EXAMPLE: Quantum efficiency and responsivity

Consider the photodiode shown in Figure 5.7. What is the QE at peak responsivity? What is the QE at 450 nm (blue)? If the photosensitive device area is 1 mm^2, what would be the light intensity corresponding to a photocurrent of 10 nA at the peak responsivity?

Solution

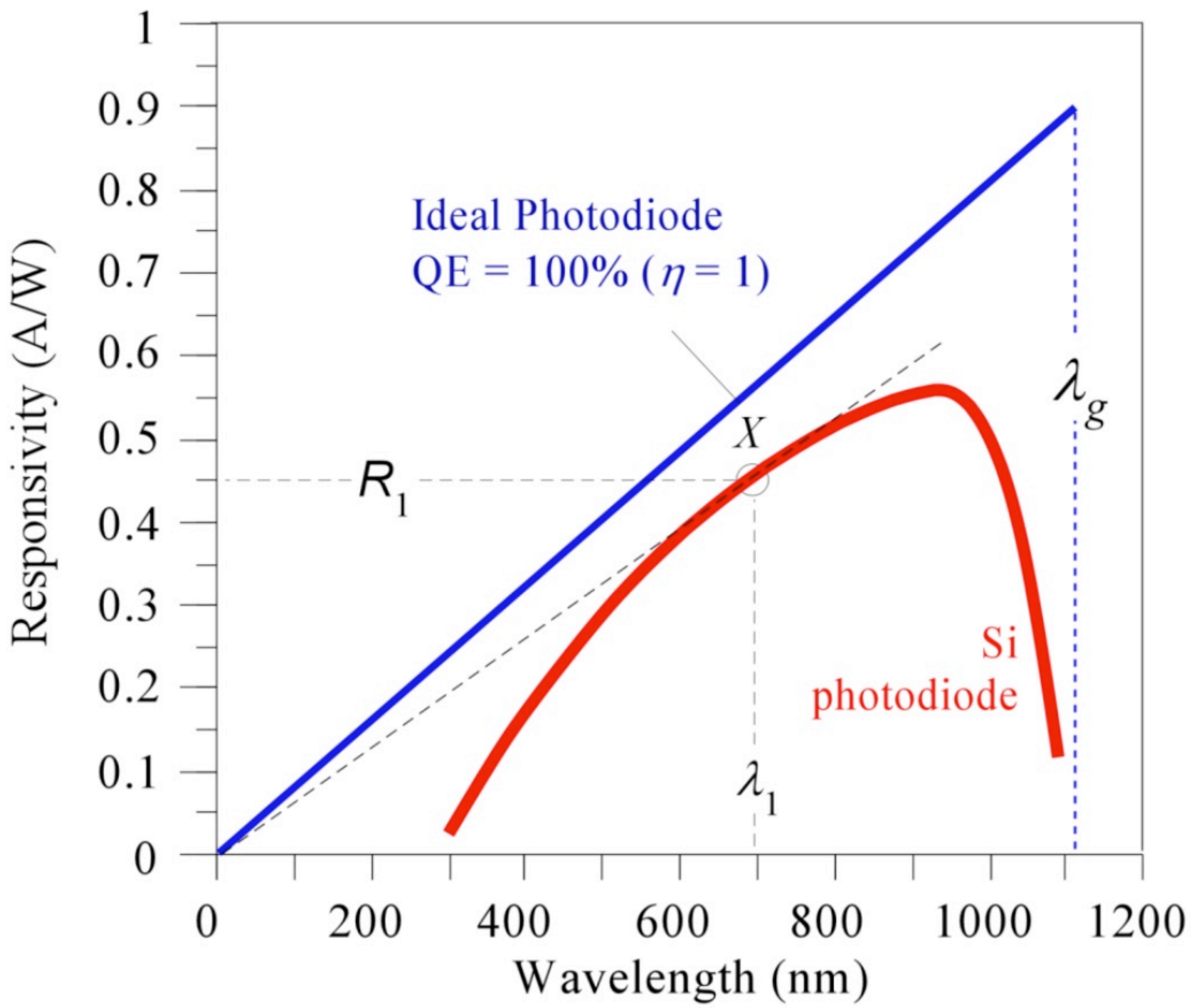
The peak responsibility in Figure 5.7 occurs at about $\lambda \approx 940$ nm where $R \approx 0.56$ A W^{-1}. Thus, from Eq. (5.4.4), that is $R = \eta_e e\lambda / hc$, we have

$$0.56\text{AW}^{-1} = \eta_e \frac{(1.6\times10^{-19}\ \text{C})(940\times10^{-9}\ \text{m})}{(6.63\times10^{-34}\ \text{J s})(3\times10^{8}\ \text{m s}^{-1})}$$

i.e. $\eta_e = 0.74$ or **74%**

We can repeat the calculation for $\lambda = 450$ nm, where $R \approx 0.24$ AW^{-1}, which gives $\eta_e = 0.66$ or 66%.

From the definition of responsivity, $R = I_{ph}/P_o$, we have 0.56 AW^{-1} = $(10 \times 10^{-9}$ A$)/P_o$, *i.e.* $P_o = 1.8 \times 10^{-8}$ W or 18 nW. Since the area is 1 mm^2 the intensity must be 18 nW mm^{-2}.



EXAMPLE: Maximum quantum efficiency

Show that a photodiode has maximum QE when

$$\boxed{\frac{dR}{d\lambda} = \frac{R}{\lambda}} \tag{5.4.5}$$

that is, when the tangent X at λ_1 in Figure 5.7 passes through the origin ($R = 0$, $\lambda = 0$). Hence determine the wavelengths where the QE is maximum for the Si photodiode in Figure 5.7

Solution

From Eq. (5.4.4) the QE is given by

$$\eta_e = \frac{hcR(\lambda)}{e\lambda} \tag{5.4.6}$$

where $R(\lambda)$ depends on λ and there is also λ in the denominator. We can differentiate Eq. (5.4.6) with respect to λ and then set to zero to find the maximum point X. Thus

$$\frac{d\eta_e}{d\lambda} = \frac{hc}{e\lambda}\frac{dR}{d\lambda} - \frac{hcR}{e}\left(\frac{1}{\lambda^2}\right) = 0$$

which leads to Eq. (5.4.5). Equation (5.4.5) represents a line through the origin that is a tangent to the R vs λ curve. This tangential point is X in Figure 5.7 , where $\lambda_1 = 700$ nm and $R_1 = 0.45$ AW^{-1}. Then, using Eq. (5.4.6), the maximum QE is

$\eta_e = (6.626 \times 10^{-34}$ J s) $(3 \times 10^8$ m s^{-1}) (0.45 A W^{-1}) / $(1.6 \times 10^{-19}$ C) $(700 \times 10^{-9}$ m)

= **0.80 or 80%**

EXAMPLE: Responsivity of a *pin* photodiode

A Si *pin* photodiode has an active light receiving area of diameter 0.4 mm. When radiation of wavelength 700 nm (red light) and intensity 0.1 mW cm^{-2} is incident, it generates a photocurrent of 56.6 nA. What is the responsivity and external QE of the photodiode at 700 nm?

Solution

The incident light intensity I = 0.1 mW cm^{-2} means that the incident power for conversion is

$$P_o = AI = [\pi(0.02\ \text{cm})^2](0.1\times10^{-3}\ \text{W cm}^{-2}) = 1.26\times10^{-7}\ \text{W or } \mathbf{0.126\ \mu W}.$$

The responsivity is

$$R = I_{ph}/P_o = (56.6\times10^{-9}\ \text{A})/(1.26\times10^{-7}\ \text{W}) = \mathbf{0.45\ A\ W^{-1}}$$

The QE can be found from

$$\eta = R\frac{hc}{e\lambda} = (0.45\ \ \text{A W}^{-1})\frac{(6.62\times10^{-34}\ \ \text{J s})(3\times10^{8}\ \ \text{m s}^{-1})}{(1.6\times10^{-19}\ \ \text{C})(700\times10^{-9}\ \ \text{m})} = 0.80 = \mathbf{80\ \%}$$

EXAMPLE: Operation and speed of a *pin* photodiode

A Si *pin* photodiode has an *i*-Si layer of width 20 μm. The p^+-layer on the illumination side is very thin (0.1 μm). The *pin* is reverse biased by a voltage of 100 V and then illuminated with a very short optical pulse of wavelength 900 nm. What is the duration of the photocurrent if absorption occurs over the whole *i*-Si layer?

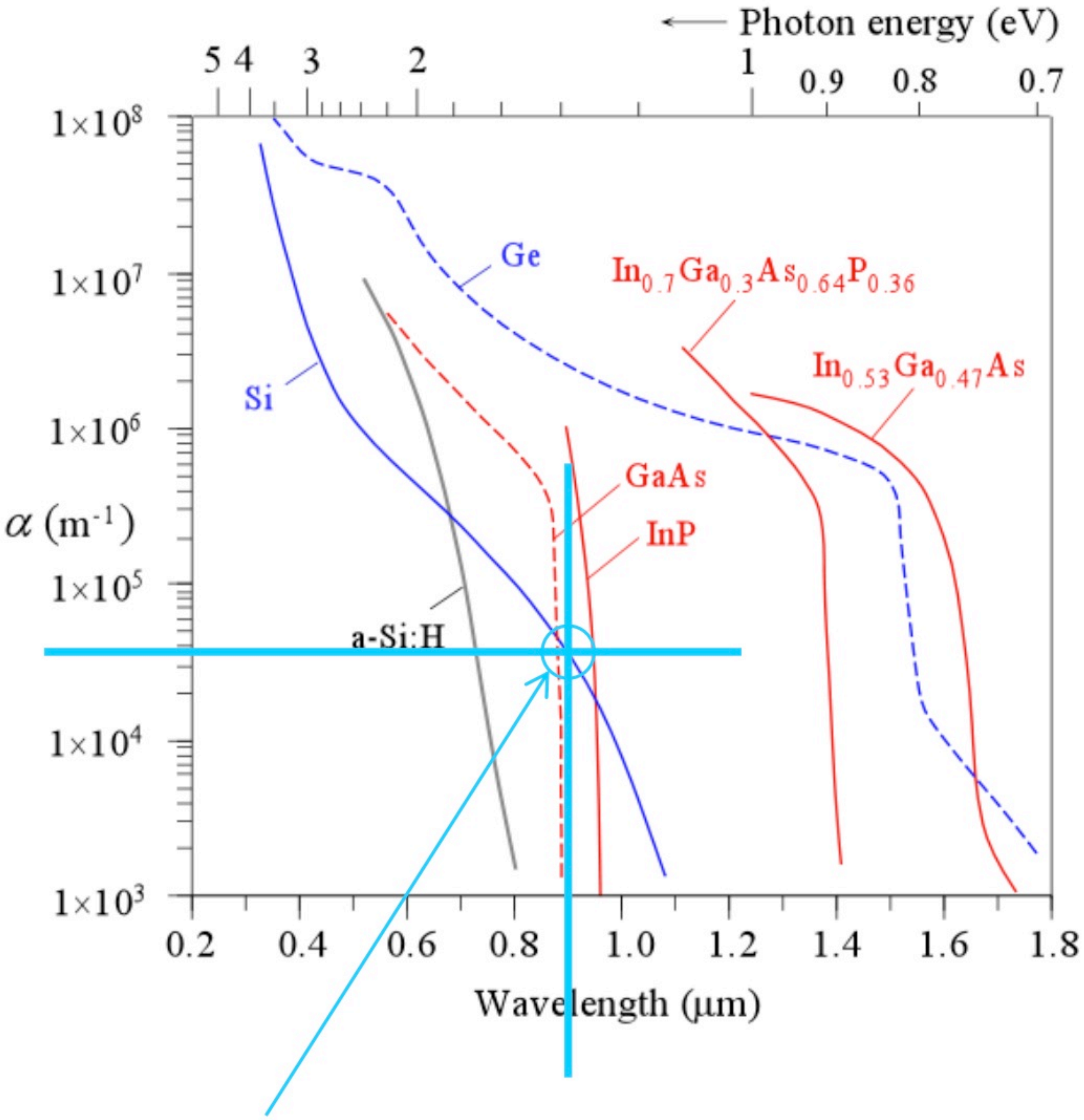
Solution

From Figure 5.5 , the absorption coefficient at 900 nm is ~ 3×10^4 m^{-1} so that the absorption depth is ~33 μm. We can assume that absorption and hence photogeneration occurs over the entire width W of the *i*-Si layer. The field in the *i*-Si layer is

$$E \approx V_r / W$$

$$= (100\text{ V})/(20\times10^{-6}\text{ m})$$

$$= 5\times10^6\text{ V m}^{-1}$$



Note: The absorption coefficient is between 3×10^4 m^{-1} and 4×10^4 m^{-1}

EXAMPLE: Steady state photocurrent in the *pin* photodiode

Solution (continued)

From Figure 5.5, at $\lambda = 900$ nm, $\alpha \approx 3 \times 10^4$ m^{-1}. Further for $\lambda = 0.90$ μm, the photon energy $h\upsilon = 1.24 / 0.90 = 1.38$ eV. Given $P_o(0) = 100$ nW, we have

$$I_{ph} \approx \frac{(1.6\times10^{-19})(1)(0.68)(100\times10^{-9})}{(1.38\times1.6\times10^{-19})}[1-\exp(-3\times10^{4}\times20\times10^{-6})] = \mathbf{22\ nA}$$

and the responsivity R = 22 nA / 100 nW = **0.22 A W^{-1}**, which is on the low-side.

Consider next, a perfect AR coating so that $T = 1$, and using Eq. (5.5.4) again, we find I_{ph} = 32.7 nA and $\boldsymbol{R}$ **= 0.33 A W^{-1}**, a significant improvement.

The factor $[1-\exp(-\alpha W)]$ is only 0.451, and can be significantly improved by making the SCL thicker. Setting W = 40 μm, gives $[1-\exp(-\alpha W)] = 0.70$ and R = 0.51, which is close to values for commercial devices.

The maximum theoretical photocurrent would be obtained by setting $\exp(-\alpha W) \approx 0$, $T = 1$, $\eta_i = 1$, which gives I_{ph} = 73 nA and R = **0.73 A W^{-1}**.

EXAMPLE: Steady state photocurrent in the *pin* photodiode

We assume these will drift through the depletion region and thereby contribute to the photocurrent. The current contribution δI_{ph} from absorption and photogeneration at x within the SCL will thus be

$$\delta I_{ph} = \frac{e\eta_i \alpha P_o(x)\delta x}{h\upsilon} = \frac{e\eta_i \alpha T P_o(0)}{h\upsilon}\exp(-\alpha x)\delta x$$

We can integrate this from x = 0 (assuming ℓ_p is very thin) to the end of $x = W$, and assuming $W >> L_h$ to find

$$I_{ph} \approx \frac{e\eta_i T P_o(0)}{h\upsilon}[1-\exp(-\alpha W)]$$

Steady state photocurrent pin photodiode (5.5.4)

where the approximate sign embeds the many assumptions we made in deriving Eq. (5.5.4). Consider a *pin* photodiode without an AR coating so that $T = 0.68$. Assume $\eta_i = 1$. The *SCL* width is 20 μm. If the device is to be used at 900 nm, what would be the photocurrent if the incident radiation power is 100 nW? What is the responsivity? Find the photocurrent and the responsivity if a perfect AR coating is used. What is the primary limiting factor? What is the responsivity if W = 40 μm?

$$I_{ph} = \frac{e\eta_i T P_o}{h\upsilon}\{\exp[-\alpha(\ell_p - L_e)] - \exp[-\alpha(\ell_p + W + L_h)]\}$$

EXAMPLE: Steady state photocurrent in the *pin* photodiode

Consider a pin photodiode that is reverse biased and illuminated, as in Figure 5.9 and operating under steady state conditions.

Assume that the photogeneration takes place inside the depletion layer of width W, and the neutral p-side is very narrow.

If the incident optical power on the semiconductor is $P_o(0)$, then $TP_o(0)$ will be transmitted, where T is the transmission coefficient.

At a distance x from the surface, the optical power $P_o(x) = TP_o(0)\exp(-\alpha x)$.

In a small volume δx at x, the absorbed radiation power (by the definition of α) is $\alpha P_o(x)\delta x$, and the number of photons absorbed per second is $\alpha P_o(x)\delta x / h\upsilon$.

Of these absorbed photons, only a fraction η_i will photogenerate EHPs, where η_i is the **internal quantum efficiency** IQE.

Thus, $\eta_i \alpha P_o(x)\delta x / h\upsilon$ number of EHP*s* will be generated per second.

EXAMPLE: Photocarrier Diffusion in a *pin* photodiode
Solution (continued)

$t_{diff} = \ell^2/(2D_e) = (1\times10^{-6}\text{ m})^2 / [2(3\times10^{-4}\text{ m}^2\text{ s}^{-1})] = 1.67\times10^{-9}$ s or **1.67 ns**.

On the other hand, once the electron reaches the depletion region, it becomes drifted across the width W of the *i*-Si layer at the saturation drift velocity since the electric field here is $E = V_r / W = 60$ V / 20 μm = 3×10^6 V m^{-1}; and at this field the electron drift velocity v_e saturates at 10^5 m s^{-1}. The drift time across the *i*-Si layer is

$$t_{drift} = W / v_e = (20\times10^{-6}\text{ m}) / (1\times10^5\text{ m s}^{-1}) = 2.0\times10^{-10}\text{ s or } \mathbf{0.2\ ns}.$$

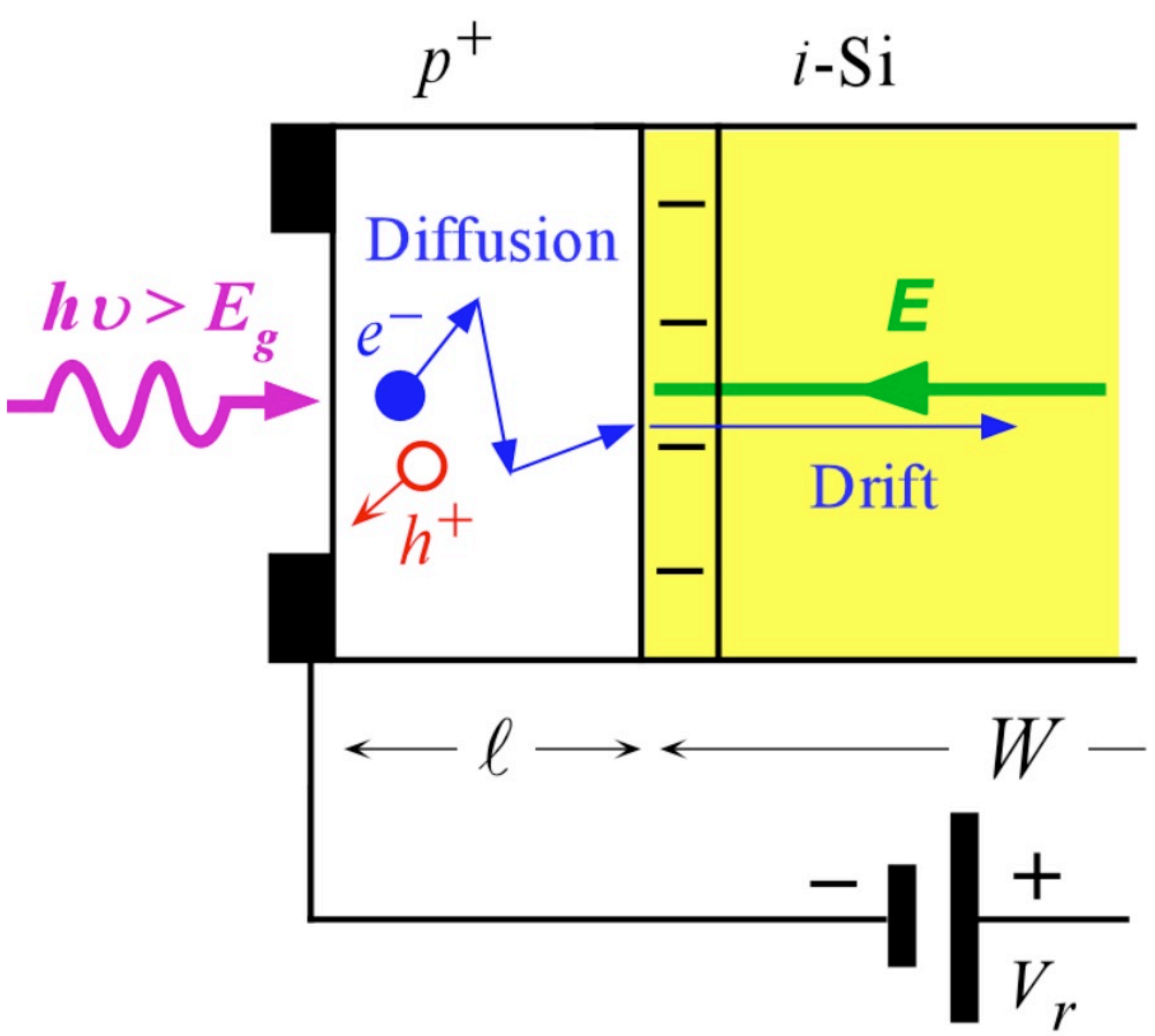
Thus, the response time of the *pin to* a pulse of short wavelength radiation that is absorbed near the surface is very roughly $t_{diff} + t_{drift}$ or 1.87 ns. Notice that the diffusion of the electron is much slower than its drift. In a proper analysis, we have to consider the **diffusion and drift of many carriers, and we have to average ($t_{diff} + t_{drift}$) for all the electrons**.

EXAMPLE : Photocarrier Diffusion in a *pin* photodiode

A reverse biased *pin* photodiode is illuminated with a short wavelength light pulse that is absorbed very near the surface. The photogenerated electron has to diffuse to the depletion region where it is swept into the *i*-layer and drifted across by the field in this region. What is the speed of response of this photodiode if the *i*-Si layer is 20 μm and the p^+-layer is 1 μm and the applied voltage is 60 V? The diffusion coefficient (D_e) of electrons in the heavily doped p^+-region is approximately 3×10^{-4} m^2 s^{-1}.

Solution

There is no electric field in the p^+-side outside the depletion region as shown in Figure 5.12 . The photogenerated electrons have to make it across to the n^+-side to give rise to a photocurrent. In the p^+-side, the electrons move by diffusion. In time *t,* an electron, on average, diffuses a distance ℓ given by



$$\ell = [2D_e t]^{1/2}$$

The *diffusion time* t_{diff} is the time it takes for an electron to diffuse across the p^+-side (of length ℓ) to reach the depletion layer and is given by

EXAMPLE: Operation and speed of a *pin* photodiode

Solution (continued)

At this field the electron drift velocity v_e is very near its saturation at 10^5 m s^{-1}, whereas the hole drift velocity v_h, $\approx 7\times10^4$ m s^{-1} as shown in Figure 5.10. Holes are slightly slower than the electrons. The transit time t_h of holes across the *i*-Si layer is

$$t_h = W/v_h = (20\times10^{-6}\text{ m})/(7\times10^4\text{ m s}^{-1})$$
$$= 2.86\times10^{-10}\text{ s or } \mathbf{0.29\ ns}$$

This is the response time of the *pin* as determined by the transit time of the slowest carriers, holes, across the *i*-Si layer. To improve the response time, the width of the *i*-Si layer has to be narrowed but this decreases the quantity of photons absorbed and hence reduces the responsivity. There is therefore a trade off between speed and responsivity.

EXAMPLE: Noise of an ideal photodetector
Solution (continued)

For an ideal photodetector, $\eta_e = 1$ which leads to Eq. (5.12.9). We note that for a bandwidth of 1Hz, NEP is numerically equal to P_1 or NEP = $2hc/\lambda$.

For an ideal photodetector operating at 1.3 μm and at 1 GHz,

$$P_1 = 2hcB/\eta_e\lambda$$
$$= 2(6.63\times10^{-34}\text{ J s})(3\times10^{8}\text{ m s}^{-1})(10^{9}\text{ Hz}) / (1)(1.3\times10^{-6}\text{ m})$$
$$= 3.1\times10^{-10}\text{ W or } \mathbf{0.31\ nW}.$$

This is the minimum signal for a SNR = 1. The noise current is due to quantum noise. The corresponding photocurrent is

$$I_{ph} = 2eB = 2(1.6\times10^{-19}\text{ C})(10^{9}\text{ Hz}) = 3.2\times10^{-10}\text{ A or } \mathbf{0.32\ nA}.$$

Alternatively we can calculate I_{ph} from $I_{ph} = \eta_e e P_1 \lambda / hc$ with $\eta_e = 1$.

EXAMPLE: Noise of an ideal photodetector

Consider an ideal photodiode with $\eta_e = 1$ (QE = 100%) and no dark current, $I_d = 0$. Show that the minimum optical power required for a signal to noise ratio (SNR) of 1 is

$$\boxed{P_1 = \frac{2hc}{\lambda}B} \qquad (5.12.9)$$

Calculate the minimum optical power for a SNR = 1 for an ideal photodetector operating at 1300 nm with a bandwidth of 1 GHz? What is the corresponding photocurrent?

Solution

We need the incident optical power P_1 that makes the photocurrent I_{ph} equal to the noise current i_n, so that SNR = 1. The photocurrent (signal) is equal to the noise current when

$$I_{ph} = i_n = [2e(I_d + I_{ph})B]^{1/2} = [2eI_{ph}B]^{1/2}$$

since $I_d = 0$. Solving the above, $I_{ph} = 2eB$

From Eqs. (5.4.3) and (5.4.4), the photocurrent I_{ph} *and* the incident optical power P_1 are related by

$$I_{ph} = \frac{\eta_e e P_1 \lambda}{hc} = 2eB$$

Thus, $\boxed{P_1 = \dfrac{2hc}{\eta_e \lambda}B}$

EXAMPLE: Avalanche multiplication in Si APDs
Solution (contiued)

We can now repeat the calculations for $E = 4.30 \times 10^5$ V cm^{-1} and again for $E = 4.38 \times 10^5$ V cm^{-1}. The results are summarized in Table 5.3 for both M and M_e. **Notice how quickly *M* builds up with the field and how a very small change at high fields causes an enormous change in *M* that eventually leads to a breakdown**. (M running away to infinity as V_r increases.) Notice also that in the presence of only electron-initiated ionization, M_e simply increases without a sharp run-away to breakdown.

E (V cm^{-1})	a_e (cm^{-1})	a_h (cm^{-1})	k	M	M_e	Comment
4.00×10^5	4.07×10^4	2.96×10^3	0.073	11.8	7.65	M and M_e not too different at low E
4.30×10^5	4.98×10^4	4.35×10^3	0.087	57.2	12.1	7.5% increase in E, large difference between M and M_e
4.38×10^5	5.24×10^4	4.77×10^3	0.091	647	13.7	1.9% increase in E

EXAMPLE: Avalanche multiplication in Si APDs

The electron and hole ionization coefficients α_e and α_h in silicon are approximately given by Eq. (5.6.4) with $A \approx 0.740 \times 10^6$ cm^{-1}, $B \approx 1.16 \times 10^6$ V cm^{-1} for electrons (α_e) and $A \approx 0.725 \times 10^6$ cm^{-1} and $B \approx 2.2 \times 10^6$ V cm^{-1} for holes (α_h). Suppose that the width w of the avalanche region is 0.5 μm. Find the multiplication gain M when the applied field in this region reaches 4.00×10^5 V cm^{-1}, 4.30×10^5 V cm^{-1} and 4.38×10^5 V cm^{-1}. What is your conclusion?

Solution

At the field of $E = 4.00 \times 10^5$ V cm^{-1}, from Eq. (5.6.4)

$$\alpha_e = A\exp(-B/E)$$
$$= (0.74 \times 10^6 \text{ cm}^{-1})\exp[-(1.16 \times 10^6 \text{ V cm}^{-1})/(4.00 \times 10^5 \text{ V cm}^{-1})]$$
$$= 4.07 \times 10^4 \text{ cm}^{-1}.$$

Similarly using Eq. (5.6.4) for holes, $\alpha_h = 2.96 \times 10^3$ cm^{-1}. Thus $k = \alpha_h/\alpha_e = 0.073$. Using this k and α_e above in Eq. (5.6.6) with $w = 0.5 \times 10^{-4}$ cm,

$$M = \frac{1-0.073}{\exp[-(1-0.073)(4.07\times10^4 \text{ cm})(0.5\times10^{-4} \text{ cm}^{-1})]-0.073} = 11.8$$

Note that if we had only electron avalanche without holes ionizing, then the multiplication would be

$$M_e = \exp(\alpha_e w) = \exp[(4.07 \times 10^4 \text{ cm}^{-1})(0.5 \times 10^{-4} \text{ cm})] = 7.65$$

EXAMPLE: Silicon APD

A Si APD has a QE of 70 % at 830 nm in the absence of multiplication, that is $M = 1$. The APD is biased to operate with a multiplication of 100. If the incident optical power is 10 nW what is the photocurrent?

Solution

The unmultiplied responsivity is given by,

$$R = \eta_e \frac{e\lambda}{hc} = (0.70)\frac{(1.6\times10^{-19}\ \mathrm{C})(830\times10^{-9}\ \mathrm{m})}{(6.626\times10^{-34}\ \mathrm{J\ s})(3\times10^{8}\ \mathrm{m\ s^{-1}})} = 0.47\ \mathrm{A\ W^{-1}}$$

The unmultiplied primary photocurrent from the definition of R *is*

$$I_{pho} = RP_o \ = (0.47\ \mathrm{A\ W^{-1}})(10\times10^{-9}\ \mathrm{W}) \ = 4.7\ \mathrm{nA}$$

The multiplied photocurrent is

$$I_{ph} = MI_{pho} \ = (100)(4.67\ \mathrm{nA}) = \mathbf{470\ nA} \text{ or } \mathbf{0.47\ \mu A}$$

EXAMPLE: InGaAs APD Responsivity

An InGaAs APD has a quantum efficiency (QE, η_e) of 60 % at 1.55 μm in the absence of multiplication ($M = 1$). It is biased to operate with a multiplication of 12. Calculate the photocurrent if the incident optical power is 20 nW. What is the responsivity when the multiplication is 12?

Solution

The responsivity at $M = 1$ in terms of the quantum efficiency is

$$R = \eta_e \frac{e\lambda}{hc} = (0.6)\frac{(1.6\times10^{-19}\ \mathrm{C})(1550\times10^{-9}\ \mathrm{m})}{(6.626\times10^{-34}\ \mathrm{J\ s})(3\times10^{8}\ \mathrm{m\ s^{-1}})} = \mathbf{0.75\ A\ W^{-1}}$$

If I_{pho} is the primary photocurrent (unmultiplied) and P_o is the incident optical power then by definition, $R = I_{pho}/P_o$ so that

$$I_{pho} = RP_o$$
$$= (0.75\ \mathrm{A\ W^{-1}})(20\times10^{-9}\ \mathrm{W})$$
$$= 1.5\times10^{-8}\ \mathrm{A}\ \text{or}\ 15\ \mathrm{nA}.$$

The photocurrent I_{ph} in the APD will be I_{pho} multiplied by M,

$$I_{ph} = MI_{pho}$$
$$= (12)(1.5\times10^{-8}\ \mathrm{A})$$
$$= 1.80\times10^{-7}\ \mathrm{A}\ \text{or}\ 180\ \mathrm{nA}.$$

The responsivity at $M = 12$ is

$$R' = I_{ph}/P_o = MR = (12) / (0.75) = \mathbf{9.0\ A\ W^{-1}}$$

EXAMPLE: Solar cell driving a load

Consider the solar cell driving a 16-Ω resistive load as in Figure 5.42 (b). Suppose that the cell has an area of 1 cm × 1 cm and is illuminated with light of intensity 900 W m^{-2} as in the figure. What are the current and voltage in the circuit? What is the power delivered to the load? What is the efficiency of the solar cell in this circuit? If you assume it is operating close to the maximum deliverable power, what is the FF?

Solution

The *I-V* characteristic of the load is the load line as described by Eq. (5.14.4), $I = -V/R$ with $R = 16\ \Omega$. This line is drawn in Figure 5.42 (b) with a slope 1/(16 Ω). It cuts the *I-V* characteristics of the solar cell at $I' \approx -31.5$ mA and $V' \approx 0.505$ V which are the current and voltage in the photovoltaic circuit of Figure 5.42 (b). In fact, from Eq. (5.14.4), V'/I' gives – 16 Ω as expected. The power delivered to the load is

$$P_{out} = |I'V'| = (31.5\times10^{-3}\ \text{A})(0.505\ \text{V}) = \mathbf{0.0159\ W} \text{ or } \mathbf{15.9\ mW}$$

This is not necessarily the maximum power available from the solar cell. The input sun-light power is

$$P_{in} = (\text{Light Intensity})(\text{Surface Area}) = (900\ \text{W m}^{-2})(0.01\ \text{m})^2$$
$$= \mathbf{0.090\ W}$$

EXAMPLE: Noise in an APD
Solution (continued)

(c) The SNR with a primary photocurrent I_{pho} in the APD is

$$\text{SNR} = \frac{\text{Signal Power}}{\text{Noise Power}} = \frac{M^2 I_{pho}^2}{\left[2e(I_{do} + I_{pho})M^{2+x}B\right]}$$

Rearranging to obtain I_{pho} we get,

$$(M^2)I_{pho}^2 - [2eM^{2+x}B(\text{SNR})]I_{pho} - [2eM^{2+x}B(\text{SNR})I_{do}] = 0$$

This is a quadratic equation in I_{pho} with defined coefficients since M, x, B, I_{do} and SNR are given. Solving this quadratic with a SNR = 10 for I_{pho} we find

$I_{pho} \approx 1.76 \times 10^{-8}$ A or 17.6 nA

While it may seem odd that I_{pho} is less than the dark noise current (33.5 nA) itself, the actual photocurrent I_{ph} however is 176 nA, since it is multiplied by M. Further the total noise current, $i_{n\text{-APD}} = [2e(I_{do} + I_{pho})M^{2+x}B]^{1/2}$ is 55.7 nA so that one can easily check that SNR = $I_{ph}^2 / i^2_{n\text{-APD}}$ is indeed 10.

By the definition of responsivity, $R = I_{pho}/P_o$, we find,

$P_o = I_{pho} / R = (1.76\times10^{-8}\text{ A})/(0.8\text{ A W}^{-1}) = 2.2\times10^{-8}$ W or **22 nW**

EXAMPLE: Noise in an APD

Consider an InGaAs APD with $x \approx 0.7$ which is biased to operate at $M = 10$. The unmultiplied dark current is 10 nA and bandwidth is 700 MHz.

(a) What is the APD noise current per square root of bandwidth?

(b) What is the APD noise current for a bandwidth of 700 MHz?

(c) If the responsivity (at $M = 1$) is 0.8 A W^{-1} what is the minimum optical power for a SNR of 10?

Solution

(a) In the absence of any photocurrent, the noise in the APD comes from the dark current. If the unmultipled dark current is I_{do} then the noise current (rms) is

$$i_{n\text{-dark}} = [2eI_{do}M^{2+x}B]^{1/2}$$

Thus,

$$\frac{i_{n\text{-dark}}}{\sqrt{\mathrm{B}}} = \sqrt{2eI_{do}M^{2+x}} = \sqrt{2(1.6\times10^{-19}\ \mathrm{C})(10\times10^{-9}\ \mathrm{A})(10)^{2+0.7}}$$

$$= 1.27 \times 10^{-12}\ \mathrm{A\ Hz^{-1/2}}\ \text{or}\ \mathbf{1.27\ pA\ Hz^{-1/2}}.$$

(b) In a bandwidth B of 700 MHz, the noise current is

$$i_{n\text{-dark}} = (700\times10^{6}\ \mathrm{Hz})^{1/2}(1.27\ \mathrm{pA\ Hz^{-1/2}})$$
$$= 3.35\times10^{-8}\ \mathrm{A}\ \text{or}\ \mathbf{33.5\ nA}.$$

EXAMPLE: SNR of a receiver
Solution (continued)

Shot noise current from the detector = $[2e(I_d + I_{ph})B]^{1/2} = 0.047$ nA

$= 1.29$ nA

Thus, the noise contribution from R_L is greater than that from the photodiode. The SNR is

$$\mathrm{SNR} = \frac{(5\times10^{-9}\ \mathrm{A})^2}{(0.047\times10^{-9}\ \mathrm{A})^2 + (1.29\times10^{-9}\ \mathrm{A})^2} = 15.0$$

Generally SNR is quoted in decibels. We need 10log(SNR), or 10log(15.0) *i.e.*, 11.8 dB. Clearly, **the load resistance has a dramatic effect on the overall noise performance**.

EXAMPLE: SNR of a receiver

Consider an InGaAs *pin* photodiode used in a receiver circuit as in Figure 5.31 with a load resistor of 10 kΩ. The photodiode has a dark current of 2 nA. The bandwidth of the photodiode and the amplifier together is 1 MHz. Assuming that the amplifier is noiseless, calculate the SNR when the incident optical power generates a mean photocurrent of 5 nA (corresponding to an incident optical power of about 6 nW since *R* is about 0.8–0.9 nA/nW at the peak wavelength of 1550 nm).

Solution

The noise generated comes from the photodetector as shot noise and from R_L *as* thermal noise. The mean thermal noise power in the load resistor R_L is $4k_BTB$. If I_{ph} is the photocurrent and i_n is the shot noise in the photodetector then

$$\mathrm{SNR} = \frac{\text{Signal Power}}{\text{Noise Power}} = \frac{I_{ph}^2 R_L}{i_n^2 R_L + 4k_BTB} = \frac{I_{ph}^2}{\left[2e(I_d + I_{ph})B\right] + 4k_BTB / R_L}$$

The term $4k_BTB/R_L$ in the denominator represents the mean square of the thermal noise current in the resistor. We can evaluate the magnitude of each noise current by substituting, $I_{ph} = 5$ nA, $I_d = 2$ nA, $B = 1$ MHz, $R_L = 10^4\ \Omega$, T = 300 K.

EXAMPLE: NEP of a Si *pin* photodiode

A Si *pin* photodiode has a quoted NEP of 1×10^{-13} W Hz$^{-1/2}$. What is the optical signal power it needs for a signal to noise ratio (SNR) of 1 if the bandwidth of operation is 1GHz?

Solution

By definition, NEP is that optical power per square root of bandwidth which generates a photocurrent equal to the noise current in the detector.

$$\mathbf{NEP} = \boldsymbol{P_1/B^{1/2}}$$

Thus,

$$P_1 = \mathrm{NEP}B^{1/2}$$

$$= (10^{-13}\ \mathrm{W\ Hz}^{-1/2})(10^9\ \mathrm{Hz})^{1/2}$$

$$= 3.16\times10^{-9}\ \mathrm{W\ or\ \mathbf{3.16\ nW}}$$

EXAMPLE: Open circuit voltage and short circuit current

Solution (continued)

In Eq. (5.14.6), the photocurrent, I_{ph}, depends on the light intensity I via, $I_{ph} = KI$. At a given temperature, then the change in V_{oc} is

$$V_{oc2} - V_{oc1} = \frac{\eta k_B T}{e} \ln\left(\frac{I_{ph2}}{I_{ph1}}\right) = \frac{\eta k_B T}{e} \ln\left(\frac{I_2}{I_1}\right)$$

Assuming $\eta = 1$, the new open circuit voltage is

$$V_{oc2} = V_{oc1} + \frac{\eta k_B T}{e} \ln\left(\frac{I_2}{I_1}\right) = 0.50 \text{ V} + (1)(0.0259 \text{ V})\ln(2) \approx \mathbf{0.52\ V}$$

NOTE: This is a ~4% increase in V_{oc} compared with the 100% increase in illumination and the short circuit current.

EXAMPLE: Open circuit voltage and short circuit current

A solar cell under an illumination of 500 W m^{-2} has a short circuit current I_{sc} of –16 mA and an open circuit output voltage V_{oc}, of 0.50 V. What are the short circuit current and open circuit voltages when the light intensity is doubled? Assume $\eta = 1$.

Solution

The general I-V characteristics under illumination is given by Eq. (5.14.3). The short circuit current corresponds to the photocurrent so that, from Eq. (5.14.2), at double the intensity the photocurrent is

$$\boxed{I_{ph2} = \left(\frac{I_2}{I_1}\right) I_{ph1}} = (16\text{ mA})(1000/500) = 32\text{ mA}$$

Setting $I = 0$ for open circuit we can obtain the open circuit voltage V_{oc},

$$I = -I_{ph} + I_o[\exp(eV_{oc}/\eta k_B T) - 1] = 0$$

Assuming that $V_{oc} >> \eta k_B T/e$, rearranging the above equation we can find V_{oc}

$$\boxed{V_{oc} = \frac{\eta k_B T}{e} \ln\left(\frac{I_{ph}}{I_o}\right)} \quad \textit{Open circuit output voltage} \qquad (5.14.6)$$

EXAMPLE: Solar cell driving a load
Solution (continued)

The efficiency is

$$\text{Efficiency} = 100 \times (P_{out} / P_{in}) = 100\ (15.9\ \text{mW} / 90\ \text{mW})$$

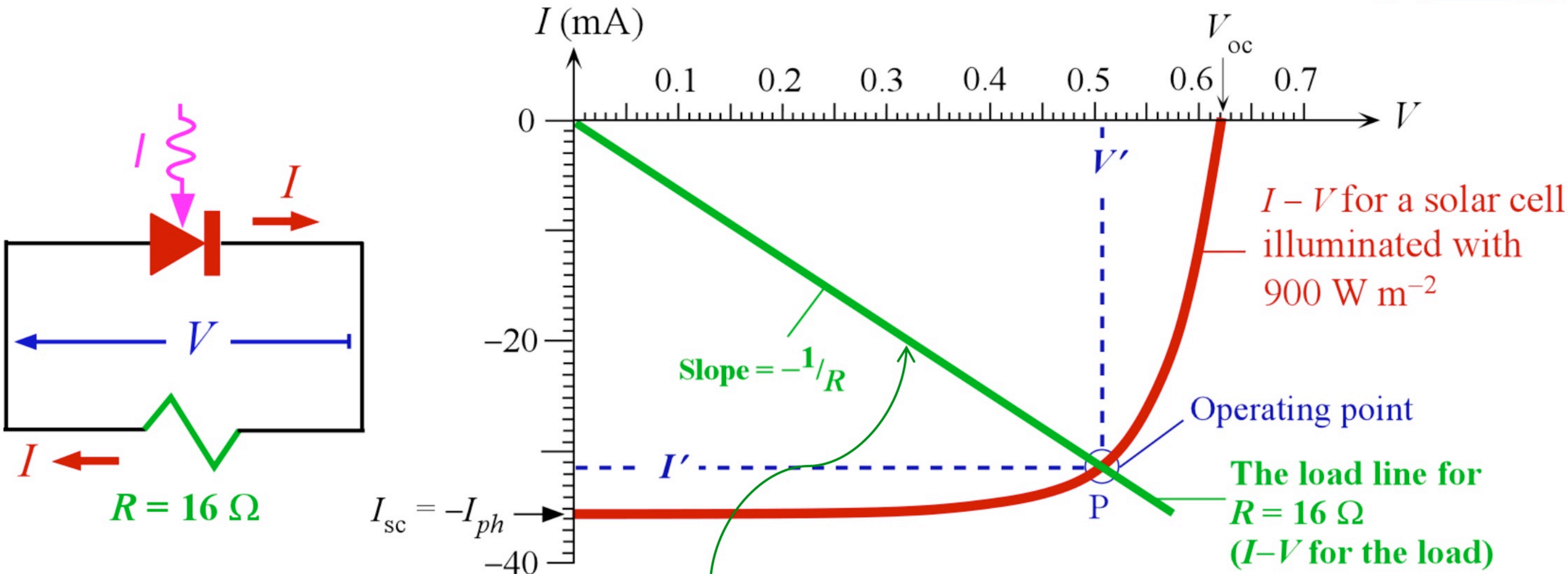
$$= \mathbf{17.7\%}$$

This will increase if the load is adjusted to extract the maximum power from the solar cell but the increase will be small as the rectangular area $I'V'$ in Figure 5.42 in it is already close to the maximum. Assuming that $|I'V'|$ is roughly the maximum power available (maximum area for the rectangle $I'V'$), then $I_m \approx I' \approx -31.5$ mA and $V_m \approx V' \approx 0.505$ V. For the solar cell in Figure 5.42 (b), $I_{sc} = -35.5$ mA and $V_{oc} = 0.62$ V. Then,

$$FF = I_m V_m / I_{sc} V_{oc} \approx (-31.5\ \text{mA})(0.505\ \text{V}) / (-35.5\ \text{mA})(0.62\ \text{V})$$

$$= \mathbf{0.72} \text{ or } \mathbf{72\%}$$

EXAMPLE: Solar cell driving a load



$$I = -\frac{V}{R}$$ **Load line**

LEFT: A solar cell driving a load R and the definitions of positive current I and voltage V. RIGHT: The load line construction for finding the operating point when a load $R_L = 16\ \Omega$ is connected across the solar cell.