Exemplo clássico

$$\underbrace{ax^2+bxy+cy^2}+dx+ey+f=0$$

coef ñ todos nulos

$$(x\ \ y)\underbrace{\begin{pmatrix}a & b/2\\ b/2 & c\end{pmatrix}}_{M}\begin{pmatrix}x\\ y\end{pmatrix}+(d\ \ e)\begin{pmatrix}x\\ y\end{pmatrix}+f=0$$

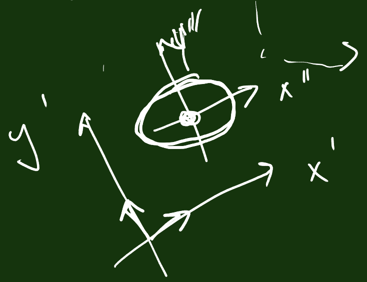
$\exists$ base $P=[v_1,v_2]$, ortonormal tq

$\exists\, D=\begin{pmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{pmatrix}$:

$$\begin{tikzcd} & \mathbb{R}^2 \arrow[r, "M"] \arrow[d, "P"'] & \mathbb{R}^2 \arrow[d, "P"] & (x,y) \arrow[l] \\ \text{ponto de vista} \arrow[r] & \mathbb{R}^2 \arrow[r, "D"'] & \mathbb{R}^2 & (x',y') \arrow[l] \end{tikzcd}$$

$$\underline{D=PMP^{-1}}$$

$$\lambda_1 x'^2+\lambda_2 y'^2+(d'\ \ e')\begin{pmatrix}x'\\ y'\end{pmatrix}=f'$$



5.1 T é diag se $\exists\ \{v_i\}$ base

$$Tv_i=\lambda_i v_i\ ,\ \forall i$$

$\exists\, D,\ \exists\, P \leftarrow$ invertível, colunas vetores da base B

$\uparrow$ diagonal

Tais que $\qquad [T]_B = D.$

Def: $T: V \to V \qquad \dim V = n$

v é autovetor se $\underline{\underline{v \neq 0}}$

e $Tv = \lambda v$, para algum $\lambda \in \mathbb{K}$

$v \leftrightarrow \lambda \qquad$ autovet / autoval. associado

$$Tv = \lambda v$$

$$Tv - \lambda I v = 0$$

$$(T - \lambda I)\, v = 0$$

v é sol. ñ nula de $\underbrace{(T - \lambda I)}_{M} X = 0$

numa base, M é quadrada,

$\exists\, v \Leftrightarrow \det (T - \lambda I) = 0$

- Exigimos $\det (T - \lambda I) = 0$

EQ. CARACTERÍSTICA

$p(x) = \det(xI - T)$ é o pol. característ.

grau $p(x) = \dim V$

mônico

→ fixamos uma base

→ outra base? $B \leftrightarrow B'$

$$p(x) = \det(xI - T_B) = \det(xI - T_{B'})$$

↑ não varia se mudamos a base

---

v é autovetor $v \leftrightarrow \lambda \in \mathbb{K}$ se e só se

$$Tv = \lambda v \quad \text{ou} \quad (\lambda I - T)v = 0$$

$\Leftrightarrow v \in \mathrm{Nuc}(\lambda I - T)$

Ex $T: \mathbb{R}^2 \to \mathbb{R}^2$, $T(x,y) = (-y, x) = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$

não há autovalores, logo $\nexists$ autovet

Ex| $p(x) = (x-\lambda)^2 (x-\gamma)$

$\exists \ 0 \neq v \in \mathrm{Nuc}(\gamma I - T) \xrightarrow{dim} 1$

$\exists \ 0 \neq v \in \mathrm{Nuc}(\lambda I - T) \xrightarrow[d]{dim} 1 \leq d \leq 2$

$\mathrm{Nuc}(\lambda I - T) = \mathrm{Aut}_T(\lambda) \longrightarrow dim = 2$, ok

$\searrow$ dim 1, $\cancel{ok}$

— n —

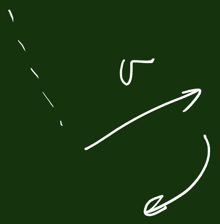
T é diagonalizável $\Leftrightarrow$ $\exists$ base de autovetores $\Leftrightarrow$ $\sum_i \dim_T(\lambda_i) = n$ onde $p_T(x) = (x-\lambda_1)^{i_1} \cdots$
$\cdots (x-\lambda_k)^{i_k}$

— n —

$m_A(\lambda)$ = multiplicidade de λ como raiz de $p(x)$

$m_G(\lambda) = \dim \mathrm{Aut}_T(\lambda)$

$$1 \leq m_G(\lambda) \leq m_A(\lambda)$$



$$\xrightarrow{T} \left(\begin{array}{cc|c} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ \hline 0 & 0 & \lambda \end{array}\right) \quad ?$$

$$p_T(x) \longmapsto p_T(T) \in L(V,V)$$

$$\dim L(V,V) = n^2$$

$\Rightarrow \exists\, k \leq n^2$ ~~tq~~ o menor inteiro tq

$\{I, T, T^2, \dots, T^k\}$ é LI mas

$\{I = T^0, T, \dots, T^{k+1}\}$ é LD

$$\Rightarrow T^{k+1} = \sum_{i=0}^{n} \lambda_i T^i$$

$$\rightarrow T^{k-1} - \sum \lambda_i T^i = 0 \in L(V,V)$$

se $\underbrace{p_m}(x) = x^{k-1} - \sum \lambda_i x^i$,

$$\Rightarrow \underline{p_m(T)} = 0 .$$

"o polinômio se anula em T"

$T \equiv 0 : V^n \to V^n$

$$T_{\mathcal{B}} = \begin{pmatrix} 0 & & \\ \vdots & \ddots & \\ 0 & & 0 \end{pmatrix}$$

$$p_T(x) = \det(xI - T) = \det\begin{pmatrix} x & & 0 \\ & \ddots & \\ 0 & & x \end{pmatrix} = x^n$$

$\{\underline{I, T}, T^2, \dots\}$

$\{I, T\}$ LD

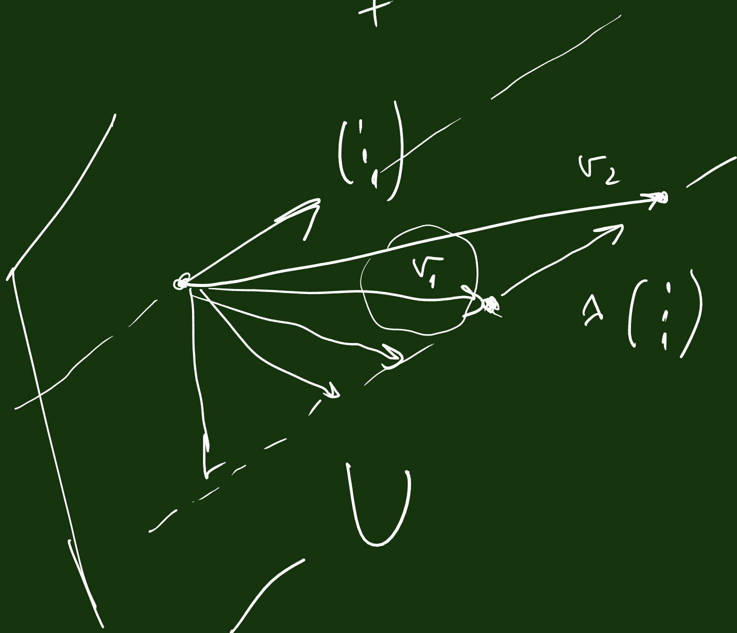
$m_T(x) = x$

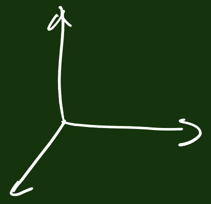
$\mathrm{Nuc}(0I - T) = V$

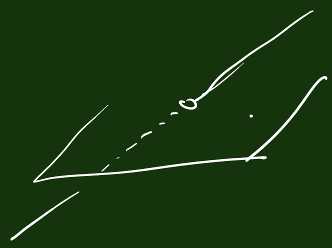
comentário sobre a nova

(1) U rep V/W se dada $\bar{v}$ $\exists!\, u$ tq $\bar{u} = \bar{v}$

(a) $\mathbb{R}^3 / \left[\begin{pmatrix}1\\1\\1\end{pmatrix}\right]$

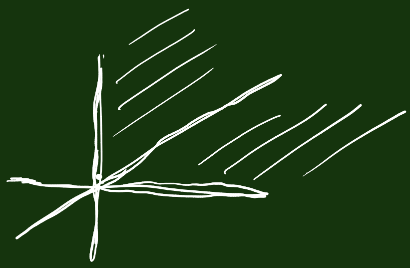






— n —

(c)



— n —

(4) $(U \cap W)^\circ = U^\circ + W^\circ$

$\supset$ trivial

$\subset$?!

argumento de dimensão

— n —

(5) $T : U \to V$ ñ sobre

$\Downarrow$

$\mathrm{Nuc}(T^t)$ não trivial

$$\begin{array}{ccc} U & \xrightarrow{T} & V \\ & \searrow & \downarrow f \\ & & \mathbb{K} \end{array}$$

$T^t = f \circ T$

$\exists v \neq Tu, \forall u$

f dual a v

$f(v) = 1$, $f(w) = 0$

w LI com u

$$\underbrace{T^t(f)}_{=0 \in U^*} = f \circ T : u \mapsto 0, \ \forall u$$

T $\bar{n}$ sobre $\Rightarrow$ Nuc (T^t) $\bar{n}$ triv.

$\boxed{P \Rightarrow Q}$

$\boxed{\sim Q \Rightarrow \sim P}$

T^t injetora $\Rightarrow$ T sobre (contrapositiva)