Exercise 4

The full calculation is provided in `ex4.nb` Mathematica notebook, it has been performed using the `HEPMath` package.

part (a)

$$\frac{e^4 Q_f^2(\cos(2\theta)+3)}{4\pi s} = Q_f^2 \frac{\pi\alpha^2}{2s}(1+\cos^2(\theta))$$

This is the same cross section for production of muon pairs from electron-positron annihilation, apart from the charge factor Q_f^2, and the omitted sum over colors.

The flavors to sum on should be those who are possible to excite at the experiment energy, and so those whose mass is below $s/2$.

The observed plateaus in the ratio between the electrons-into-hadrons and electron-into-muons indeed gave two fundamental pieces of information for the birth of the quark model:

- the number of active flavors is increasing with the energy
- the plateaus and hadrons charges can be explained with attributing the known charges to the quarks, but then a factor 3 is still missing in the ratio: together with the hadron multiplets structure this has been a crucial clue of a further hidden quantum number, the color

part (b)

$$\frac{\frac{1}{64}\left(8\sin^4(\theta_W)-4\sin^2(\theta_W)+1\right)\left(A_f^2+V_f^2\right)(\cos(2\theta)+3)+\frac{1}{4}(1-4\sin^2(\theta_W))A_f V_f\cos(\theta)}{128\pi s}$$

The $\cos(\theta)$ distribution is split in two parts:

- the first one still proportional to $\cos(2\theta)+3=2(1+\cos^2(\theta))$, whose coefficient is proportional to $A_f^2+V_f^2$, i.e. it gets contribution from all the bits in the square amplitudes that contain an even number of γ^5
- the second one it's now proportional to $\cos(\theta)$ instead of $\cos(2\theta)\to\cos^2(\theta)$, and its coefficient is proportional to $A_f V_f$, so it is the contribution of the terms that are odd in γ^5 (i.e. that contain a single γ^5)