Seja $T = \{(x, y, z) \in \mathbb{R}^3 ; x \geq 0, y \geq 0, z \geq 0, x + y + z \leq 1\}$. Mostre que:

$$Vol(T) = \int_0^1 dx \int_0^{1-x} dz \int_0^{1-y-z} dy = 1/6$$

$$Vol(T) = \int_0^1 dx \int_0^{1-x} dz \int_0^{1-y-z} dy = \int_0^1 dx \int_0^{1-x} dz \, [y] \Big|_0^{1-x-z} =$$

$$\int_0^1 dx \int_0^{+1-x} [1-x-z-0] \, dz = \int_0^1 dx \int_0^{1-x} (1-x-z) \, dz =$$

$$\int_0^1 \left[(1-z)z - \frac{z^2}{2} \right] \Big|_0^{1-x} dx = \int_0^1 \left[(1-x)(1-x) - \frac{(1-x)^2}{2} - 0 - 0 \right] dx$$

$$\int_0^1 \left[(1-x)^2 - \frac{(1-x)^2}{2} \right] dx = \int_0^1 \left(\frac{2(1-x)^2 - (1-x)^2}{2} \right) dx = \frac{1}{2} \int_0^1 (1-x)^2 dx =$$

$$\frac{1}{2} \left[\frac{(1-x)^3}{-3} \right] \Big|_0^1 = \frac{1}{2} \left[\frac{0}{-3} + \frac{1}{3} \right] = \frac{1}{6}$$, então:

$$\int_0^1 dx \int_0^{1-x} dz \int_0^{1-x-z} dy = 1/6$$