

# Chapter 3 Semiconductor Science and Light-Emitting Diodes



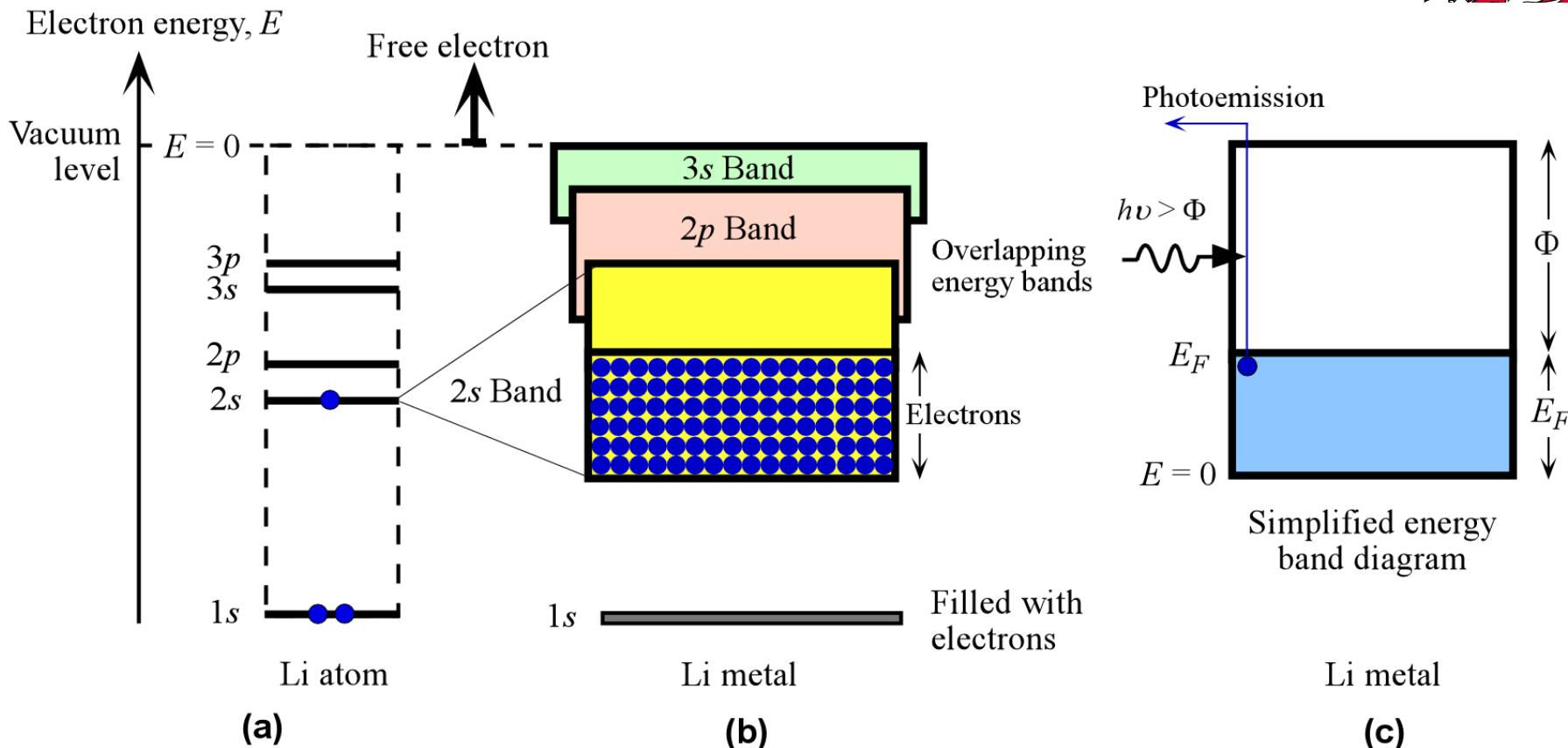
*I wanted visible light, because I knew that if I could get red light, other colors would be possible.*

—Nick Holonyak Jr.



Nick Holonyak Jr. carried out the early work in the development of practical light-emitting diodes (LEDs) in the visible spectrum during the 1960s while working as a consulting research scientist for General Electric Co. in Syracuse. He made his first visible laser-LED in 1962, which emitted red light.<sup>1</sup> It served as the basis for the concept of the “ultimate lamp.” Since 1963, he has been at the University of Illinois at Urbana-Champaign, where he currently holds the John Bardeen Endowed Chair. In the February 1963 issue of *Readers Digest*, Nick Holonyak Jr. suggested that the incandescent light bulb will eventually be replaced by the LED. The above photo was taken around 1970–1975. (Courtesy of University of Illinois at Urbana-Champaign.)

# Energy Bands in Metals

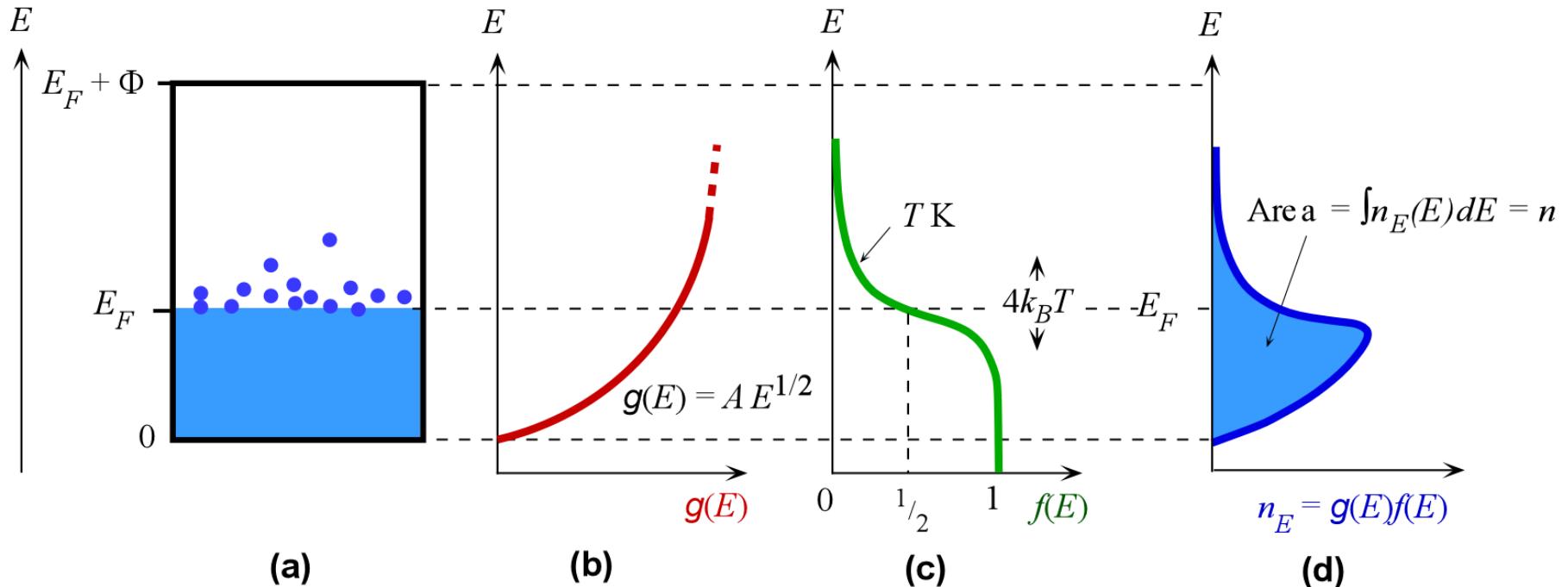


(a) Energy levels in a Li atom are discrete. (b) The energy levels corresponding to outer shells of isolated Li atoms form an energy band inside the crystal, for example the 2s level forms a 2s band.

Energy levels form a quasi continuum of energy within the energy band. Various energy bands overlap to give a single band of energies that is only partially full of electrons. There are states with energies up to the vacuum level where the electron is free. (c) A simplified energy band diagram and the photoelectric effect.



# Energy Bands in Metals



- (a) Above 0 K, due to thermal excitation, some of the electrons are at energies above  $E_F$ .
- (b) The density of states,  $g(E)$  vs.  $E$  in the band.
- (c) The probability of occupancy of a state at an energy  $E$  is  $f(E)$ . The product  $g(E)f(E)$  is the number of electrons per unit energy per unit volume or electron concentration per unit energy. The area under the curve with the energy axis is the concentration of electrons in the band,  $n$ .



# Energy Bands in Metals

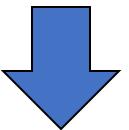
Density of states

$$g(E) = 4\pi(2m_e)^{3/2} h^{-3} E^{1/2} = AE^{1/2}$$

Fermi-Dirac function

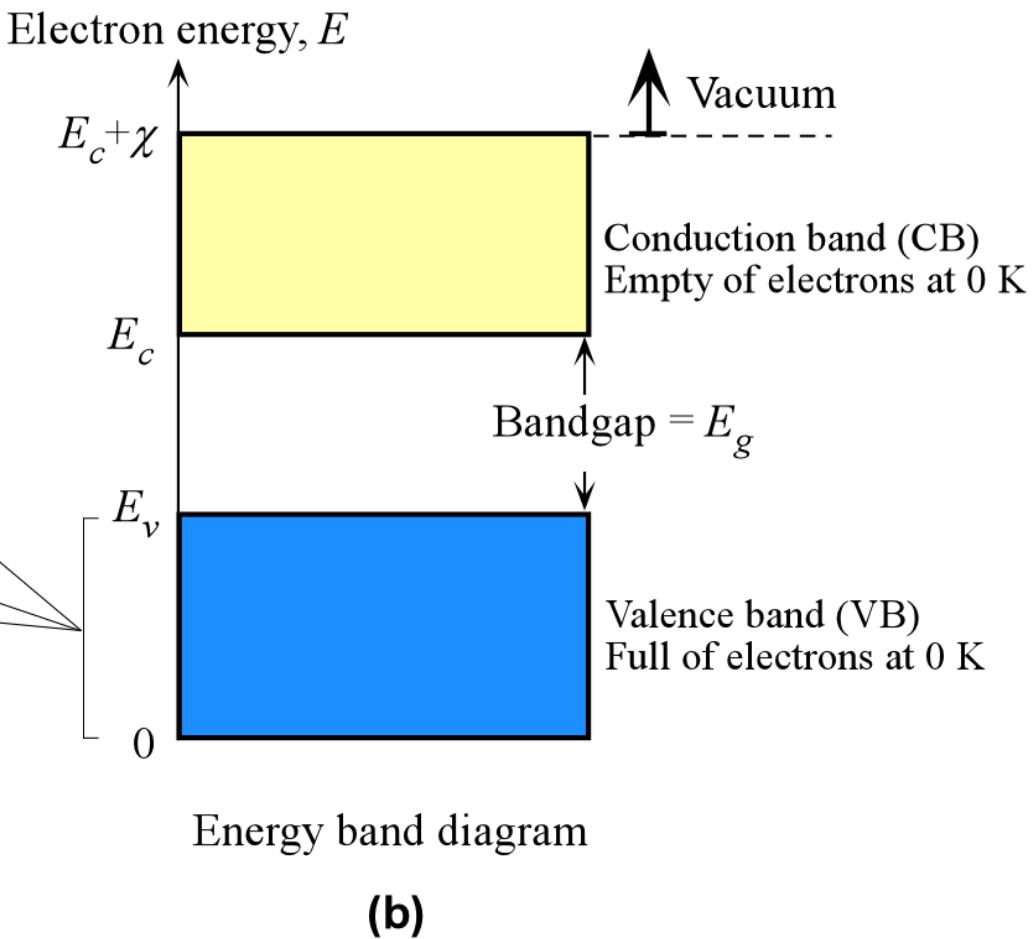
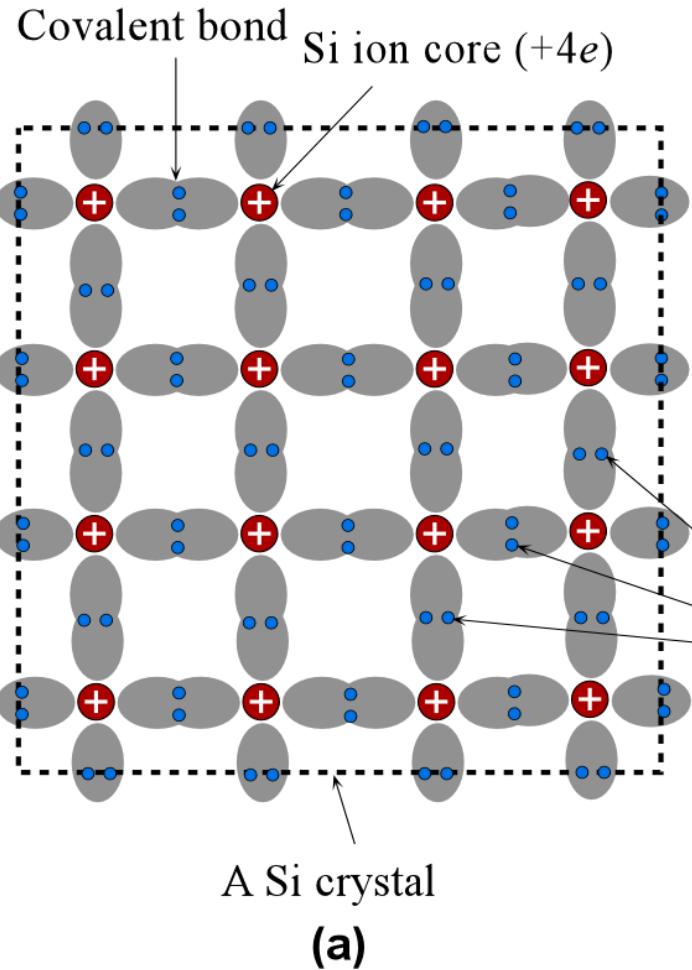
$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{k_B T}\right)}$$

$$n = \int_0^{E_F + \Phi} g(E) f(E) dE$$



$$E_{FO} = \left( \frac{h^2}{8m_e} \right) \left( \frac{3n}{\pi} \right)^{2/3}$$

# Energy Bands in Semiconductors

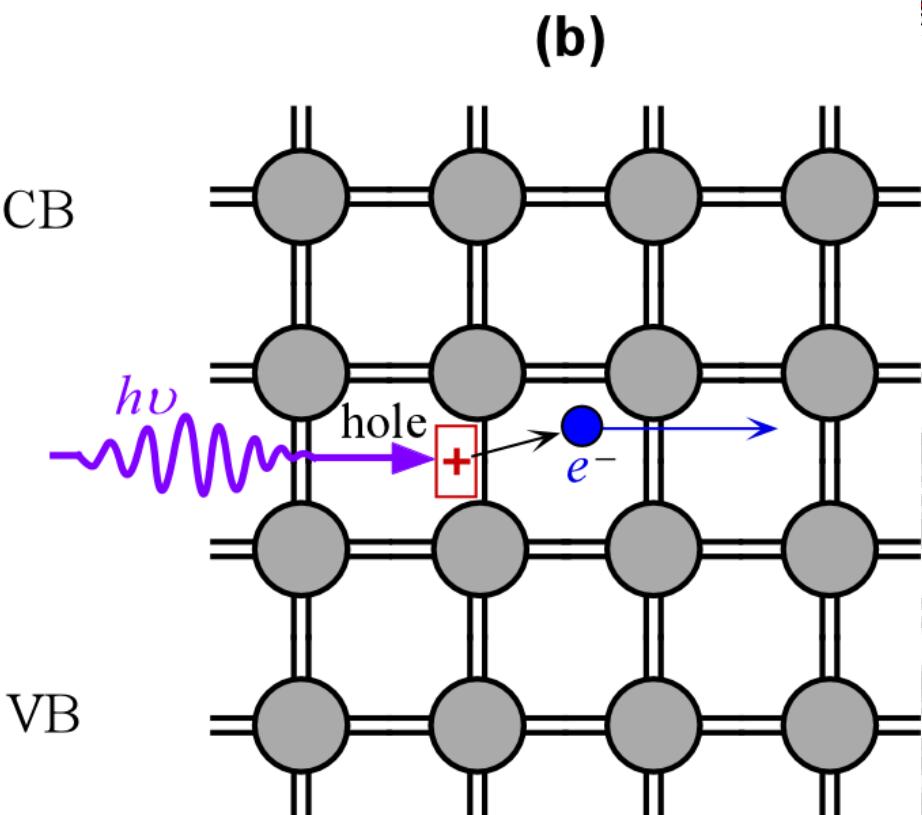
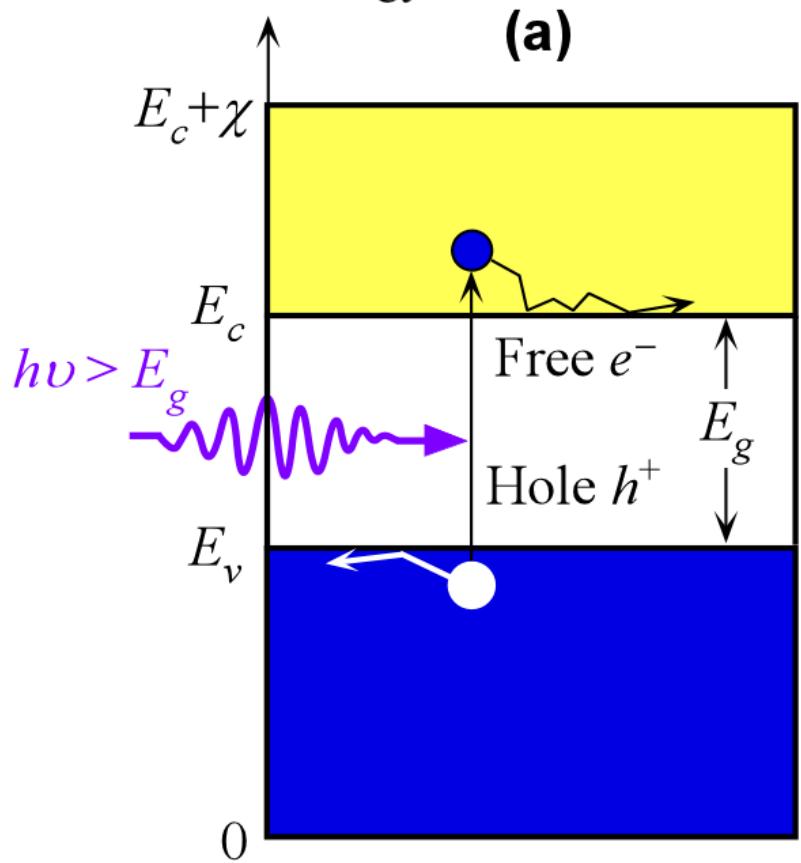


(a) A simplified two dimensional view of a region of the Si crystal showing covalent bonds. (b) The energy band diagram of electrons in the Si crystal at absolute zero of temperature. The bottom of the VB has been assigned a zero of energy.

# Energy Bands in Semiconductors

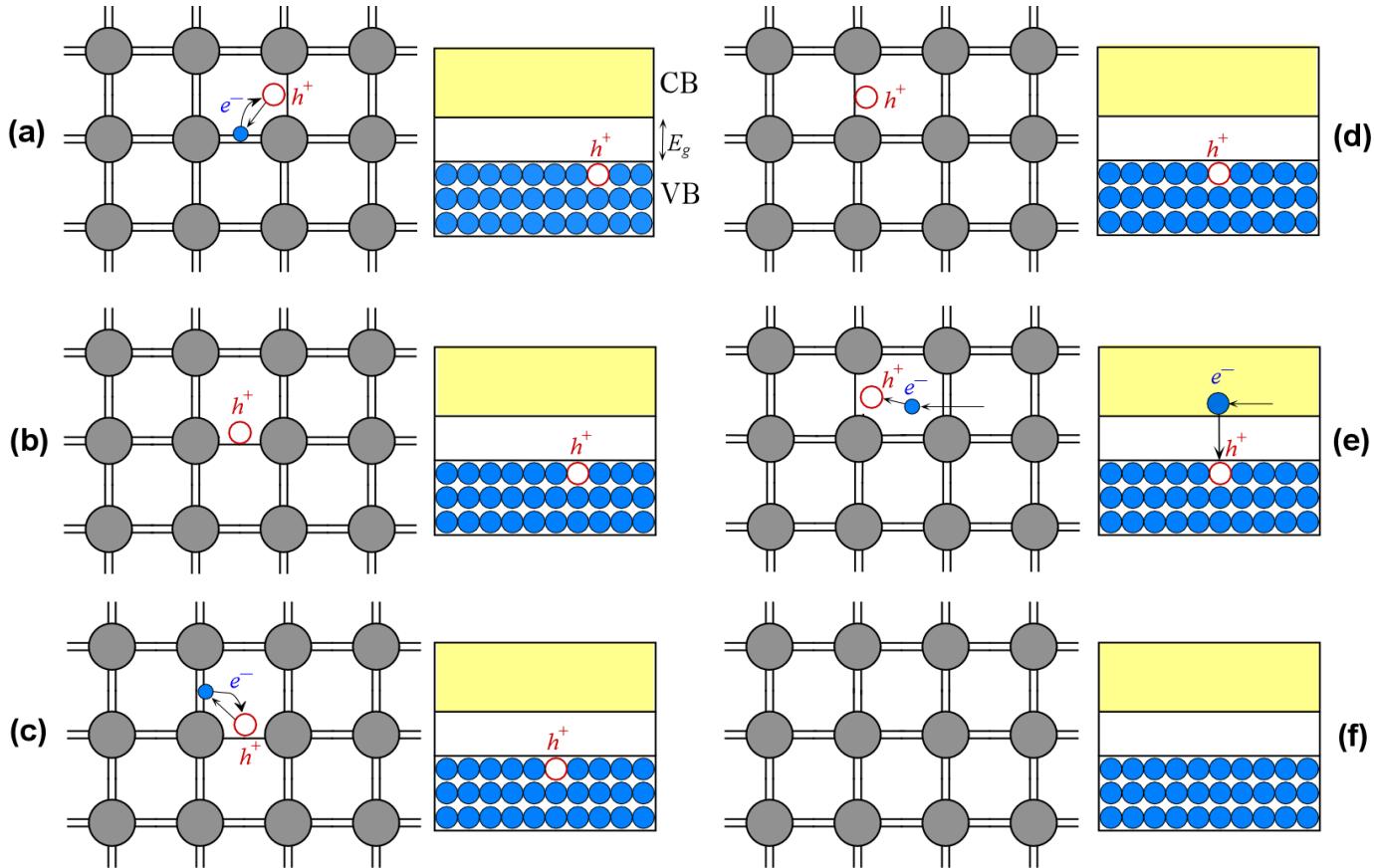


Electron energy,  $E$



(a) A photon with an energy  $h\nu$  greater than  $E_g$  can excite an electron from the VB to the CB. (b) Each line between Si-Si atoms is a valence electron in a bond. When a photon breaks a Si-Si bond, a free electron and a hole in the Si-Si bond is created. The result is the photogeneration of an electron and a hole pair (EHP)

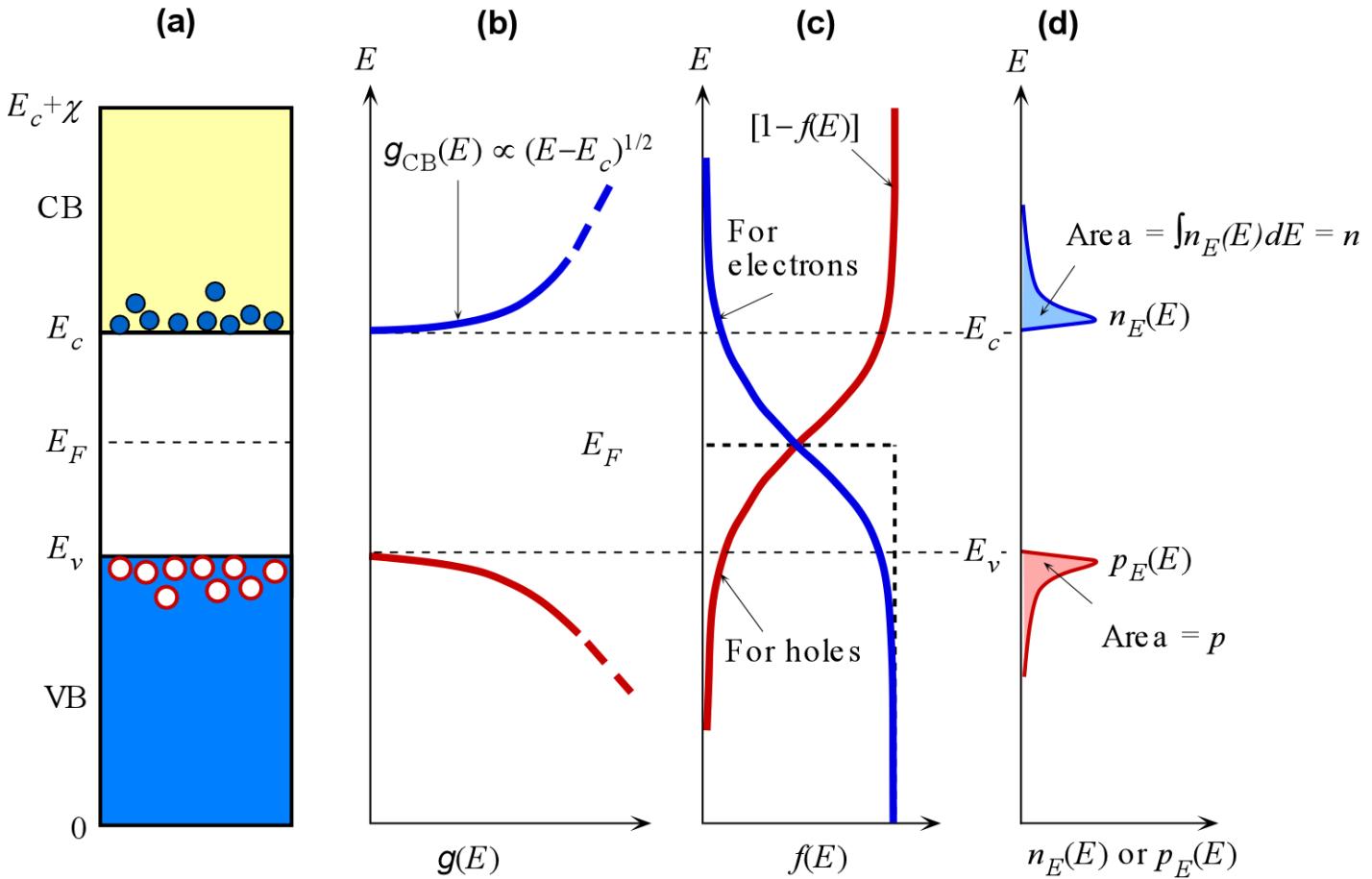
# Hole Motion in a Semiconductor



A pictorial illustration of a hole in the valence band (VB) wandering around the crystal due to the tunneling of electrons from neighboring bonds; and its eventual recombination with a wandering electron in the conduction band. A missing electron in a bond represents a hole as in (a). An electron in a neighboring bond can tunnel into this empty state and thereby cause the hole to be displaced as in (a) to (d). The hole is able to wander around in the crystal as if it were free but with a different effective mass than the electron. A wandering electron in the CB meets a hole in the VB in (e), which results in the recombination and the filling of the empty VB state as in (f).



# Semiconductor Statistics



(a)

Energy band diagram.

(b)

Density of states (number of states per unit energy per unit volume).

(c)

Fermi-Dirac probability function (probability of occupancy of a state).

(d)

The product of  $g(E)$  and  $f(E)$  is the energy density of electrons in the CB (number of electrons per unit energy per unit volume). The area under  $n_E(E)$  versus  $E$  is the electron concentration.

# Electron Concentration in CB



$$n = \int_{E_c}^{E_c + \chi} g_{\text{CB}}(E) f(E) dE$$

$$\begin{aligned} g_{\text{CB}}(E) &= 4\pi(2m_e)^{3/2} h^{-3} (E - E_c)^{1/2} \\ &= A(E - E_c)^{1/2} \end{aligned}$$

Density of states in the CB

$$f(E) \approx \exp\left(-\frac{E - E_F}{k_B T}\right)$$

Fermi-Dirac function  $\approx$   
Boltzmann function

$$n = N_c \exp\left[-\frac{(E_c - E_F)}{k_B T}\right]$$

# Electron Concentration in CB



$$n = N_c \exp\left[-\frac{(E_c - E_F)}{k_B T}\right]$$

$n$  = Electron concentration in the CB

$N_c$  = Effective density of states at the CB edge

$E_c$  = Conduction band edge,  $E_F$  = Fermi energy

$k_B$  = Boltzmann constant,  $T$  = Temperature (K)

## Effective Density of States at CB Edge

$$N_c = 2 \left( \frac{2\pi m_e^* k_B T}{h^2} \right)^{3/2}$$

$N_c$  = Effective density of states at the CB edge,  $m_e^*$  = Effective mass of the electron in the CB,  $k$  = Boltzmann constant,  $T$  = Temperature,  $h$  = Planck's constant



# Hole Concentration in VB

$$p = N_v \exp\left[-\frac{(E_F - E_v)}{k_B T}\right]$$

$p$  = Hole concentration in the VB

$N_v$  = Effective density of states at the VB edge

$E_v$  = Valence band edge,  $E_F$  = Fermi energy

$k_B$  = Boltzmann constant, ,  $T$  = Temperature (K)

## Effective Density of States at VB Edge

$$N_v = 2\left(\frac{2\pi m_h^* k_B T}{h^2}\right)^{3/2}$$

$N_v$  = Effective density of states at the VB edge,  $m_h^*$  = Effective mass of a hole in the VB,  $k$  = Boltzmann constant,  $T$  = Temperature,  $h$  = Planck's constant

# Mass Action Law



$$np = n_i^2 = N_c N_v \exp\left(-\frac{E_g}{k_B T}\right)$$

$n_i$  = intrinsic concentration

The  $np$  product is a “constant”,  $n_i^2$ , that depends on the material properties  $N_c$ ,  $N_v$ ,  $E_g$ , and the temperature. If somehow  $n$  is increased (e.g. by doping),  $p$  must decrease to keep  $np$  constant.

**Mass action law applies**

**in thermal equilibrium**

**and**

**in the dark (no illumination)**



# Fermi Level in Intrinsic Semiconductors

$$E_{Fi} = E_v + \frac{1}{2} E_g - \frac{1}{2} k_B T \ln\left(\frac{N_c}{N_v}\right)$$

$E_{Fi}$  = Fermi level in the intrinsic semiconductor

$E_v$  = Valence band edge,  $E_g = E_c - E_v$  = Bandgap energy

$k_B$  = Boltzmann constant,  $T$  = temperature

$N_c$  = Effective density of states at the CB edge

$N_v$  = Effective density of states at the VB edge

$$E_{Fi} = E_v + \frac{1}{2} E_g - \frac{3}{4} k_B T \ln\left(\frac{m_e^*}{m_h^*}\right)$$

$m_e^*$  = Electron effective mass (CB),  $m_h^*$  = Hole effective mass (VB)

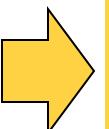
**Fermi energy is the highest energy that could take an electron at 0K**  
Fermi level is the 50% point due to the distribution of carriers at T>0K

# Average Electron Energy in CB



Average is found from

$$\bar{E}_{\text{CB}} = \frac{\int_{E_c}^{\infty} E g_{\text{CB}}(E) f(E) dE}{\int_{E_c}^{\infty} g_{\text{CB}}(E) f(E) dE}$$



$$\bar{E}_{\text{CB}} = E_c + \frac{3}{2} k_B T$$

$\bar{E}_{\text{CB}}$  = Average energy of electrons in the CB,  $E_c$  = Conduction band edge,  $k_B$  = Boltzmann constant,  $T$  = Temperature

$(3/2)k_B T$  is also the average kinetic energy per atom in a monatomic gas (kinetic molecular theory) in which the gas atoms move around freely and randomly inside a container.

The electron in the CB behaves as if it were “free” with a mean kinetic energy that is  $(3/2)k_B T$  and an effective mass  $m_e^*$ .

# Fermi level

**Fermi level** is a convenient way to represent free carrier concentrations ( $n$  in the CB and  $p$  in the VB) on the energy band diagram.

However, the most useful property of  $E_F$  is in terms of a change in  $E_F$ .

Any change  $\Delta E_F$  across a material system represents electrical work input or output per electron.

$$\Delta E_F = eV$$

For a semiconductor system in equilibrium, in the dark, and with no applied voltage or no emf generated,  $\Delta E_F = 0$  and  $E_F$  must be uniform across the system.

For readers familiar with thermodynamics, its rigorous definition is that  $E_F$  is the *chemical potential of the electron*, that is Gibbs free energy per electron. The definition of  $E_F$  above is in terms of a change in  $E_F$ .

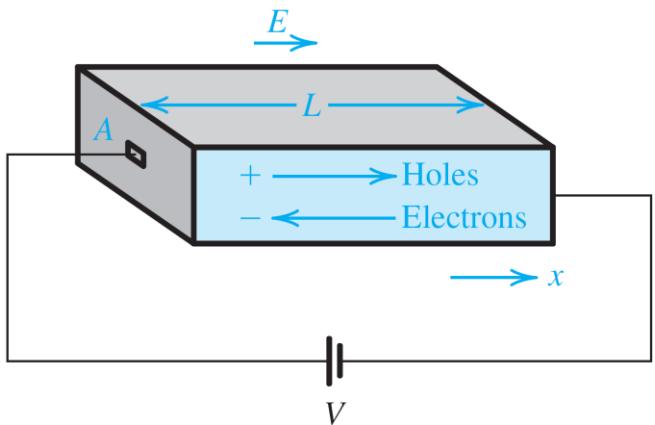


# Electron and Hole Drift Velocities

$$v_{de} = \mu_e E_x$$

$$v_{dh} = \mu_h E_x$$

$v_{de}$  = Drift velocity of the electrons,  $\mu_e$  = Electron drift mobility,  $E_x$  = Applied electric field,  $v_{dh}$  = Drift velocity of the holes,  $\mu_h$  = Hole drift mobility

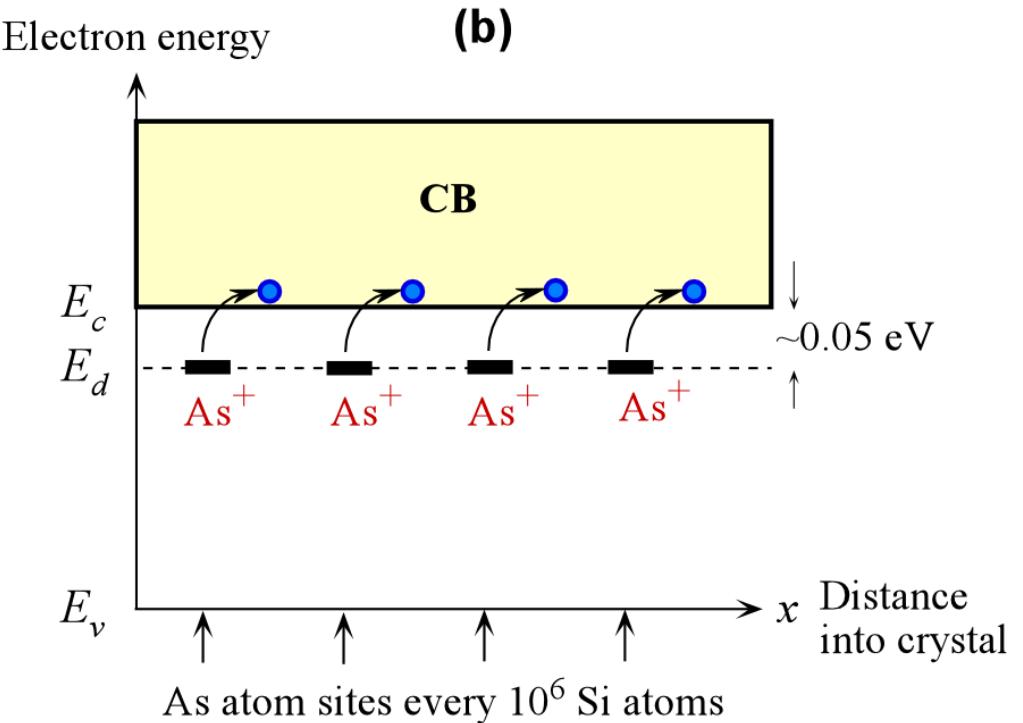
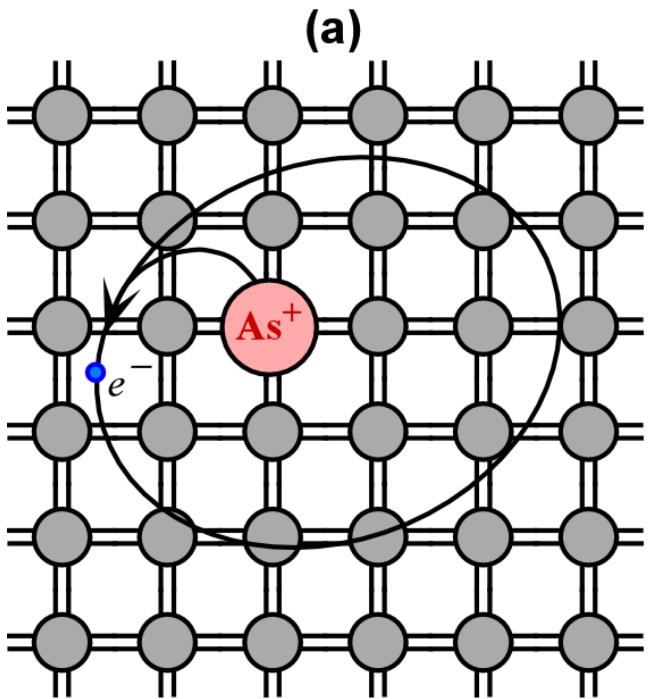


## Conductivity of a Semiconductor

$$\sigma = en\mu_e + ep\mu_h$$

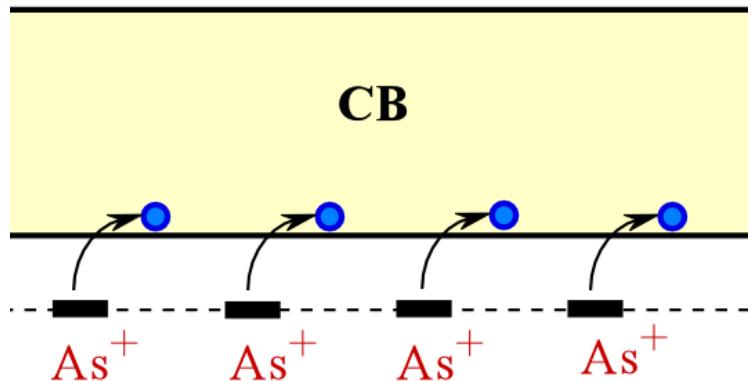
$\sigma$  = Conductivity,  $e$  = Electronic charge,  $n$  = Electron concentration in the CB,  $\mu_e$  = Electron drift mobility,  $p$  = Hole concentration in the VB,  $\mu_h$  = Hole drift mobility

# Extrinsic Semiconductors: *n*-Type



(a) The four valence electrons of As allow it to bond just like Si but the fifth electron is left orbiting the As site. The energy required to release to free fifth-electron into the CB is very small. (b) Energy band diagram for an *n*-type Si doped with 1 ppm As. There are donor energy levels just below  $E_c$  around As<sup>+</sup> sites.

# Extrinsic Semiconductors: *n*-Type



$N_d \gg n_i$ , then at room temperature, the electron concentration in the CB will nearly be equal to  $N_d$ , i.e.  $n \approx N_d$

A small fraction of the large number of electrons in the CB recombine with holes in the VB so as to maintain  $np = n_i^2$

$$np = n_i^2$$

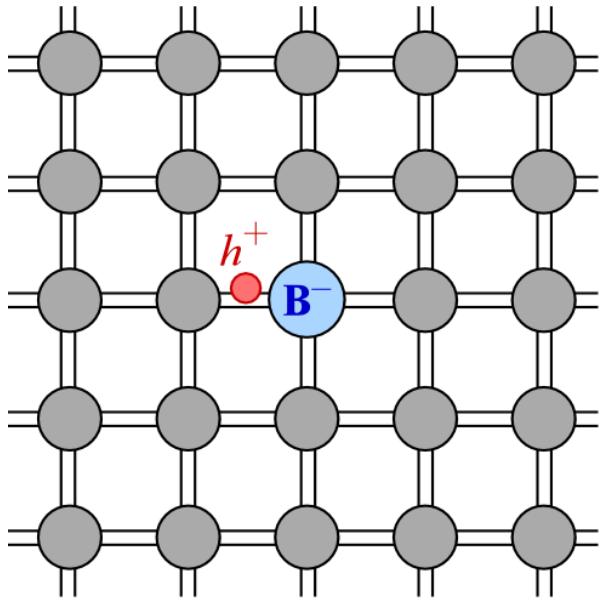
$$n = N_d \text{ and } p = n_i^2/N_d$$

$$\sigma = eN_d\mu_e + e\left(\frac{n_i^2}{N_d}\right)\mu_h \approx eN_d\mu_e$$

# Extrinsic Semiconductors: *p*-Type

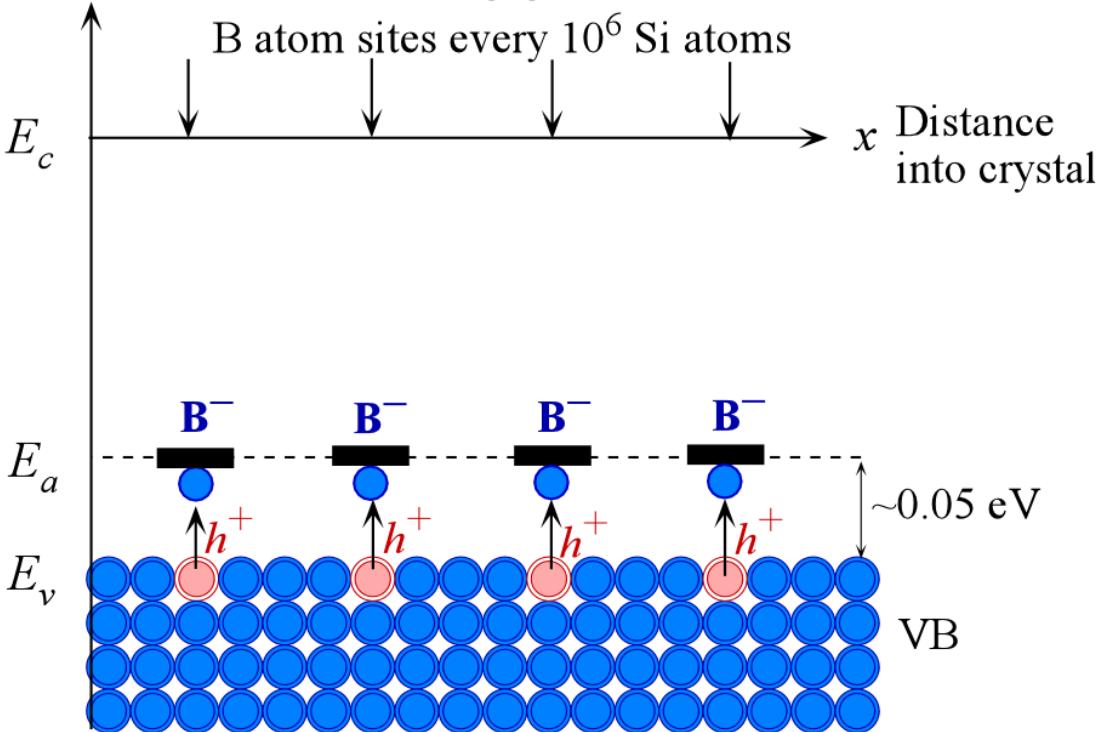


(a)



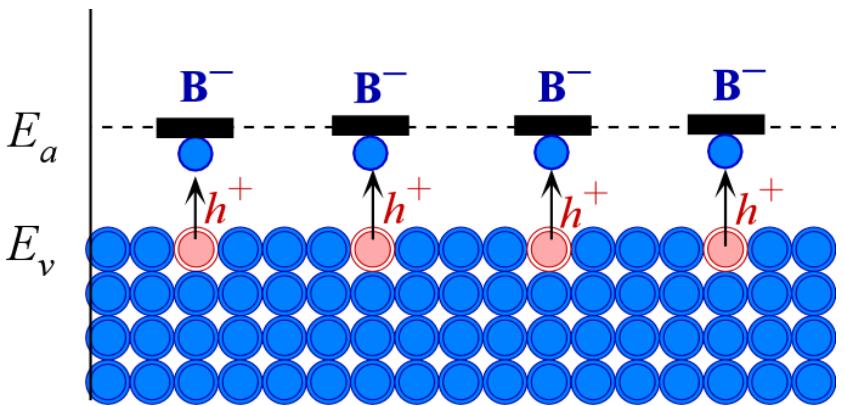
Electron energy

(b)



(a) Boron doped Si crystal. B has only three valence electrons. When it substitutes for a Si atom one of its bonds has an electron missing and therefore a hole. (b) Energy band diagram for a p-type Si doped with 1 ppm B. There are acceptor energy levels just above  $E_v$  around  $B^-$  sites. These acceptor levels accept electrons from the VB and therefore create holes in the VB.

# Extrinsic Semiconductors: *n*-Type



$N_a \gg n_i$ , then at room temperature, the hole concentration in the VB will nearly be equal to  $N_a$ , i.e.  $p \approx N_d$

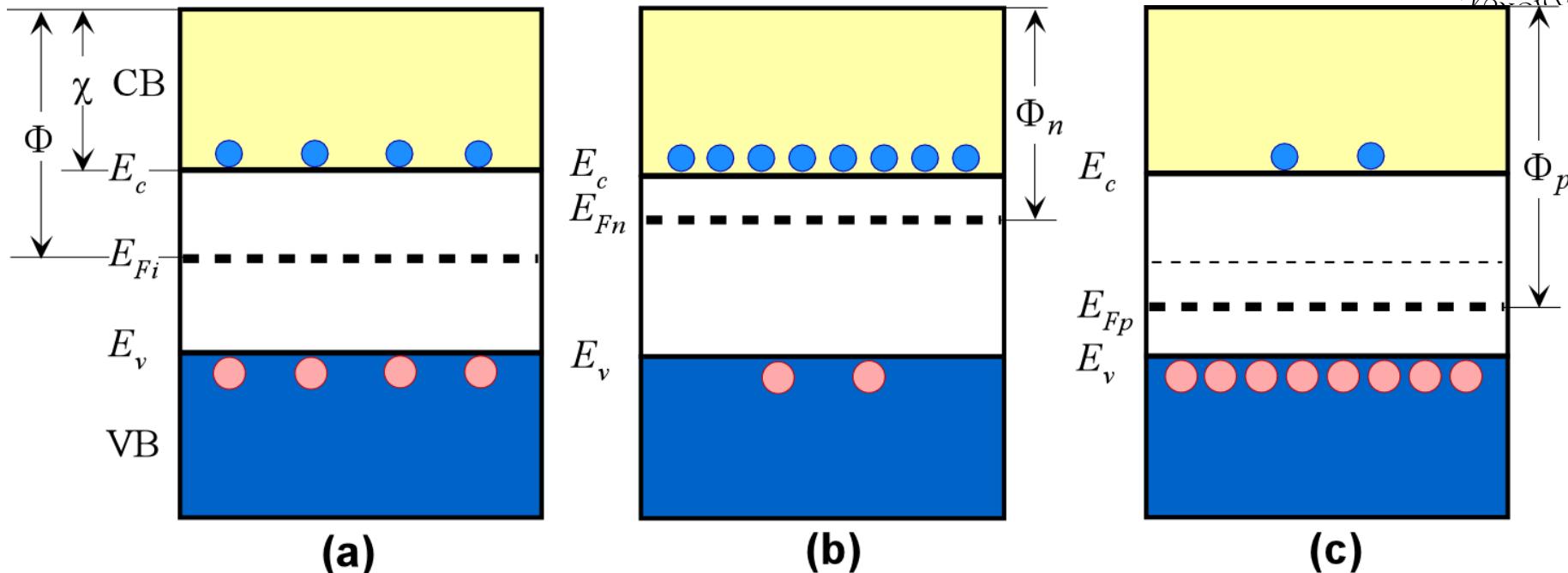
A small fraction of the large number of holes in the VB recombine with electrons in the CB so as to maintain  $np = n_i^2$

$$np = n_i^2$$

$$p = N_a \text{ and } n = n_i^2/N_a$$

$$\sigma = eN_a\mu_h + e\left(\frac{n_i^2}{N_a}\right)\mu_e \approx eN_a\mu_h$$

# Semiconductor energy band diagrams



Intrinsic, *i*-Si

$$n = p = n_i$$

*n*-type

$$n = N_d$$

$$p = n_i^2/N_d$$

$$np = n_i^2$$

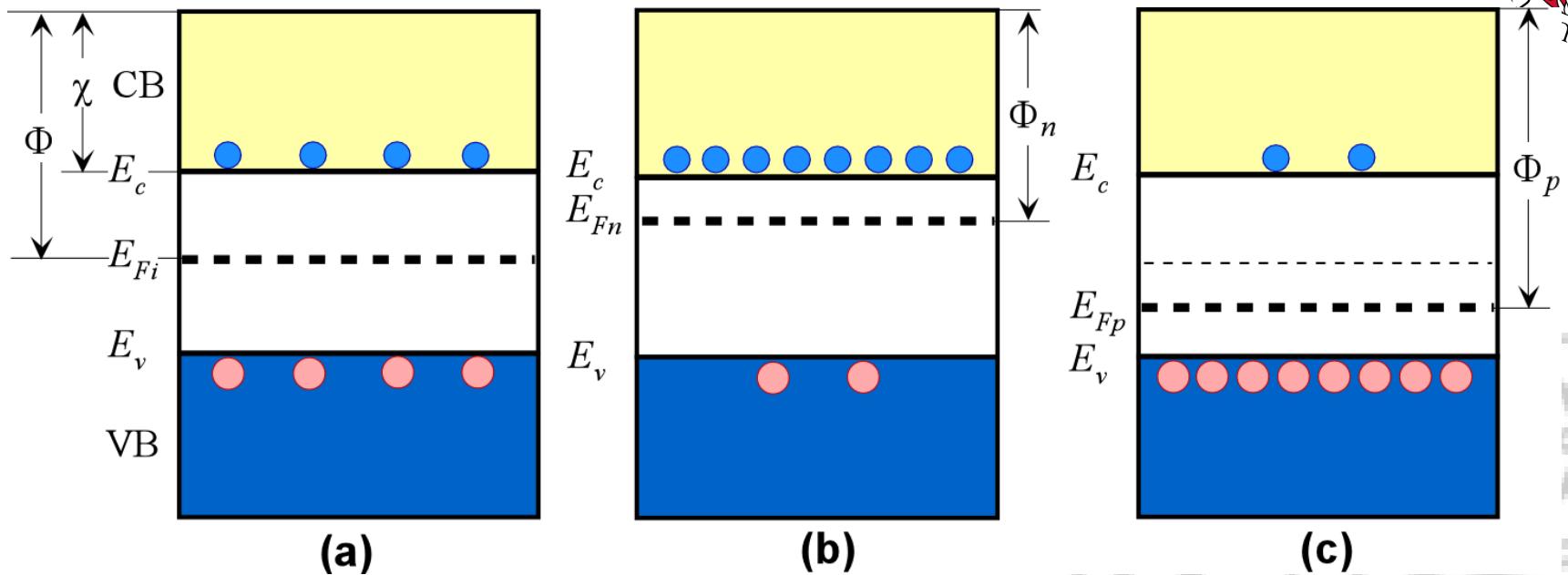
*p*-type

$$p = N_a$$

$$n = n_i^2/N_a$$

$$np = n_i^2$$

# Semiconductor energy band diagrams



Energy band diagrams for (a) intrinsic (b)  $n$ -type and (c)  $p$ -type semiconductors. In all cases,  $np = n_i^2$ . Note that donor and acceptor energy levels are not shown. CB = Conduction band, VB = Valence band,  $E_c$  = CB edge,  $E_v$  = VB edge,  $E_{Fi}$  = Fermi level in intrinsic semiconductor,  $E_{Fn}$  = Fermi level in  $n$ -type semiconductor,  $E_{Fp}$  = Fermi level in  $p$ -type semiconductor.  $\chi$  is the electron affinity.  $\Phi$ ,  $\Phi_n$  and  $\Phi_p$  are the work functions for the intrinsic,  $n$ -type and  $p$ -type semiconductors

# Semiconductor Properties



**TABLE 3.1** Selected typical properties of various semiconductors at 300 K

	$a$ (nm)	$E_g$ (eV)	$\chi$ (eV)	$N_c$ ( $\text{cm}^{-3}$ )	$N_v$ ( $\text{cm}^{-3}$ )	$n_i$ ( $\text{cm}^{-3}$ )	$\epsilon_r$	$\mu_e$ ( $\text{cm}^2 \text{V}^{-1} \text{s}^{-1}$ )	$\mu_h$ ( $\text{cm}^2 \text{V}^{-1} \text{s}^{-1}$ )
Ge (DI)	0.5650	0.66 (I)	4.13	$1.04 \times 10^{19}$	$6.0 \times 10^{19}$	$2.3 \times 10^{13}$	16.0	3900	1900
Si (DI)	0.5431	1.11 (I)	4.05	$2.8 \times 10^{19}$	$1.2 \times 10^{19}$	$1.0 \times 10^{10}$	11.8	1450	490
InP (ZB)	0.5868	1.35 (D)	4.50	$5.2 \times 10^{17}$	$1.1 \times 10^{19}$	$3.0 \times 10^7$	12.6	4600	150
GaAs (ZB)	0.5653	1.42 (D)	4.07	$4.7 \times 10^{17}$	$7.0 \times 10^{18}$	$2.1 \times 10^6$	13.0	8500	400
AlAs (ZB)	0.5661	2.17 (I)	3.50	$1.5 \times 10^{19}$	$1.7 \times 10^{19}$	10	10.1	200	100

*Notes:* Data combined from a number of sources. I and D represent indirect and direct bandgap. DI, diamond crystal; ZB, zinc blend;  $a$ , lattice constant. (Note that there are variations in the values of certain properties among books, for example,  $n_i$  for Si,  $\epsilon_r$  for GaAs, etc. Most commonly used or recent values have been selected.)



# Compensation Doping

**Compensation doping** describes the doping of a semiconductor with both donors and acceptors to control the properties.

**Example:** A *p*-type semiconductor doped with  $N_a$  acceptors can be converted to an *n*-type semiconductor by simply adding donors until the concentration  $N_d$  exceeds  $N_a$ .

The effect of donors compensates for the effect of acceptors.

The electron concentration  $n = N_d - N_a > n_i$

When both acceptors and donors are present, electrons from donors recombine with the holes from the acceptors so that the mass action law  $np = n_i^2$  is obeyed.

We cannot simultaneously increase the electron and hole concentrations because that leads to an increase in the recombination rate which returns the electron and hole concentrations to values that satisfy  $np = n_i^2$ .

$$N_d > N_a$$

$$n = N_d - N_a$$

$$p = n_i^2 / (N_d - N_a)$$

# Summary of Compensation Doping



**More donors than acceptors**

$$N_d - N_a \gg n_i$$

$$n = N_d - N_a$$

$$p = \frac{n_i^2}{n} = \frac{n_i^2}{N_d - N_a}$$

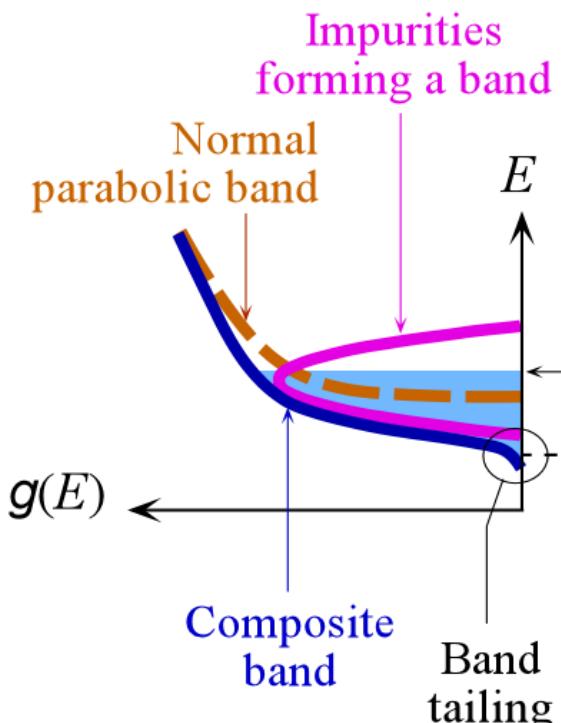
**More acceptors than donors**

$$p = N_a - N_d$$

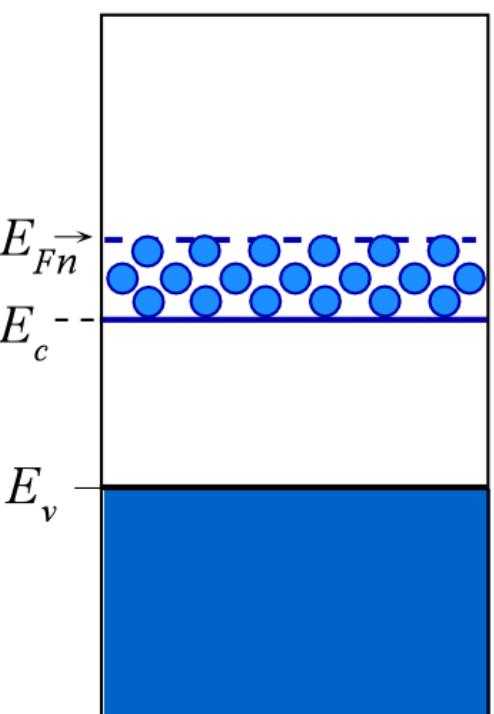
$$N_a - N_d \gg n_i$$

$$n = \frac{n_i^2}{p} = \frac{n_i^2}{N_a - N_d}$$

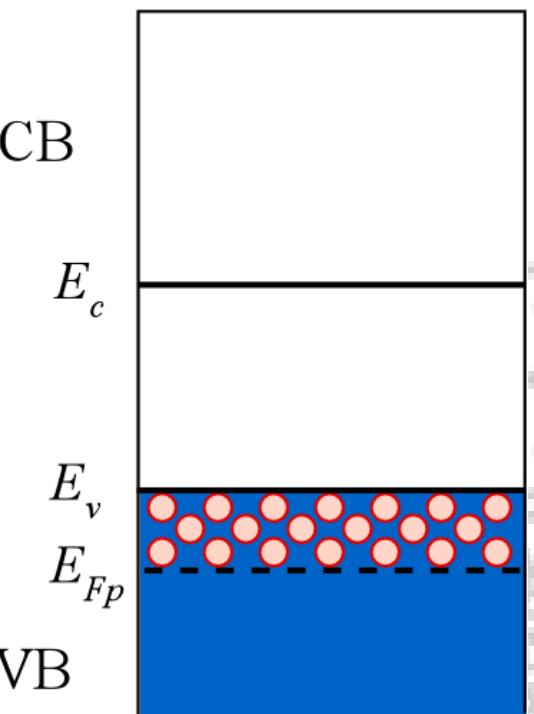
# Degenerate Semiconductors



(a)



(b)



(a) Degenerate  $n$ -type semiconductor. Large number of donors form a band that overlaps the CB.  $E_c$  is pushed down and  $E_{Fn}$  is within the CB. (b) Degenerate  $p$ -type semiconductor.



## Example: Fermi levels in semiconductors

An *n*-type Si wafer has been doped uniformly with  $10^{16}$  phosphorus (P) atoms  $\text{cm}^{-3}$ . Calculate the position of the Fermi energy with respect to the Fermi energy  $E_{Fi}$  in intrinsic Si. The above *n*-type Si sample is further doped with  $2 \times 10^{17}$  boron atoms  $\text{cm}^{-3}$ . Calculate position of the Fermi energy with respect to the Fermi energy  $E_{Fi}$  in intrinsic Si at room temperature (300 K), and hence with respect to the Fermi energy in the *n*-type case above.

### Solution

P (Group V) gives *n*-type doping with  $N_d = 10^{16} \text{ cm}^{-3}$ , and since  $N_d \gg n_i$  ( $= 10^{10} \text{ cm}^{-3}$  from Table 3.1), we have  $n = N_d = 10^{16} \text{ cm}^{-3}$ . For intrinsic Si,

$$n_i = N_c \exp[-(E_c - E_{Fi})/k_B T]$$

whereas for doped Si,

$$n = N_c \exp[-(E_c - E_{Fn})/k_B T] = N_d$$

where  $E_{Fi}$  and  $E_{Fn}$  are the Fermi energies in the intrinsic and *n*-type Si. Dividing the two expressions

$$N_d/n_i = \exp[(E_{Fn} - E_{Fi})/k_B T]$$

so that

$$E_{Fn} - E_{Fi} = k_B T \ln(N_d/n_i) = (0.0259 \text{ eV}) \ln(10^{16}/10^{10}) = 0.358 \text{ eV}$$



## Example: Fermi levels in semiconductors

### Solution (Continued)

When the wafer is further doped with boron, the acceptor concentration,  $N_a = 2 \times 10^{17} \text{ cm}^{-3} > N_d = 10^{16} \text{ cm}^{-3}$ . The semiconductor is compensation doped and compensation converts the semiconductor to a *p*-type Si. Thus,

$$p = N_a - N_d = 2 \times 10^{17} - 10^{16} = 1.9 \times 10^{17} \text{ cm}^{-3}.$$

For intrinsic Si,

$$p = n_i = N_v \exp[-(E_{Fi} - E_v)/k_B T],$$

whereas for doped Si,

$$p = N_v \exp[-(E_{Fp} - E_v)/k_B T] = N_a - N_d$$

where  $E_{Fi}$  and  $E_{Fp}$  are the Fermi energies in the intrinsic and *p*-type Si respectively. Dividing the two expressions,

$$p/n_i = \exp[-(E_{Fp} - E_{Fi})/k_B T]$$

so that

$$\begin{aligned} E_{Fp} - E_{Fi} &= -k_B T \ln(p/n_i) \\ &= -(0.0259 \text{ eV}) \ln(1.9 \times 10^{17}/1.0 \times 10^{10}) = -0.434 \text{ eV} \end{aligned}$$



## Example: Conductivity of *n*-Si

Consider a pure intrinsic Si crystal. What would be its intrinsic conductivity at 300K? What is the electron and hole concentrations in an *n*-type Si crystal that has been doped with  $10^{16} \text{ cm}^{-3}$  phosphorus (P) donors. What is the conductivity if the drift mobility of electrons is about  $1200 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$  at this concentration of dopants.

### Solution

The intrinsic concentration  $n_i = 1 \times 10^{10} \text{ cm}^{-3}$ , so that the intrinsic conductivity is

$$\begin{aligned}\sigma &= en_i(\mu_e + \mu_h) = (1.6 \times 10^{-19} \text{ C})(1 \times 10^{10} \text{ cm}^{-3})(1450 + 490 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}) \\ &= 3.1 \times 10^{-6} \Omega^{-1} \text{ cm}^{-1} \text{ or } 3.1 \times 10^{-4} \Omega^{-1} \text{ m}^{-1}\end{aligned}$$

Consider *n*-type Si.  $N_d = 10^{16} \text{ cm}^{-3} > n_i (= 10^{10} \text{ cm}^{-3})$ , the electron concentration  $n = N_d = 10^{16} \text{ cm}^{-3}$  and

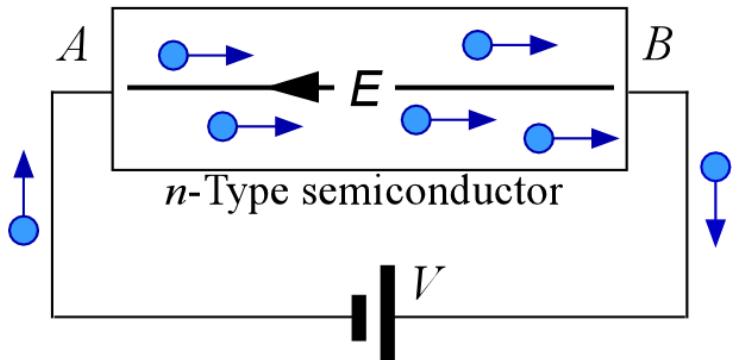
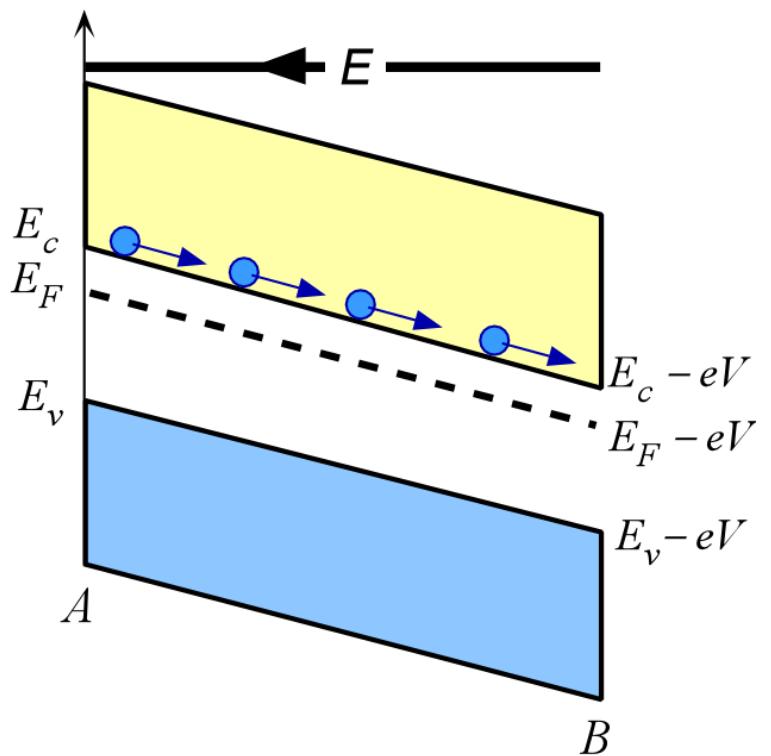
$$p = n_i^2/N_d = (10^{10} \text{ cm}^{-3})^2/(10^{16} \text{ cm}^{-3}) = 10^4 \text{ cm}^{-3}$$

and negligible compared to  $n$ .

The conductivity is

$$\sigma = eN_d\mu_e = (1.6 \times 10^{-19} \text{ C})(1 \times 10^{16} \text{ cm}^{-3})(1200 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}) = 1.92 \Omega^{-1} \text{ cm}^{-1}$$

Electron energy



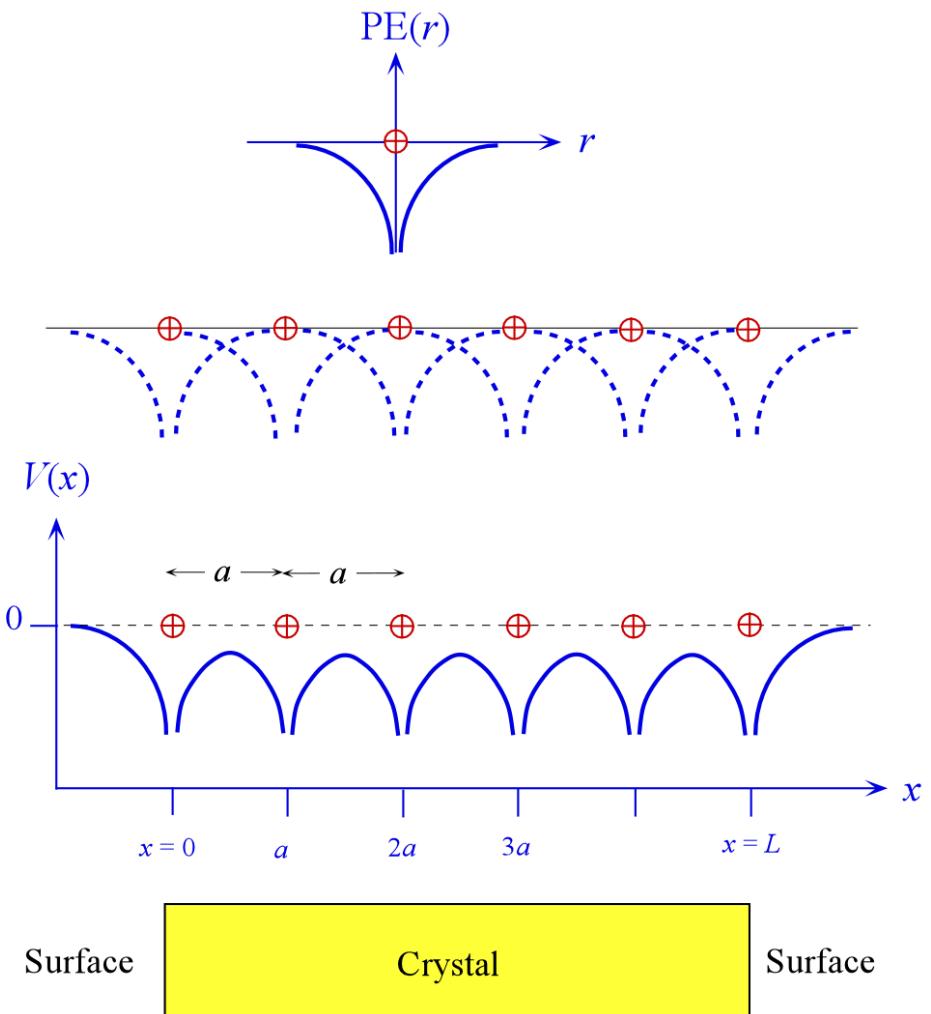
# Energy bands bend in an applied field

Energy band diagram of an *n*-type semiconductor connected to a voltage supply of  $V$  volts.

The whole energy diagram tilts because the electron now also has an electrostatic potential energy.



# Electrons in Semiconductor Crystals



PE of the electron around an isolated atom

When  $N$  atoms are arranged to form the crystal then there is an overlap of individual electron PE functions.

PE of the electron,  $V(x)$ , inside the crystal is periodic with a period  $a$ .

The electron *Potential Energy (PE)*,  $V(x)$ , inside the crystal is periodic with the same periodicity as that of the Crystal,  $a$ . Far away outside the crystal, by choice,  $V = 0$  (the electron is free and  $PE = 0$ ).



# Electrons in Semiconductor Crystals

## Periodic Potential Energy

$$V(x) = V(x + ma) ; \quad m = 1, 2, 3\dots$$

$$\frac{d^2\psi}{dx^2} + \frac{2m_e}{\hbar^2} [E - V(x)]\psi = 0$$

$$\psi_k(x) = U_k(x)\exp(jkx)$$

Bloch electron wavefunction

Electron wave vector  
Quantized



# Electrons in Semiconductor Crystals

$\Psi_k(x,t)$

Periodic Potential Energy

$$\psi_k(x) = U_k(x) \exp(jkx) \times \exp(-j\omega t)$$

$k$  is quantized. Each  $k$  represents a particular  $\psi_k$

Each  $k$  and  $\psi_k$  have an energy  $E_k$

Electron's crystal momentum =  $\hbar k$

External force =  $d(\hbar k) / dt$



# Electrons in Semiconductor Crystals

## Periodic Potential Energy in 3 D

$$V(\mathbf{r}) = V(\mathbf{r} + m\mathbf{T}) ; \quad m = 1, 2, 3\dots$$

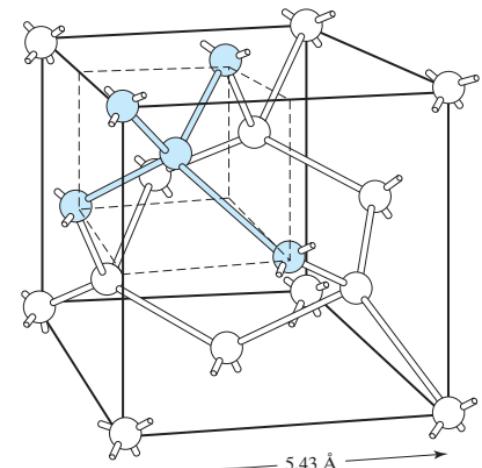
**Crystal structure is very important**

$\mathbf{k}$  is quantized. Each  $\mathbf{k}$  represents a particular  $\psi_{\mathbf{k}}$

$$\psi_{\mathbf{k}}(x) = U_{\mathbf{k}}(\mathbf{r}) \exp(j\mathbf{k} \cdot \mathbf{r})$$

$$E_k \text{ VS } \hbar\mathbf{k}$$

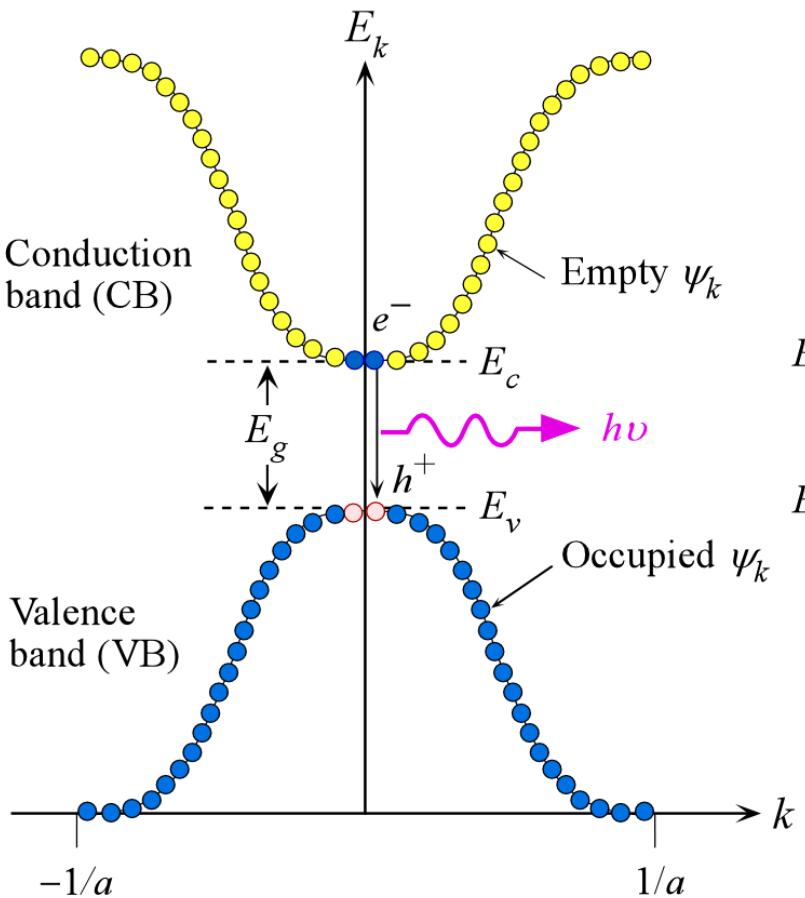
Crystal momentum



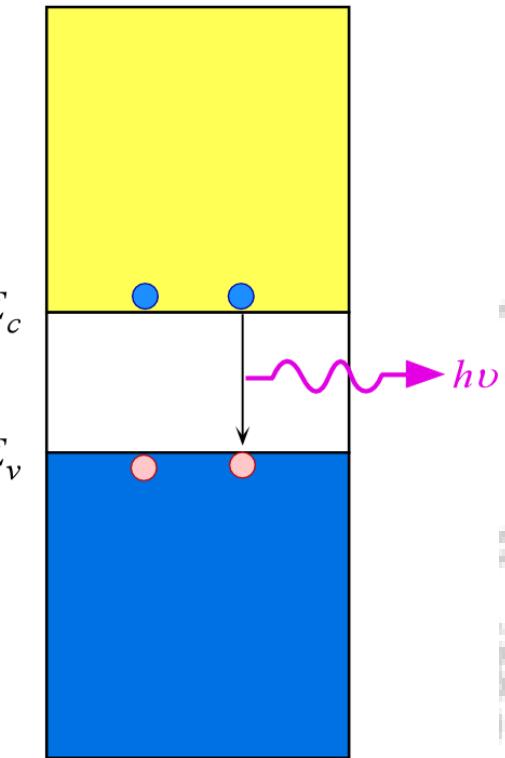
# *E* vs. *k* Diagrams



The *E*-*k* diagram

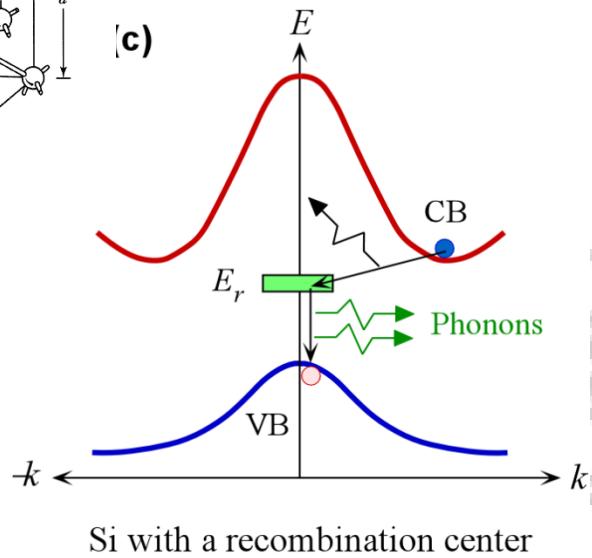
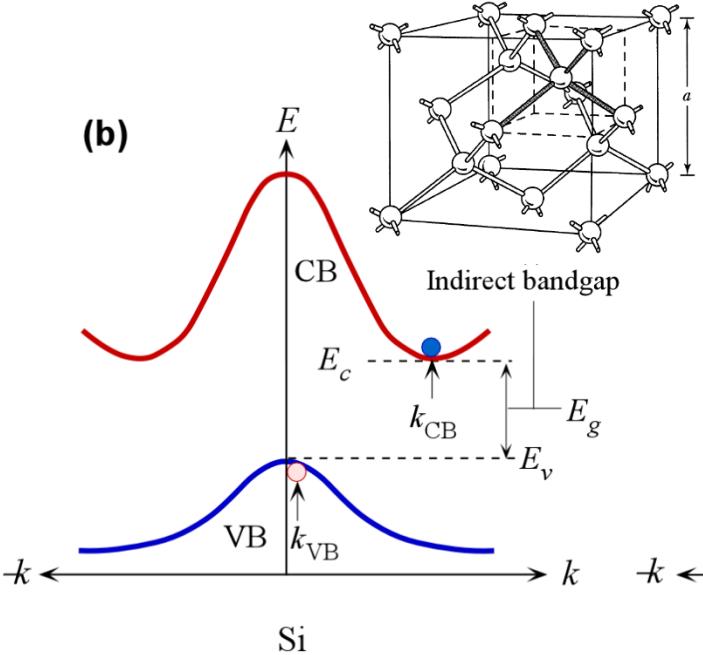
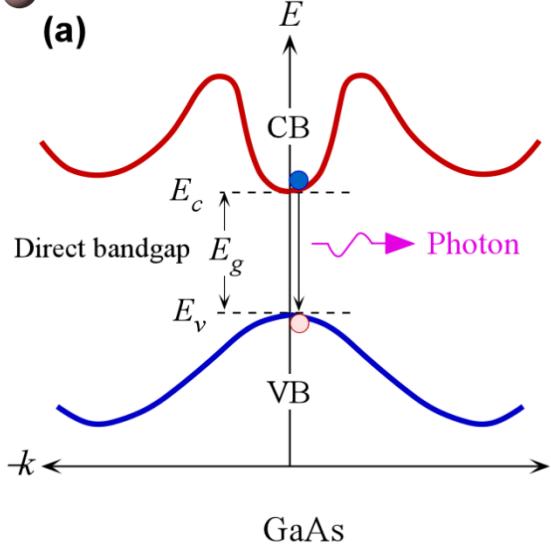
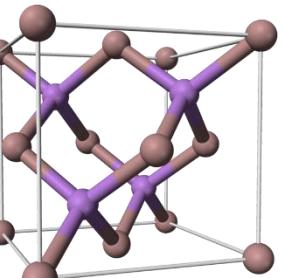


The energy band diagram



The *E*-*k* diagram of a direct bandgap semiconductor such as GaAs. The *E*-*k* curve consists of many discrete points with each point corresponding to a possible state, wavefunction  $\psi_k(x)$ , that is allowed to exist in the crystal. The points are so close that we normally draw the *E*-*k* relationship as a continuous curve. In the energy range  $E_v$  to  $E_c$  there are no points, *i.e.* no  $\psi_k(x)$  solutions

# $E$ vs. $k$ Diagrams and Direct and Indirect Bandgap Semiconductors



(a) In GaAs the minimum of the CB is directly above the maximum of the VB. GaAs is therefore a direct bandgap semiconductor. (b) In Si, the minimum of the CB is displaced from the maximum of the VB and Si is an indirect bandgap semiconductor. (c) Recombination of an electron and a hole in Si involves a recombination center .

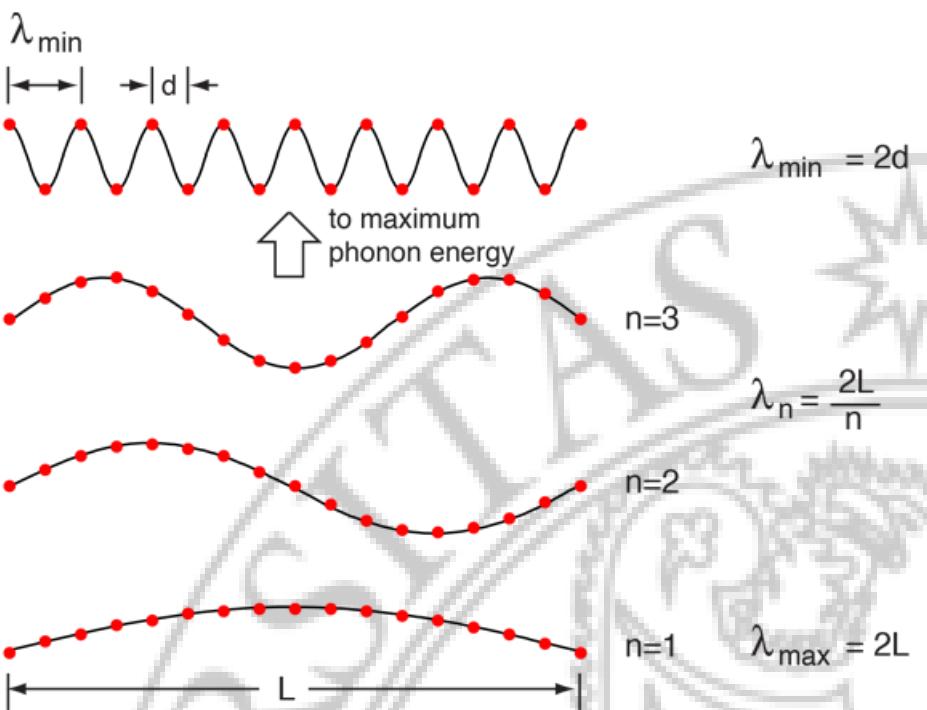
# Some words on phonons

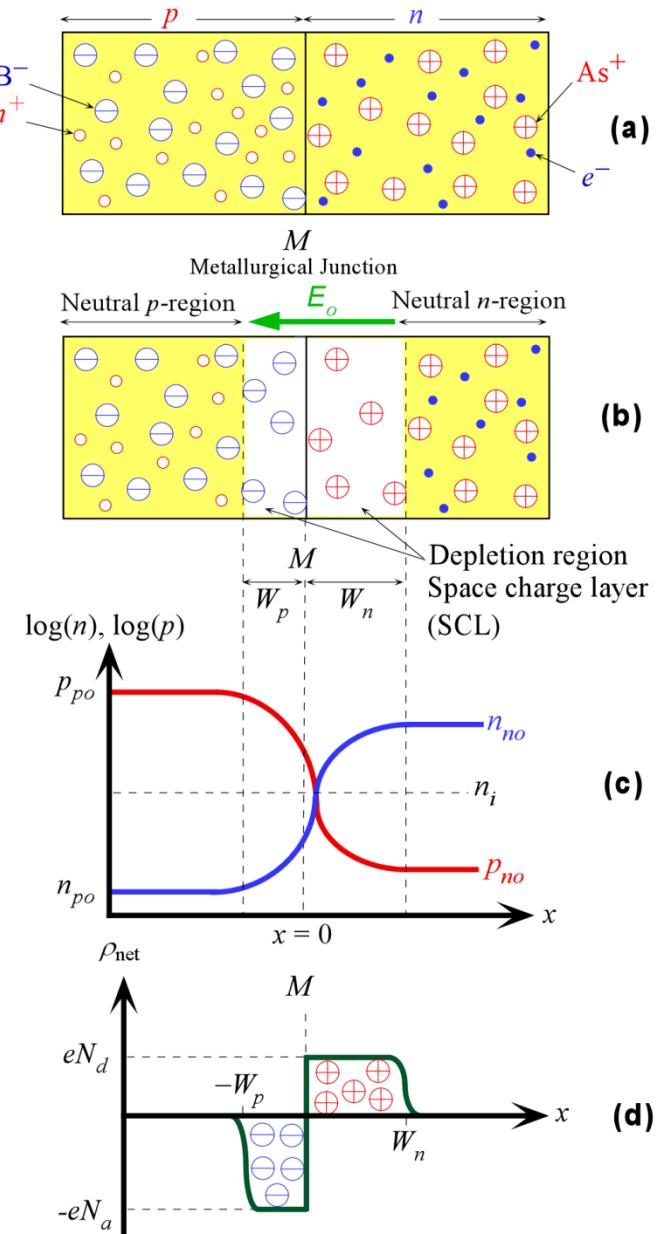
Energy

$$h\nu = \frac{h\nu_s}{\lambda} = \frac{h\nu_s n}{2L}$$

$V_s$  = speed of sound

Depends on structure of the crystal





# *pn* Junction

Properties of the *pn* junction

- (a) The *p*- and *n*- sides of the *pn* junction before the contact.
- (b) The *pn* junction after contact, in equilibrium and in open circuit.
- (c) Carrier concentrations along the whole device, through the *pn* junction. At all points,  $n_{po}p_{po} = n_{no}p_{no} = n_i^2$ .
- (d) Net space charge density  $\rho_{net}$  across the *pn* junction.

**Note that most of voltage drop is around the Space Charge Layer or Depletion Layer**



# One step back

- Remember that:

$$\nabla D = \rho$$

- And that:  $D = \epsilon E$

- The electrostatic potential (volts):

$$E = -\nabla \psi$$

- We obtain that:

$$\epsilon \nabla^2 \psi = -\rho$$

- In other words:

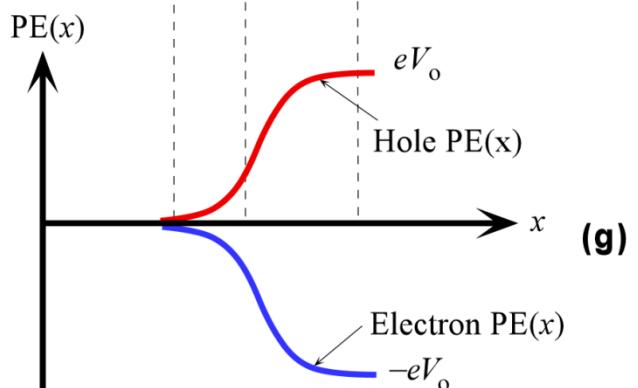
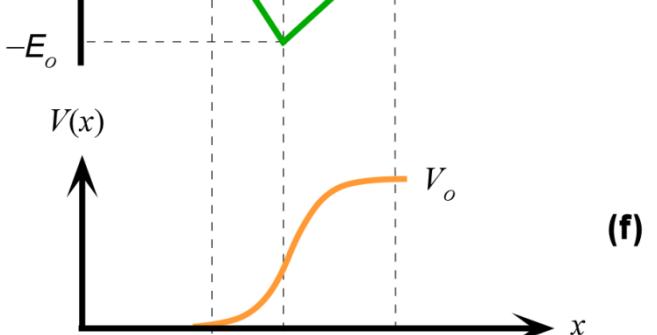
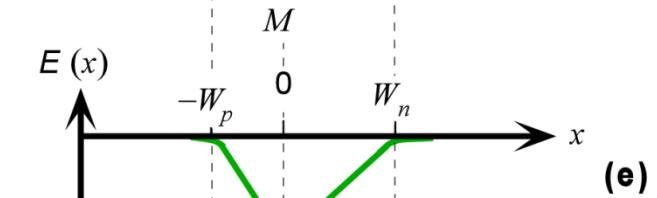
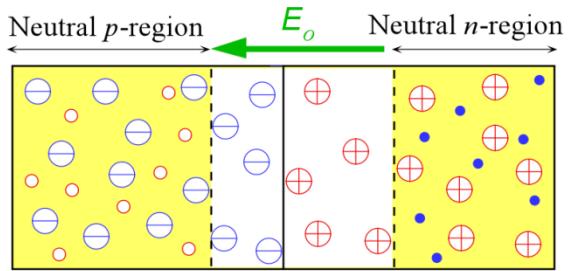
$$\epsilon \nabla^2 \psi = -q[n(x) - p(x) + NA - ND]$$

Also called Poisson equation,

This equation define the electrostatic potential as a function of the distribution of charge



## *pn Junction*



(e) The electric field across the *pn* junction is found by integrating  $\rho_{\text{net}}$  in (d).

(f) The potential  $V(x)$  across the device. Contact potentials are not shown at the semiconductor-metal contacts now shown.

(g) Hole and electron potential energy (PE) across the *pn* junction. Potential energy is charge  $\times$  potential =  $q V$

*Note that most of voltage drop is around the Space Charge Layer*



# Ideal $pn$ Junction

Depletion Widths

Acceptor concentration      Donor concentration

$$N_a W_p = N_d W_n$$

Field ( $E$ ) and net space charge density

$$\frac{dE}{dx} = \frac{\rho_{\text{net}}(x)}{\epsilon}$$

Field in depletion region

Net space charge density

Permittivity of the medium

$$E(x) = \frac{1}{\epsilon} \int_{-W_p}^x \rho_{\text{net}}(x) dx$$

# Ideal $pn$ Junction



## Built-in field

$$E_o = -\frac{eN_d W_n}{\epsilon}$$

$\epsilon = \epsilon_0 \epsilon_r$

## Built-in voltage

$$V_o = \frac{kT}{e} \ln \left( \frac{N_a N_d}{n_i^2} \right)$$

$n_i$  is the intrinsic concentration

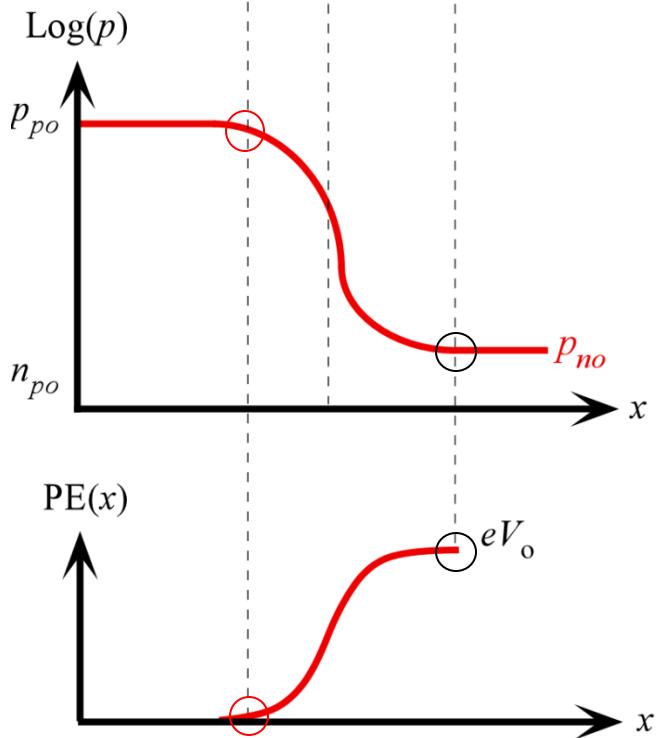
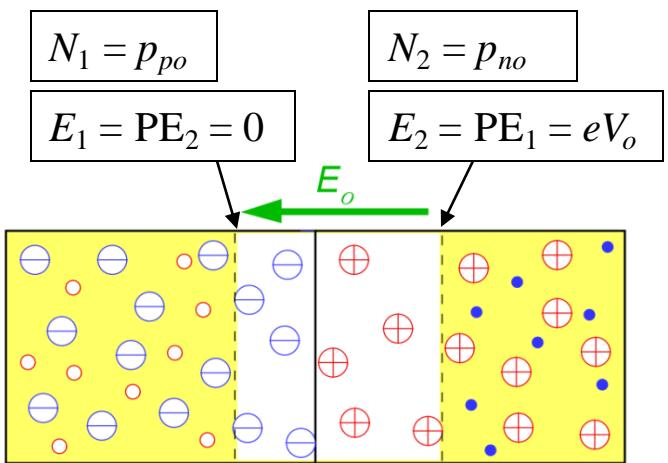
## Depletion region width

$$W_o = \left[ \frac{2\epsilon(N_a + N_d)V_o}{eN_a N_d} \right]^{1/2}$$

where  $W_o = W_n + W_p$  is the total width of the depletion region under a zero applied voltage

# Law of the Junction

Apply Boltzmann Statistics (can only be used with nondegenerate semiconductors)



$$\frac{N_2}{N_1} = \exp\left[-\frac{(E_2 - E_1)}{k_B T}\right]$$

$$\frac{p_{no}}{p_{po}} = \exp\left[-\frac{(eV_o - 0)}{k_B T}\right]$$

$$\frac{p_{no}}{p_{po}} = \exp\left(-\frac{eV_o}{k_B T}\right)$$



# Law of the Junction

## Apply Boltzmann Statistics

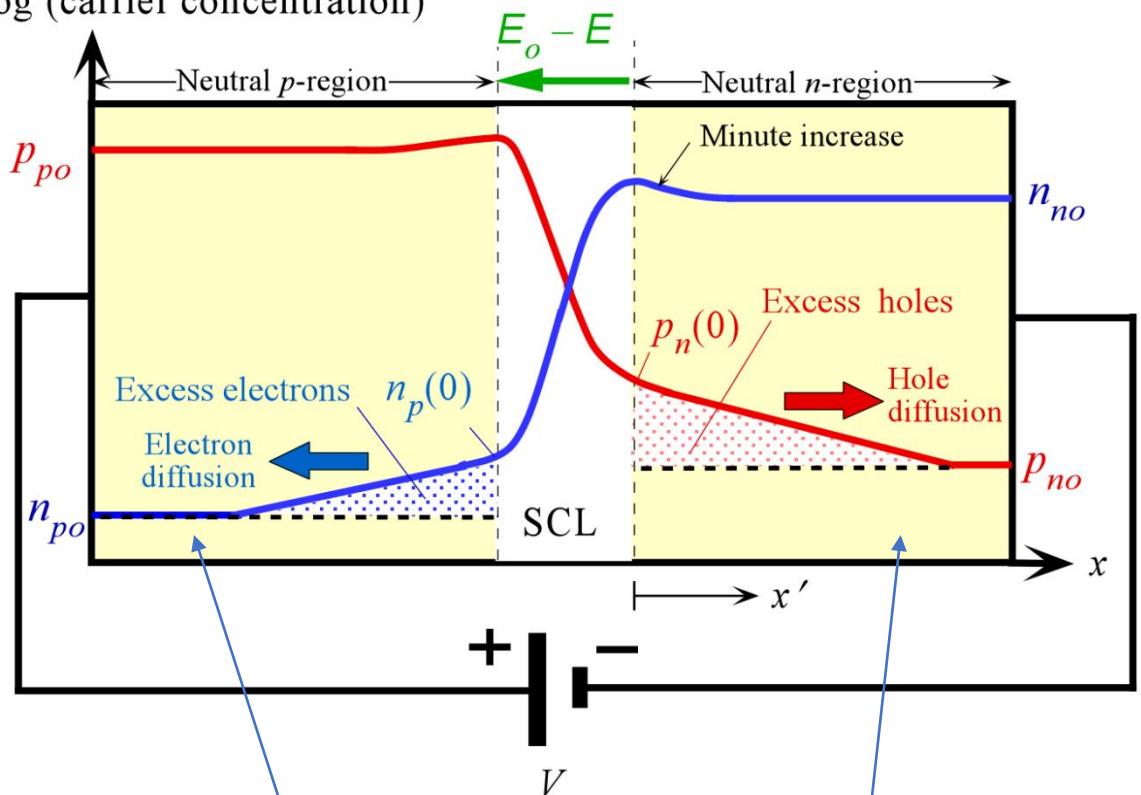
$$\frac{p_{no}}{p_{po}} = \exp\left(-\frac{eV_o}{k_B T}\right)$$

$$\frac{n_{po}}{n_{no}} = \exp\left(-\frac{eV_o}{k_B T}\right)$$

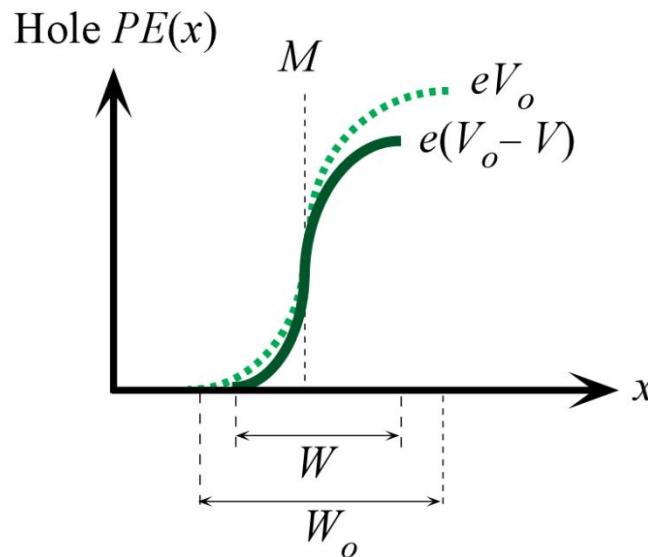
$$V_o = \frac{k_B T}{e} \ln\left(\frac{p_{po}}{p_{no}}\right) = \frac{k_B T}{e} \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

# Forward Biased *pn* Junction

Log (carrier concentration)



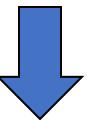
Large conductivity because  
of majority carriers



Forward biased *pn* junction and the injection of minority carriers (a) Carrier concentration profiles across the device under forward bias. (b). The hole potential energy with and without an applied bias.  $W$  is the width of the SCL with forward bias

# Forward Bias: Apply Boltzmann Statistics

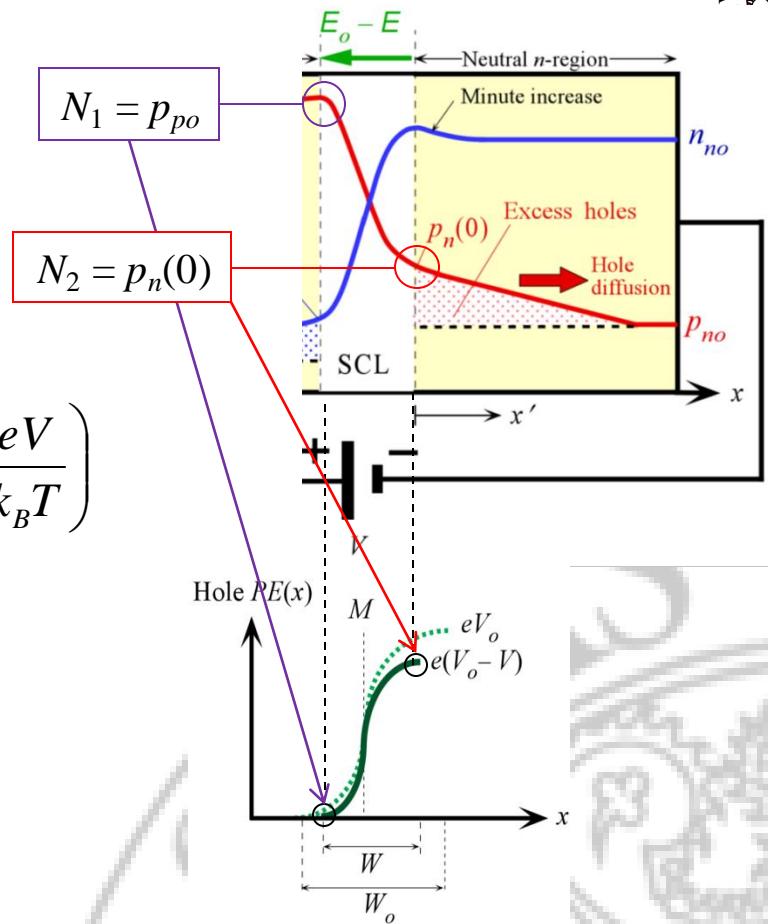
$$\frac{N_2}{N_1} = \exp\left[-\frac{(E_2 - E_1)}{k_B T}\right]$$



$$\frac{p_n(0)}{p_{po}} = \exp\left[-\frac{e(V_o - V)}{k_B T}\right] = \exp\left(-\frac{eV_o}{k_B T}\right) \exp\left(\frac{eV}{k_B T}\right)$$



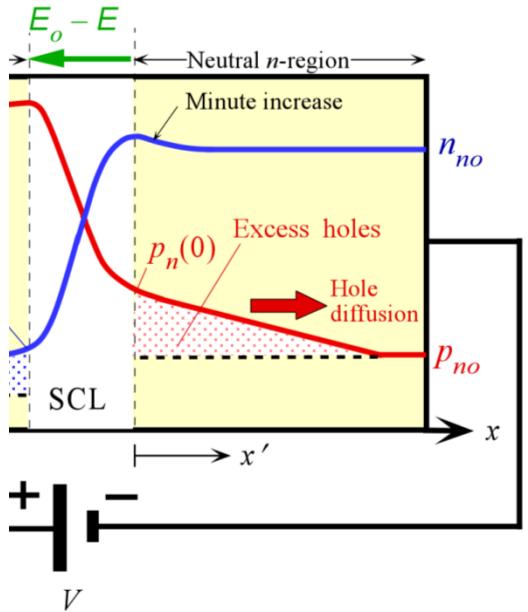
$$p_n(0) = p_{no} \exp\left(\frac{eV}{k_B T}\right)$$



**Note:**  $p_n(0)$  is the hole concentration just outside the depletion region on the *n*-side

# Forward Bias

## Law of the Junction: Minority Carrier Concentrations and Voltage



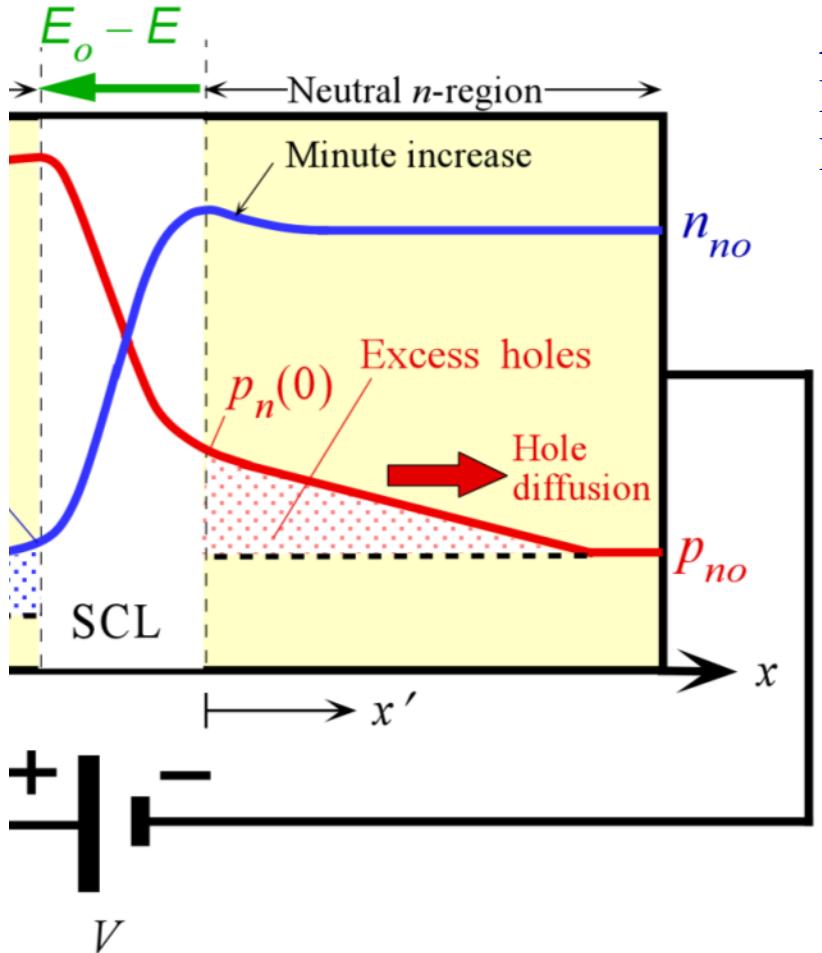
$$p_n(0) = p_{no} \exp\left(\frac{eV}{k_B T}\right)$$

$$n_p(0) = n_{po} \exp\left(\frac{eV}{k_B T}\right)$$

$p_n(0)$  is the hole concentration just outside the depletion region on the *n*-side

$n_p(0)$  is the electron concentration just outside the depletion region on the *p*-side

# Forward Bias: Minority Carrier Profile



## Assumptions

- No field in the neutral region (pure diffusion)
- Neutral region is very long

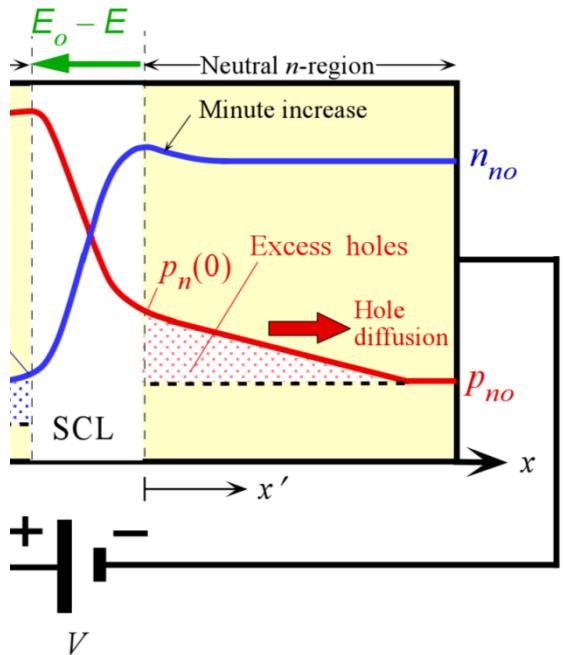
$1/L_h$  = Mean probability of recombination per unit distance; or mean distance diffused

Change in excess hole concentration due to recombination over a small distance  $\delta x'$  would be

$$\delta \Delta p_n(x') = -\Delta p_n(x') \frac{\delta x'}{L_h}$$

$$\Delta p_n(x') = \Delta p_n(0) \exp\left(-\frac{x'}{L_h}\right)$$

# Forward Bias: Diffusion Current



$$J_{D,\text{hole}} = -eD_h \frac{dp_n(x')}{dx'} = -eD_h \frac{d\Delta p_n(x')}{dx'}$$

$$J_{D,\text{hole}} = \left( \frac{eD_h}{L_h} \right) \Delta p_n(0) \exp\left(-\frac{x'}{L_h}\right)$$

$$J_{D,\text{hole}} = \left( \frac{eD_h n_i^2}{L_h N_d} \right) \left[ \exp\left(\frac{eV}{k_B T}\right) - 1 \right]$$

# Forward Bias: Diffusion Current

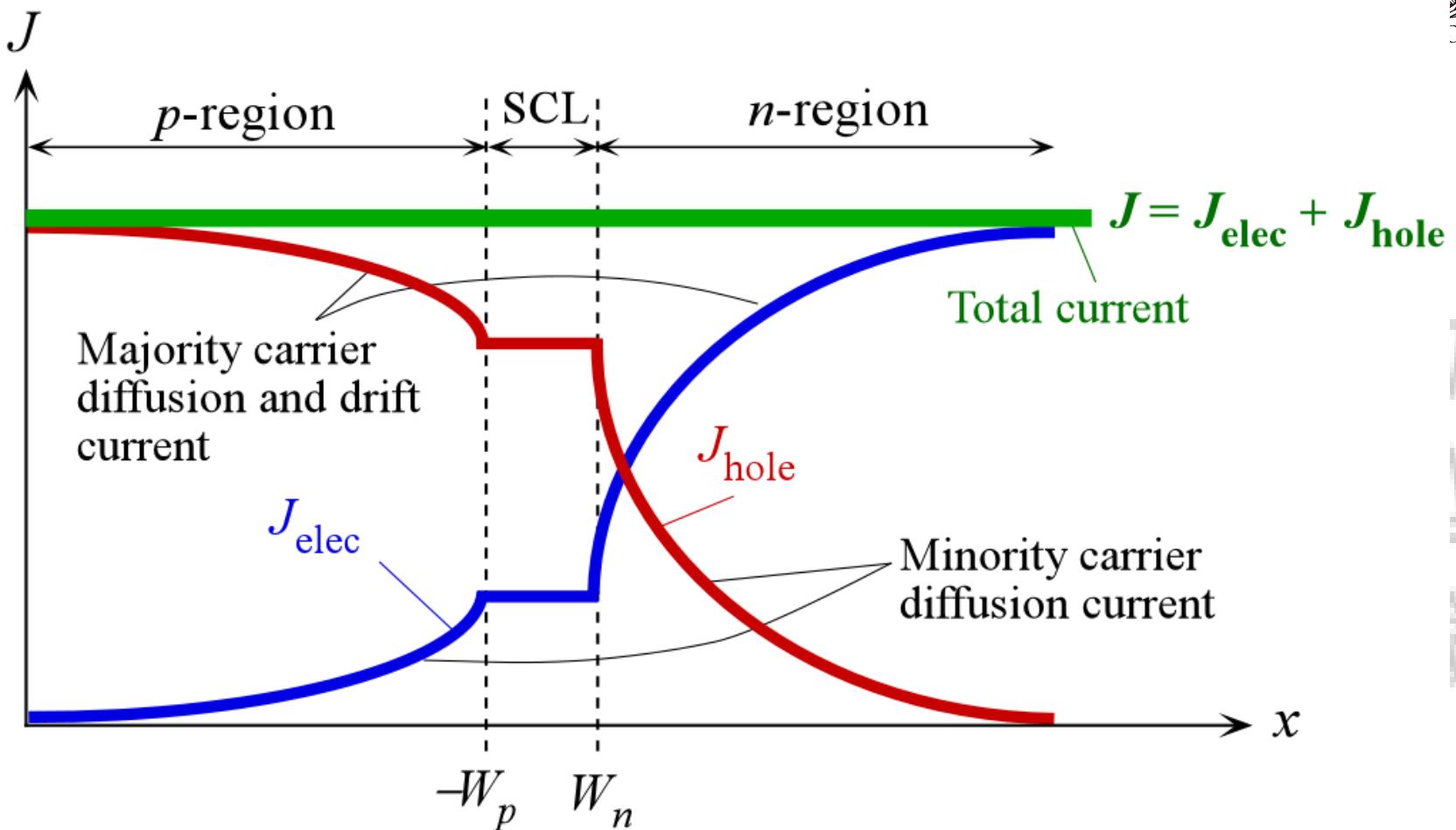


$$J_{D,\text{hole}} = -eD_h \frac{dp_n(x')}{dx'} = -eD_h \frac{d\Delta p_n(x')}{dx'}$$

$$J_{D,\text{hole}} = \left( \frac{eD_h}{L_h} \right) \Delta p_n(0) \exp\left(-\frac{x'}{L_h}\right)$$

$$J_{D,\text{hole}} = \left( \frac{eD_h n_i^2}{L_h N_d} \right) \left[ \exp\left(\frac{eV}{k_B T}\right) - 1 \right]$$

# Forward Bias: Total Current



The total current anywhere in the device is constant. Just outside the depletion region it is due to the diffusion of minority carriers.



# Forward Bias: Diffusion Current

Hole diffusion current in *n*-side in the neutral region

$$J_{D,\text{hole}} = \left( \frac{e D_h n_i^2}{L_h N_d} \right) \left[ \exp\left( \frac{eV}{k_B T} - 1 \right) \right]$$

There is a similar expression for the electron diffusion current density  $J_{D,\text{elec}}$  in the *p*-region.



# Forward Bias: Diffusion Current

Ideal diode (Shockley) equation

$$J = J_{so} \left[ \exp\left(\frac{eV}{k_B T}\right) - 1 \right]$$

Reverse saturation current

$$J_{so} = \left[ \left( \frac{eD_h}{L_h N_d} \right) + \left( \frac{eD_e}{L_e N_a} \right) \right] n_i^2$$

# Forward Bias: Diffusion Current



## Ideal diode (Shockley) equation

$J_{so}$  depends strongly on the material (e.g. bandgap) and temperature

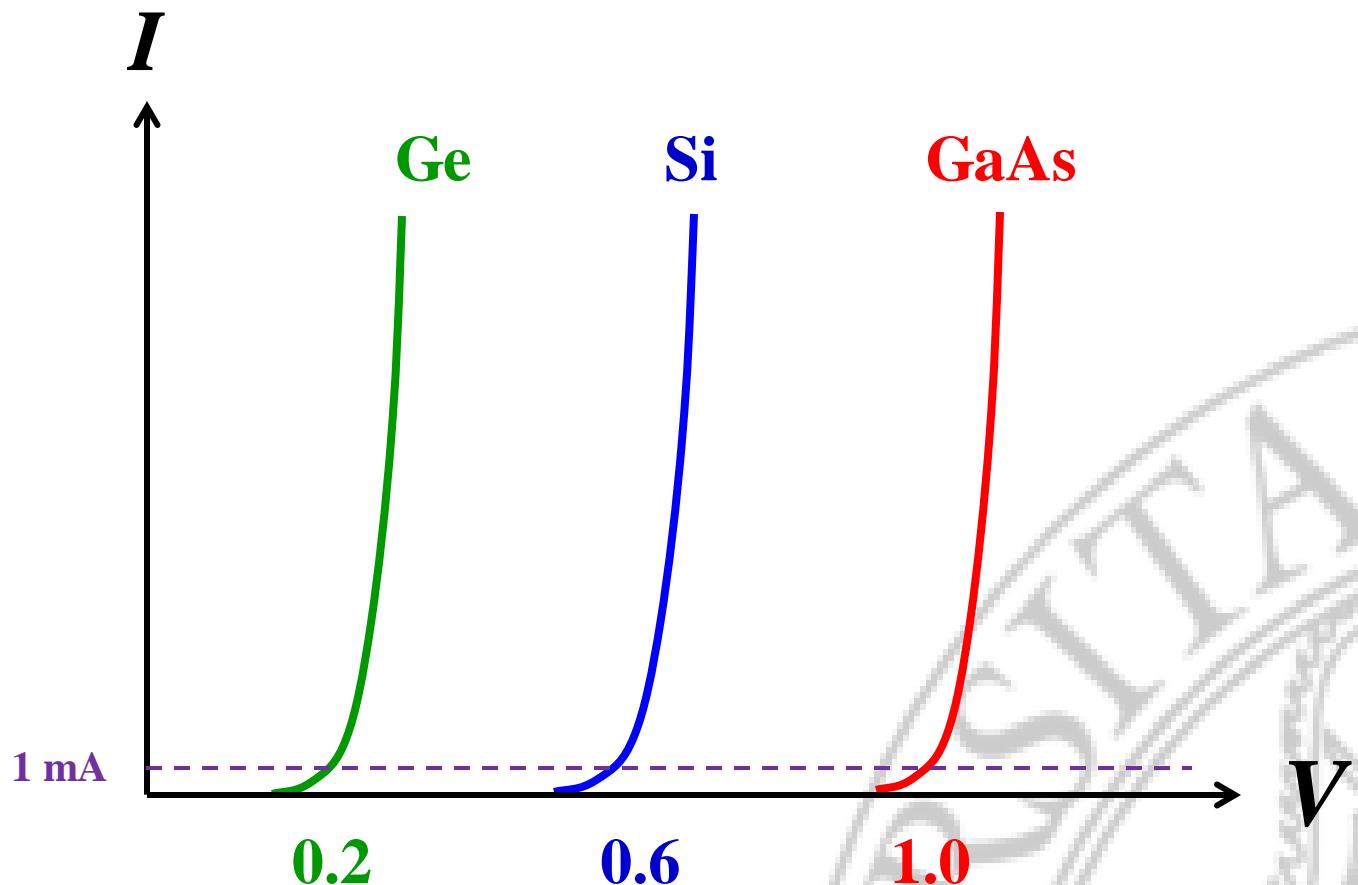
$$J = \left( \frac{eD_h}{L_h N_d} + \frac{eD_e}{L_e N_a} \right) n_i^2 \left[ \exp\left(\frac{eV}{k_B T}\right) - 1 \right]$$

Intrinsic concentration

$$n_i^2 = N_c N_v \exp\left[-\frac{E_g}{k_B T}\right]$$

$n_i$  depends strongly on the material (e.g. bandgap) and temperature

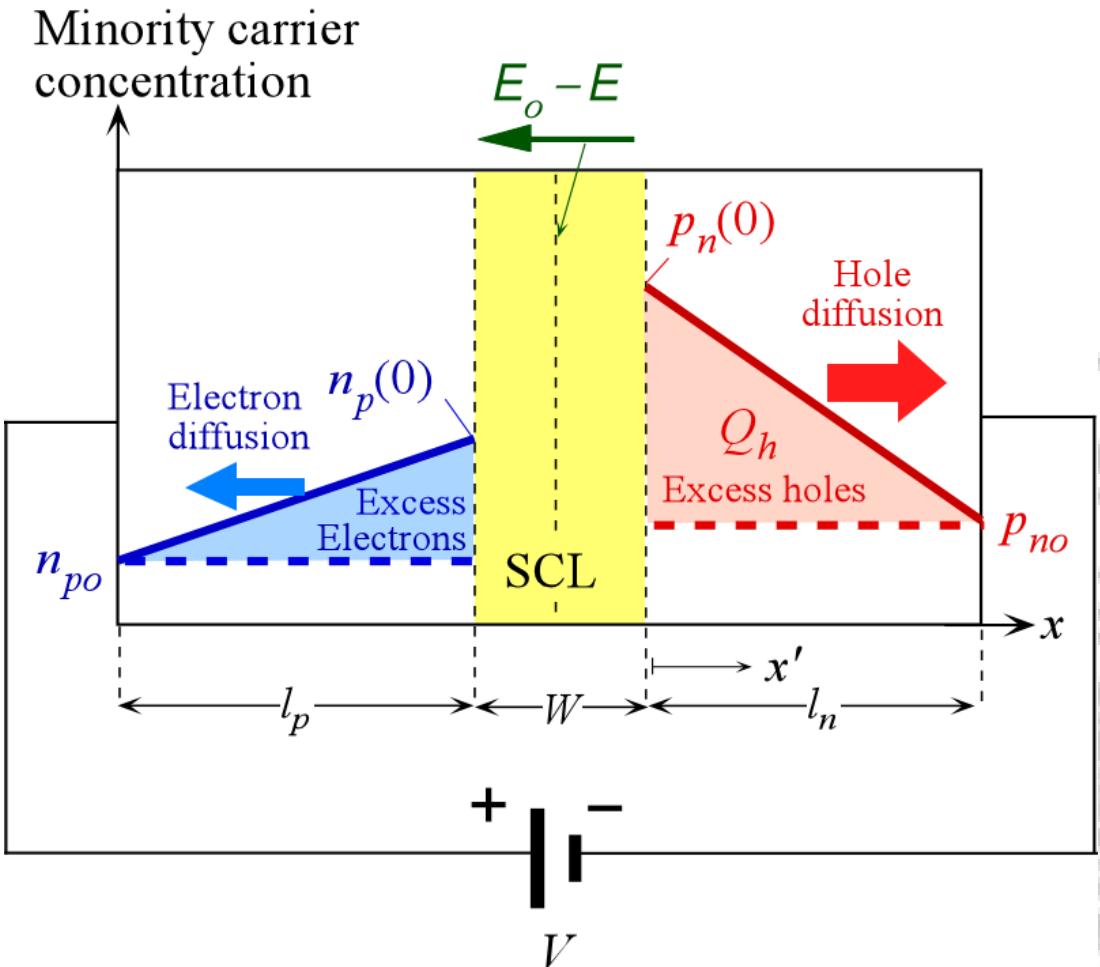
# Ge, Si and GaAs Diodes: Typical Characteristics



Schematic sketch of the  $I$ - $V$  characteristics of Ge, Si, and GaAs pn junctions.

# Forward Bias: Diffusion Current

## Short Diode



$l_n$  and  $l_p$  represent the lengths of the neutral  $n$ - and  $p$ -regions outside the depletion region.



# Forward Bias: Diffusion Current

## Short Diode

$$J = \left( \frac{eD_h}{l_n N_d} + \frac{eD_e}{l_p N_a} \right) n_i^2 \left[ \exp\left(\frac{eV}{k_B T}\right) - 1 \right]$$

$l_n$  and  $l_p$  represent the lengths of the neutral  $n$ - and  $p$ -regions outside the depletion region.



# Difference between short and long diode

- Short Diode

$$J = \left( \frac{eD_h}{l_n N_d} + \frac{eD_e}{l_p N_a} \right) n_i^2 \left[ \exp\left(\frac{eV}{k_B T}\right) - 1 \right]$$

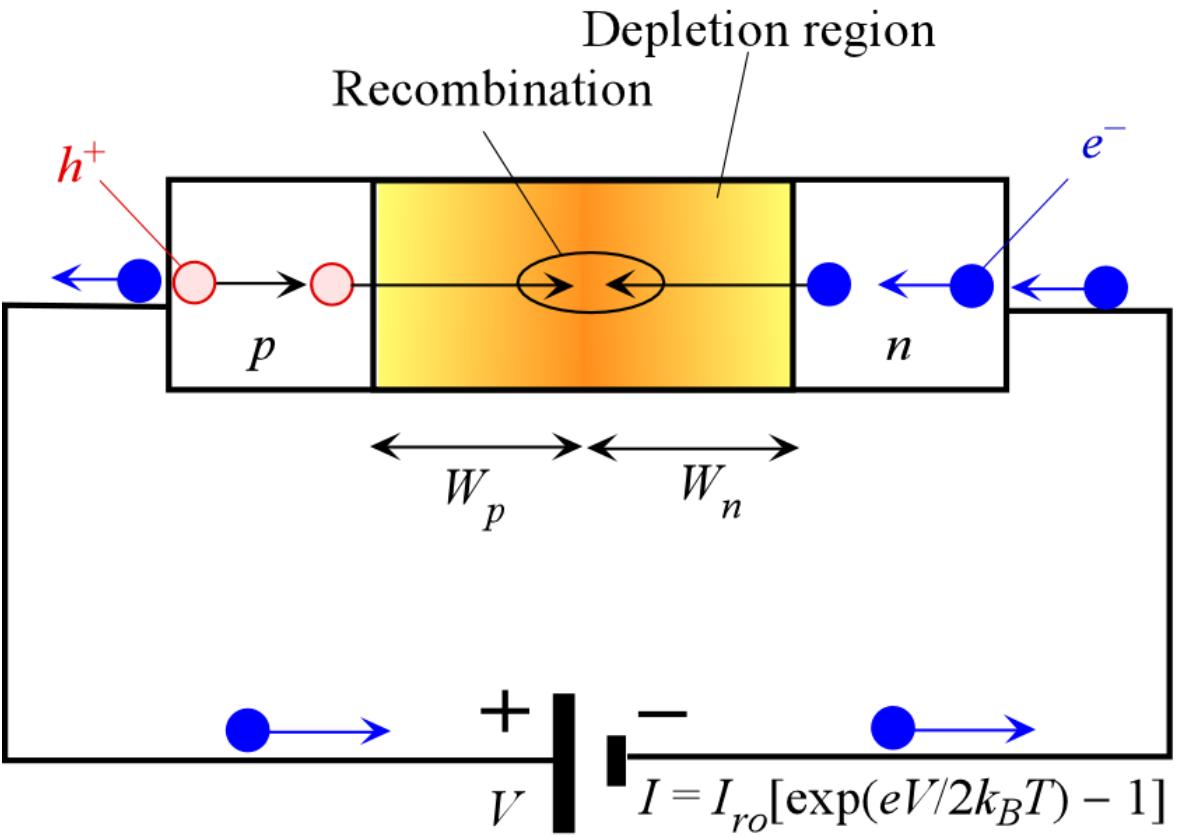
- Long diode

$$J = \left( \frac{eD_h}{L_h N_d} + \frac{eD_e}{L_e N_a} \right) n_i^2 \left[ \exp\left(\frac{eV}{k_B T}\right) - 1 \right]$$

- Excess holes exit the neutral n-side after crossing  $l_n$
- Transit time  $\tau_t v$
- $\tau_h = l_n^2 / 2D_h$  due to random walk of carriers
- Then charge  $Q_h$  should be replenished at a rate  $I = Q_h / \tau_t$

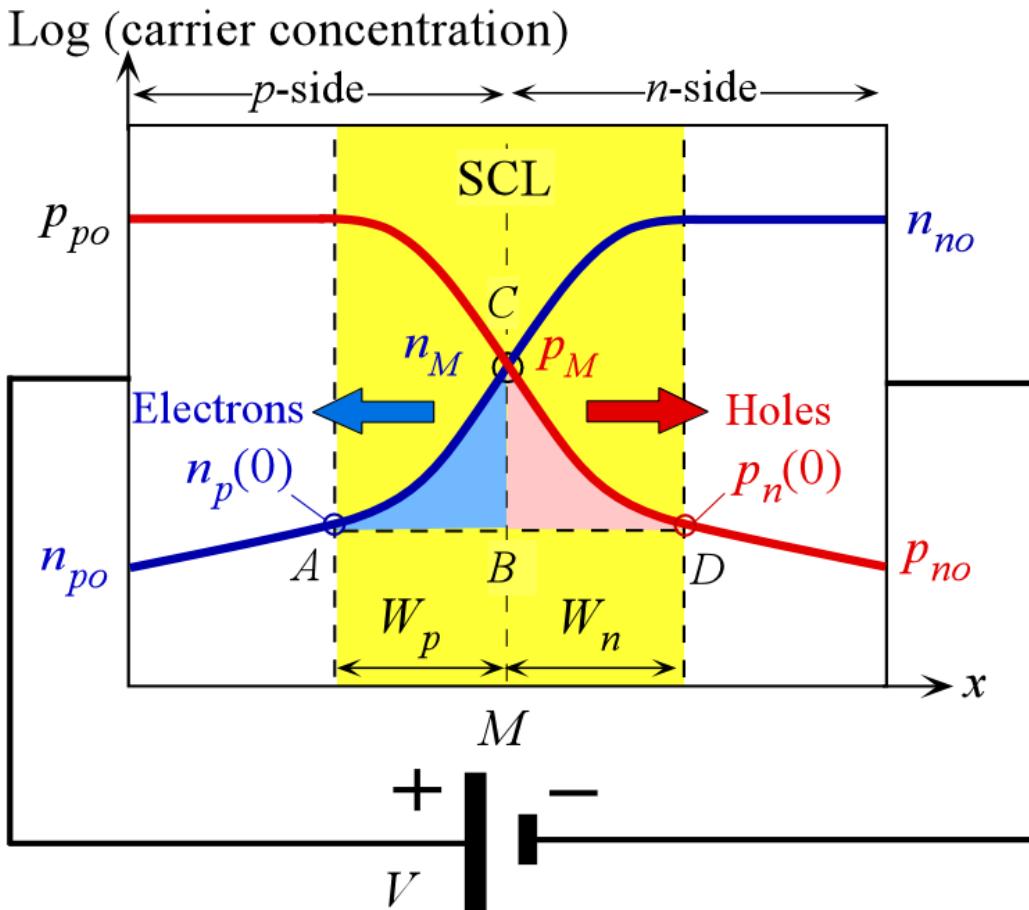
- Excess holes recombine in the neutral n-side after  $\tau_h$  seconds
- $L_h = (D_h \tau_h)^{1/2}$
- $\tau_h$  lifetime of holes
- Then charge  $Q_h$  should be replenished at a rate  $I = Q_h / \tau_h$

# Forward Bias: Recombination Current



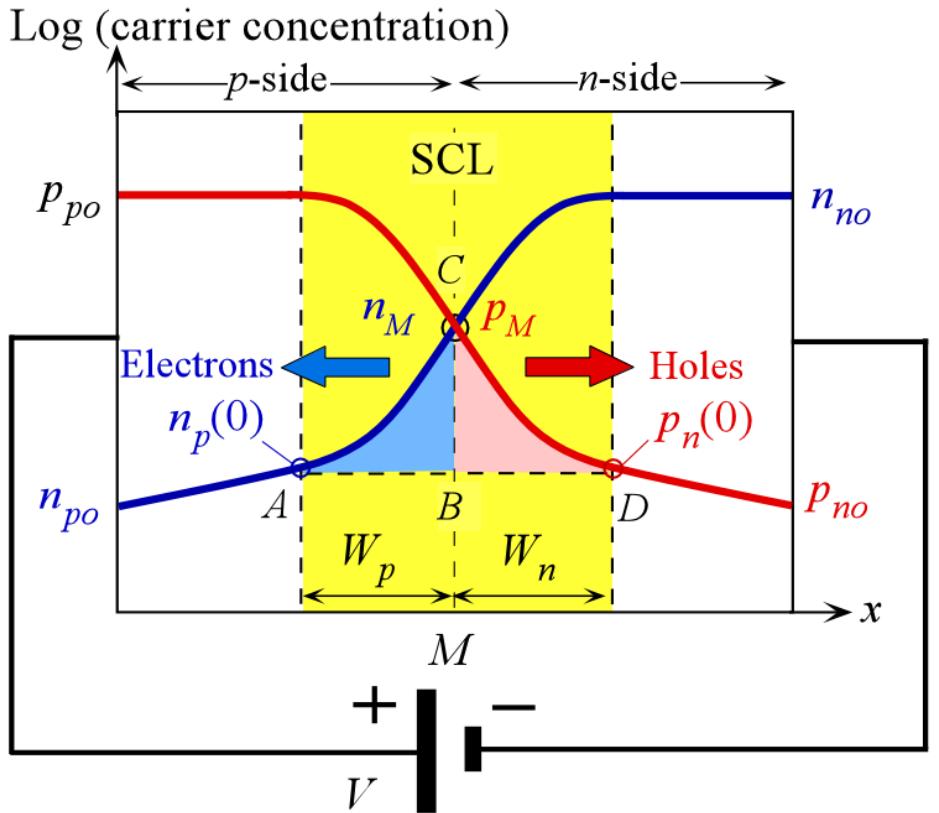
Forward biased *pn* junction, the injection of carriers and their recombination in the SCL.  
 Recombination of electrons and holes in the depletion region involves the current supplying the carriers.

# Forward Bias: Recombination Current



A symmetrical  $pn$  junction for calculating the recombination current.

# Forward Bias: Recombination Current



$$J_{\text{recom}} \approx \frac{eABC}{\tau_e} + \frac{eBCD}{\tau_h}$$

$$\frac{p_M}{p_{po}} = \exp\left[-\frac{e(V_o - V)}{2k_B T}\right]$$

$$p_M = n_i \exp\left(\frac{eV}{2k_B T}\right)$$

$$J_{\text{recom}} \approx \frac{en_i}{2} \left[ \frac{W_p}{\tau_e} + \frac{W_n}{\tau_h} \right] \exp\left(\frac{eV}{2k_B T}\right)$$



# Forward Bias: Recombination and Total Current

## Recombination Current

$$J_{\text{recom}} = J_{ro} [\exp(eV/2k_B T) - 1] \quad \text{where } J_{ro} = \frac{en_i}{2} \left( \frac{W_p}{\tau_e} + \frac{W_n}{\tau_h} \right)$$

Total diode current = diffusion + recombination

$$J = J_{so} \exp\left(\frac{eV}{k_B T}\right) + J_{ro} \exp\left(\frac{eV}{2k_B T}\right)$$

$$V > \frac{k_B T}{e}$$

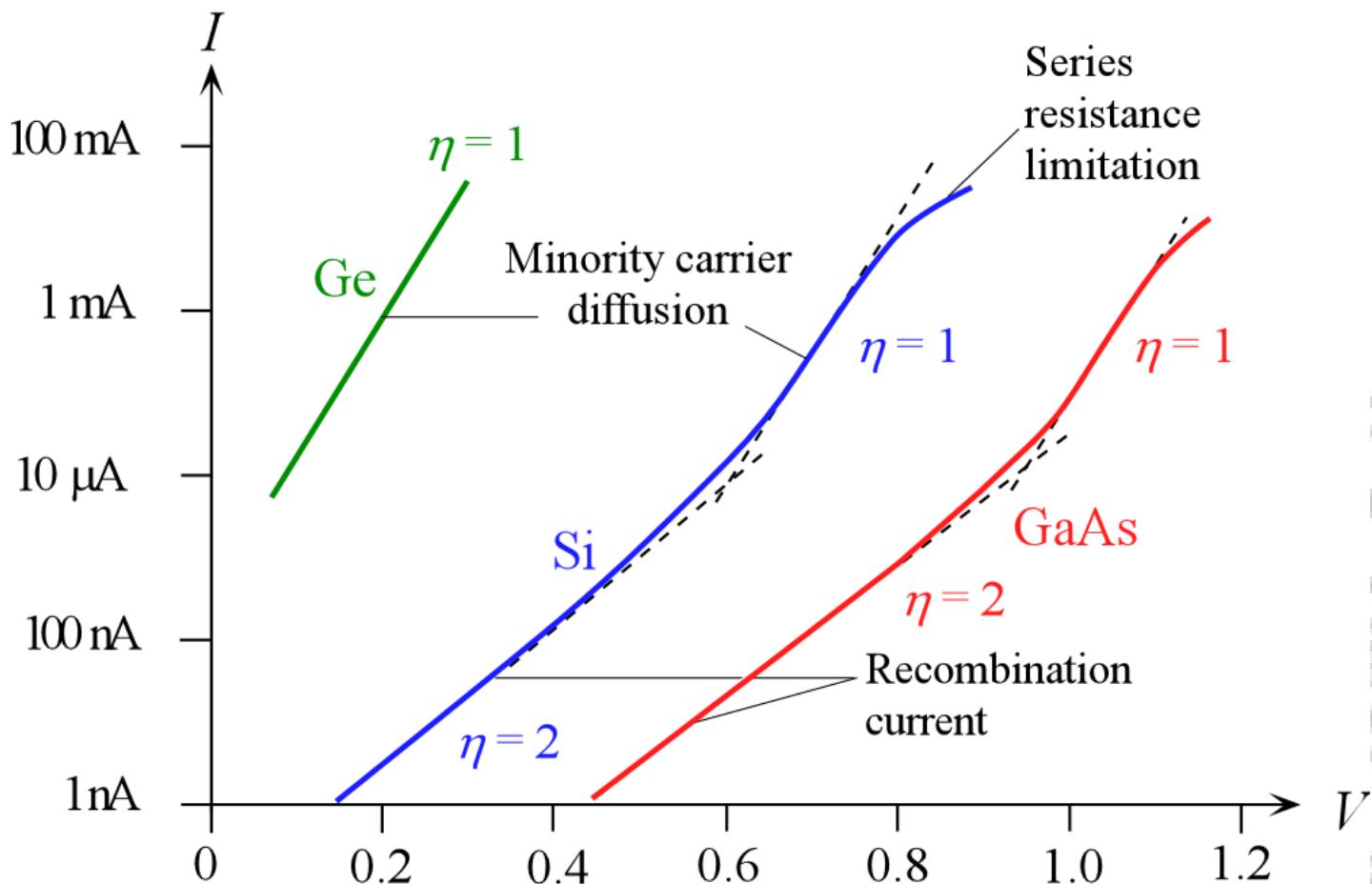
The diode equation

$$J = J_o \left[ \exp\left(\frac{eV}{\eta k_B T}\right) - 1 \right]$$

General diode equation

where  $J_o$  is a new constant and  $\eta$  is an ideality factor

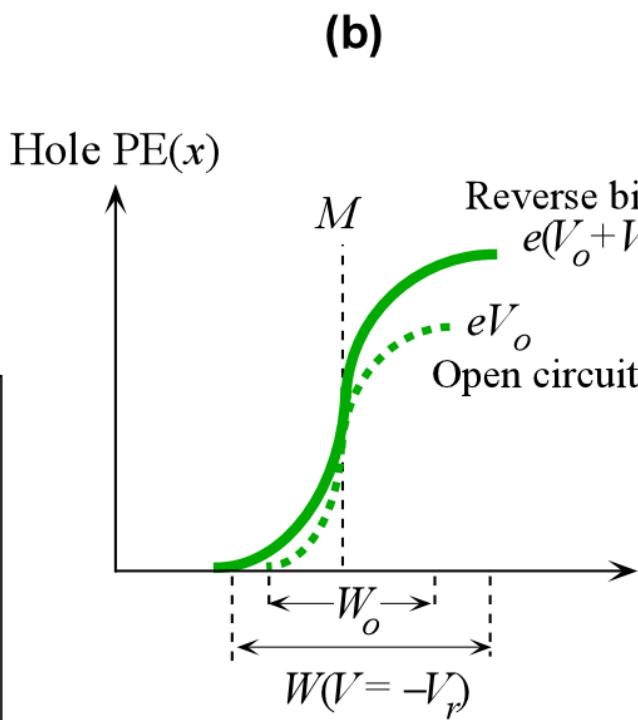
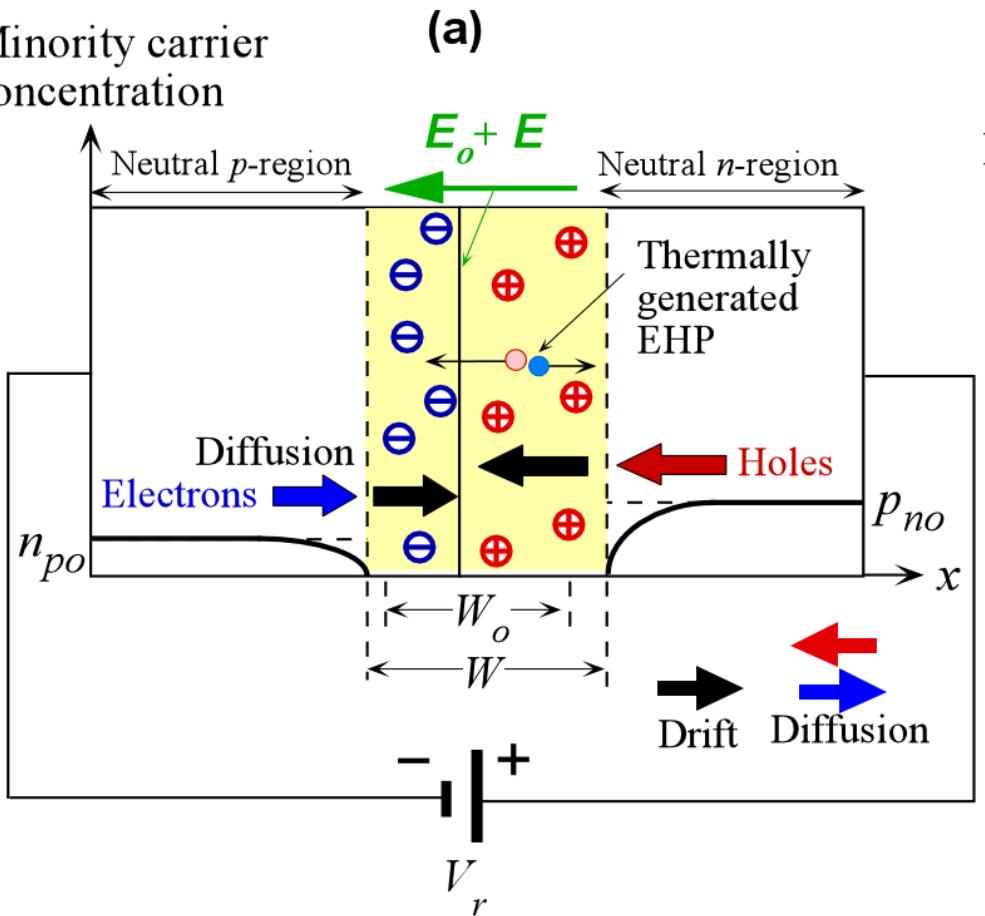
## Typical $I$ - $V$ characteristics of Ge, Si and GaAs $pn$ junctions



Typical  $I$ - $V$  characteristics of Ge, Si and GaAs  $pn$  junctions as  $\log(I)$  vs.  $V$ . The slope indicates  $e/(\Omega k_B T)$

# Reverse Bias

Minority carrier concentration



Reverse biased *pn* junction. (a) Minority carrier profiles and the origin of the reverse current. (b) Hole PE across the junction under reverse bias

# Reverse Bias



## Total Reverse Current

$$J_{\text{rev}} = \left( \frac{eD_h}{L_h N_d} + \frac{eD_e}{L_e N_a} \right) n_i^2 + \frac{eWn_i}{\tau_g}$$

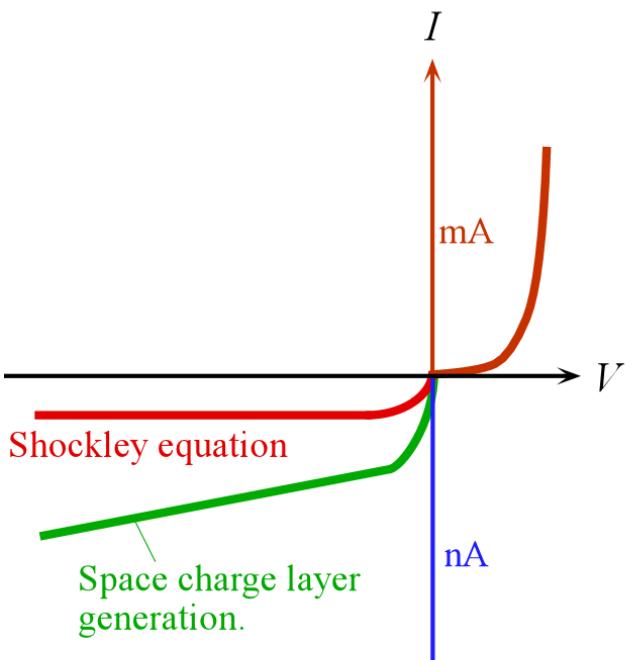
Diffusion current in neutral regions  
the Shockley reverse current

Considered long diode

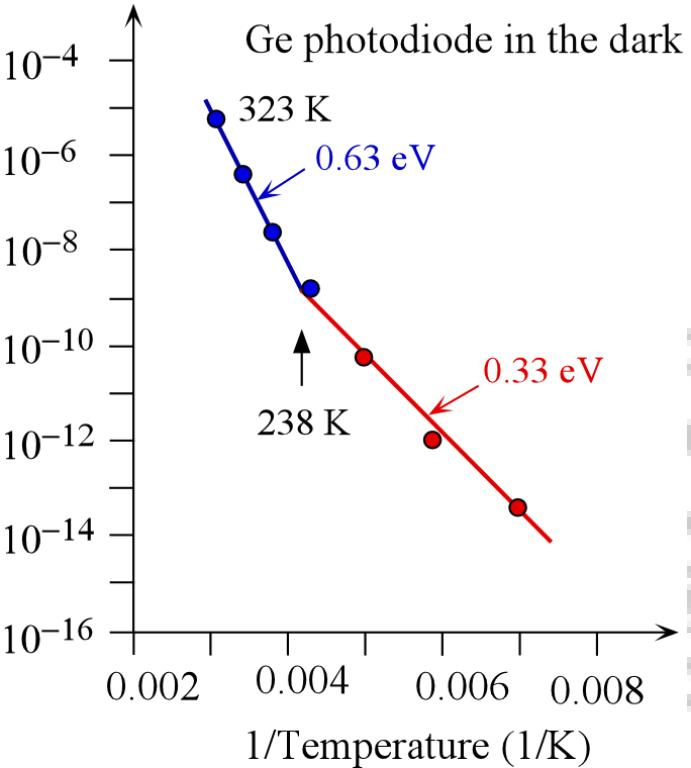
Thermal generation in depletion region

Mean thermal generation time

# Reverse Current



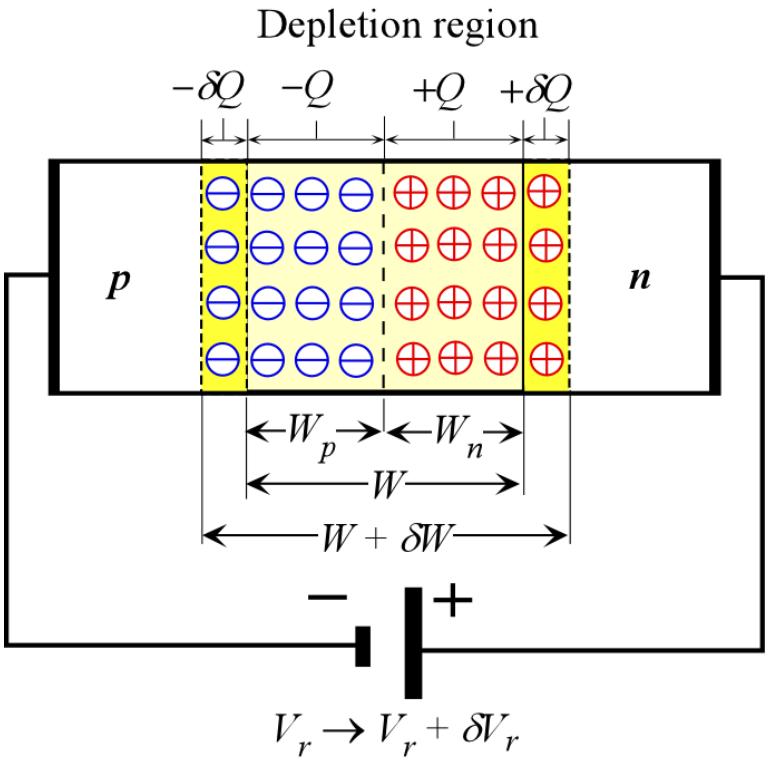
Reverse diode  
current (A) at  $V = -5$  V



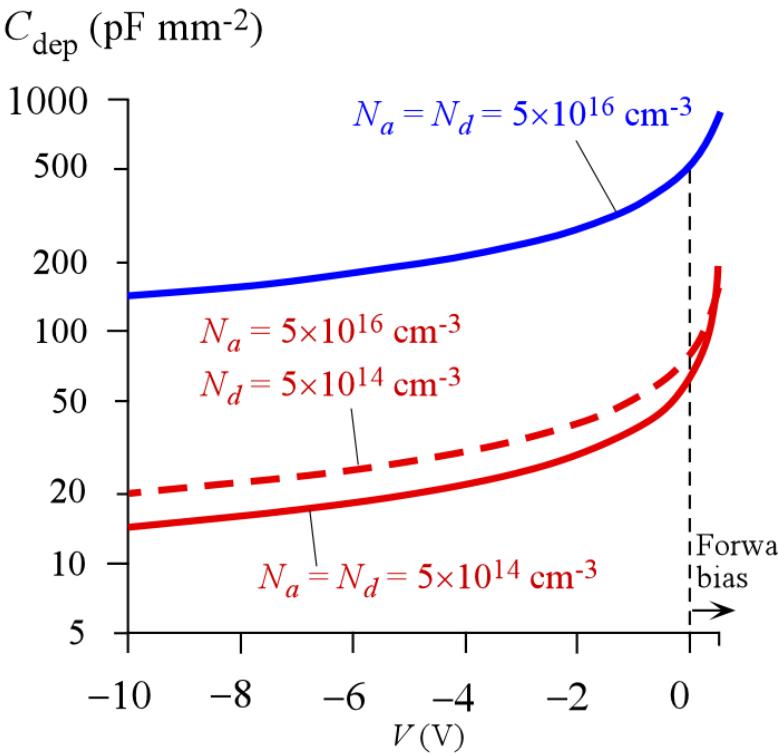
Left: Reverse  $I$ - $V$  characteristics of a  $pn$  junction (the positive and negative current axes have different scales). Right: Reverse diode current in a Ge  $pn$  junction as a function of temperature in a  $\ln(I_{rev})$  vs  $1/T$  plot. Above 238 K,  $I_{rev}$  is controlled by  $n_i^2$  and below 238 K it is controlled by  $n_i$ . The vertical axis is a logarithmic scale with actual current values. (From D. Scansen and S.O. Kasap, *Cnd. J. Physics.*

**70**, 1070-1075, 1992.)

# Depletion Capacitance: $C_{\text{dep}}$



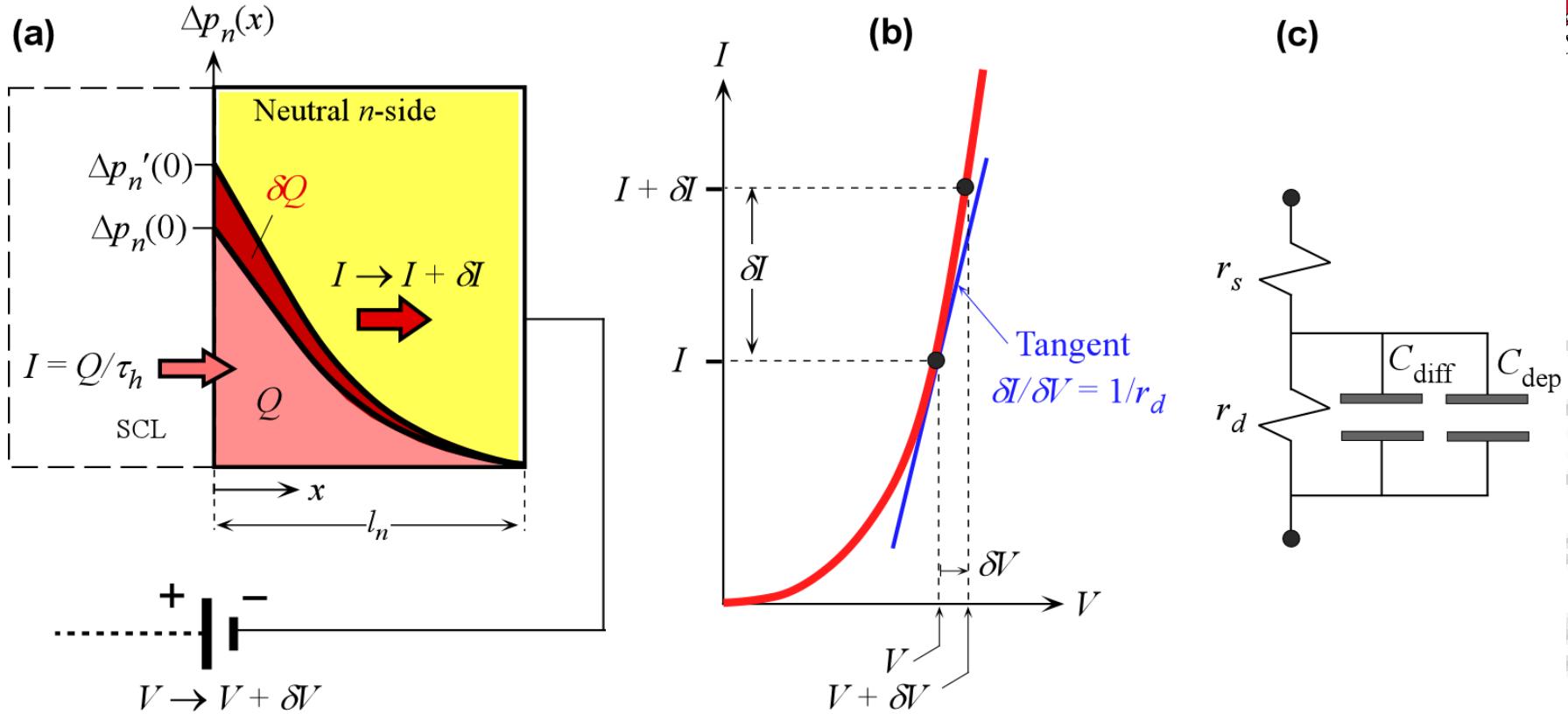
(a)



(b)

(a) Depletion region has negative ( $-Q$ ) charges in  $W_p$  and positive ( $+Q$ ) charges in  $W_n$ , which are separated as in a capacitor. Under a reverse bias  $V_r$ , the charge on the *n*-side is  $+Q$ . When the reverse bias is increased by  $\delta V_r$ , this charge increases by  $\delta Q$ . (b)  $C_{\text{dep}}$  vs voltage across an abrupt *pn* junction for 3 different set of dopant concentrations. (Note that the vertical scale is logarithmic.)

# Dynamic Resistance and Capacitance



(a) The forward voltage across a *pn* junction increases by  $\delta V$ , which leads to further minority carrier injection and a larger forward current, which increases by  $\delta I$ . Additional minority carrier charge  $\delta Q$  is injected into the *n*-side. The increase  $\delta Q$  in charge stored in the *n*-side with  $\delta V$  appears as though there is a capacitance across the diode. (b) The increase  $\delta V$  results in an increase  $\delta I$  in the diode current. The dynamic or incremental resistance  $r_d = \delta V / \delta I$ . (c) A simplified equivalent circuit for a forward biased *pn* junction for small signals.



# Forward Bias Dynamic Resistance and Capacitance

*pn* junction current

$$I = I_o \exp(eV/k_B T)$$

$$I = Q/\tau_h$$

Dynamic resistance

$$r_d = \frac{dV}{dI} = \frac{V_{\text{th}}}{I}$$

$$V_{\text{th}} = k_B T/e$$

Diffusion capacitance

$$C_{\text{diff}} = \frac{dQ}{dV}$$

Diffusion capacitance

$$C_{\text{diff}} = \frac{\tau_h I}{V_{\text{th}}} = \frac{\tau_h}{r_d}$$



# Forward Bias Diffusion Capacitance

Diffusion capacitance

$$C_{\text{diff}} = \frac{dQ}{dV}$$

Diffusion capacitance,  
long diode (ac)

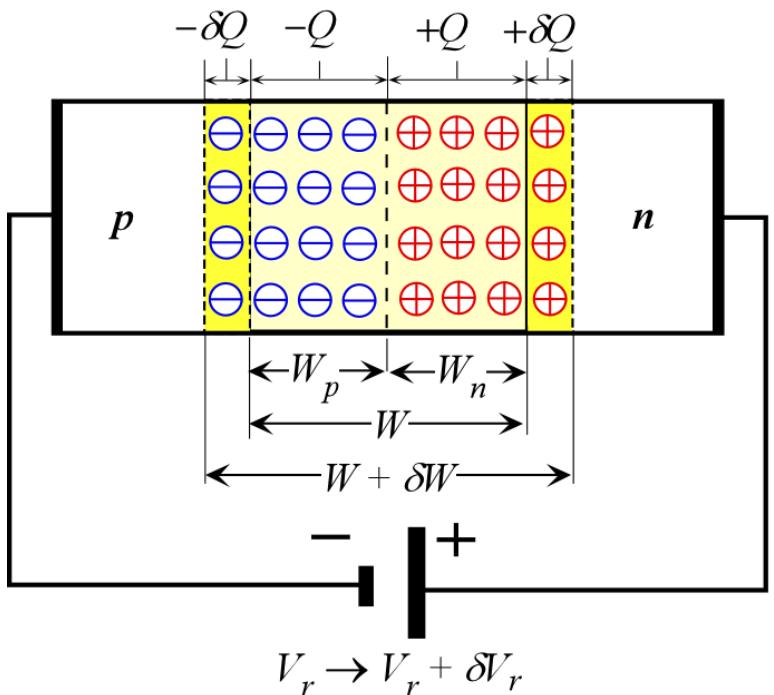
$$C_{\text{diff}} = \frac{\tau_h I}{2V_{\text{th}}} = \frac{\tau_h}{2r_d}$$

Diffusion capacitance, short  
diode (ac)

$$C_{\text{diff}} = \frac{\tau_t I}{V_{\text{th}}} = \frac{\tau_t}{r_d}$$

Minority carrier transit time  
 $\tau_t = l_n^2 / 2D_h$

# Depletion Capacitance: $C_{\text{dep}}$



$$C_{\text{dep}} = \left| \frac{\delta Q}{\delta V} \right| = \left| \frac{dQ}{dV} \right|$$

Definition

$$C_{\text{dep}} = \frac{\epsilon A}{W} = \frac{A}{(V_o - V)^{1/2}} \left[ \frac{e \epsilon (N_a N_d)}{2(N_a + N_d)} \right]^{1/2}$$

$$C_{\text{dep}} = A \left[ \frac{e \epsilon N_d}{2(V_o - V)} \right]^{1/2}$$

**Depletion layer capacitance for  $p^+n$**



## EXAMPLE : A direct bandgap *pn* junction

Consider a symmetrical GaAs *pn* junction which has the following properties.  $N_a$  (*p*-side doping) =  $N_d$  (*n*-side doping) =  $10^{17} \text{ cm}^{-3}$  (or  $10^{23} \text{ m}^{-3}$ ), direct recombination coefficient  $B \approx 2 \times 10^{-16} \text{ m}^3 \text{ s}^{-1}$ , cross sectional area  $A = 1 \text{ mm}^2$ . Suppose that the forward voltage across the diode  $V = 0.80 \text{ V}$ . What is the diode current due to minority carrier diffusion at  $27^\circ\text{C}$  (300 K) assuming direct recombination. If the mean minority carrier lifetime in the depletion region were to be the same as this lifetime, what would be the recombination component of the diode current? What are the two contributions at  $V = 1.05 \text{ V}$ ? What is your conclusion? Note that at 300 K, GaAs has an intrinsic concentration ( $n_i$ ) of  $2.1 \times 10^6 \text{ cm}^{-3}$  and a relative permittivity ( $\varepsilon_r$ ) of 13.0. The hole drift mobility ( $\mu_h$ ) in the *n*-side is  $250 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$  and electron drift mobility ( $\mu_e$ ) in the *p*-side is  $5000 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$  (these are at the doping levels given).



# EXAMPLE : A direct bandgap *pn* junction

## Solution

Assuming weak injection, we can calculate the recombination times  $\tau_e$  and  $\tau_h$  for electrons and holes recombining in the neutral p and n-regions respectively. Using S.I. units throughout, and taking  $k_B T/e = 0.02585 \text{ V}$ , and since this is a symmetric device,

$$\tau_h = \tau_e \approx \frac{1}{BN_a} = \frac{1}{(2.0 \times 10^{-16} \text{ m}^3 \text{s}^{-1})(1 \times 10^{23} \text{ m}^{-3})} = 5.00 \times 10^{-8} \text{ s}$$

To find the Shockley current we need the diffusion coefficients and lengths. The Einstein relation gives the diffusion coefficients as

$$D_h = \mu_h k_B T/e = (250 \times 10^{-4}) (0.02585) = 6.46 \times 10^{-4} \text{ m}^2 \text{ s}^{-1},$$

$$D_e = \mu_e k_B T/e = (5000 \times 10^{-4}) (0.02585) = 1.29 \times 10^{-2} \text{ m}^2 \text{ s}^{-1}.$$

## Solution (continued)

where  $k_B T/e$  was taken as 0.02585 V. The diffusion lengths are easily calculated as

$$L_h = (D_h \tau_h)^{1/2} = [(6.46 \times 10^{-4} \text{ m}^2 \text{ s}^{-1})(50.0 \times 10^{-9} \text{ s})]^{1/2} = 5.69 \times 10^{-6} \text{ m},$$

$$L_e = (D_e \tau_e)^{1/2} = [(1.29 \times 10^{-2} \text{ m}^2 \text{ s}^{-1})(50.0 \times 10^{-9} \text{ s})]^{1/2} = 2.54 \times 10^{-5} \text{ m}.$$

Notice that the electrons diffuse much further in the *p*-side. The reverse saturation current due to diffusion in the neutral regions is

$$I_{so} = A \left( \frac{D_h}{L_h N_d} + \frac{D_e}{L_e N_a} \right) e n_i^2$$

$$= (10^{-6}) \left[ \frac{6.46 \times 10^{-4}}{(5.69 \times 10^{-6})(10^{23})} + \frac{1.29 \times 10^{-2}}{(2.54 \times 10^{-5})(10^{23})} \right] (1.6 \times 10^{-19})(2.1 \times 10^{12})^2$$

$$\approx 4.4 \times 10^{-21} \text{ A}$$

Thus, the forward diffusion current at  $V = 0.80 \text{ V}$  is

$$I_{\text{diff}} = I_{so} \exp\left(\frac{eV}{k_B T}\right)$$

$$= (4.4 \times 10^{-21} \text{ A}) \exp\left[\frac{0.80 \text{ V}}{0.02585 \text{ V}}\right] = 1.2 \times 10^{-7} \text{ A or } 0.12 \mu\text{A}$$



## Solution (continued)

Recombination component of the current is quite difficult to calculate because we need to know the mean electron and hole recombination times in the SCL.

Direct recombination time within the SCL is actually very long (carrier concentrations are small) and the main recombination mechanism is due to indirect recombination, which depends on defects and impurities. As a worst case assumption, we can take  $\tau_e$  and  $\tau_h$  due to indirect recombination in the SCL to be roughly the same as direct recombination lifetimes in the neutral regions above, though they are likely to be longer in practice.

First, we need the width  $W$  of the depletion region and hence  $V_o$ .

The built-in voltage  $V_o$

$$V_o = \frac{k_B T}{e} \ln \left( \frac{N_a N_d}{n_i^2} \right) = (0.02585) \ln \left[ \frac{10^{23} 10^{23}}{(2.1 \times 10^{12})^2} \right] = 1.27 \text{ V}$$



## Solution (continued)

Depletion layer width  $W$  is

$$\begin{aligned} W &= \left[ \frac{2\epsilon(N_a + N_d)(V_o - V)}{eN_a N_d} \right]^{1/2} \\ &= \left[ \frac{2(13)(8.85 \times 10^{-12} \text{ F m}^{-1})(10^{23} + 10^{23} \text{ m}^{-3})(1.27 - 0.80 \text{ V})}{(1.6 \times 10^{-19} \text{ C})(10^{23} \text{ m}^{-3})(10^{23} \text{ m}^{-3})} \right]^{1/2} \\ &= 1.16 \times 10^{-7} \text{ m, or } 0.116 \text{ } \mu\text{m.} \end{aligned}$$

As this is a symmetric diode,  $W_p = W_n = W/2$ . The pre-exponential  $I_{ro}$  is

$$\begin{aligned} I_{ro} &= \frac{Aen_i}{2} \left[ \frac{W_p}{\tau_e} + \frac{W_n}{\tau_h} \right] = \frac{Aen_i}{2} \left( \frac{W}{\tau_e} \right) \\ &= \frac{(10^{-6})(1.6 \times 10^{-19})(2.1 \times 10^{12})}{2} \left( \frac{1.16 \times 10^{-7}}{5.00 \times 10^{-8}} \right) \approx 3.9 \times 10^{-13} \text{ A} \end{aligned}$$



## Solution (continued)

$$= \frac{(10^{-6})(1.6 \times 10^{-19})(2.1 \times 10^{12})}{2} \left( \frac{1.16 \times 10^{-7}}{5.00 \times 10^{-8}} \right) \approx 3.9 \times 10^{-13} \text{ A}$$

so that at  $V = 0.8 \text{ V}$ ,

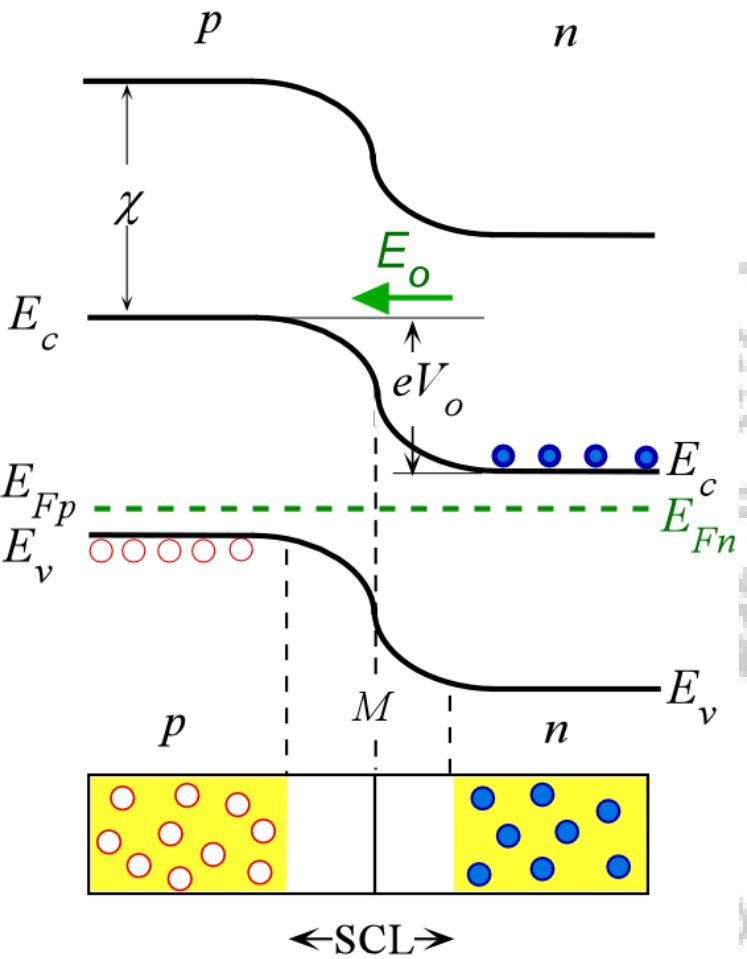
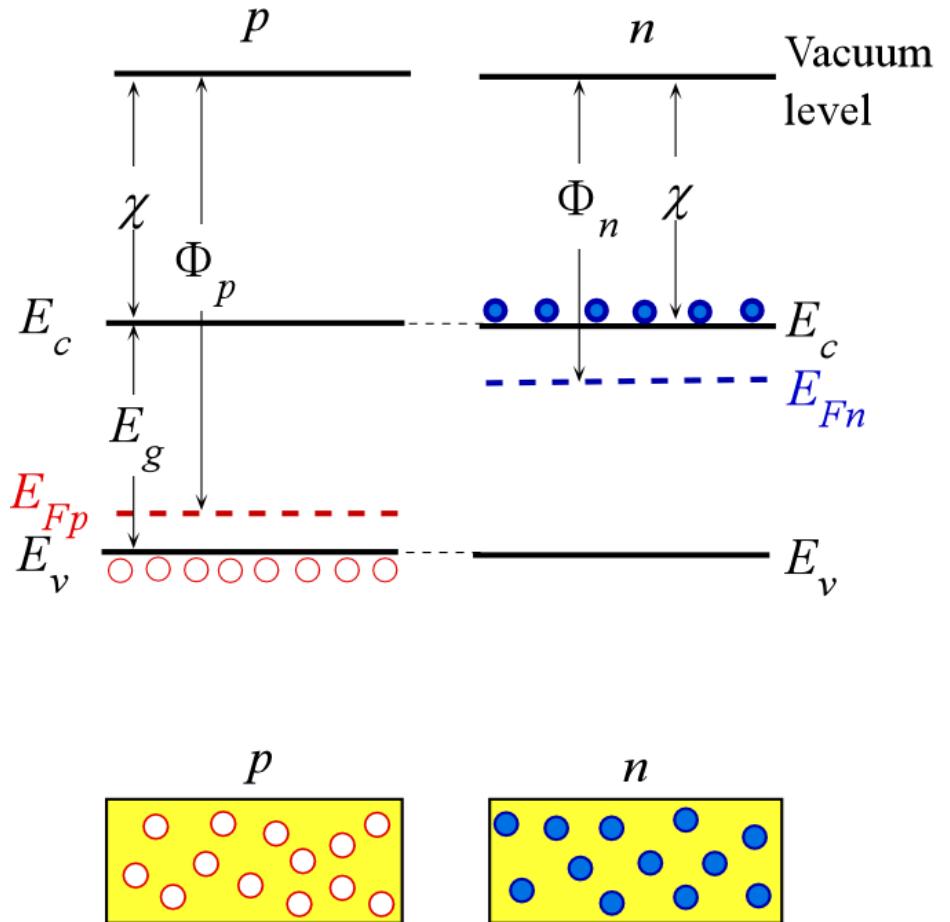
$$\begin{aligned} I_{\text{recom}} &\approx I_{ro} \exp\left(\frac{eV}{2k_B T}\right) \\ &\approx (3.9 \times 10^{-13} \text{ A}) \exp\left[\frac{0.8 \text{ V}}{2(0.02585 \text{ V})}\right] = 2.0 \times 10^{-6} \text{ A or } 2.1 \mu\text{A} \end{aligned}$$

The recombination current is more than an order of magnitude greater than the diffusion current. If we repeat the calculation for a voltage of 1.05 V across the device, then we would find  $I_{\text{diff}} = 1.9 \text{ mA}$  and  $I_{\text{recom}} = 0.18 \text{ mA}$ , where  $I_{\text{diff}}$  dominates the current. Thus, as the voltage increases across a GaAs *pn* junction, the ideality factor  $\eta$  is initially 2 but then becomes 1 as shown in Figure 3.20.

The EHP recombination that occurs in the SCL and the neutral regions in this GaAs *pn* junction case would result in photon emission, with a photon energy that is approximately  $E_g$ . This direct recombination of injected minority carriers and the resulting emission of photons represent the principle of operation of the Light Emitting Diode (LED).

# recombination

# Back to the band diagram



LEFT: Consider *p*- and *n*-type semiconductor (same material) before the formation of the *p*-*n* junction,, separated from each and not interacting. RIGHT: After the formation of the *p*-*n* junction, there is a built-in voltage across the junction.

## EXAMPLE : The built-in voltage from the band diagram

Derive the expression for the built-in voltage  $V_o$  using the energy band diagram.

### Solution

The extent of band bending from  $n$  gives the PE barrier  $eV_o$ , thus

$$eV_o = \Phi_p - \Phi_n = E_{Fn} - E_{Fp} \text{ (before contact)}$$

Before the contact, on the  $n$ -side we have

$$n = N_c \exp[-(E_c - E_{Fn})/k_B T] = N_d$$

so that

$$E_c - E_{Fn} = -k_B T \ln(N_d / N_c) \quad (3.9.1)$$

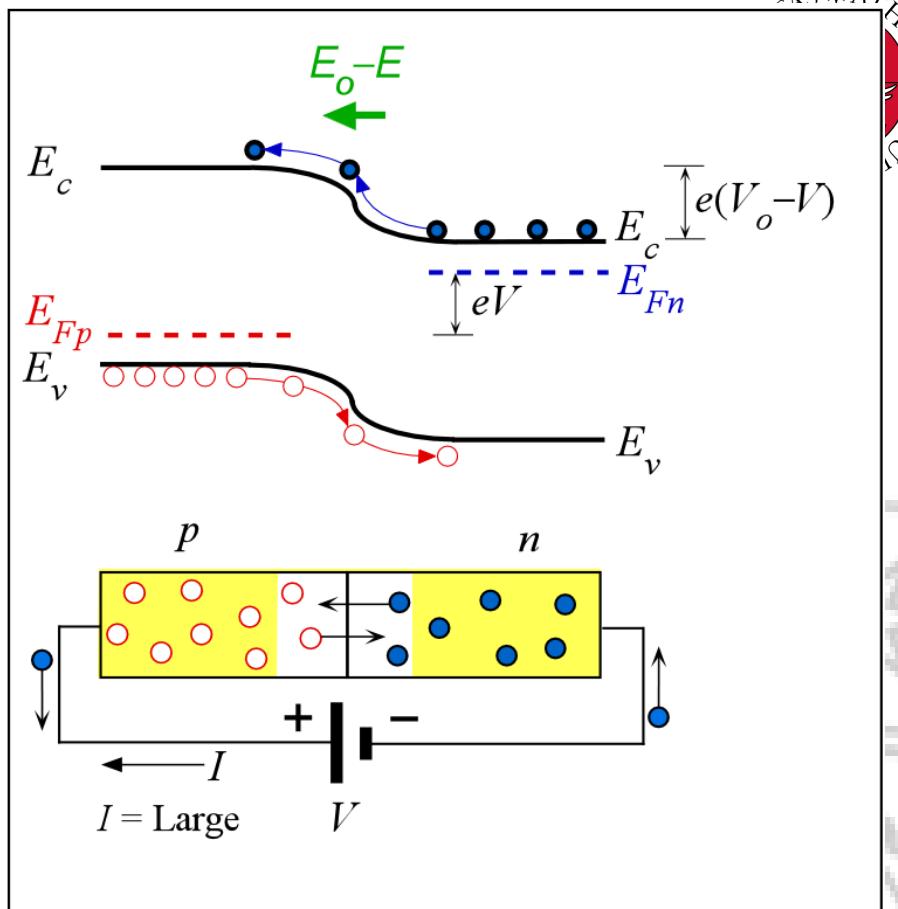
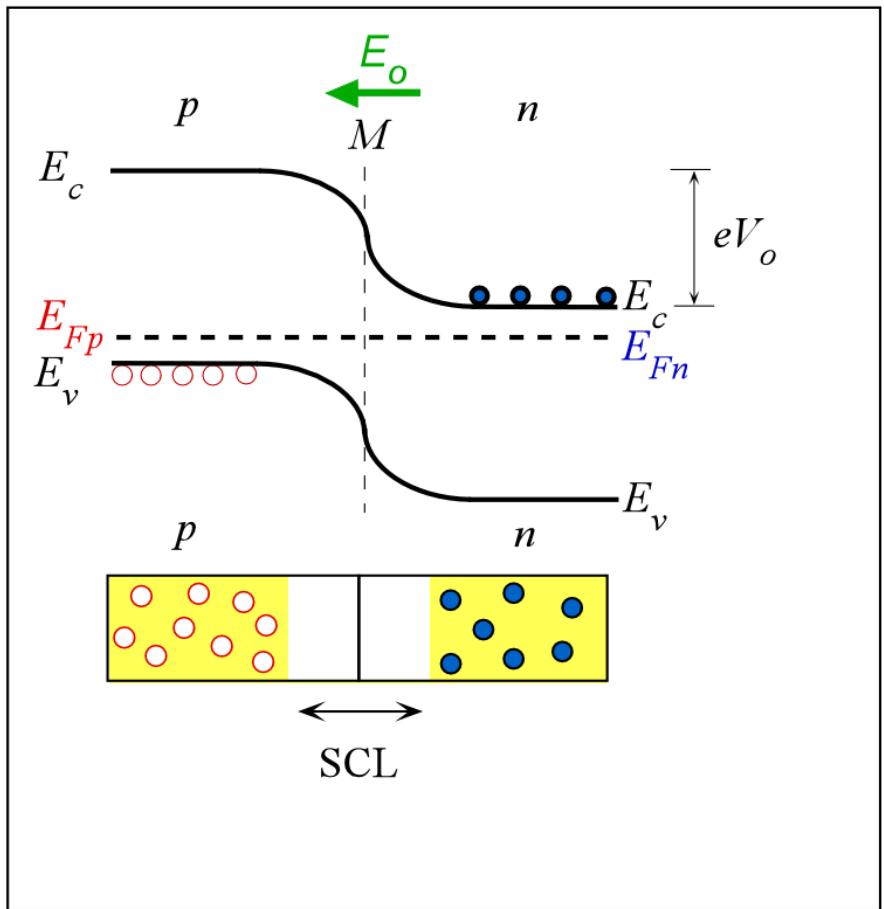
On the  $p$ -side  $n = N_c \exp[-(E_c - E_{Fp})/k_B T] = n_i^2 / N_a$

so that  $E_c - E_{Fp} = -k_B T \ln[n_i^2 / (N_a N_c)] \quad (3.9.2)$

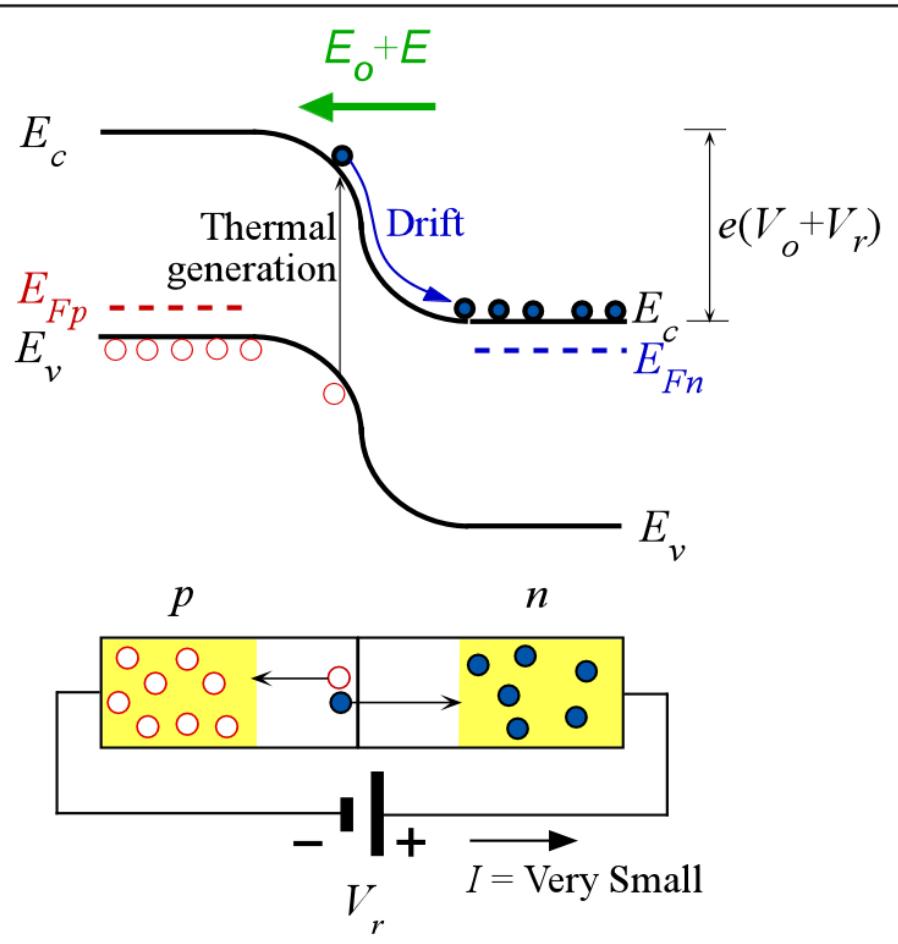
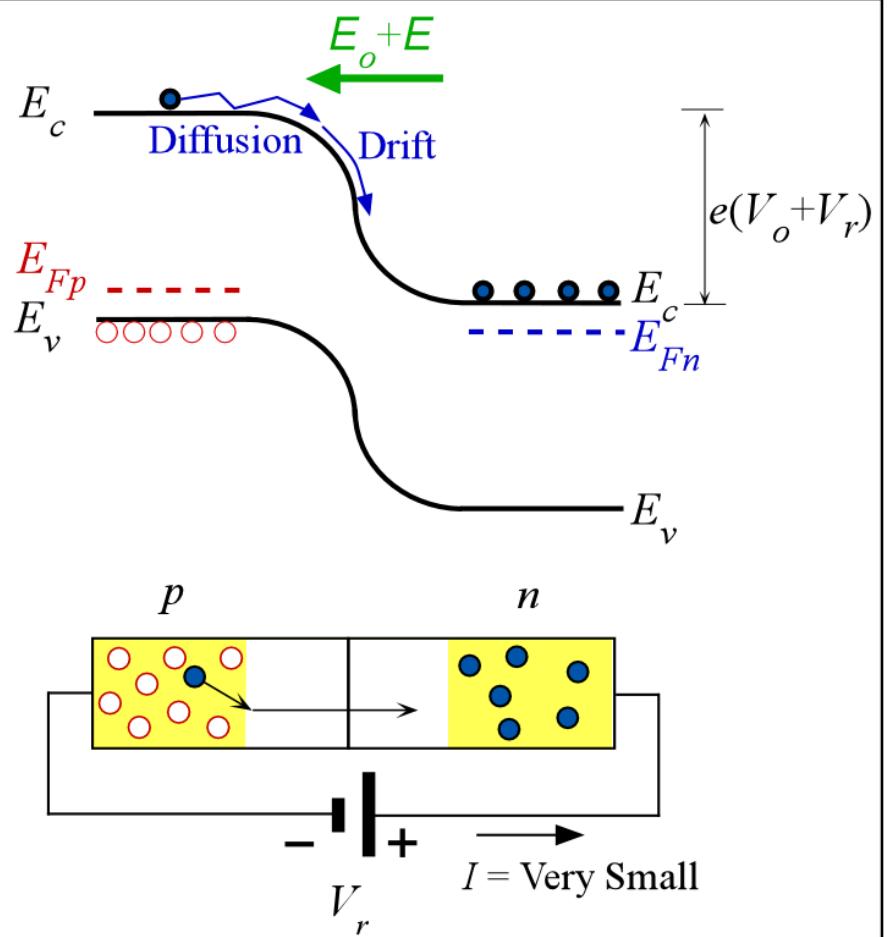
Subtracting Eq. (3.9.2) from (3.9.1) gives

$$eV_o = E_{Fn} - E_{Fp} = k_B T \ln[(N_a N_d) / n_i^2]$$





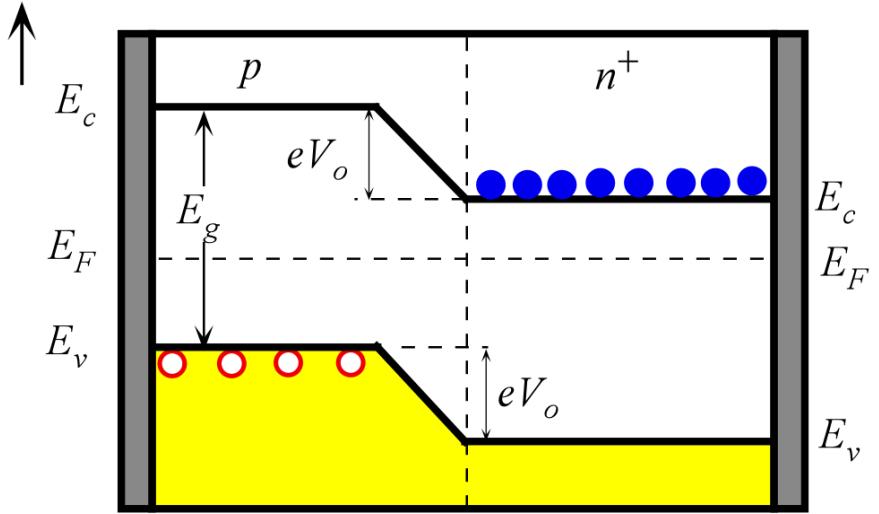
**Energy band diagrams for a *pn* junction under  
(a) open circuit and (b) forward bias**



**Energy band diagrams for a *pn* junction under reverse bias**  
**LEFT: Shockley model**  
**RIGHT: Thermal generation in the depletion region**

# Basic LED Principle

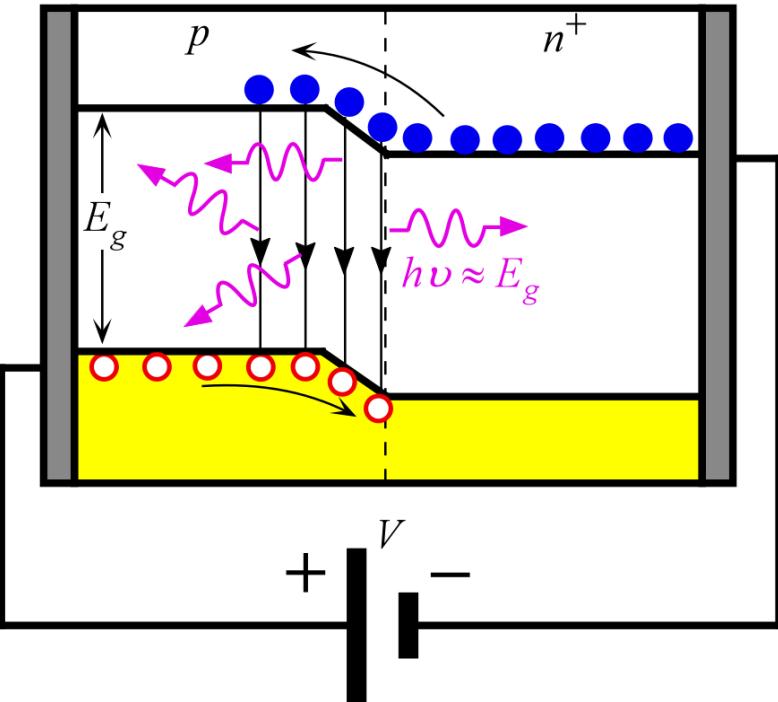
(a) Electron energy



→ Distance into device

- Electron in CB
- Hole in VB

(b)

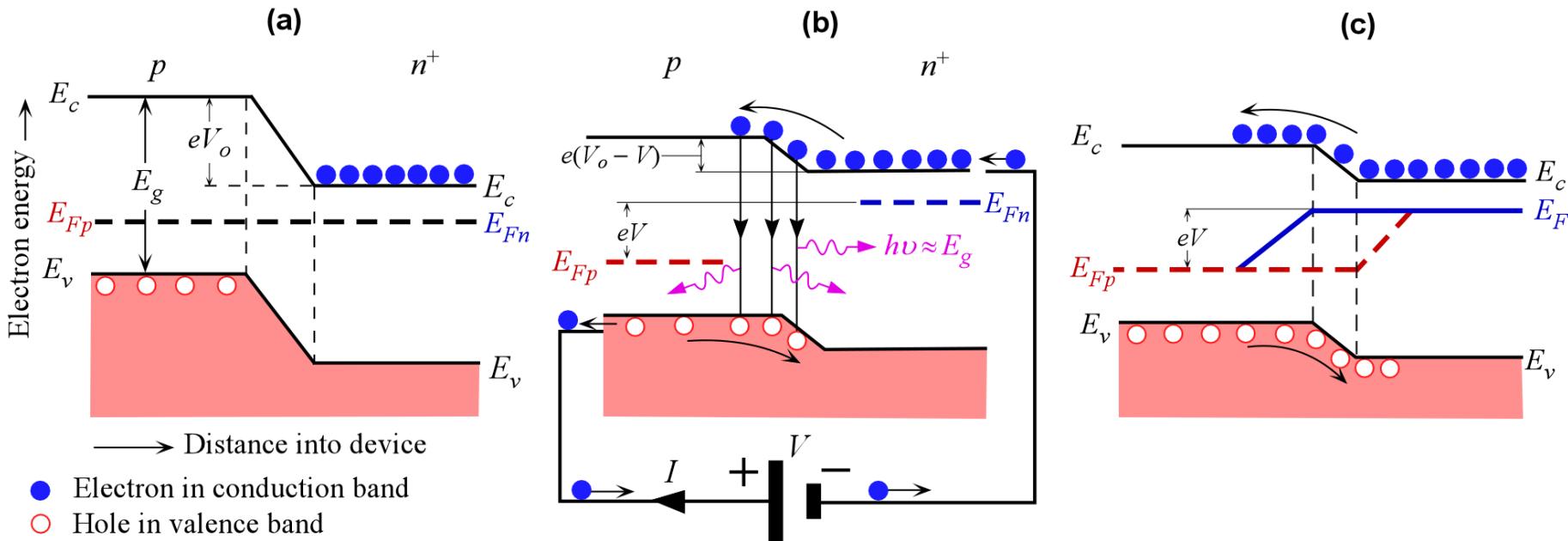


(a) The energy band diagram of a  $pn^+$  (heavily  $n$ -type doped) junction without any bias.

Built-in potential  $V_o$  prevents electrons from diffusing from  $n^+$  to  $p$  side.

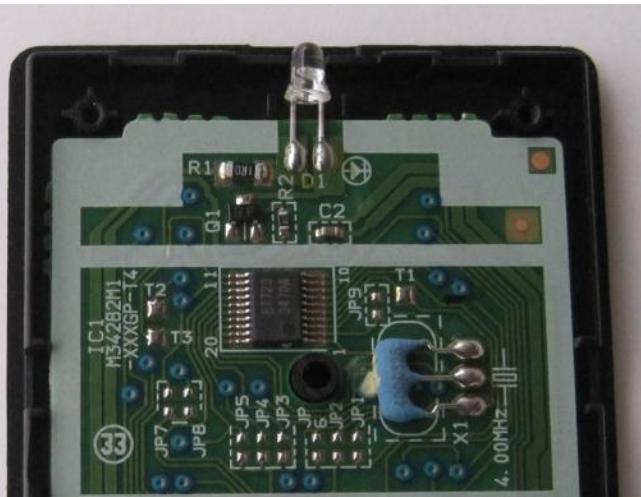
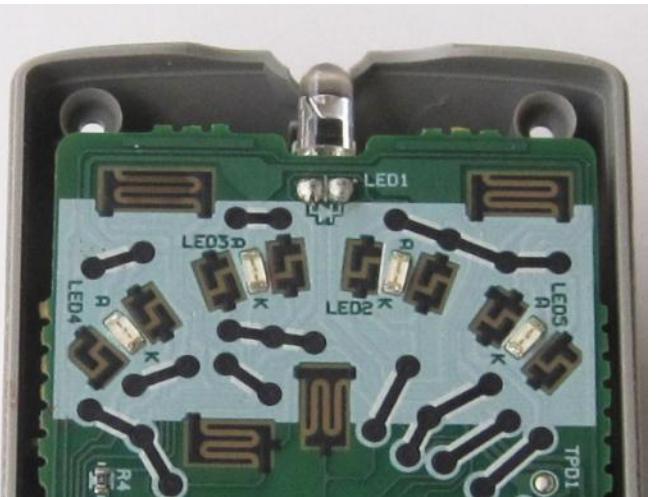
(b) The applied bias reduces  $V_o$  and thereby allows electrons to diffuse, be injected, into the  $p$ -side. Recombination around the junction and within the diffusion length of the electrons in the  $p$ -side leads to photon emission.

# Light Emitting Diodes



(a) The energy band diagram of a  $pn^+$  (heavily  $n$ -type doped) junction without any bias. Built-in potential  $V_o$  prevents electrons from diffusing from  $n^+$  to  $p$  side. (b) The applied bias potential  $V$  reduces  $V_o$  and thereby allows electrons to diffuse, be injected, into the  $p$ -side. Recombination around the junction and within the diffusion length of the electrons in the  $p$ -side leads to spontaneous photon emission. (c) Quasi-Fermi levels  $E_{Fp}$  and  $E_{Fn}$  for holes and electrons across a forward biased  $pn^-$  junction.

# Light Emitting Diodes



(Photos by SK)



UV LED

Courtesy of Chris Collins

# LED Light Bulbs



(Photos by SK)

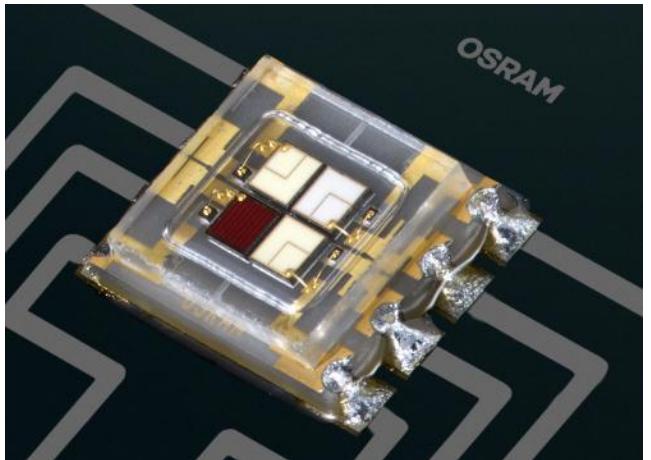
# Light Emitting Diodes



(Photo by SK)



# Light Emitting Diodes



This multichip LED from Osram is used in microprojectors. It is based on thin film GaN technology. (Courtesy of Osram)

Snap LED emitting in the amber for automotive signaling applications. The lamp includes the driver under the LED (Courtesy of Philips Lumileds)

LUXEON Rebel ES white emitting LED (Courtesy of Philips Lumileds)

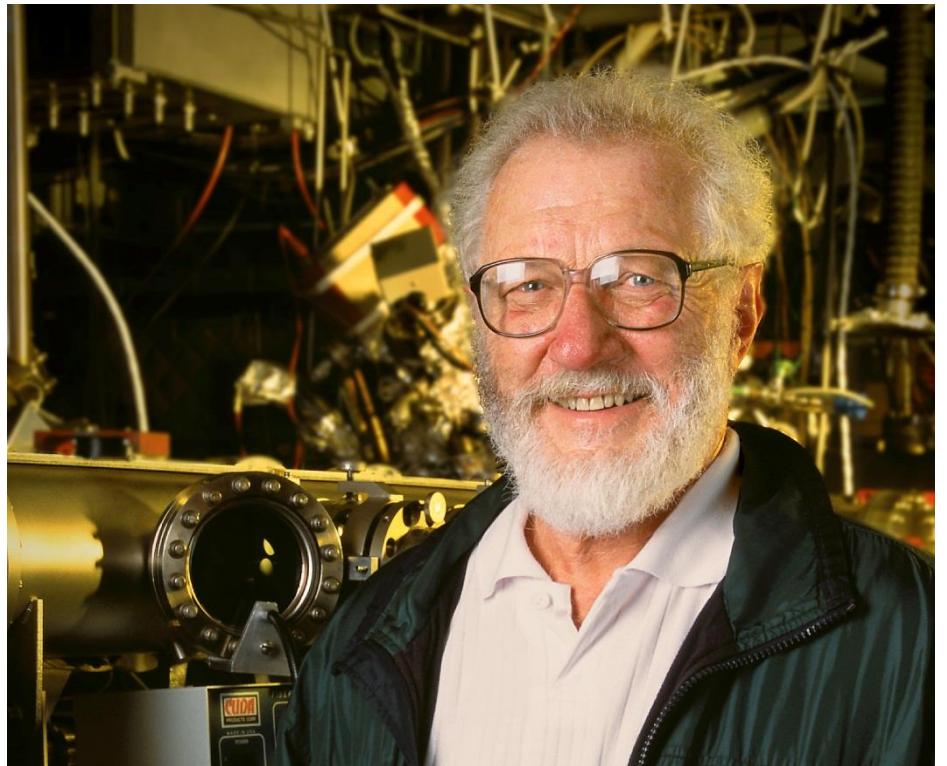


# Microprojector using LEDs



A handheld microprojector MPro120 from 3M for projecting photos and videos on a wall uses LEDs for its projection light. (Courtesy fo 3M)

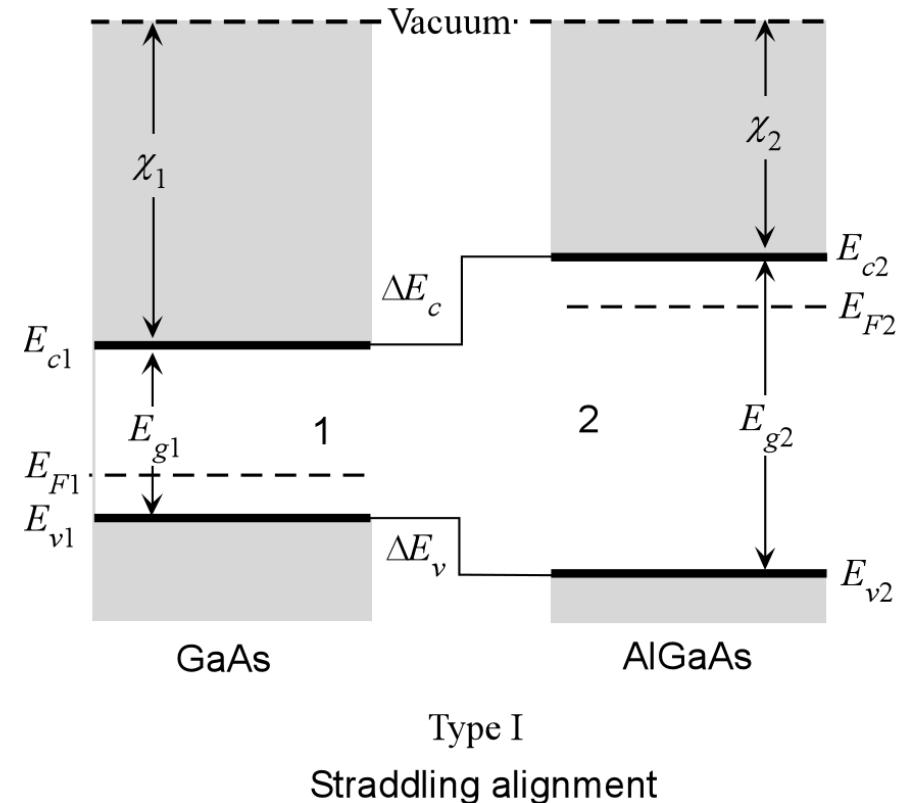
# Semiconductor Heterostructures



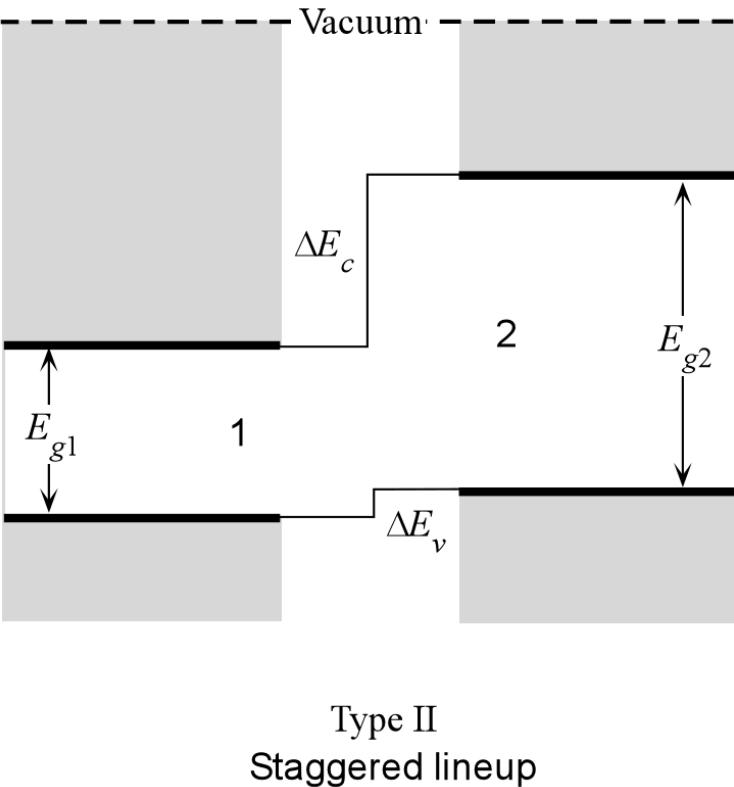
Herbert Kroemer (left), along with Zhores Alferov (See Ch. 4), played a key role in the development of semiconductor heterostructures that are widely used in modern optoelectronics. Herbert Kroemer was also well-recognized for his experimental work on the fabrication of heterostructures by using an atomic layer-by-layer crystal growth technique called Molecular Beam Epitaxy (MBE); the equipment shown behind Professor Kroemer in the photo. Since 1976, Professor Kroemer has been with the University of California, Santa Barbara where he continues his research. Herbert Kroemer and Zhores Alferov shared the Nobel Prize in Physics (2000) with Jack Kilby. Their Nobel citation is "*for developing semiconductor heterostructures used in high-speed-and opto-electronics*" (Courtesy of Professor Herbert Kroemer, University of California, Santa Barbara)

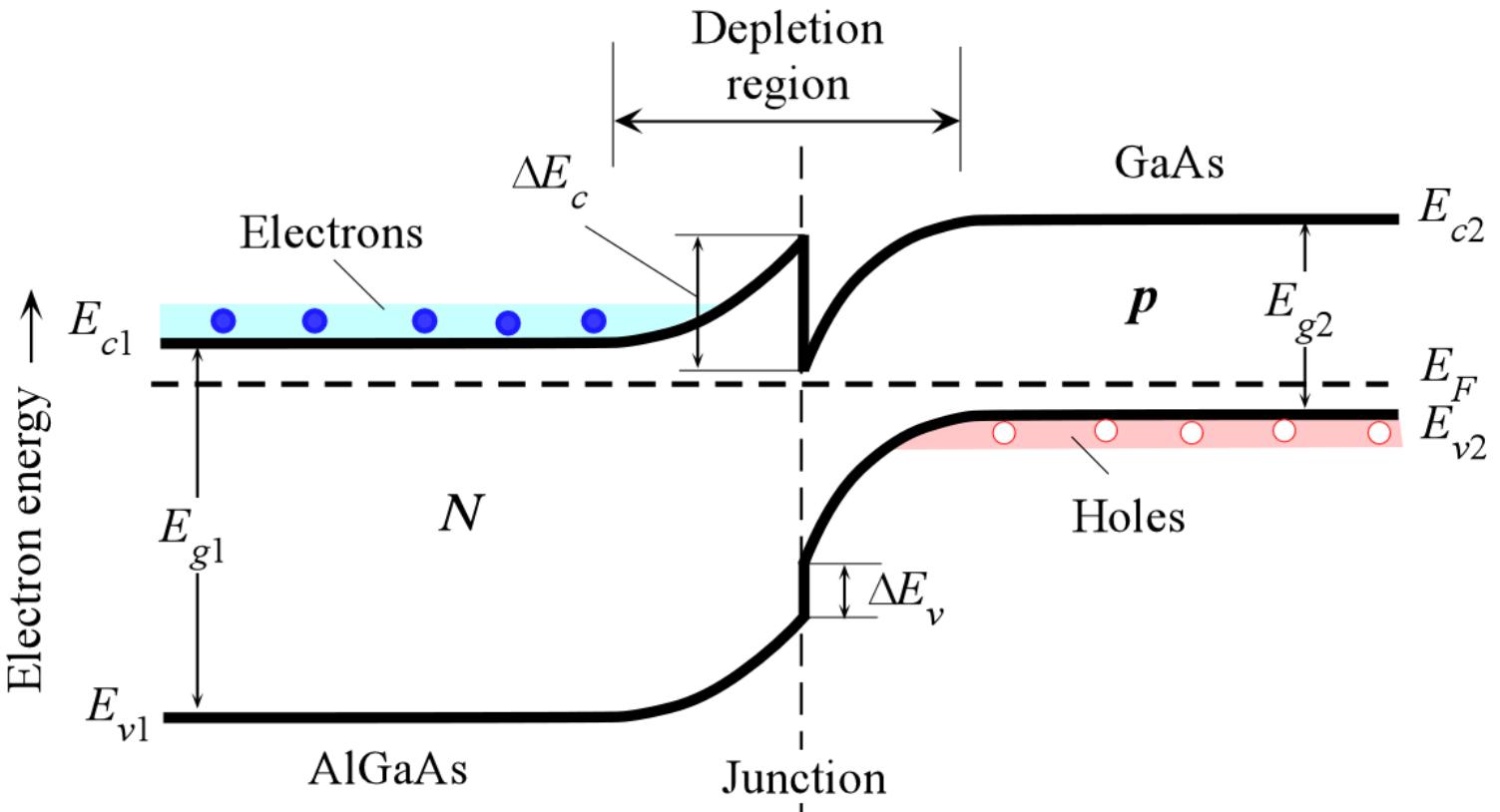
See H. Kroemer, *Rev. Mod. Phys.*, **73**, 783, 2001

# Heterojunctions

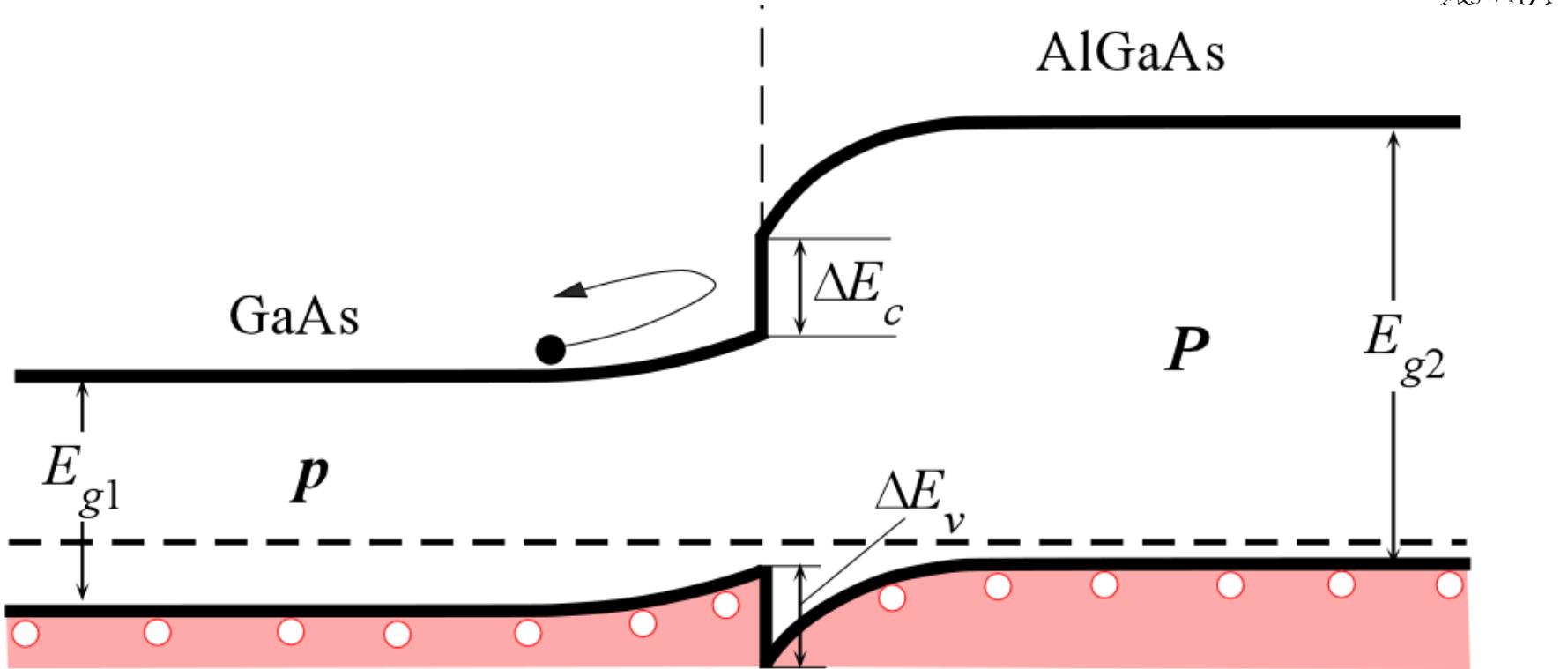


Two types of heterojunction and the definitions of band offsets, Type I and Type II between two semiconductor crystals 1 and 2. Crystal 1 has a narrower bandgap  $E_{g1}$  than  $E_{g2}$  for crystal 2. Note that the semiconductors are not in contact so that the Fermi level in each is different. In this example, crystal 1 (GaAs) is *p*-type and crystal 2 (AlGaAs) is *N*-type.



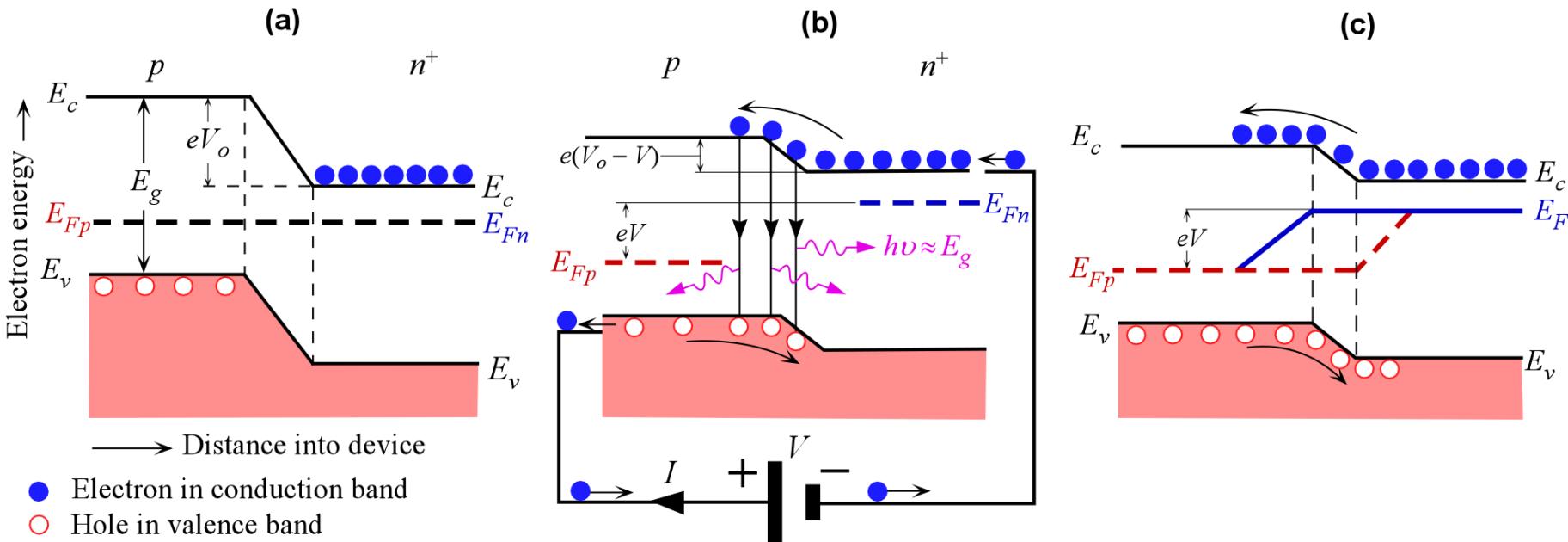


*Np* heterojunction energy band diagram. Under open circuit and equilibrium conditions, the Fermi level  $E_F$  must be uniform, *i.e.* continuous throughout the device. If  $E_F$  is close to the conduction band (CB) edge,  $E_c$ , it results in an *n*-type, and if it is close to the valence band (VB) edge,  $E_v$ , it results in a *p*-type semiconductor. There is a discontinuity  $\Delta E_c$  in  $E_c$ , and  $\Delta E_v$  in  $E_v$ , right at the junction.

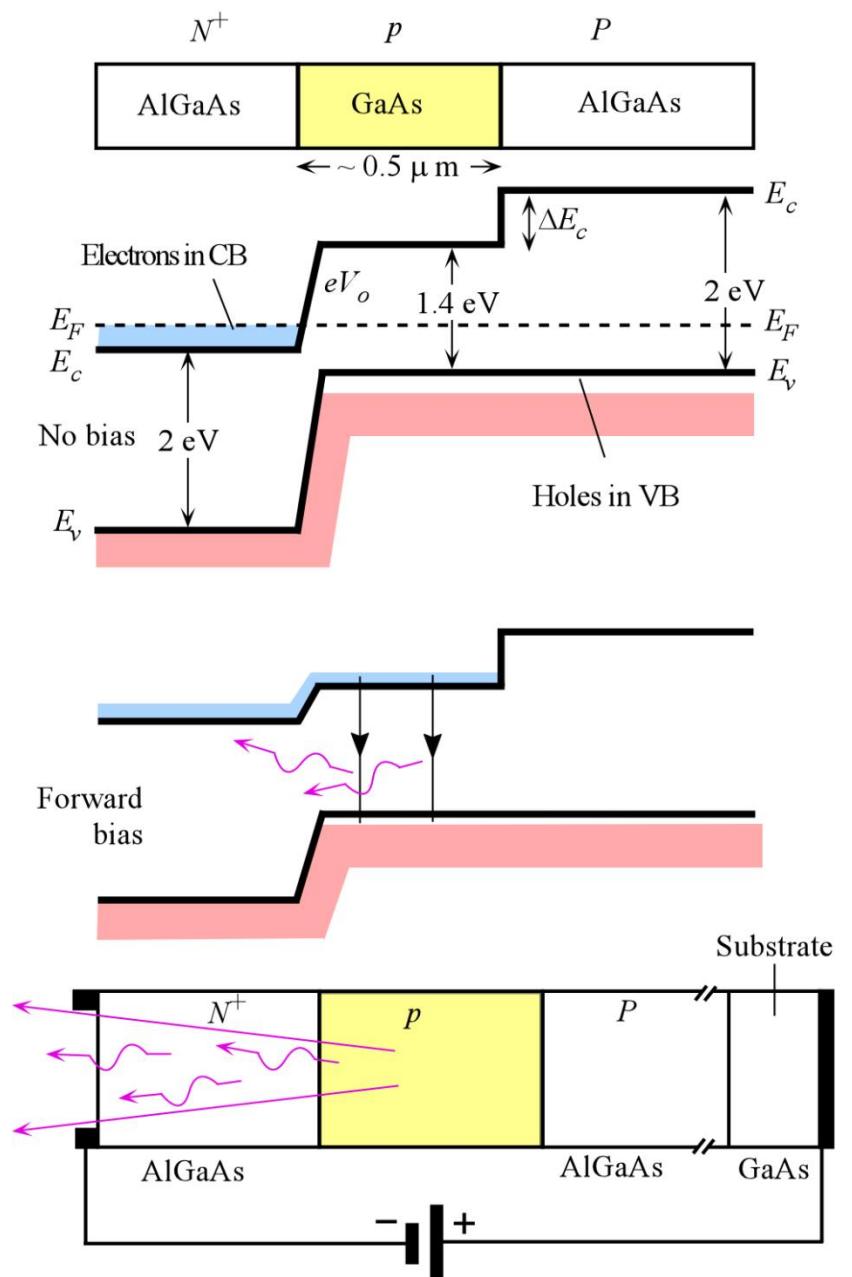


$pP$  heterojunction energy band diagram. (Schematic only to illustrate general features). Under open circuit and equilibrium conditions, the Fermi level  $E_F$  must be uniform, *i.e.* continuous throughout the device. If  $E_F$  is close to the conduction band (CB) edge,  $E_c$ , it results in an  $n$ -type, and if it is close to the valence band (VB) edge,  $E_v$ , it results in a  $p$ -type semiconductor. There is a discontinuity  $\Delta E_c$  in  $E_c$ , and  $\Delta E_v$  in  $E_v$ , right at the junction.

# Light Emitting Diodes



(a) The energy band diagram of a  $pn^+$  (heavily  $n$ -type doped) junction without any bias. Built-in potential  $V_o$  prevents electrons from diffusing from  $n^+$  to  $p$  side. (b) The applied bias potential  $V$  reduces  $V_o$  and thereby allows electrons to diffuse, be injected, into the  $p$ -side. Recombination around the junction and within the diffusion length of the electrons in the  $p$ -side leads to spontaneous photon emission. (c) Quasi-Fermi levels  $E_{Fp}$  and  $E_{Fn}$  for holes and electrons across a forward biased  $pn^-$  junction.



**(a)** A double heterostructure diode has two heterojunctions which are between two different bandgap semiconductors (GaAs and AlGaAs)

**(b)** A simplified energy band diagram with exaggerated features.  $E_F$  must be uniform

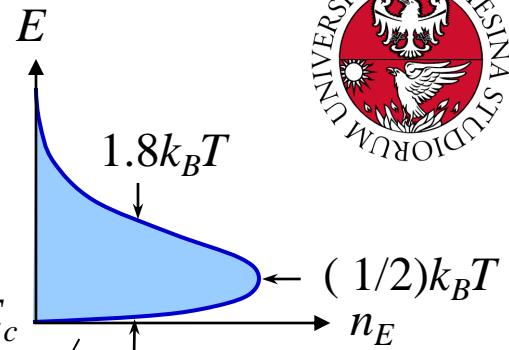
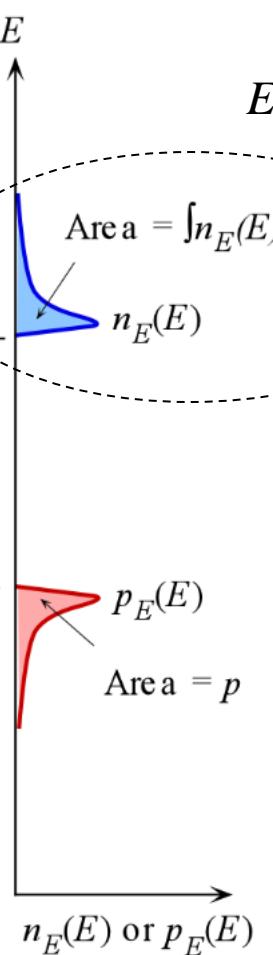
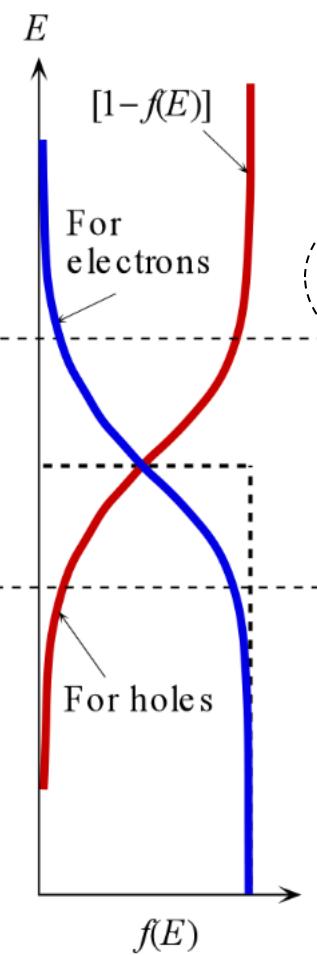
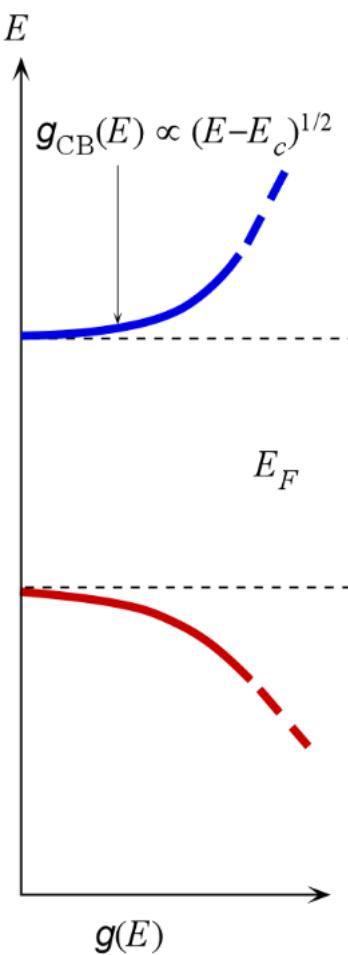
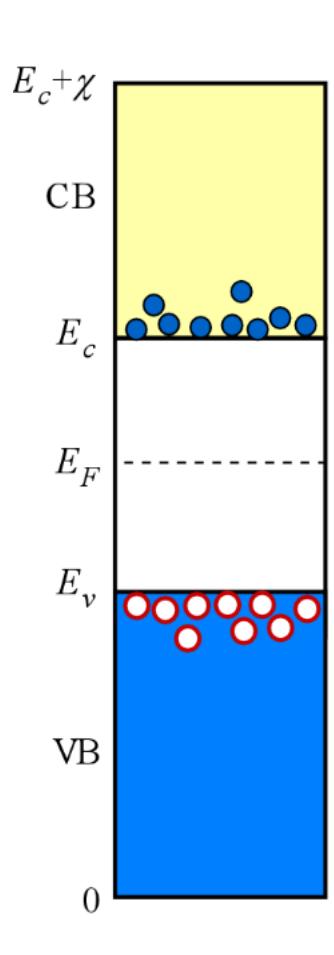
**(c)** Forward biased simplified energy band diagram

**(d)** Forward biased LED. Schematic illustration of photons escaping reabsorption in the AlGaAs layer and being emitted from the device. Layers are grown on a GaAs crystal substrate

# Remember?

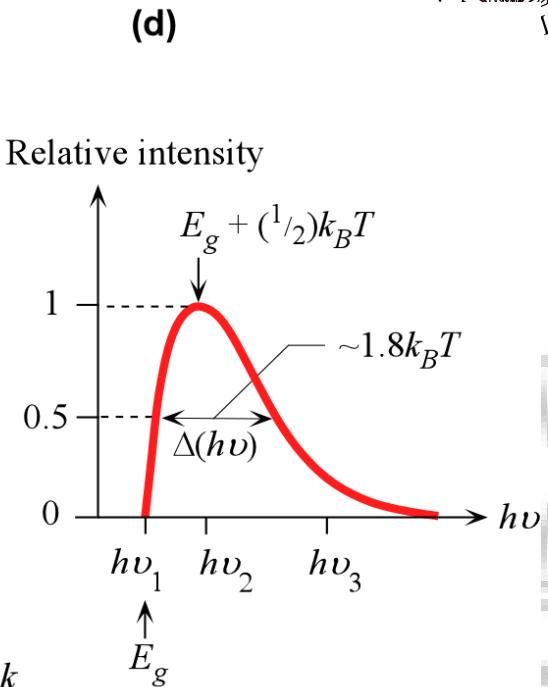
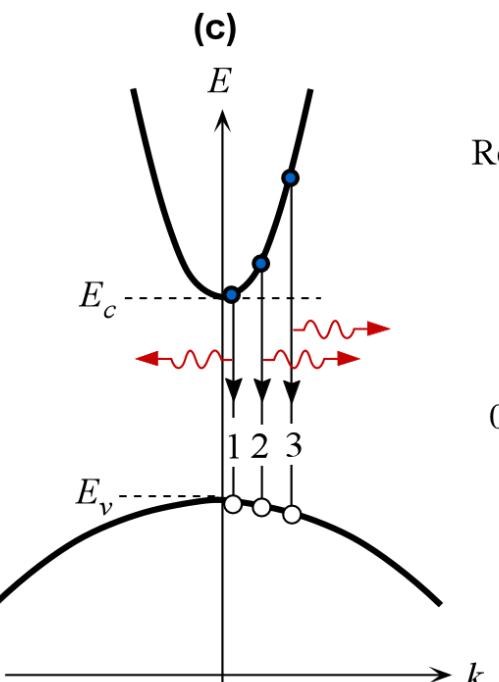
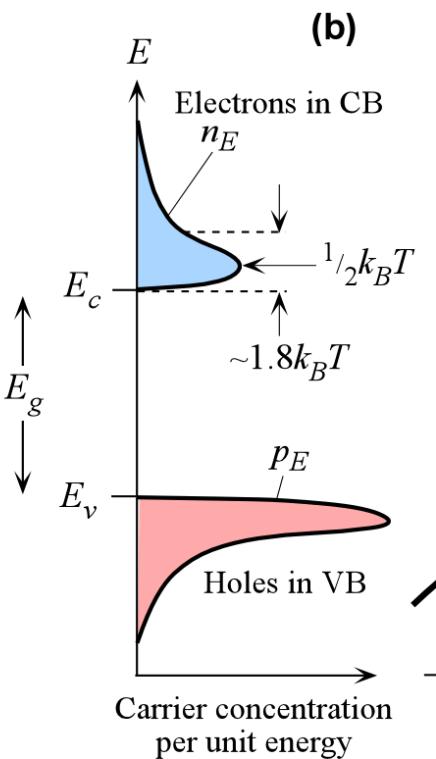
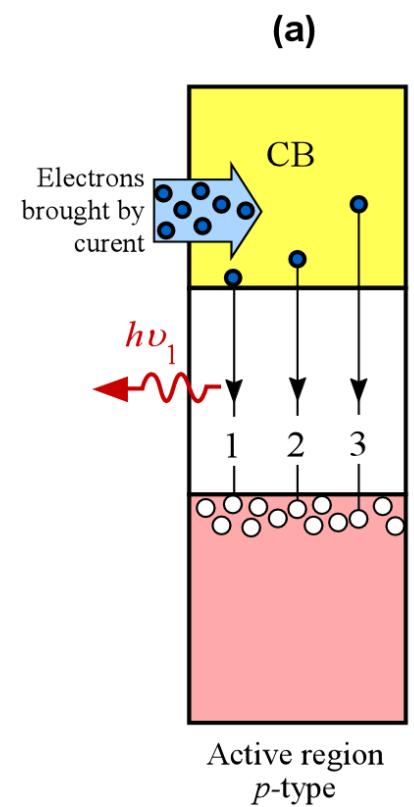


Energy distribution of electrons in the CB



Density of states      Fermi-Direct function      Energy distribution of carriers

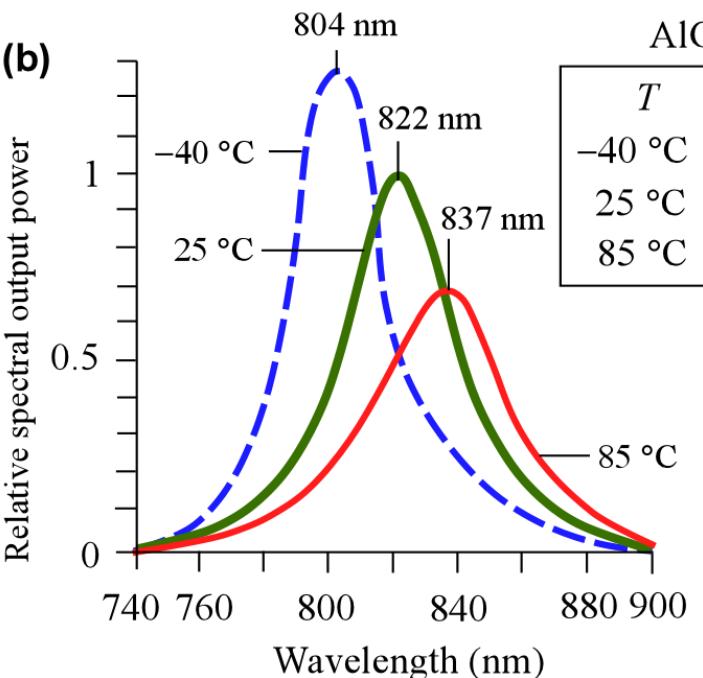
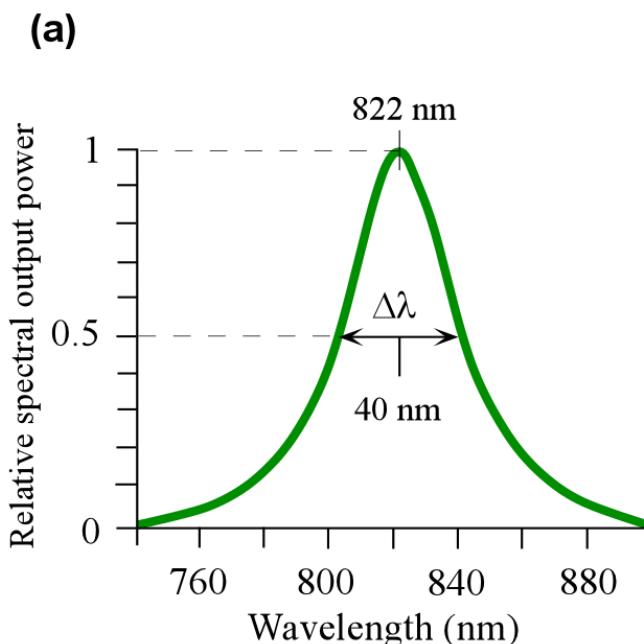
# Emission Spectrum



$$h\nu_o \approx E_g + \frac{1}{2} k_B T$$

$$h\Delta\nu = m k_B T$$

# Emission Spectrum

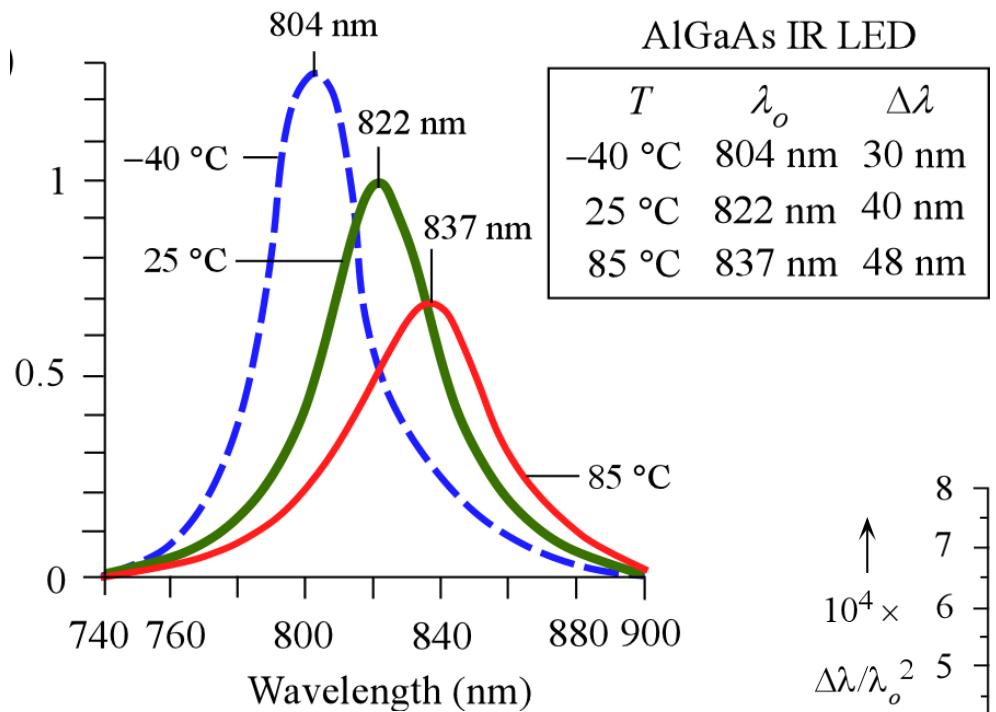


- (a) A typical output spectrum (relative intensity vs. wavelength) from an IR (infrared) AlGaAs LED.
- (b) The output spectrum of the LED in (a) at 3 temperatures: 25 °C, -40 °C and 85 °C. Values normalized to peak emission at 25 °C. The spectral widths are FWHM.

$$h\nu_o \approx E_g + \frac{1}{2} k_B T$$

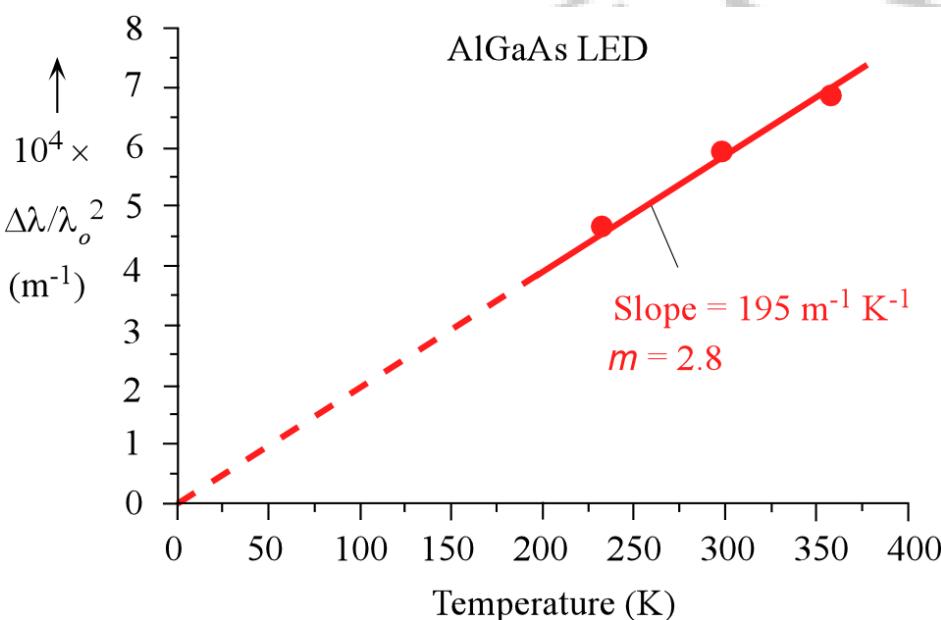
$$h\Delta\nu = m k_B T$$

# Emission Spectrum



The plot of  $\Delta\lambda/\lambda_o^2$  vs.  $T$  for an AlGaAs infrared LED, using the peak wavelength  $\lambda_o$  and spectral width  $\Delta\lambda$  at three different temperatures

The output spectrum at 3 temperatures: 25 °C, -40 °C and 85 °C. Values normalized to peak emission at 25 °C. The spectral widths are FWHM





## EXAMPLE: LED spectral linewidth

We know that a spread in the output wavelengths is related to a spread in the emitted photon energy. The emitted photon energy  $h\nu = hc/\lambda$ . Assume that the spread in the photon energies  $\Delta(h\nu) \approx 3k_B T$  between the half intensity points. Show that the corresponding linewidth  $\Delta\lambda$  between the *half intensity points* in the output spectrum is

$$\Delta\lambda = \lambda_o^2 \frac{3k_B T}{hc} \quad \text{LED spectral linewidth} \quad (3.11.3)$$

where  $\lambda_o$  is the peak wavelength. What is the spectral linewidth of an optical communications LED operating at 1310 nm and at 300 K?

### Solution

First consider the relationship between the photon frequency  $\nu$  and  $\lambda$ ,

$$\lambda = \frac{c}{\nu} = \frac{hc}{h\nu}$$

in which  $h\nu$  is the photon energy. We can differentiate this

$$\frac{d\lambda}{d(h\nu)} = -\frac{hc}{(h\nu)^2} = -\frac{\lambda^2}{hc} \quad (3.11.4)$$

The negative indicates that increasing the photon energy decreases the wavelength. We are only interested in changes, thus  $\Delta\lambda/\Delta(h\nu) \approx |d\lambda/d(h\nu)|$ , and this spread should be around  $\lambda = \lambda_o$ , so that Eq. (3.11.4) gives,



## EXAMPLE : LED spectral linewidth

### Solution (continued)

$$\Delta\lambda = \frac{\lambda_o^2}{hc} \Delta(h\nu) = \lambda_o^2 \frac{3k_B T}{hc}$$

where we used  $\Delta(h\nu) = 3k_B T$ . We can substitute  $\lambda = 1310 \text{ nm}$ , and  $T = 300 \text{ K}$  to calculate the linewidth of the 1310 nm LED

$$\begin{aligned}\Delta\lambda &= \lambda^2 \frac{3k_B T}{hc} = (1310 \times 10^{-9})^2 \frac{3(1.38 \times 10^{-23})(300)}{(6.626 \times 10^{-34})(3 \times 10^8)} \\ &= \mathbf{1.07 \times 10^{-7} \text{ m or } 107 \text{ nm}}\end{aligned}$$

The spectral linewidth of an LED output is due to the spread in the photon energies, which is fundamentally about  $3k_B T$ . The only option for decreasing  $\Delta\lambda$  at a given wavelength is to reduce the temperature. The output spectrum of a laser, on the other hand, has a much narrower linewidth.

# EXAMPLE: LED spectral width

Consider the three experimental points in Figure 3.32 (b) as a function of temperature. By a suitable plot find  $m$  and verify

$$\Delta\lambda = \lambda_o^2 \frac{3k_B T}{hc} \quad \text{LED spectral linewidth} \quad (3.11.3)$$

## Solution

From Example 3.11.1, we can use the Eq. (3.11.3). with  $m$  instead of 3 as follows

$$\frac{\Delta\lambda}{\lambda_o^2} = \left( \frac{mk_B}{hc} \right) T \quad \text{LED linewidth and temperature} \quad (3.11.5)$$

and plot  $\Delta\lambda/\lambda_o^2$  vs.  $T$ . The slope of the best line forced through zero should give  $mk/hc$  and hence  $m$ . Using the three  $\lambda_o$  and  $\Delta\lambda$  values in the inset of Figure 3.32(b), we obtain the graph in Figure 3.34. The best line is forced through zero to follow Eq. (3.11.5), and gives a slope of  $1.95 \times 10^{-7} \text{ nm}^{-1} \text{ K}^{-1}$  or  $195 \text{ m}^{-1} \text{ K}^{-1}$ . Thus,

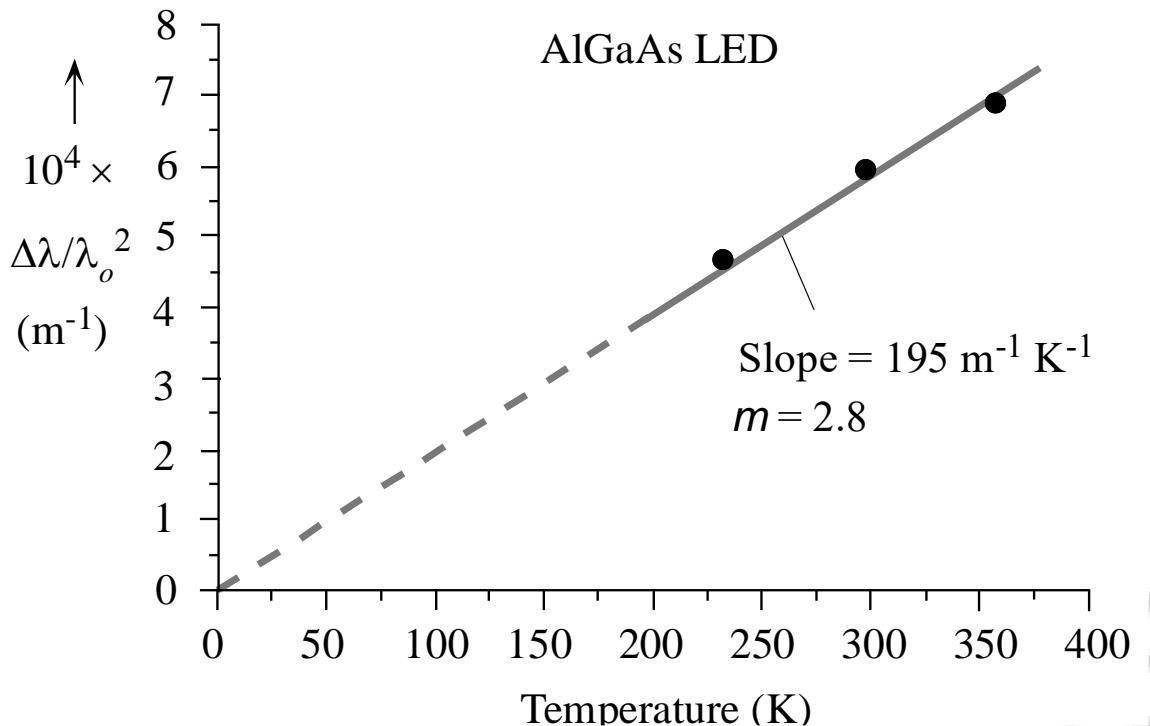
$$\text{slope} = 195 \text{ m K}^{-1} = \frac{m(1.38 \times 10^{-23} \text{ J K}^{-1})}{(6.626 \times 10^{-34} \text{ Js})(3 \times 10^8 \text{ m s}^{-1})}$$

so that

$$m = 2.81$$

## EXAMPLE: LED spectral width

### Solution (continued)



AlGaAs IR LED

$T$	$\lambda_o$	$\Delta\lambda$
-40°C	804 nm	30 nm
25°C	822 nm	40 nm
85°C	837 nm	48 nm

The plot of  $\Delta\lambda/\lambda_o^2$  vs.  $T$  for an AlGaAs infrared LED, using the peak wavelength  $\lambda_o$  and spectral width  $\Delta\lambda$  at three different temperatures, using the data shown in the table.



# EXAMPLE: Dependence of the emission peak and linewidth on temperature

Using the Varshni equation, Eq. (3.11.2),  $E_g = E_{go} - AT^2/(B + T)$ , find the shift in the peak wavelength ( $\lambda_o$ ) emitted from a GaAs LED when it is cooled from 25 °C to -25 °C. The *Varshni constants* for GaAs are,  $E_{go} = 1.519 \text{ eV}$ ,  $A = 5.41 \times 10^{-4} \text{ eV K}^{-1}$ ,  $B = 204 \text{ K}$ .

## Solution

At  $T = 298 \text{ K}$ , using the Varshni equation

$$\begin{aligned}E_g &= E_{go} - AT^2/(B + T) \\&= 1.519 \text{ eV} - (5.41 \times 10^{-4} \text{ eV K}^{-1})(298 \text{ K})^2/(204 \text{ K} + 298 \text{ K}) = 1.423 \text{ eV}.\end{aligned}$$

At 298 K,  $(1/2)k_B T = 0.0128 \text{ eV}$ . The peak emission is at  $h\nu_o \approx E_g + (1/2)k_B T$ . Using  $\nu_o = c/\lambda_o$ , we get

$$\lambda_o = \frac{ch}{(E_g + \frac{1}{2}k_B T)} = \frac{(3 \times 10^8 \text{ m s}^{-1})(6.626 \times 10^{-34} \text{ Js}) / 1.602 \times 10^{-19} \text{ eV J}^{-1}}{(1.4223 \text{ eV} + 0.0128 \text{ eV})} = 864.2 \text{ nm}$$

At -25 °C or , 248 K,  $(1/2)k_B T = 0.0107 \text{ eV}$ , repeating the above calculation,

$$E_g = 1.519 \text{ eV} - (5.41 \times 10^{-4} \text{ eV K}^{-1})(248 \text{ K})^2/(204 \text{ K} + 248 \text{ K}) = 1.445 \text{ eV}$$

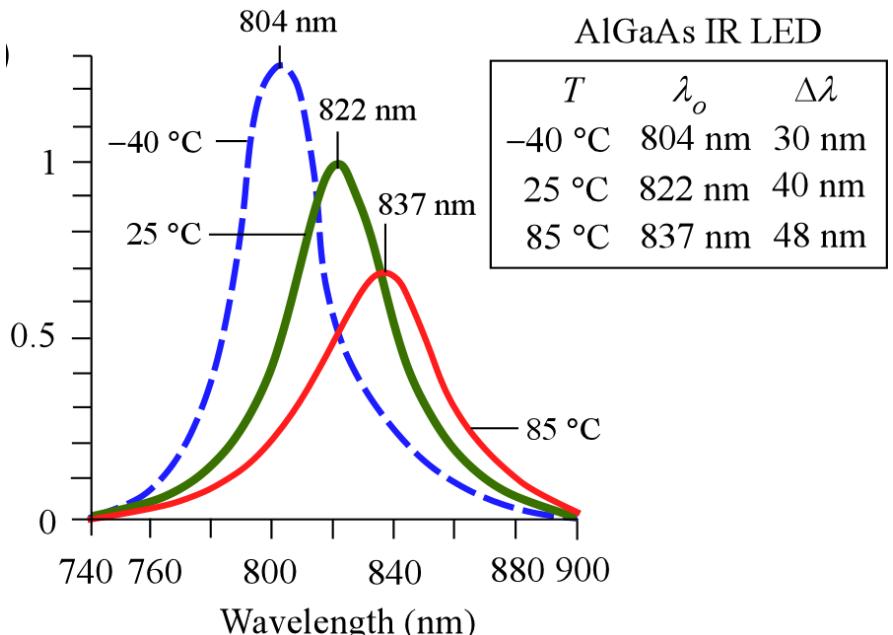
# EXAMPLE: Dependence on the emission peak and linewidth on temperature

## Solution (continued)

and the new peak emission wavelength  $\lambda'_o$  is

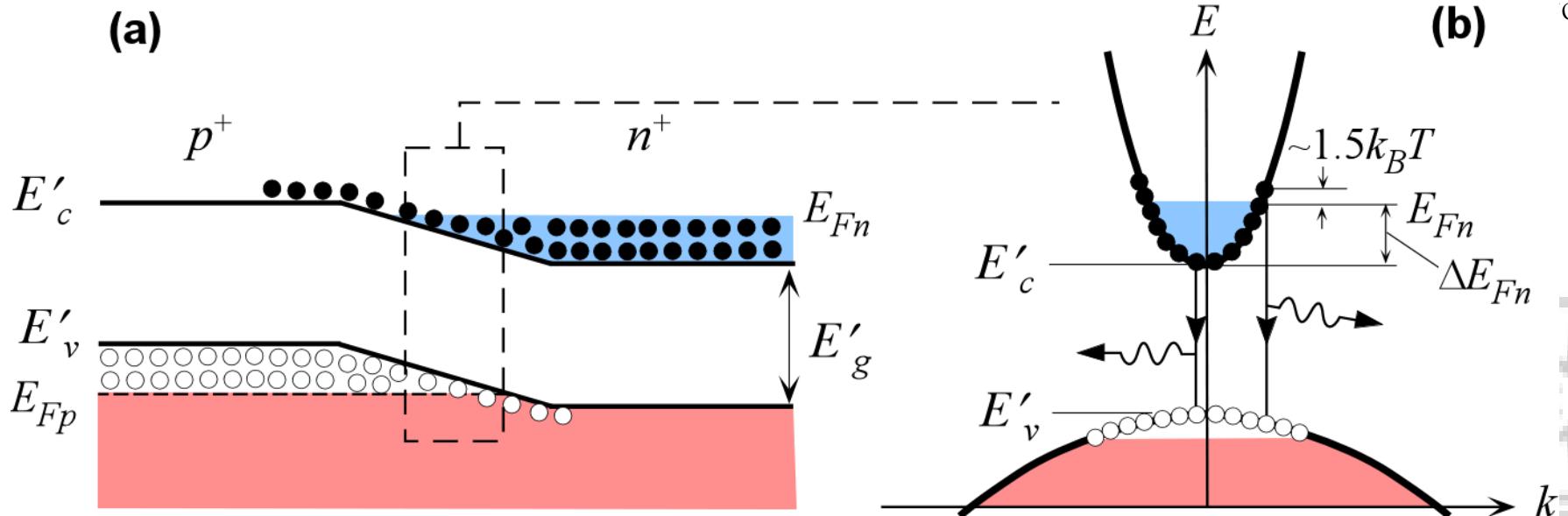
$$\lambda'_o = \frac{(3 \times 10^8 \text{ m s}^{-1})(6.626 \times 10^{-34} \text{ Js}) / (1.602 \times 10^{-19} \text{ eV J}^{-1})}{(1.445 \text{ eV} + 0.01069 \text{ eV})} = 852.4 \text{ nm}$$

The change  $\Delta\lambda = \lambda_o - \lambda'_o = 864.2 - 852.4 = 11.8 \text{ nm}$  over  $50 \text{ }^\circ\text{C}$ , or  $0.24 \text{ nm} / \text{ }^\circ\text{C}$ .



The examination of the Figure 3.32(b) shows that the change in the peak wavelength per unit temperature in the range  $-40 \text{ }^\circ\text{C}$  to  $85 \text{ }^\circ\text{C}$  is roughly the same. Because of the small change, we kept four significant figures in  $E_g$  and  $\lambda_o$  calculations.

# Emission Spectrum in degenerate SC

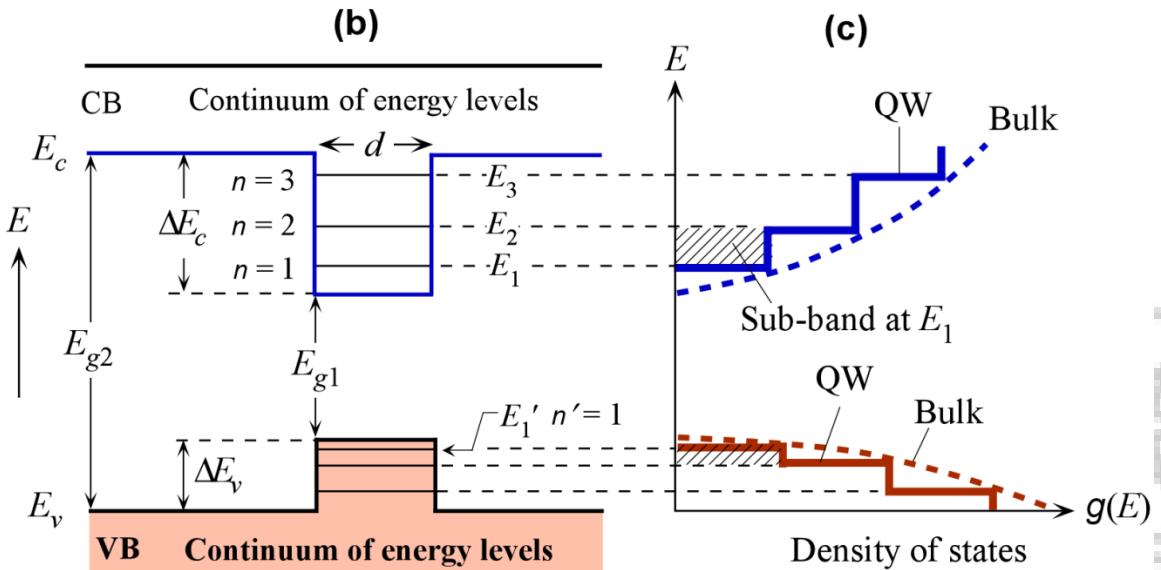
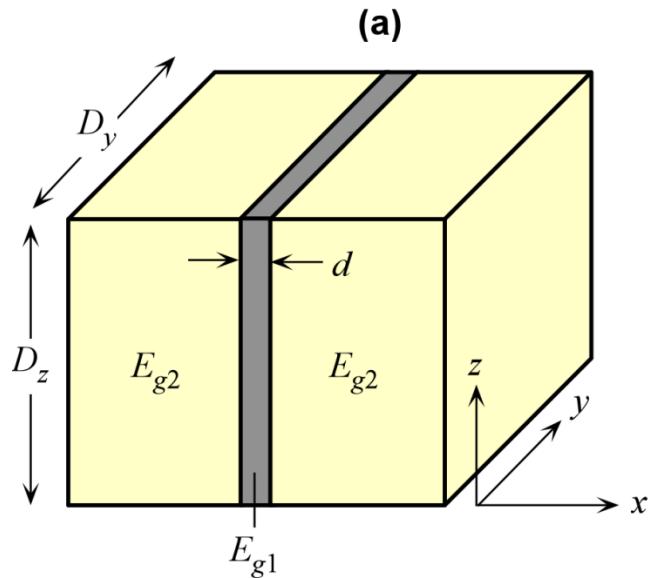


(a) Forward biased degenerately doped *pn* junction.  $E'_c$  is lower than  $E_c$  in the bulk and  $E'_v$  is higher than  $E_v$  in the bulk and the bandgap  $E'_g$  is narrower than in the bulk. The quasi-Fermi levels  $E_{Fn}$  and  $E_{Fp}$  overlap around the junction. (b) The transitions involved in a degenerately doped *pn* junction.

Emission spectral width in frequency or  $\Delta(h\nu)$

$$\approx (E_g + \Delta E_{Fn} + 1.5 k_B T) - E_g = \Delta E_{Fn} + 1.5 k_B T$$

# Quantum Wells

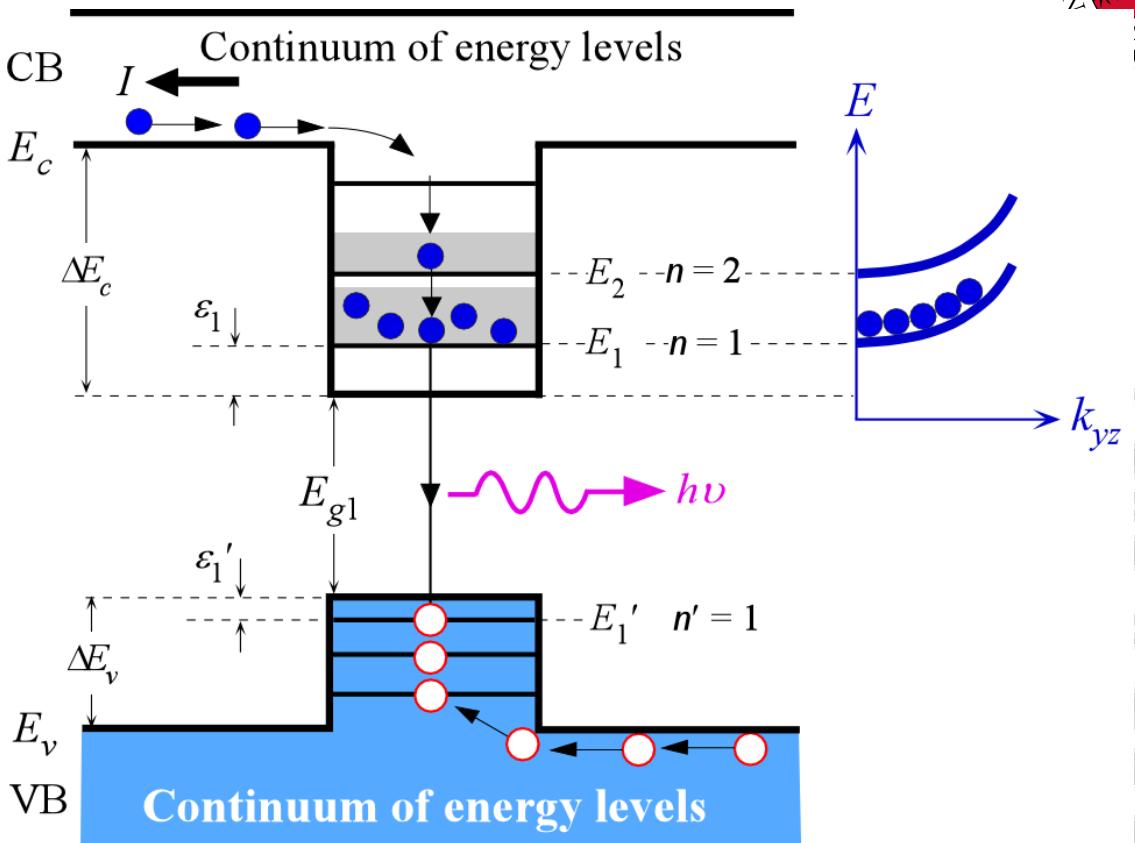


(a) A single quantum well (SQW) of a smaller bandgap material ( $E_{g1}$ ) of thickness  $d$  along  $x$  surrounded by a thicker material of wider bandgap ( $E_{g2}$ ). (b) The electron energy levels associated with motion along  $x$  are quantized as  $E_1, E_2, E_3$  etc. Each level is characterized by a quantum number  $n$ . (c) The density of states for a bulk semiconductor and a QW.

# Quantum Wells



A QW structure that shows the energy levels in the wells and how charge carriers that are brought in by the current fall into the lowest energy level in the well and then recombine, emitting a photon. The electrons at a particular energy level also have kinetic energies in the  $yz$  plane, which is not quantized. The electrons are therefore spread in energy above  $E_n$  as shown. The same notion also applies to holes in the  $\Delta E_v$  well.



$$E_n = E_c + \frac{h^2 n^2}{8m_e^* d^2} + \frac{\hbar^2 k_y^2}{2m_e^*} + \frac{\hbar^2 k_z^2}{2m_e^*}$$

## EXAMPLE: Energy levels in the quantum well

Consider a GaAs QW sandwiched between two  $\text{Al}_{0.40}\text{Ga}_{0.60}\text{As}$  layers. Suppose that the barrier height  $\Delta E_c$  is 0.30 eV, the electron effective mass in the well is  $0.067m_e$  and the width of the QW ( $d$ ) is 12 nm. Calculate the energy levels  $E_1$  and  $E_2$  from the bottom of the well ( $E_c$ ) assuming an infinite PE well as in Eq. (1). Compare these with the calculations for a finite PE well that give 0.022 eV, 0.088 and 0.186 for  $n = 1, 2$  and 3.

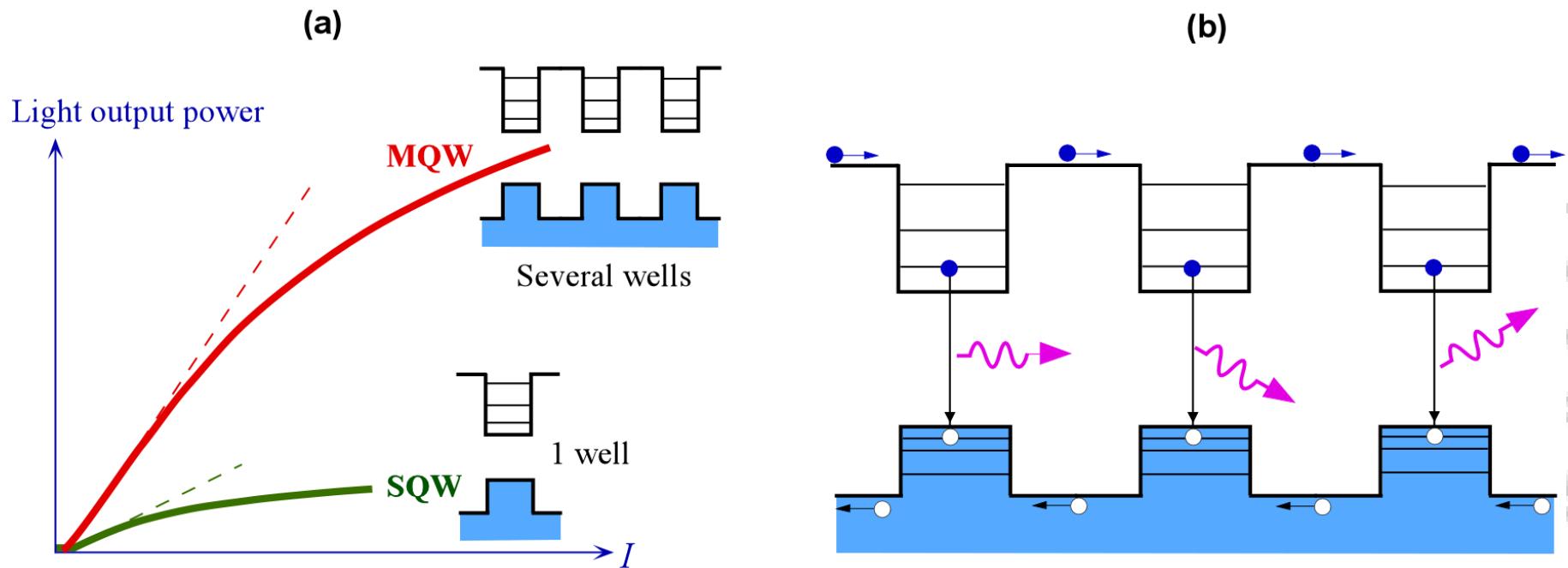
### Solution

We use Eq. (1) with  $m_e^* = 0.067m_e$ ,  $d = 12 \times 10^{-9} \text{ nm}$ , so that for  $n=1$ ,

$$\Delta E_n = E_n - E_c = \frac{\hbar^2 n^2}{8m_e^* d^2} = \frac{(6.624 \times 10^{-34} \text{ Js})^2 (1)^2 / (1.602 \times 10^{-19} \text{ J eV}^{-1})}{8(0.067 \times 9.1 \times 10^{-31} \text{ kg})(12 \times 10^{-9} \text{ m})^2} = 0.039 \text{ eV}$$

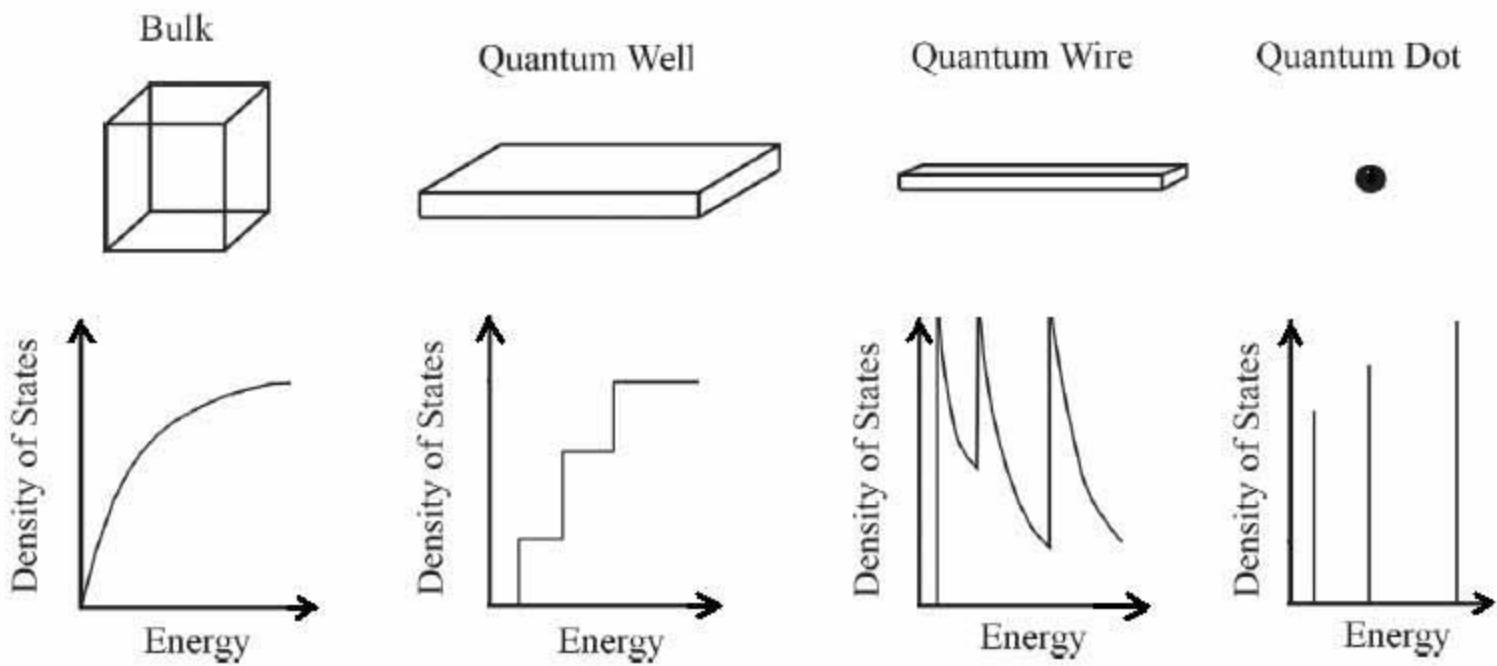
We can repeat the above calculation for  $n = 2$  and 3 to find  $\Delta E_2 = 0.156 \text{ eV}$  and  $\Delta E_3 = 0.351 \text{ eV}$ . The third level will be above the well depth ( $\Delta E_c = 0.3 \text{ eV}$ ). Clearly, the infinite QW predicts higher energy levels, by a factor of 1.8, and puts the third level inside the well not outside. The finite QW calculation is not simple, and involves a numerical solution.

# Multiple Quantum Wells



A schematic illustration of the comparison of light power output vs. current characteristics for a SQW and a MQW LED

# Other confinements

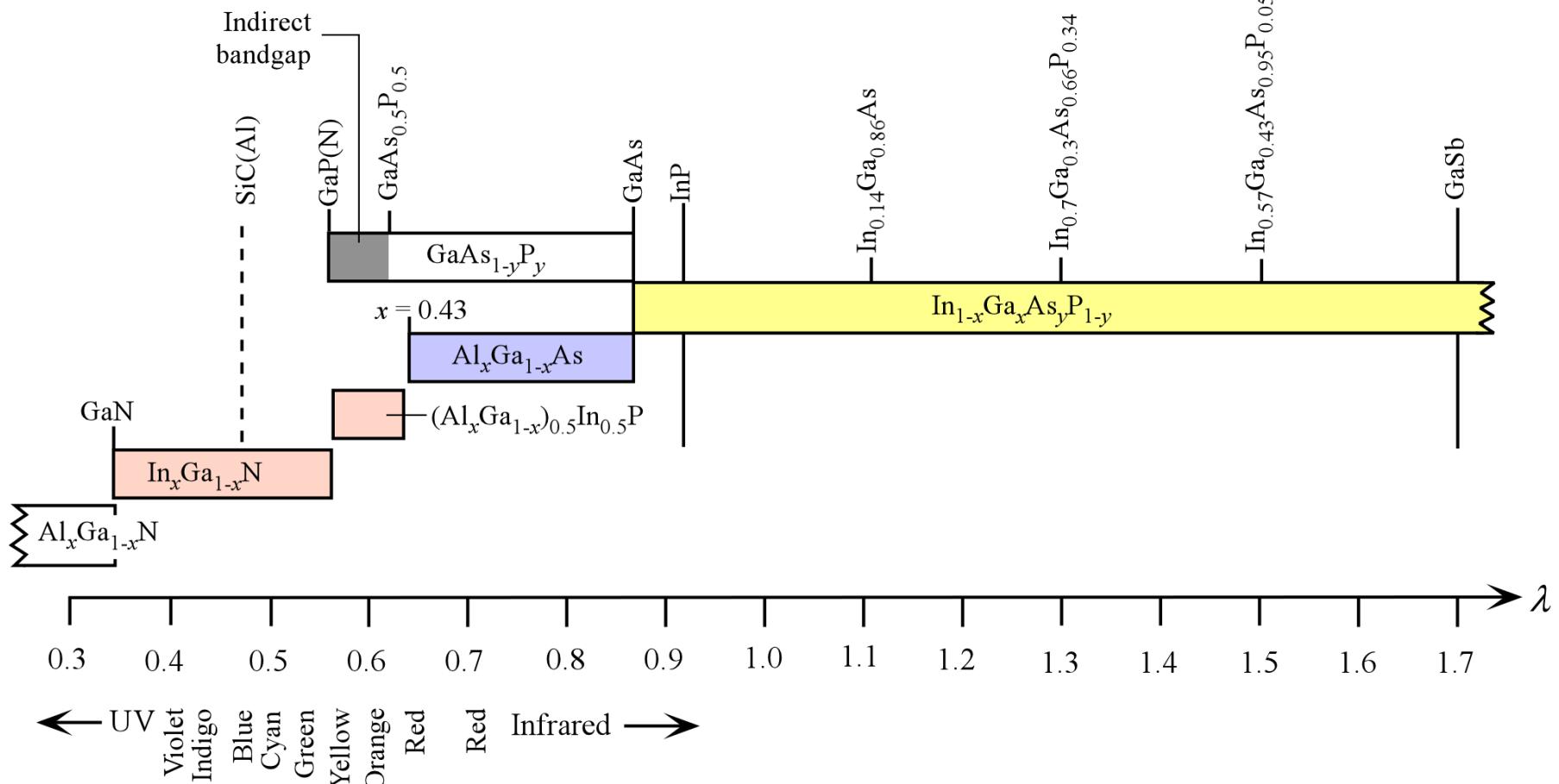


# Other confinements

Degrees of freedom	Dispersion (kinetic energy)	Density of states	Effective density of states
3 (bulk)	$E = \frac{\hbar^2}{2m^*} (k_x^2 + k_y^2 + k_z^2)$	$\rho_{\text{DOS}}^{\text{3D}} = \frac{1}{2\pi^2} \left( \frac{2m^*}{\hbar^2} \right)^{\frac{3}{2}} \sqrt{E - E_C}$	$N_c^{\text{3D}} = \frac{1}{\sqrt{2}} \left( \frac{m^* k T}{\pi \hbar^2} \right)^{\frac{3}{2}}$
2 (slab)	$E = \frac{\hbar^2}{2m^*} (k_x^2 + k_y^2)$	$\rho_{\text{DOS}}^{\text{2D}} = \frac{m^*}{\pi \hbar^2} \sigma(E - E_C)$	$N_c^{\text{2D}} = \frac{m^*}{\pi \hbar^2} k T$
1 (wire)	$E = \frac{\hbar^2}{2m^*} (k_x^2)$	$\rho_{\text{DOS}}^{\text{1D}} = \frac{m^*}{\pi \hbar} \sqrt{\frac{m^*}{2(E - E_C)}}$	$N_c^{\text{1D}} = \sqrt{\frac{m^* k T}{2\pi \hbar^2}}$
0 (box)	–	$\rho_{\text{DOS}}^{\text{0D}} = 2\delta(E - E_C)$	$N_c^{\text{0D}} = 2$

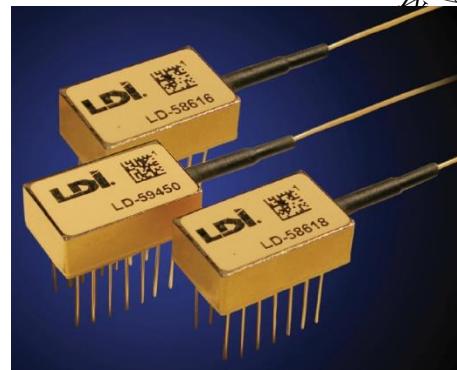
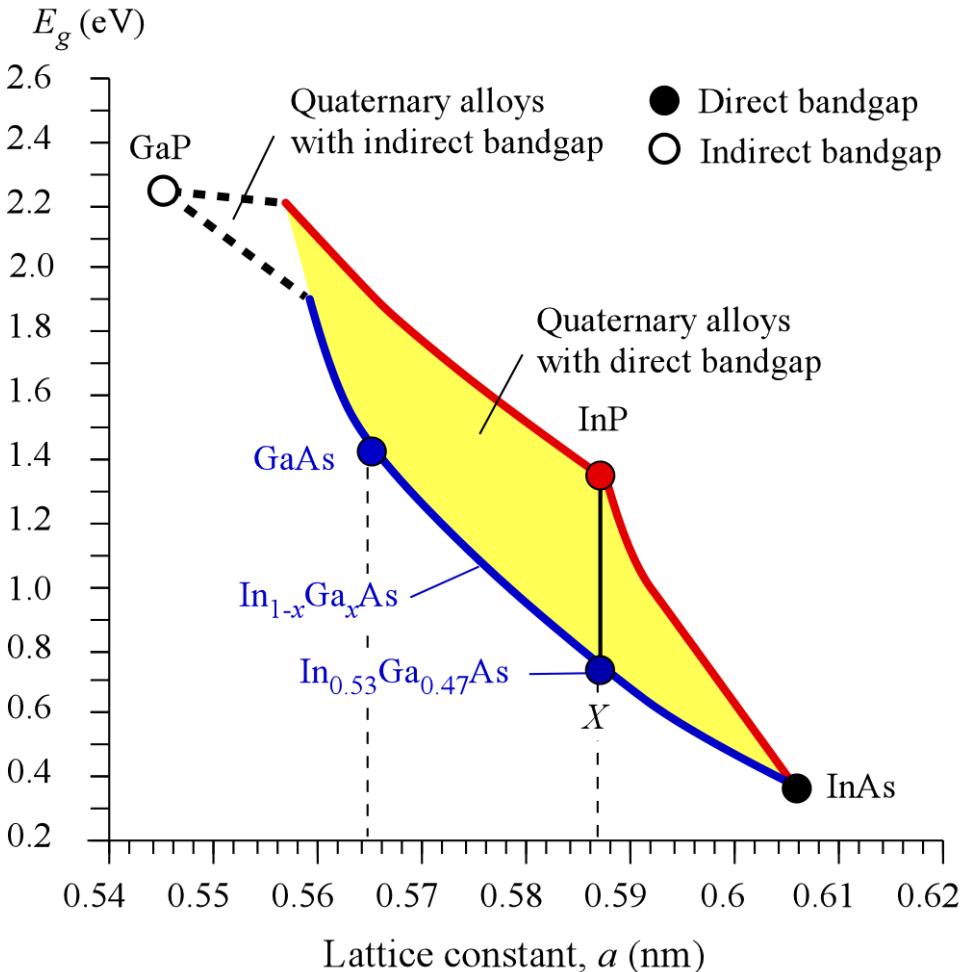
**Table 12.1** Density of states for semiconductor with 3, 2, 1, and 0 degrees of freedom for propagation of electrons. The dispersion relations are assumed to be parabolic. The formulas can be applied to anisotropic semiconductors if the effective mass  $m^*$  is replaced by the density-of-states effective mass  $m_{\text{DOS}}^*$ . If the semiconductor has a number of  $M_c$  equivalent minima, the corresponding density of states must be multiplied by  $M_c$ . The bottom of the band is denoted as  $E_C$  and  $\sigma(E)$  is the step-function.

# LED Materials



Free space wavelength coverage by different LED materials from the visible spectrum to the infrared including wavelengths used in optical communications. Hatched region and dashed lines are indirect  $E_g$  materials. Only material compositions of importance have been shown.

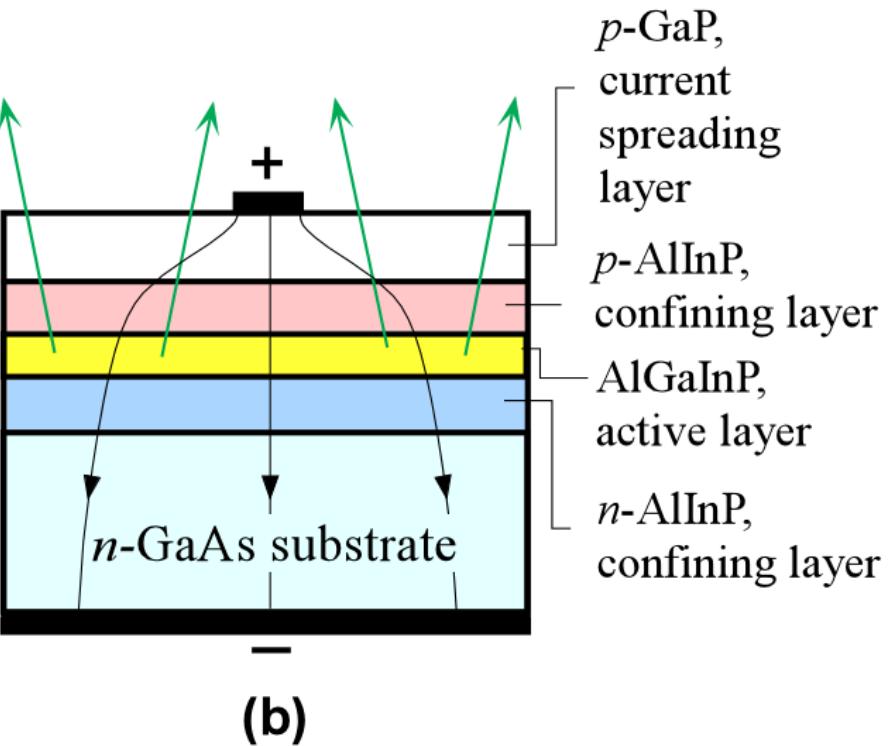
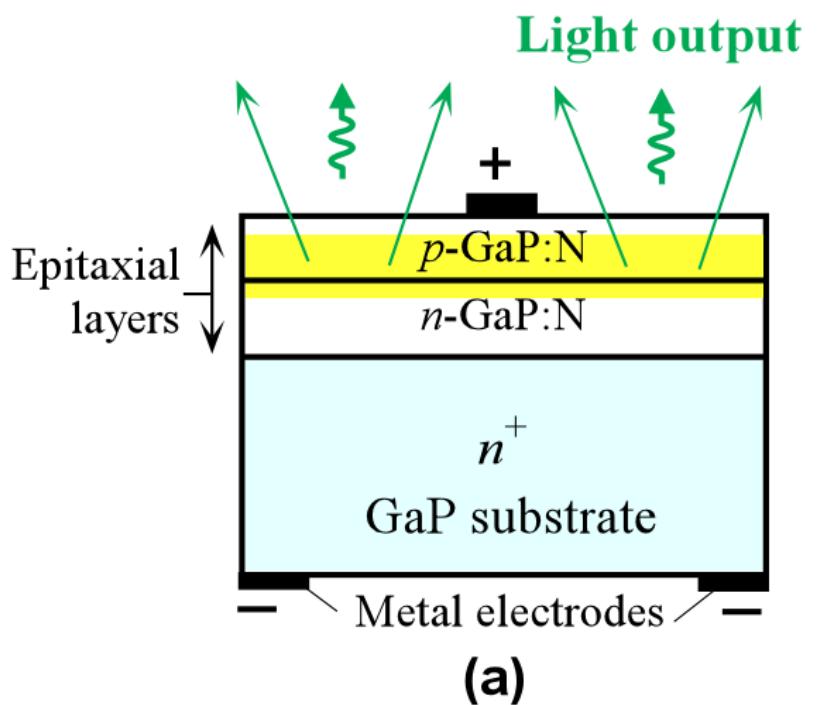
# LED Materials



InGaAsP 1300nm LED emitters, each pigtailed to an optical fiber for use in ruggedized optical communication modems and lower speed data / analog transmission systems.  
(Courtesy of OSI Laser Diode, Inc)

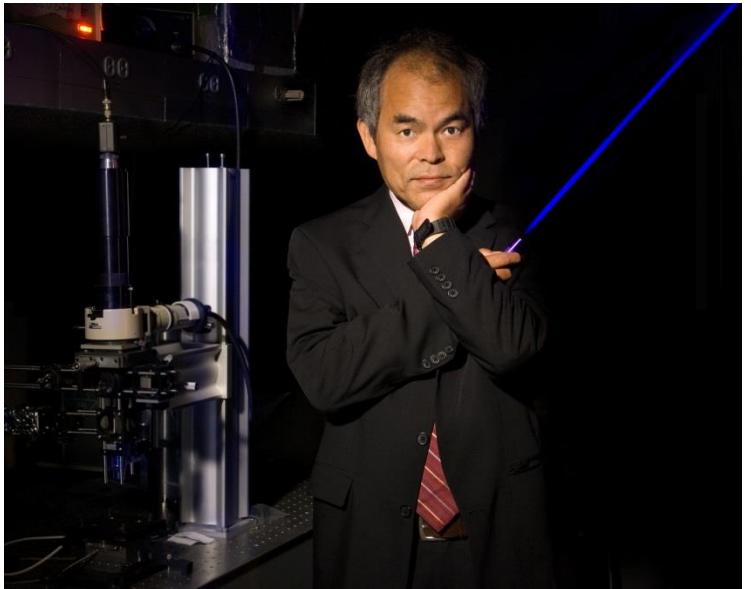


# Typical LEDs



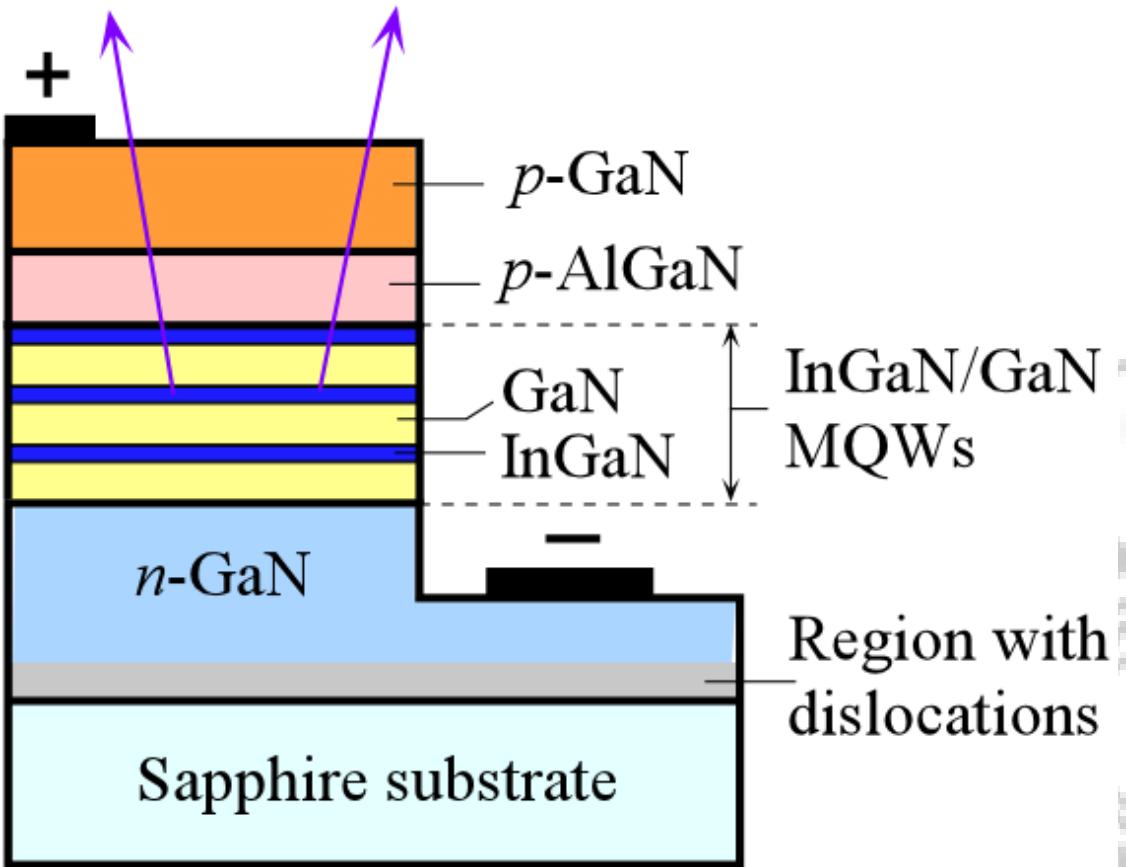
A schematic illustration of various typical LED structures. (a) A planar surface emitting homojunction green GaP:N LED (b) AlGaInP high intensity heterostructure LED.

# III-Nitride LEDs



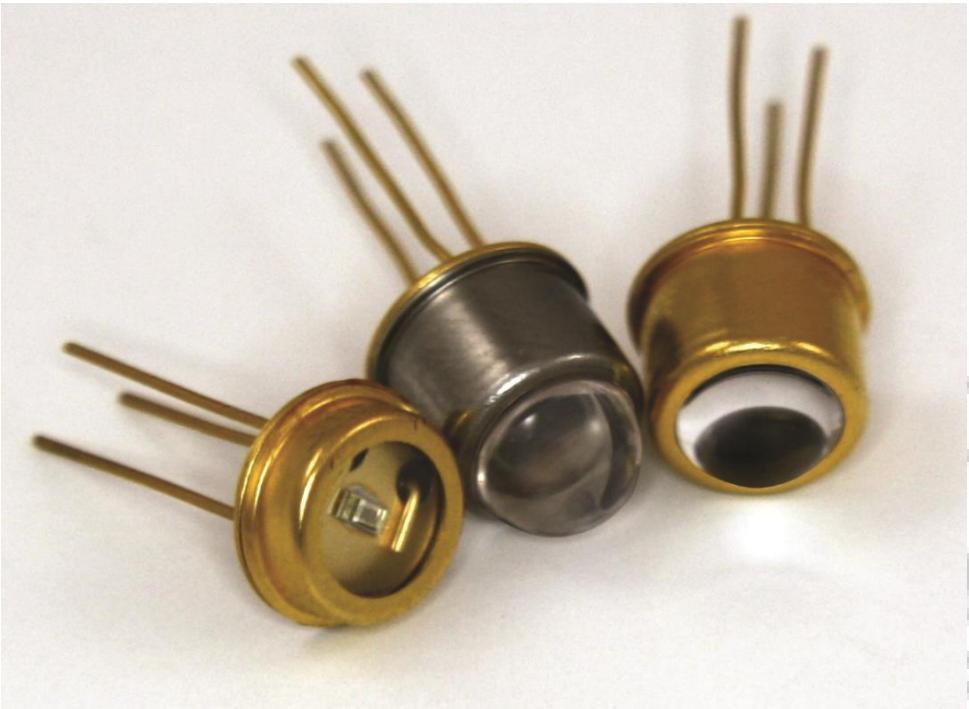
Shuji Nakamura, obtained his PhD from the University of Tokushima in Japan, and is currently a Professor at the University of California at Santa Barbara and the Director of Solid State Lighting and Energy Center. He has been credited with the pioneering work that has led to the development of GaN and  $In_xGa_{1-x}N$  based blue and violet light emitting diodes and laser diodes. He discovered how III-Nitrides could be doped p-type, which opened the way to fabricating various UV, violet, blue and green LEDs. He holds the violet laser diode in the left side picture. He is holding a blue laser diode that is turned on. (Courtesy of Randy Lam, University of California, Santa Barbara)

# III-Nitride LEDs



A schematic illustration of various typical LED structures. III-Nitride based (GaN/InGaN) MQW LED for emission from the UV to green.

# UV LEDS

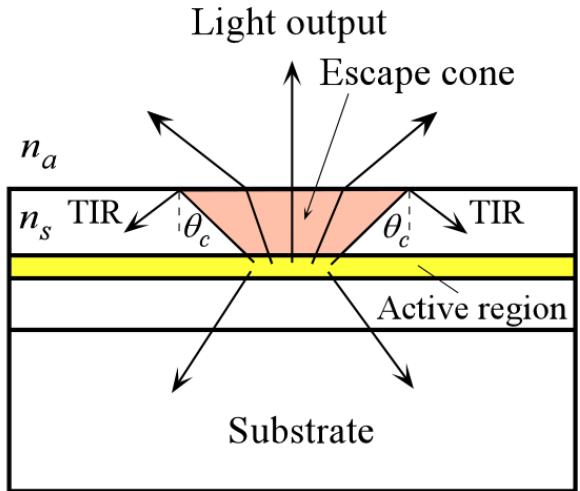


UV emitting LEDs (Courtesy of Sensor Electronic Technology Inc.)

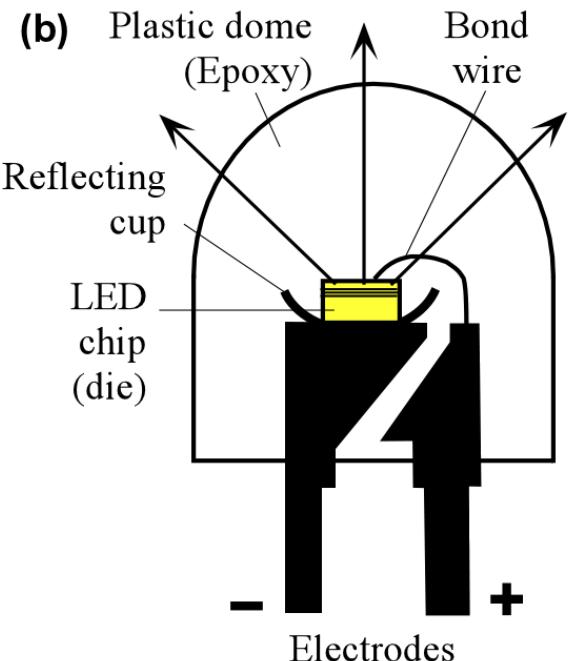
# Light Extraction



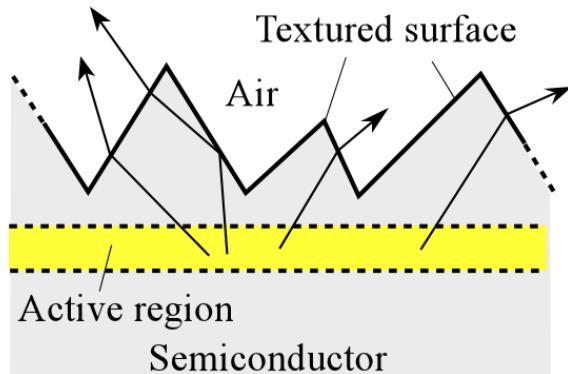
(a)



(b)

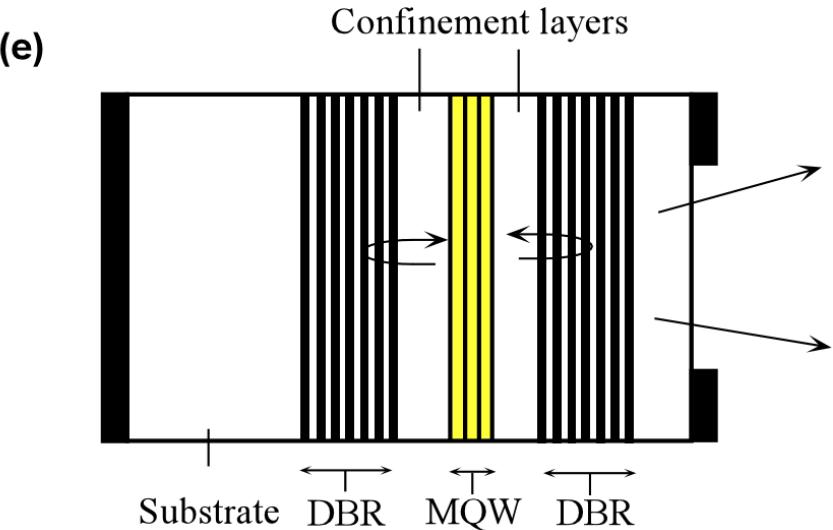
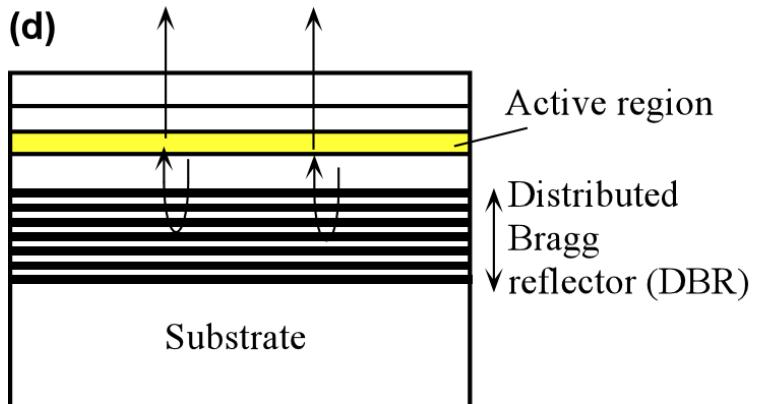


(c)



(a) Some of the internally generated light suffers total internal reflection (TIR) at the semiconductor/air interface and cannot be emitted into the outside. (b) A simple structure that overcomes the TIR problem by placing the LED chip at the centre of a hemispherical plastic dome. The epoxy is refractive index matched to the semiconductor and the rays reaching the dome's surface do not suffer TIR. (b) An example of a textured surface that allows light to escape after a couple of (or more) reflections (highly exaggerated sketch).

# DBR LED and RCLED



(d) A distributed Bragg reflector (DBR), that is a dielectric mirror, under the confining layer (below the active region in grey) acts as a dielectric mirror, and increases the extraction ratio. (e) An RCLED is an LED with an optical resonant cavity (RC) formed by two DBRs has a narrower emission spectrum.

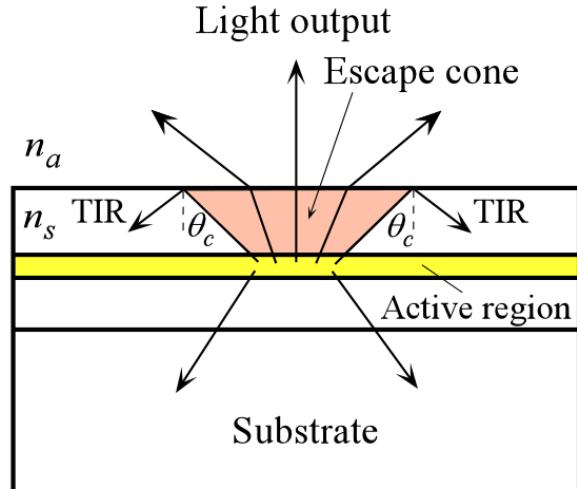
## EXAMPLE: Light extraction from a bare LED chip

As shown in (a), due to total internal reflection (TIR) at the semiconductor-air surface, only a fraction of the emitted light can escape from the chip. The critical angle  $\theta_c$  is determined by  $\sin \theta_c = n_a / n_s$  where  $n_a$  and  $n_s$  are the refractive indices of the ambient (e.g. for air,  $n_a = 1$ ) and the semiconductor respectively. The light within the **escape cone** defined by  $\theta_c$  can escape into the ambient without TIR as indicated in (a). To find the fraction of light within the escape cone we need to consider solid angles, which leads to  $(1/2)[1 - \cos \theta_c]$ .

Further, suppose that  $T$  is the average light transmittance of the  $n_s$ - $n_a$  interface for those rays within the escape cone, then for a simple bare chip,

$$\text{Light extraction ratio} \approx (1/2)[1 - \cos \theta_c] \times T \quad (1)$$

Estimate the extraction ratio for a GaAs chip with  $n_s = 3.4$  and air as ambient ( $n_a = 1$ ) and then with epoxy dome with  $n_a = 1.8$ .



## Solution

First note that  $\theta_c = \arcsin(n_a/n_s) = \arcsin(1/3.4) = 17.1^\circ$ . For  $T$  we will assume near-normal incidence (somewhat justified since the angle  $17.1^\circ$  is not too large) so that from Chapter 1,

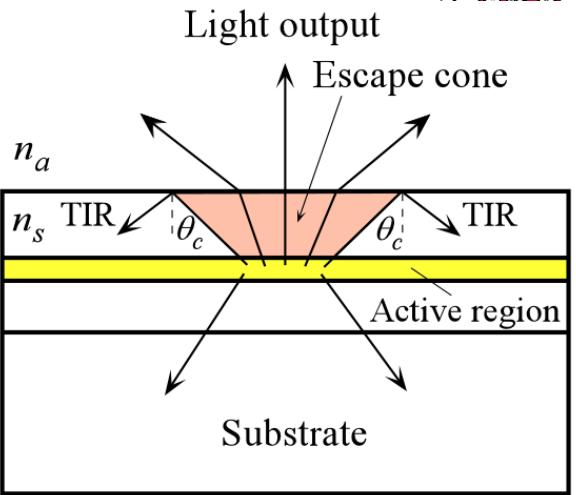
$$T = 4n_s n_a / (n_s + n_a)^2 = 4(3.4)(1)/(3.4 + 1)^2 = 0.702$$

Using Eq.(1),

Light extraction ratio

$$\begin{aligned} &\approx (1/2)[1 - \cos \theta_c] \times T = (1/2)[1 - \cos(17.1^\circ)] \times 0.702 \\ &\approx \mathbf{0.0155 \text{ or } 1.6\%} \end{aligned}$$

It is clear that only 1.6% of the generated light power is extracted from a bare chip, which is disappointingly small. The technological drive is therefore to improve light extraction as much as possible. If we now repeat the calculation for  $n_a = 1.8$ , we would find,  $\theta_c = 32^\circ$ , and 6.9% light extraction.



# LED PARAMETERS



## Internal quantum efficiency (IQE) $\eta_{\text{IQE}}$

$$\eta_{\text{IQE}} = \frac{\text{Rate of radiative recombination}}{\text{Total rate of recombination (radiative and nonradiative)}}$$

$$\eta_{\text{IQE}} = \frac{\tau_r^{-1}}{\tau_r^{-1} + \tau_{nr}^{-1}}$$

$$\eta_{\text{IQE}} = \frac{\text{Photons emitted per second}}{\text{Total carriers lost per second}} = \frac{\Phi_{\text{ph}}}{I/e} = \frac{P_{o(\text{int})}/h\nu}{I/e}$$

# EXTRACTION EFFICIENCY



**Extraction ratio**, or the **extraction efficiency** (EE),  $\eta_{\text{EE}}$

$$\eta_{\text{EE}} = \frac{\text{Photons emitted externally from the device}}{\text{Photons generated internally by recombination}}$$

$$P_o = \eta_{\text{EE}} P_{o(\text{int})} = h\nu\eta_{\text{EE}}\eta_{\text{IQE}}(I/e)$$

# External Quantum Efficiency



**External quantum efficiency (EQE)**  $\eta_{\text{EQE}}$  of an LED represents the efficiency of conversion from electrical quanta, *i.e.* electrons, that flow in the LED to optical quanta, *i.e.* photons, that are emitted into the outside world.

**Actual optical power emitted to the ambient = Radiant flux =  $P_o$**   
**( $\Phi_e$  is also used)**

**$P_o/h\nu$  is the number of emitted photons per second**

**$I/e$  is the number of electrons flowing into the LED**

$$\eta_{\text{EQE}} = \frac{P_o / h\nu}{I / e}$$

# POWER CONVERSION EFFICIENCY



## Power conversion efficiency (PCE) Power efficiency

$\eta_{\text{PCE}}$

Efficiency of conversion from the input of electrical power to the output of optical power

$$\eta_{\text{PCE}} = \frac{\text{Optical output power}}{\text{Electrical input power}} = \frac{P_o}{IV} \approx \eta_{\text{EQE}} \left( \frac{E_g}{eV} \right)$$



# LED BRIGHTNESS

**Luminous flux**  $\Phi_v$  is a measure of *visual brightness*, in lumens (lm), and is defined by

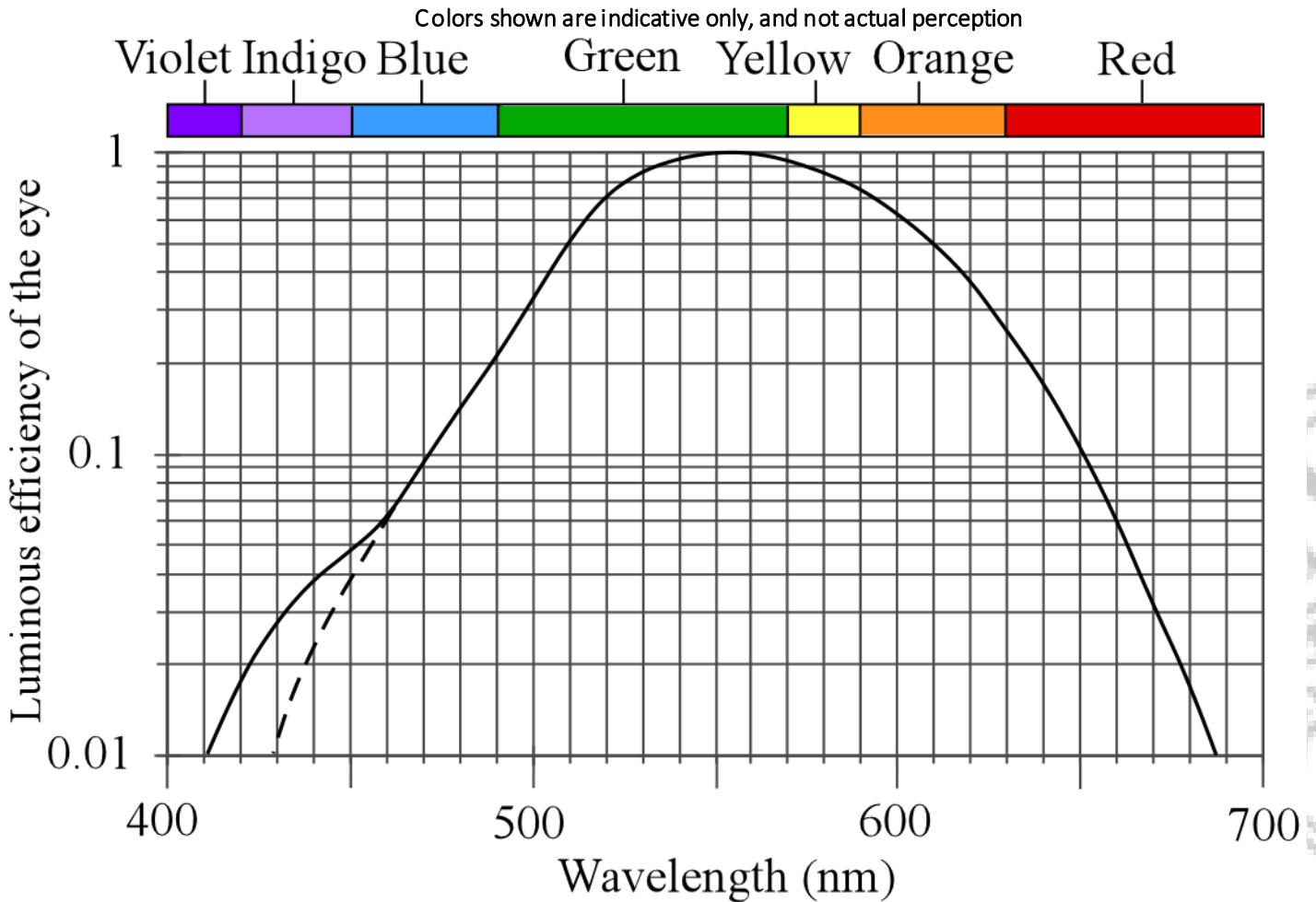
$$\Phi_v = P_o \times (683 \text{ lm W}^{-1}) \times V(\lambda)$$

$V(\lambda)$  = **relative luminous efficiency** or the relative sensitivity of an average light-adapted (photopic) eye, which depends on the wavelength

$V(\lambda)$  = **luminosity function** and the **visibility function**

$V(\lambda)$  is a Gaussian-like function with a peak of unity at 555

# The luminous efficiency of the eye



The luminous efficiency  $V(\lambda)$  of the light-adapted (photopic) eye as a function of wavelength. The solid curve is the Judd-Vos modification of the CIE 1924 photopic photosensitivity curve of the eye. The dashed line shows the modified region of the original CIE 1924 curve to account for its deficiency in the blue-violet region. (The vertical axis is logarithmic)

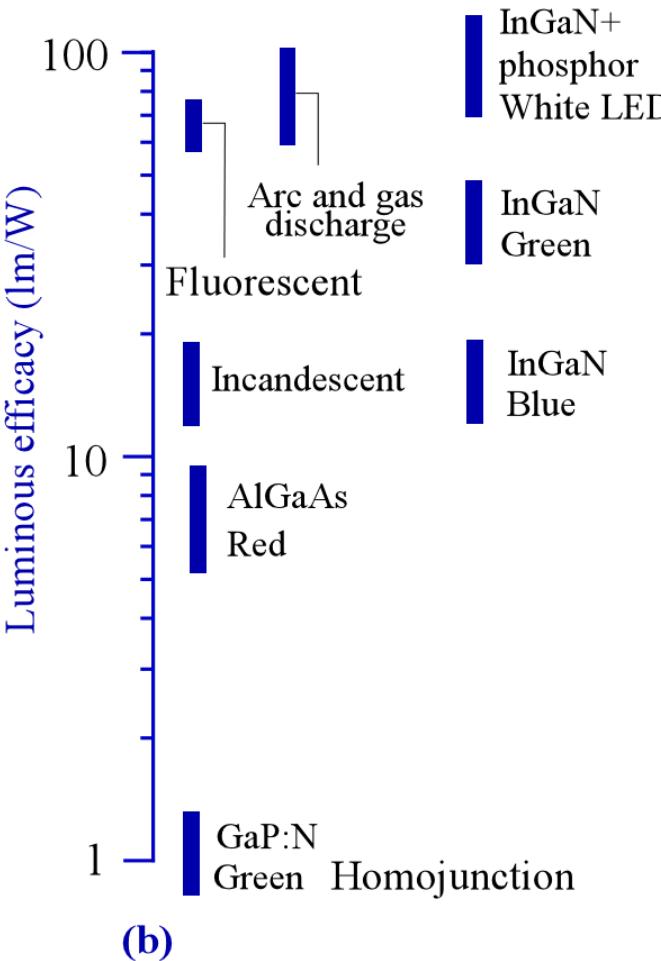
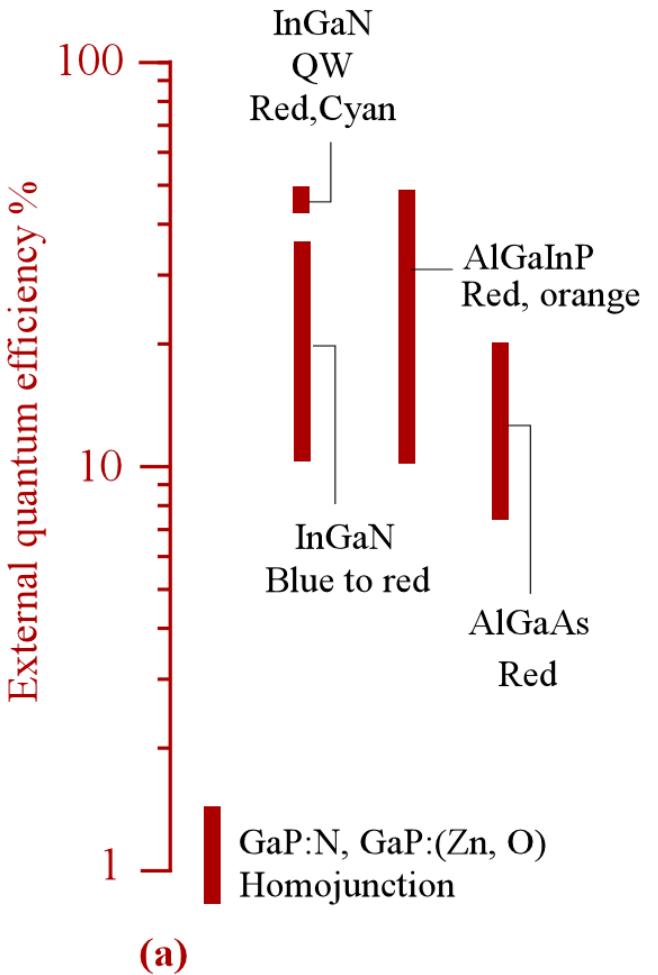
# LED BRIGHTNESS AND EFFICACY



## Luminous efficacy

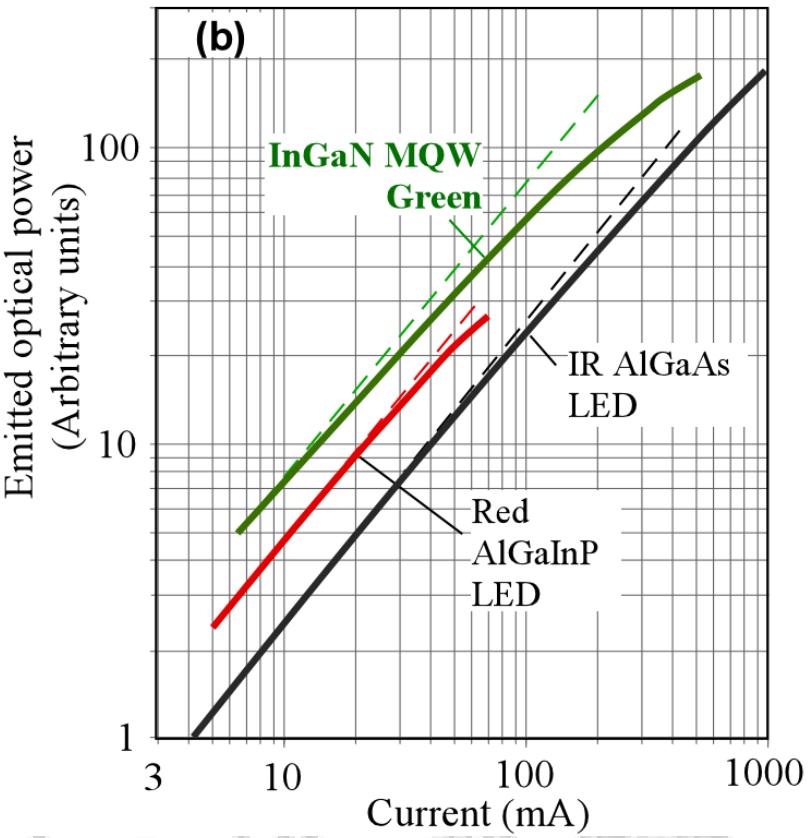
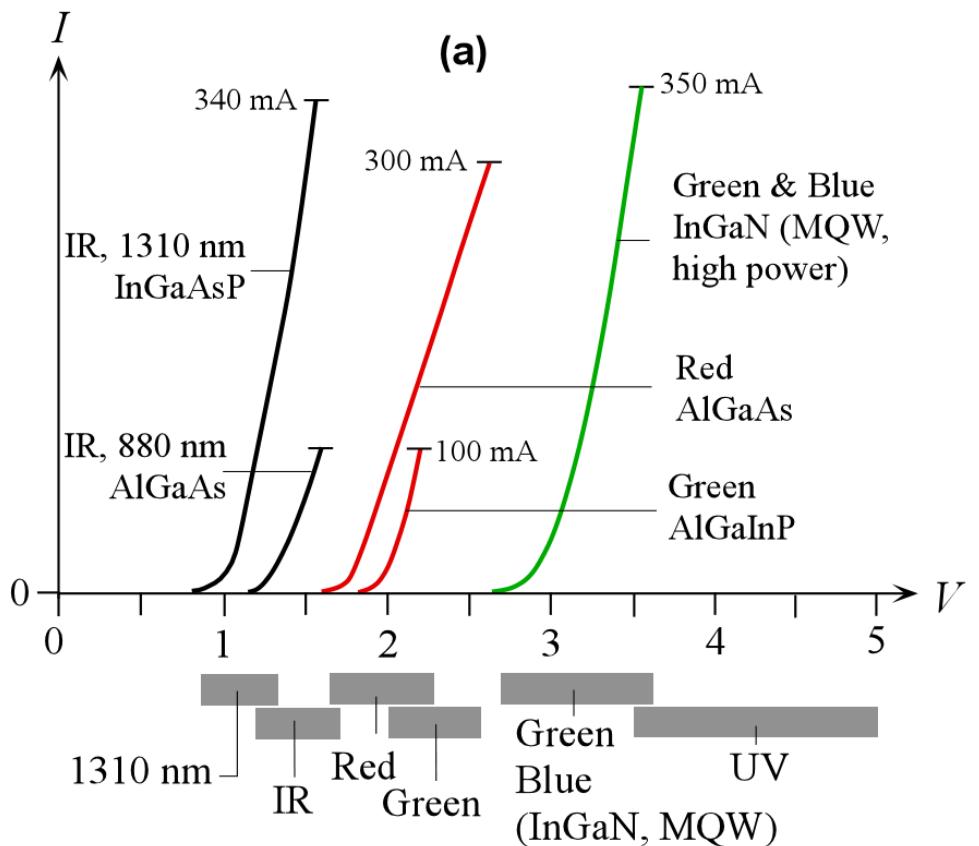
$$\eta_{\text{LE}} = \frac{\Phi_v}{IV}$$

# Luminous Efficacy



Typical (a) external quantum efficiency and (b) luminous efficacy of various selected LEDs, and how they stand against other light sources such as the fluorescent tube, arc and gas discharge lamps and the incandescent lamp.

# LED Characteristics: $I$ vs. $V$



(a) Current-Voltage characteristics of a few LEDs emitting at different wavelengths from the IR to blue. (b) Log-log plot of the emitted optical output power vs. the dc current for three commercial devices emitting at IR (890 nm), Red and Green. The vertical scale is in arbitrary unit and the curves have been shifted to show the dependence of  $P_o$  on  $I$ . The ideal linear behavior  $P_o \propto I$  is also shown.

## EXAMPLE: LED brightness LED brightness

Consider two LEDs, one red, with an optical output power (radian flux) of  $10 \text{ mW}$  emitting at  $650 \text{ nm}$ , and the other, a weaker  $5 \text{ mW}$  green LED, emitting at  $532 \text{ nm}$ . Find the luminous flux emitted by each LED.

### Solution

For the **red LED**, at  $\lambda = 650 \text{ nm}$ , Figure 3.41 gives  $V \approx 0.10$  so that from Eq. (3.14.8)

$$\begin{aligned}\Phi_v &= P_o \times (633 \text{ lm W}^{-1}) \times V \\ &= (10 \times 10^{-3} \text{ W})(683 \text{ lm W}^{-1})(0.10) = \mathbf{0.68 \text{ lm}}\end{aligned}$$

For the **green LED**,  $\lambda = 532 \text{ nm}$ , Figure 3.41 gives  $V \approx 0.87$  so that from Eq. (3.14.8)

$$\begin{aligned}\Phi_v &= P_o \times (633 \text{ lm W}^{-1}) \times V \\ &= (5 \times 10^{-3} \text{ W})(683 \text{ lm W}^{-1})(0.87) = \mathbf{3.0 \text{ lm}}\end{aligned}$$

Clearly the **green LED** at half the power is 4 times brighter than the **red LED**.

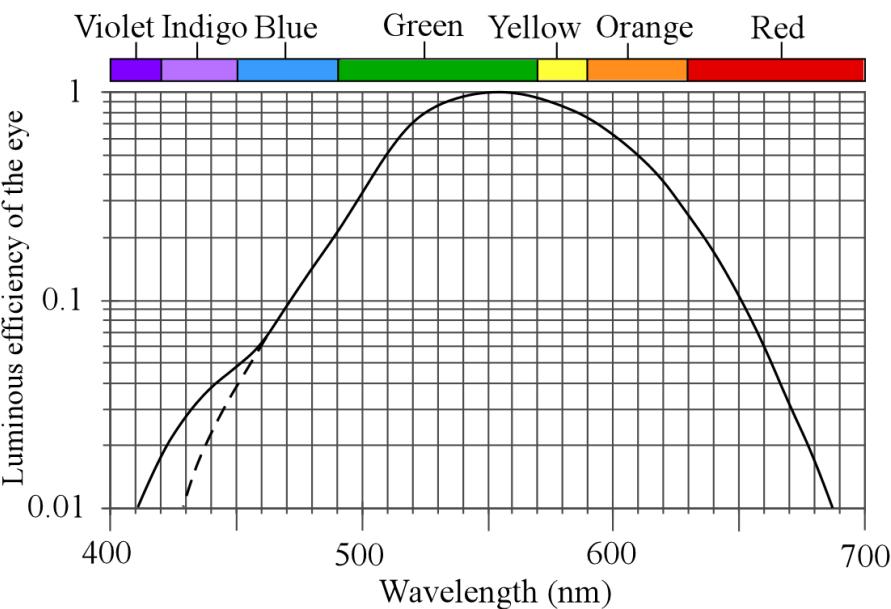
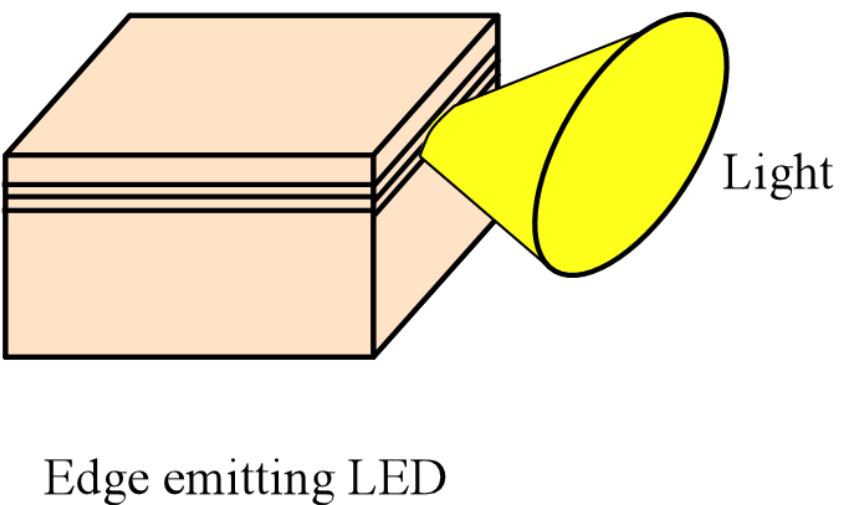
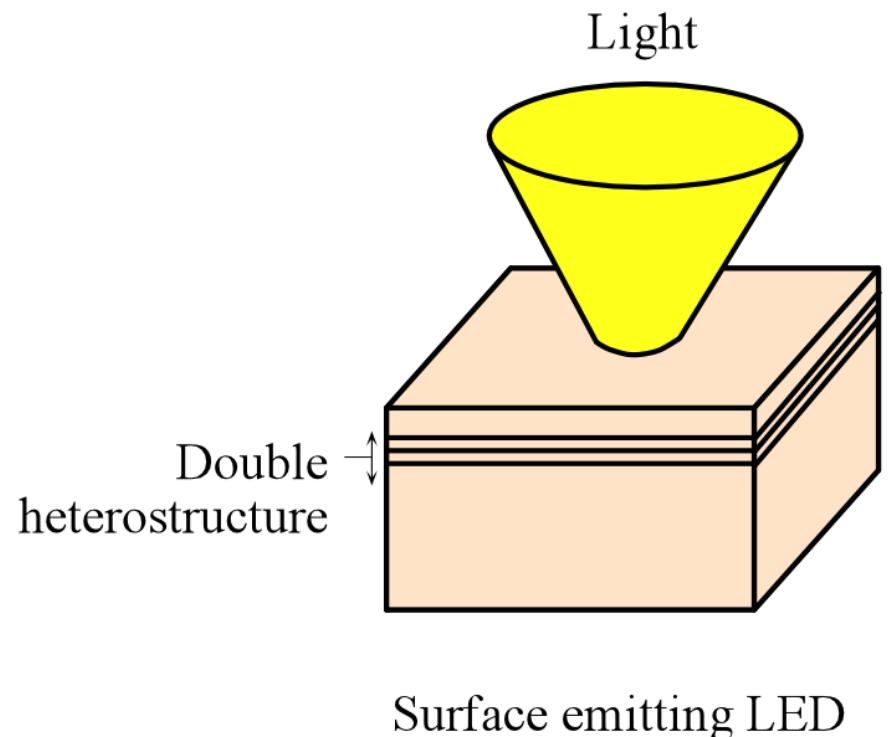
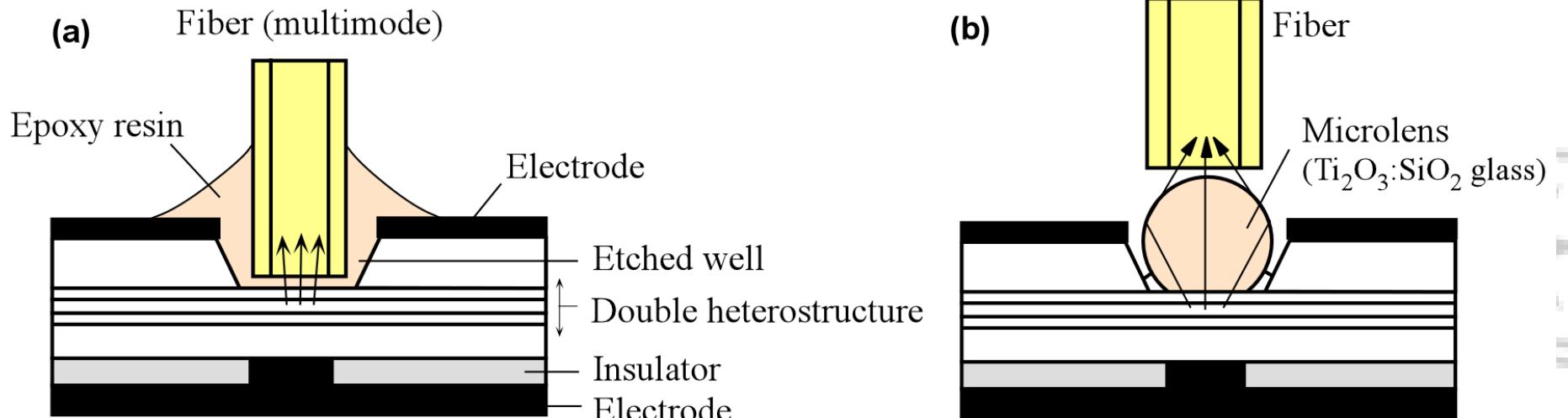


Figure 3.41

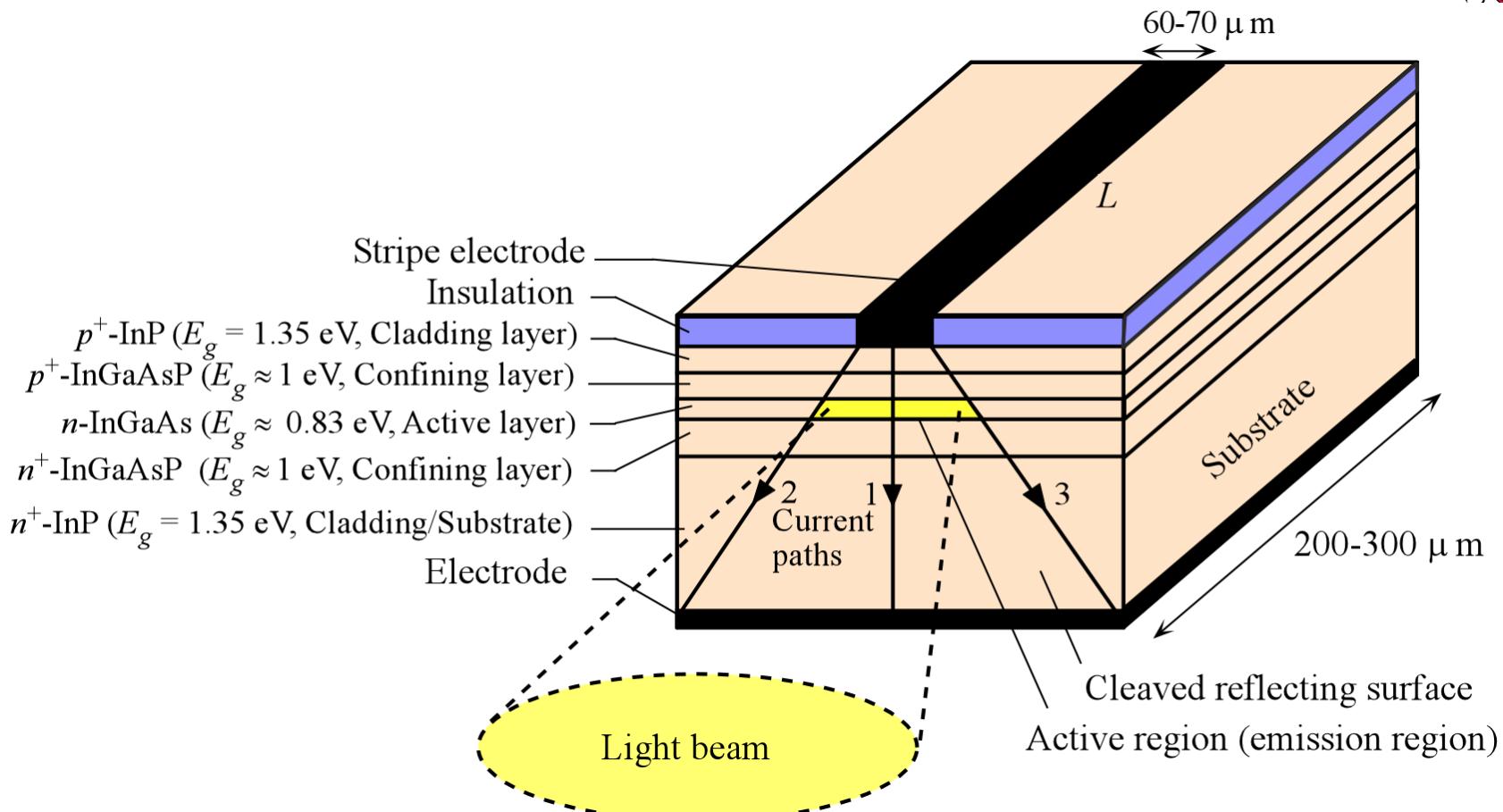
# LED TYPES



# SLED

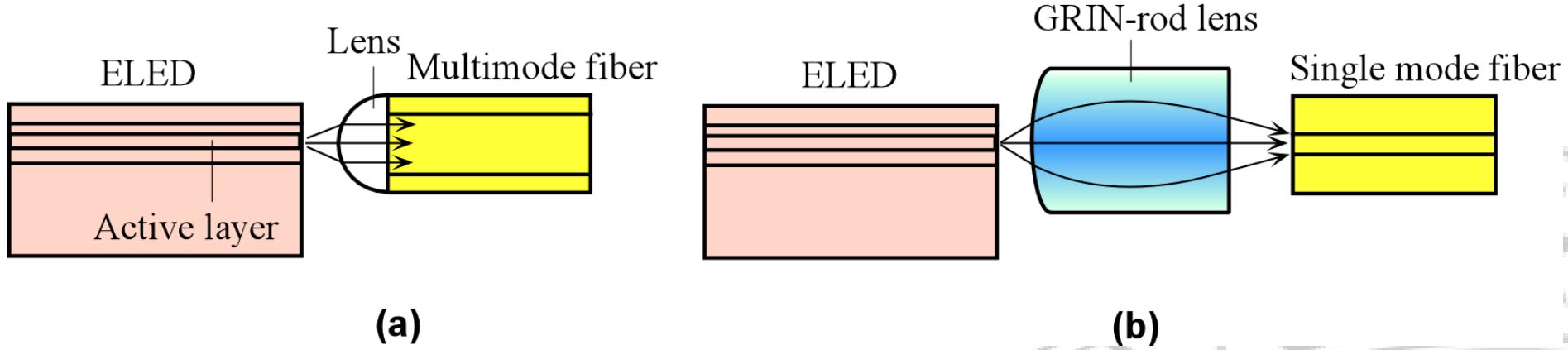


Coupling of light from LEDs into optical fibers. (a) Light is coupled from a surface emitting LED into a multimode fiber using an index matching epoxy. The fiber is bonded to the LED structure. (b) A microlens focuses diverging light from a surface emitting LED into a multimode optical fiber.



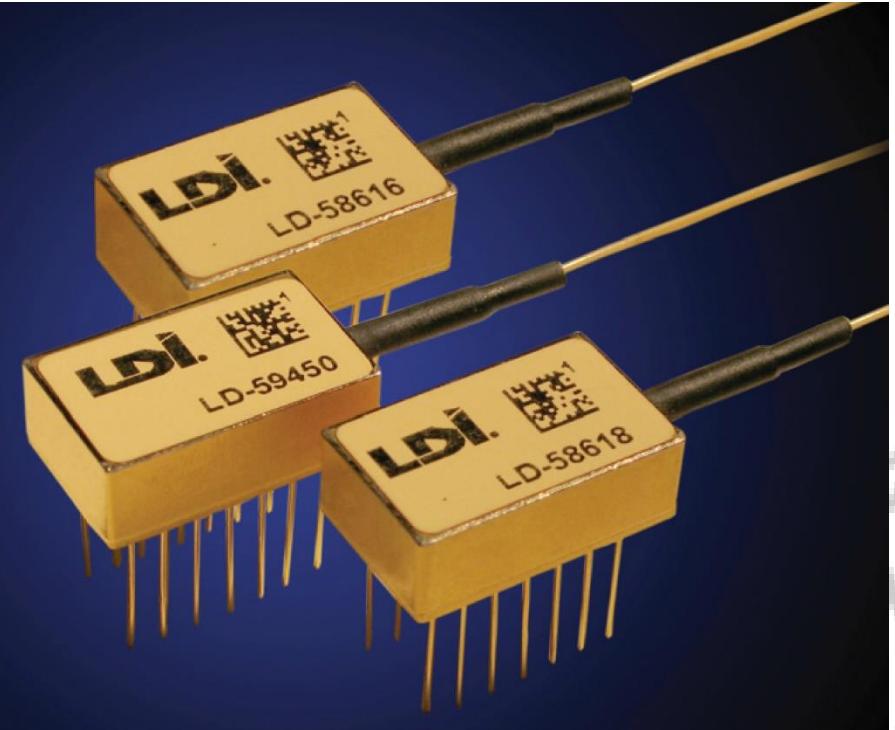
Schematic illustration of the structure of a double heterojunction stripe contact edge emitting LED. (Upper case notation for a wider bandgap semiconductor is not used as there are several layers with different bandgaps.)

# ELED Coupling into Fibers

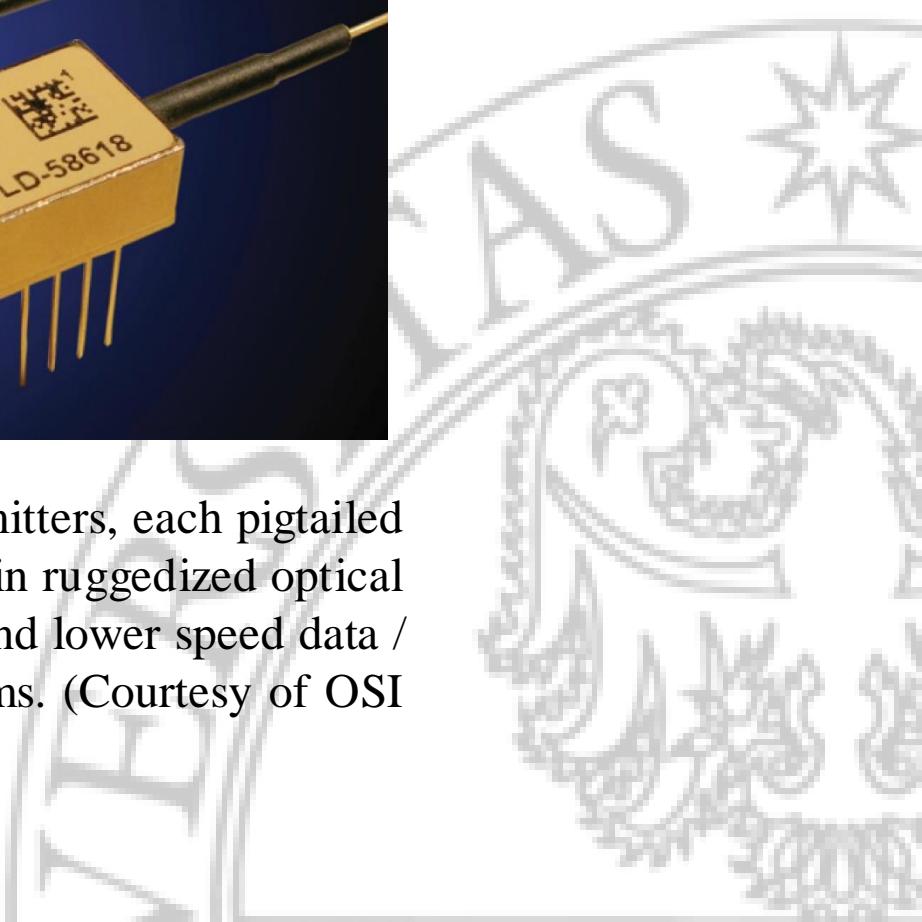


Light from an edge emitting LED is coupled into a fiber typically by using a lens or a GRIN rod lens.

# Pigtailed LEDs



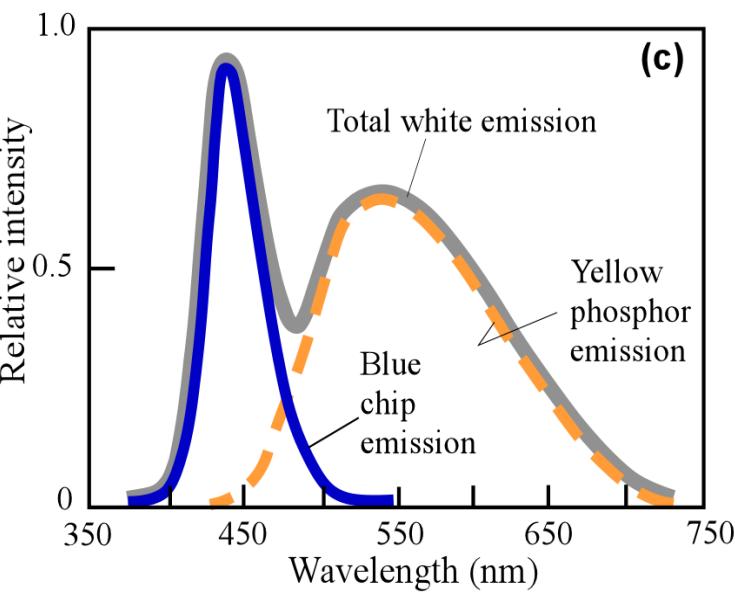
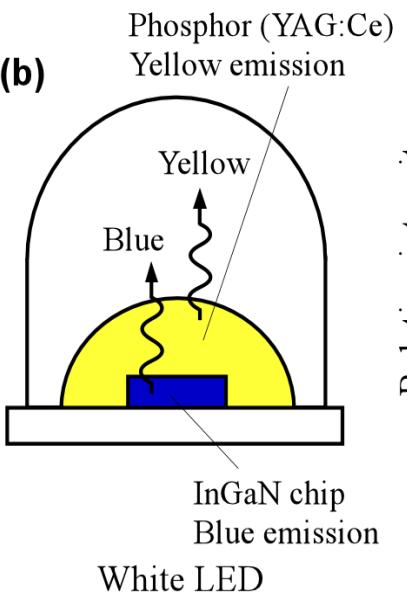
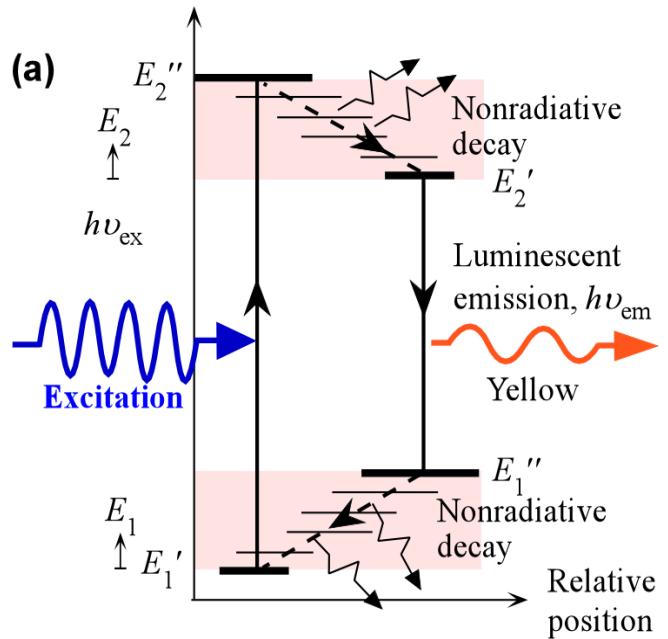
InGaAsP 1300nm LED emitters, each pigtailed to an optical fiber for use in ruggedized optical communication modems and lower speed data / analog transmission systems. (Courtesy of OSI Laser Diode, Inc)



# White LEDs



Energy of luminescent center in host



(a) A simplified energy diagram to explain the principle of photoluminescence. The activator is pumped from  $E_1'$  to  $E_2''$ . It decays nonradiatively down to  $E_2'$ . The transition from  $E_2'$  down to  $E_1''$ . (b) Schematic structure of a blue chip yellow phosphor white LED (c) The spectral distribution of light emitted by a white LED. Blue luminescence is emitted by GaInN chip and "yellow" phosphorescence is produced by phosphor. The combined spectrum looks "white". (Note: Orange used for yellow as yellow does not show well.)

# White LEDs



**Photoluminescence** is the emission of light by a material, called a **phosphor**, that has been first excited by light of higher frequency. Higher energy photons are first absorbed, and then lower energy photons are emitted.

Typically the emission of light occurs from certain dopants, impurities or even defects, called luminescent or **luminescence centers**, purposefully introduced into a **host matrix**, which may be a crystal or glass.

# White LEDs



The luminescent center is also called an **activator**. Many phosphors are based on activators doped into a host matrix.

**Eu<sup>3+</sup>** (europium ion) in a **Y<sub>2</sub>O<sub>3</sub>** (yttrium oxide, called **yttria**) matrix is a widely used modern phosphor. When excited by UV radiation, it provides an efficient luminescence emission in the red (around 613 nm). It is used as the red-emitting phosphor in color TV tubes and in modern tricolor fluorescent lamps.

Another important phosphor is **Ce<sup>3+</sup>** in **Y<sub>3</sub>Al<sub>5</sub>O<sub>12</sub>** (**YAG**), written as **Y<sub>3</sub>Al<sub>5</sub>O<sub>12</sub>:Ce<sup>3+</sup>**, which is used in white LEDs. **YAG:Ce<sup>3+</sup>** can absorb blue radiation, and emit yellow light.

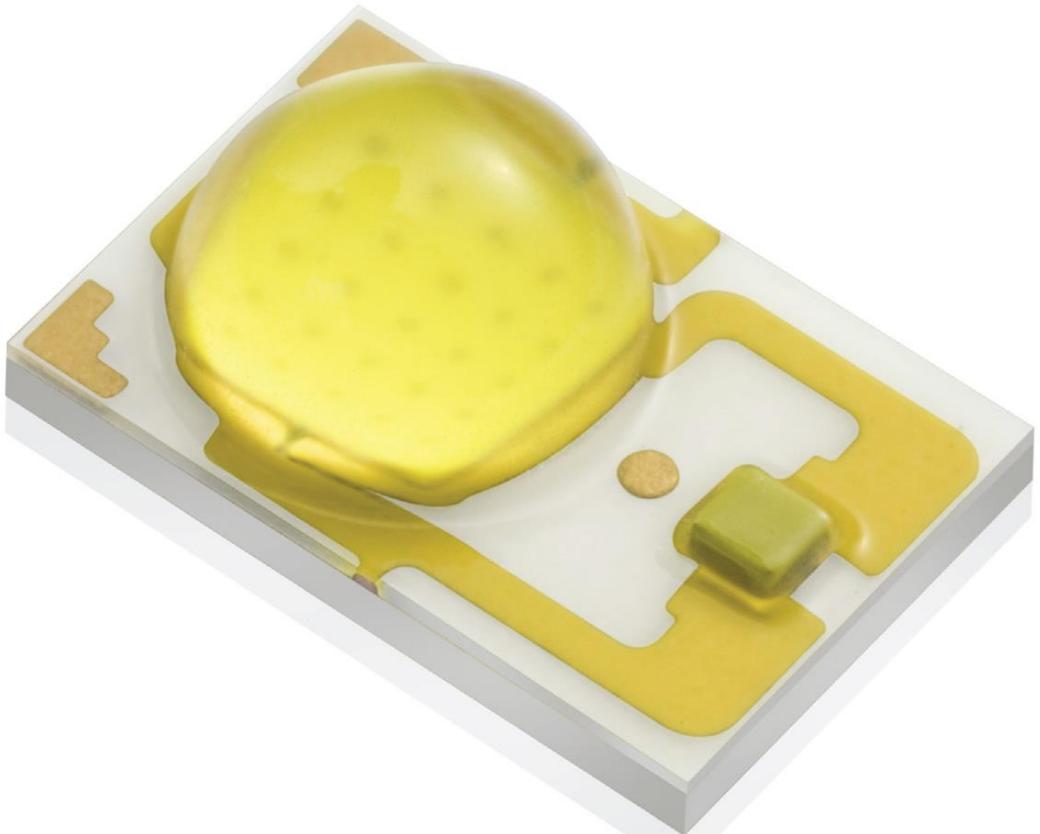
# White LEDs



(Photo by SK)

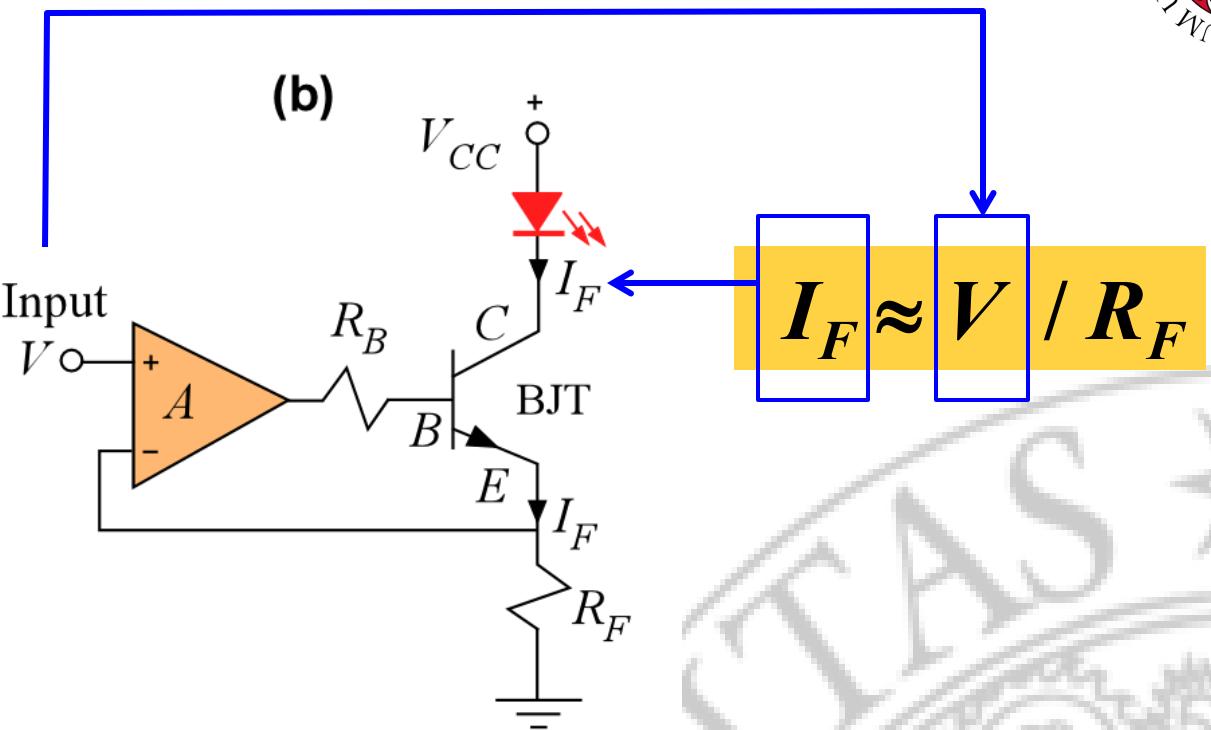
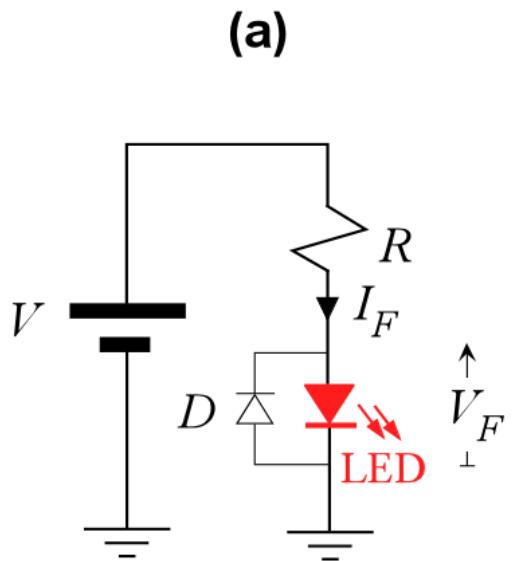


# White LEDs



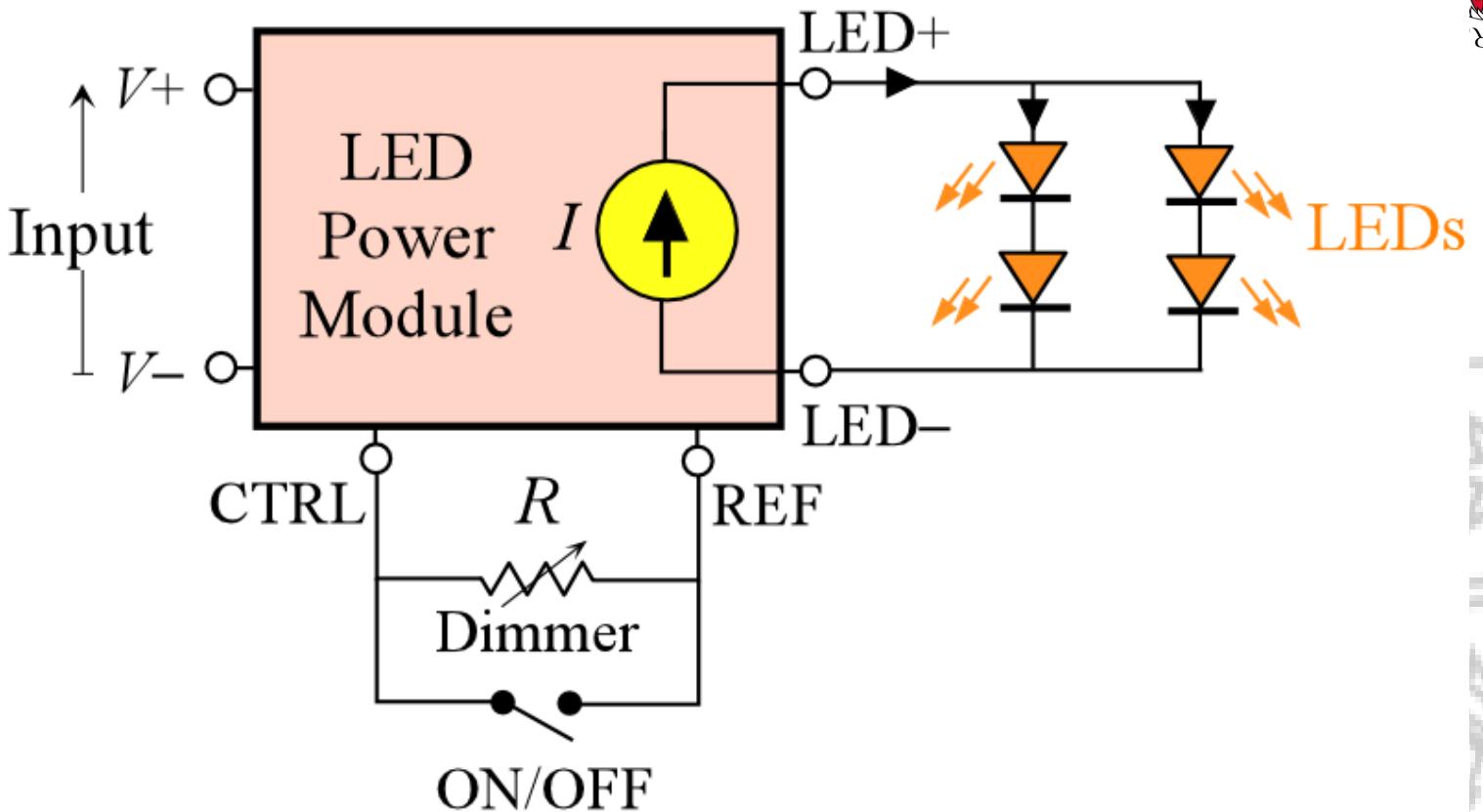
LUXEON Rebel ES white emitting LED (Courtesy of Philips Lumileds)

# LED Electronics: Analog



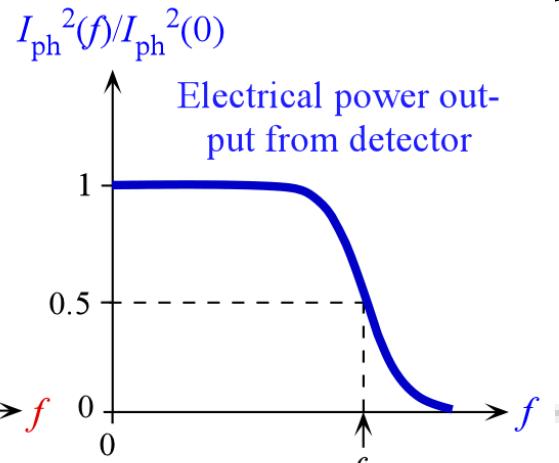
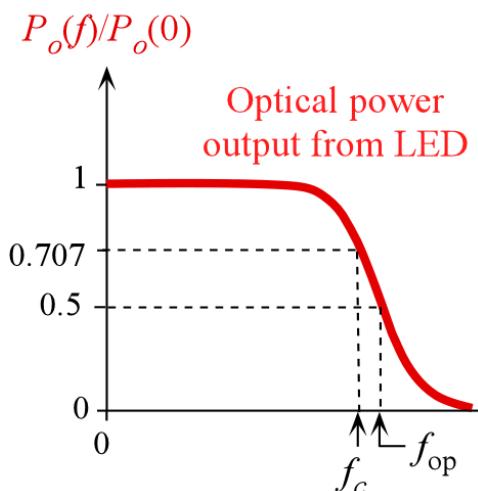
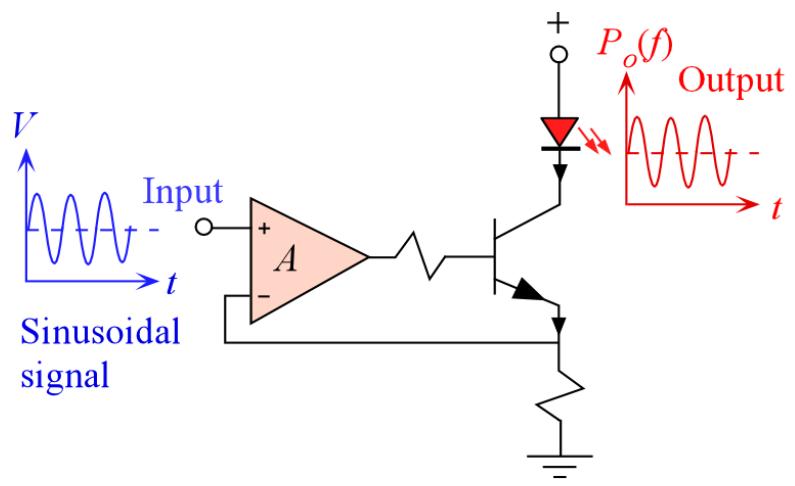
(a) The simplest circuit to drive an LED involves connecting it to a voltage supply ( $V$ ) through a resistor  $R$ . (b) Bipolar junction transistors are well suited for supplying a constant current. Using an IC and negative feedback, the current is linearly controlled by  $V$ . (c) There are various commercial LED driver modules that can be easily configured to drive a number of LEDs in parallel and/or series. The example has a module driving 4 LEDs, has a dimmer ( $R$ ) and an on/off switch.

# LED Electronics: Drive Modules



There are various commercial LED driver modules that can be easily configured to drive a number of LEDs in parallel and/or series. The example has a module driving 4 LEDs, has a dimmer ( $R$ ) and an on/off switch.

# LED Electronics



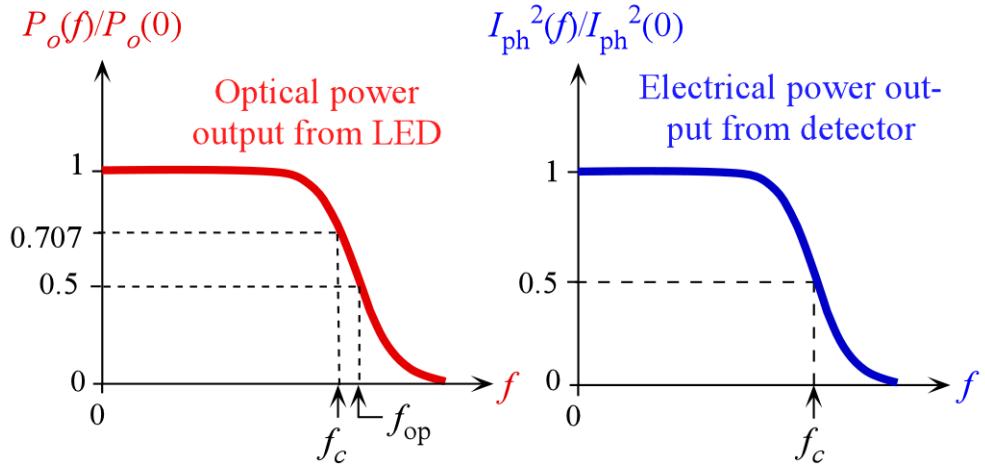
$$P_o(\omega) / P_o(0) = 1 / \sqrt{[1 + (\omega\tau)^2]}$$

$\tau$  = Net minority carrier recombination time

$$1/\tau = 1/\tau_r + 1/\tau_{nr}$$

(a) Sinusoidal modulation of an LED. (b) The frequency response where  $f_c$  is the cut-off frequency at which  $P_o(f)/P_o(0)$  is 0.707. (c) The electrical power output from the detector as a function of frequency. At  $f_c$ ,  $[I_{ph}(f)/I_{ph}(0)]^2$  is 0.5. However, it is 0.707 at a lower frequency  $f'_c$ .

# LED Electronics



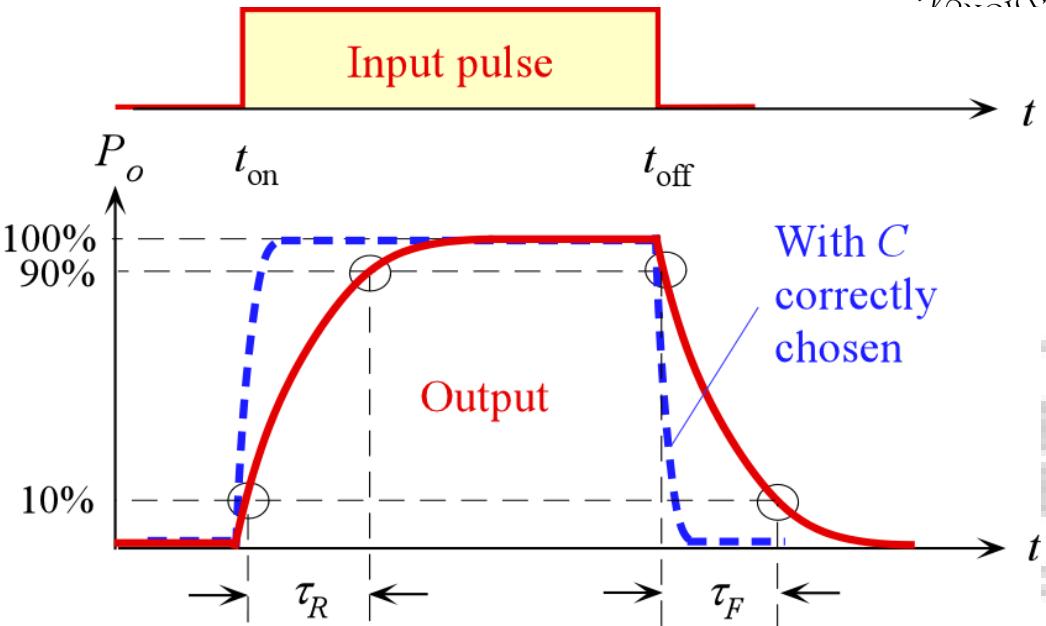
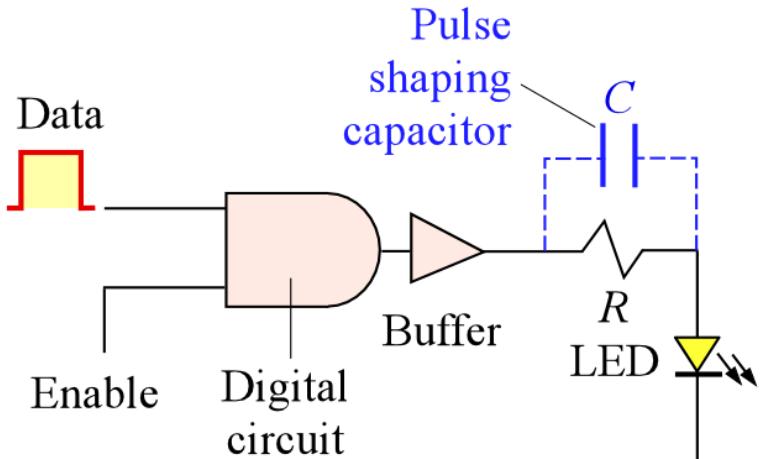
$$P_o(\omega) / P_o(0) = 1 / \sqrt{[1 + (\omega\tau)^2]}$$

$$f_{op} = \frac{\sqrt{3}}{2\pi\tau}$$

## Optical Bandwidth

$$f = f_{op} \longrightarrow P_o(f_{op}) / P_o(0) = 1/2$$

# LED Electronics: Digital

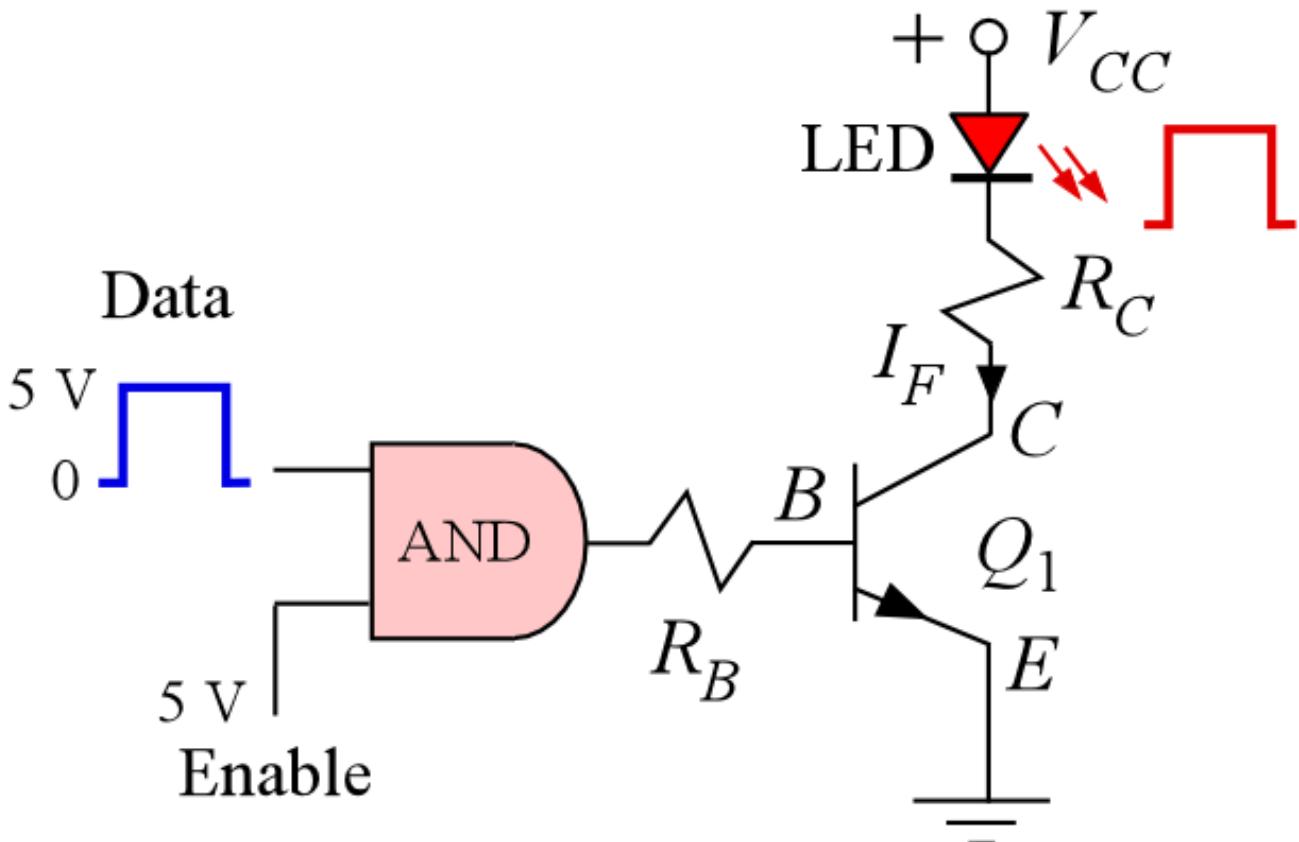


$$\tau_R = \tau_F = 2.2 \tau$$

$$1/\tau = 1/\tau_r + 1/\tau_{nr}$$

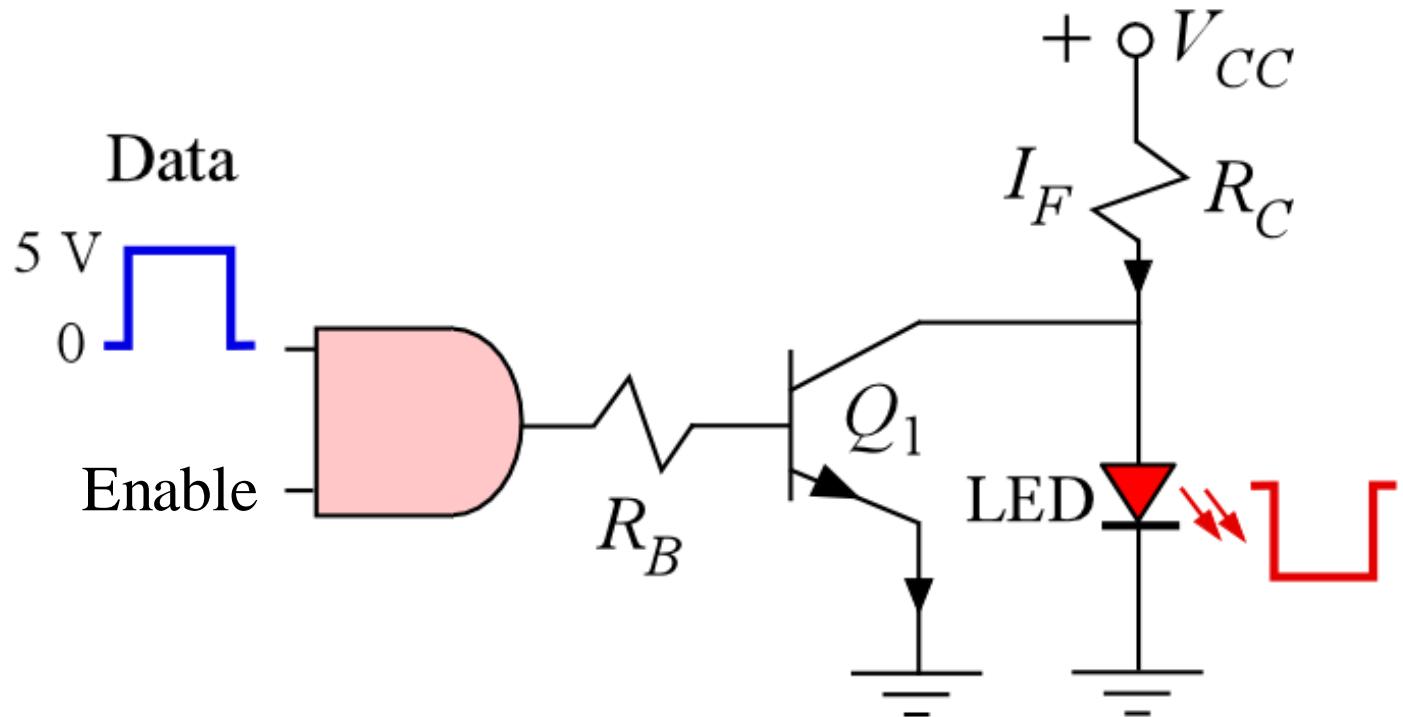
A LED in a digital circuit is turned on and off by a logic gate, assumed to have a buffered output as shown, to avoid being loaded. A BJT can be used after the logic gate to drive the LED as well (not shown). Definitions of rise and fall times are shown in the light output pulse.

# LED Electronics: Digital



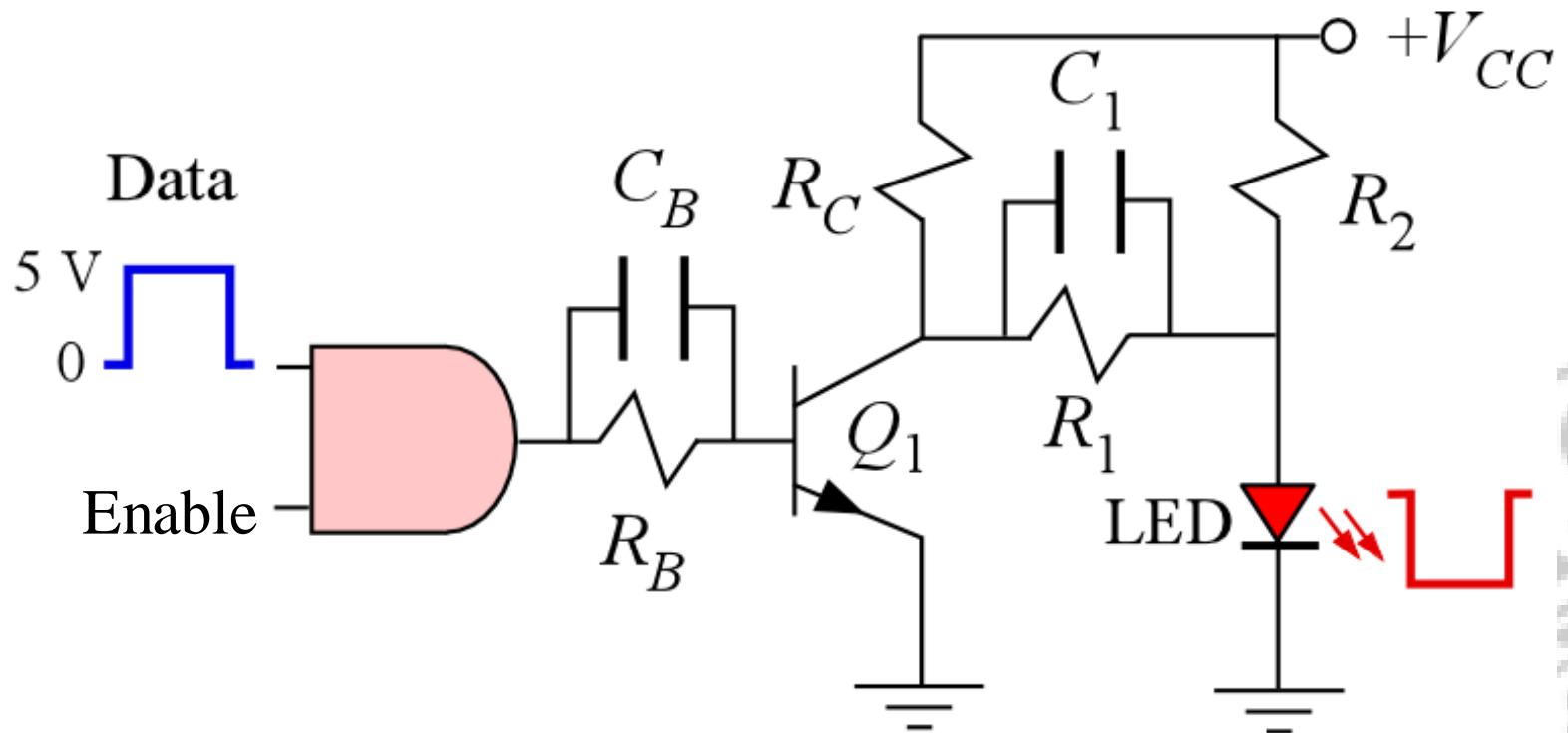
A CMOS logic circuit (or a gate) drives an LED through a BJT  $Q_1$ .

# LED Electronics: Digital



The logic gate drives  $Q_1$ , which shunts the current from the LED and switches it off.

# LED Electronics: Digital



Speeding capacitors  $C_B$  and  $C_1$  help turn the LED on and off more quickly. In all three cases, the inputs to the logic circuit are the same.