Can $\overline{\text{MS}}$ PDF be negative?

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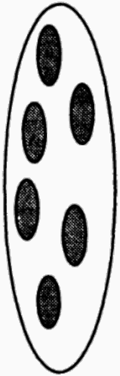
Parton model

The *parton model* consist in a model of the proton structure as a bunch of free components, collectively called *partons*:

- in principle any elementary particle
- in practice mostly **quarks** and **gluons**



It has been historically formulated as a model before the theory of quarks, just assuming **point-like constituents** for the proton, now it has a special role as a model because some of its properties[1]can be deduced from field theory and Standard Model.

[1]The actual structure of the proton has a non-perturbative origin, so it cannot be completely understood by perturbative QFT

LO PDF definition

Since they are free the main property of each parton is the fraction of the total momentum it carries.

The probability distribution of finding a parton p with momentum fraction x it's encoded in its *Parton Density Function*[2], $f_p(x)$.

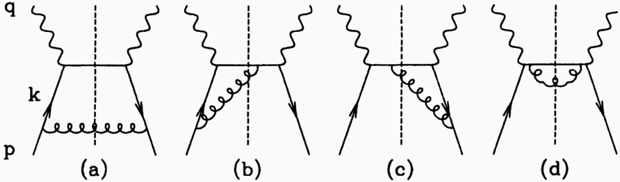
Plot LO PDFs

[2] In general PDFs also depend on the energy scale Q^2, but at LO they scale (see *Björken scaling*). This statement can be explained by perturbative QCD and systematically improve, including the dependency through α_s.

PDF @ NLO: factorization scheme

NLO divergences

As well known at NLO divergences start to appear, both in virtual and real contributions.



There are 3 kinds of divergences:

- **virtual**, due to loop integrals
- **soft**, due to emission of an extra soft particle
- **collinear**, due to

The first two kind are known to cancel in sufficiently inclusive observables[3].

[3] e.g.: the KLN theorem is one of the results that guarantee this cancellation.

NLO collinear divergences

On the other hand collinear divergences have *a different origin* that can be related to the asymptotic freedom of strong interactions:

The collinear limit corresponds to a long range (soft) part of the strong interaction, which is not calculable in perturbation theory.

These divergences must be treated in a different way, defining a suitable **factorization scheme**.

Notice that collinear divergences are also responsible for the appearance of the characteristic $\alpha_s \log(Q^2)$, that will introduce the PDF dependency on Q^2.

Factorization Scheme

The way to deal with these divergences is offered by the previous interpretation: we can hide them in a non-perturbative object, the PDF.

$$\hat{F}_{2,0}(x, Q^2) = e_q^2 x \left[\delta(1-x) + \frac{\alpha_s}{2\pi}\left(P(x)\log\left(\frac{Q^2}{\kappa^2}\right) + C(x)\right) + \dots\right]$$

$$\begin{aligned} F_2(x, Q^2) &= \sum_{q,\bar{q}} q_0 \otimes \hat{F}_{2,0}(x, Q^2) = \sum_{q,\bar{q}} \int_x^1 \frac{\mathrm{d}\xi}{\xi} q_0(\xi) \hat{F}_{2,0}\left(\frac{x}{\xi}, Q^2\right) \\ &= \sum_{q,\bar{q}} q \otimes \hat{F}_2(x, Q^2) \end{aligned}$$

$$q(x, \mu_F^2) = q_0(x) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{\mathrm{d}\xi}{\xi} q_0(\xi) \left\{P\left(\frac{x}{\xi}\right) \log\left(\frac{\mu_F^2}{\kappa^2}\right) + C\left(\frac{x}{\xi}\right)\right\} + \dots$$

This will result in the effective *subtraction* of the collinear divergence, the definition of a ”renormalized” PDF and the appearance of a new unphysical energy scale: μ_F, the factorization scale (which is on the same ground of μ_R).

The formula for a cross section in a generic factorization scheme can be written as an additional counterterm contribution $\mathrm{d}\sigma_a^C$, in this case coming from the PDF redefinition:

$$\sigma_a^{NLO}(p;\mu_F^2) = \int_{m+1} \mathrm{d}\sigma_a^R(p) + \int_m \mathrm{d}\sigma_a^V(p) + \int_m \mathrm{d}\sigma_a^C(p;\mu_F^2)$$

$$\mathrm{d}\sigma_a^C(p;\mu_F^2) = -\frac{\alpha_s}{2\pi}\frac{1}{\Gamma(1-\epsilon)}\sum_b \int_0^1 \mathrm{d}z \left[-\frac{1}{\epsilon}\left(\frac{4\pi\mu^2}{\mu_F^2}\right)^{\epsilon} P^{ab}(z) + K^{ab}(z)\right] \mathrm{d}\sigma_b^B(zp)$$

in dimensional regularization[4].

This will be useful in what follows, because it is exactly by choosing a specific factorization scheme (i.e. a specific K^{ab} matrix) that we can analyze the property of the frequent $\overline{\mathrm{MS}}$ choice.

[4] The counterterm is such that $K^{ab} = 0$ for $\overline{\mathrm{MS}}$ factorization scheme

An intrinsic positive scheme

DIS scheme and similar.

Defined on physical observables.

Coefficient functions NLO behaviour

We can play this game because we know in advance that the relevant structure (the one related to the collinear subtraction) is universal.

How we switch scheme and K properties

A bunch of nontrivial positivity schemes

POS, MPOS, DPOS

Is $\overline{\mathrm{MS}}$ negative?

N-space positivity $\neq$ x-space positivity

The easy way in *N-space* and Why we need an argument in *x-space*

Argument from MPOS -> MSbar

Why positivity?

My opinion?

Reduce PDF variance limiting the hypothesis space.

Thanks for your attention