Pricing Total Return Swap

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Abstract

Total return swap (TRS) involves a pricing dilemma: Libor discounting of its premium leg forces upfront payment of future funding premium, and yet replacing Libor with a firm's own funding rate falls into the well-known FVA debate trap. We consider TRS hedge financing from a repo market perspective and apply post-crisis derivatives valuation with collateralization and funding to TRS. We find that the financing cost of the TRS hedge should be reflected on the security leg, and the funding premium can only be discounted in conjunction with the TRS as a whole, depending on margining schemes. An easy to implement, recursive tree model is developed to value TRS with repo-style margining or defaultable underlying, together with any value adjustments.

Keywords: total return swap, TRS pricing dilemma, funding costs, funding valuation adjustment, liability-side pricing, bond forward

1. Introduction

A total return swap (TRS) consists of a security leg and a premium or funding leg. The security leg pays the total return of a reference (underlying) security during a specified payment period and the funding leg pays interest accrued during the period at the rate of a floating interest rate index plus a spread premium. The party pays the total return is the payer or seller as paying price return equates to shorting the security. Accordingly the party pays premium is the receiver or buyer. When the security is defaultable, such as a corporate

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bond, TRS is considered a credit derivative as the payer essentially shorts the bond and buys protection from the receiver.

A standalone TRS contract is sometime called an arbitrage TRS as it allows an investor to synthetically gain exposure to a security, without actually buying and holding or shorting the security. A dealer-bank taking the other side will need to hedge the TRS by taking a position in the security. The hedged TRS no longer bears primary risk and leaves behind only funding operations. Indeed, TRS when combined with an offsetting physical position is a funding TRS, a popular security financing instrument. Especially if the buyer of the TRS will take physical delivery of the security with a return of the TRS notional, it looks every bit like a repurchase agreement (repo), with the payer as the financer and the buyer as the borrower. The TRS premium then reflects the financing cost of the security in the same way a repo spread does. In fact, TRS and repo are considered securities financing in different formats, one synthetic and the other in cash form.

As a form of OTC derivatives, TRS requires fair value or mark-to-market (MTM) accounting. Traditionally, a TRS is valued by discounting its funding leg cash flow at the Libor curve, while the security leg prices exactly at its current market. Repo's spread, however, is not discounted, as it is predominantly treated as a secured loan product in the banking book and goes with accrual accounting. This creates a simple asymmetry: on the first day of the trade (day 1): repo is valued at par, but TRS has a positive net present value (*npv*). Now that TRS are typically cash collateralized under ISDA Credit Support Annex (CSA), the day 1 *npv* has to be posted to the lender, effectively forcing the borrower to pay the future funding premium of the whole duration upfront. A repo of course pays only accrued borrowing interest on a periodic basis, much like a coupon bond. This has been an issue with repo minded buyers.

Derivatives minded traders, on the other hand, want to see their internal funding costs reflected in TRS valuation. A trader on the lending side may get fund from his treasury department and lend out at a higher rate to make a profit for his desk. The reverse is true for the trader on the borrowing side: he will only enter a TRS at a premium lower than his internal funding rate. In this sense, the TRS is used as a funding arbitrage tool. If both traders use their own internal rates to discount the premium leg, then both will show a positive *npv* on day 1, and both will claim a variation margin against each other. But wait, shouldn't a fully collateralized TRS be discounted at the OIS? A TRS pricing dilemma thus arises: neither repo nor derivatives minded traders like Libor discounting, yet replacing Libor discounting is just as problematic.

Prior to the financial crisis, repo's margin rule is adopted as a workaround to override the standard fair value based variation margin. As is with a repo, the margin amount is computed based on the current security price change and accrued interest. Operational risk is induced as it requires manual operations outside of derivatives trading, risk, and collateral management system. There is also legal risk involved as some might have treated it as a collateral dispute resolution mechanism rather than a formal legal construct in a CSA. The impact of such a margin on TRS discounting and valuation, however, has not been fully studied.

The TRS dilemma has also drawn from the post-crisis development in discounting and FVA (funding valuation adjustment), a controversial topic yet to garner consensus between academic and industry (Duffie 2017). Fries and Lichtner (2016), for example, reason that since the TRS payer funds the premium leg at his firm's senior unsecured rate, he would incur an FVA cost. Similarly, the receiver gets an FVA benefit because TRS premium is lower than her internal funding rate. The well-established OIS discounting standard for fully collateralized OTC derivatives is thus ignored. The law of one price is violated, for two parties now see different premium leg valuations. The authors suggest to agree to the mid of parties' 'exogenously' determined collateral calculations. Some have proposed to use Treasury repo rates rather than a firm's own funding rate so that there is one price and no collateral dispute, at least on the surface. Unfortunately, there is no existing consensus as how to correctly discount the premium leg, so the dilemma continues.

This paper's main contribution is to derive TRS valuation equations or formulae under different collateralization with consideration of TRS hedging financing constraints. Our main findings are that cost of financing the TRS hedge in the repo market is reflected on the security leg, not the premium leg, and that the correct rate to discount the TRS, including the premium leg, is determined by its collateralization and margining schemes. With full CSA, TRS shall remain to be discounted at the cash deposit rate, e.g., the OIS curve for US dollar, and that there is counterparty risk valuation adjustments, CVA or FVA. We show that for TRS there is another form of funding value adjustment (fva) that corresponds to funding costs of TRS hedges, rather than the TRS itself as a derivative. For practical TRS pricing, TRS par premium can be designed to offset fva, which is governed by the underlying security repo spread. At-the-issue (ATI) swaps now can have zero *npv*, and implied repo spread can be inferred from TRS premium by setting the TRS *npv* to zero.

Additionally, repo-style margin rule is incorporated into TRS valuation. An easy to implement, recursive binomial or trinomial tree model is developed. CVA and FVA of uncollateralized or partially collateralized TRS can be computed along with TRS valuation on the tree. New result for TRS with defaultable underlying asseta is also derived.

The rest of the paper proceeds as follows. Section 2 reviews the risk neutral TRS valuation and derives a new valuation model incorporating the funding cost of the hedge, i.e., the underlying security. Section 3 develops a binomial/trinomial tree model to compute the npv of a TRS or solve for the fair TRS premium. Section 4 presents valuation of uncollateralized TRS and related valuation adjustments in line with the liability-side pricing principle. Section 5 considers TRS referencing defaultable bond and haircut's role in TRS valuation. Computational results are shown in Section 6 and Section 7 concludes.

2. Funding Cost in Total Return Swap

Consider a bank B entering a total return swap agreement with a customer C where B pays the periodic total return of a reference security to C, in exchange for receiving a stream of cash flow from C. B is the (total return) payer and C is the receiver[2]. Or in the context of securities financing, B is the lender and C is the borrower of cash.

Let S_t denote the reference security's market price, T swap maturity, t_i, $i=1,2, \ldots, n$, the payment dates when the return of the underlying and the funding interest are to be paid simultaneously. t_0 is the previous payment date. r_f is the floating rate applied to a nominal (funding) amount of M_0, normally a floating rate index such as 3 month Libor plus a fixed spread s_f.

2. 1 Standard pre-crisis valuation

[2] Formally, the buyer is documented in TRS confirm as a floating rate payer and the seller as the fixed amount (or rate) payer.

To begin, let's consider a one-period TRS, i.e., $n=1$. It trades at t_0, matures at T, and is valued at t, $t_0 \leq t < T$. From the payer's perspective, the payoff of the TRS is $M_0 * r_f * (T - t_0) - (S_T - S_0)$, assuming that the underlying does not pay dividend. This payoff is valid for physical settled TRS as well, where the underlying security is delivered against payment. In fact, it can be rewritten as $[M_0*(1+r_f *(T- t_0))-S_T] - [M_0-S_0]$. The first square bracket term is the value of the physical settlement (exchange of principal and accrued interest for the underlying) at maturity, while the second bracket is the value of initial physical settlement, zero if $M_0=S_0$, as is often the case.

According to the risk neutral pricing theory, the fair value of the TRS is the risk-free discounted payoff in the risk neutral world, or Q-measure,

$$V(t) = E_t^Q[e^{-\int_t^T rdu}(M_0 r_f(T - t_0) - (S_T - S_0))] \tag{1}$$

where r is the risk free rate, $D(t,T) = E_t^Q[e^{-\int_t^T rdu}]$ is the discount factor. Assuming $M_0=S_0$, and r_f is preset, equation (1) works out simply,

$$V(t) = S_0\left(1 + r_f(T - t_0)\right)D(t,T) - S_t \tag{2}$$

Equation (2) carries the usual interpretation of decomposing TRS valuation into the funding leg present value (pv) and the asset leg pv which is simply the current market price of the asset. Note that $V(t)$ given above is dirty in that it contains the accrued interest of $M_0 r_f(t - t_0)$.

A special case is at-the-issue TRS when the initial Libor rate l and the discount factor are related by $\left(1 + l(T - t_0)\right)D(t_0,T) = 1$. If $r_f=l+s_f$, it follows

$$V_0 = s_f(T - t_0)S_0 D(t_0,T)$$

which is exactly the present value of the funding premium s_f. Under the CSA, V_0 amount of cash needs to be posted to the payer, leading to the dilemma claim by repo traders.

If the stock pays dividend, the one-period payoff function has to include it as the swap pays total return including any coupon or dividend. Let q denote the underlying's continuous dividend yield. $e^{\int_t^T qdu}$ is the share growth factor over time period from t to T, and equation (1) is revised as follows,

$$V(t) = E_t^Q[e^{-\int_t^T rdu}(M_0 r_f(T - t_0) - \left(e^{\int_t^T qdu}S_T - S_0\right))] \tag{3}$$

Since in the risk neutral world, stock's expected rate of return is r-q, (3) leads to

$$V(t) = (M_0 r_f (T - t_0) + S_0) D(t, T) - S_t \quad (4)$$

Note that the share growth factor from t_0 to T is $e^{\int_{t_0}^{T} q du}$, but the realized part (from t_0 up to t) has to be excluded, because stock dividend, once paid on the stock, is paid out of the TRS, so that past dividend wouldn't need to be accounted for in the terminal payoff.

The risk neutral return of r-q assumes the risk free stock financing, which is neither realistic nor necessary (Lou, 2017). Repo financing rate is the correct choice, as shown below.

2. 2 TRS hedge financing cost

Suppose that the bilateral TRS is transacted as a credit derivative under an ISDA Master Agreement with a version of Credit Support Annex (CSA) that stipulates variation margin. When party B has a positive exposure to party C ($V_t > 0$, the derivative is a receivable to B), C posts L_t amount of cash collateral to B, $L_t \leq V_t$. Otherwise, party B posts to C. To simplify, the CSA has zero threshold and zero minimum transfer amount. The amount of cash collateral does not necessarily equal to the exposure amount, creating an unsecured amount W_t, $W_t = V_t - L_t$. Following Lou (2015), W_t is financed by the liability-side, here party C when B has positive exposure. In general, $W_t = W_t^+ - W_t^-$, W_t^+ is the cash amount deposited or posted by party C to B that pays C's senior unsecured bond interest rate $r_c(t)$, and W_t^- is by B to C earning B's rate $r_b(t)$. Or collectively, $r_w(t) = r_b I(W(t) \leq 0) + r_c I(W(t) > 0)$, where $I(\,)$ is the indicator function.

The bank dynamically hedges the TRS by trading in the underlying stock with price S_t and finances the stock purchases in the repo market. The wealth equation of the TRS economy from party B's perspective is

$$\pi_t = M_t + (1 - \Gamma_t)(V_t - L_t - W_t - N_t + \Delta_t S_t - L_t^s) \quad (5)$$

where π_t denotes the net wealth of the segregated TRS economy, $1-\Gamma_t$ is B and C's joint survival indicator at time t, L_t^s a stock repo account with zero haircut on Δ_t shares of stock, $L_t^s = \Delta_t S_t$, at the repo rate of $r_s(t)$, $r_s(t) \geq r(t)$. M_t is a bank account that earns the cash deposit (or risk free) rate $r(t)$. N_t is B's debt account that issues short term rolled debt at par rate $r_N(t)$, $r_N(t) \geq r(t)$.

Pre-default self-financing activities include dynamic hedging, stock financing, and collateral rebalancing. Briefly, during an infinitesimal time period dt, the cash collateralized account and unsecured account will have additional incoming amounts dL_t and dW_t posted while paying out accrued interest amounts of rL_tdt and r_wW_tdt; buying $d\Delta_t$ more shares at $t+dt$ to rehedge pays out $d\Delta_t(S_t + dS_t)$, and holding Δ_t shares earns dividend $\Delta_t S_t q dt$; as the stock price change, the repo account delivers more cash dL_t^s, while paying out repo interest of $r_s L_t^s dt$; debt account interest $r_N N_t dt$ is cleared, new issuance dN_t is raised; finally the net cash inflow is deposited into the bank account M_t while earning interest at the cash deposit rate on its balance. Collecting these terms together leads to the self-financing equation,

$$dM_t = rM_t dt + (1 - \Gamma_t)[dW_t - r_w W_t dt + dN_t - r_N N_t dt + dL_t - rL_t dt$$
$$+\Delta_t S_t q dt - d\Delta_t(S_t + dS_t) + dL_t^s - r_s L_t^s dt] \quad (6)$$

At $t=0$, the wealth reduces to $\pi_0 = M_0 + V_0 - L_0 - W_0 - N_0$. Since $V_t=W_t+L_t$ for all t, we set $M_0= N_0=\pi_0=0$. For $t>0$, keeping the pre-default wealth to 0 leads to $N_t=0$ and $M_t=0$, trivially.

Now assume that party C defaults at time τ, $\tau<T$. Unwinding the stock financing gets the stock back and sells at the market to pay back cash amount of $\Delta_\tau S_\tau$. The TRS fair value V_τ is fully covered by the combination of the liability-side deposit W_τ and the cash collateral L_τ, There is no jump in cash flow at default and equation (6) is indeed the financing equation covering both pre-default and post-default. This is also true when party B defaults first.

Differentiate equation (5) and plug in equation (6) to obtain,

$$dV_t - r_w W_t dt - rL_t dt + \Delta_t dS_t - (r_s - q)\Delta_t S_t dt = 0 \quad (7)$$

Apply Ito's lemma, assume delta hedge ($\Delta_t = -\frac{\partial V(t,S)}{\partial S}$) under the usual geometric Brownian motion stock price ($dS=\mu S dt+\sigma S dZ$), and set dt term to zero, we arrive at the following partial differential equation (PDE),

$$\frac{\partial V}{\partial t} + (r_s - q)S\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} - rL - r_w(V - L) = 0 \quad (8)$$

The ratio of cash collateral to the fair value μ_t, i.e., $\mu_t = {}^{L_t}/_{V_t}$, can be reasonably assumed an adapted process, allowing us to rewrite equation (8),

$$\frac{\partial V}{\partial t} + (r_s - q)S\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 y}{\partial S^2} - (\mu r + (1-\mu)r_w)V = 0 \tag{9}$$

Applying the Feynman-Kac formula results in the familiar valuation equation,

$$V(t) = E_t^Q[e^{-\int_t^T r_e du} H(T)], \tag{10}$$

$$r_e(t) = \mu r + (1-\mu)[r_b(t)I(W(t) \leq 0) + r_c(t)I(W(t) > 0)] \tag{11}$$

The above discount rate formulation is similar to Lou (2017), which also covers non-cash collateral. For ease of exposition, we assume all short rates and spreads are assumed deterministic, although equations (10&11) remain valid as a form of backward stochastic differential equation (BSDE) with properly behaved stochastic rates.

2.3. Fully collateralized TRS

For the one period TRS with continuous dividend yield q, the payoff function is given by,

$$H(T) = M_0 r_f (T - t_0) - \left(e^{\int_t^T q du} S_T - S_0\right) \tag{12}$$

If we consider full cash collateral, $L_t=V_t$, thus $W_t=0$, when $r_e=r$, equation (10) becomes (3). Also as shown in equation (9), under the Q-measure, the stock rate of return is r_s-q, or r-q', $q'=(q-r_s+r)$, so that $E_t^Q\left[e^{-\int_t^T r du} S_T\right] = S_t e^{-\int_t^T q' du}$. This brings equations (10 & 12) to

$$V(t) = (M_0 r_f (T - t_0) + S_0)D(t,T) \quad - S_t e^{\int_t^T (r_s - r) du} \tag{13}$$

In the above, it's clear that the funding leg premium and the asset leg as well are discounted at the cash rate or the OIS rate. This is consistent with the understanding that for fully cash collateralized OTC derivatives, the discount rate is the OIS for US dollars and EONIA (Euro over-night index average) for Euros (Piterbarg, 2010).

Obviously, (13) reduces to (4), if $r_s=r$, i.e., when stock could be repo financed at the risk free rate. Otherwise, it shows a value reduction of the amount,

$$fva = (e^{\int_t^T (r_s - r) du} - 1)S_t \tag{14}$$

Because $r_s \geq r$, we must have $fva \geq 0$ and it is a pure cost. fva reflects the present value of the cost of hedge financing for the remaining term of the TRS, as $r_s - r$ is the excess funding spread paid to repo financers. It is a funding valuation adjustment due to cost of financing hedges, distinguished from the capitalized "FVA", an acronym reserved for the cost of financing uncollateralized swaps and derivatives.

It should be noted that when the stock financing rate is built into the risk neutral dynamics, it carries implication of being a market rate available to all market participants. If a firm only sees a funding cost particular to itself, or a private funding cost, it is best to be incorporated into pre-trade pricing instead of post trade valuation.

Strictly speaking, a total rate of return swap (TRORS) is different from a total return swap, in that it refers to the rate of return in a period and applies to any fixed notional amount. The payoff function of a one-period TRORS given a notional of M_0 is $M_0\left(\frac{S_T}{S_0}-1\right)$, which is $(S_T-S_0)\frac{M_0}{S_0}$, equivalent to TRS on $\frac{M_0}{S_0}$ number of shares. With multiple periods, the number of shares will change or reset after each payment of return, as the notional remains the same. Sometimes, a "TRS+Asset" package can be priced as a single product. The above derivation can be trivially modified as needed.

Equation (13) displays a linear relationship between TRS fair value and the underlying stock price S_t. For this reason, equity swap is considered a linear product, similar to equity forward and futures. The equity forward price can be trivially obtained, $F=S_te^{\int_t^T(r_s-q)du}$, recognizing its payoff function is $H(T)=S_T-F$. This agrees with usual no-arbitrage argument with a layout of a forward hedged by buying a share with borrowed money. *fva* in Equation (14) can now be understood as a result of a disparity in forward financing rate and TRS discount rate: financing at r_s, yet discounting at r.

The derivation above indeed seems heavy handed given its linear nature of the product, as the same formula can be derived by simply discounting the forward price at the riskfree rate. Riskfree discounting, however, shall not be taken for granted, see discussions in Lou (2017). Moreover, the linearity is only guaranteed by full collateralization: it may not persist, once other collateralization schemes such as repo-style margin are kicked in.

2.4. Repo style margin in TRS

In a standard repo, the daily margin mechanism ensures that the repo principal and the accrued but unpaid interest is fully collateralized. Specifically, let h be a constant haircut on the collateral security with market value S_t, M_t repo principal, A_t the accrued interest, then the bilateral margin account balance L_t is calculated as $L_t=M_t + A_t - S_t(1-h)$[3]. If L is positive, the borrower has to post the margin. Otherwise, the lender posts to the borrower.

As discussed earlier, traditional TRS valuation has a positive *npv* by discounting the funding premium of the full TRS term at the Libor curve. A cash collateral call in the same amount follows and has to be posted by the borrower on the first day of the trade. Since the collateral is not segregated, it amounts to an upfront payment of financing charges. This is different from repos which are typically accounted as banking book positions and funding premium is taken in only periodically on an accrual basis, exactly like bond coupons. Even before the financial crisis, some firms had chosen not to perform premium leg discounting. Such a valuation twist effectively enforces the repo margin rule and voids the upfront payment, and the usual CSA variation margin as well. At least for securities financing business purposes, the TRS as the synthetic form and repo as the cash form have the same economics and similar operational aspects.

A more rigorous approach would be to carve out a CSA subset where the variation margin is explicitly calculated following the repo-margin rule and let the unscathed valuation model handle the rest. The collateral amount then consists of the price return amount $S_0 - S_t$ and the accrued interest amount $r_F M_0(t - t_0)$,

$$L_t = S_0 - S_t + r_F M_0(t - t_0) \tag{15}$$

PDE (8) still applies, with L_t specified above and residual exposure, $W_t = V_t - L_t$. It is possible that L_t in (15) is greater than V_t depending on the level of TRS funding premium and the repo rate. Parties therefore could actually overcollateralize, i.e., $L_t>V_t$ and W_t is negative. This is not an issue as the overcollateralized portion becomes an exposure to the other party and equation (9) still stands. For a repo in a trading book, (15) is its exact margin rule, because repos are governed by GMRA instead of ISDA. The modeling framework would also apply to repo MTM.

Assuming deterministic rates and spreads, and under a special case of $r_b=r_c$ (thus $r_w=r_b$), PDE (9) can be solved directly for one period TRS that originally starts from t_0,

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[3] For a fixed income instrument, one could argue that the haircut, as a measure of mitigating collateral market price volatility, should apply to the clean price rather than the dirty price, so that $L_t=M_t+A_t-S_t(1-h)-I_t$, where I_t is debt interest accrued and S_t is understood to be a clean price. Repo legal documentation standard, e.g., GMRA (global master repurchase agreement), applies haircut to market value, which means dirty price.

with funding notional M_0, initial security price S_0, and maturity T. In fact, the Feynman-Kac formula shows the fair value of the TRS at time t,

$$V(t) = E_t^Q[(M_0 r_f(T-t_0) + S_0 - S_T)e^{-\int_t^T r_b du} + \int_t^T e^{-\int_t^u r_b ds}(r_b - r)(S_0 - S_u + r_F M_0(u-t_0))du] \quad (16)$$

which can be worked out explicitly to arrive at,

$$V(t) = -S\left(\frac{r_b - r - (r_s - r)D_{T,r_b - r_s}}{r_b - r_s}\right) + \left(\mathrm{S}_0 - \mathrm{t}_0\mathrm{M}_0\mathrm{r}_f\right)\left(1 - \mathrm{r}\frac{1-D_{T,r_b}}{r_b}\right) + \mathrm{M}_0\mathrm{r}_f[TD_{T,r_b}\frac{r}{r_b} + (t + \frac{1-D_{T,r_b}}{r_b})\frac{r_b - r}{r_b}] \quad (17)$$

where $D_{T,x} = e^{-\int_t^T x du}$ is a discount factor with x as the placeholder instantaneous rate. Equation (17) is valid if dividend yield q is non-zero, and it can be extended to handle discrete future dividends. Also, this analytic solution can be readily extended to a multi-period TRS. Generally $r_b \neq r_c$, the solution is no longer analytically tractable and is left to the following tree model.

3. Trinomial Tree Model

In the fully collateralized case, a TRS retains a linear pricing with respect to the current asset price and has analytic pricing formula (13) available. For other types of collateralization, such as repo-style margin or no collateralization, analytic solutions are not generally guaranteed and numerical solutions have to be sought.

Equations (10&11) synthesize derivative collateralization effect into one derivative financing rate $r_e(t)$, allowing us to bypass valuation adjustments to compute the fair value directly. $r_e(t)$ switches depending on the sign of the unsecured amount of the fair value, so the fair value formula in (10) is generally recursive. This type of coupling between the discount rate and the fair value reflects the nonlinear nature of pricing with counterparty risk[4] and becomes the key to numerical solutions.

[4] Unlike Burgard and Kjaer (2011), where the coupling binds the unknown fair value and various valuation adjustments through counterparty exposure amounts, in our construct, the only nonlinearity appears in the effective discount rate.

For TRS on a portfolio of underlying assets or a portfolio of TRS with the same counterparty (thus a netting set), Monte Carlo simulation with a regression procedure on the switching rate could be developed, see for example, Lou (2016a). For a standalone TRS trade, or a group of TRS on the same underlying, i.e., a hedging set, the finite difference method (Lou 2015) can be used to solve for the fair value of the TRS. Note that in the explicit finite difference scheme, the switching rate is naturally decoupled. In the binomial tree model for uncollateralized options (Lou, 2017), for example, the sign of W_t can be decided at a time step prior to rolling back in time to the next time step, conveniently decoupling the discount rate switch and the fair value.

To extend to TRS valuation, a new challenge is that the price return at τ_i depends on security price on the previous reset date τ_{i-1}. Let's first consider a single period TRS, where the terminal payoff is already preset, so the binomial tree procedure proposed in Lou (2017) can be used with little modification. A slight extension to trinomial tree is shown in Figure 1. Under the trinomial tree method, the stock price has three possible moves: an up, down, and middle move, each with probabilities p_u, p_d, and p_m respectively defined below,

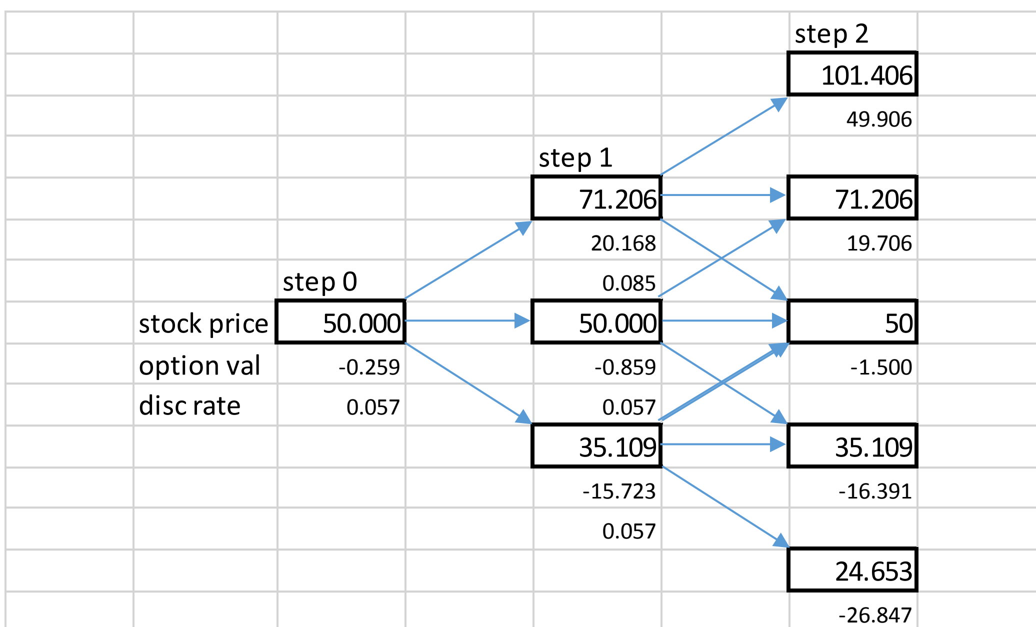

Figure 1. Illustrative diagram showing a one period trinomial tree solving an uncollateralized TRS, $S=50$, $\sigma=50\%$, $dt=0.25$, $T=0.5$, $r=0.05$, $r_f=0.06$, $M_0=S_0=50$.

$$p_u = \left(\frac{e^{(r_s-q)\Delta t/2} - e^{-\sigma\sqrt{\Delta t/2}}}{e^{\sigma\sqrt{\Delta t/2}} - e^{-\sigma\sqrt{\Delta t/2}}}\right)^2,$$

$$p_d = \left(\frac{e^{\sigma\sqrt{\Delta t/2}} - e^{(r_s-q)\Delta t/2}}{e^{\sigma\sqrt{\Delta t/2}} - e^{-\sigma\sqrt{\Delta t/2}}}\right)^2$$

$$p_m = 1 - p_u - p_d$$

The proportional moves u, d, or m to the current node's price are defined to build a recombining tree, $u = e^{\sigma\sqrt{2\Delta t}}, d = \frac{1}{u}, m = 1$. Note that the tree is built with the repo rate r_s and the discount rate to use is given by (11).

A regular TRS of multiple pay periods involves many price fixings and floating rate resets. Fix a node at t_i, there are only countable many paths that lead to the node, a combinatorial exercise easily complicated by asymmetric up and down probabilities. Here we propose a recursive pricing function to facilitate easy implementation.

- Start from the root node at current valuation date t_0 and S_0, with previous reset time τ_0 and price $S\tau_0$. For ATI trades, $\tau_0 = t_0$.
- If there is no more reset, compute TRS payoff at T and apply the one period binomial or trinomial tree procedure with switching rate to roll back to t_0.
- On next reset time τ_i, for each node j, fetch its price fixing $S_{i,j}$,
  - Call the recursive pricing function to compute the fair value at the node,
  - Add the price return from τ_{i-1} to τ_i, knowing previous reset level and current level of $S_{i,j}$.
  - Apply the binomial or trinomial procedure with switching rate to roll back step by step until reaching to the previous reset time τ_{i-1}.
    - at each time step, roll back to get fair value,
    - compute cash collateral amount and unsecured amount,
    - deduct funding costs for hedges and unsecured exposure.

This procedure is easy to implement and flexible enough to handle both the fixed position TRS (or cusip TRS) and the fixed loan TRS. The fixed loan TRS can be on a cusip, although generally, it has a pool of similar securities and allows substitutions. When security market price fixings go up on a reset day, the TRS borrower takes out a portion of the securities in the TRS facility. When fixings go down, more securities will be delivered into the pool in lieu of return payments. This underlying security position change can be captured at the reset step.

For a fixed position TRS referencing a security throughout its entire duration, when the security's price goes up, the lender needs to pay the return to the borrower. This amount of payment, however, is treated as additional lending and will start to earn funding premium in the next interest payment period. When the cusip price goes down, the negative return entails a payment from the borrower to the lender, which is treated as a return of principal. So for the next period, financing interest is calculated on a reduced principal amount. In between the payments, the CSA margin account kicks in to mitigate the exposure left behind by price moves since last payment.

TRS transaction features like these can be accommodated at ease. Repo-style margin rule can be plugged in straightforwardly, for instance. If the underlying is a debt instrument, coupon payments can be made at a time step nearest to the actual payment date. For defaultable debts where substitution is allowed upon an existing security default, there is no economic difference and the procedure remains valid. If an underlying default triggers an early termination of the TRS, then default time modeling must be included. This situation is separately examined in a later section.

4. Uncollateralized TRS and Valuation Adjustments

The modeling framework presented in section 2 applies to bilateral TRS with imperfect collateralization, including no-collateralization and partial collateralization. The former relates to dealer-banks' trades with non-covered customers, and the latter applies to product features such as repo-style margin rule. Let V_t^* denote the counterparty riskfree or OIS discounted TRS fair value, and $U_t=V_t^*-V_t$ the total counterparty risk valuation adjustment. Then U_t satisfies the following PDE, with $U_T=0$,

$$\frac{\partial U}{\partial t}+(r_s-q)S\frac{\partial U}{\partial S}+\frac{1}{2}\sigma^2S^2\frac{\partial^2 U}{\partial S^2}-r_wU-(r_w-r)(V^*-L)=0 \tag{18}$$

with the exact solution given by,

$$U(t)=E_t^Q\left[\int_t^T(r_w-r)(V^*(s)-L(s))e^{-\int_t^s r_w du}ds\ \right] \tag{19}$$

In this formula, we see that r_w is both the discount rate (appearing in the exponent) and the financing rate as r_w-r is a financing spread applied to the counterparty risk-free price V_t^* minus the cash collateral amount. Obviously, for OIS discounted TRS, $r_w=r$, U_t is zero. To be clear, in the full CSA case, there is no valuation adjustment, neither CVA nor FVA, regardless what the funding premium is. Uncollateralized TRS has the maximum counterparty exposure as $L_t=0$. Because the repo-style margin leaves out a small packet of unsecured exposure, counterparty valuation adjustments could apply, but will be much less than uncollateralized trades.

This CRA represents total valuation adjustment made to take into account of unsecured counterparty exposure, in both credit and funding aspects. Following Lou (2015) where CRA for a fully uncollateralized OTC derivatives trade is decomposed into credit valuation adjustment (cva[5]), debt valuation adjustment (dva), credit funding adjustment (cfa), and debt funding adjustment (dfa), equation (19) can be subsequently split into four components, $U_t = cva_t - dva_t + cfa_t - dfa_t$,

$$cva(t) = E_t^Q[\int_t^T s_c I\{V(s) > 0\}(V^*(s) - L(s))e^{-\int_t^s r_w du} ds \quad], \qquad (20.a)$$

$$dva(t) = -E_t^Q[\int_t^T s_b I\{V(s) \le 0\}(V^*(s) - L(s))e^{-\int_t^s r_w du} ds \quad], \qquad (20.b)$$

$$cfa(t) = E_t^Q[\int_t^T \mu_c I\{V(s) > 0\}(V^*(s) - L(s))e^{-\int_t^s r_w du} ds \quad], \qquad (20.c)$$

$$dfa(t) = -E_t^Q[\int_t^T \mu_b I\{V(s) \le 0\}(V^*(s) - L(s))e^{-\int_t^s r_w du} ds \quad]. \qquad (20.d)$$

These formulae are based on the decomposition of the credit spread. Party B's credit spread with respect to the OIS rate, for example, is decomposed into a credit component s_b that associates to B's CDS premium, and a (funding) basis component μ_b that reflects B's bond and CDS basis. By definition, $r_b - r = s_b + \mu_b$ and $r_c - r = s_c + \mu_c$, this scheme avoids any overlapping among these xvas.

Such a decomposition conforms to the law of one price, as party B's *cva* (*cfa*) is exactly C's *dva* (*dfa*), so there will be no disagreement in fair value and collateral amount. In particular, *fva* refers to the liability-side's funding basis, in parallel with *cva* which refers to its default risk premium. Measuring funding cost's impact on fair value from the

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[5] We use lower case acronyms as their definitions are different from conventional CVA and FVA. The key is only CRA is precisely defined.

liability-side's perspective also conforms to the accounting standard (IFRS 13) as it creates a scenario where market participants in a principal market compete for the issuer's product, albeit limited in certain market sectors, to eventually reach an equilibrium which becomes the exit price.

A perennial confusion about traditional CVA and FVA (Hull and White 2012, Lou, 2017) calculation has been the choice of the effective discount rate, varying between the riskfree rate, the riskfree rate plus one party's hazard rate or both parties' hazard rates, or hypothetical zero recovery zero coupon bond yields. This leads to a prominent example of not being able to reproduce a corporate bond's market price when CVA and FVA adjustments are added back to bond's riskfree price, see Morini and Prampolini (2011)[6]. In equations (20), the discount rate to use is the senior unsecured cash bond interest rate, although its precise application depends on the local fair value. If we treat the bond as a derivative and plug it in, then $V_t>0$, $r_w=r_c$, it is straightforward to verify that a bond's price is fully recovered by adding *cva* and *cfa* back to V_t^*.

The binomial or trinomial tree procedure outlined in section 3 can be easily enhanced to compute V_t* and xva's alongside with V_t, since the discount rate used for them is the same as the rate for V_t, which is already specified at each node. A rollback procedure for xva can then be implemented similarly (Lou, 2017). The rule of TRS discounting also follows Lou (2017), that is, fully collateralized TRS gets on OIS discounting, uncollateralized TRS discounts on the liability-side's senior unsecured rate, and partial collateralization settles on a rate mixed of OIS and senior unsecured rates.

5. TRS on Defaultable Bond

TRS can reference an equity underlying or a debt underlying security. With the former, it is also named equity swaps, either on price return only or on total return that includes stock dividend. Equity based TRS usually don't explicitly consider equity issuer default, but debt based TRS is subject to debt default. A cusip debt TRS refers specifically

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[6] Contributing to this has been the traditional riskless close-out of ISDA 1992, which has been superseded by the risky close-out rule of ISDA 2000 or 2009 Close-out Protocol and adopted by all major dealer-banks, but is still alive due to implementation delays.

to one bond from start to end. If the bond defaults prior to TRS maturity, the TRS is either physically settled or cash settled based on a default settling mechanism (e.g., dealer poll) that determines the settlement value of the defaulted debt. A TRS facility allows debt substitution when a referenced debt no longer satisfies certain pre-agreed eligibility criteria or defaults outright. The substitution debt will have to meet those criteria. From a modeling perspective, the defaulted debt is cured with a similar-in-kind debt, such that the reference asset possesses same credit quality and similar price behavior. Therefore, default has no economic impact on the TRS seller or lender, so the equity based TRS model presented in previous sections can be applied.

Cusip TRS require a different model as the TRS are early terminated upon underlying debt default. Let $\lambda(t)$ be the default intensity of a Cox process defined in a probability space (Ω,P,F). A bond issuer defaults when the Cox process has its first jump at time τ. Let $\Gamma_t=I[\tau\leq t]$ denote the default indicator, 1 if default before time t, 0 otherwise. Then the survival probability at time T as seen from time t denoted by $q(t,T)$ is

$$q(t,T) = \Pr(\Gamma_T = 0) = E_t^Q[e^{-\int_t^T \lambda(u)du}] \tag{21}$$

where E^Q is an expectation taken under the risk-neutral measure Q.

The bank hedges the TRS by holding one unit of the underlying bond priced at B_t, and finances the bond purchase in the repo market. The wealth equation of the TRS economy from party B's perspective is

$$\pi_t = M_t + (1-\Gamma_t)(V_t - L_t - W_t - N_t + B_t - L_t^b) \tag{22}$$

where L_t^b a bond repo account with haircut h, $L_t^b = (1-h)B_t$, at the repo rate of r_s. For simplicity, we start with a zero coupon bond and will add back coupon payments.

Pre-default self-financing equation includes bond financing and collateral rebalancing cash flows detailed as follows,

$$\begin{aligned} dM_t &= rM_t dt + (1-\Gamma_t)[dW_t - r_w W_t dt + dN_t - r_N N_t dt + dL_t - rL_t dt \\ &+ dL_t^b - r_s L_t^b dt] \end{aligned} \tag{23}$$

At $t=0$, the wealth reduces to $\pi_0 = M_0 - N_0 + hB_0$. We set $M_0=\pi_0=0$, $N_0=hB_0$. For $t>0$, keeping the pre-default wealth to 0 leads to $N_t= hB_t$ and $M_t=0$.

Now assume that the underlying bond defaults at time τ, $\tau<T$. Unwinding the bond financing pays the repo lender $L^b(\tau\text{-})$ and gets the bond back to be sold in the market at its

post default value, i.e., its recovery value R. τ- denotes time immediately before default but not including the default. Here $B(t)$ and others are pre-default processes, thus continuous. The liability-side deposit W_τ and the cash collateral L_τ are released in exchange for the TRS default settlement amount of $B_0 - R$. Note, $B(\tau\text{-}) = W(\tau\text{-}) + L(\tau\text{-})$, and $B_0 - R = [B_0 - B(\tau\text{-})] + [B(\tau\text{-}) - R]$, the first bracket being the price return immediately prior to default and the second bracket the jump caused by default. The net default settlement is $B_0 - B(\tau\text{-}) - V(\tau\text{-})$ and the self-financing equation covering both pre-default and after default becomes,

$$dM_t = rM_t dt + (1 - \Gamma_t)[dV_t - (r_w W_t + rL_t)dt + dB_t - (hr_N + (1 - h)r_s)B_t dt] + d\Gamma_t[B_0 - B(\tau_-) - V(\tau_-)] \tag{24}$$

Write the wealth in the differential form,

$$\begin{aligned} d\pi_t = (1 - \Gamma_t)[dV_t - (r_w W_t + rL_t)dt + dB_t - (hr_N + (1 - h)r_s)B_t dt \\ + \lambda(B_0 - B_t - V_t)dt] \\ + (d\Gamma_t - (1 - \Gamma_t)\lambda dt)[B_0 - B(\tau_-) - V(\tau_-)] \end{aligned} \tag{25}$$

Noting that the last row is a martingale. Assuming a continuous intensity process ($d\lambda = adt + bdZ$), the bond price can be shown to satisfy the following PDE,

$$\frac{\partial B}{\partial t} + a\frac{\partial B}{\partial \lambda} + \frac{1}{2}b^2\frac{\partial^2 B}{\partial \lambda^2} - (\overline{r_s} + \lambda)B + \lambda R = 0 \tag{26}$$

where $\overline{r_s} = (1 - h)r_s + hr_N$is the all-in funding cost for the bond. Now apply Ito's lemma to both $V(t,\lambda)$ and $B(t,\lambda)$ in equation (25), set dt term to zero, we obtain the following,

$$\frac{\partial V}{\partial t} + a\frac{\partial V}{\partial \lambda} + \frac{1}{2}b^2\frac{\partial^2 V}{\partial \lambda^2} - \lambda V - r_w W - rL + \lambda(B_0 - R) = 0 \tag{27}$$

Let T_b denote bond's maturity, T_i its i-th discrete coupon date, and c_i coupon rate, the bond pricing formulae is given by,

$$B(t) = E_t^Q[e^{-\int_t^{T_b}(\overline{r_s}+\lambda)du} + \int_t^{T_b}\lambda R e^{-\int_t^{\tau}(\overline{r_s}+\lambda)du}d\tau + \sum_{i,T_i>t} c_i e^{-\int_t^{T_i}(\overline{r_s}+\lambda)du}] \tag{28}$$

Unlike the bond price, the TRS fair value will depend on collateralization scheme, i.e., a specification of L_t in (27). Assuming full collateralization, $L_t = V_t$, the TRS *npv* formula is

$$V(t) = E_t^Q[\int_t^T \lambda(B_0 - R)e^{-\int_t^{\tau}(r+\lambda)du}d\tau + \left(M_0 r_f T + B_0 - B_T\right)e^{-\int_t^T(r+\lambda)du} - \sum_{i,T_i\le T} c_i e^{-\int_t^{T_i}(r+\lambda)du}] \tag{29}$$

The first term is due to early termination settlement (payoff), the second term is the discounted period end (maturity) payoff which is subject to non-default of the reference bond, and the intermittent coupon payments are included in the third and last term.

Working out the last term of (29) utilizing (28), we arrive at

$$V(t) = M_0 r_f T \overline{D}(t,T) + B_0 \left(\overline{D}(t,T) + E_t^Q \left[\int_t^T \lambda e^{-\int_t^\tau (r+\lambda) du} d\tau \right] \right) - B_t - fva \qquad (30)$$

where $\gamma(t,T)$ is the repo funding cost factor, and $\overline{D}(t,T)$ the risky discount factor (product of the riskfree discounting and the survival probability, conceptually),

$$\gamma(t,T) = e^{\int_t^T (\overline{r_s} - r) du} - 1,$$

$$\overline{D}(t,T) = E_t^Q [e^{-\int_t^T (r+\lambda) du}], \qquad (31)$$

The first term in (30) is simply the risky discounted funding interest. The funding leg premium is now discounted at the OIS rate subject to the survival probability. The second term is the value of a zero-coupon, defaultable yet zero loss note with notional of B_0. These form the funding leg pricing. The third term is the usual asset price and the last term *fva* is a funding cost value adjustment to the asset leg,

$$fva = \gamma(t,T) B_t - \sum_{i, T \geq T_i > t} c_i \overline{D}(t, T_i) \gamma(t,T) - E_t^Q \left[\int_t^T \lambda R \gamma(\tau, T) e^{-\int_t^\tau (r+\lambda) du} d\tau \right] \quad (32)$$

Comparing to (14), *fva* now has more terms: $\gamma(t,T)B_t$ is the cost on the current price covering the remaining maturity of the TRS, savings of repo financing cost due to bond coupon payments (second term in (32)), and savings due to default recovery payment. The funding cost compounding factor $\gamma(t,T)$ appears in the same way as is in equity swaps, e.g., equation (14).

With deterministic interest rate and repo rate, the two integrals in equations (30 and 32) can be written out in terms of discount factor $D(t,T)$and survival probability $q(t,T)$ to facilitate computation, see Appendix A, where multi-period TRS valuation formula is also given.

For a bond forward with maturity *T* and forward price *F*, its payoff function is simply *F*- B_T. This is essentially the TRS payoff with zero funding interest rate on the funding leg, see equation (12) with r_f and *q* to zero and S_0 replaced by *F*. Equations (29 & 30) then immediately give formulae for MTM of a forward contract with B_0 replaced by the forward price *F*. To price a forward, one sets *V(t)* to zero to solve for *F*.

A trinomial tree on λ can be set up to price the bond up to its maturity T_b. Because we are solving the pre-default price, there is no need to simulate bond default on the tree. Bond coupon payments can be easily accommodated. Then the same recursive procedure from section 3 can be used to price the TRS on the bond.

The model above has not included wrong way risk (WWR) modeling. Although not difficult to quantify with tools from the credit correlation literature, WWR has proven to be elusive, because of a severe lack of historical data in correlated defaults to support model choices and estimation, and non-availability of capital market instruments to perform WWR risk hedging with any proven reliability and efficiency. The best approach remains to rely on due diligence and other risk management processes to eradicate any specific or structural WWR.

Fortunately for TRS, the general WWR is expected to be weak, because of the prevalent application of haircut or independent amount (IA). IA is an amount paid by the receiver or borrower at the start of a TRS transaction, often in the form of a fixed percentage of the TRS notional. It serves the purpose of providing protection against sudden market moves following a counterparty credit event. Economically, it is the same as haircut in standard repo transactions. Lou (2016b) finds that repo pricing is not sensitive to correlation between the borrower credit and the collateral asset price process and that general WWR capturing can be safely left for the future.

6. Results and Discussions

To start, we show some comparisons of the tree model and the analytical results. Table 1 exhibits the *npv* or fair value of one year maturity TRS under full cash collateral and zero repo financing cost, to show the accuracy of the tree model. Binomial tree model has the same results as the trinomial tree model up to 12 digits.

Table 1. Comparisons of analytic and binomial tree results for TRS on a non-dividend paying stock with current price of 100, volatility of 50%, riskfree rate of 10%, and zero repo spread, except for the last column with 5%.

Test cases	ATI, one period	ATI, one period, K=80	one stub, t0=0.25	ATI, 1 year, 4 period	4 period, t0=0.1	4 period, t0=0.1, repo=5%
Analytic	0	-1.9032516	2.53151205	-0.35977699	0.71606575	0.73916662
Binomial Tree	1.42E-13	-1.9032516	2.53151205	-0.35977699	0.71606575	0.73916662

The special case of same counterparty credit spreads (or funding rates) can be used to test repo-style margined TRS valuation. Table 2 shows that with typical one-day margin (approximately 250 days in a year, thus 250 steps for T=1 TRS), *npv* from the tree is -0.52336251 vs analytic result of -0.52327778, a relative error of 1.62E-4. Results in general don't seem to be as highly accurate as in Table 1, understandably because the cash collateral amount L_t has to be set at the previous time step, so it is expected to be more accurate as time step becomes smaller, or number of steps greater as shown in Table 2.

Table 2. Repo-style TRS valuation converges to analytical results with $r_b=r_c=2\%$. TRS has 1 year maturity, one period ATI, 5% repo spread. Other parameters see Table 1.

Time steps	100	250	500	1000	Analytic
Npv	-0.52348862	-0.52336251	-0.52332021	-0.52329901	-0.52327778

For an uncollateralized TRS, the trinomial tree model is used to compute fair value and XVAs, with results shown in Table 3. Because of the asymmetry in B and C's funding rates (r_b=12%, r_c=15%), the bid side (payer TRS) and ask side (receiver TRS) have different fair values, e.g., to enter a payer at -0.7577, or a receiver at 0.3509. The difference of -0.4068 attributes to XVAs shown in fourth and fifth columns of the Table.

Fix the TRS premium at 12.131%, in order to achieve zero *npv*, the implied repo spread is 1.26723% for the payer TRS, and 1.65867% for the receiver. Alternatively, fixing repo spread at 2%, these non-zero day 1 *npv* can be eliminated by setting fair funding leg premium, resulting in payer fair premium of 12.9541%, and receiver fair premium of 12.5136%. A bid/ask spread of 0.44% thus is naturally induced.

The last two columns of Table 3 show XVAs under the repo-style margin rule. Its effect of reducing XVAs is clear as XVAs are reduced by an order of magnitude, for instance, cva reduced from 0.2760 to 0.0168 for the payer TRS. Subsequently, the bid/ask sdie *npvs* are closer, -0.5742 vs 0.5384, a difference of only -0.0385.

Table 3. XVAs of a TRS on the buyer/receiver or seller/payer side. $S=100, K=100, \sigma=0.5, q=0, T=1, r=0.1, r_s=0.02, r_f = 12.131\%, r_b=0.12, r_c=0.15$.

	spread	Xva	payer no CSA	receiver no CSA	payer repo margin	receiver repo margin
sb	1.7%	Dva	0.1199	0.1120	0.0137	0.0069
sc	4.2%	Cva	0.2760	0.2948	0.0168	0.0334
mb	0.3%	dfa	0.0212	0.0198	0.0024	0.0012
mc	0.8%	cfa	0.0526	0.0561	0.0032	0.0064
rb-r	2.0%	vstar	-0.5701	0.5701	-0.5701	0.5701
rc-r	5.0%	v	-0.7577	0.3509	-0.5742	0.5384

The critical input to the TRS valuation model is the repo rate. Even in the most liquid Treasuries repo market, quoted repo rates seldom go beyond 3 month tenor. And only for Treasuries going on special, repo rates are quoted per security; otherwise GC (general collateral) tri-party repo rates apply. In the bilateral repo market, more granularity exists, but data is much less transparent as haircuts and repo rates are privately negotiated and generally not subject to reporting requirements. An analytical model to predict repo rates becomes the only viable solution at present time.

Treating repo as a debt product, Lou (2016b) considers an economic capital approach for the repo gap risk during a margin period of risk which is neither hedgeable nor diversifiable. Gap risk pricing introduces a capital valuation adjustment (KVA) for economic capital charge. A repo break-even rate formulae is derived, $r_p - r = RoE * E_c + \mu_0 + \lambda El$, where E_c is repo economic capital, *RoE* return on equity, μ_0 cost of fund, *El* is expected gap loss, and λ borrower's hazard rate. λEl is very small, and can be safely ignored compared to E_c*RoE* term, even under marginal haircuts. At zero haircut, for instance, a one-year repo with 10 day MPR on US main equities could command about 50 bp KVA for a 'BBB' rated borrower. Increased haircut reduces KVA, e.g., to 4 bp at 10% haircut. The repo rate is then specified once E_c is computed per asset class and μ_0 is given, see details in Lou (2016b).

A funding TRS is a synthetic kin of repo. Naturally one would expect TRS premium to be comparable to repo spread on the same asset or asset class. In fact, our model can be used to imply a repo rate for the same underlying security, if we choose to price an at-the-issue TRS at zero, while fixing its premium. A zero npv ATI swap makes sense as it is free to enter, typical of rate and credit default swaps. In this way, repo and TRS are made economically equivalent, while they both serve the purpose of security financing.

7. Conclusions

As the most popular tool of securities financing in the derivative form, total return swaps (TRS) has a long standing pricing and valuation dilemma: its funding premium discounted at the Libor forces funding interest to be paid upfront rather than on accrual; discounting with a firm's own funding curve leads to collateral dispute and violation of established finance theories. We show that, by considering TRS hedge financing cost from a repo market perspective and by adopting post-crisis collateral and funding sensitive derivative pricing to TRS, this dilemma can be resolved.

We find that the cost of financing the TRS hedge should be reflected on the security leg rather than the funding leg. We confirm that under full CSA, TRS shall remain to be discounted at the cash deposit rate, e.g., the OIS curve for US dollar, and there is no CVA and FVA. The funding premium can't be separately discounted: its discounting rate is exactly same as whatever is applicable to the TRS as a whole, governed by its collateralization scheme. Uncollateralized TRS, for example, shall discount at the liability-side counterparty's senior unsecured rate.

Our model handles different collateralization schemes, including full collateralization, uncollateralized TRS with retail customers, and customized, repo-style margined TRS that can be adopted to eliminate the upfront payment. A binomial/trinomial tree model is presented to compute the fair value as the valuation problem is no longer linear and without analytic formula. The periodic reset feature is implemented via a recursive pricing tree scheme, with minimal implementation cost. A total counterparty risk valuation adjustment can be computed precisely on uncollateralized or partially collateralized TRS, and decomposed into CVA and FVA as needed. Extension to TRS on defaultable debt and bond forward is provided, which can also be implemented on the tree.

As is with other types of swaps, fair TRS premium can now be determined by setting the net present value of the TRS to zero, or implied repo rate can be solved from traded TRS premium. The synthetic form and cash form of securities financing are now on par with each other.

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Appendix A

Assuming deterministic interest rate and repo rate, the two integrals in equations (30 and 32) can be written as follow,

$$dpv(t,T) = E_t^Q\left[\int_t^T \lambda e^{-\int_t^\tau (r+\lambda)du} d\tau\right] = -\int_t^T D(\tau,T)dq(\tau,T) \tag{A.1}$$

$$E_t^Q\left[\int_t^T \lambda R\gamma(\tau,T) e^{-\int_t^\tau (r+\lambda)du} d\tau\right] = -R\int_t^T \gamma(\tau,T)D(\tau,T)dq(\tau,T) \tag{A.2}$$

where $D(t,T)$ is the OIS discount factor and $q(t,T)$ is the survival probability.

For a regular coupon bond, the risky bond forward collateralized under a full CSA is given by,

$$\widetilde{D}(t,T) = E_t^Q[e^{-\int_t^T (\bar{r}_s+\lambda)du}] \tag{A.3}$$

$$F(t,T) = \frac{B_t}{\widetilde{D}(t,T)} - \int_t^T \lambda e^{\int_\tau^T (\bar{r}_s+\lambda)du} d\tau - \sum_{i,T_i\le T} c_i \frac{\widetilde{D}(t,T_i)}{\widetilde{D}(t,T)} \tag{A.4}$$

A special case is non-defaultable bond, such as government bonds. Letting λ be zero, the bond forward for Treasuries is now shown to be,

$$F(t,T) = B_t e^{\int_t^T \bar{r}_s du} - \sum_{i,T_i\le T} c_i e^{\int_{T_i}^T \bar{r}_s du} \tag{A.5}$$

For a multi-period TRS, with payment dates on t_j, $j=0, 1, 2, \ldots$ The cash flow in j-th period (t_j, t_{j+1}] consists of a (possible) bond coupon c_i at time T_i, and TRS payment of $B_j - B_{j+1} + M_j r_{f,j}\Delta t_j$. A fully collateralized TRS valuation formula is,

$$V(t) = \sum_{j=0}\{\left(M_j r_{f,j}\Delta t_j + F_j - F_{j+1}\right)D\left(t,t_{j+1}\right) - c_i D(t,T_i) + \left(F_j - R\right)\left(dpv\left(t,t_{j+1}\right) - dpv(t,t_j)\right)\} \tag{A.6}$$

where F_j is a short form of $F(t, t_j)$. If there are multiple coupon payments during a TRS period, simply treat the term $c_i D(t,T_i)$ as a sum of all coupons included.

In equation (A.6), we have not specified the funding leg notional M_j. For a TRS with fixed funding amount, $M_j = M_0$. If a TRS resets its notional in accordance with the then bond price at t_j, then replace M_j with F_j.