Example on Waveguide Modes

Consider a planar dielectric guide with a core thickness 20 μm, n_1 = 1.455, n_2 = 1.440, light wavelength of 900 nm. Find the modes

TIR phase change ϕ_m for TE mode

$$\tan\left(\tfrac{1}{2}\phi_m\right) = \frac{\left[\sin^2\theta_m - \left(\dfrac{n_2}{n_1}\right)^2\right]^{1/2}}{\cos\theta_m}$$

TE mode

Waveguide condition

$$\left[\frac{2\pi n_1(2a)}{\lambda}\right]\cos\theta_m - \phi_m = m\pi$$

Waveguide condition

$$\phi_m = 2ak_1\cos\theta_m - m\pi$$

Optical Fiber Parameters

$n = (n_1 + n_2)/2 =$ **average refractive index**

$\Delta =$ **normalized index difference**

$\Delta = (n_1 - n_2)/n_1 \approx (n_1{}^2 - n_2{}^2)/2n_1{}^2$

V-number $$V = \frac{2\pi a}{\lambda}\left(n_1^2 - n_2^2\right)^{1/2} \approx \frac{2\pi a n_1}{\lambda}(2\Delta)^{1/2}$$

$V < 2.405$ only 1 mode exists. Fundamental mode

$V < 2.405$ or $\lambda > \lambda_c$ Single mode fiber

$V > 2.405$ Multimode fiber

Number of modes $$M \approx \frac{V^2}{2}$$

Group Velocity and Group Delay

Consider a single mode fiber with core and cladding indices of 1.4480 and 1.4400, core radius of 3 μm, operating at 1.5 μm. *What are the group velocity and group delay at this wavelength*?

$$b = \frac{(\beta / k) - n_2}{n_1 - n_2} \quad \Rightarrow \quad \beta = n_2 k[1 + b\Delta]$$

$k = 2\pi/\lambda = 4{,}188{,}790$ m^{-1} and $\omega = 2\pi c/\lambda = 1.255757\times10^{15}$ rad s^{-1}

$$V = (2\pi a/\lambda)(n_1^2 - n_2^2)^{1/2} = 1.910088$$

$1.5 < V < 2.5$

$$\boxed{b \approx \left(1.1428 - \frac{0.996}{V}\right)^2}$$

$b = 0.3860859$, and $\beta = 6.044796\times10^6$ m^{-1}.

Increase wavelength by 0.1% and recalculate. Values in the table

Analysis of OTDR Traces

Example: 100 km fiber is probed with an OTDR having a peak output power of 13 dBm and the pulsewidth is 10 μsec.

Calculate the backscattered power returning from the far end of the fiber having attenuation of
0.33 dB/km, a scattering coeff. α of 0.3 dB/km and capture factor S=10^{-3} at 1300 nm.

$$P_s(L) = S \cdot \alpha_s \cdot W \cdot P_0 \cdot e^{-2\alpha L}$$

$$P_s(100km) = 0.001 \times (0.3 \times 0.23) \times 2 \times 20mW \times e^{-2\times(0.33\times0.23)\times100} =$$
$$= 35.3 \times 10^{-12} \times 20mW = 0.75\,pW = -91.5dBm$$

$$W = \tau \cdot v_g = 2km$$

Example of Time Averaging

*A 10 km fiber link is tested with an OTDR at 1300 nm. Compute the noise reduction that can be achieved by signal averaging compared to a single shot measurement within the first second and after 3 min, considering that **10%** of the time is needed for processing overhead.*

Round trip time for a 10 km fiber: $T = 10\dfrac{\mu s}{km} \times 20 km = 200\mu s$

Number of averages in 1 sec: $N_{1s} = \dfrac{1}{200 \times 10^{-6}} \times 0.9 = 4500$

Number of averages in 3 min: $N_{3\min} = 4500 \times 180 = 810000$

The noise reduction is proportional to the square-root of N:

$$\Delta SNR_{1s} = 10 \times \log(\sqrt{N_{1s}}) = 5 \times \log(4500) = 18.3 dB$$

$$\Delta SNR_{3\min} = 10 \times \log(\sqrt{N_{3\min}}) = 5 \times \log(810000) = 29.5 dB$$

FBG at 1550 nm

EXAMPLE

A silica fiber based FBG is required to operate at 1550 nm. What should be the periodicity of the grating Λ? If the amplitude of index variation Δn is 10^{-4} and total length of the FBG is 10 mm, what is the reflectance at the Bragg wavelength and the spectral width of the reflected light? Assume that the effective refractive index is 1.4550

SOLUTION

We can use

$$\lambda_B = 2\bar{n}\Lambda$$

1550 nm = 2(1.450)(Λ) so that the grating periodicity Λ = **534.5 nm**

The coupling coefficient κ is by

$$\kappa = \pi\Delta n/\lambda = \pi(10^{-4})/(1550\times10^{-9}\text{ m}) = \mathbf{202.7\ m^{-1}}.$$

Thus, $\kappa L = 2.027$, so the FBG is a strong grating. The reflectance is

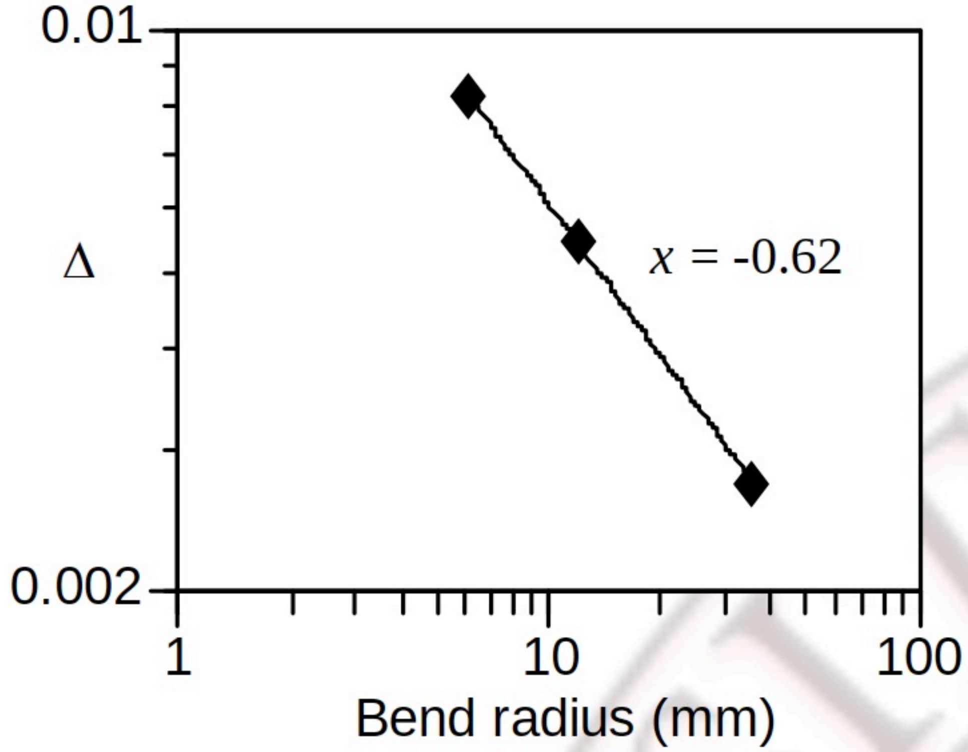
$$R = \tanh^2(\kappa L) = \tanh^2(2.027) = 0.933 \text{ or } \mathbf{93.3\%}$$

$$\Delta\lambda_{\text{strong}} = \frac{4\kappa\lambda_B^2}{\pi\bar{n}} = \frac{4(202.7)(1.55\times10^{-6})^2}{\pi(1.450)} = \mathbf{0.428\ nm}$$

The reflectance and spectral width values are approximate inasmuch as above equations contain a number of assumptions.

Example: Microbending loss It is found that for a single mode fiber with a cut-off wavelength λ_c = 1180 nm, operating at 1300 nm, the microbending loss reaches 1 dB m^{-1} when the radius of curvature of the bend is roughly 6 mm for Δ = 0.00825, 12 mm for Δ = 0.00550 and 35 mm for Δ = 0.00275. Explain these findings.

Solution:



Log-log plot of the results of experiments on Δ vs. bend radius R for 1 dB/m microbending loss

$$\alpha \propto \exp\left(-\frac{R}{R_c}\right) \propto \exp\left(-\frac{R}{\Delta^{-3/2}}\right)$$

R_c is a constant ("a critical radius type of constant") that is proportional to $\Delta^{-3/2}$. Taking logs,

$\ln\alpha = -\Delta^{3/2}R$ + constant

We are interested in the Δ vs R behavior at a constant α. We can lump the constant into $\ln\alpha$ and obtain,

$\Delta \propto R^{-2/3}$

The plot in the figure gives an index close this value.

Example: Microbending loss It is found that for a single mode fiber with a cut-off wavelength λ_c = 1180 nm, operating at 1300 nm, the microbending loss reaches 1 dB m^{-1} when the radius of curvature of the bend is roughly 6 mm for $\Delta = 0.00825$, 12 mm for $\Delta = 0.00550$ and 35 mm for $\Delta = 0.00275$. Explain these findings.

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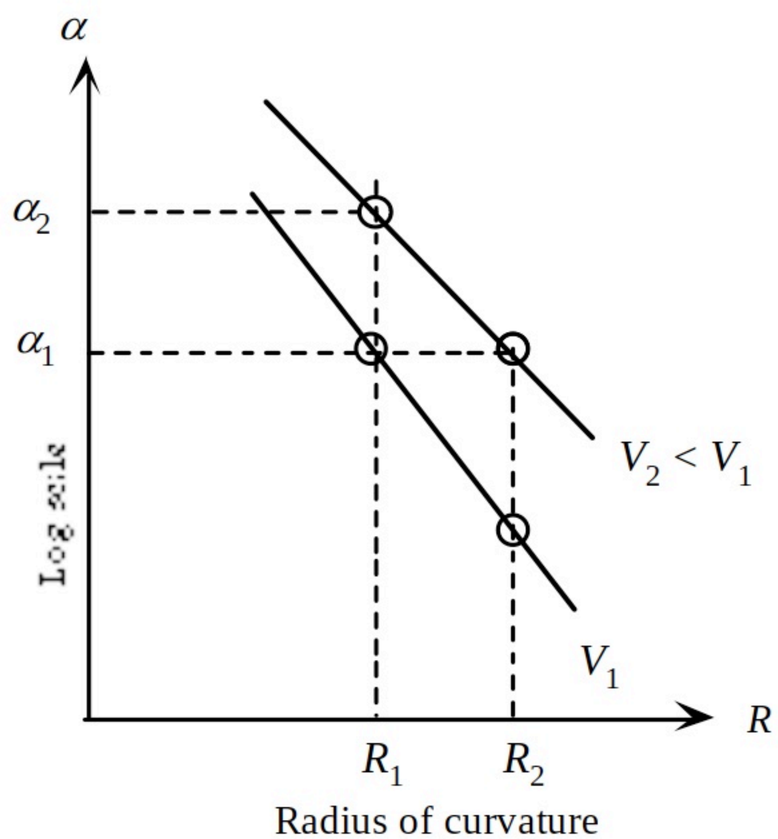

Microbending loss α decreases sharply with the bend radius R. (Schematic only.)

Given $\alpha = \alpha_1$, *R increases* from R_1 to R_2 when V *decreases* from V_1 to V_2.

<u>Expected</u> $R\uparrow$ with $V\downarrow$

Equivalently at one $R = R_1$, $\alpha\uparrow$ with $V\downarrow$

We can *generalize* by noting that the penetration depth into the cladding $\delta \propto 1/V$.

<u>Expected</u> $R\uparrow$ with $\delta\uparrow$

Equivalently at one $R = R_1$, $\alpha\uparrow$ with $\delta\uparrow$

Thus, microbending loss α gets worse when penetration δ into cladding increases; intuitively correct. Experiments show that for a given $\alpha = \alpha_1$, R increases with decreasing Δ. We know from basic optics $\delta\uparrow$ with $\Delta\downarrow$ i.e. δ increases with decreasing Δ.

Thus <u>expected</u> $R\uparrow$ with $\delta\uparrow$ with $\Delta\downarrow$ as observed

Bending Loss Example

Example: Experiments on a standard SMF operating around 1550 nm have shown that the bending loss is 0.124 dB/turn when the bend radius is 12.5 mm and 15.0 dB/turn when the bend radius is 5.0 mm. What is the loss at a bend radius of 10 mm?

Solution

Apply

$$\boldsymbol{\alpha_B = A\exp(-R/R_c)}$$

$$0.124 \text{ dB/turn} = A\exp[-(12.5 \text{ mm})/R_c]$$

$$15.0 \text{ dB/turn} = A\exp[-(5.0 \text{ mm})/R_c]$$

Solve for A and R_c. Dividing the first by the second and separating out R_c we find,

$$R_c = (12.5 \text{ mm} - 5.0 \text{ mm}) / \ln(15.0/0.124) = 1.56 \text{ mm}$$

and $$A = (15.0 \text{ dB/turn}) \exp[(5.0 \text{ mm})/(1.56 \text{ mm}] = 370 \text{ dB/turn}$$

At $R = 10$ mm

$$\boldsymbol{\alpha_B = A\exp(-R/R_c) = (370)\exp[-(10 \text{ mm})/(1.56 \text{ mm})]}$$

$$\boldsymbol{= 0.61 \text{ dB/turn.}}$$

The experimental value is also **0.61 dB/turn** to within two decimals.

Rayleigh Scattering

Solution

We can repeat the above calculations at $\lambda = 1.31$ μm. However, **we do not need** to add α_{FIR}.

$$\alpha_R = A_R/\lambda^4 = (0.90 \text{ dB km}^{-1}\ \mu\text{m}^4)/(1.31\ \mu\text{m})^4 = \mathbf{0.306\ dB\ km^{-1}}$$

and using the NA based for A_R,

$$\alpha_R = A_R/\lambda^4 = (0.918 \text{ dB km}^{-1}\ \mu\text{m}^4)/(1.31\ \mu\text{m})^4 = \mathbf{0.312\ dB\ km^{-1}}.$$

Both close to the measured value.

Rayleigh Scattering

Solution

Rayleigh scattering at 1550 nm, the simplest equation with $A_R = 0.9$ dB km^{-1} µm^4, gives

$$\alpha_R = A_R/\lambda^4 = (0.90 \text{ dB km}^{-1}\ \mu\text{m}^4)/(1.55\ \mu\text{m})^4 = 0.178 \text{ dB km}^{-1}$$

This equation is basically a rule of thumb. The total attenuation is then

$$\alpha_R + \alpha_{FIR} = 0.178 + 0.02 = \mathbf{0.198\ dB\ km^{-1}}.$$

The current fiber has NA = 0.14.

$\therefore \quad A_R = 0.63 + 2.06\times\text{NA} = 0.63 + 2.06\times 0.14 = 0.918 \text{ dB km}^{-1}\ \mu\text{m}^4$

i.e. $\quad \alpha_R = A_R/\lambda^4 = (0.918 \text{ dB km}^{-1}\ \mu\text{m}^4)/(1.55\ \mu\text{m})^4 = 0.159 \text{ dB km}^{-1}$,

which gives a total attenuation of 0.159 + 0.020 or **0.179 dB km^{-1}**.

Rayleigh Scattering

Example: Consider a single mode step index fiber, which has a numerical aperture of 0.14. Predict the expected attenuation at 1.55 μm, and compare your calculation with the reported (measured) value of 0.19 - 0.20 dB km^{-1} for this fiber. Repeat the calculations at 1.31 μm, and compare your values with the reported 0.33 - 0.35 dB km^{-1} values.

Solution

First, we should check the fundamental infrared absorption at 1550 nm.

$$\alpha_{\mathrm{FIR}} = A\exp(-B/\lambda) = 7.8\times10^{11}\exp[-(48.5)/(1.55)]$$

$$= 0.020 \text{ dB km}^{-1}, \text{ very small}$$

Example: Rayleigh scattering limit

What is the attenuation due to Rayleigh scattering at around the λ = 1.55 μm window given that pure silica (SiO_2) has the following properties: T_f = 1730°C (softening temperature); $\beta_T = 7\times10^{-11}$ m^2 N^{-1} (at high temperatures); n = 1.4446 at 1.5 μm.

Solution

We simply calculate the Rayleigh scattering attenuation using

$$\alpha_R \approx \frac{8\pi^3}{3\lambda^4}(n^2-1)^2\beta_T k_B T_f$$

$$\alpha_R \approx \frac{8\pi^3}{3(1.55\times10^{-6})^4}(1.4446^2-1)^2(7\times10^{-11})(1.38\times10^{-23})(1730+273)$$

$\alpha_R = 3.276\times10^{-5}$ m^{-1} or 3.276×10^{-2} km^{-1}

Attenuation in dB per km is

$\alpha_{dB} = 4.34\alpha_R = (4.34)(3.735\times10^{-2}$ km^{-1}) = **0.142 dB km^{-1}**

This represents the lowest possible attenuation for a silica glass core fiber at 1.55 μm.

Example: Combining intermodal and intramodal dispersions

Consider a graded index fiber with a core diameter of 30 μm and a refractive index of 1.474 at the center of the core and a cladding refractive index of 1.453. Suppose that we use a laser diode emitter with a spectral linewidth of 3 nm to transmit along this fiber at a wavelength of 1300 nm. Calculate, the total dispersion and estimate the bit-rate × distance product of the fiber. The material dispersion coefficient D_m at 1300 nm is –7.5 ps nm^{-1} km^{-1}.

Solution

The normalized refractive index difference $\Delta = (n_1 - n_2)/n_1 = (1.474 - 1.453)/1.474 = 0.01425$. Modal dispersion for 1 km is

$$\sigma_{\text{intermode}} = Ln_1\Delta^2/[(20)(3^{1/2})c] = 2.9\times10^{-11}\ \text{s}^{1} \text{ or } 0.029 \text{ ns.}$$

The material dispersion is

$$\Delta\tau_{1/2} = LD_m\Delta\lambda_{1/2} = (1\text{ km})(7.5\text{ ps nm}^{-1}\text{ km}^{-1})(3\text{ nm}) = 0.0225\text{ ns}$$

Assuming a Gaussian output light pulse shape,

$$\sigma_{\text{intramode}} = 0.425\Delta\tau_{1/2} = (0.425)(0.0225\text{ ns}) = 0.0096\text{ ns}$$

Total dispersion is

$$\sigma_{\text{rms}} = \sqrt{\sigma^2_{\text{intermode}} + \sigma^2_{\text{intramode}}} = \sqrt{0.029^2 + 0.0096^2} = 0.030\ \text{ ns}$$

Assume L = 1 km

$B = 0.25/\Delta\tau_{\text{rms}}$ = **8.2 Gb**

Example: Dispersion in a graded-index fiber and bit rate

Consider a graded index fiber whose core has a diameter of 50 μm and a refractive index of n_1 = 1.480. The cladding has n_2 = 1.460. If this fiber is used at 1.30 μm with a laser diode that has very a narrow linewidth what will be the bit rate × distance product? Evaluate the *BL* product if this were a multimode step index fiber.

Solution

The normalized refractive index difference $\Delta = (n_1 - n_2)/n_1 = (1.48 - 1.46)/1.48 = 0.0135$.
Dispersion for 1 km of fiber is

$$\sigma_{\text{intermode}}/L = n_1\Delta^2/[(20)(3^{1/2})c] = 2.6\times10^{-14}\ \text{s m}^{-1} \text{ or } 0.026\ \text{ns km}^{-1}.$$

$$BL = 0.25/\sigma_{\text{intermode}} = \mathbf{9.6\ Gb\ s^{-1}\ km}$$

We have ignored any material dispersion and, further, we assumed the index variation to perfectly follow the optimal profile which means that in practice *BL* will be worse. (For example, a 15% variation in γ from the optimal value can result in $\sigma_{\text{intermode}}$ and hence *BL* that are more than 10 times worse.)

If this were a multimode step-index fiber with the same n_1 and n_2, then the full dispersion (total spread) would roughly be 6.67×10^{-11} s m^{-1} or 66.7 ns km^{-1} and BL = 12.9 **Mb s^{-1} km**

Note: Over long distances, the bit rate × distance product is not constant for multimode fibers and typically $B \propto L^{-\gamma}$ where γ is an index between 0.5 and 1. The reason is that, due to various fiber imperfections, there is mode mixing which reduces the extent of spreading.

Example: Dispersion in a GRIN Fiber and Bit Rate

Solution continued

If this were a multimode step-index fiber with the same n_1 and n_2, then the full dispersion (total spread) would roughly be

$$\frac{\Delta\tau}{L} \approx \frac{n_1 - n_2}{c} = \frac{n_1\Delta}{c} = \frac{(1.475)(0.01)}{3\times10^8}$$

$$= 4.92\times10^{-11} \text{ s m}^{-1} \text{ or } 49.2 \text{ ns km}^{-1}$$

To calculate the BL we use $\sigma_{\text{intermode}} \approx 0.29\Delta\tau$

$$BL \approx \frac{0.25L}{\sigma_{\text{intermode}}} = \frac{0.25}{0.29(\Delta\tau/L)} = \frac{0.25}{(0.29)(49.2\times10^{-9}\text{ s km}^{-1})} = 17.5 \text{ Mb s}^{-1}\text{ km}$$

LANs now use graded index MMFs, and the step index MMFs are used mainly in low speed instrumentation

Example: Dispersion in a GRIN Fiber and Bit Rate

Solution continued

$$B = \frac{0.25}{\sigma} = \frac{0.25}{0.408\Delta\tau} = \frac{0.61}{\Delta\tau} = \frac{0.61}{(53.0\times10^{-12}\,\mathrm{s})} = \mathbf{11.5\ Gb\ s^{-1}}$$

Optical bandwidth $f_{op} = 0.99B =$ **11.4 GHz**

This is the upper limit since we assumed that the graded index fiber is perfectly optimized with $\sigma_{\text{intermode}}$ being minimum. Small deviations around the optimum γ cause large increases in $\sigma_{\text{intermode}}$, which would sharply reduce the bandwidth.

Example: Dispersion in a GRIN Fiber and Bit Rate

Solution continued

$$\frac{\sigma_{\text{intermode}}}{L} \approx \frac{n_1}{20\sqrt{3}c}\Delta^2 = \frac{1.4750}{20\sqrt{3}(3\times10^8)}(0.010)^2$$

$$= 14.20\times10^{-15}\text{ s m}^{-1}\text{ or }14.20\text{ ps km}^{-1}$$

Assuming a triangular output light pulse (max bandwidth) and the relationship between σ and $\Delta\tau_{1/2}$ given in Table 2.4, the intermodal spread $\Delta\tau_{\text{intermode}}$ (FWHM) in the group delay over 1 km is

$$\Delta\tau_{\text{intermode}} = (6^{1/2})\sigma_{\text{intermode}} = (2.45)(14.20\text{ ps}) = 34.8\text{ ps}$$

We also need the material dispersion at the operating wavelength over 1 km, which makes up the intramodal dispersion $\Delta\tau_{\text{intramode}}$ (FWHM)

$$\Delta\tau_{\text{intramode}} = L|D_{ch}|\,\Delta\lambda_{1/2} = (1\text{ km})(-100\text{ ps nm}^{-1}\text{ km}^{-1})(0.40\text{ nm}) = 40.0\text{ ps}$$

$$\Delta\tau^2 = \Delta\tau^2_{\text{intermode}} + \Delta\tau^2_{\text{intramode}} = (34.8)^2 + (40.0)^2 \Rightarrow \Delta\tau = 53.0\text{ ps}$$

Example: Dispersion in a GRIN Fiber and Bit Rate

Graded index fiber. Diameter of 50 μm and a refractive index of $n_1 = 1.4750$, $\Delta = 0.010$.

The fiber is used in LANs at 850 nm with a vertical cavity surface emitting laser (VCSEL) that has very a narrow linewidth that is about 0.4 nm (FWHM). Assume that the chromatic dispersion at 850 nm is −100 ps nm^{-1} km^{-1} as shown in Table 2.5. Assume the fiber has been optimized at 850 nm, and find the minimum rms dispersion. How many modes are there? What would be the upper limit on its bandwidth? What would be the bandwidth in practice?

Solution

Given Δ and n_1, we can find n_2 from

$\Delta = 0.01 = (n_1 - n_2)/n_1 = (1.4750 - n_2)/1.4750.$

$\therefore \; n_2 = 1.4603.$

The V-number is then

$V = [(2\pi)(25\ \mu\text{m})/(0.850\ \mu\text{m})(1.4750^2 - 1.4603^2)^{1/2} = 38.39$

For the number of modes we can simply take $\gamma = 2$ and use

$M = (V^2/4) = (38.39^2/4) = 368$ modes

The lowest intermodal dispersion for a profile optimized graded index fiber for a 1 km of fiber, $L = 1$ km, is

Example: Bit rate and dispersion

Consider an optical fiber with a chromatic dispersion coefficient 8 ps km^{-1} nm^{-1} at an operating wavelength of 1.5 μm. Calculate the bit rate distance product (*BL*), and the optical and electrical bandwidths for a 10 km fiber if a laser diode source with a FWHP linewidth $\Delta\lambda_{1/2}$ of 2 nm is used.

Solution

For FWHP dispersion,

$$\Delta\tau_{1/2}/L = |D_{ch}|\Delta\lambda_{1/2} = (8 \text{ ps nm}^{-1} \text{ km}^{-1})(2 \text{ nm}) = 16 \text{ ps km}^{-1}$$

Assuming a Gaussian light pulse shape, the RTZ bit rate × distance product (*BL*) is

$$BL = 0.59L/\Delta\tau_{1/2} = 0.59/(16 \text{ ps km}^{-1}) = 36.9 \text{ Gb s}^{-1} \text{ km}$$

The optical and electrical bandwidths for a 10 km fiber are

$$f_{\mathrm{op}} = 0.75B = 0.75(36.9 \text{ Gb s}^{-1} \text{ km}) / (10 \text{ km}) = 2.8 \text{ GHz}$$

$$f_{\mathrm{el}} = 0.70 f_{op} = 1.9 \text{ GHz}$$

Example: A single mode fiber and cut-off wavelength

A typical single mode optical fiber has a core of diameter 8 μm and a refractive index of 1.460. The normalized index difference is 0.3%. The cladding diameter is 125 μm. Calculate the numerical aperture and the total acceptance angle of the fiber. What is the single mode cut-off frequency λ_c of the fiber?

Solution

The numerical aperture

$$NA = (n_1^2 - n_2^2)^{1/2} = [(n_1 + n_2)(n_1 - n_2)]^{1/2}$$

Substituting $(n_1 - n_2) = n_1\Delta$ and $(n_1 + n_2) \approx 2n_1$, we get

$$NA \approx [(2n_1)(n_1\Delta)]^{1/2} = n_1(2\Delta)^{1/2} = 1.46(2\times0.003)^{1/2} = \mathbf{0.113 \text{ or } 11.3\ \%}$$

The acceptance angle is given by

$$\sin\alpha_{max} = NA/n_o = 0.113/1 \text{ or } \alpha_{max} = \mathbf{6.5°}, \text{ and } 2\alpha_{max} = \mathbf{13°}$$

The condition for single mode propagation is $V \le 2.405$. At cut-off,

$$V = (2\pi a/\lambda_c)(n_1^2 - n_2^2)^{1/2} = 2.405$$

$$\therefore\ \lambda_c = [2\pi a NA]/2.405 = [(2\pi)(4\ \mu m)(0.113)]/2.405 = \mathbf{1.18\ \mu m}$$

Wavelengths shorter than 1.18 μm will result in multimode operation.

Example: A multimode fiber and total acceptance angle

A step index fiber has a core diameter of 100 μm and a refractive index of 1.480. The cladding has a refractive index of 1.460. Calculate the numerical aperture of the fiber, acceptance angle from air, and the number of modes sustained when the source wavelength is 850 nm.

Normalized refractive index
$\Delta = (n_1 - n_2) / n_1 = \mathbf{0.0135}$ **or 1.35%**

Solution

The numerical aperture is

$$NA = (n_1^2 - n_2^2)^{1/2} = (1.480^2 - 1.460^2)^{1/2} = \mathbf{0.2425 \text{ or } 24.3\%}$$

From, $\sin\alpha_{max} = NA/n_o = 0.2425/1$

Acceptance angle $\alpha_{max} = \mathbf{14°}$

Total acceptance angle $2\alpha_{max} = \mathbf{28°}$

V-number in terms of the numerical aperture can be written as,

$$V = (2\pi a/\lambda)NA = [(2\pi 50 \text{ μm})/(0.85 \text{ μm})](0.2425) = 89.62$$

The number of modes, $M \approx V^2/2 = \mathbf{4016}$

Solution (continued)

Mode field diameter MFD from the Marcuse Equation is

$$\boxed{\begin{aligned} 2w &= 2a(0.65 + 1.619V^{-3/2} + 2.879V^{-6}) \\ &= 2(4.1)[0.65 + 1.62(2.30)^{-3/2} + 2.88(2.30)^{-6}] \end{aligned}}$$

$\mathbf{2w = 9.30\ \mu m}$ **86% of total power is within this diameter**

$\mathbf{2w = (2a)(2.6/V) = 2(4.1)(2.6/2.30) = 9.28\ \mu m}$

$\mathbf{2w = 2a[(V+1)/V] = 11.8\ \mu m}$

This is for a planar waveguide, and the definition is different than that for an optical fiber

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Example: Single mode cut-off wavelength

Calculate the cut-off wavelength for single mode operation for a fiber that has a core with diameter of 8.2 μm, a refractive index of 1.4532, and a cladding of refractive index of 1.4485. What is the V-number and the mode field diameter (MFD) for operation at $\lambda = 1.31$ μm?

Solution

For single mode operation,

$$V = (2\pi a/\lambda)(n_1^2 - n_2^2)^{1/2} \leq 2.405$$

Substituting for a, n_1 and n_2 and rearranging we get,

$$\lambda > [2\pi(4.1\ \mu\text{m})(1.4532^2 - 1.4485^2)^{1/2}]/2.405 = 1.251\ \mu\text{m}$$

Wavelengths shorter than 1.251 μm give multimode propagation.

At $\lambda = 1.31$ μm,

$$V = 2\pi[(4.1\ \mu\text{m})/(1.31\ \mu\text{m})](1.4532^2 - 1.4485^2)^{1/2} = 2.30$$

Mode field diameter MFD

Example: A single mode fiber

What should be the core radius of a single mode fiber which has a core of $n_1 = 1.4680$, cladding of $n_2 = 1.447$ and it is to be used with a source wavelength of 1.3 μm?

Solution

For single mode propagation, $V \leq 2.405$. We need,

$$V = (2\pi a/\lambda)(n_1^2 - n_2^2)^{1/2} \leq 2.405$$

or

$$[2\pi a/(1.3\ \mu\text{m})](1.468^2 - 1.447^2)^{1/2} \leq 2.405$$

which gives $\boldsymbol{a \leq 2.01\ \mu\text{m}}$.

Rather thin for easy coupling of the fiber to a light source or to another fiber; a is comparable to λ which means that the geometric ray picture, strictly, cannot be used to describe light propagation.

Example: A multimode fiber

Calculate the number of allowed modes in a multimode step index fiber which has a core of refractive index of 1.468 and diameter 100 μm, and a cladding of refractive index of 1.447 if the source wavelength is 850 nm.

Solution

Substitute, $a = 50$ μm, $\lambda = 0.850$ μm, $n_1 = 1.468$, $n_2 = 1.447$ into the expression for the V-number,

$$V = (2\pi a/\lambda)(n_1^2 - n_2^2)^{1/2} = (2\pi 50/0.850)(1.468^2 - 1.447^2)^{1/2}$$

$$= 91.44.$$

Since $V >> 2.405$, the number of modes is

$$M \approx V^2/2 = (91.44)^2/2 = 4181$$

which is large.

Group Velocity and Group Delay

Calculation →	***V***	***k* (m^{-1})**	**ω (rad s^{-1})**	***b***	**β (m^{-1})**
λ = 1.500000 μm	1.910088	4188790	1.255757×10^{15}	0.3860859	6.044818×10^{6}
λ' = 1.50150 μm	1.908180	4184606	1.254503×10^{15}	0.3854382	6.038757×10^{6}

$$v_g = \frac{d\omega}{d\beta} = \frac{\omega' - \omega}{\beta' - \beta} = \frac{(1.254503 - 1.255757)\times 10^{15}}{(6.038757 - 6.044818)\times 10^{6}} \approx 2.07\times 10^{8}\ \mathrm{m\,s^{-1}}$$

The group delay τ_g over 1 km is 4.83 μs