Matrizes similares

$$P^{-1} A P = \underbrace{P^{-1} P}_{I} . B = B$$

$B = \underbrace{P^{-1}}_{I} A P$, se $A = PB$ então:

$$A P P^{-1} = P B . P^{-1}$$

$A = P B P^{-1}$, $B = P^{-1} A P$, P é inversível.

1) traço de B = traço de A
2) $\det(B) = \det(A)$
3) $\lambda_B = \lambda_A$
4) $p_T(B) = p_T(A)$
5) $m_T(B) = m_T(A)$

diagonal $i=j$

Temos que $C = AB \dots$, $c_{ij} = \sum_{k=1}^{N} a_{ik} b_{jk}$

1) Temos que $tr(AB) = \sum (AB)_{ii}$

$$tr(AB) = \sum_{i=1}^{N} \sum_{k=1}^{N} a_{ik} b_{ik}$$

$$tr(BA) = \sum_{i=1}^{N} \sum_{k=1}^{N} b_{ki} a_{ik}$$

Embora $AB \neq BA \rightarrow tr(AB) = tr(BA)$

Agora p/ matrizes similares:

$$tr(B) = tr(P^{-1}AP) = tr(AP.P^{-1}) =$$

permutar

$$tr(A.I) = tr(A)$$

2) $det(B) = det(A)$., obs: $det(ABC) = det(A).det(B).det(C)$

$$det(B) = det(P^{-1}).det(A).det(P)$$

$$det(B) = \frac{1}{det(P)}.det(A).det(P)$$

permutar n°

$$det(B) = det(A).det(P).\frac{1}{det(P)}$$

$$det(B) = det(A)$$

3) $\lambda_B = \lambda_A \rightarrow p^+(B) = p^+(A)$ e $mt(B) = mt(A)$

$B = P^{-1}AP$, $Bx = P^{-1}APx$

Supondo que λ seja autovalor de B,

$$PBx = \underbrace{PP^{-1}}_{I}APx = \lambda Px$$

$$PBx = A\underbrace{Px}_{y} = \lambda \underbrace{Px}_{y}$$

$$PBx = Ay = \lambda y$$

Então λ é autovalor de A, $A(y) = \lambda y$.
Mas λ é autovalor de B. Assim: $\lambda_B = \lambda_A$.
Apesar dos autovalores ser iguais, os autovetores são diferentes.
Se $\lambda_B = \lambda_A$ e $\vec{v}_B = \vec{v}_A$, então A é idêntica a B.

Ex: $\begin{cases} x + 2y = 3 \\ 2y = 2 \end{cases}$, $A = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$.

Escolher uma base qualquer:
$M = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$, com $u_1 = (1,2)$, $u_2 = (1,1)$

$$AM = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 4 & 2 \end{bmatrix}$$

$$M^{-1} = \begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix}, \quad B = M^{-1} A M$$

$$B = \begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} = \overset{M^{-1}A}{\begin{bmatrix} -1 & 0 \\ 2 & 2 \end{bmatrix}} \cdot \overset{A}{\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}}$$

$$B = \begin{bmatrix} -1 & -1 \\ 6 & 4 \end{bmatrix}, \quad C = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Se $A\vec{x} = \vec{c}$, $\overbrace{MBM^{-1}}^{A}x = \vec{c}$ ∴

$\underline{M^{-1}x}$: $y = ?$ Base diferente

$$\underbrace{M^{-1}M}_{I} B \underbrace{M^{-1}x}_{y} = M^{-1}\vec{c} \;\therefore$$

$$By = M^{-1}\vec{c} \;\therefore$$

$$\overset{B}{\begin{bmatrix} -1 & -1 \\ 6 & 4 \end{bmatrix}} \cdot \overset{?}{\begin{bmatrix} x \\ y \end{bmatrix}} = \overset{M^{-1}}{\begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix}} \cdot \overset{C}{\begin{bmatrix} 3 \\ 2 \end{bmatrix}} = \begin{bmatrix} -1 \\ 4 \end{bmatrix} \rightarrow \begin{matrix} y_1 = 0, \\ y_2 = 1 \end{matrix}$$

$$y = M^{-1}x \rightarrow My = MM^{-1}x \;\therefore\; x = My,$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$