$$33.1 \quad \int_0^{\infty} 4\pi \left(\frac{M}{2\pi RT}\right)^{3/2} v^2 e^{\left(-\frac{Mv^2}{2RT}\right)} dv = 1$$

$$u = \frac{Mv^2}{2RT} \Rightarrow v = \sqrt{\frac{u2RT}{M}} \quad // \quad du = \frac{Mv}{RT} dv \Rightarrow v\,dv = \frac{RT}{M} du$$

$$\Rightarrow 2^2 \pi \frac{M^{3/2}}{(2\pi RT)^{3/2}} \int_0^{\infty} \sqrt{\frac{u2RT}{M}}\, e^{-u} \frac{RT}{M} du = \frac{2^2}{2^{3/2}} \frac{\pi}{\pi^{3/2}} \frac{M^{3/2}}{M^{1/2}} \frac{R^{1/2}}{R^{3/2}} \frac{T^{1/2}}{T^{3/2}} \frac{RT\, 2^{1/2}}{M} \int_0^{\infty} e^{-u} \sqrt{u}\, du$$

$$= \frac{2}{\sqrt{\pi}} \left(\frac{MRT}{MRT}\right)^{3/2} \int_0^{\infty} e^{-u} \sqrt{u}\, du = \frac{2}{\sqrt{\pi}} \underbrace{\int_0^{\infty} e^{-u} \sqrt{u}\, du}_{\frac{\sqrt{\pi}}{2}} = 1$$

$\frac{\sqrt{\pi}}{2} \Rightarrow$ Aproximar con Gauss-Laguerre

$$3. \quad v_{avg} = \int_0^{\infty} v \rho(v) dv = \int_0^{\infty} 4\pi \left(\frac{M}{2\pi RT}\right)^{3/2} v^3 e^{\left(\frac{-Mv^2}{2RT}\right)} dv$$

$$u = \frac{Mv^2}{2RT} \Rightarrow v^2 = \frac{u2RT}{M} \quad // \quad du = \frac{M}{RT} v\,dv \Rightarrow v\,dv = \frac{RT}{M} du$$

$$v_{avg} = \int_0^{\infty} 4\pi \left(\frac{M}{2\pi RT}\right)^{3/2} \frac{u2RT}{M} e^{-u} \frac{RT}{M} du = \frac{2^3}{2^{3/2}} \frac{\pi}{\pi^{3/2}} \left(\frac{M}{RT}\right)^{3/2} \left(\frac{RT}{M}\right)^2 \int_0^{\infty} e^{-u} u\, du$$

$$= 2\sqrt{\frac{2RT}{\pi M}} \underbrace{\int_0^{\infty} e^{-u} u\, du}_{1} = \sqrt{\frac{8RT}{\pi M}}$$

$1 \Rightarrow$ Aproximar con Gauss-Laguerre

$$4\ V_{rms} = \sqrt{\int_0^\infty v^2 f(v)dv} = \sqrt{\int_0^\infty 4\pi\left(\frac{M}{2\pi RT}\right)^{3/2} v^4 e^{\left(-\frac{Mv^2}{2RT}\right)} dv}$$

$$u = \frac{Mv^2}{2RT} \Rightarrow v^2 = \frac{u2RT}{M} \quad // \quad du = \frac{M}{RT} v\,dv \Rightarrow v\,dv = \frac{RT}{M} du$$

$$V_{rms} = \sqrt{\int_0^\infty 4\pi\left(\frac{M}{2\pi RT}\right)^{3/2} \frac{u2RT}{M}\sqrt{\frac{u2RT}{M}}\ e^{-u}\ \frac{RT}{M} du}$$

$$= 4^{1/2}\pi^{1/2}\left(\frac{M}{2\pi RT}\right)^{3/4}\left(\frac{2RT}{M}\right)^{1/2}\left(\frac{2RT}{M}\right)^{1/4}\left(\frac{RT}{M}\right)^{1/2}\sqrt{\int_0^\infty e^{-u} u^{3/2} du}$$

$$= \frac{2}{\pi^{1/4}}\sqrt{\frac{RT}{M}}\ \underbrace{\sqrt{\int_0^\infty e^{-u} u^{3/2} du}}_{\frac{3\sqrt{\pi}}{4} \Rightarrow \text{Aproximar con Gauss-Laguerre}} = \frac{2}{\pi^{1/4}}\sqrt{\frac{RT}{M}}\ \frac{\sqrt{3}\ \pi^{1/4}}{2} = \sqrt{\frac{3RT}{M}}$$

$$5\ \mathcal{E}_{cinética} = \frac{1}{2} m \langle v\rangle^2 \quad \left|\ v^2_{avg} = \frac{8RT}{\pi M},\ v^2_{rms} = \frac{3RT}{M}\right\} v^2_{Avg} \approx v^2_{rms}$$

$$\Rightarrow \mathcal{E} = \frac{1}{2} m v^2_{avg} \approx \frac{1}{2} m v^2_{rms} = \frac{1}{2} m \frac{3RT}{M} \qquad // \quad \underset{\text{Masa Molar}}{M} = \frac{m \rightarrow \text{Masa molecular}}{n \rightarrow \#\text{ Moles}}$$

$$\Rightarrow \boxed{\mathcal{E}_{int} \approx \frac{3}{2} nRT}$$