fosse $l_1 = 2$, $S^2 w_1 = 0$ e φ ~~…~~ não pode ser iso

$P_T(x)$, $m_T(x)$

$(x - \lambda_1)^{m_1}$, $(x - \lambda_1)^{d_1}$

Em U_1 ($\leftrightarrow \lambda_1$), existe subesp T-inv de dim d_1:

$U_1 = W_1^{d_1} \oplus W_2$ e $\exists v_1$:

$$V = U_1^{m_1} \oplus W_1 \quad T\text{-inv}$$

$$\exists\, v_1 : U_1 = [v_1, Tv_1, \dots, T^{m_1-1} v_1]$$

$T|_{W_1}$ é m_2-nilp $(m_2 \leq m_1)$

$\Rightarrow$ $W_1 = U_2^{m_2} \oplus W_2$, $\exists\, v_2 : [v_2, \dots, T^{m_2-1} v_2]$

$\vdots$

$T|_{W_2}$ é m_3-nilp, $m_3 \leq m_2$

$\vdots$