Coeficientes fourier

Ejecicio a mano

$$T = T_0$$

$$x(t) = \boxed{x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & T_1 < |t| < \frac{T_0}{2} \end{cases}}$$

$$a_K = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t)\, e^{-jk\omega_0 t}\, dt$$

Sustituir x(t) en la integral

$$a_K = \frac{1}{T_0} \int_{\boxed{}}^{\boxed{}} \boxed{}\; e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{T_0}\left[\int_{-\frac{T_0}{2}}^{-T_1} \underbrace{x(t)}_{x(t)=0} e^{-jk\omega_0 t}\, dt + \int_{-T_1}^{T_1} \underbrace{x(t)}_{x(t)=1} e^{-jk\omega_0 t}\, dt + \int_{T_1}^{\frac{T_0}{2}} \underbrace{x(t)}_{x(t)=0} e^{-jk\omega_0 t}\, dt \right]$$

Propiedad de Axel

$$a_K = \frac{1}{T_0}\int_{-T_1}^{T_1} e^{-jk\omega_0 t}\,dt$$ Propiedad de Erick

$$\omega_0 = \frac{2\pi}{T_0}$$

1 $\quad k=0 \qquad\qquad k\neq 0$

$$a_0 = \frac{1}{T_0}\int_{-T_1}^{T_1} 1\,dt = \frac{1}{T_0}(T_1+T_1)$$

$$\boxed{a_0 = \frac{2T_1}{T_0}}$$

$$a_K = \frac{1}{T_0}\int_{-T_1}^{T_1} e^{\boxed{-jk\omega_0 t}}\,dt$$

$k \neq 0$

$$a_K = \frac{1}{T_0}\int_{-T_1}^{+T_1} e^{-jk\omega_0 t}\,dt = -\frac{e^{-jk\omega_0 t}}{T_0\, jk\omega_0}\Bigg|_{\text{de } t=T_1 \to t=-T_1}$$

$$= -\frac{1}{T_0\, jk\omega_0}\left(e^{-jk\omega_0 T_1} - e^{jk\omega_0 T_1}\right)$$

$$= \frac{1}{T_0\, jk\omega_0}\left(e^{jk\omega_0 T_1} - e^{-jk\omega_0 T_1}\right)$$

Propiedad de Axel

Recordando que $Im(z) = \frac{z-\bar{z}}{2i}$, entonces:

$$= \frac{2}{T_0 k\omega_0}\, Im\left(e^{jk\omega_0 T_1}\right)$$

$$= \frac{2}{T_0 k\omega_0}\, sen(k\omega_0 T_1)$$

$$= \frac{2}{T_0 k\omega_0}\, \sin(k\omega_0 T_1)$$

$$= \frac{2T_1}{T_0}\boxed{\frac{\sin(k\omega_0 T_1)}{k\omega_0 T_1}}$$

$$\boxed{a_k = \frac{2T_1}{T_0}\,\text{sinc}\{k\omega_0 T_1\}}$$

$k \neq 0$

Ploteo onda cuadrada

```
N=100;
k=-N:N;

A=8;
T0=2;
T1=T0/A;

w0=2*pi/T0;
```

$$\boxed{a_k = \frac{2T_1}{T_0}\,\mathrm{sinc}\{k\omega_0 T_1\}}$$

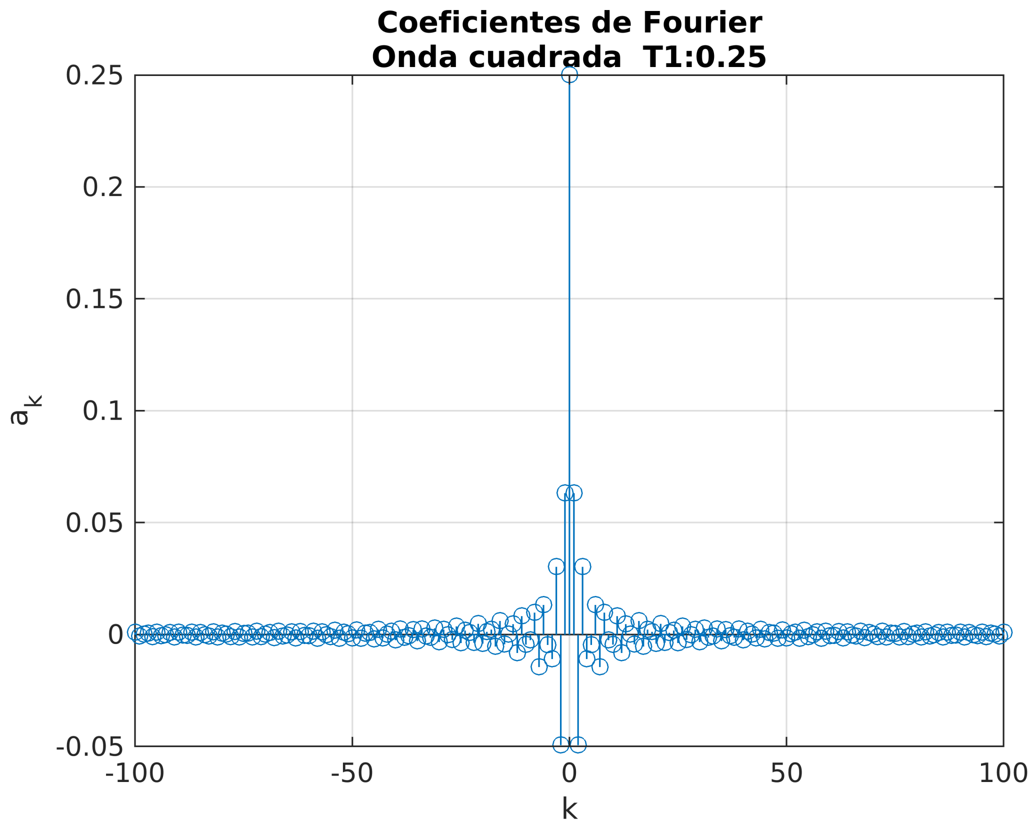
$k \neq 0$

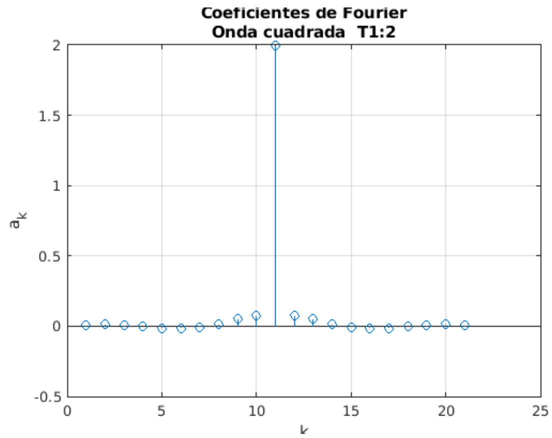
$$\boxed{a_0 = \frac{2T_1}{T_0}}$$

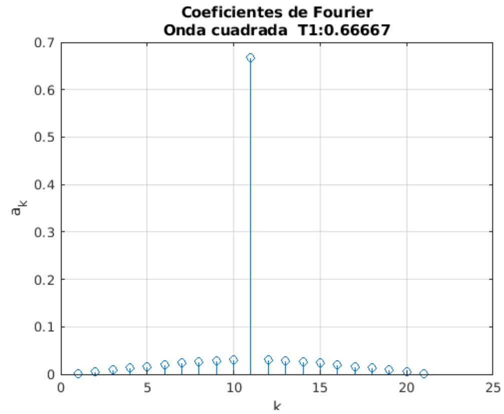
```
a_k=(2*T1/T0)*sinc(k*w0*T1);
a_0=2*T1/T0;

%La ecuación de a_k general no funciona
%Para k=0, por eso sustituimos un valor
%previamente calculado.
a_k(k==0)=a_0;

figure
stem(k,a_k)
```

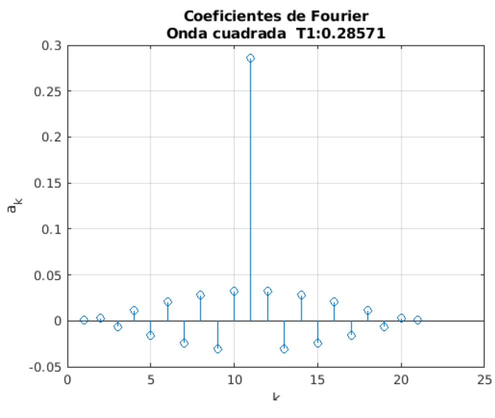
Warning: MATLAB has disabled some advanced graphics rendering features by switching to software OpenGL. For more information, click here.

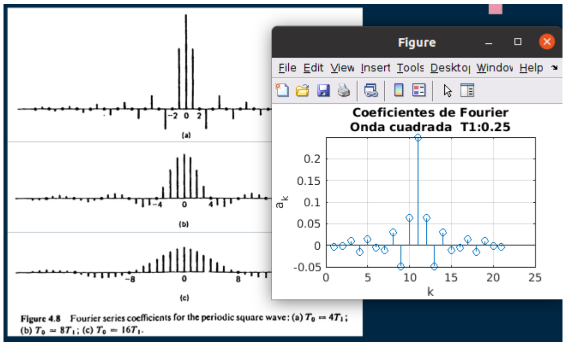
```
xlabel("k")
ylabel("a_k")
grid on
title(["Coeficientes de Fourier";"Onda cuadrada  T1:"+T1])
```


Coeficientes de Fourier
Onda cuadrada T1:0.25
a_k
k


Coeficientes de Fourier
Onda cuadrada T1:2
a_k
k


Coeficientes de Fourier
Onda cuadrada T1:0.66667
a_k
k

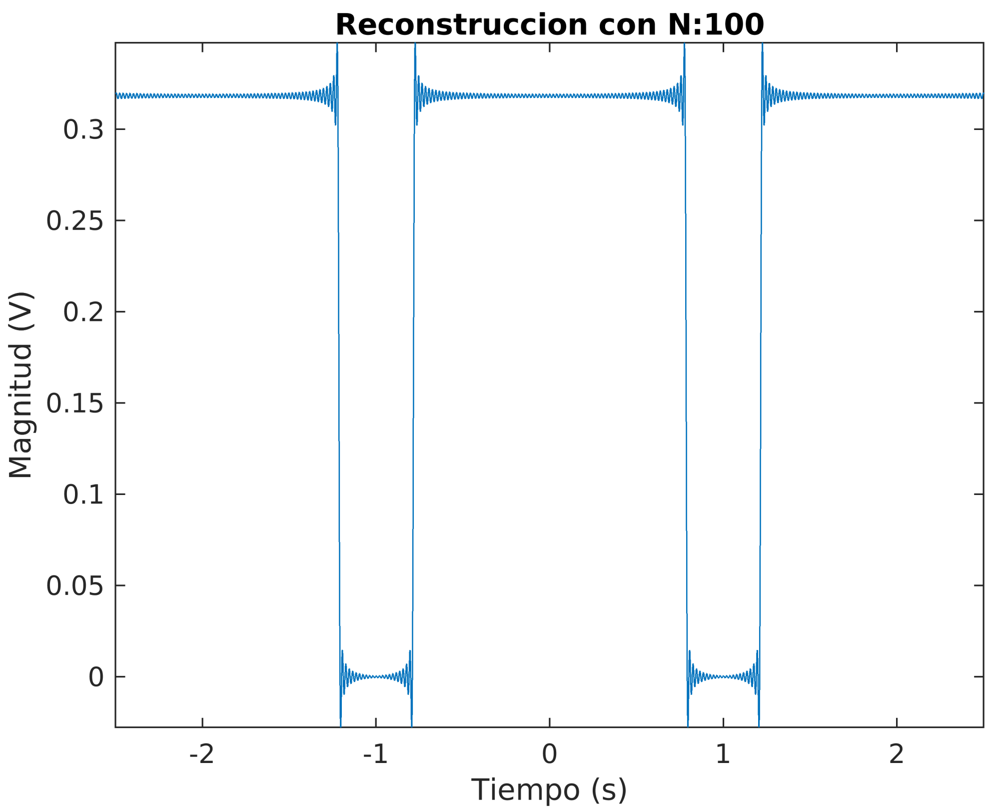

Coeficientes de Fourier
Onda cuadrada T1:0.28571
a_k
k

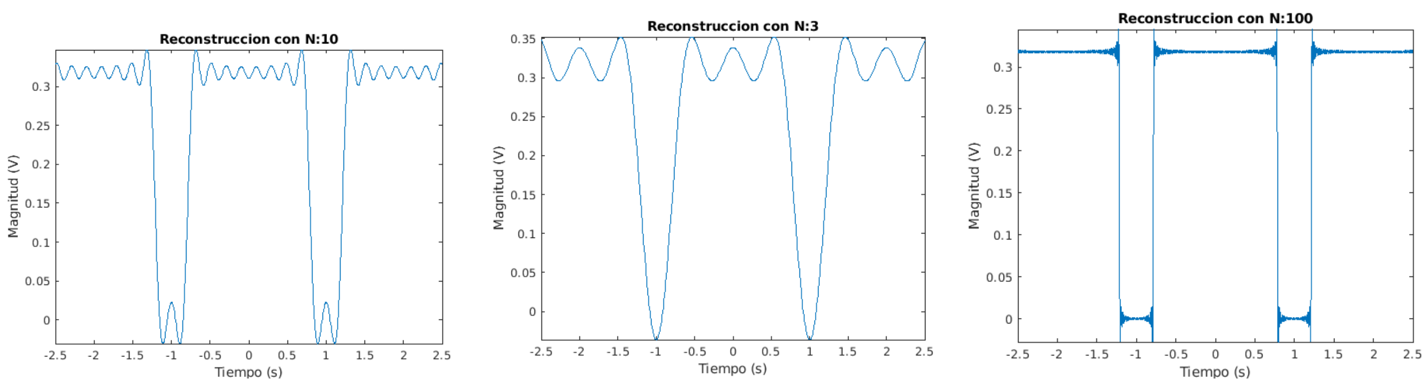


Síntesis

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 t}, \quad t \in [t_0, t_0 + T],$$

```
syms t
n=-3:3;
% %Numero de elementos
% numel()
% %Tamaño de la matriz en n X m
% size()
% %Tamaño más grande de la matriz
% length()

x=sum(a_k.*exp(1i*k*w0*t));


figure
fplot(x,[-2.5 2.5])
xlabel("Tiempo (s)")
ylabel("Magnitud (V)")
title("Reconstruccion con N:"+N)
```





Automatizacion de coeficientes

- Calcular la potencia del intervalo (Parseval)
- Calcular una integral de exponenciales complejas
- Generalizar la fórmula de cálculo
- Aprovechar ese loop para generar la base armónica
- Hacer una función que calule todos los coeficientes que uno pida
- Reconstruir

```
clear
syms t
x=exp(-t);
t0=0;
tf=3;
T0=tf-t0;
w0=2*pi/T0;
```

$$\frac{1}{T_0}\int_{T_0} |x(t)|^2\,dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

```
sympref('FloatingPointOutput',true);
```

true=Decimales; false=Fracciones

```
%Potencia de la señal en el intervalo
P=(1/T0)*int(x^2,t0,tf)
```

P =

$$\frac{1}{6}-\frac{e^{-6}}{6}$$

```
P=double(P)
```

P = 0.1663

$$a_k = \frac{1}{T}\int_{t_0}^{t_0+T} x(t)e^{-jk\Omega_0 t}dt,$$

```
%Integral de un coeficiente
k=3;
ee_test=exp(-1i*k*w0*t);
a_test=(1/T0)*int(x*ee_test,t0,tf)
```

a_test =

$$-\frac{e^{-3}\,(e^3-1)\,i}{3\,(2\,\pi-i)}$$

```
a_test=double(a_test)
```

a_test = 0.0078 - 0.0492i

```
%Generalizar calculo con ciclo for
N_rec=40;
k=-N_rec:N_rec;
[ee,a]=fcc(x,t0,T0,N_rec);
```

```
Coeficiente k=-40
Coeficiente k=-39
Coeficiente k=-38
Coeficiente k=-37
Coeficiente k=-36
Coeficiente k=-35
Coeficiente k=-34
Coeficiente k=-33
Coeficiente k=-32
Coeficiente k=-31
Coeficiente k=-30
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Coeficiente k=-14
Coeficiente k=-13
Coeficiente k=-12
Coeficiente k=-11
Coeficiente k=-10
Coeficiente k=-9
Coeficiente k=-8
Coeficiente k=-7
Coeficiente k=-6
Coeficiente k=-5
Coeficiente k=-4
Coeficiente k=-3
Coeficiente k=-2
Coeficiente k=-1
Coeficiente k=0
Coeficiente k=1
Coeficiente k=2
Coeficiente k=3
Coeficiente k=4
Coeficiente k=5
Coeficiente k=6
Coeficiente k=7
Coeficiente k=8
Coeficiente k=9
Coeficiente k=10
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Coeficiente k=13
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Coeficiente k=15
Coeficiente k=16
Coeficiente k=17
Coeficiente k=18
Coeficiente k=19
Coeficiente k=20
Coeficiente k=21
Coeficiente k=22
Coeficiente k=23
```
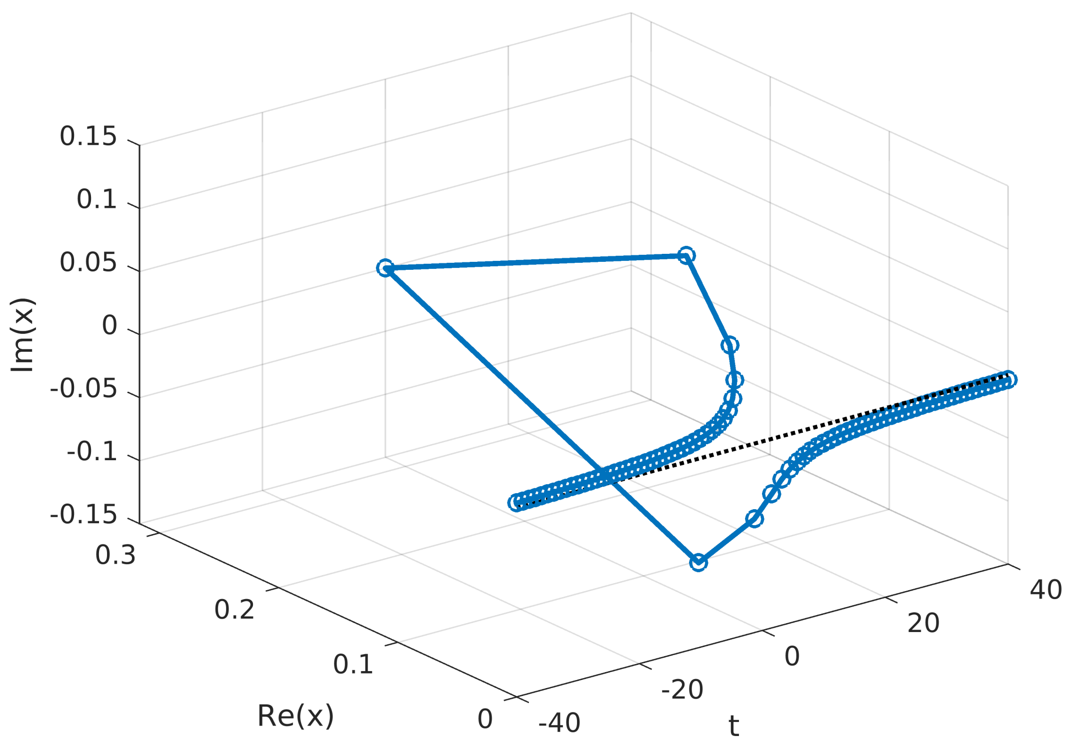

```
Coeficiente k=24
Coeficiente k=25
Coeficiente k=26
Coeficiente k=27
Coeficiente k=28
Coeficiente k=29
Coeficiente k=30
Coeficiente k=31
Coeficiente k=32
Coeficiente k=33
Coeficiente k=34
Coeficiente k=35
Coeficiente k=36
Coeficiente k=37
Coeficiente k=38
Coeficiente k=39
Coeficiente k=40
```

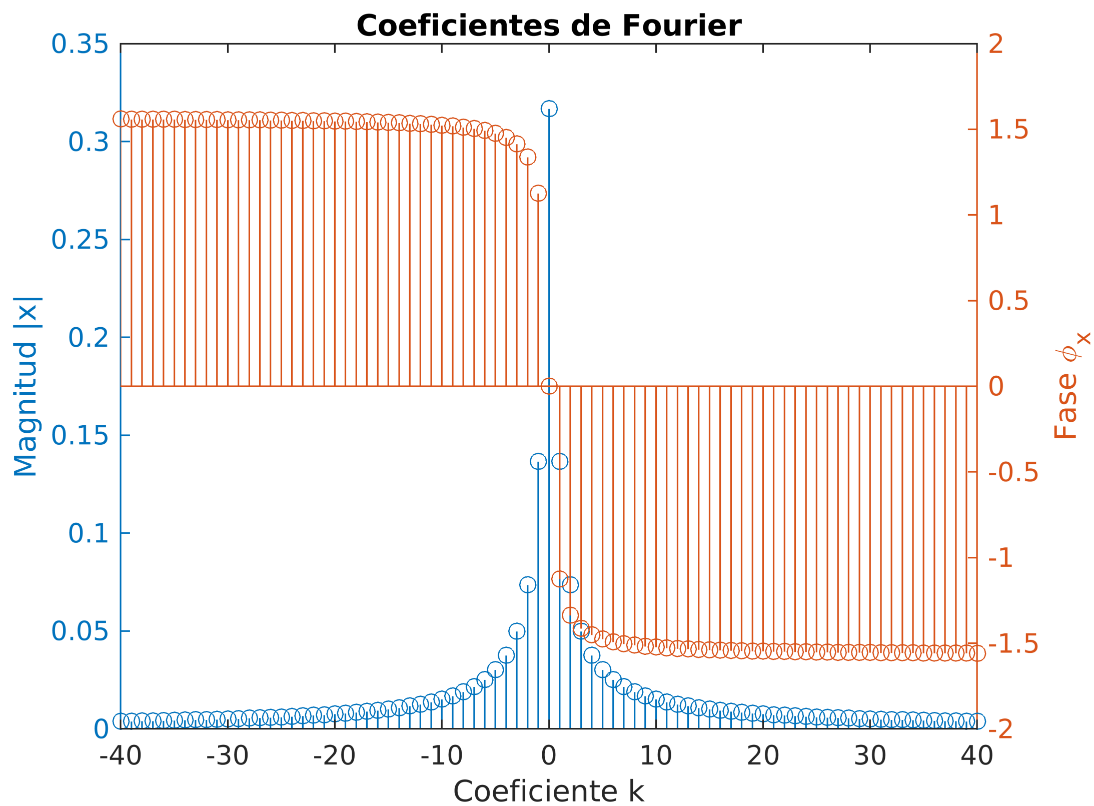
Reconstrucción

```
%Coeficientes
%Complex 3d
figure
complex3Dplot(k,a,[-N_rec N_rec])
```

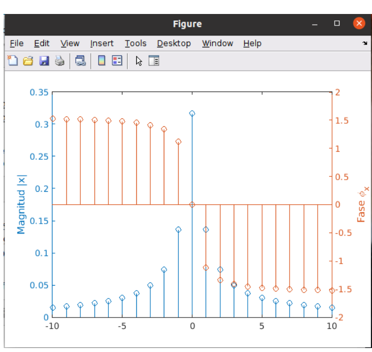


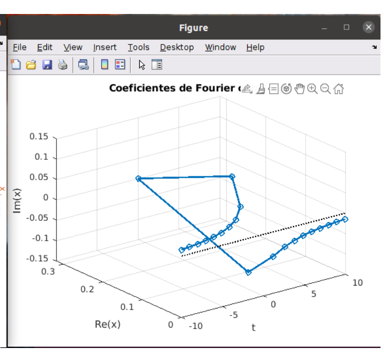

```
title("Coeficientes de Fourier")

%Real
```

```
figure
yyaxis left
stem(k,abs(a))
ylabel("Magnitud |x|")
yyaxis right
stem(k,angle(a))
ylabel("Fase \phi_x")
title("Coeficientes de Fourier")
xlabel("Coeficiente k")
```




```
%Reconstruccion
x_rec=sum(a.*ee);
figure
fplot([x x_rec],[0 5])
legend("Original","Reconstruccion")
```





Parseval

$$\frac{1}{T_0}\int_{T_0}|x(t)|^2\,dt = \sum_{k=-\infty}^{+\infty}|a_k|^2$$

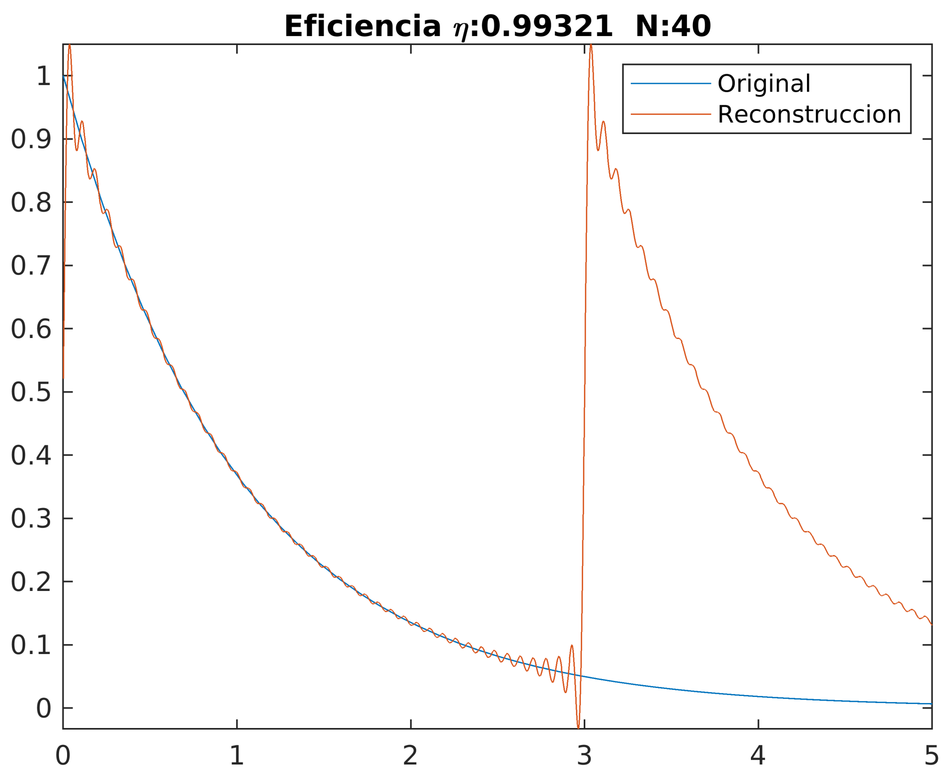
Eficiencia

$$\eta = \sum_{-\infty}^{+\infty}|a_k|^2 \ / \ P$$

```
eta=sum(abs(a).^2)/P
```

```
eta = 0.9932
```

```
title("Eficiencia \eta:"+eta+"  N:"+N_rec)
```



- Utilizar las funciones para calcular trigonométricas y coseno
- Plotear los coeficientes

```
%Tarea moral
```

- Aproximar la función compleja

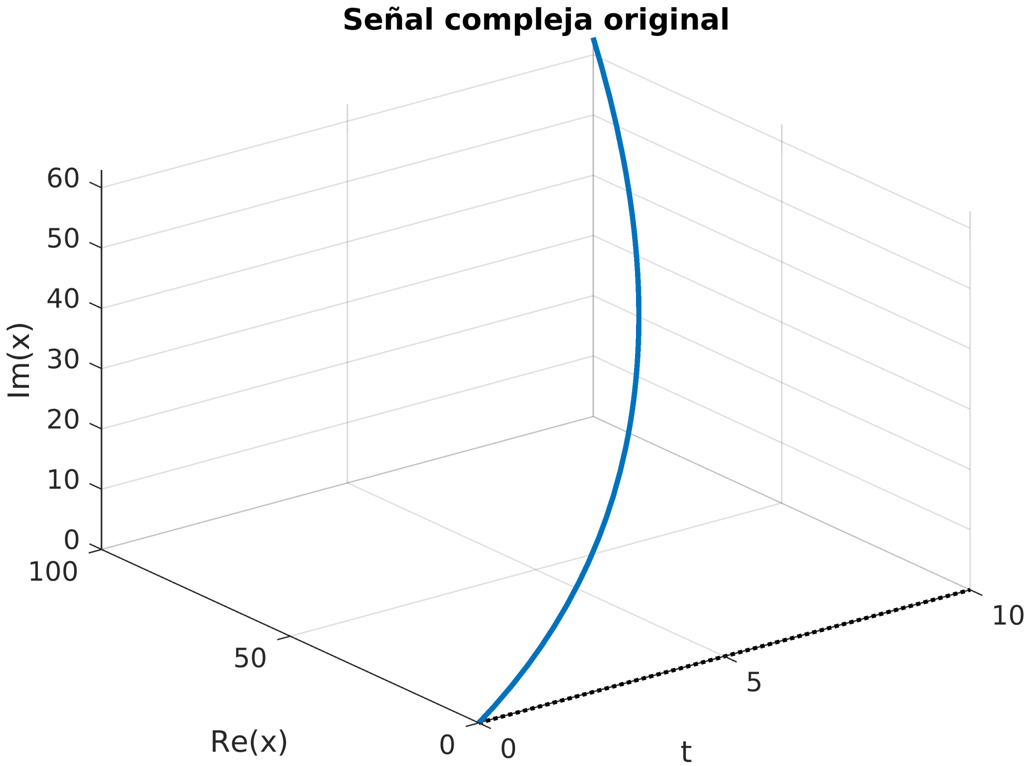
```
clear
syms t
%Visualizar función compleja
x=t^2+1i*2*pi*t
```

x = $t^2 + 2\,\mathrm{i}\,\pi\,t$

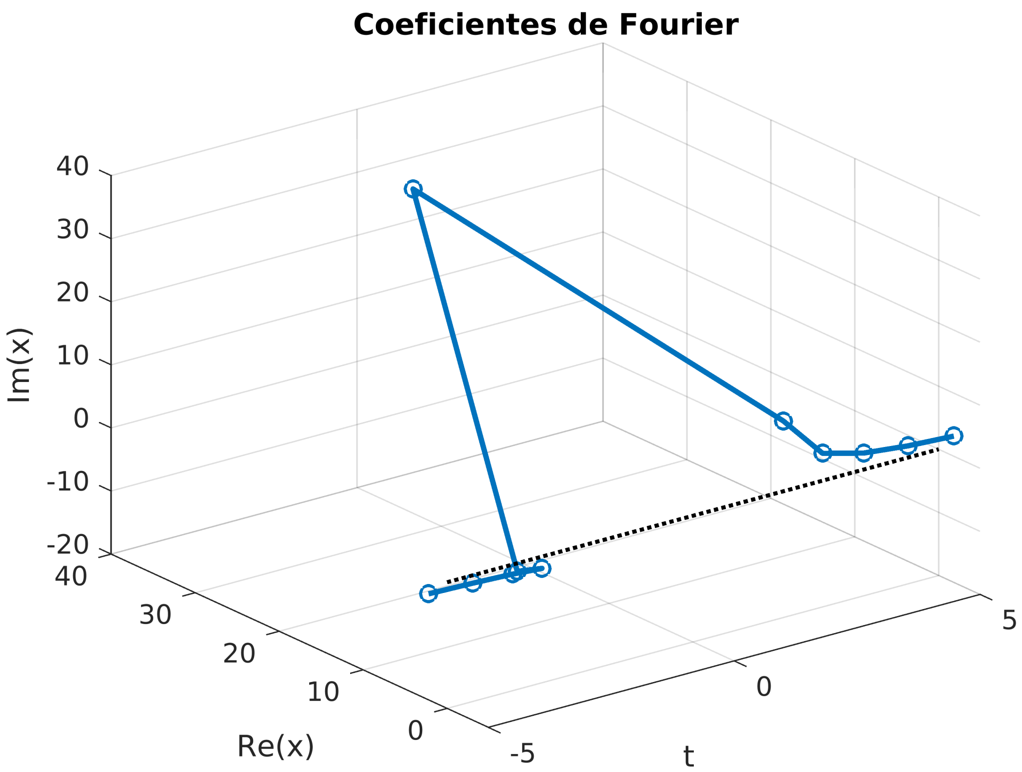
```
t0=0;
t1=10;
T0=t1-t0;
w0=2*pi/T0;
%Calcular coeficientes (5)
N=5;
k=-N:N;
[ee,a]=fcc(x,t0,T0,N);
```

```
Coeficiente k=-5
Coeficiente k=-4
Coeficiente k=-3
Coeficiente k=-2
Coeficiente k=-1
Coeficiente k=0
Coeficiente k=1
Coeficiente k=2
Coeficiente k=3
Coeficiente k=4
Coeficiente k=5
```

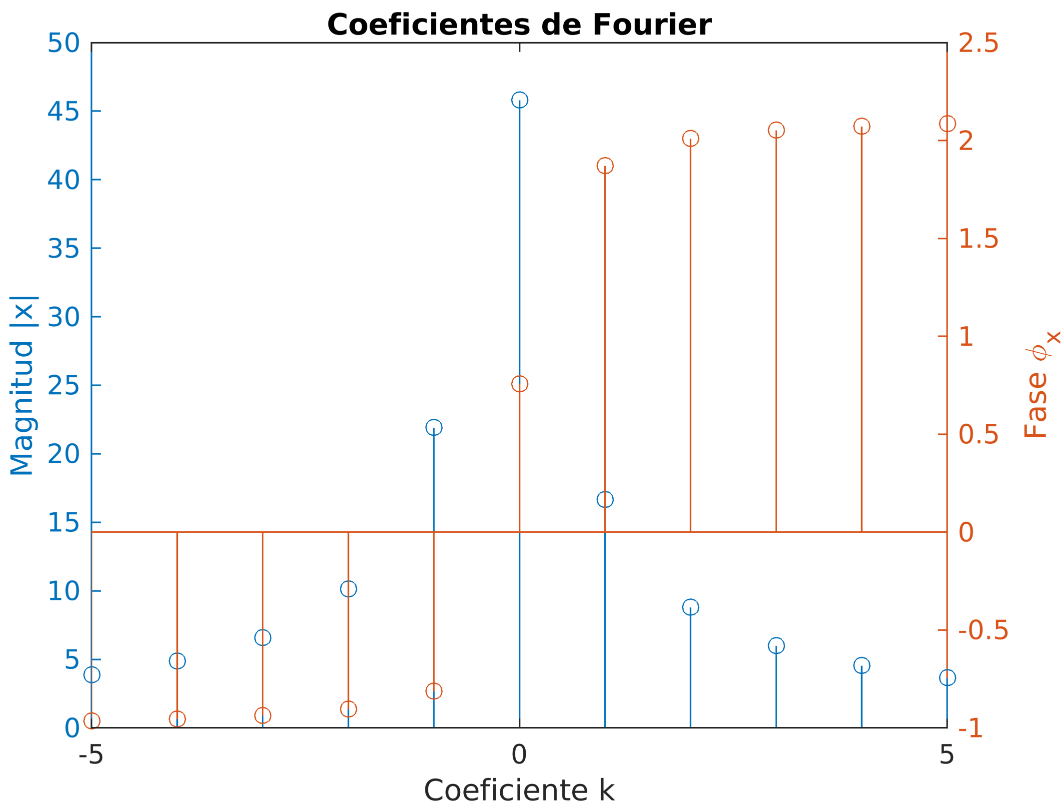
```
%Visualizar función compleja
figure
complex3Dplot(t,x,[t0 t1])
title("Señal compleja original")
```




```
%Visualizar coeficientes 3D y magnitud fase
figure
complex3Dplot(k,a)
title("Coeficientes de Fourier")
```




```
figure
yyaxis left
stem(k,abs(a))
ylabel("Magnitud |x|")
yyaxis right
stem(k,angle(a))
ylabel("Fase \phi_x")
xlabel("Coeficiente k")
title("Coeficientes de Fourier")
```




```
%Reconstrucción con eficiencia
P=(1/T0)*int(abs(x)^2,t0,t1)
```

P =

$$\frac{400\,\pi^2}{3} + 2000$$

```
P=double(P)
```

P = 3.3159e+03

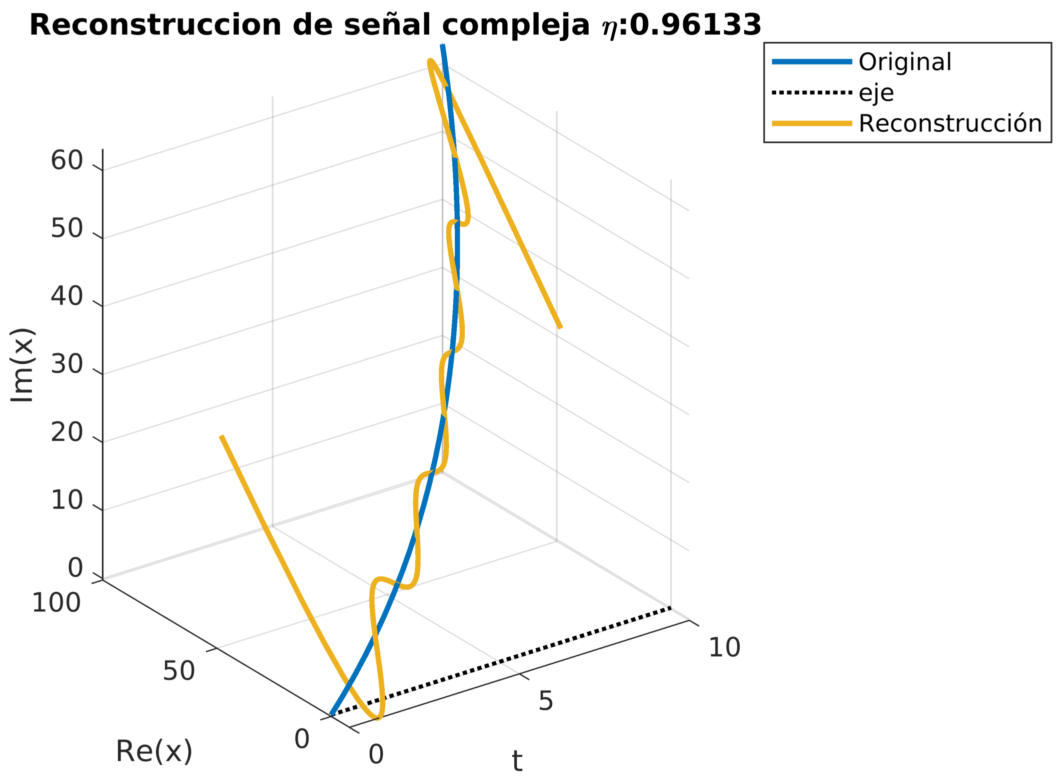
```
P_a=sum(abs(a).^2)
```

P_a = 3.1877e+03

```
eta=P_a/P
```

eta = 0.9613

```
figure
complex3Dplot(t,x,[t0 t1])
hold on
complex3Dplot(t,sum(a.*ee),[t0 t1])
hold off
legend("Original","eje","Reconstrucción")
title("Reconstruccion de señal compleja \eta:"+eta)
```


Reconstruccion de señal compleja η:0.96133
Original
eje
Reconstrucción
60
50
40
30
20
10
0
100
50
0
Im(x)
Re(x)
0
5
10
t