

Next, we add the two equations:

$$x^2 - xy + x + C_1(y) + y^2 - xy - y + C_2(x)$$

Simplifying and collecting the terms involving x and y:

$$x^2 + y^2 + x - y + (C_2(x) + C_1(y)) = 0$$

We want this equation to be equal to the integrating factor, which is $e^{\int P(x)dx + \int Q(y)dy}$. Therefore:

$$x^2 + y^2 + x - y + (C_2(x) + C_1(y)) = e^{\int P(x)dx + \int Q(y)dy}$$

Comparing the left-hand side of the equation with the integrating factor, we have:

$$C_2(x) + C_1(y) = 0$$

This implies that $C_2(x) = -C_1(y) = C$

Therefore, the integrating factor becomes:

$$IF = e^{(x^2 + y^2 + x - y + C)}$$

Now, we multiply the original differential equation by the integrating factor (IF):

$$e^{(x^2 + y^2 + x - y + C)} * [(2x - y + 1)dx + (2y - x - 1)dy] = 0$$

By distributing the integrating factor and simplifying, the equation becomes:

$$(2x - y + 1)e^{(x^2 + y^2 + x - y + C)}dx + (2y - x - 1)e^{(x^2 + y^2 + x - y + C)}dy = 0$$

Now, we integrate this equation. Notice that the left-hand side is in the form of the total differential of a function, so integrating it will give us the desired solution.

Let $F(x, y)$ be the function such that $\partial F/\partial x = (2x - y + 1)e^{(x^2 + y^2 + x - y + C)}$ and $\partial F/\partial y = (2y - x - 1)e^{(x^2 + y^2 + x - y + C)}$.

Integrating the first equation with respect to x gives:

$$\begin{aligned} F(x, y) &= \int (2x - y + 1)e^{(x^2 + y^2 + x - y + C)}dx \\ &= \int \partial F/\partial x \, dx \end{aligned}$$