Exercise 4

The full calculation is provided in `ex4.nb` Mathematica notebook, it has been performed using the `HEPMath` package.

part (a)

$$\frac{e^4 Q_f^2(\cos(2\theta)+3)}{4\pi s} = Q_f^2 \frac{\pi\alpha^2}{2s}(1+\cos^2(\theta))$$

This is the same cross section for production of muon pairs from electron-positron annihilation, apart from the charge factor Q_f^2, and the omitted sum over colors and flavors.

The flavors to sum on should be those who are possible to excite at the experiment energy, and so those whose mass is below $s/2$.

The observed plateaus in the ratio between the electrons-into-hadrons and electron-into-muons indeed gave two fundamental pieces of information for the birth of the quark model:

- the number of active flavors is increasing with the energy
- the plateaus and hadrons charges can be explained with attributing the known charges to the quarks, but then a factor 3 is still missing in the ratio: together with the hadron multiplets structure this has been a crucial clue of a further hidden quantum number, the color

part (b)

$$\frac{\frac{1}{64}\left(8\sin^4(\theta_W) - 4\sin^2(\theta_W) + 1\right)\left(A_f^2 + V_f^2\right)(\cos(2\theta)+3) + \frac{1}{4}(1-4\sin^2(\theta_W))A_f V_f \cos(\theta)}{128\pi s}$$

The $\cos(\theta)$ distribution is split in two parts:

- the first one still proportional to $\cos(2\theta)+3 = 2(1+\cos^2(\theta))$, whose coefficient is proportional to $A_f^2 + V_f^2$, i.e. it gets contribution from all the bits in the square amplitudes that contain an even number of γ^5
- the second one it's now proportional to $\cos(\theta)$ instead of $\cos(2\theta) \to \cos^2(\theta)$, and its coefficient is proportional to $A_f V_f$, so it is the contribution of the terms that are odd in γ^5 (i.e. that contain a single γ^5)