Internal Assignment - 1

Name : Aldino Fernandes

Program : BCA

Subject : Basic Mathematics

1. **State inclusion-exclusion principle. In a group of 50 people, 35 speak Hindi, 25 speak both English and Hindi and all the people speak at least one of the two languages. How many people speak only English and not Hindi ? How many people speak English**

   The inclusion-exclusion principle is a counting principle used to calculate the size of a set by considering the sizes of its subsets and their intersections. In this case, we can use the inclusion-exclusion principle to determine the number of people who speak only English and not Hindi.

   Let's denote the following:

   E = Number of people who speak English.

   H = Number of people who speak Hindi.

   We are given the following information:

   $E \cup H = 50$ (All the people speak at least one of the two languages)

   $H = 35$ (Number of people who speak Hindi)

   $H \cap E = 25$ (Number of people who speak both English and Hindi)

   If we consider

   $E - (H \cap E)$ = Number of people who speak only English and not Hindi

   $E - 25$ = Number of people who speak only English and not Hindi

   To find the number of people who speak English,

   $(E - 25) + (H \cap E) = E$

   $E - 25 + 25 = E$

   $E = 50$

Therefore, the number of people who speak only English and not Hindi is 25, and the total number of people who speak English is 50.

2. **Simplify $z = \frac{(cos\,\theta + isin\theta)^5}{(cos\,\theta - isin\theta)^4}$ into $x + iy$ form and find its modulus and the amplitude.**

Rewriting the denominator using the conjugate property:

(cos θ - i sin θ) = (cos θ + i sin θ)^*

Using the De Moivre's theorem, we can simplify the numerator and denominator:

(cos θ + i sin θ)^5 = cos(5θ) + i sin(5θ)

Now using the formula,

(cos θ + i sin θ)^* = cos(-θ) + i sin(-θ) = cos θ - i sin θ

Substituting these simplified expressions back into the original equation:

z = (cos(5θ) + i sin(5θ)) / (cos θ - i sin θ)

Rationalising the denominator we get,

z = [(cos(5θ) + i sin(5θ)) * (cos θ + i sin θ)] / [(cos θ - i sin θ) * (cos θ + i sin θ)]

Expanding the numerator and denominator:

z = [cos(5θ)cosθ + i sin(5θ)cosθ + i cos(5θ)sinθ - sin(5θ)sinθ] / [cos^2(θ) + sin^2(θ)]

Simplifying:

z = [(cos(5θ)cosθ - sin(5θ)sinθ) + i (sin(5θ)cosθ + cos(5θ)sinθ)] / 1

Using the trigonometric identities:

z = [cos(6θ) + i sin(6θ)] / 1

Therefore, z simplifies to:

z = cos(6θ) + i sin(6θ)

Hence we have the expression in the form x + iy, where x = cos(6θ) and y = sin(6θ).

To find the modulus of z, we can use Euler's formula:
|z| = √[cos^2(6θ) + sin^2(6θ)] = √1 = 1

The modulus of z is 1.

To find the amplitude (argument), we can use the formula:
arg(z) = arctan(y/x) = arctan(sin(6θ)/cos(6θ)) = arctan(tan(6θ)) = 6θ

The amplitude of z is 6θ.

3. **A. Solve:** $\int_{0}^{\pi/2} \sqrt{1 + \sin sin\, 2x}\, dx.$

Integrating the given function:
∫(1 + sin(2x)) dx

Integrating 1 with respect to x gives x, and integrating sin(2x) gives (-1/2)cos(2x) using the chain rule. So we have:
∫(1 + sin(2x)) dx = ∫1 dx + ∫sin(2x) dx
= x - (1/2)cos(2x) + C,   where C is the constant of integration.

Now, further evaluating the definite integral for the given limits :
∫[0 to π/2] (1 + sin(2x)) dx = [x - (1/2)cos(2x)] [0 to π/2]
= [π/2 - (1/2)cos(2(π/2))] - [0 - (1/2)cos(2(0))]
= [π/2 - (1/2)cos(π)] - [0 - (1/2)cos(0)]
= [π/2 - (1/2)(-1)] - [0 - (1/2)(1)]
= [π/2 + 1/2] - [0 - 1/2]
= π/2 + 1/2 - (-1/2)
= π/2 + 1/2 + 1/2
= π/2 + 1
= π/2 + 2/2

$= \pi/2 + 2/2$

$= \pi/2 + 1$

$= \pi/2 + 1$

Therefore, the value of the definite integral of $(1 + \sin(2x))$ with respect to x from 0 to $\pi/2$ is $\pi/2 + 1$.

B. Solve the differential equation

$$(2x - y + 1)dx + (2y - x - 1)dy = 0$$

To solve the given differential equation: $(2x - y + 1)dx + (2y - x - 1)dy = 0$, we can follow these steps:

If we calculate the partial derivatives of $(2x - y + 1)$ with respect to x and $(2y - x - 1)$ with respect to y:

$\partial(2x - y + 1)/\partial x = 2$

$\partial(2y - x - 1)/\partial y = 2$

Since these partial derivatives are not equal, the equation is not exact.

To make the equation exact, we need an integrating factor. We can calculate it using the formula:

Integrating factor (IF) = e ^ $(\int P(x)dx + \int Q(y)dy)$

Where $P(x)$ and $Q(y)$ are the coefficients of dx and dy, respectively. In this case, $P(x) = 2x - y + 1$ and $Q(y) = 2y - x - 1$.

$\int P(x)dx = \int(2x - y + 1)dx = x^2 - xy + x + C1(y)$

$\int Q(y)dy = \int(2y - x - 1)dy = y^2 - xy - y + C2(x)$

Here, $C1(y)$ and $C2(x)$ represent constants of integration that depend on y and x, respectively.

Next, we add the two equations:
$x^2 - xy + x + C1(y) + y^2 - xy - y + C2(x)$

Simplifying and collecting the terms involving x and y:
$x^2 + y^2 + x - y + (C2(x) + C1(y)) = 0$

We want this equation to be equal to the integrating factor, which is $e^{(\int P(x)dx + \int Q(y)dy)}$. Therefore:
$x^2 + y^2 + x - y + (C2(x) + C1(y)) = e^{(\int P(x)dx + \int Q(y)dy)}$

Comparing the left-hand side of the equation with the integrating factor, we have:
$C2(x) + C1(y) = 0$

This implies that $C2(x) = -C1(y) = C$

Therefore, the integrating factor becomes:
$IF = e^{(x^2 + y^2 + x - y + C)}$

Now, we multiply the original differential equation by the integrating factor (IF):
$e^{(x^2 + y^2 + x - y + C)} * [(2x - y + 1)dx + (2y - x - 1)dy] = 0$

By distributing the integrating factor and simplifying, the equation becomes:
$(2x - y + 1)e^{(x^2 + y^2 + x - y + C)}dx + (2y - x - 1)e^{(x^2 + y^2 + x - y + C)}dy = 0$

Now, we integrate this equation. Notice that the left-hand side is in the form of the total differential of a function, so integrating it will give us the desired solution.

Let $F(x, y)$ be the function such that $\partial F/\partial x = (2x - y + 1)e^{(x^2 + y^2 + x - y + C)}$ and $\partial F/\partial y = (2y - x - 1)e^{(x^2 + y^2 + x - y + C)}$.

Integrating the first equation with respect to x gives:
$F(x, y) = \int(2x - y + 1)e^{(x^2 + y^2 + x - y + C)}dx$
$\quad = \int \partial F/\partial x \, dx$

$= \int dx\ [e^{(x^2 + y^2 + x - y + C)}]$

$= e^{(x^2 + y^2 + x - y + C)} + g(y)$ (integrating with respect to x)

Here, g(y) represents a constant of integration that depends on y.

Now, we differentiate F(x, y) with respect to y and set it equal to the second equation:

$\partial F/\partial y = \partial/\partial y\ (e^{(x^2 + y^2 + x - y + C)} + g(y))$

$= (2y - x - 1)e^{(x^2 + y^2 + x - y + C)}$

Comparing the coefficients of $e^{(x^2 + y^2 + x - y + C)}$, we have:

$2y - x - 1 = 0$

Rearranging this equation gives:

$2y = x + 1$

$y = (1/2)x + 1/2$

Therefore, the solution to the differential equation $(2x - y + 1)dx + (2y - x - 1)dy = 0$ is $y = (1/2)x + 1/2$.

4. **A. By using truth tables, check whether the propositions $\sim(p \wedge q)$ and $(\sim p) \vee (\sim q)$ are logically equivalent or not?**

To check the logical equivalence of the propositions $\sim(p \wedge q)$ and $(\sim p) \vee (\sim q)$, we can create truth tables for both propositions and compare their outputs.

The truth values for p and q and for $\sim(p \wedge q)$ is :

p	**q**	**p∧q**	**~(p∧q)**
T	T	T	F
T	F	F	T
F	T	T	T
F	F	T	T

Now, let's define the truth values for p and q and construct the truth table for (~p)∨(~q):

p	**q**	**~p**	**~q**	**(~p)∨(~q)**
T	T	T	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

By comparing the outputs of the two truth tables, we can see that ~(p∧q) and (~p)∨(~q) have the same truth values for all possible combinations of p and q. Therefore, we can conclude that ~(p∧q) and (~p)∨(~q) are logically equivalent.

B. Consider the set $G = \{1, 5, 7, 11, 13, 17\}$ under multiplication modulo 18 as a group. Construct the multiplication table for *G* and find the inverse of each element of *G.*

To construct the multiplication table for the group G = {1, 5, 7, 11, 13, 17} under multiplication modulo 18, we need to perform the multiplication operation on each pair of elements in G. Let's denote the operation as *.

Multiplication Table for G:

*	**1**	**5**	**7**	**11**	**13**	**17**
1	1	5	7	11	13	17
5	5	7	11	13	17	1
7	7	11	13	17	1	5
11	11	13	17	1	5	7

13	13	17	1	5	7	11
17	17	1	5	7	11	13

Now let's find the inverse of each element in G.

Inverse of 1:
To find the inverse of 1, we need to find an element in G that, when multiplied by 1, gives the result 1 (identity element). In this case, the inverse of 1 is itself.

Inverse of 5:
To find the inverse of 5, we need to find an element in G that, when multiplied by 5, gives the result 1. Checking the multiplication table, we see that the inverse of 5 is 11.

To find the inverse of 17, we need to find an element in G that, when multiplied by 17, gives the result 1. Checking the multiplication table, we see that the inverse of 17 is 17 itself.

Similarly, we can find the inverses of the remaining elements

Therefore, the inverses of the elements in G are:
Inverse of 1: 1
Inverse of 5: 11
Inverse of 7: 13
Inverse of 11: 5
Inverse of 13: 7
Inverse of 17: 17