14) $\lim \sqrt[n]{n!} = +\infty$

Tome $y_n = \ln x_n$

$y_n = \ln \sqrt[n]{n!}$

$\ln \sqrt[n]{n!} = \frac{1}{n} \cdot (\ln 1 + \ln 2 + \dots + \ln n)$ I

Opção 1

$\lim x_n = \lim e^{y_n} \geq \lim \exp \left[\frac{1}{n} \int_1^n \ln x \, dx \right]$

$= \lim \exp \frac{n \cdot \ln n - n + 1}{n} \geq \lim \exp (\ln n - 1) = +\infty$

Opção 2

$y_n = \frac{1}{n} (\ln 1 + \ln 2 + \dots + \ln n)$

$= \lim y_n = \lim \frac{\ln (n+1)}{(n+1) - n} = \lim \ln (n+1) = \infty$

$\lim x_n = \lim e^{y_n} = \infty$