Encontre a forma canônica de Jordan da matriz.

$$A = \begin{pmatrix} -2 & -2 & -4 \\ 0 & -2 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

Temos: $|A-\lambda I| = \begin{vmatrix} -2-\lambda & -2 & -4 \\ 0 & -2-\lambda & -1 \\ 0 & 0 & 1-\lambda \end{vmatrix}$

$p_T(\lambda) = -\lambda^3 - 3\lambda^2 + 4 = -(\lambda-1).(\lambda^2+4\lambda+4) = -(\lambda-1).(\lambda+2)^2 = 0$

Portanto os autovalores são: $\lambda_1 = 1$ e $\lambda_2 = -2$.

1) Para $\lambda_1 = 1$, temos multiplicidade algébrica $\mu_{a1} = 1$

$$|A - 1I|.x = \begin{vmatrix} -3 & -2 & -4 \\ 0 & -3 & -1 \\ 0 & 0 & 0 \end{vmatrix} . \begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$$

$(x,y,z) = \{(-\frac{10}{9}, -\frac{1}{3}, 1)\}$

Portanto o autovetor de $\lambda_1 = \{(-\frac{10}{9}, -\frac{1}{3}, 1)\}$

2) Para $\lambda_2 = -2$, $\mu_{a2} = 2$

$|A-(-2)I|^2 . x$, então $(x,y,z) = \{(0,1,0)\}$

$[|A+2I|.x].(0,1,0) = (-2,0,0)$

Portanto temos 3 autovetores: $(x,y,z) = \{(-\frac{10}{9}, -\frac{1}{3}, 1)$, $(-2,0,0)$, $(0,1,0)\}$ que forma a matriz M dos autovetores.

$$M = \begin{vmatrix} -10/9 & -2 & 0 \\ -1/3 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix}$$

A matriz na forma de Jordan é formada pelos autovalores, sendo em dois blocos dado que $\lambda_1 \neq \lambda_2$.

$$J = \left|\begin{array}{c|cc} 1 & 0 & 0 \\ \hline 0 & -2 & 1 \\ 0 & 0 & -2 \end{array}\right|$$

Assim: $A = MDM^{-1} = MJM^{-1}$

$$\begin{vmatrix} -2 & -2 & -4 \\ 0 & -2 & -1 \\ 0 & 0 & 1 \end{vmatrix} = \underbrace{\begin{vmatrix} -10/9 & -2 & 0 \\ -1/3 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} \cdot \begin{vmatrix} 1 & 0 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -2 \end{vmatrix}} \cdot \begin{vmatrix} 0 & 0 & 1 \\ -1/2 & 0 & -5/9 \\ 0 & 1 & 1/3 \end{vmatrix}$$

$$= \underbrace{\begin{vmatrix} -\frac{10}{9} & 4 & -2 \\ -\frac{1}{3} & 0 & -2 \\ 1 & 0 & 0 \end{vmatrix} \cdot \begin{vmatrix} 0 & 0 & 1 \\ -\frac{1}{2} & 0 & -\frac{5}{9} \\ 0 & 1 & \frac{1}{3} \end{vmatrix}}$$

$$= \underbrace{\begin{vmatrix} -2 & -2 & -4 \\ 0 & -2 & -1 \\ 0 & 0 & 1 \end{vmatrix}}$$

Portanto determinamos a matriz de Jordan.

$$J = \begin{vmatrix} 1 & 0 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -2 \end{vmatrix}$$