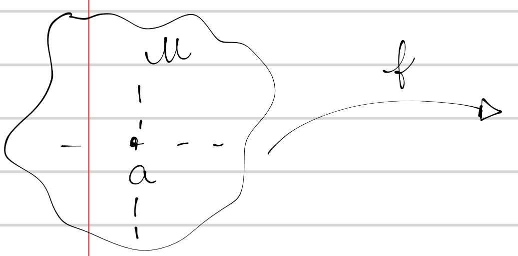
Funções de n Variáveis : $f : U \to \mathbb{R}$, se $U \in \mathbb{R}^n$

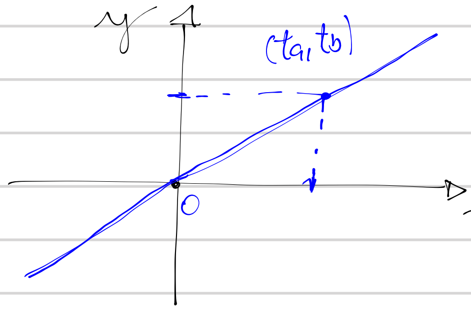


$R \quad \bullet \; \dfrac{\partial f}{\partial x_i}(a) = \lim\limits_{t \to 0} \dfrac{f(a+t\,) - f(a)}{t}$

$\bullet \; \dfrac{\partial f}{\partial x_i} = \dfrac{\partial f}{\partial y_i} = \dfrac{\partial f}{\partial z_i}$

Exemplo : $f : \mathbb{R}^2 \to \mathbb{R}$

$f(x,y) = \dfrac{xy}{x^2+y^2}$, (se $(x,y) \neq (0,0)$)



$f(0,0) = 0$

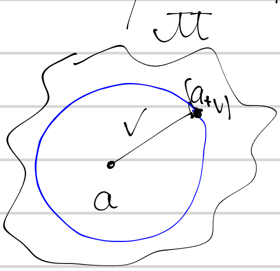
$\dfrac{\partial f}{\partial x}(0,0) = \dfrac{\partial f}{\partial y}(0,0) = 0$

$\bullet \; f : U \to \mathbb{R}$, e $f \in C^1$ se $\dfrac{\partial f}{\partial x_1}, \dots, \dfrac{\partial f}{\partial x_n} : U \to \mathbb{R}$ contínuas

$\bullet \; f : U \to \mathbb{R}$ é diferenciável no ponto $a \in U$, 1) $\exists$ todas as derivadas parciais no ponto a, $v = 1, \dots, n$

2) $f(a+v) - f(a) =$

$\sum \dfrac{\partial f(a)}{\partial x_i} . \alpha_1 + r(v)$ e $V = (\alpha_1, \dots, \alpha_n)$



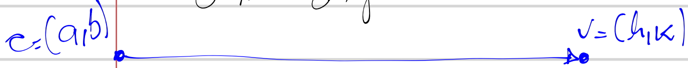
onde $\lim\limits_{v \to 0} \dfrac{r(v)}{|v|} = 0$

Teorema : Toda função de classe C^1 é diferenciável.

$\dfrac{\partial f}{\partial x}(x,y)$, $\dfrac{\partial f}{\partial y}(x,y)$ existem em todos os pontos $(x,y) \in U \in \mathbb{R}^2$

Tomamos o ponto $(a,b) \in \mathbb{R}^2$ e as derivadas parciais são contínuas.,

$\dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y} : U \to \mathbb{R}$ contínua



$r(v) = f(a+h, b+k) - f(a,b) -$

$\in U \quad \dfrac{\partial f}{\partial x}(a,b).h - \dfrac{\partial f}{\partial y}(a,b).k$

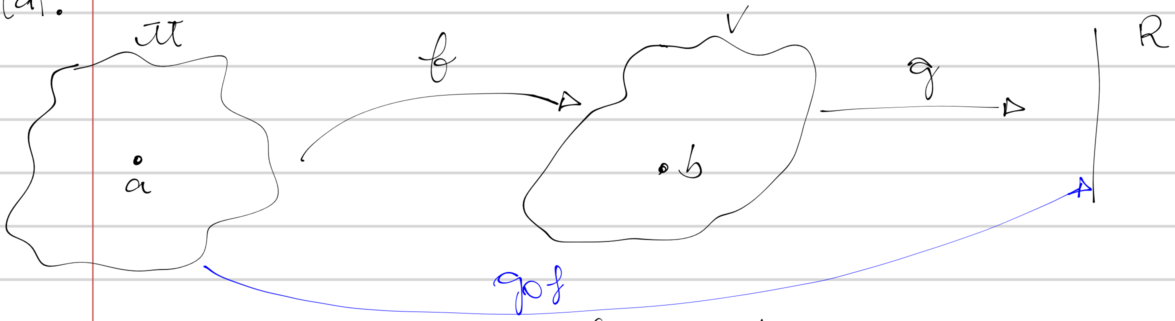
$r(h,k) = \overbrace{f(a+h, b+k) - f(a, b+k)} + f(a, b+k) - f(a,b) - \dfrac{\partial f}{\partial x}(a,b).h - \dfrac{\partial f}{\partial y}(a,b).k$

$= \dfrac{\partial f}{\partial x}(a + \theta_1 a, b+k) + \dfrac{\partial f}{\partial y}(a, b+\theta_2 k) - \dfrac{\partial f}{\partial x} h \, \partial x - \dfrac{\partial f}{\partial y}.k \, \partial y$

$0 < \theta < 1$

Toda função de classe C^1 é contínua,

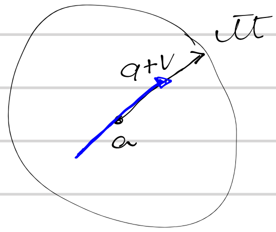
Dada a função $f: U \to V$, sendo U e V abertos no $\mathbb{R}^n$, e $f = (f_1, \dots, f_n)$ as coordenadas de f. $(f_1, \dots, f_n)$ possuem derivadas parciais no ponto $a \in U$, $b = f(a)$.



Dada uma $g: V \to \mathbb{R}$ diferenciável no ponto $b = f(a)$. Então $g \circ f: U \to \mathbb{R}$ possui derivadas parciais no ponto a, e tem-se:

$$\frac{\partial f(g \circ f)}{\partial x_i} = \sum_{k=1}^{m} \frac{\partial g}{\partial y_k} \cdot \frac{\partial f_k}{\partial x_i}$$

$f: U \to \mathbb{R}$, $a \in U \subset \mathbb{R}^n$ e $v \in \mathbb{R}$, $\frac{\partial f}{\partial v}(a) \in \mathbb{R}$



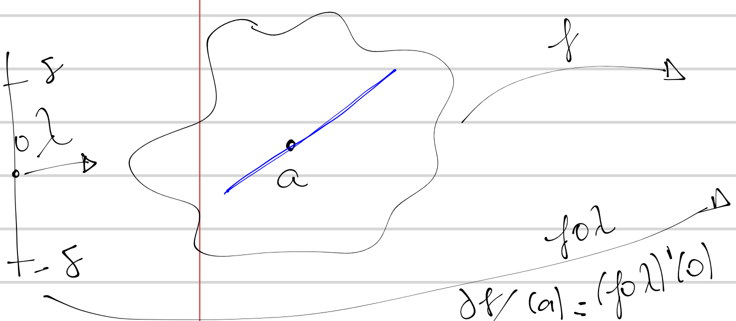
$$\frac{\partial f}{\partial v}(a) = \lim_{t \to 0} \frac{f(a + t_v) - f(a)}{t}$$

Se $w = \alpha . v \longrightarrow \frac{\partial f}{\partial w}(a) = \alpha . \frac{\partial f}{\partial v}(a) = \lim_{t \to 0} \frac{f(a + tw) - f(a)}{t} =$

$$\lim_{t \to 0} \frac{f(a + \alpha t v) - f(a)}{\alpha t} . \alpha = \frac{\partial f}{\partial v}(a) . \alpha$$

Se $f: U \to \mathbb{R}$ é diferenciável no ponto $a \in U$ então $\forall v \in \mathbb{R}^n$, $\forall v = (\alpha_1, \dots, \alpha_n)$ tem-se: $\frac{\partial f}{\partial v}(a) = \sum_{i=1}^{n} \alpha_i \frac{\partial f}{\partial x_i}(a)$ "A derivada direcional é a derivada de uma função composta"

Seja λ um caminho e $\lambda = (-1, 1) \to \mathbb{R}^n$, $\lambda(t) = a + tv$ e $t \in (-\delta, \delta) \to a + tv \in U$



$f \circ \lambda : t \longrightarrow f(a + tv)$ $\quad \lambda_i(t) = a_i + t\alpha_i$

$$f \circ \lambda'(0) = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i}(a) . \lambda_i'(0) = \sum_{i=1}^{n} \alpha_i \frac{\partial f}{\partial x_i}(a)$$

$\lambda_i'(0) = \alpha_i$

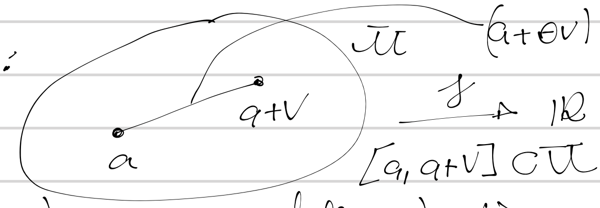
$\frac{\partial f}{\partial v}(a) = (f \circ \lambda)'(0)$

$f \circ \lambda : (-\delta, \delta) \longrightarrow \mathbb{R}$

Vetor gradiente: é o vetor formado com as coordenadas (produto interno de $\vec{V}$ com as derivadas parciais), $= \langle \text{gradiente}(a), V \rangle$

gradiente $f(a) = \left(\frac{\partial f}{\partial x_1}(a), \ldots, \frac{\partial f}{\partial x_n}(a) \right)$, contém informações crescimento e decrescimento da função

• Se $f: U \to \mathbb{R}$ é diferenciável:



Então: $f(a+v) - f(a) = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i}(a+\theta v)\alpha_i = \langle \text{grad}\, f(a+\theta v), V \rangle$