

Coeficientes fourier

Ejecicio a mano

$$T = T_0$$

$$x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & T_1 < |t| < \frac{T_0}{2} \end{cases}$$

$$a_K = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t) e^{-jK\omega_0 t} dt$$

Sustituir x(t) en la integral

$$a_K = \frac{1}{T_0} \int_{\square}^{\square} \square e^{-jK\omega_0 t} dt$$

$$= \frac{1}{T_0} \left[\int_{-\frac{T_0}{2}}^{-T_1} \underbrace{x(t)}_{=0} e^{-jK\omega_0 t} dt + \int_{-T_1}^{T_1} \underbrace{x(t)}_{=1} e^{-jK\omega_0 t} dt + \int_{T_1}^{\frac{T_0}{2}} \underbrace{x(t)}_{=0} e^{-jK\omega_0 t} dt \right]$$

Propiedad de Axel

$$a_k = \frac{1}{T_0} \int_{-T_1}^{T_1} e^{-jk\omega_0 t} dt$$

Propiedad de Erick

$$\omega_0 = \frac{2\pi}{T_0}$$

1

$k=0$

$k \neq 0$

$$a_0 = \frac{1}{T_0} \int_{-T_1}^{T_1} 1 dt = \frac{1}{T_0} (T_1 + T_1) \quad a_k = \frac{1}{T_0} \int_{-T_1}^{T_1} e^{-jk\omega_0 t} dt$$

$$a_0 = \frac{2T_1}{T_0}$$

$$\begin{aligned} k \neq 0 \\ a_k &= \frac{1}{T_0} \int_{-T_1}^{+T_1} e^{-jk\omega_0 t} dt = \frac{e^{-jk\omega_0 t}}{T_0 (-jk\omega_0)} \Big|_{t=-T_1}^{t=T_1} \\ &= -\frac{1}{T_0 jk\omega_0} (e^{-jk\omega_0 T_1} - e^{jk\omega_0 T_1}) \\ &= \frac{1}{T_0 jk\omega_0} (e^{jk\omega_0 T_1} - e^{-jk\omega_0 T_1}) \end{aligned}$$

Propiedad de Axel

Recordando que $\text{Im}(z) = \frac{z - \bar{z}}{2j}$, entonces:

$$\begin{aligned} &= \frac{2}{T_0 k\omega_0} \text{Im}(e^{jk\omega_0 T_1}) \\ &= \frac{2}{T_0 k\omega_0} \sin(k\omega_0 T_1) \end{aligned}$$

$$\begin{aligned} &= \frac{2}{T_0 k\omega_0} \sin(k\omega_0 T_1) \\ &= \frac{2T_1}{T_0} \frac{\sin(k\omega_0 T_1)}{k\omega_0 T_1} \end{aligned}$$

$$a_k = \frac{2T_1}{T_0} \text{sinc}\{k\omega_0 T_1\} \quad k \neq 0$$

Ploteo onda cuadrada

```
N=100;
k=-N:N;

A=8;
T0=2;
T1=T0/A;

w0=2*pi/T0;
```

$$a_k = \frac{2T_1}{T_0} \operatorname{sinc}\{k\omega_0 T_1\}$$

$k \neq 0$

$$a_0 = \frac{2T_1}{T_0}$$

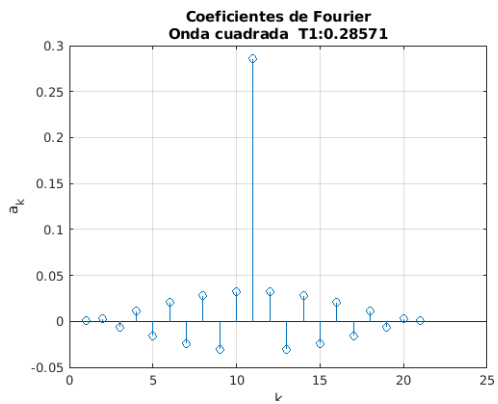
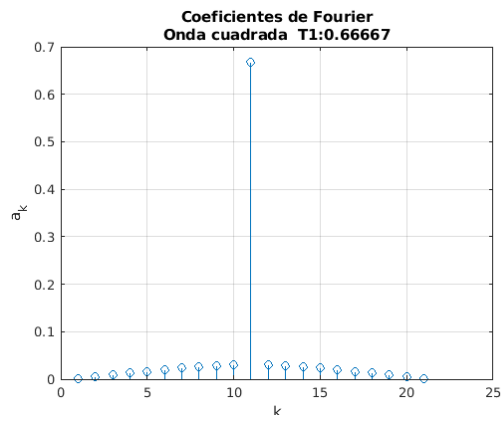
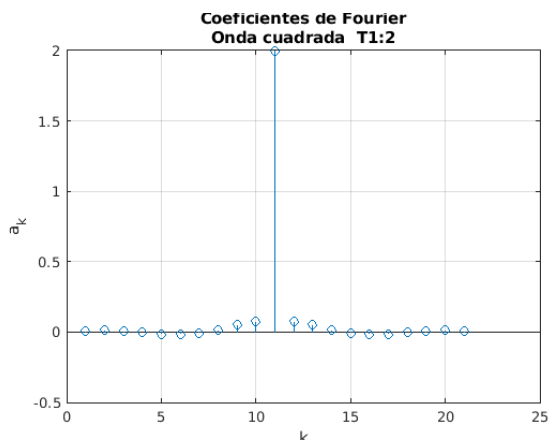
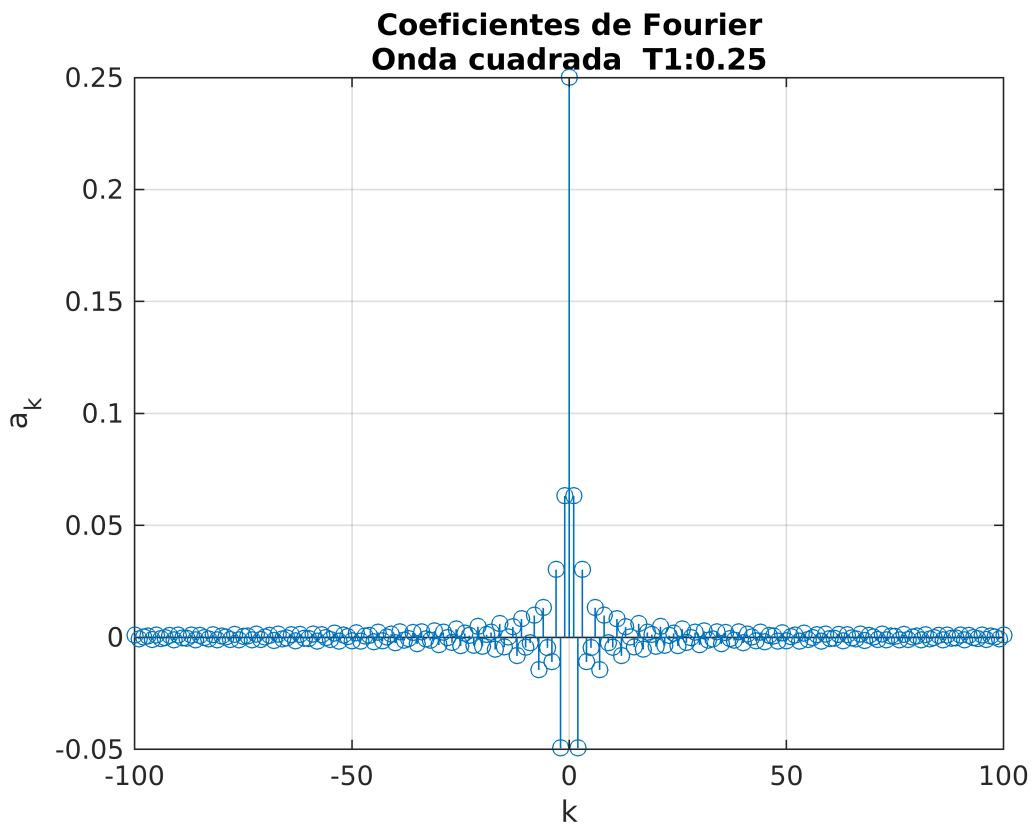
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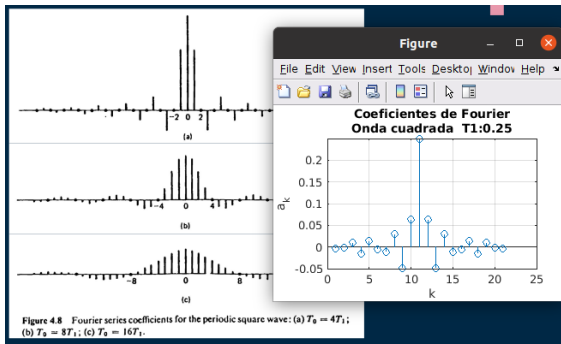
a_k=(2*T1/T0)*sinc(k*w0*T1);
a_0=2*T1/T0;

%La ecuación de a_k general no funciona
%Para k=0, por eso sustituimos un valor
%previamente calculado.
a_k(k==0)=a_0;

figure
stem(k,a_k)
xlabel("k")
ylabel("a_k")
grid on
title(["Coeficientes de Fourier";"Onda cuadrada T1:"+T1])

```





Síntesis

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 t}, \quad t \in [t_0, t_0 + T],$$

```
syms t
n=-3:3;
% %Numero de elementos
% numel()
% %Tamaño de la matriz en n X m
% size()
% %Tamaño más grande de la matriz
% length()

x=sum(a_k.*exp(1i*k*w0*t));

figure
fplot(x,[-2.5 2.5])
xlabel("Tiempo (s)")
ylabel("Magnitud (V)")
title("Reconstruccion con N:"+N)
```

